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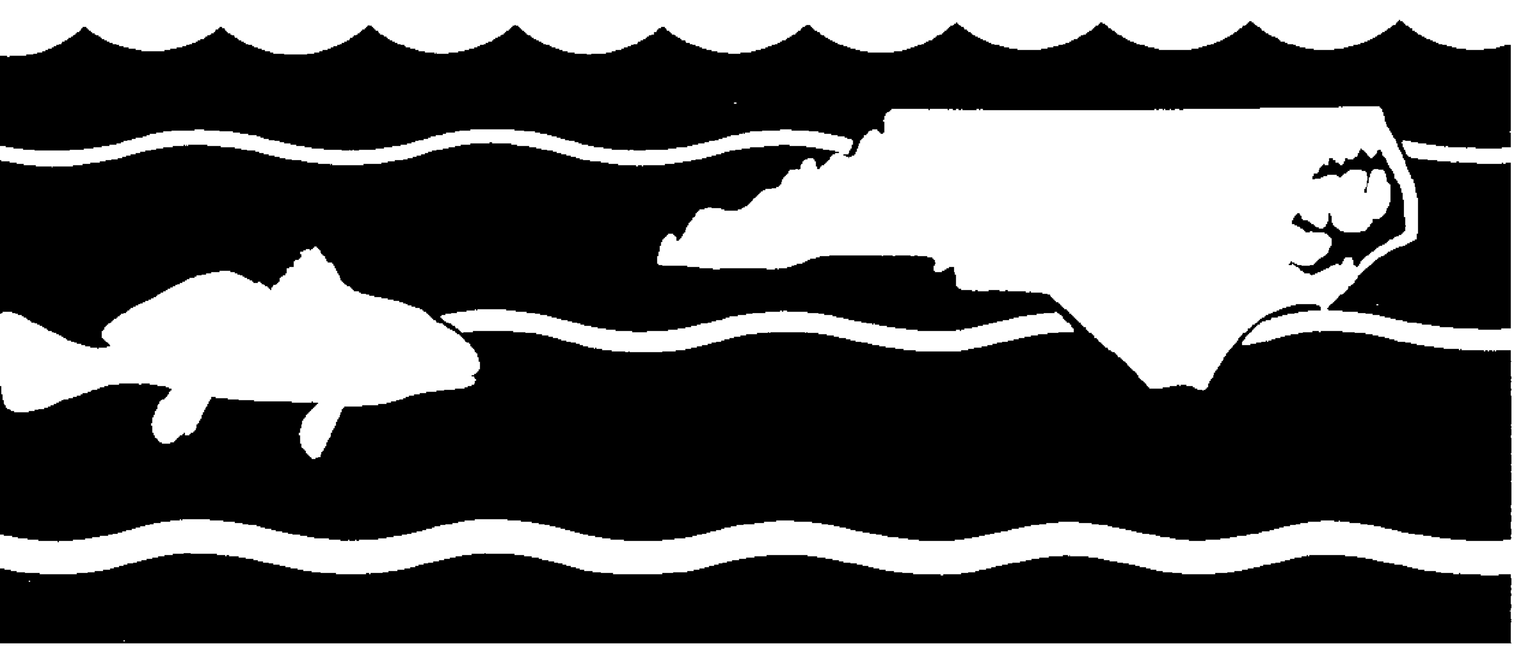

ON THE STATISTICAL PROPERTIES

OF WAVE FORCE

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INTRODUCTION

In evaluating wave force on cylinder, the Morison's formula is often used. That is, wave force is assumed to consist of two parts. The first part, referred to as the inertia force, is linearly proportional to fluid particle acceleration. The second part, referred to as the drag force, is proportional to the square and in the direction of fluid particle velocity.

Due to fluctuation of the free surface, points on the cylinder may rise above or fall below the water surface. At instants when the point is not submerged, it experiences no wave force. This is recognized, in the case of single wave (Ippen, 1966). For random waves, however, such consideration has so far been ignored (Borgman, 1971).

It is the purpose of this paper to derive expressions for some statistical quantities of wave field kinematics and wave forces on elements of a vertical cylinder in a random wave field with due regard to the phenomenon mentioned above and to examine its effects on these quantities. Specifically, expressions for the mean and standard deviation of horizontal component of fluid particle velocity, acceleration and wave force on element of cylinder are derived. Numerical results are obtained for these quantities and presented graphically. For simplicity, only the case of unidirectional gravity wave field in deep water is considered.

SPECIFICATION OF THE RANDOM SEA

The description of a random wave field has been discussed by many authors (Borgman, 1971, Phillips, 1969). For easy reference, a brief summary is given below.

Consider a coordinate system with the z axis directed vertically upward and origin at the equilibrium surface. The position of the free surface is specified by $z = \eta(x,t)$ in which x is the horizontal position coordinate in the direction of the waves and t is time. It is assumed that $\eta(x,t)$ is a zero mean, Gaussian random process stationary in time and homogeneous in space and can be represented as (Phillips, 1969, Huang, 1971)

$$\eta(x,t) = \int_k \int_n dB(k,n) e^{i(kx-nt)} \tag{1}$$

in which i is the imaginary unit, dB(k,n) is a zero mean complex random function of wave-number k and frequency n. The integration in Eq. (1) is over all wave-number, frequency space in the gravity wave range. It can be shown that

$$\begin{aligned} E[dB(k,n)\, dB^*(k,n)] &= o \qquad \text{if } k \neq k',\ n \neq n' \\ &= X(k,n)\, dk dn \text{ if } k = k',\ n = n' \end{aligned} \tag{2}$$

in which $dB^*(\cdot,\cdot)$ is the complex conjugate of $dB(\cdot,\cdot)$, $E[\cdot]$ is the expected value of the random quantity enclosed in the bracket and X(k,n) is the wave-number, frequency spectrum of the surface waves.

Under the assumptions that the fluid is inviscid, incompressible and the motion of the wave field is irrotational the associated velocity potential $\phi(x,z,t)$ that satisfies Laplace's equation

$$\nabla^2 \phi(x,z,t) = 0 \tag{3}$$

together with the linearized kinematic and dynamic boundary conditions is given, in deep water, by (Huang, 1971)

$$\phi(x,z,t) = -i \int_k\int_n \frac{n}{|k|}\, dB(k,n)\, e^{|k|z}\, e^{i(kx-nt)} \tag{4}$$

in which the frequency

$$n = \pm(gk)^{1/2} \tag{5}$$

g being gravitational acceleration.

The horizontal components of fluid particle velocity and acceleration are respectively given by (Huang, 1971)

$$V(x,z,t) = \frac{\partial\phi(x,z,t)}{\partial x}$$

$$= \int_k\int_n \frac{nk}{|k|}\, dB(k,n)\, e^{|k|z}\, e^{i(kx-nt)} \tag{6}$$

and

$$A(x,z,t) = \frac{\partial V(x,z,t)}{\partial t}$$

$$= -i \int_k\int_n \frac{n^2 k}{|k|}\, dB(k,n)\, e^{|k|z}\, e^{i(kx-nt)} \tag{7}$$

correct to the first order of approximation.

It is recognized that, to this order, V(x,z,t) and A(x,z,t) are zero mean, Gaussian, stationary in time and homogeneous in space and Eqs. (6) and (7) holds everywhere below the free surface. Stated explicitly, the horizontal components of fluid particle velocity and acceleration should be expressed as

$$\bar{V}(x,z,t) = V(x,z,t)\ H\ (\eta(x,t) - z) \tag{8}$$

and

$$\bar{A}(x,z,t) = A(x,z,t)\ H\ (\eta(x,t) - z) \tag{9}$$

in which $H(\cdot)$ is the heaviside unit function. From Eqs. (8) and (9) it is immediately clear that the horizontal components of fluid particle velocity and acceleration, being nonlinear functions of Gaussian processes, are no longer Gaussian. The difference between the representation of wave field kinematics given by Eqs. (8) and (9) and that by Eqs. (6) and (7), accounts for the significant discrepancy observed in the statistical properties of these quantities to be discussed later in the paper. It is to the derivation of the mean and standard deviation of the quantities $\bar{V}(x,z,t)$ and $\bar{A}(x,z,t)$ that the next section is devoted.

MEAN AND STANDARD DEVIATION OF WAVE FIELD KINEMATICS

In subsequent derivation of the mean and standard deviation, for brevity, the arguments z,x, and t in the quantities $\eta(x,t)$, $V(x,z,t)$, $\bar{V}(x,z,t)$, $A(x,z,t)$ and $\bar{A}(x,z,t)$ are dropped.

To obtain the mean of $\bar{V}$, take the expected value of both sides of Eq. (8). That is

$$E[\bar{V}] = E[VH(\eta-z)]. \tag{10}$$

Eq. (1) can be written as (Papoulis, 1965)

$$E[\bar{V}] = E[VH(\eta-z)] = E[H(\eta-z)E[V|\eta]] \tag{11}$$

in which $E[\cdot|\cdot]$ is conditional expected value. Noting that V and η are jointly Gaussian, (Papoulis, 1965)

$$E[V|\eta] = (r\sigma_V / \sigma_\eta)\eta \tag{12}$$

in which σ_η and σ_V are respectively standard deviations of η and V given by

$$\sigma_\eta = [\int_n S(n)dn]^{1/2} \tag{13}$$

and

$$\sigma_V = [\int_n n^2 S(n)\ e^{2|k|z}\ dn]^{1/2} \tag{14}$$

as can be derived from Eqs. (1) and (6) together with Eq. (2). S(n) is the frequency spectrum of the surface waves. In Eq. (12), r is the correlation coefficient of V and η

$$r = \frac{1}{\sigma_\eta \sigma_V} \int_n |n| S(n)\ e^{|k|z}\ dn. \tag{15}$$

Substituting Eq. (12) into Eq. (11),

$$E[\bar{V}] = \frac{r\sigma_V}{\sigma_\eta}\ E[H(\eta - z)\eta]. \tag{16}$$

Since η is zero mean Gaussian whose probability density function is

$$f_\eta(\eta) = \frac{1}{2\Pi\sigma_\eta}\ \exp(-\frac{\eta^2}{2\sigma_\eta^2}) \tag{17}$$

the expected value

$$E[H(z+\eta)\eta] = \int_{-\infty}^{\infty} H(\eta - z)\eta f_\eta(\eta)d\eta$$

$$= \int_z^{\infty} \eta f_\eta\ (\eta)d\,\eta = \sigma_\eta Z(\frac{z}{\sigma_\eta}) \tag{18}$$

in which

$$Z(\xi) = \frac{1}{\sqrt{2\Pi}} \exp(-\frac{\xi^2}{2}). \tag{19}$$

Thus,

$$E[\bar{V}] = r\sigma_V Z(\frac{z}{\sigma_\eta}). \tag{20}$$

It is noted here that $E[\bar{V}]$ is a first order quantity and as z approaches negative infinity, $E[\bar{V}] = 0$.

The standard deviation of $\bar{V}$ may be obtained from the mean square $E[\bar{V}^2]$ of $\bar{V}$ which can be determined by squaring both sides of Eq. (8) and taking their expected values. That is, (Papoulis, 1965)

$$E[\bar{V}^2] = E[V^2 H(\eta-z)] = E[H(\eta-z)E[V^2|\eta]] \tag{21}$$

in which (Papoulis, 1965)

$$E[V^2|\eta] = \sigma_{V|\eta}^2 + m_{V|\eta}^2 . \tag{22}$$

The quantities $m_{V|\eta}$ and $\sigma_{V|\eta}$ are respectively the conditional mean and standard deviation of V given η

$$m_{V|\eta} = E[V|\eta] = (r\sigma_V/\sigma_\eta)\eta \tag{23}$$

as given in Eq. (12), and

$$\sigma_{V|\eta} = \sigma_V (1-r^2)^{1/2}. \tag{24}$$

Using Eqs. (22), (23), and (24), Eq. (21) becomes

$$E[\bar{V}^2] = \sigma_V^2 (1-r^2) E[H(\eta-z)] + \frac{r^2\sigma_V^2}{\sigma_\eta^2} E[\eta^2 H(\eta-z)]. \tag{25}$$

The expected values $E[H(\eta-z)]$ and $E[\eta^2 H(\eta-z)]$ can all be readily determined using Eq. (17). That is,

$$E[H(\eta-z)] = Q\left(\frac{z}{\sigma_\eta}\right) \tag{26}$$

in which

$$Q(u) = \int_u^\infty Z(\xi)\, d\xi \tag{27}$$

and

$$E[\eta^2 H(\eta-z)] = \sigma_\eta^2 \left[\frac{z}{\sigma_\eta} Z\left(\frac{z}{\sigma_\eta}\right) + Q\left(\frac{z}{\sigma_\eta}\right)\right]. \tag{28}$$

The expression for $E[\bar{V}^2]$ is therefore, from Eq. (25),

$$E[\bar{V}^2] = \sigma_V^2 \left[Q\left(\frac{z}{\sigma_\eta}\right) + r^2 \frac{z}{\sigma_\eta} Z\left(\frac{z}{\sigma_\eta}\right)\right] \tag{29}$$

and the standard deviation $\sigma_{\bar{V}}$ of $\bar{V}$ is

$$\sigma_{\bar{V}} = \{E[\bar{V}^2] - E^2[\bar{V}]\}^{1/2}. \tag{30}$$

It is seen that as z approaches negative infinity, $E[\bar{V}^2]$ approaches ${\sigma_V}^2$ and with $E[\bar{V}]$ becoming vanishingly small, $\sigma_{\bar{V}}$ converges to σ_V.

The mean of $\bar{A}$ is obtained from Eq. (9). That is

$$E[\bar{A}] = E[AH(\eta-z)]. \tag{31}$$

Since A and η are Gaussian and uncorrelated with each other, as can be verified from Eqs. (1) and (7), they are statistically independent. Thus

(Papoulis, 1965).

$$E[\bar{A}] = E[A]E[H(\eta-z)]. \quad (32)$$

But, from Eq. (7), $E[A] = 0$, giving

$$E[\bar{A}] = 0. \quad (33)$$

The mean square value of $\bar{A}$ can also be obtained simply from Eq. (9) using the same argument leading to Eq. (32). That is,

$$E[\bar{A}^2] = E[A^2\ H(\eta-z)] = E[A^2]\ E[H(\eta-z)]$$
$$= \sigma_A^2\ Q(\frac{z}{\sigma_\eta}) \quad (34)$$

in which, σ_A is the standard deviation of A, and, from Eq. (7),

$$\sigma_A = \{\int_n n^4 S(n) e^{2|k|z}\ dn\}^{1/2}. \quad (35)$$

With the mean $E[\bar{A}] = 0$, the standard deviation $\sigma_{\bar{A}}$

of $\bar{A}$ is

$$\sigma_{\bar{A}} = \{E[\bar{A}^2]\}^{1/2}. \quad (36)$$

Again, it is observed that $\sigma_{\bar{A}}$ approaches σ_A when the point under consideration is far below the mean water level.

MEAN AND STANDARD DEVIATION OF WAVE FORCE

According to Morison's formula, the wave force on an element of a vertical cylinder, of unit length, located at z distance from equilibrium surface is

$$\bar{F}(x,z,t) = C_D\ V|V|H(\eta-z) + C_M AH(\eta-z) \quad (37)$$

in which $C_D = \rho k_D D$, $C_M = \rho k_M \frac{\pi D^2}{4}$, k_D and k_M are respectively drag and inertia coefficients, ρ is density of water and D is diameter of the cylinder. This expression of wave force differs from

$$F(x,z,t) = C_D V|V| + C_M A \tag{38}$$

that is ordinarily used (Borgman, 1965) in that in Eq. (37) the heaviside unit function is introduced.

The mean value of $\bar{F}$ (arguments x,z, and t are again dropped for brevity) is, from Eq. (37),

$$E[\bar{F}] = C_D\ E[V|V|H(\eta-z)] + C_M E[AH(\eta-z)]. \tag{39}$$

In Eq. (39), the second expected value on the right hand side is equal to zero (Eq. (33)). The term $E[V|V|H(\eta-z)]$ is (Papoulis, 1965)

$$E[V|V|H(\eta-z)] = E[H(\eta-z)\ E[V|V|\ \big|\eta]]. \tag{40}$$

But

$$E[V|V|\big|\eta] = \int_{-\infty}^{\infty} V|V| f_{V|\eta}(V)dV \tag{41}$$

in which $f_{V|\eta}(V)$ is the conditional probability density function of V given η (Papoulis, 1965)

$$f_{V|\eta}(V) = \frac{1}{\sqrt{2\pi}\,\sigma_{V|\eta}} \exp[-\frac{1}{2}(\frac{V-m_{V|\eta}}{\sigma_{V|\eta}})^2]. \tag{42}$$

$m_{V|\eta}$ and $\sigma_{V|\eta}$ being given by Eqs. (23) and (24). Substituting Eq. (42) into Eq. (41) and carrying out the integration by parts,

$$E[V|V|\big|\eta] = \sigma^2_{V|\eta}\ [2\lambda Z(\lambda) + 2Q(\lambda) -1] + 4m_{V|\eta}\sigma_{V|\eta}Z(\lambda) + 2m^2_{V|\eta}Q(\lambda) - m^2_{V|\eta} \tag{43}$$

in which

$$\lambda = -m_{V|\eta} / \sigma_{V|\eta} = \frac{-r\eta}{\sigma_\eta(1-r^2)^{1/2}}. \tag{44}$$

From Eqs. (43) and (40),

$$E[V|V|H(\eta-z)] = \int_{-\infty}^{\infty} H(\eta- z) [\sigma^2_{V|\eta} (2\lambda Z(\lambda) + 2Q(\lambda)-1) + 4m_{V|\eta}\sigma_{V|\eta}Z(\lambda) +$$

$$2m^2_{V|\eta}Q(\lambda) - m^2_{V|\eta}] f_\eta(\eta)d\eta \tag{45}$$

in which, it is noted that λ and $m_{V|\eta}$ are both functions of η.

Integrating Eq. (45) by parts,

$$E[V|V|H(\eta-z)] = \sigma^2_V \{-Q(\frac{3}{\sigma_\eta}) + 2L(0, \frac{z}{\sigma_\eta}, r) + 2r(1-r^2)^{1/2} \; Z(\frac{z}{\sigma_\eta(1-r^2)^{1/2}}) \Big/ \sqrt{2\pi}$$

$$+ r^2 \frac{3}{\sigma_\eta} Z(\frac{z}{\sigma_\eta}) [2Q(\frac{-rz}{\sigma_\eta(1-r^2)^{1/2}})-1]\} \tag{46}$$

in which (Abramowitz, 1968)

$$L(a,b,r) = \int_a^{\infty} Z(\xi)d\xi \int_b^{\infty} Z(y)dy; \; w = \frac{b-r\xi}{(1-r^2)^{1/2}} . \tag{47}$$

It can be verified that $E[V|V|H(\eta-z)]$ and hence $E[\bar{F}]$ approaches $E[F] = 0$ (Borgman, 1965) as z becomes increasingly large.

The mean square value $E[\bar{F}^2]$ of $\bar{F}$ is, from Eq. (37),

$$E[\bar{F}^2] = C_D^2E[V^4H(\eta-z)] + C_M^2E[A^2H(\eta-z)] + 2C_DC_ME[V|V|AH(\eta-z)]. \tag{48}$$

The third term on the right hand side of Eq. (48) is (Papoulis, 1965)

$$E[V|V|H(\eta-z)]E[A] = 0 \tag{49}$$

since $E[A] = 0$. The term $E[A^2H(\eta-z)]$ is given by Eq. (34). The remaining term to be evaluated in Eq. (48) is $E[V^4H(\eta-z)]$ which can be done in much the same manner as the term $E[V|V|H(z+\eta)]$ in Eq. (40) was carried out. That is,

$$E[V^4H(\eta-z)] = E[H(\eta-z)E[V^4|\eta]]. \tag{50}$$

But (Papoulis, 1965)

$$E[V^4|\eta] = \int_{-\infty}^{\infty} V^4 f_{V|\eta}(V)dV = 3\sigma_{V|\eta}^4 + 6\sigma_{V|\eta}^2 m_{V|\eta}^2 + m_{V|\eta}^4. \tag{51}$$

Therefore,

$$E[V^4H(\eta-z)] = \int_{-\infty}^{\infty} H(\eta-z)\ (3\sigma_{V|\eta}^4 + 6\sigma_{V|\eta}^2 m_{V|\eta}^2 + m_{V|\eta}^4) f_\eta(\eta)d\eta =$$

$$\sigma_V^4[3Q(\frac{z}{\sigma_\eta}) + Z(\frac{z}{\sigma_\eta})\ \frac{z}{\sigma_\eta}\ r^2(6 + r^2\ \frac{z^2}{\sigma_\eta^2} - 3r^2)]. \tag{52}$$

Substituting Eq. (52) and Eq. (34) into Eq. (48), the quantity $E[\bar{F}^2]$ is obtained and the standard deviation $\sigma_{\bar{F}}$ of $\bar{F}$ is given by

$$\sigma_{\bar{F}} = \{E[\bar{F}^2] - E^2[\bar{F}]\}^{1/2}. \tag{53}$$

Far below the mean water level, the quantities $E[A^2H(\eta-z)]$ and $E[V^4 H(\eta-z)]$ in Eq. (48) respectively approach $\sigma_A{}^2$ and $3\sigma_V{}^4$ giving,

$$E[\bar{F}^2] = 3C_D^2\sigma_V{}^4 + C_M^2\sigma_A^2 \tag{54}$$

which concurs with the expression of $E[F^2]$ previously derived by Borgman (Borgman 1965).

NUMERICAL RESULTS

Numerical results are obtained for all the quantities discussed above. The frequency spectrum S(n) is taken to be that of the one-sided Kitaigorodskii-Pierson-Moskowitz spectrum

$$S(n) = \frac{\alpha g^2}{n^5} \exp[-\beta(\frac{n_o}{n})^4] \quad (55)$$

in which $\alpha = 0.81 \times 10^{-2}$, $\beta = 0.74$, $n_o = g/W$, W being the mean wind speed. The cut-off frequency for S(n) is determined from the condition $E^{1/2} \{[\nabla_h \eta(x,t)]^2\} << 1$, (Phillips, 1960) in which ∇_h denotes horizontal gradient.

For wave force computations, the values of $k_M = 1.4$, $k_D = 0.5$, $\rho = 2.0$ #-s^2/ft.ft^3 and D = 1 ft are used. The value of W = 40 miles per hour is chosen for all the computations in this study.

Figure 1 gives the quantities $E[\bar{V}]$, $\sigma_{\bar{V}}$, $\sigma_{\bar{A}}$ together with σ_V and σ_A as function of z. It is seen that $E[\bar{V}]$ becomes insignificantly small for $|z|>3\sigma_\eta$. Also, $\sigma_{\bar{V}}$ and $\sigma_{\bar{A}}$ coincide respectively with σ_V and σ_A when $z<-3\sigma_\eta$ but deviate drastically from σ_V and σ_A as the point is above and farther removed from the mean water level. More significantly, while $\sigma_{\bar{V}}$ and $\sigma_{\bar{A}}$ converge to zero as z approaches infinity, σ_V and σ_A diverge indefinitely.

In Figure 2, the quantities $E[\bar{F}]$, $\sigma_{\bar{F}}$ and σ_F are presented as function of z. $E[\bar{F}]$ exhibits the same characteristics as $E[\bar{V}]$ in Figure 1 except that $E[\bar{F}]$ approaches zero less rapidly as z approaches negative infinity. Examination of $\sigma_{\bar{F}}$ and σ_F indicates that the two quantities converge

Figure 1: Mean and Standard Deviation of Fluid Partical Velocity and Acceleration.

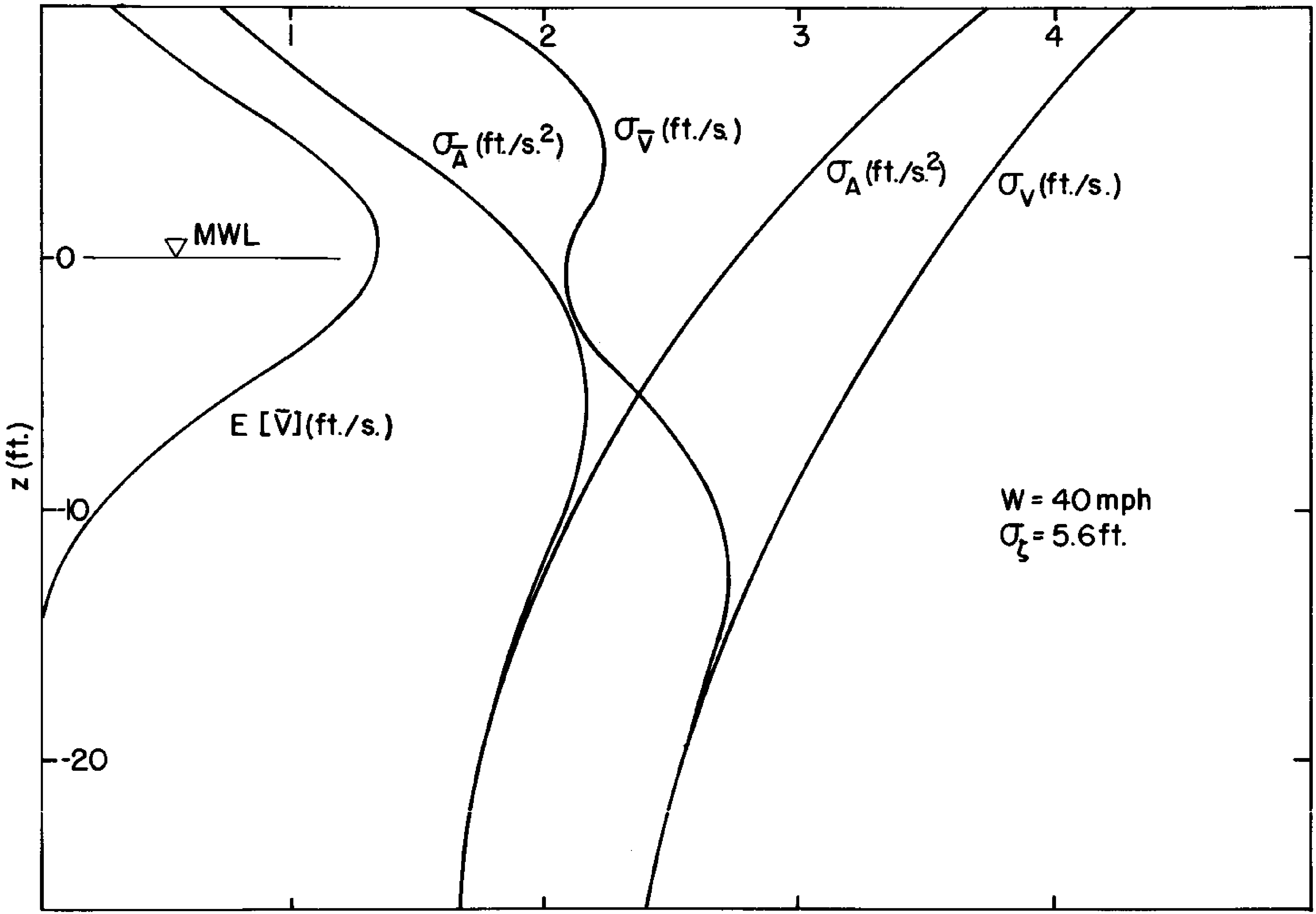
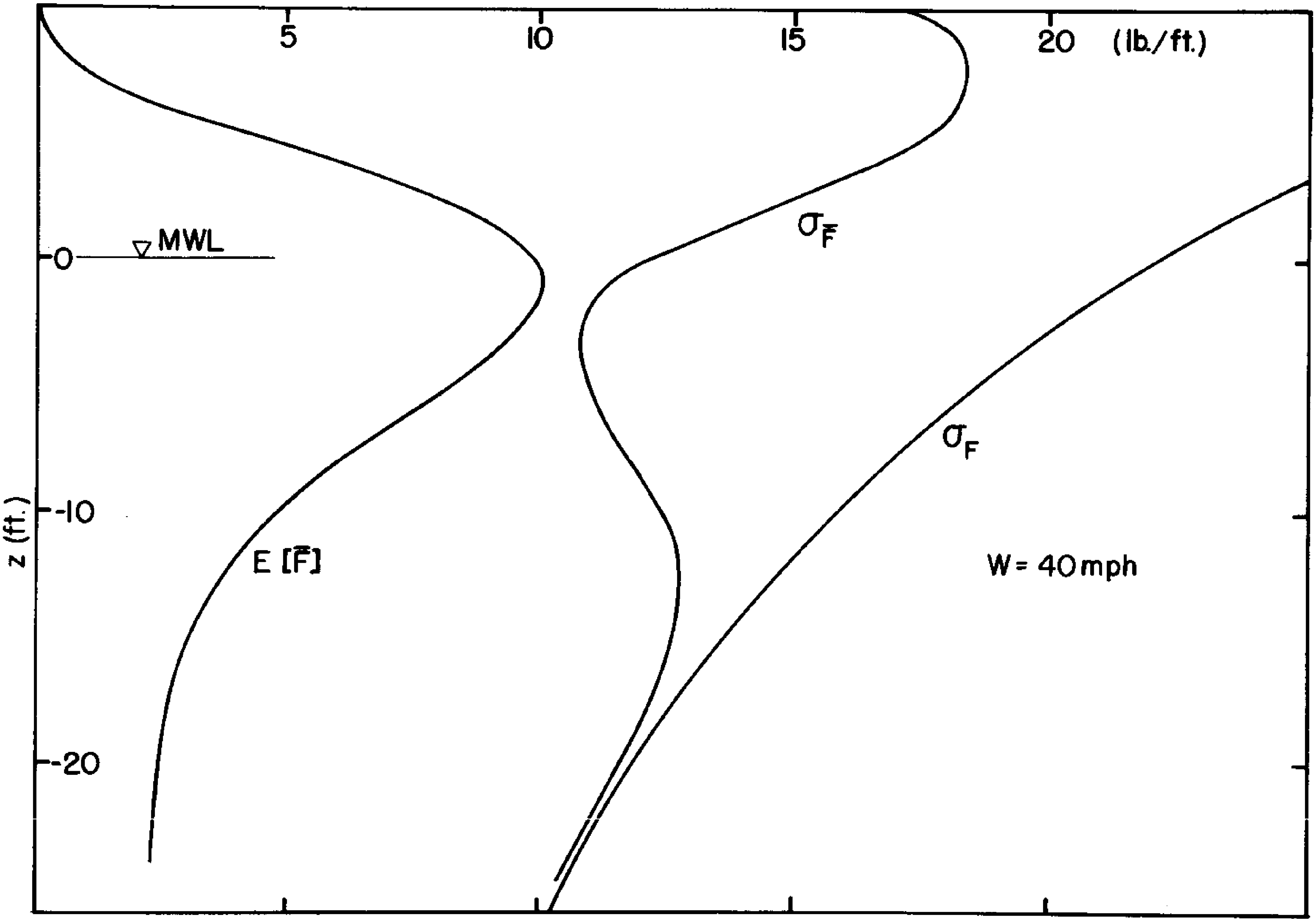


Figure 2: Mean and Standard Deviation of Wave Force.



when the element under consideration is far below the mean water level but they diverge from each other as the element is above and away from the mean water level.

CONCLUDING REMARKS

Results of this investigation indicates that, in a random wave field, when wave kinematics and wave force are expressed as given in Eqs. (8), (9) and (37), taking into account the possibility that the point under consideration may rise above the water surface,

1. the mean values of horizontal component of fluid particle velocity and associated wave force are non-zero except when the point is far removed from the mean water level, and
2. the standard deviations of horizontal fluid particle velocity and acceleration and associated wave force deviate from those commonly used to an appreciably extent. The departure is neglegible at points far below the mean water level but beomes significant around and above the mean water level.

In as much as most of the water movement in a wave field is concentrated around and above the mean water level at which the discrepancy observed above is most pronounced, the results of this study has obvious implications in design considerations of marine structures that protrude above the water. In this connection, it should be mentioned that while vorticity, nonlinear wave-wave interactions all affect water movement, these effects are of higher order. The results of this study, based on potential theory, carried to first order should therefore be considered generally satisfactory (Kinsman, 1965).

For simplicity, only unidirectional deep water waves are considered. However, results can be obtained without undue difficulty and the general characteristics of the quantities examined here are believed to hold, for directional and intermediate water waves as well.

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APPENDIX II. - NOTATION

The following symbols are used in this paper:

$A(x,z,t)$, $\bar{A}(x,z,t)$ = horizontal component of fluid particle acceleration (Eqs. (7), (9));

a = argument (Eq. (47));

b = argument (Eq. (47));

C_D, C_M = coefficients (Eq. (37));

D = diameter of cylinder;

$dB(k,n)$, $dB^*(k,n)$ = complex random function and its complex conjugate (Eqs. (1), (2));

$E[\cdot]$ = expected value of random quantity enclosed in the bracket;

$F(x,z,t)$, $\bar{F}(x,z,t)$ = wave force on element of cylinder (Eqs. (37), (38));

$f_\eta(\cdot)$ = probability density function of surface elevation $\eta(x,t)$ (Eq. (17));

$f_{V|\eta}(\cdot)$ = conditional probability density function of $V(x,z,t)$ given $\eta(x,t)$ (Eq. (42));

g = gravitational acceleration;

$H(\cdot)$ = Heaviside unit function;

i = imaginary unit;

k,k' = wave-number;

k_D,k_M = drag and inertia coefficients (Eq. (37));

$L(a,b,r)$ = function defined in Eq. (47);

$m_{V|\eta}$ = conditional mean of $V(x,z,t)$ given $\eta(x,t)$ (Eq. (23));

n,n' = frequency;

n_o = g/W (Eq. (55));

$Q(\cdot)$ = function defined in Eq. (27);

r = correlation coefficient of $V(x,z,t)$ and $\eta(x,t)$ (Eq. (15));

$S(n)$ = frequency spectrum of surface elevation;

t = time;

u = dummy variable;

$V(x,z,t)$, $\bar{V}(x,z,t)$ = horizontal component of fluid particle velocity (Eqs. (6), (8));

W = mean wind speed (Eq. (55));

w = dummy variable;

$X(k,n)$ = wave-number, frequency spectrum of surface elevation;

x = horizontal position coordinate;

y = dummy variable;

$Z(\cdot)$ = function defined in Eq. (19);

z = vertical position coordinate;

α,β = constants appearing in Eq. (55);

∇_h = horizontal gradient;

$\eta(x,t)$ = surface elevation (Eq. (1));

λ = quantity defined in Eq. (44);

ξ = dummy variable;

ρ = density of water (Eq. (37));

σ = standard deviation of the quantity in the subscript;

$\sigma_{V|\eta}$ = conditional standard deviation of $V(x,z,t)$ given $\eta(x,t)$; and

$\phi(x,z,t)$ = velocity potential (Eq. (4)).

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