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Final Report on a Study of Ship Wave Resistance

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FINAL REPORT ON A STUDY OF
SHIP WAVE RESISTANCE

by

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RELATED REPORT

ALTERNATING EXPRESSIONS FOR THE GREEN FUNCTION OF THE THEORY OF SHIP WAVE RESISTANCE. MITSG 79-23. Cambridge: Massachusetts Institute of Technology, 1979. 41pp. $4.00.

ABSTRACT

A proof of convergence of the sequence of slender-ship low-Froude-number approximations $r_{\ell F}^{(n)}$, $n \geq 0$, defined in the slender-ship theory exposed in Noblesse [1] is given for the particular case of ship hulls in the form of vertical cylinders with elliptical waterlines. More precisely, it is shown that we have

$$r_{\ell F}^{(n)} = [1-\{b/(1+b)\}^{n+1}]^2 \, r_{\ell F} \,, \quad n \geq 0,$$

where b is the beam/length ratio of the elliptical cylinder, and $r_{\ell F}$ is the low-Froude-number wave-resistance approximation associated with the exact zero-Froude-number potential.

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INTRODUCTION

This research project was concerned with the wave resistance experienced by a ship in rectilinear motion with constant speed in a calm sea. More precisely, the project focused on a new analytical theory for predicting the wave resistance of a ship. Two parallel and complementary objectives of the project were: (i) to perform calculations for various idealized geometrical ship forms for the purpose of systematically evaluating and testing the theory, and (ii): to perform theoretical studies with the view of developing numerical procedures for the ultimate practical application of the theory to usual real ship forms.

Work has been performed along these two lines, and it is believed that the above-stated objectives have been approximately met. The largest part of this work in fact has already been published or presented in conferences, so that this work will only be referenced here. The work performed within the first of the two above-defined objectives of the project has essentially been presented in three numerical studies performed by graduate students (P. Koch and C.Y. Chen) under the supervision of the principal investigator. The precise references to these three studies are given below:

1. "Wave resistance of the Wigley and Inui hull forms predicted by two simple slender-ship wave-resistance formulas", by P. Koch & F. Noblesse, Proc. of the Workshop on Ship Wave-Resistance Computations, David W. Taylor Naval Ship Research & Development Center, Bethesda, MD, Nov. 1979, pp. 339-353.
2. "A note on the waterline integral and thin-ship approximation", by P. Koch & F. Noblesse, Wave Resistance Meeting at Izu Shuzenji, Japan, May 1980; included in the Proc. of the Workshop on Ship Wave-Resistance Computations, David W. Taylor Naval Ship Research & Development Center, Bethesda, MD, pp. 515-522.
3. "A numerical investigation of a low-Froude-number slender-ship wave-resistance formula", by C.Y. Chen & F. Noblesse, Proc. of the Continued Workshop on Ship Wave-Resistance Computations, Izu Shuzenji, Japan, Oct. 1980, pp. 39-63.

The work performed within the second objective of the project mainly consists in a study of the Green function of the theory of ship wave resistance. This study was presented in a M.I.T. Sea Grant Report. The precise reference to this study is:

4. "Alternative expressions for the Green function of the theory of ship wave resistance", by F. Noblesse, M.I.T. Sea Grant College Program, Rep. No. MITSG 79-23, Sep. 1979, 41 pp.

Additional work was performed on the subject. This led to a revised and extended version of the above Sea Grant Report, due to appear in the July 1981 issue of the Journal of Engineering Mathematics. The reference to this study is:

5. "Alternative integral representations for the Green function of the theory of ship wave resistance", by F. Noblesse, Journal of Engineering Mathematics, July 1981.

The present report presents additional recent results related to the first of the two above-defined objectives of the project. A slightly-modified form of this report will be submitted for publication to the Journal of Engineering Mathematics. Specifically, the main object of this study is to prove the convergence of the sequence of slender-ship approximations defined in the slender-ship theory of wave resistance exposed in Noblesse [1]. Convergence is demonstrated for the particular case of the sequence of slender-ship low-Froude-number approximations $r_{\ell F}^{(n)}$, $n \geq 0$, and for ship hulls in the form of vertical cylinders with elliptical waterlines. The approximations $r_{\ell F}^{(n)}$ are shown to converge to the low-Froude-number approximation $r_{\ell F}$ (obtained by using the zero-Froude-number potential as an approximation to the velocity potential in the expression for the Kochin free-wave amplitude function) as $n \to \infty$. More precisely, it is proved that we have

$$r_{\ell F}^{(n)} = [1-\{b/(1+b)\}^{n+1}]^2 \; r_{\ell F} \, , \; n \geq 0 \, ,$$

where b is the beam/length (thickness) ratio of the elliptical cylinder. Furthermore, the low-Froude-number wave-resistance approximation for a vertical elliptical cylinder, $r_{\ell F}$, is examined in the limiting case b=1, corresponding to a circular cylinder, and in the thin-ship ($b \to 0$) and low-Froude-number ($F \to 0$) limits. In particular, it is shown that we have

$$r_{\ell F} \sim (21\pi/64)F^6/b^2 + 2\pi^{1/2}(F^7/b^2)\sin(2/F^2 + \pi/4) \text{ for } F \ll b \ll 1 \ ,$$

$$r_{\ell F} \sim \pi b^2 F^2 - 2\pi^{1/2} b^2 F^3 \sin(2/F^2 + \pi/4) \text{ for } b \ll F \ll 1 \ .$$

1. CONVERGENCE OF THE SEQUENCE OF SLENDER-SHIP LOW-FROUDE-NUMBER APPROXIMATIONS $r_{\ell F}^{(n)}$

The low-Froude-number wave-resistance approximation, $r_{\ell F}$, is defined in Noblesse [1] as the approximation obtained by using the zero-Froude-number (double-hull) potential as an approximation to the velocity potential of the flow caused by the ship in the expression for the Kochin free-wave amplitude function. This low-Froude-number wave-resistance approximation is essentially identical to the low-speed approximations proposed by Guevel, Vaussy, and Kobus [2], Baba [3], Maruo [4], and Kayo [5], as is shown in [1].

The zero-Froude-number potential, ϕ_0, is given by the solution of an integral equation of the form

$$\phi_0(\vec{x}) = f_0(\vec{x}) - L(\vec{x};\phi_0) \ . \tag{1}$$

In this equation, $f_0(\vec{x})$ is the potential defined as

$$f_0(\vec{x}) = \int_h G_0(\vec{x},\vec{\xi})n_x(\vec{\xi})da(\vec{\xi}) \ , \tag{2}$$

where h is the portion of the ship hull surface below the plane z=0 of the mean sea surface, da is the differential element of area of h, n_x is the component along the x axis — taken along the ship course and pointing towards the ship bow — of the unit outward normal vector $\vec{n}$ to h, and $G_0(\vec{x},\vec{\xi})$ is the zero-Froude-number Green function given by $4\pi G_0(\vec{x},\vec{\xi}) = -1/[(x-\xi)^2 + (y-\eta)^2 + (z-\zeta)^2]^{1/2} - 1/[(x-\xi)^2 + (y-\eta)^2 + (z+\zeta)^2]^{1/2}$. The term $L(\vec{x};\phi_0)$ in equation (1) is the linear transform of the potential ϕ_0 defined by

$$L(\vec{x};\phi_0) = \int_h [\phi_0(\vec{x}) - \phi_0(\vec{\xi})]\nabla_x G_0(\vec{x},\vec{\xi})\cdot\vec{n}(\vec{\xi})da(\vec{\xi}) \ , \tag{1b}$$

where ∇_x represents the differential operator $(\partial/\partial x,\ \partial/\partial y,\ \partial/\partial z)$. The integral equation (1) thus expresses the potential $\phi_0(\vec{x})$ as the sum of the term $f_0(\vec{x})$, which is given explicitly in terms of the hull shape, and of the term $L(\vec{x};\phi_0)$, which is evidently unknown a priori.

The integral equation (1) may be solved by using an iterative procedure based on the recurrence relation

$$\phi_0^{(n+1)}(\vec{x}) = f_0(\vec{x}) - L(\vec{x};\phi_0^{(n)}) \ , \ n \geq 0 \ , \qquad (2)$$

where the initial (zeroth) approximation $\phi_0^{(0)}$ may simply be taken as $\phi_0^{(0)} \equiv 0$. The recurrence relation (2) thus defines a sequence of iterative approximations $\phi_0^{(n)}$ to the zero-Froude-number potential ϕ_0. In particular, the first-order approximation $\phi_0^{(1)}$ is given by $\phi_0^{(1)} = f_0$. Convergence of the above-defined sequence of iterative approximations $\phi_0^{(n)}$ is proved in Noblesse and Triantafyllou [6] for the particular case of ellipsoidal hull forms, for which we have the remarkable property that the approximations $\phi_0^{(n)}$ are proportional to the exact zero-Froude-number potential ϕ_0; we thus have

$$\phi_0^{(n)}(\vec{x}) = \sigma^{(n)}\phi_0(\vec{x}) \ , \qquad (3)$$

where the constant of proportionality $\sigma^{(n)}$ depends on the beam/length and draft/length ratios. Convergence of the sequence of approximations $\phi_0^{(n)}$ actually is quite rapid for usual slender ship forms. Indeed, the first-order approximation $\phi_0^{(1)} \equiv f_0$ already provides a fairly good slender-ship approximation to the zero-Froude-number potential ϕ_0. For instance, for an ellipsoidal hull form with beam/length and draft/length ratios equal to .15 and .05, respectively, we have $\sigma^{(1)} \approx .97$, so that the first-order slender-ship approximation $\phi_0^{(1)}$ is smaller than the exact potential ϕ_0 by about 3%.

In this study, we consider the particular case when the hull is a vertical elliptical cylinder, with beam/length (thickness) ratio b. Specifically, the elliptical waterlines are defined by the equation $x^2 + y^2/b^2 = 1$, or by the parametric equations $x = \cos\theta$, $y = b\sin\theta$. On the surface of the elliptical cylinder, the zero-Froude-number potential ϕ_0 is given by

$$\phi_0(\vec{x}) = -bx = -b\cos\theta \ ,$$

as is well known. The iterative approximations $\phi_0^{(n)}$ are given by equation (3). We thus have

$$\phi_0^{(n)}(\vec{x}) = -\sigma^{(n)}b\cos\theta \ , \quad n \geq 0 \ , \qquad (4)$$

where the constant of proportionality $\sigma^{(n)}$ is given by

$$\sigma^{(n)} = 1 - \gamma^n \ , \quad n \geq 0 \ , \qquad (4a)$$

with γ defined as

$$\gamma = b/(1+b) \ . \qquad (4b)$$

The relative error, $\varepsilon^{(n)}$ say, associated with the approximation $\phi_0^{(n)}$ is given by

$$\varepsilon^{(n)} = (\phi_0 - \phi_0^{(n)})/\phi_0 = \gamma^n , \quad n \geq 0 . \tag{5}$$

A sequence of slender-ship low-Froude-number wave-resistance approximations, $r_{\ell F}^{(n)}$ say, may be associated to the sequence of slender-ship zero-Froude-number potentials $\phi_0^{(n)}$ by using the potentials $\phi_0^{(n)}$, $n \geq 0$, as successive approximations to the velocity potential in the expression for the Kochin free-wave amplitude function. The wave resistance is given by the classical Havelock formula, which takes the form

$$r_{\ell F}^{(n)} \equiv R_{\ell F}^{(n)}/\rho U^2 L^2 = (16/\pi)F^2 \int_0^\infty |K_{\ell F}^{(n)}(\tau)|^2 (\nu^2 + \tau^2)^{1/2} d\tau , \tag{6}$$

for a hull having both port-and-starboard and fore-and-aft symmetry. In equation (6), ρ is the density of water, U is the speed of the ship, L is a characteristic length for the ship (in the present case of a hull in the shape of a vertical elliptical cylinder, L is taken as half the length of the cylinder), $F \equiv U/(gL)^{1/2}$ is the Froude number, $\nu \equiv 1/F$ is the inverse of the Froude number, $K_{\ell F}^{(n)}$ is the n^{th} order slender-ship low-Froude-number approximation to the Kochin free-wave amplitude function, and $R_{\ell F}^{(n)}$ and $r_{\ell F}^{(n)}$ are the corresponding dimensional and nondimensional approximations to the wave resistance, respectively.

In the particular case of a vertical cylinder, the zero-Froude-number potentials $\phi_0^{(n)}$ are independent of the vertical coordinate z, and the expression for the Kochin spectrum function takes the form

$$K_{\ell F}^{(n)}(\tau) = -i \int_c \{[t_y^2 - \nu^2/(\nu^2 + \tau^2) + t_x \partial\phi_0^{(n)}/\partial\ell] t_y \sin\alpha \cos\beta$$
$$+ \nu\tau(\nu t_x \sin\alpha \sin\beta - \tau t_y \cos\alpha \cos\beta)\phi/(\nu^2 + \tau^2)^{1/2}\} \, d\ell. \tag{7}$$

In this equation, α and β are defined as

$$\alpha = \nu(\nu^2 + \tau^2)^{1/2} x , \quad \beta = \tau(\nu^2 + \tau^2)^{1/2} y ; \tag{7a,b}$$

furthermore, c represents the portion of the mean waterline (the intersection curve between the vertical cylindrical hull and the mean sea plane $z=0$) situated in the first quadrant $x \geq 0$, $y \geq 0$, $d\ell$ is the differential element of arc length along c, and t_x, t_y are the components of the unit tangent vector

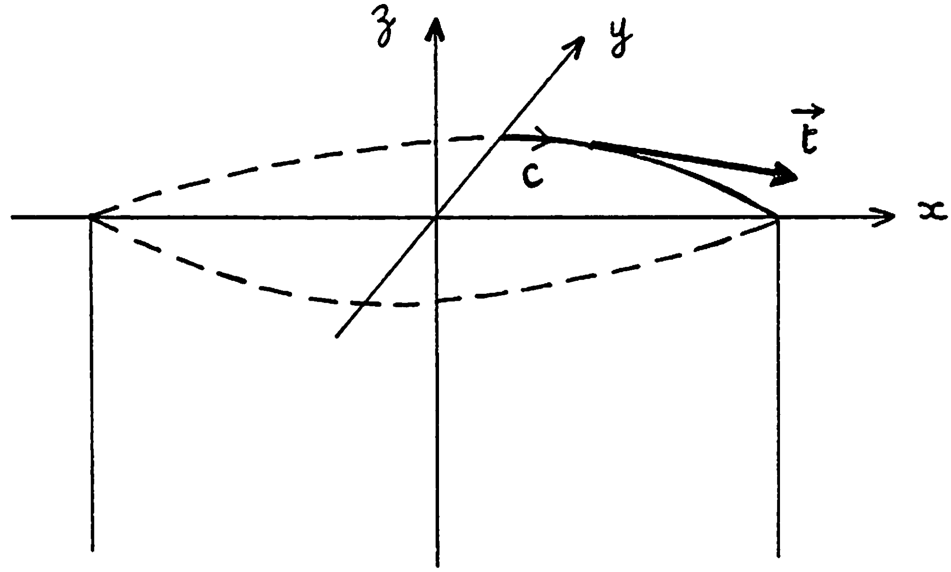

Figure 1 - Definition sketch for a hull in the shape of a vertical cylinder having both port-and-starboard and fore-and-aft symmetry.

$\vec{t}$ to c, pointing towards the ship bow (c is oriented in the clockwise direction), along the x and y axes, as is shown in figure 1.

In the special case of the elliptical cylinder defined by the equations $x = \cos\theta$ and $y = b\sin\theta$, we have $d\ell = -(\sin^2\theta + b^2\cos^2\theta)^{1/2}d\theta$, $t_x = \sin\theta/(\sin^2\theta + b^2\cos^2\theta)^{1/2}$, $t_y = -b\cos\theta/(\sin^2\theta + b^2\cos^2\theta)^{1/2}$, $\phi_0^{(n)} = -\sigma^{(n)}b\cos\theta$ as was given previously by equation (4), and $\partial\phi_0^{(n)}/\partial\ell = -\sigma^{(n)}b\sin\theta/(\sin^2\theta + b^2\cos^2\theta)^{1/2}$. Equation (7) then becomes

$$K_{\ell F}^{(n)}(\tau) = ib\int_0^{\pi/2}[\{(b^2\cos^2\theta - \sigma^{(n)}b\sin^2\theta)/(b^2\cos^2\theta + \sin^2\theta) - \nu^2/(\nu^2+\tau^2)\}\sin\alpha\cos\beta$$

$$+ \sigma^{(n)}\nu\tau(\nu\sin\theta\sin\alpha\sin\beta + b\tau\cos\theta\cos\alpha\cos\beta)/(\nu^2+\tau^2)^{1/2}]\cos\theta\, d\theta \,, \tag{8}$$

where α and β are given by

$$\alpha = \nu(\nu^2+\tau^2)^{1/2}\cos\theta \,, \qquad \beta = b\tau(\nu^2+\tau^2)^{1/2}\sin\theta \,. \tag{8a,b}$$

After extensive transformations of equation (8), given in the appendix, equations (6) and (8) for the wave-resistance approximations $r_{\ell F}^{(n)}$ can be expressed in the form of the following equations:

$$r_{\ell F}^{(n)} = (16/\pi)b^2(1+\sigma^{(n)}b)^2\int_0^\infty k^2(\tau)(\nu^2+b^2\tau^2)(\nu^2+\tau^2)^{-3/2}d\tau \,, \quad n \geq 0 \,, \tag{9}$$

where the function k(τ) is given by the integral

$$k(\tau) = \int_0^1 \sin(\rho u)u(1-u^2)^{1/2}[N(u)/D(u)]du \,; \tag{10}$$

in equation (10), ρ, N(u), and D(u) are defined as

$$\rho = (\nu^2+\tau^2)^{1/2}(\nu^2+b^2\tau^2)^{1/2} \,, \tag{10a}$$

$$N(u) = b^2(\nu^2-3\tau^2)/(\nu^2+b^2\tau^2)+(1-b^2)(1-u^2) \,, \tag{10b}$$

$$D(u) = b^4(\nu^2+\tau^2)^2/(\nu^2+b^2\tau^2)^2+2b^2(1-b^2)(\nu^2-\tau^2)(\nu^2+b^2\tau^2)^{-1}(1-u^2)+(1-b^2)^2(1-u^2)^2 \,. \tag{10c}$$

It may be seen from equations (4a,b) that we have $\sigma^{(n)} \to 1$ as $n \to \infty$. Equation (9) then yields

$$r_{\ell F} = (16/\pi)b^2(1+b)^2 \int_0^\infty k^2(\tau)(\nu^2+b^2\tau^2)(\nu^2+\tau^2)^{-3/2}d\tau \ . \qquad (11)$$

Equations (9) and (11) show that we have $r_{\ell F}^{(n)} = r_{\ell F}(1+\sigma^{(n)}b)^2/(1+b)^2$. By using equations (4a,b), we may finally obtain

$$r_{\ell F}^{(n)} = (1-\gamma^{n+1})^2 r_{\ell F} \ , \quad n \geq 0 \ . \qquad (12)$$

Convergence of the sequence of slender-ship low-Froude-number approximations $r_{\ell F}^{(n)}$, $n \geq 0$, is immediately established by equation (12). More precisely, the relative error, $\eta^{(n)}$ say, associated with the approximation $r_{\ell F}^{(n)}$ is given by

$$\eta^{(n)} = (r_{\ell F} - r_{\ell F}^{(n)})/r_{\ell F} = 2\gamma^{n+1}(1-\gamma^{n+1}/2) \ . \qquad (13)$$

It is interesting that the relative error $\eta^{(n)}$ associated with the wave-resistance approximation $r_{\ell F}^{(n)}$, which is obtained by using the approximation $\phi_0^{(n)}$ for the velocity potential, is of the same order of magnitude as the error $\varepsilon^{(n+1)}$ associated with the approximation $\phi_0^{(n+1)}$ to the potential. As a matter of fact, we have

$$\eta^{(n)} = 2\varepsilon^{(n+1)}(1-\varepsilon^{(n+1)}/2) \ , \qquad (14)$$

as may be seen from equations (5) and (13). The above result indicates that it may be more advantageous to determine the wave resistance of a ship by means of the Havelock and Kochin formulas for the energy contained in the waves following the ship, rather than by direct integration of the pressure acting upon the hull. Indeed, use of the approximation of order n to the potential, $\phi_0^{(n)}$, yields a Havelock-Kochin wave-resistance approximation comparable to the wave-resistance approximation which could be obtained by hull-integration of the pressure given by the approximation of order n+1 to the potential, $\phi_0^{(n+1)}$.

2. WAVE RESISTANCE OF A VERTICAL CIRCULAR CYLINDER

It is interesting to examine the limiting case when the beam/length (thickness) ratio b of the vertical elliptical cylinder considered in the previous section is equal to 1, corresponding to a circular cylinder. Equation (4b) then yields $\gamma = 1/2$, so that equation (12) becomes

$$r_{\ell F}^{(n)} = (1-1/2^{n+1})^2 \, r_{\ell F} \ , \quad n \geq 0 \ . \tag{15}$$

By putting b = 1 in equations (10) and (11), we may obtain the following expression for the low-Froude-number wave-resistance approximation $r_{\ell F}$:

$$r_{\ell F} = (64/\pi) \int_0^\infty k^2(\tau)[(\nu^2 - 3\tau^2)/(\nu^2 + \tau^2)]^2 (\nu^2 + \tau^2)^{-1/2} d\tau \ , \tag{16}$$

where the function $k(\tau)$ is given by the integral

$$k(\tau) = \int_0^1 \sin[(\nu^2 + \tau^2)u]u(1-u^2)^{1/2} du \ . \tag{16a}$$

By performing the change of variable $u = \sin\theta$ in equation (16a), we may obtain

$$k(\tau) = \int_0^{\pi/2} \sin[(\nu^2 + \tau^2)\sin\theta]\cos^2\theta \, \sin\theta \, d\theta = -(1/3) \int_0^{\pi/2} \sin[(\nu^2 + \tau^2)\sin\theta] d \cos^3\theta \ .$$

Integration by parts then yields

$$k(\tau) = [(\nu^2 + \tau^2)/3] \int_0^{\pi/2} \cos[(\nu^2 + \tau^2)\sin\theta]\cos^4\theta \, d\theta \ .$$

By using equation 3.715(10) page 401 in Gradshteyn and Ryzhik [7], we may finally obtain

$$k(\tau) = (\pi/2)[J_2(\nu^2 + \tau^2)]/(\nu^2 + \tau^2) \ , \tag{17}$$

where J_2 is the Bessel function of the first kind of the second order. Use of equation (17) in equation (16) then yields

$$r_{\ell F} = 16\pi \int_0^\infty [J_2(\nu^2 + \tau^2)]^2 [(\nu^2 - 3\tau^2)/(\nu^2 + \tau^2)]^2 (\nu^2 + \tau^2)^{-5/2} d\tau \ . \tag{18}$$

It is interesting to consider the low-Froude-number limit $F \to 0$ ($\nu \equiv 1/F \to \infty$) of the above-defined wave-resistance approximation $r_{\ell F}$. Equation (9.2.1) page 364 in Abramowitz and Stegun [8] yields

$$J_2(\nu^2+\tau^2) = -(2/\pi)^{1/2}(\nu^2+\tau^2)^{-1/2}\cos(\nu^2+\tau^2-\pi/4) + O(1/\nu^3) \text{ as } \nu \to \infty .$$

By using this asymptotic approximation in equation (18), we may obtain

$$r_{\ell F} \sim 32\int_0^\infty (\nu^2-3\tau^2)^2(\nu^2+\tau^2)^{-11/2}\cos^2(\nu^2+\tau^2-\pi/4)d\tau \text{ as } F \to 0 .$$

This equation may be expressed in the form

$$r_{\ell F} \sim 16(I_1+I_2) \text{ as } F \to 0 , \qquad (19)$$

where I_1 and I_2 are the integrals

$$I_1 = \int_0^\infty (\nu^2-3\tau^2)^2(\nu^2+\tau^2)^{-11/2}d\tau , \qquad (19a)$$

$$I_2 = \int_0^\infty (\nu^2-3\tau^2)^2(\nu^2+\tau^2)^{-11/2}\sin\{2(\nu^2+\tau^2)\}\, d\tau . \qquad (19b)$$

By performing the change of variable $\tau = \nu\tan\theta$ in equation (19a), we may obtain

$$I_1 = F^6\int_0^{\pi/2}(1-3\tan^2\theta)^2\cos^9\theta\, d\theta = F^6\int_0^{\pi/2}(4\cos^2\theta-3)^2\cos^5\theta\, d\theta .$$

This integral can easily be evaluated analytically, and it may be found that we have

$$I_1 = (104/315)F^6 . \qquad (20)$$

By performing the change of variable $\tau = \nu(u-1)^{1/2}$ in equation (19b), we may obtain

$$I_2 = (F^6/2)\int_1^\infty [\sin(2\nu^2u)](4-3u)^2u^{-11/2}(u-1)^{-1/2}\, du .$$

The low-Froude-number limit of the Fourier integral I_2 is given by

$$I_2 = (F^7/2)(\pi/2)^{1/2}\sin(2/F^2+\pi/4) + O(F^9) \text{ as } F \to 0 , \qquad (21)$$

as may be obtained, for instance, from equation (4) page 48 in Erdélyi [9] .

By using equations (20) and (21) in equation (19), we may finally obtain

$$r_{\ell F} = (1664/315)F^6 + 4(2\pi)^{1/2}F^7 \sin(2/F^2 + \pi/4) + O(F^8) \text{ as } F \to 0 , \qquad (22)$$

in agreement with the result obtained previously (in a different manner) by Guevel, Vaussy, and Kobus [2] and Baba [3] .

3. THE MICHELL THIN-SHIP APPROXIMATION

The classical Michell thin-ship approximation, r_M say, may readily be obtained from equation (9) by putting b=0 in the integrand and $\sigma^{(n)} = 0$. This yields

$$r_M = (16/\pi)b^2\nu^2 \int_0^\infty k^2(\tau)(\nu^2+\tau^2)^{-3/2}\, d\tau\ , \tag{23}$$

where the function $k(\tau)$ is given by equation (10) in which b is taken as b=0. We thus have

$$k(\tau) = \int_0^1 \sin[\nu(\nu^2+\tau^2)^{1/2}u]u(1-u^2)^{-1/2}\, du\ . \tag{23a}$$

By performing the change of variable $u = \sin\theta$ in equation (23a), we may obtain

$$k(\tau) = \int_0^{\pi/2} \sin[\nu(\nu^2+\tau^2)^{1/2}\sin\theta]\sin\theta\, d\theta = -\int_0^{\pi/2} \sin[\nu(\nu^2+\tau^2)^{1/2}\sin\theta]d\cos\theta\ .$$

Integration by parts then yields

$$k(\tau) = \nu(\nu^2+\tau^2)^{1/2}\int_0^{\pi/2} \cos[\nu(\nu^2+\tau^2)^{1/2}\sin\theta]\cos^2\theta\, d\theta\ .$$

By using equation 3.715(10) page 401 in [7], we may finally obtain

$$k(\tau) = (\pi/2)J_1\{\nu(\nu^2+\tau^2)^{1/2}\}\ , \tag{24}$$

where J_1 is the Bessel function of the first kind of the first order. Use of equation (24) in equation (23) then yields

$$r_M = 4\pi b^2\nu^2 \int_0^\infty [J_1\{\nu(\nu^2+\tau^2)^{1/2}\}]^2(\nu^2+\tau^2)^{-3/2}\, d\tau\ . \tag{25}$$

In the low-Froude-number limit, we have

$$J_1\{\nu(\nu^2+\tau^2)^{1/2}\} = -\ (2/\pi\nu)^{1/2}\ (\nu^2+\tau^2)^{-1/4}\ \cos\{\nu(\nu^2+\tau^2)^{1/2}+\pi/4\} + O(1/\nu^3)$$

as $\nu \to \infty$,

as may be obtained from equation (9.2.1) page 364 in [8]. By using this asymptotic expansion in equation (25), we may obtain

$$r_M \sim 8b^2\nu \int_0^\infty (\nu^2+\tau^2)^{-2}\cos^2\{\nu(\nu^2+\tau^2)^{1/2}+\pi/4\}\,d\tau \text{ as } F \to 0 .$$

This equation may be expressed in the form

$$r_M \sim 4b^2\nu\,(I_1 - I_2) \text{ as } F \to 0 , \tag{26}$$

where I_1 and I_2 are given by the integrals

$$I_1 = \int_0^\infty (\nu^2+\tau^2)^{-2}d\tau , \quad I_2 = \int_0^\infty (\nu^2+\tau^2)^{-2}\sin\{2\nu(\nu^2+\tau^2)^{1/2}\}\,d\tau , \tag{26a,b}$$

The integral I_1 can be evaluated analytically; we have

$$I_1 = (\pi/4)F^3 . \tag{27}$$

By performing the change of variable $\tau = \nu(u^2-1)^{1/2}$ in equation (26b), we may obtain

$$I_2 = F^3 \int_1^\infty [\sin(2\nu^2u)](u+1)^{-1/2}u^{-3}(u-1)^{-1/2}\,du .$$

The low-Froude-number limit of this Fourier integral is given by

$$I_2 = (F^4/2)\pi^{1/2}\sin(2/F^2+\pi/4) + 0(F^6) \text{ as } F \to 0 , \tag{28}$$

as may be obtained from equation (4) page 48 in [9]. By using equations (27) and (28) in equation (26), we may finally obtain

$$r_M/b^2 = \pi F^2 - 2\pi^{1/2}F^3\sin(2/F^2+\pi/4) + 0(F^4) \text{ as } F \to 0 , \tag{29}$$

in agreement with equation (72) in Maruo [4] .

4. THE LOW-FROUDE-NUMBER LIMIT

We now consider the low-Froude-number asymptotic behavior of the low-Froude-number wave-resistance approximation for a vertical elliptical cylinder given by equations (11) and (10). Equation (10) may be expressed in the form

$$k(\tau) = \int_0^1 \sin(\rho u)\phi(u)(1-u)^{-1/2}\, du , \tag{30}$$

where the function $\phi(u)$ is defined as

$$\phi(u) = u(1-u)(1+u)^{1/2} N(u)/D(u) . \tag{30a}$$

The low-Froude-number limit of the Fourier integral (30) is given by

$$k(\tau) \sim -(\pi/2)^{1/2}(\nu^2 - 3\tau^2)(\nu^2 + b^2\tau^2)^{1/4} \sin(\rho+\pi/4)/b^2(\nu^2 + \tau^2)^{11/4} \text{ as } F \to 0 , \tag{31}$$

as may be obtained from equation (6) page (48) in [9]. By using the asymptotic approximation (31) in equation (11), we may obtain

$$r_{\ell F} \sim 8[(1+b)/b]^2 \int_0^\infty (\nu^2 - 3\tau^2)^2(\nu^2 + b^2\tau^2)^{3/2}(\nu^2 + \tau^2)^{-7} \sin^2(\rho+\pi/4)d\tau \text{ as } F \to 0 .$$

By performing the change of variable $\tau = \nu t$, we may express the above equation in the form

$$r_{\ell F} \sim 4F^6[(1+b)/b]^2[I(b) + J(b,F)] \text{ as } F \to 0 , \tag{32}$$

where I and J are given by the integrals

$$I(b) = \int_0^\infty (1-3t^2)^2(1+b^2t^2)^{3/2}(1+t^2)^{-7}\, dt , \tag{32a}$$

$$J(b,F) = \int_0^\infty (1-3t^2)^2(1+b^2t^2)^{3/2}(1+t^2)^{-7} \sin[2\nu^2(1+t^2)^{1/2}(1+b^2t^2)^{1/2}]dt . \tag{32b}$$

By performing the change of variable $u = (1+t^2)^{1/2}(1+b^2t^2)^{1/2}$, which yields $t = (1+b^2)^{1/2}[\{1+4b^2(u^2 - 1)/(1+b^2)^2\}^{1/2} - 1]^{1/2}/2^{1/2}b$, we may express the function J(b,F) in the form

$$J(b,F) = \int_1^\infty \sin(2\nu^2 u)\phi(u)(u-1)^{-1/2}\, du , \tag{33}$$

where the function $\phi(u)$ is given by

$$\phi(u) = (1-3t^2)^2(1+b^2t^2)^{3/2}(1+t^2)^{-7}(u-1)^{1/2}(dt/du) \ . \quad (33a)$$

The low-Froude-number limit of the Fourier integral (33) is given by

$$J(b,F) \sim [\pi^{1/2}/2(1+b^2)^{1/2}]F \sin(2/F^2 + \pi/4) \quad \text{as } F \to 0 \ . \quad (34)$$

Use of the asymptotic approximation (34) in equation (32) then yields

$$r_{\ell F} \sim 4F^6[(1+b)/b]^2 \ I(b) + 2\pi^{1/2}[(1+b)^2/b^2(1+b^2)^{1/2}]F^7 \sin(2/F^2+ \pi/4) \text{ as } F \to 0, \quad (35)$$

where the function $I(b)$ is defined by the integral (32a). In the limit $b=1$, we have $I(1) = 104/305$, and the asymptotic approximation (35) becomes identical to equation (22).

We have $I(0) = 21\pi/256$, and equation (35) yields

$$r_{\ell F} \sim (21\pi/64)F^6/b^2 + 2\pi^{1/2}(F^7/b^2)\sin(2/F^2+ \pi/4) \text{ as } F \to 0 \ \& \ b \to 0 \ . \quad (36)$$

This asymptotic approximation however is not valid in the limit $b=0$. Indeed, equation (36) assumes $F \ll b$. The low-Froude-number thin-ship approximation (36) should be compared to the thin-ship low-Froude-number approximation given by equation (29). The latter approximation may be expressed in the form

$$r_{\ell F} \sim \pi b^2F^2- 2\pi^{1/2}b^2F^3 \sin(2/F^2+ \pi/4) \text{ as } b \to 0 \ \& \ F \to 0 \ . \quad (37)$$

The approximation (37), which assumes $b \ll F$, is quite different from the approximation (36), valid if $F \ll b$. This difference stems from the square-root singularity that develops at $u=1$ in the integrand of equation (10) in the limit $b \to 0$. If the Froude number F and the beam/length ratio b are both small and of the same order of magnitude, ε say, both equations (36) and (37) yield $r_{\ell F} = O(\varepsilon^4)$ as $\varepsilon \to 0$.

APPENDIX : THE KOCHIN FREE-WAVE AMPLITUDE FUNCTION FOR A VERTICAL ELLIPTICAL CYLINDER IN THE LOW-FROUDE-NUMBER APPROXIMATION

Let $\gamma_\pm$ be defined as

$$\gamma_\pm = \alpha \pm \beta ,$$

where α and β are given by equations (8a,b). We then have

$$\gamma_\pm = (\nu \cos\theta \pm b\tau \sin\theta)(\nu^2 + \tau^2)^{1/2} . \qquad \text{(A1)}$$

Differentiation of the above equation yields

$$d\gamma_\pm/d\theta = -(\nu \sin\theta \mp b\tau \cos\theta)(\nu^2 + \tau^2)^{1/2} .$$

The expression

$$2(\nu \sin\theta \sin\alpha \sin\beta + b\tau \cos\theta \cos\alpha \cos\beta)$$

may be written in the form

$$\nu \sin\theta(\cos\gamma_- - \cos\gamma_+) + b\tau \cos\theta(\cos\gamma_- + \cos\gamma_+) =$$

$$(\nu \sin\theta + b\tau \cos\theta)\cos\gamma_- - (\nu \sin\theta - b\tau \cos\theta)\cos\gamma_+ =$$

$$[\cos\gamma_+(d\gamma_+/d\theta) - \cos\gamma_-(d\gamma_-/d\theta)]/(\nu^2 + \tau^2)^{1/2} =$$

$$[d(\sin\gamma_+ - \sin\gamma_-)/d\theta]/(\nu^2 + \tau^2)^{1/2} .$$

The integral

$$2(\nu^2 + \tau^2)^{1/2} \int_0^{\pi/2} (\nu \sin\theta \sin\alpha \sin\beta + b\tau \cos\theta \cos\alpha \cos\beta)\cos\theta \, d\theta$$

may then be expressed in the form

$$\int_0^{\pi/2} [d(\sin\gamma_+ - \sin\gamma_-)/d\theta]\cos\theta \, d\theta .$$

Integration by parts finally yields

$$\int_0^{\pi/2} (\sin\gamma_+ - \sin\gamma_-)\sin\theta \, d\theta$$

since the expression $(\sin\gamma_+ - \sin\gamma_-)\cos\theta$ vanishes for both $\theta=0$ and $\theta=\pi/2$.

Equation (8) may then be expressed in the form

$$2K_{\ell F}^{(n)} = ib \int_0^{\pi/2} [\{(b^2\cos^2\theta-\sigma^{(n)}b\sin^2\theta)/(b^2\cos^2\theta+\sin^2\theta)-\nu^2/(\nu^2+\tau^2)\} \cos\theta(\sin\gamma_+ + \sin\gamma_-) + \sigma^{(n)}\nu\tau(\nu^2+\tau^2)^{-1}\sin\theta(\sin\gamma_+ - \sin\gamma_-)]d\theta \ .$$

By grouping the terms multiplying $\sin\gamma_+$ and $\sin\gamma_-$, we may express the above equation in the form

$$2K_{\ell F}^{(n)} = ib \int_0^{\pi/2} (c_+\sin\gamma_- + c_- \sin\gamma_+)d\theta \ , \qquad \text{(A2)}$$

where the factors c_+ and c_- are given by

$$c_\pm = (b^2\cos^2\theta - \sigma^{(n)}b\sin^2\theta)\cos\theta/(b^2\cos^2\theta + \sin^2\theta)-(\nu^2\cos\theta \pm \sigma^{(n)}\nu\tau\sin\theta)/(\nu^2+\tau^2) \ .$$

Let ρ and λ be defined as

$$\rho = (\nu^2+\tau^2)^{1/2}(\nu^2+b^2\tau^2)^{1/2} \ , \qquad \text{(A3)}$$

$$\cos\lambda = \nu/(\nu^2+b^2\tau^2)^{1/2} \ , \quad \sin\lambda = b\tau/(\nu^2+b^2\tau^2)^{1/2} \ . \qquad \text{(A4a,b)}$$

Equation (A1) defining $\gamma_\pm$ may then be written in the form

$$\gamma_\pm = \rho \cos(\theta \mp \lambda) \ .$$

Equation (A2) may now be expressed in the form

$$2K_{\ell F}^{(n)} = ib \int_0^{\pi/2} [c_+(\theta;\lambda)\sin\{\rho \cos(\theta+\lambda)\} + c_-(\theta;\lambda)\sin\{\rho \cos(\theta-\lambda)\}]d\theta \ , \qquad \text{(A5)}$$

where $c_\pm(\theta;\lambda)$ is given by

$$c_\pm(\theta;\lambda) = (b^2\cos^2\theta - \sigma^{(n)}b\sin^2\theta)\cos\theta/(b^2\cos^2\theta + \sin^2\theta) -(b^2\cos\lambda \cos\theta \pm \sigma^{(n)}b\sin\lambda \sin\theta)\cos\lambda/(b^2\cos^2\lambda + \sin^2\lambda) \ . \qquad \text{(A5a)}$$

By performing the changes of variables $u = \theta\pm\lambda$ in equation (A5), we may obtain

$$2K_{\ell F}^{(n)} = ib\{ \int_\lambda^{\pi/2+\lambda} c_+(u-\lambda;\lambda)\sin(\rho \cos u)du + \int_{-\lambda}^{\pi/2-\lambda} c_-(u+\lambda;\lambda)\sin(\rho \cos u)du \} \ .$$

This equation may be expressed in the form

$$2K_{\ell F}^{(n)} = ib(I_1 + I_2 + I_3) \ , \tag{A6}$$

where I_1, I_2, and I_3 are defined as the following integrals:

$$I_1 = \int_0^{\pi/2} [c_+(u-\lambda;\lambda)+c_-(u+\lambda;\lambda)]\sin(\rho \cos u)du \ , \tag{A6a}$$

$$I_2 = \int_\lambda^0 c_+(u-\lambda;\lambda)\sin(\rho \cos u)du + \int_{-\lambda}^0 c_-(u+\lambda;\lambda)\sin(\rho \cos u)du \ , \tag{A6b}$$

$$I_3 = \int_{\pi/2}^{\pi/2+\lambda} c_+(u-\lambda;\lambda)\sin(\rho \cos u)du + \int_{\pi/2}^{\pi/2-\lambda} c_-(u+\lambda;\lambda)\sin(\rho \cos u)du \ . \tag{A6c}$$

Equation (A6b) may be expressed in the form

$$I_2 = -\int_0^\lambda c_+(u-\lambda;\lambda)\sin(\rho \cos u)du + \int_0^\lambda c_-(-u+\lambda;\lambda)\sin(\rho \cos u)du$$

$$= -\int_0^\lambda [c_+(u-\lambda;\lambda)-c_-(-u+\lambda;\lambda)]\sin(\rho \cos u)du \ .$$

Equation (A5a) then shows that we have

$$I_2 = 0 \ . \tag{A7a}$$

By performing the changes of variables $u = \pi/2 + v$ and $u = \pi/2-v$ in equation (A6c), we may obtain

$$I_3 = \int_0^\lambda c_+(\pi/2+v-\lambda;\lambda)\sin(-\rho \sin v)dv + \int_0^\lambda c_-(\pi/2-v+\lambda;\lambda)\sin(\rho \sin v)(-dv)$$

$$= -\int_0^\lambda [c_+(\pi/2+v-\lambda;\lambda)+ c_-(\pi/2-v+\lambda;\lambda)]\sin(\rho \sin v)dv \ .$$

Equation (A5a) shows that we also have

$$I_3 = 0. \tag{A7b}$$

By using equation (A6a) and equations (A7a,b) in equation (A6), we may obtain

$$2K_{\ell F}^{(n)} = ib \int_0^{\pi/2} c(\theta)\sin(\rho \cos \theta)d\theta \ , \tag{A8}$$

where $c(\theta) \equiv c_+(\theta-\lambda;\lambda)+c_-(\theta+\lambda;\lambda)$ is given by

$$c(\theta) = [b^2\cos^2(\theta+\lambda)-\sigma^{(n)}b\sin^2(\theta+\lambda)]\cos(\theta+\lambda)/[b^2\cos^2(\theta+\lambda)+\sin^2(\theta+\lambda)]$$
$$+[b^2\cos^2(\theta-\lambda)-\sigma^{(n)}b\sin^2(\theta-\lambda)]\cos(\theta-\lambda)/[b^2\cos^2(\theta-\lambda)+\sin^2(\theta-\lambda)]$$
$$-\ 2\nu\ \cos\theta(\nu\ \cos\lambda\ -\ \sigma^{(n)}\tau\ \sin\lambda)/(\nu^2+\ \tau^2)\ . \qquad \text{(A8a)}$$

Let u and v be defined as

$$u = \cos\theta\ , \qquad v = (1-u^2)^{1/2} = \sin\theta\ . \qquad \text{(A9a,b)}$$

Use of equations (A9a,b) and (A4a,b) yields

$$\cos(\theta+\lambda) = (\nu u - b\tau v)/(\nu^2+ b^2\tau^2)^{1/2}\ , \qquad \cos(\theta-\lambda)=(\nu u + b\tau v)/(\nu^2+ b^2\tau^2)^{1/2}\ ,$$
$$\sin(\theta+\lambda) = (\nu v + b\tau u)/(\nu^2+ b^2\tau^2)^{1/2}\ , \qquad \sin(\theta-\lambda)=(\nu v - b\tau u)/(\nu^2+ b^2\tau^2)^{1/2}\ .$$

The function $c(\theta)$ defined by equation (A8a) then becomes the function $c(u)$ given by the equation

$$(\nu^2+ b^2\tau^2)^{1/2}c(u) = (\nu u-b\tau v)[b^2(\nu u-b\tau v)^2-\sigma^{(n)}b(\nu v+b\tau u)^2]/[b^2(\nu u-b\tau v)^2+(\nu v+b\tau u)^2]$$
$$+(\nu u+b\tau v)[b^2(\nu u+b\tau v)^2-\sigma^{(n)}b(\nu v-b\tau u)^2]/[b^2(\nu u+b\tau v)^2+(\nu v-b\tau u)^2]$$
$$-2\nu u(\nu^2- \sigma^{(n)}b\tau^2)/(\nu^2+ \tau^2)\ .$$

The change of variable $u = \cos\theta$ in equation (A8) then yields

$$2K_{\ell F}^{(n)}= ib \int_0^1 \sin(\rho u)c(u)(1-u^2)^{-1/2}du\ .$$

This equation may be expressed in the form

$$2(\nu^2+ b^2\tau^2)^{1/2}\ K_{\ell F}^{(n)}= ib \int_0^1 \sin(\rho u)f(u)(1-u^2)^{-1/2}du\ , \qquad \text{(A10)}$$

where the function $f(u)$ is given by

$$f(u) = (\nu u-b\tau v)A(u,v)/B(u,v)+(\nu u+b\tau v)A(u,-v)/B(u,-v)-2\nu u(\nu^2- \sigma^{(n)}b\tau^2)/(\nu^2+ \tau^2);$$

in this equation, the functions $A(u,v)$ and $B(u,v)$ are defined as

$$A(u,v) = b^2(\nu u-b\tau v)^2- \sigma^{(n)}b(b\tau u+\nu v)^2\ , \quad B(u,v) = b^2(\nu u-b\tau v)^2+ (b\tau u+\nu v)^2\ .$$

For simplicity, the notation

$$A \equiv A(u,v)\ , \quad B \equiv B(u,v)\ , \quad A^- \equiv A(u,-v)\ , \quad B^- \equiv B(u,-v)\ ,$$

will be used. The above-defined function f(u) then becomes

$$f(u) = [\nu u(AB^- + A^-B)+b\tau v(A^-B-AB^-)]/BB^- - 2\nu u(\nu^2 - \sigma^{(n)}b\tau^2)/(\nu^2+\tau^2) .$$

Let the functions A(u,v) and B(u,v) be expressed in the form

$$A = A^e - A^o , \quad B = B^e + B^o ,$$

where A^e, A^o, B^e, B^o are given by

$$A^e = b^4\tau^2 - \sigma^{(n)}b\nu^2 + (b^2+\sigma^{(n)}b)(\nu^2 - b^2\tau^2)u^2 , \quad A^o = 2b(b^2+\sigma^{(n)}b)\nu\tau uv , \quad \text{(A11a,b)}$$

$$B^e = \nu^2 + b^4\tau^2 - (1-b^2)(\nu^2-b^2\tau^2)u^2 , \quad B^o = 2b(1-b^2)\nu\tau uv . \quad \text{(A11c,d)}$$

We then have $A^- = A^e + A^o$, $B^- = B^e - B^o$, and

$$AB^- + A^-B = 2(A^eB^e + A^oB^o) , \quad A^-B-AB^- = 2(A^eB^o + A^oB^e) , \quad BB^- = (B^e)^2-(B^o)^2 ,$$

as may easily be verified. The function f(u) then becomes

$$f(u)/2 = [\nu(A^eB^e + A^oB^o)u+b\tau(A^eB^o + A^oB^e)v]/[(B^e)^2-(B^o)^2]-\nu u(\nu^2 - \sigma^{(n)}b\tau^2)/(\nu^2+\tau^2) .$$

Equations (A11) yield

$$A^eB^o + A^oB^e = 2b\nu\tau uv[(1-b^2)A^e + (b^2+\sigma^{(n)}b)B^e] = 2b^3\nu\tau uv(1+\sigma^{(n)}b)(\nu^2+b^2\tau^2) .$$

Let C and D be defined as

$$C = A^eB^e + A^oB^o , \quad D = (B^e)^2-(B^o)^2 . \quad \text{(A12a,b)}$$

We may then obtain

$$f(u)/2\nu u = C/D - (\nu^2 - \sigma^{(n)}b\tau^2)/(\nu^2+\tau^2) + 2b^4(1+\sigma^{(n)}b)\tau^2(\nu^2+b^2\tau^2)(1-u^2)/D ,$$

where equation (A9b) was used. This yields

$$f(u)/2\nu u = [(\nu^2+\tau^2)C-(\nu^2 - \sigma^{(n)}b\tau^2)D]/(\nu^2+\tau^2)D + 2b^4(1+\sigma^{(n)}b)\tau^2(\nu^2+b^2\tau^2)(1-u^2)/D .$$

It may be shown from equations (A12) and (A11) that we have

$$D = (\nu^2+b^4\tau^2)^2 - 2(1-b^2)(\nu^2+b^2\tau^2)(\nu^2-b^4\tau^2)u^2 + (1-b^2)^2(\nu^2+b^2\tau^2)^2u^4 , \quad \text{(A13)}$$

$$C = 4b^2(1-b^2)(b^2+\sigma^{(n)}b)\nu^2\tau^2u^2(1-u^2) + A^eB^e .$$

The function f(u) may then be expressed in the form

$$f(u)/2\nu u = 2b^2[b^2(1+\sigma^{(n)}b)(\nu^2+b^2\tau^2) + 2(1-b^2)(b^2+\sigma^{(n)}b)\nu^2u^2]\tau^2(1-u^2)/D$$
$$+[(\nu^2+\tau^2)A^eB^e-(\nu^2-\sigma^{(n)}b\tau^2)D]/(\nu^2+\tau^2)D . \quad \text{(A14)}$$

The expression $(\nu^2+\tau^2)A^eB^e-(\nu^2-\sigma^{(n)}b\tau^2)D$ may be written in the form

$$(\nu^2+\tau^2)A^eB^e-(\nu^2-\sigma^{(n)}b\tau^2)D = -P + Qu^2 - Ru^4 \ , \qquad \text{(A15)}$$

where the factors P, Q, R are given by

$$P = (\nu^2+b^4\tau^2)[(\nu^2+\tau^2)(\sigma^{(n)}b\nu^2-b^4\tau^2)+(\nu^2-\sigma^{(n)}b\tau^2)(\nu^2+b^4\tau^2)] \ , \qquad \text{(A15a)}$$

$$Q = (\nu^2+\tau^2)(\nu^2-b^2\tau^2)[(1-b^2)(\sigma^{(n)}b\nu^2-b^4\tau^2)+(b^2+\sigma^{(n)}b)(\nu^2+b^4\tau^2)]$$
$$+2(1-b^2)(\nu^2+b^2\tau^2)(\nu^2-\sigma^{(n)}b\tau^2)(\nu^2-b^4\tau^2) \ , \qquad \text{(A15b)}$$

$$R = (1-b^2)[(b^2+\sigma^{(n)}b)(\nu^2+\tau^2)(\nu^2-b^2\tau^2)^2+(1-b^2)(\nu^2-\sigma^{(n)}b\tau^2)(\nu^2+b^2\tau^2)^2] \ . \qquad \text{(A15c)}$$

We have

$$P + R - Q \equiv 0 \ , \qquad \text{(A16)}$$

as may be verified after fairly lengthy transformations. By using equation (A16) in equation (A15), we may then obtain

$$(\nu^2+\tau^2)A^eB^e-(\nu^2-\sigma^{(n)}b\tau^2)D = -(1-u^2)(P-Ru^2) \ .$$

Equation (A14) may then be expressed in the form

$$(\nu^2+\tau^2)D\ f(u)/2\ \nu u(1-u^2) = S + Tu^2 \ , \qquad \text{(A17)}$$

where S and T are defined as

$$S = 2b^4(1+\sigma^{(n)}b)(\nu^2+\tau^2)(\nu^2+b^2\tau^2)\tau^2 - P \ , \qquad \text{(A17a)}$$

$$T = 4b^2(1-b^2)(b^2+\sigma^{(n)}b)(\nu^2+\tau^2)\nu^2\tau^2 + R \ . \qquad \text{(A17b)}$$

By using equation (A15c), we may express equation (A17b) in the form

$$T = (1+\sigma^{(n)}b)(1-b^2)(\nu^2+b^2\tau^2)^3 \ . \qquad \text{(A18a)}$$

Furthermore, by using equation (A15a) in equation (A17a), and after fairly lengthy transformations, we may obtain

$$S = -(1+\sigma^{(n)}b)(\nu^2+b^2\tau^2)^2(\nu^2-2b^2\tau^2-b^4\tau^2) \ . \qquad \text{(A18b)}$$

Use of equations (A18a,b) in equation (A17) then yields

$$(\nu^2+\tau^2)D\ f(u)/2\ \nu u(1-u^2) = -(1+\sigma^{(n)}b)(\nu^2+b^2\tau^2)^3N(u) \ , \qquad \text{(A19)}$$

where N(u) is defined as

$$N(u) = [\nu^2-2b^2(1+b^2/2)\tau^2]/(\nu^2+b^2\tau^2)-(1-b^2)u^2 \ . \qquad \text{(A19a)}$$

Let the function D(u) defined by equation (A13) be expressed in the form

$$D(u) = (\nu^2 + b^2\tau^2)^2\, \overline{D}(u) \;, \tag{A20}$$

where $\overline{D}(u)$ thus is given by

$$\overline{D}(u) = [(\nu^2 + b^4\tau^2)/(\nu^2 + b^2\tau^2)]^2 - 2(1-b^2)[(\nu^2 - b^4\tau^2)/(\nu^2 + b^2\tau^2)]u^2 + (1-b^2)^2u^4 . \tag{A20a}$$

Use of equation (A20) in equation (A19) then yields

$$f(u) = -2\nu(1+\sigma^{(n)}b)[(\nu^2 + b^2\tau^2)/(\nu^2 + \tau^2)]u(1-u^2)N(u)/\overline{D}(u) \;. \tag{A21}$$

By using equation (A21) in equation (A10), we may obtain the following expression for the Kochin free-wave amplitude function:

$$K^{(n)}_{\ell F}(\tau;\nu,b) = -ib\nu(1+\sigma^{(n)}b)[(\nu^2 + b^2\tau^2)^{1/2}/(\nu^2 + \tau^2)]k(\tau;\nu,b) \;, \tag{A22}$$

where the function $k(\tau;\nu,b)$ is defined as

$$k(\tau;\nu,b) = \int_0^1 \sin(\rho u)u(1-u^2)^{1/2}[N(u)/\overline{D}(u)]du \;. \tag{A22a}$$

In equation (A22a), ρ is given by equation (A3), which is identical to equation (10a) in section 1, and the functions N(u) and $\overline{D}(u)$ are given by equations (A19a) and (A20a), respectively. The functions N(u) and $\overline{D}(u)$ can be expressed in the form of equations (10b,c) in section 1, so that equation (A22a) is identical to equation (10). Equation (9) can then finally be obtained by using equation (A22) in equation (6).

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