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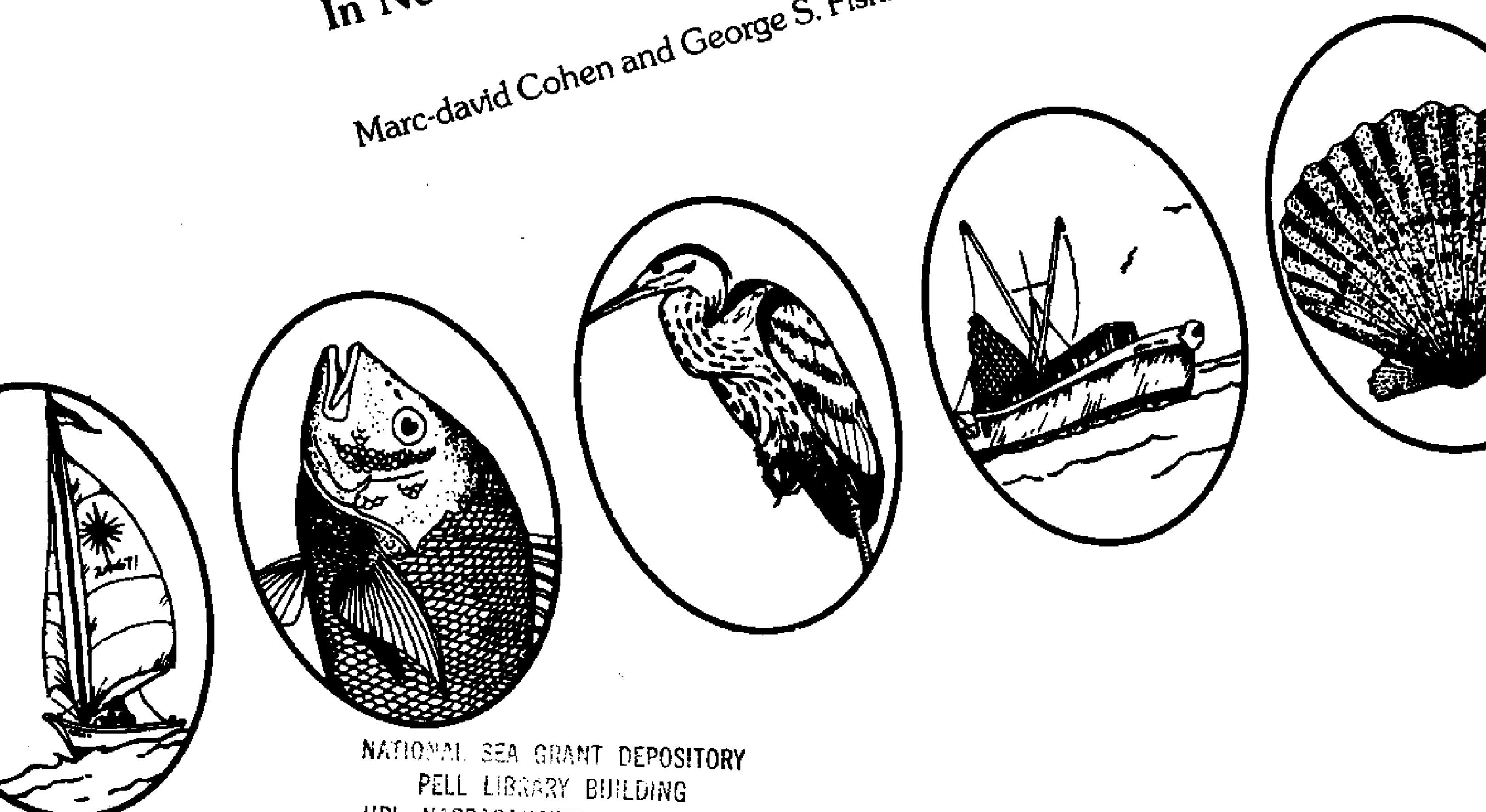
Modeling the Relationship Between Biomass and Revenue In a Regional Setting

With an Example from the Brown Shrimp Fishery In North Carolina

Marc-david Cohen and George S. Fishman



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EXECUTIVE SUMMARY

Since the passage of the Fisheries Conservation and Management Act (FCMA) in 1976 there has been increasing attention to the problems associated with managing marine resources. By establishing a fishery conservation zone that extends 200 nautical miles from the coastline into the sea, the FCMA not only increased the zone of jurisdiction but also increased the responsibilities for managing the marine resources found in that zone. To facilitate management, the act established eight Regional Fishery Councils to "prepare, monitor and revise" management plans to achieve the specific management goals that are outlined in the act.

It is this mandate for managing our fishery resources that has encouraged research in areas that would help to provide a scientific basis for management. These efforts have taken several different directions. The most general of these directions is in direct response to the FCMA's call for a systematic approach to developing management plans. Broad plans have been proposed by the regional councils for developing comprehensive management systems (see Eldridge and Goldstein 1975). Based on an assessment of the issues, they have targeted specific areas for research and called for a completely new orientation toward solving management problems. As Eldridge states with regard to the management of the southeast shrimp fishery,

> The present management regime has evolved over approximately 50 years and is based largely on biological knowledge, experience, and intuition. In fact, the major limitation of the present regime is the lack of methodology to adequately evaluate management decisions (Eldridge and Goldstein 1975, p. 10).

In this document the South Atlantic Committee for Shrimp Management calls for the development of a methodology based on biological models that would enable managers to evaluate the impact of management decisions on the fishing industries. The purpose of the present paper is to introduce such a methodology and to demonstrate its value using data from the North Carolina brown shrimp fishery.

This work principally is concerned with developing a methodology for evaluating the impact of fishery management decisions on catch biomass, catch revenue, and catch profit. In particular, it presents models relating the character of the fish population to the distribution of each measure of catch. It also shows with two examples how these can be used with decision analysis techniques to determine optimal fishery management policies. In both examples a manager must determine mesh size and fishing season length for a shrimp population. In one case he has perfect information about the cost of fishing; in the other case he has incomplete information. The analysis shows how to determine the optimal decision, based on maximizing profit, in both of these cases. It also demonstrates how one determines the value of obtaining more accurate fishing cost information when such information is incomplete. The proposed models focus on the relationship between the size of an arbitrary member of the fish population, the size of the fishing mesh in the net, and the resulting catch revenue. These models form an integral part of a comprehensive multiple age-class model for management policy analysis.

Since variation in revenue and profit is important to fishing institutions and consequently is an important element in fishery management decisions, the proposed models explicitly acknowledge the structural and random variation in revenue and profit. Both components of variation in profit and revenue can be identified with sources of variation in the marketplace, in the abundance of catch, and in the size of the members of catch. In turn, the variation in catch biomass can be identified with sources of variation in the abundance of the fish population, in the size of its members, and in a contribution resulting from the mesh size of the fishing net. Since stochastic simulation models are particularly amenable to comprehensive, detailed modeling, we focus our attention on these types of models.

Two models emerge in the paper: one relates revenue to catch biomass, and the other relates the weight of captured fish to fishing net mesh size and the character of the fish population. Each incorporates the influences of both structural and random variation. In turn these are used to describe how a particular mesh size (as characterized by a mesh selectivity curve) affects the catch biomass, catch revenue, and catch profit.

The relationship between revenue and catch biomass depends on the characterization of price in terms of the mean and variance of the weight of a captured fish. In the setting of the North Carolina brown shrimp fishery we consider three sources of variation in this relationship. The first source reflects the dependence of price on the size of shrimp in the catch, directly proportional to shrimp size. The second source represents a temporal component that reflects changes with time in the relationship between price and size. The third source accounts for the random variation that results from factors exogenous to the regional fishery. Because the fishery is a regional fishery, which accounts for a small portion of total catch, price is independent of regional catch abundance. Consequently, the relationship between price and regional catch abundance is not examined.

The size of the fishing mesh affects catch in two ways. First, the size distribution of captured fish depends both on the size distribution of fish in the population and on the mesh size. Therefore, by changing the mesh size one also changes the distribution of fish size in the catch and consequently the price. Second, by changing mesh size one also changes the number of fish captured, which in turn affects revenue. We develop a model of net selectivity (based on a continuous mesh selection curve) that accounts for each of these factors.

Although the models are presented within the framework of the North Carolina brown shrimp fishery, they have greater generality. In this setting, where data exist, techniques of parameter estimation are given, along with parameter estimates and residual analysis. Algorithms for parameter estimation and for computer simulation sampling of catch revenue are also exhibited.

The purpose of developing comprehensive models is to provide the fishery manager with tools that aid in policy analysis. The models we exhibit provide the basis of a methodology for accomplishing this goal as is amply demonstrated in the examples.

Eldridge, P.J. and S.A. Goldstein, ed. (1975). The Shrimp Fishery of the South Atlantic United States: A Regional Management Plan. South Carolina Marine Resources Center Technical Report No. 8. Charleston, South Carolina.

Abstract

The ability of marine fishery managers to evaluate policy plans before implementation improves management by identifying new potentially useful management techniques. Typically, management policy is evaluated with a measure of harvest value such as catch revenue or catch profit. This report addresses modeling the relationship between catch biomass and revenue as a part of a larger stochastic fishery-simulation model that is designed for policy analysis. The usefulness of this model in addressing management policy issues is illustrated in an example in Section 4. This is accomplished in the setting of the brown shrimp (Penaeus aztecus) fishery of Pamlico Sound, North Carolina. The report also presents a model of the relationship between fishing net mesh size and the character of catch biomass that is an extension of the Beverton and Holt (1957) model of mesh selectivity.

Since these models are part of a larger stochastic simulation model, probabilistic structure is important. Consequently, the report characterizes the distribution of catch revenue for a given catch biomass. However, because the character of the catch biomass is affected by both mesh size and population age structure, these play a role in the distribution of revenue. In particular, we include an analysis of the affects of mesh size on the distribution of catch revenue for a single age class population. Furthermore, the report presents several algorithms for sampling catch revenue.

An example demonstrates the use of these models in management policy analysis. It shows how alternative mesh sizes and fishery closing

dates affect profit in a hypothetical fishery. Furthermore, since fishing cost is a factor in profit, the relationship between fishing cost and the optimal mesh size and closing date is found. This enables an analysis of the value of fishing cost information.

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Introduction

Managers of marine fisheries have a variety of techniques for fishery regulation. Fishing season restrictions and net-mesh size limitations are two examples. The purpose of this report is two-fold: 1) to model the relationship between catch biomass and revenue in a way that is useful for studying the effects of management regulations, and 2) to demonstrate the technique by examining the effects of season restrictions and mesh size limitations on catch revenue.

Traditional biomass-revenue studies focus on how price per unit biomass relates to biomass supplied. Although this focus is useful for determining a fishery supply curve, additional aspects of this relationship need attention, particularly when modeling a regional segment of a national fishery. This paper describes these additional aspects, which concern the structural components of supply and demand on the national and regional levels, and incorporates them into a regional model for evaluating dockside revenue as a function of the character of the biomass caught in a regional setting.

In many fisheries the quantity of biomass landed in particular regions has little effect on the national price structure. The North Carolina brown shrimp fishery exemplifies this phenomenon. This fishery is a segment of a national shrimp fishery that includes the southeast Atlantic states and the Gulf states. Although important to North Carolina, the effect of North Carolina shrimp landings on price is sufficiently small so as to regard price as a given quantity. Therefore, an analysis of revenue can focus on the variation in the weight characteristics of landings without the need to delve into the interaction between local price and local catch.

Two factors principally influence the weight distribution of captured fish: 1) the age structure of the fish population and 2) the mesh size of the fishing net. By selecting a large mesh size, one skews the distribution of catch to a larger and older segment of the fish population. However, the age structure determines the total quantity of fish caught for a specified mesh size. In particular, note that an age structure skewed to younger fish limits the magnitude of biomass caught when using a large mesh size.

Although Beverton and Holt (1957) address the mesh-weight relationship in their discussion of the experimental work of Davis (1934) and Jensen (1949), room remains to extend their analysis. Beverton and Holt use data on controlled experiments to determine net selection curves. For each length fish caught, a selection curve shows the ratio of catches from two different mesh sizes. They suggest that selection curves be represented by the ratio of the integrals of two normal curves with different means and common variances. We extend this model in a way that insures its compatibility with our biomass-revenue model.

This technical report extends the Beverton and Holt model and incorporates it and the biomass-revenue model into a stochastic characterization of the dockside value of catch. This is accomplished by first characterizing dockside revenues based upon the distributional parameters of captured individuals without explicitly acknowledging the mesh size effects. Then a characterization of dockside revenue is exhibited which explicitly accounts for mesh effects. In particular, the report:

(1) Presents a stochastic model of the biomass-revenue relationship.

(2) Presents a stochastic model of the mesh-weight relationship.

(3) Describes how the two models characterize the dockside revenue of catch.

(4) Illustrates the estimation techniques using data on 1978 brown shrimp landings in Pamlico Sound collected by the National Marine Fisheries Service and the North Carolina Division of Marine Fisheries.

(5) Presents an example of how the dockside revenue model can be used as a management tool to compare revenue obtained from fishing with two nets having differing mesh size.

These models are components of a larger study of the Pamlico Sound brown shrimp fishery. The goal of the larger study is to develop comprehensive methods for evaluating fishery management policies in a single year class fishery. Accordingly, the models presented here reflect the concerns of the larger problem as is illustrated in the example.

1. The Weight-Price Relationship

The characterization of revenue is based on a model that relates the weight of a captured individual fish to price in a regional setting. No widely accepted models of this relationship exist in the fisheries literature. Since one of our goals is to characterize revenue in the North Carolina brown shrimp fishery, we use data on that fishery to illustrate the concepts that we have in mind.

Description of Industry Practices and Data

Before a catch is priced, the shrimp are sorted according to size, either by machine or by hand. This grading process is done either on the vessel or at dockside and either on a heads-on or heads-off basis. Regardless of the procedure each shrimp falls into one of 12 possible categories or grades. Assignment is based on count per unit weight, usually count per pound. A typical scenario includes unloading the catch from the vessel and removing the head of each shrimp as it is placed in a box according to grade. As a result each box (category) includes shrimp in a range of weights. Regardless, the price per unit biomass is fixed for each box and is a function of its grade.

To represent this structure we partition weight by the points $\{\omega_i : i=0,1,\ldots,12\}$, where $\omega_0 > \omega_1 > \cdots \omega_{12} > 0$, and define the kth category to include all shrimp whose count per unit biomass is within the interval[1] $[1/\omega_{k-1}, 1/\omega_k)$. For the purpose of modeling we choose a representative weight factor w_k for each class $k=1,\ldots,12$ such that[2] $1/w_k \in [1/\omega_{k-1}, 1/\omega_k)$. For example, if $1/w_k$ is the midpoint of the kth interval then $w_k = 2\omega_{k-1}\omega_k/(\omega_{k-1} + \omega_k)$. Table 1 lists the range of count per pound that defines each of the 12 categories used for describing shrimp landings in North Carolina waters.[3] These are taken from trip interview forms established jointly by the National Oceanic and Atmospheric Administration and the North Carolina Division of Marine Fisheries in 1978, these forms also include heads-off price per pound by grade category, biomass landed by grade category, landing date, shrimp species, gear used, location of the catch, time spent fishing and dealer identification. Except for dealer identification all these data were made available to us for the 1978 season. Table 2 lists the geographical subdivisions of Pamlico Sound and adjacent waters for which data have been included in the analysis.

[1]The half open interval $[a,b)$ includes all points x such that $a \le x < b$.
[2]The expression $x \in [a,b)$ means that the point x is such that $a \le x < b$.
[3]See the "South Atlantic Regional Shrimp Trip Interview Form Instructions," published by the State/Federal Shrimp Management Committee, May 1, 1979.

Table 1

Shrimp Grading Parameters

k	Grade Category Range Count Per Pound †	ω_k grams	ω_k pounds	w_k grams	w_k pounds
0		453.6	1.		
1	1 to 15	30.4	.067	64.80	.143
2	15 to 20	22.7	.050	25.92	.057
3	20 to 25	18.1	.040	20.16	.044
4	25 to 30	15.1	.034	16.49	.036
5	30 to 35	12.9	.028	13.96	.031
6	35 to 40	11.3	.025	12.09	.027
7	40 to 45	10.1	.022	10.67	.024
8	45 to 50	9.1	.020	9.55	.021
9	50 to 55	8.2	.018	8.64	.019
10	55 to 60	7.5	.017	7.89	.017
11	60 to 70	6.5	.014	6.98	.015
12	70 and up	5.3	.012	6.05	.013

†These are taken from the "South Atlantic Regional Shrimp Trip Form Instructions," published by the State/Federal Shrimp Management Committee, May 1, 1979.

Table 2

Subdivisions of Pamlico Sound

and Adjacent Waters

Alligator River
Roanoke Sound
Croatan Sound
Pamlico Sound
- Pamlico Sound - East of Bluff Shoal
- Stumpy Point Bay
- Long Shoal River
- Pamlico Sound - West of Bluff Shoal
- Juniper Bay
- Swanquarter Bay
- Rose Bay
- Jones Bay
- Bay River
- West Bay

Pamlico River
- Pungo River
- South Creek
- Goose Creek
- Oyster Creek

Mattamuskeet Lake
Inland Waterway
- Alligator River to Pungo River
- Goose Creek to Bay River

To remove the day-of-week effect, daily data were aggregated on a weekly basis. Since the fishery is seasonal, typically opening in early summer and closing in early winter, this aggregation allows one to preserve the seasonal pattern. Although we recognize the need to account for between-year variation, only one year of data was available, which is insufficient to accomplish this.

Data Analysis

The data analysis focuses on the relationship between price per pound and grade. A data point consists of an observation of the brown shrimp price per pound for a given grade recorded from a dockside transaction. Let the subscripts j and k denote week and grade, respectively, J the set of weekly indices with more than one observation and K_j the set of grade indices with at least one observation.[4] Then for week j and grade k let N_{jk} be the number of observations and $p_{jk\ell}$ the ℓth observation of price per pound. For each week $j \epsilon J$ the maximum observed price is $\hat{Y}_j = \max \{p_{jk\ell} : \ell = 1, \ldots, N_{jk}; k \epsilon K_j\}$ so that $\Delta p_{jk\ell} = \hat{Y}_j - p_{jk\ell}$ denotes the deviation from the maximum observed price for grade k on transaction ℓ. Then

$$\bar{S}_{jk} = \frac{1}{\tilde{N}_{jk}} \sum_{\ell=1}^{N_{jk}} \Delta p_{jk\ell} \qquad k \epsilon K_j \qquad j \epsilon J \; ,$$

where $\tilde{N}_{jk}$ is the number of points for which $p_{jk\ell} \neq \hat{Y}_j$, is the average deviation from the maximum price per pound for week j and grade k. The restriction that $p_{jk\ell} \neq \hat{Y}_j$ is included because of the estimation procedures used as is discussed in section 1.1.

[4]This restriction is required because of the estimation procedures used.

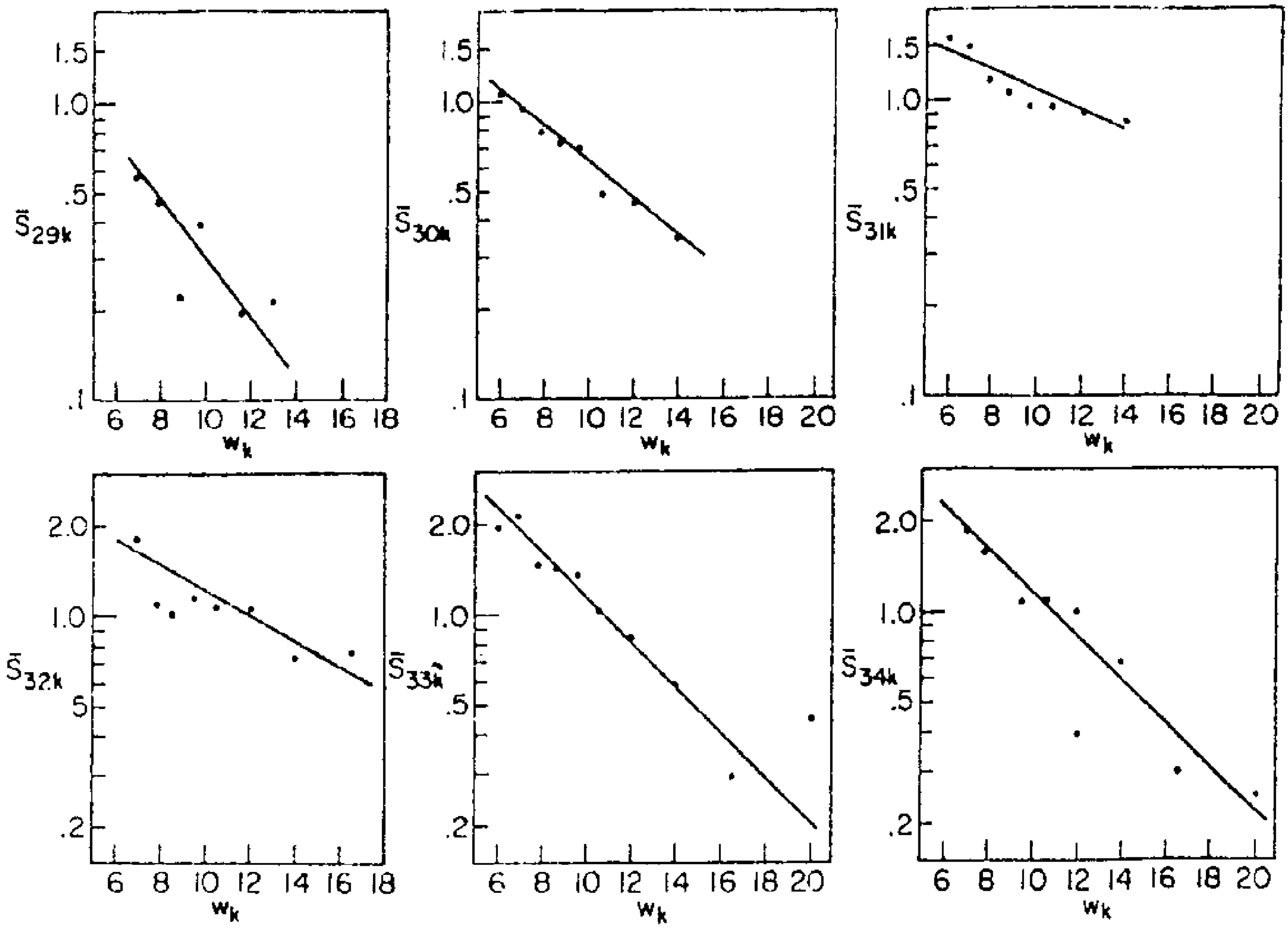

Figure 1

Plot of the 1978 North Carolina Brown Shrimp
Average Deviation from Maximum Price Versus Weight,
and the Estimated Regression Line (weeks 29 through 34)

Figure 1 shows plots of the points $\{(\bar{S}_{jk}, w_k); k \in K_j\}$ on a semilogarithmic scale for selected values of j (weeks). The plots reveal a linear assocation implying an exponential relationship between $\bar{S}_{jk}$ and w_k for $k \in K_j$. Furthermore, for each j, visual examination of the set of deviations $\{\ln (\Delta p_{jk\ell} - \bar{S}_{jk}): \ell = 1,\ldots,\tilde{N}_{jk}; k \in K_j\}$

suggests that it is a set of independent observations of a normally distributed random variable. These observations and the fact that $\bar{S}_{jk}$ is the average deviation from the maximum observed price per pound suggest that we explore the formulation

$$p_{jk\ell} = \gamma_j - \beta_j e^{-\delta_j w_k} \eta_{jk\ell} \qquad \ell=1,\ldots,N_{jk},\ k\epsilon K_j,\ j\epsilon J. \tag{1.1}$$

Here $\beta_j > 0$ and $\delta_j > 0$ are scale parameters, and $\gamma_j > 0$ is a location parameter which represents the maximum possible price per pound. Moreover, if the deviations $\{\ln(\Delta p_{jk\ell} - \bar{S}_{jk}): k\epsilon K_j;\ \ell=1,\ldots,\tilde{N}_{jk}\}$ are normally distributed, then in the exponential model (1.1), $\{\eta_{jk\ell}: \ell=1,\ldots,N_{jk};\ k\epsilon \hat{K}_j\}$ is a sequence of independent and identically distributed (i.i.d.) lognormal random variables. Furthermore, since the elements of this sequence are identically distributed, we write the mean and variance of $\eta_{jk\ell}$ as μ_{η_j} and $\sigma^2_{\eta_j}$ respectively. Moreover, we assume that $\ln \eta_{jk\ell}$ has zero mean. This assumption ensures that β_j and μ_{η_j} are identifiable. The validity of these assumptions is examined after a discussion of parameter estimation.

This characterization of $\eta_{jk\ell}$ implies that for each $\ell=1,\ldots, N_{jk}$, $k\epsilon K_j$ and $j\epsilon J$, $\eta_{jk\ell} > 0$ with probability one and thus $p_{jk\ell} < \gamma_j$ with probability one. Although $p_{jk\ell}$ can be negative, in this setting it occurs with small probability. It also implies that the price per pound $p_{jk\ell}$, has mean

$$\mu_{p_{jk}} = \gamma_j - \beta_j e^{-\delta_j w_k} \mu_{\eta_j} \tag{1.2a}$$

and variance

$$\sigma^2_{p_{jk}} = \beta_j^2 e^{-2\delta_j w_k} \sigma^2_{n_j} \tag{1.2b}$$

Note that (1.2a) and (1.2b) hold for all observations $\ell=1,\ldots,N_{jk}$.

Moreover, $\ln(\gamma_j - p_{jk\ell})$ has a normal distribution with mean and variance

$$\xi_{jk} = \frac{1}{2} \ln\left[\frac{(\gamma_j-\mu_{p_{jk}})^4}{\sigma^2_{p_{jk}} + (\gamma_j - \mu_{p_{jk}})^2}\right] \tag{1.3a}$$

and

$$\psi^2_{jk} = \ln\left[\frac{\sigma^2_{p_{jk}}}{(\gamma_j-\mu_{p_{jk}})^2} + 1\right] \tag{1.3b}$$

respectively. Now, since

$$Z = \frac{\ln(\gamma_j-p_{jk\ell}) - \xi_{jk}}{\psi_{jk}} \tag{1.4}$$

is a standardized normal random variable, we know that

probability (price per pound in week j for grade $k \le y$) =

$$1 - \Phi\left(\frac{\ln(\gamma_j-y) - \xi_{jk}}{\psi_{jk}}\right),$$

where

$$\Phi(x) = \int_{-\infty}^{x} \phi(s)\, ds$$

and

$$\phi(s) = \frac{1}{\sqrt{2\pi}} e^{-s^2/2} \qquad -\infty < s < \infty .$$

1.1 Estimating the Parameters of the Weight-Price Relationship

To examine the assumptions of model (1.1) and to profit from the characterization it provides we require estimates of γ_j, β_j, μ_{n_j} and $\sigma^2_{n_j}$ for each $j \in J$. The problem of estimating these parameters is related to the problem of estimating the parameters of a three-parameter lognormal random variable. A random variable X has this distribution if there exists a number θ, called the location parameter, such that the random variable $\ln(\theta - X)$ has a normal distribution. Examination of (1.4) shows that $p_{jk\ell}$ has a three-parameter lognormal distribution with location parameter γ_j and parameters ξ_{jk} and ψ_{jk}.

Estimating γ_j

Consider the problem of estimating the location parameter γ_j for fixed $k \in K_j$ and $j \in J$ in the setting presented by (1.4). Johnson and Kotz (1970) provide a comprehensive discussion of several approaches to this problem. They note that although there is no well accepted solution, any estimate of γ_j must be bounded below by $\max\{p_{jk\ell} : \ell=1,\ldots,N_{jk};\ k\epsilon K_j\}$ for $j\epsilon J$. This results from the fact that $pr(p_{jk\ell} > \gamma_j) = 0$, as can be seen in (1.4). One approach is to estimate γ_j by $\hat{\gamma}_j$, the maximum observed price per pound. We use this estimator even though it is a biased estimator of γ_j because it is easily calculated.

Estimating β_j, δ_j *and* $\sigma^2_{\epsilon_j}$ *Using the Linear Least Squares Method*

Difficulties associated with estimating the parameters of (1.1) with limited data can be minimized by conditioning on a priori knowledge of γ_j. This enables one to linearize (1.1) so that conventional estimation techniques can be used. First, without loss of generality, one orders and then renames the elements of the sequence $\{p_{jk\ell} : \ell=1,\ldots,N_{jk}\}$ so that for $k\epsilon K_j$ and $j\epsilon J$, $\ell_1 < \ell_2$ implies that $p_{jk\ell_1} \leq p_{jk\ell_2}$. Then one transforms (1.1) to

$$\ln \Delta p_{jk\ell} = \ln \beta_j - \delta_j w_k + \epsilon_{jk\ell} \qquad \begin{array}{l} \ell=1,\ldots,\tilde{N}_{jk} \\ k\epsilon K_j \\ j\epsilon J , \end{array} \qquad (1.5)$$

by subtracting both sides of (1.1) from $\hat{\gamma}_j$ and taking logarithms. This results in a linear model with $\epsilon_{jk\ell} = \ln \eta_{jk\ell}$. Since for $j\epsilon J$, $\{\eta_{jk\ell} : \ell=1,\ldots,\tilde{N}_{jk};\ k\epsilon K_j\}$ is a sequence of i.i.d. lognormal random variables, $\{\epsilon_{jk\ell} : \ell=1,\ldots,\tilde{N}_{jk};\ k\epsilon K_j\}$ is a sequence of i.i.d. normal random variables. As for $\eta_{jk\ell}$, the variance of $\epsilon_{jk\ell}$ is structurally dependent on week (j), but independent of grade (k). Furthermore, since $\varepsilon_{jk\ell} = \ln \eta_{jk\ell}$ and we assumed that $\ln \eta_{jk\ell}$ has zero mean, (1.5) can be regarded as a linear regression model. In this case we write the parameters of the distribution of $\eta_{jk\ell}$ in terms of the variance $\sigma^2_{\varepsilon_j}$ of $\varepsilon_{jk\ell}$ as

$$\mu_{\eta_j} = e^{\sigma^2_{\varepsilon_j}/2} \tag{1.6a}$$

and

$$\sigma^2_{\eta_j} = e^{\sigma^2_{\varepsilon_j}}(e^{\sigma^2_{\varepsilon_j}}-1) \ . \tag{1.6b}$$

The linear regression model (1.5) enables one to estimate the parameters δ_j, $\ln \beta_j$ and $\sigma^2_{\varepsilon_j}$ using the linear least squares method. Let $y_{jk\ell} = \ln \Delta p_{jk\ell}$ for $\ell=1,\ldots,\tilde{N}_{jk}$, $k\epsilon K_j$, and $j\epsilon J$. Then define

$$\tilde{N}_{j.} = \sum_{k\epsilon K_j} \tilde{N}_{jk} \qquad j\epsilon J \ ,$$

$$\bar{w}_j = \frac{1}{\tilde{N}_{j.}} \sum_{k\epsilon K_j} \tilde{N}_{jk} w_k \qquad j\epsilon J \ ,$$

$$\bar{y}_{jk} = \frac{1}{\tilde{N}_{jk}} \sum_{\ell=1}^{\tilde{N}_{jk}} y_{jk\ell} \qquad k\epsilon K_j \text{ and } j\epsilon J \ ,$$

and

$$\bar{y}_j = \frac{1}{\tilde{N}_{j.}} \sum_{k\epsilon K_j} \sum_{\ell=1}^{\tilde{N}_{jk}} y_{jk\ell}$$

The least squares estimates are

$$\hat{\delta}_j = \frac{\sum_{k\epsilon K_j} \left[\tilde{N}_{jk} \ (w_k - \bar{w}_j) \ \sum_{\ell=1}^{\tilde{N}_{jk}} \ (y_{jk\ell} - \bar{y}_j) \right]}{\left[\sum_{k\epsilon K_j} \tilde{N}_{jk} \ w_k^2 - \bar{w}_j^2 \ \tilde{N}_{j\cdot} \right]} \tag{1.7a}$$

$$\widehat{\ln \beta_j} = \frac{1}{\tilde{N}_{j\cdot}} \sum_{k\epsilon K_j} \left(\sum_{\ell=1}^{\tilde{N}_{jk}} y_{jk\ell} + \hat{\delta}_j \ \tilde{N}_{jk} \ w_k \right) \tag{1.7b}$$

$$\hat{\sigma}^2_{\varepsilon_j} = \frac{1}{\tilde{N}_{j\cdot}-2} \sum_{k\epsilon K_j} \sum_{\ell=1}^{\tilde{N}_{jk}} (y_{jk\ell} - \widehat{\ln \beta_j} + \hat{\delta}_j w_k)^2 \ . \tag{1.7c}$$

Because $\{\varepsilon_{jk\ell}: \ell=1,\ldots, \tilde{N}_{jk} ; k\epsilon K_j\}$ is normal, these estimates are identical to the maximum likelihood estimates with the exception of $\hat{\sigma}^2_{\varepsilon_j}$. To be a maximum likelihood estimate $\hat{\sigma}^2_{\varepsilon_j}$ must be multiplied by $\tilde{N}_j = (\tilde{N}_{j\cdot}-2)/\tilde{N}_{j\cdot}$, which for large $\tilde{N}_{j\cdot}$ is a small adjustment. If γ_j were known then these estimates would be unbiased and because of their relationship to maximum likelihood estimates they would be asymptotically minimum variance unbiased estimates. In this application we assume that $\gamma_j = \hat{\gamma}_j$ and accept these estimates for the parameters of the model described by (1.5).

Evaluating expressions (1.6) using the estimates (1.7) yields maximum likelihood estimates for μ_{η_j}, $\sigma^2_{\eta_j}$ and β_j . They are

$$\hat{\mu}_{\eta_j} = \exp(\tilde{N}_j \hat{\sigma}^2_{\varepsilon_j}/2) \tag{1.8a}$$

$$\hat{\sigma}^2_{\eta_j} = \exp(\tilde{N}_j \hat{\sigma}^2_{\varepsilon_j})[\exp(\tilde{N}_j \hat{\sigma}^2_{\varepsilon_j}) - 1] \tag{1.8b}$$

and

$$\hat{\beta}_j = \exp \widehat{\ln \beta_j} \tag{1.8c}$$

Table 3

Estimates for the Parameters of the Weight-Price Relationship

j	Week	$\sum_{k=1}^{12} \tilde{N}_{jk}$	$\hat{\beta}_j$	$\hat{\delta}_j$	$\hat{\sigma}^2_{\epsilon_j}$	$\hat{Y}_j$
28	7/10 - 7/16	4	1.56	62.50	.0043	2.15
29	7/17 - 7/23	41	2.80	100.69	.1713	2.15
30	7/24 - 7/30	167	2.57	65.49	.0518	2.55
31	7/31 - 8/6	75	2.26	34.42	.0258	3.00
32	8/7 - 8/13	89	3.36	46.58	.1050	3.20
33	8/14 - 8/20	50	6.22	77.20	.1003	3.45
34	8/21 - 8/27	25	6.33	76.43	.0768	3.45
35	8/28 - 9/3	9	5.03	63.86	.1297	3.45
36	9/4 - 9/10	0	-	-	-	3.20
37	9/11 - 9/17	13	6.11	75.93	.2072	3.65
38	9/18 - 9/24	9	3.88	66.58	.0082	3.65
39	9/25 - 10/1	5	6.73	88.76	.0207	3.45
40	10/2 - 10/8	9	5.75	70.48	.0015	3.65
41	10/9 - 10/15	16	4.93	66.72	.0026	3.65
42	10/16 - 10/22	10	4.93	65.13	.0015	3.65

Table 3 lists values for $\hat{Y}_j$, $\hat{\beta}_j$, $\hat{\delta}_j$ and $\hat{\sigma}^2_{\epsilon_j}$ for $j \epsilon J$ estimated from the Pamlico Sound brown shrimp data. In this case $j = 28,\ldots,42$. Figure 1 shows the relationship between the estimated regression line and the points $\{(\bar{S}_{jk}, w_k);\ k \epsilon K_j\}$ for $j = 29,\ldots,34$. The lines

$$S = \hat{\beta}_j e^{-\hat{\delta}_j w} \qquad \text{for } j = 29,\ldots,34,$$

are graphed on a semilogarithmic scale where the w coordinate (corresponding to the independent variable in model (1.1)) is plotted along the abscissa and the S coordinate (corresponding to the difference between Y_j and the dependent variable in model (1.1) is plotted along the ordinate. The random dispersion of the data points about these lines supports the use of this linear regression model.

1.2 Examining Residuals

An alternative procedure for evaluating how well a model fits data results from the probabilistic structure of the sequence $\{\varepsilon_{jk\ell} : \ell=1,\ldots,\tilde{N}_{jk};\ k\epsilon K_j\}$ of random variables. By writing (1.5) as

$$\varepsilon_{jk\ell} = \ln \Delta p_{jk\ell} - \ln \beta_j + \delta_j w_k$$

one sees that $\{(\ln \Delta p_{jk\ell} - \ln \beta_j + \delta_j w_k) : \ell=1,\ldots,\tilde{N}_{jk};\ k\epsilon K_j\}$ is a sequence of i.i.d. normal random variables with mean zero and variance $\sigma^2_{\varepsilon_j}$. Hence $\{(\ln \Delta p_{jk\ell} - \ln \beta_j + \delta_j w_k)/\sigma_{\varepsilon_j} : \ell=1,\ldots,\tilde{N}_{jk};\ k\epsilon K_j\}$ is a sequence of i.i.d. standardized normal random variables. Although we cannot observe this sequence directly, for each $j\epsilon J$ we are able to compute the sequence of residuals

$$R_{jk\ell} = \ln \Delta p_{jk\ell} - T_{jk} \qquad \begin{matrix} \ell=1,\ldots,\tilde{N}_{jk} \\ k\epsilon K_j \end{matrix}$$

where for $k \epsilon K_j$ and $j \epsilon J$

$$T_{jk} = \widehat{\ln} \beta_j - \hat{\delta}_j w_k$$

is a predicted value of $\ln \Delta p_{jk\ell}$ determined from the regression line. Consequently, if the model is appropriate one expects the sequence of normalized residuals

$$\hat{R}_{jk\ell} = \frac{R_{jk\ell}}{\sqrt{\hat{\sigma}_{\epsilon_j}}} \qquad \begin{array}{l} \ell=1,\ldots,\tilde{N}_{jk} \\ k \epsilon K_j \\ j \epsilon J \end{array}$$

to approximate a sample of $\tilde{N}_{j.} = \sum_{k \epsilon K_j} \tilde{N}_{jk}$ independent observations from a standardized normal distribution.

The Residual Plot

For each $j \epsilon J$ a plot of the points $\{(\hat{R}_{jk\ell}, T_{jk}): \ell=1,\ldots,\tilde{N}_{jk};\ k \epsilon K_j\}$ is called a residual plot. It can be used to check for two types of departures from model (1.5):

1. $\{\epsilon_{jk\ell} : \ell=1,\ldots, \tilde{N}_{jk};\ k \epsilon K_j\}$ are not independent.
2. $\{\epsilon_{jk\ell} : \ell=1,\ldots, \tilde{N}_{jk};\ k \epsilon K_j\}$ do not have constant variance.

If a visual inspection of a residual plot reveals dependence between $\hat{R}_{jk\ell}$ and T_{jk} one suspects that a departure of one or both of these types is present. Failure to observe dependence provides credibility for the model as an appropriate represention of the data. Figure 2 shows residual

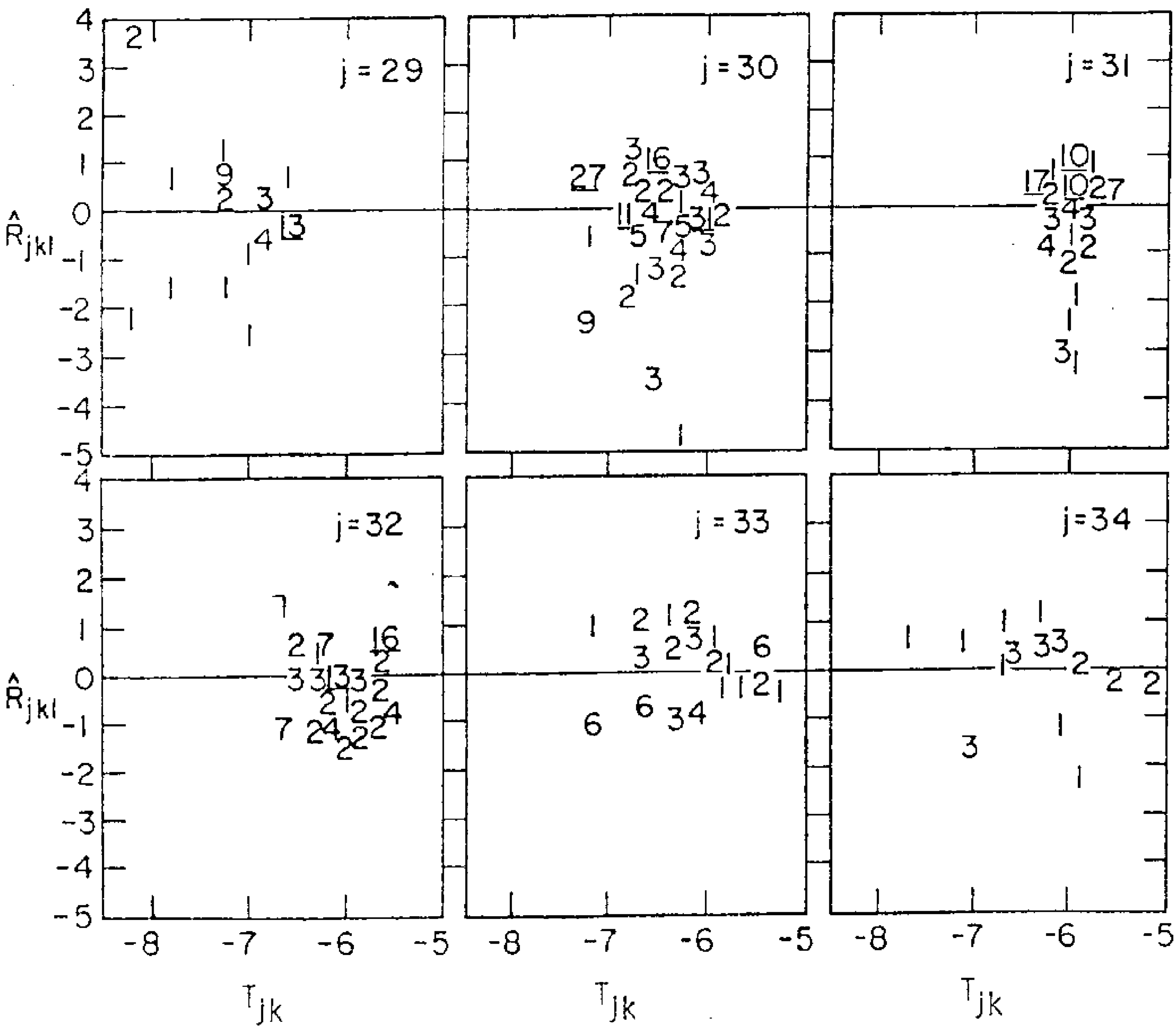

Figure 2

Residual Plot for 1978 North Carolina Brown Shrimp Price-Weight Relationship plots for j = 29,...,34. The numbers in the graphs denote the frequency of observations at each point. All two digit numbers are underlined. Although the residuals in weeks 29 through 31 appear to be somewhat skewed below the axis, we feel this does not represent a serious departure from the model. Furthermore, since the plots reveal no visual dependence between $\hat{R}_{jk}$ and T_{jk} there is no evidence to suspect departures of either type from the assumptions of model (1.5).

Further Analysis

To check the residuals for departures from normality we define

$$M_{jn}^{+} = \sum_{k \in K_j} \sum_{\ell=1}^{\tilde{N}_{jk}} I_{[n-1,n)} (\hat{R}_{jk\ell}) \tag{1.9a}$$

to be the number of normalized residuals in the interval $[n-1,n)$ and

$$M_{jn}^{-} = \sum_{k \in K_j} \sum_{\ell=1}^{\tilde{N}_{jk}} I_{[-n,1-n)} (\hat{R}_{jk\ell}) \tag{1.9b}$$

to be the number of normalized residuals in the interval $[-n,1-n)$ for each $n = 1,\ldots,5$ and each $j \in J$. Here,

$$I_{[a,b)}(x) = \begin{cases} 1 & \text{if } x \in [a,b) \\ 0 & \text{otherwise} \end{cases}$$

is the indicator function. Under the assumptions of model (1.5), the normalized residuals approximate a sample from a standard normal distribution. Hence one expects M_{jn}^{-} and M_{jn}^{+} not to differ significantly. In fact, in this case EM_{jn}^{+}, the expected value of M_{jn}^{+}, equals EM_{jn}^{-}, the expected value of M_{jn}^{-}. Thus if M_{jn}^{+} and M_{jn}^{-} differ by a significant amount one suspects a departure from the model's normality assumption.

Table 4

Statistics for Examining Residuals

Week j	Statistics for Week j and Associated Expected Value†	Interval n: 1	2	3	4	5	$\sum_{n=1}^{5} M^+_{jn}$	$\sum_{n=1}^{5} M^-_{jn}$	$\sum_{n=1}^{5} E\,M^+_{jn}$
	M^+_{jn}	16	1	0	2	0	17		
29	M^-_{jn}	18	2	2	0	0		22	
	$E\,M^+_{jn}$	17.2	2.8	0.4	0.0	0.0			19.5
	M^+_{jn}	72	3	0	0	0	75		
30	M^-_{jn}	73	6	9	3	1		92	
	$E\,M^+_{jn}$	69.8	11.3	1.7	0.1	0.0			83.5
	M^+_{jn}	37	0	0	0	0	37		
31	M^-_{jn}	32	1	1	4	0		38	
	$E\,M^+_{jn}$	31.5	5.1	0.8	0.0	0.0			37.5
	M^+_{jn}	41	7	0	0	0	48		
32	M^-_{jn}	39	2	0	0	2		43	
	$E\,M^+_{jn}$	38.2	6.2	0.9	0.0	0.0			45.5
	M^+_{jn}	23	0	3	0	0	26		
33	M^-_{jn}	23	0	0	0	0		23	
	$E\,M^+_{jn}$	20.6	3.3	0.5	0.0	0.0			24.5
	M^+_{jn}	16	0	0	0	0	16		
34	M^-_{jn}	4	4	1	0	0		9	
	$E\,M^+_{jn}$	10.5	1.7	0.3	0.0	0.0			12.5

†$M^+_{jn} = \sum_{k \in K_j} \sum_{\ell=1}^{\tilde{N}_{jk}} I_{[n-1,n)}(\hat{R}_{jk\ell})$ and $M^-_{jn} = \sum_{k \in K_j} \sum_{\ell=1}^{\tilde{N}_{jk}} I_{[-n,1-n)}(\hat{R}_{jk\ell})$.

Values for EM^+_{jn} are calculated under the assumption that the normalized residuals have a standard normal distribution.

Under the normality assumption one can show that

$$E M_{jn}^{+} = \{\Phi(n) - \Phi(n-1)\} \tilde{N}_j . \qquad \text{for } n=1,\ldots,5 \quad j \in J$$

This results from considering $(M_{j1}^{+}, M_{j2}^{+}, \ldots, M_{j5}^{+})$ as a multinomial variate. See Johnson and Kotz (1969) for a complete description of the multinomial distribution and related applications.

Table 4 shows values of M_{jn}^{+}, M_{jn}^{-} and $E(M_{jn}^{+})$ for $j=29,\ldots,34$ and for $n=1,\ldots,5$ as well as the sums $\sum_{n=1}^{5} M_{jn}^{+}$, $\sum_{n=1}^{5} M_{jn}^{-}$ and $\sum_{n=1}^{5} E(M_{jn}^{+})$. Although we observe an apparently unusual discrepancy between M_{341}^{+} and M_{341}^{-}, the significance of this difference is questionable because of the small value of $\tilde{N}_{34}$. For $n > 2$ the table shows that neither M_{jn}^{+} nor M_{jn}^{-} dominates systematically for all j. With regard to a comparison of M_{jn}^{+} and M_{jn}^{-} with $E M_{jn}^{+}$, the table provides little evidence for rejecting the normality.

2. The Biomass-Revenue Relationship

This section extends the model in (1.1) to characterize the relationship between catch biomass and revenue, our goal being to develop a model that can serve as a fishery management tool. In particular, we consider the biomass-revenue relationship as an integral part of a larger model that accounts for growth and population dynamics, as well as fishing dynamics, and is directed toward the study of management strategies.

Modeling the Price Per Pound for Individual Shrimp

In contrast to the previous sections, where we regarded price per pound as a function of grade category, here we treat price per pound as a function of shrimp weight. One way to accomplish this is to grade each captured shrimp and use (1.1) to determine an associated price per pound. In week j let M_j denote the number of shrimp captured and let $W_{j\ell}$ denote the weight of captured shrimp ℓ. Let $\{W_{j\ell}: \ell=1,\ldots,M_j\}$ be a set of i.i.d. random variables with mean μ_{W_j}, variance $\sigma^2_{W_j}$ and unknown distribution function $F_{W_j}(\cdot)$. This formulation follows the growth model described in Cohen and Fishman (1980).

One represents the grading process by a function that relates weight $W_{j\ell}$ to the set of grade categories $\{w_k: k=1,\ldots,12\}$ using the partition $\{\omega_k: k=0,\ldots,12\}$ described in Section 1. Let

$$K(W_{j\ell}) = \begin{cases} \sum_{k=1}^{12} k I_{[1/\omega_{k-1},\, 1/\omega_k)}(1/W_{j\ell}), & \text{for } \frac{1}{W_{j\ell}} \in \left[\frac{1}{\omega_0}, \frac{1}{\omega_{12}}\right) \\ 0 & \text{otherwise}. \end{cases} \tag{2.1}$$

Here $K(W_{j\ell})$ can assume grade values 1 through 12, inclusive, determined entirely by the weight $W_{j\ell}$.

2.1 The Distribution of Revenue Given Number of Captured Shrimp

Using (2.1) we modify (1.1) to represent $p_{j\ell}$ by

$$p_{j\ell} = \gamma_j - \beta_j \exp(-\delta_j w_{K(W_{j\ell})}) \cdot \eta_{j\ell} \qquad \ell=1,\ldots,M_j . \tag{2.2}$$

where $\{\eta_{j\ell} : \ell=1,\ldots,M_j\}$ is a sequence of i.i.d. lognormal random variables with mean μ_{η_j} and variance $\sigma^2_{\eta_j}$. Multiplying $p_{j\ell}$ by

$W_{j\ell}$ yields the revenue due to shrimp ℓ ,

$$v_{j\ell} = W_{j\ell} \, [\gamma_j - \beta_j e^{-\delta_j w} K(W_{j\ell}) \cdot \eta_{j\ell}] \, . \qquad \ell=1,\ldots,M_j \qquad (2.3)$$

so that revenue of a catch with biomass

$$B_j = \sum_{\ell=1}^{M_j} W_{j\ell} \qquad (2.4a)$$

is

$$V_j = \sum_{\ell=1}^{M_j} v_{j\ell} \, . \qquad (2.4b)$$

The convenience of representation (2.4) becomes apparent when we consider the magnitude of $\hat{M}_j$. Since $\{v_{j\ell}: \ell=1,\ldots,M_j\}$ is a sequence of i.i.d. random variables with finite variance, $V_j/\sqrt{M_j}$ converges (as $M_j \to \infty$) in probability to a normally distributed random variable with mean $\mu_{v_j} < \infty$, and variance $\sigma^2_{v_j}$, as a result of the central limit theorem (Feller 1968, p. 244). Formally,

$$\lim_{M_j \to \infty} \mathrm{pr}(V_j/\sqrt{M_j} \le a) = \Phi\left(\frac{a-\mu_{v_j}}{\sigma_{v_j}}\right), \qquad (2.5)$$

so that for large M_j, V_j is approximately normally distributed with mean $M_j\mu_{v_j}$ and variance $M_j\sigma^2_{v_j}$. Similarly, the biomass B_j is approximately normally distributed with mean $M_j\mu_{w_j}$ and variance $M_j\sigma^2_{w_j}$.

One can express μ_{v_j} and $\sigma^2_{v_j}$ in terms of μ_{w_j} and $\sigma^2_{w_j}$. In particular,

$$\mu_{v_j} \doteq \mu_{w_j} \{\gamma_j - \beta_j e^{-\delta_j \mu_{w_j}} \mu_{n_j} (1 + \frac{\delta_j^2 \sigma_{w_j}^2}{2} - \frac{\delta_j \sigma_{w_j}^2}{\mu_{w_j}}) \} , \quad (2.6a)$$

$$Ev_{j\ell}^2 \doteq (\mu_{w_j}^2 + \sigma_{w_j}^2) \gamma_j^2 - 2\gamma_j \beta_j \mu_{n_j} e^{-\delta_j \mu_{wj}} [\mu_{w_j}^2 + \sigma_{w_j}^2 (1 - 2\mu_{w_j}\delta_j +$$

$$\mu_{w_j}^2 \delta_j/2] + \beta_j^2 (\sigma_{n_j}^2 + \mu_{n_j}^2) e^{-\delta_j \mu_{w_j}} [\mu_{w_j}^2 + \sigma_{w_j}^2 (1-4\mu_{w_j}\delta_j + 2\mu_{w_j}^2 \delta_j^2)] \quad (2.6b)$$

so that one approximates $\sigma_{v_j}^2$ using (2.6a), (2.6b) and the identity

$$\sigma_{v_j}^2 = Ev_j^2 - \mu_{v_j}^2 . \quad (2.6c)$$

See Appendix A for details.

To summarize, we have represented B_j , catch biomass in week j , as a normally distributed random variable with mean $M_j \mu_{w_j}$ and variance $M_j \sigma_{w_j}^2$, where M_j is the number of shrimp captured in week j and μ_{w_j} and $\sigma_{w_j}^2$ are the mean and variance of the weight of a shrimp captured in week j . We have characterized V_j , revenue associated with catch B_j , as a normal random variable with mean $M_j \mu_{v_j}$ and variance $M_j \sigma_{v_j}^2$ where μ_{v_j} and $\sigma_{v_j}^2$ can be approximated in terms of the estimable parameters of the weight-price relationship, and μ_{w_j} and $\sigma_{w_j}^2$. Expressions (2.6a), (2.6b) and (2.6c) provide the basis for computing these means and variances. Note that V_j can be characterized without knowledge of the distribution of the weight of a captured shrimp. However, since the characterization of revenue includes the variance $\sigma_{w_j}^2$, it reflects any uncertainties affecting the dispersion of captured shrimp weight. These include uncertainties associated with environmental variation, through its affects on migration patterns, growth, mortality rates and fishing efficiencies, as well as the uncertainties associated with fish escapement through the fishing net.

2.2 The Distribution of Revenue Given Catch Biomass

In addition to their dependence on μ_{w_j} and $\sigma^2_{w_j}$, V_j(revenue) and B_j (biomass) also depend on M_j, the number of captured shrimp. Since one is more likely to have information about B_j then about M_j, we describe M_j in terms of B_j, μ_{w_j} and $\sigma^2_{w_j}$, and then we characterize V_j using relation (2.6). In this way revenue is functionally related to catch biomass.

Let Z denote a normal random variable having zero mean and unit variance, commonly called a standardized normal random variable. Since B_j is asymptotically normally distributed, one writes

$$Z \approx \frac{B_j - M_j \mu_{w_j}}{\sigma_{w_j}\sqrt{M_j}} , \qquad (2.7)$$

where $\approx$ indicates approximation. Then one can use the inverse Gaussian distribution (Johnson 1970, p. 137) to approximate the conditional distribution of M_j given B_j as

$$f_{M_j}(x|B_j) = \frac{1}{\sqrt{2\pi x^3 \sigma^2_{w_j}}} \sigma^2_{w_j} e^{-(B_j - x\mu_{w_j})^2/2x\sigma^2_{w_j}} \qquad 0 \le x .$$

Moreover, $E(M_j|B_j) = B_j/\mu_{w_j}$ and $var(M_j|B_j) = \mu^3_{w_j} \sigma^2_{w_j}/B_j$. Note that (2.7) implies

$$M_j = \frac{1}{2\mu^2_{w_j}} [2B_j \mu_{w_j} + \sigma^2_{w_j} Z^2 + C\sqrt{4B_j \mu_{w_j} + \sigma^2_{w_j} Z^2}] \qquad (2.8)$$

where

$$C = \begin{cases} Z\sigma_{w_j} & \text{for} \quad Z > 0 \\ -Z\sigma_{w_j} & \text{for} \quad Z \le 0 \end{cases} .$$

This expression provides a convenient way of sampling M_j for given B_j in a simulation experiment. Algorithm R1 describes a procedure for sampling V_j .

<u>Algorithm R1</u>

Given: B_j , μ_{w_j} and $\sigma^2_{w_j}$.

1. Sample Z_1 from a standardized normal distribution.
2. Sample Z_2 from a standardized normal distribution.
3. Evaluate M_j from (2.8) with $Z = Z_1$.
4. Evaluate μ_{v_j} from (2.6a).
5. Evaluate $\sigma^2_{v_j}$ from (2.6b) and (2.6c) .
6. $V_j \leftarrow M_j\mu_{v_j} + \sigma_{v_j} \cdot Z_2 \cdot \sqrt{M_j}$.

While the distribution of V_j is not easily obtained, one can show that

$$E(V_j|B_j) = B_j \, \mu_{v_j}/\mu_{w_j}$$

and

$$\mathrm{var}(V_j|B_j) = B_j\sigma^2_{V_j}/\mu_{W_j} + \mu^2_{V_j}\mu^3_{W_j}\sigma^2_{W_j}/B_j \; .$$

This description of V_j facilitates evaluating the effects of alternative management strategies on revenue. First, with the aid of a population model, one determines how a management strategy affects catch biomass B_j, and the character of catch biomass as described by μ_{W_j} and $\sigma^2_{W_j}$. Then, one applies Algorithm R1 to sample catch revenue V_j from the fishery operating under this management scheme. Hence, this technique can be used to compare management strategies using performance measures based on revenue and functions of revenue, such as profit. By a management strategy we mean a set of rules which regulate the behavior of the harvesting sector. For example, in the shrimp fishery one management strategy requires fishermen to use a one-inch mesh in the cod-end of the commonly used trawl net, while another may require fishermen to use a one-and-one-half-inch mesh in the cod-end (North Carolina Fisheries Regulations for Coastal Waters, p. 20, 1978, and McKenzie 1974). In the example in Section 4 we examine these two alternatives. However, before we can proceed to that example we must address the specific issues regarding the effects of mesh size on B_j, μ_{W_j}, and $\sigma^2_{W_j}$, and consequently on V_j.

3. The Mesh-Weight Relationship

This section examines the effect of mesh size on M_j and on μ_{W_j}

and $\sigma^2_{w_j}$, the mean and variance of the weight of an arbitrary captured shrimp. To simplify the exposition we examine the mesh-weight relationship on a population of single age-class fish. Generalization to a multiple age-class population presents no technical difficulties but clouds the issues with which this section is principally concerned. For the remainder of this paper let M_j , μ_{w_j} and $\sigma^2_{w_j}$ denote for week j , the number of captured fish and the mean and variance of the weight of a captured fish, respectively, from a population of age t (in weeks) fish. Thus t and j are related by $t = j - j_0$ where j_0 is a fixed date that defines the cohort.

Because shrimp are principally bottom dwellers, commercial shrimpers rely on an otter trawl net to capture shrimp from the sandy or muddy substrate. This device consists of:

1. A cone-shaped bag into which the catch is funneled, commonly called the cod-end of the net.
2. Wings on each side of the bag for herding the shrimp into the cod-end.
3. Trawl doors, also called otter boards, at the ends of each wing to hold the net open while under tow.
4. Tow lines connecting the trawl doors to the vessel.

Typically the commercial mesh size ranges from one inch to two inches when measured along the diagonal of a collapsed (although not forcibly stretched) square in the cod-end. The open end of a single net ranges from 50 to 120 feet wide (McKenzie 1974).

3.1 Traditional Models of Net Selectivity

Beverton and Holt (1957) discuss the problem of modeling the selectivity of a trawl net for alternative mesh sizes. The selectivity of a net is a measure of its ability to capture fish differentially as a function of fish length. Intuitively one expects that the smaller the fish the more likely it is to escape through the mesh. This has been empirically verified, and Beverton and Holt show empirically derived selection curves for the bottom dwelling flat fish plaice. A selection curve is a curve fit to data obtained from controlled experiments. The data consist of the ratios of captures of a given length fish from two alternatively sized nets versus length. One net is chosen so that its mesh is small enough to capture fish of any size in the fishable population, while the other net is chosen so that its mesh is the size whose selectivity is being measured. By taking the ratio of captures for the large mesh net to those for the small mesh net one adjusts or normalizes the selectivity of the large mesh net for the existing size distribution in the sample population. In this case, for each length the ratio is an estimate of the probability of selection given that the fish is the specified length. Alternatively, one minus the ratio is an estimate of the probability that a fish of that length escapes through the mesh.

The reader should note the distinction between the concepts of selectivity and fishing effort. In the literature, fishing effort measures the resources devoted to harvesting. Hence, we may consider it as a measure of the capitalization of the harvesting sector times the amount of time that that capital is used for harvesting. For the shrimp fishery we define a week's fishing effort as the number of feet-hours of trawl net devoted to harvesting for that week (Ricker 1978). Since a fish can be selected only if it is in the net, the probability of capturing a given fish is the product of the probability of selecting that fish given it is in the net and the probability that it is in the net. This second probability, and not the first, is related to fishing effort. In this section we discuss modeling the selectivity of a net and the associated selection probability, and when we refer to 'selection probability' we mean the probability of capture given the fish is at risk in the net.

Beverton and Holt's selection curves are empirically derived in the sense that they are smooth free hand curves drawn through a plot of the data. These curves are S-shaped. Because of this shape, Beverton and Holt consider modeling the selection process by the ratio of the integral of two normal curves. However, they do not fit such a model to data or explore its implications. Although Beverton and Holt discuss the notion of a selection range, they do not formally define it. We define a selection range by the interval of lengths over which the probability of a selection increases from .05 to .95. Beverton and Holt show four plaice selection curves each corresponding to a different mesh size. The length of the selection range increases monotonically with increasing mesh size from approximately 50 to 150mm. They also study the relationship between the center of the selection range and mesh size. The center of their selection range is the length at which the probability of selection is .5. They plot these points against mesh size in mm for alternative cod-end mesh sizes for plaice, and observe that a straight line through the origin fits these data. They define b_s , the slope of this line, as a selection factor that is dependent on the fish species under investigation. For each species the equation which describes the average behavior represented by the fit is

$$(.50 \text{ selection length}) \doteq b_s \times (\text{mesh size}) \ . \tag{3.1}$$

Furthermore, they site estimated values for b_s of 2.18 for plaice, and 3.33 and 2.90 for haddock (Beverton and Holt 1957, p. 225-229).

Another approach to the problem of modeling mesh size is to assume knife-edge selectivity. In this case the selection range is zero and selection occurs at a single length. If the selection length is ℓ , then any fish smaller than ℓ is selected with probability zero and any fish larger than ℓ is selected with probability one. In light of the empirical evidence presented by Beverton and Holt this is a crude approximation to actual net selectivity. Beverton and Holt acknowledge this and suggest approximating the empirical selection curve with a linear function or a step function (Beverton and Holt 1957, pp. 75-79). Although any desired accuracy can be achieved with a model based on a step function since the representation is not continuous, it often leads to computational difficulties. In the next section we propose a representation of net selectivity that addresses these issues.

3.2 Modeling Net Selectivity

Here we propose a characterization of net selectivity that extends the Beverton and Holt model and is consistent with the model of growth described by Cohen and Fishman (1980). The proposed representation of net selectivity forms an additional part of a fishery model for use as a management tool as is demonstrated in the example in Section 4.

In particular, the proposed selection model addresses three issues. First, it describes an S-shape selection curve similar to that observed by Beverton and Holt that provides a tractable representation, consistent with the other fishery models discussed in this report. Second, the proposed model describes a continuous selection curve, overcoming the limitations of both the knife-edge and the step function models. Third, the model describes a smooth selection curve, overcoming any limitations of linear piecewise linear models of selectivity.

Analytic Description

Let L_j be a random variable denoting the length of an arbitrary shrimp at time j and let $pr(S|L_j = \ell)$ be the selection probability of a shrimp with length ℓ. Consider the formulation

$$pr(S|L_j = \ell) = (1 - \frac{1}{h^{\ell/\ell^*}})^h \qquad (3.2)$$

where ℓ^* is a positive real-valued parameter and h is a positive integer-valued parameter. We restrict h to integer values for computational reasons which become apparent in the next section. One uses (3.2) to represent alternative nets by adjusting the values of h and ℓ^*.

Another parameterization of (3.2), that clarifies the roles of h and ℓ^*, is obtained by writing ℓ as $\alpha\ell^*$. In this way ℓ is represented as the α proportion of ℓ^*, and (3.2) becomes

$$pr(S|L_j = \alpha\ell^*) = (1 - \frac{1}{h^\alpha})^h \quad . \qquad (3.3)$$

Figure 3 shows graphs of $pr(S|L_j = \alpha\ell^*)$ as a function of α for selected values of h. The reader should note the characteristic S-shape, similar to the Beverton and Holt empirical selection curves. Also, Table 5 shows value of $pr(S|L_j = \alpha\ell^*)$ for selected values of α and h. Furthermore, for a given selection probability p_s, one can solve (3.3) for α. For given p_s and h denote the solution to

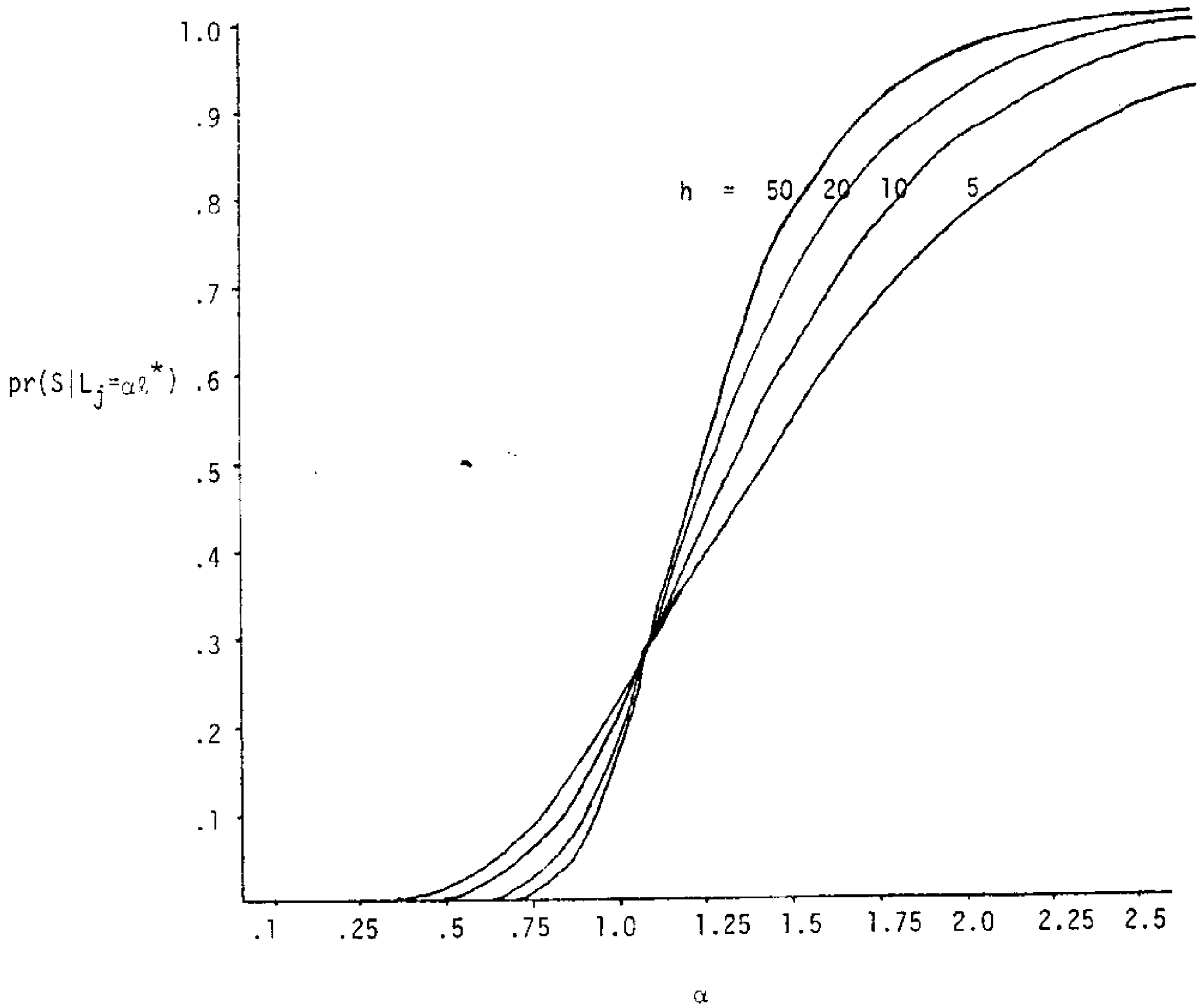

Figure 3

Graph of the Selection Probability

for Alternative Values of h

$$pr(S|L_j = \alpha \ell^*) = p_s$$

by $\alpha(p_s, h)$, then

$$\alpha(p_s, h) = \frac{-\ln(1-(p_s)^{1/h})}{\ln h} \qquad \text{for } 0 < p_s < 1$$
$$h > 0 \text{ , integer .}$$

In this notation, the selection range has limits $\ell^*\alpha(.05, h)$ and $\ell^*\alpha(.95, h)$, and length $\ell^*[\alpha(.95, h) - \alpha(.05, h)]$.

Table 5

Selection Probabilities for Selected Values of h and α

$pr(S|L_j = \alpha\ell^*)$

α \ h	2	5	10	20	30	40	50
.10	.004	.000	.000	.000	.000	.000	.000
.25	.025	.004	.003	.000	.000	.000	.000
.50	.086	.052	.022	.006	.002	.001	.000
.75	.164	.169	.141	.107	.087	.074	.065
1.00	.250	.327	.349	.358	.362	.363	.364
1.25	.336	.488	.561	.620	.650	.671	.686
1.50	.418	.626	.725	.799	.833	.853	.868
1.75	.494	.735	.836	.899	.925	.939	.940
2.00	.562	.815	.904	.951	.967	.975	.980
2.25	.624	.873	.945	.977	.986	.990	.992
2.50	.678	.914	.969	.989	.994	.996	.997

For increasing values of h the length of the selection range decreases. Table 5 shows evidence of this observation; for increasing h, $pr(S|L_j = \alpha\ell^*)$ increases for $\alpha > 1.00$ and decreases for $\alpha < 1.00$. In fact, for $0 < p_s < 1$, the limiting value of $\alpha(p_s, h)$ as h increases without bound is one. Therefore, for large h, (3.2) approaches the knife-edge representation of selectivity with selection at ℓ^*. Consequently in (3.2) ℓ^* is a location parameter for controlling the selection size, and h is a parameter for adjusting the length of the selection range.

Description of Nets Used in Example of Section 4.

In practice, one estimates values for these parameters for a particular net and fish species then examines the model's fit to the data. With regard to the shrimp fishery, data suitable for use in estimating the parameters ℓ^* and h are not currently available. For demonstrative purposes, in the example in Section 4 we *a priori* choose two sets of values for representing two alternative nets. We identify net I by $\ell^* = 50$ mm and $h = 10$ and net II by $\ell^* = 75$ mm and $h = 20$. These values are picked with two considerations in mind. First, the computation required to calculate the descriptors μ_{W_j} and $\sigma^2_{W_j}$ using model (3.2) is proportional to h. Thus, $h = 10$ or $h = 20$ is a compromise between obtaining a selection curve which is too flat to adequately represent net selectivity (this would occur for small h) and having a large h which would inflate the computation time necessary to evaluate μ_{W_j} and $\sigma^2_{W_j}$ in a general model. In Table 5 observe that as h decreases the selection curve flattens and has an increasing selection range. More importantly, the probability of selecting the smallest commercially sized shrimp (105 mm is the commonly accepted commercial minimum) is .92 for net I and .74 for net II. Hence the probability that net I fails to select the minimal commercial size shrimp is .08 as compared to .26 for net II. Hence, these nets are within a range of sizes of which managers may be interested in studying.

Since equation (3.1) relates $\alpha(.50, h)\ell^*$, the midpoint of the selection range, to mesh size, it is instructive to use it to compare the mesh size of nets I and II. For the three alternative selection factors published by Beverton and Holt, expression (3.1) shows that the mesh size of net I ranges from .79 to 1.05 inches and the mesh size of net II ranges from .98 to 1.53 inches. Since commercial nets have mesh size in the range of one to two inches this analysis provides evidence that the characterization of net selectivity with the particular choice of parameters associated with nets I and II is consistent with traditional models.

3.3 The Mean and Variance of the Weight of a Selected Fish

The effect of net selectivity on the biomass-revenue relationship is expressed through μ_{w_j} and $\sigma^2_{w_j}$, the mean and variance of weight of a selected shrimp as well as the selection probability. In this section we discuss the characterization of these three factors in the context of a single age-class population. We continue to assume a population of age t shrimp in week j. In principle, generalizing to a multiple age-class population presents no difficulties. This generalization depends on the proportion of each age-class in the population which in turn reflects each specific year's environmental conditions and how these conditions affect migration, mortality and abundance. These topics are discussed in another report.

Expressions for μ_{W_j} and $\sigma^2_{W_j}$ depend on a characterization of the joint distribution of selection (S) and the length (L_j) of an arbitrary population member in week j. Since we restrict our attention to a single age-class population we focus on the evaluation of the mean and variance of $\tilde{W}_{tk}$, the weight of a selected shrimp of age t and sex k ($k = 1$ for female and 2 for male), and we write $\mu_W(t)$ for its mean and $\sigma^2_W(t)$ for its variance. Let $\tilde{L}_{tk}$ denote the length of a selected shrimp. For the weight-length relationship we take

$$\tilde{W}_{tk} = a(k)\,\tilde{L}_{tk}^{b(k)}\,\epsilon_k \quad , \tag{3.4}$$

where $a(k)$ and $b(k)$ are species-related parameters and ϵ_k is a lognormal random variable with mean $e^{\gamma^2(k)/2}$ and variance $e^{\gamma(k)}(e^{\gamma(k)}-1)$. This formulation is the weight-length relationship as discussed in Cohen and Fishman (1980). The characterization differs from theirs in that it applies to a selected rather than to an arbitrary shrimp in the population at large. Although as presented in Cohen and Fishman (1980) L_j has a normal distribution with mean $\mu_L(t,k)$ and variance $\sigma^2_L(t,k)$ (for shrimp of sex k), $\tilde{L}_{tk}$ does not have a normal distribution. However, one can characterize the moments of $\tilde{L}_{tk}$.

First, let us consider the joint probability of selecting a shrimp of sex k with length $L_j \epsilon (c,d)$

$$pr(S,\ c < L_j < d) = \int_c^d pr(S|L_j = \ell)\ \phi\left(\frac{\ell-\mu_L(t,k)}{\sigma_L(t,k)}\right) d\ell\ .$$

Hence, the probability of selecting an arbitrary age t , sex k shrimp is

$$pr(S|t,k) = \int_{-\infty}^{\infty} pr(S|L_j = \ell)\ \phi\left(\frac{\ell-\mu_L(t,k)}{\sigma_L(t,k)}\right) d\ell$$

$$= \sum_{m=0}^{h} \binom{h}{m} (-1)^m \exp[-\lambda(m)\ \mu_L(t,k) + \lambda(m)^2\ \sigma_L^2(t,k)/2] \tag{3.5}$$

where $\lambda(m) = \frac{m}{\ell^*} \ln h$. See Appendix B for the details. Then $\tilde{L}_{tk}$ has mean

$$\mu_{\tilde{L}}(t,k) = E\ \tilde{L}_{tk} = \frac{1}{pr(S|t,k)} \sum_{m=0}^{h} \binom{h}{m}(-1)^m\ \Lambda(m)\ \Theta(m) \tag{3.6}$$

where

$$\Lambda(m) = \mu_L(t,k) - \lambda(m)\ \sigma_L^2\ (t,k)$$

$$\Theta(m) = \exp[-\lambda(m)\ \mu_L(t,k) + \lambda(m)^2 \sigma_L^2(t,k)/2]\ .$$

Furthermore, the nth central moment of $\tilde{L}_{tk}$ is

$$E\{(\tilde{L}_{tk} - \mu_{\tilde{L}}(t,k))^n\} =$$

$$\frac{1}{pr(S|t,k)} \sum_{s=0}^{n} \sum_{m=0}^{h} \binom{n}{s}\binom{h}{m}(-1)^m \left(-\mu_{\tilde{L}}(t,k)^{n-s}\right) \Theta(m)\int_{-\infty}^{\infty} \ell^s \phi\left(\frac{\ell - \Lambda(m)}{\sigma_L(t,k)}\right) d\ell \quad (3.7)$$

Table 6 gives expressions for the integral in (3.7) for $s = 1,\ldots,6$. For reading ease we suppress the subscripts and functional notation on $\sigma_L^2(t,k)$ and $\Lambda(m)$ in Table 6.

Table 6

Expressions for the Integral in (3.7)

s	$\int_{-\infty}^{\infty} \ell^s \phi\left(\frac{\ell - \Lambda(m)}{\sigma_L(t,k)}\right) d\ell$
1	Λ
2	$\Lambda^2 + \sigma^2$
3	$\Lambda^3 + 3\sigma^2 \Lambda$
4	$\Lambda^4 + 6\sigma^2 \Lambda^2 + 3\sigma^4$
5	$\Lambda^5 + 10\sigma^2 \Lambda^3 + 15\sigma^4 \Lambda$
6	$\Lambda^6 + 15\sigma^2 \Lambda^4 + 45\sigma^4 \Lambda^2 + 15\sigma^2$

Continuing to parallel the development in Cohen and Fishman (1980) we approximate the mean and variance of $\tilde{W}_{tk}$ by

$$\mu_w(t,k) = E\tilde{W}_{tk} \doteq a(k)e^{\gamma(k)^2/2}\,[\mu_{\tilde{L}}(t,k)^{b(k)} + q(b(k))] \tag{3.8a}$$

$$\sigma_w^2(t,k) = \operatorname{var} \tilde{W}_{tk} \doteq \tag{3.8b}$$

$$a(k)^2 e^{\gamma(k)^2}\{e^{\gamma(k)^2}[\mu_{\tilde{L}}(t,k)^{2b(k)} + q(2b(k))] - (\mu_{\tilde{L}}(t,k)^{b(k)} + q(b(k)))^2\},$$

respectively, where

$$q(b) = \sum_{n=2}^{\lfloor b \rfloor} g_n(b)\, E\{(\tilde{L}_{tk} - \mu_{\tilde{L}}(t,k))^n\} \quad ,$$

$$g_n(b) = \frac{g_{n-1}(b)(b-n+1)}{n\mu_{\tilde{L}}(t,k)} \quad , \qquad g_0(b) = \mu_{\tilde{L}}(t,k)^b$$

and $\lfloor b \rfloor$ is the largest integer less than or equal to b .

Let ρ be the probability that an arbitrary population member is male. Then

$$pr(S|t) = \rho\, pr(S|t,1) + (1 - \rho)\, pr(S|t,2) \tag{3.9a}$$

is the probability of selecting a shrimp of age t , and

$$\rho' = pr(S|t,1)/pr(S|t) \tag{3.9b}$$

is the probability that an arbitrarily selected shrimp is male. Thus for a uniform (0,1) variate U , the weight of an arbitary selected shrimp of age t is

$$\tilde{W}_t = \tilde{W}_{t,1}\, I_{(0,\rho']}(U) + \tilde{W}_{t,2}\, I_{(\rho',1]}(U) \tag{3.10}$$

with mean

$$\mu_W(t) = \rho'\mu_W(t,1) + (1-\rho')\,\mu_W(t,2) \tag{3.11a}$$

and variance

$$\sigma_W^2(t) = \rho'(\sigma_W^2(t,1) + \mu_W^2(t,1)) + (1-\rho')(\sigma_W^2(t,2) + \mu_W^2(t,2)) - \mu_W^2(t) \quad . \tag{3.11b}$$

Here the indicator function is defined as

$$I_{(a,b]}(x) = \begin{cases} 1 & \text{if } a < x \le b \\ 0 & \text{otherwise .} \end{cases}$$

Since in week j the population is age t , we have that $\mu_{W_j} = \mu_W(t)$ and $\sigma_{W_j}^2 = \sigma_W^2(t)$. This is not the case for the general multiple age-class population. Furthermore, the notation in (3.10) emphasizes the model's applicability to computer simulation sampling. In particular, the notation suggests a procedure for sampling $\tilde{W}_t$. Briefly, one first samples a uniform deviate U , then if $U \leq \rho'$ one samples $\tilde{W}_{t,1}$ otherwise one samples $\tilde{W}_{t,2}$.

In summary, we have characterized the mean (3.11a) and variance (3.11b) of the weight of an arbitrary shrimp selected from a population of age t fish as well as the probability of selecting (3.9a) an arbitrary fish from this population.

3.4 The Biomass-Revenue Relationship When Accounting for Net-Characteristics

In this section we show how one can incorporate the effects of net mesh into the characterization of revenue as a function of captured biomass, as described in Algorithm R1. While parallel to the development in Sections 2.1 and 2.2 on the biomass-revenue relationship, here our attention focuses on a homogeneous population of age t shrimp.

A Characterization of Catch Biomass

The probability of catching an arbitrary fish is decomposable into the product of pr(S) , the probability of selecting (catching) an arbitrary fish, given it is at risk, and the probability that an arbitrary fish in the population is at risk. The former probability is the selection probability while the latter is related to fishing effort. The distinction between these is important. For a given level of fishing effort in week j the number of shrimp at risk, $\tilde{M}_j$, is independent of mesh size. Then the number of the $\tilde{M}_j$ shrimp actually selected depends upon the mesh size of the net and the distribution of lengths. Let $\tilde{W}_{t\ell}$ denote the weight of selected shrimp $\ell = 1,\ldots,M_j$. Then B_j, the biomass catch in week j can be represented for simulation sampling purposes as

$$B_j = \sum_{\ell=1}^{\tilde{M}_j} \bar{W}_{t\ell} \; I_{[0,\ pr(S|t))}(U_\ell) \tag{3.12}$$

where $\{U_\ell: \ell=1,\ldots,\tilde{M}_j\}$ is a sequence of independent and identically distributed uniform (0,1) random variables.

For large $\tilde{M}_j$ the distribution of B_j is approximately normal with mean

$$E\,B_j = \tilde{M}_j\ \mu_w(t)\ pr(S|t) \tag{3.13a}$$

and variance

$$\text{var}\ B_j = \tilde{M}_j\ \{[\sigma_w^2(t) + \mu_w^2(t)]\ pr(S|t) - [\mu_w(t)\ pr(S|t)]^2\}\ . \tag{3.13b}$$

Expression (3.12) differs from (2.4a) in two ways. First, (3.12) is for a single age-class population while (2.4a) is for a multiple age-class population. However, (3.12) can be extended to a multi age-class population. Second, (3.12) explicitly accounts for the effects of alternative mesh size on capture biomass and thus is a more useful description for evaluating management policies.

An Algorithm for Simulating Sampling Catch Biomass and Catch Revenue

Algorithm R2 enables one to evaluate the parameters of the distribution of V_j (revenue) and B_j (biomass) for a population of age t fish with explicit accounting for mesh size. The procedure requires that $\tilde{M}_j$, ℓ^* and h be specified. The algorithm is presented so that a sequence of Q independent observations of V_j is made and the qth such observation is denoted $V(j,q)$.

Algorithm R2

Given: $\tilde{M}_j$, ℓ^* , h , $\mu_L(t,k)$, $\sigma_L^2(t,k)$, ρ and Q .

1. $k \leftarrow 1$.
2. Evaluate $pr(S|t,k)$ from (3.5).
3. Evaluate $\mu_L^{\sim}(t,k)$ from (3.6).
4. $n \leftarrow 1$.
5. Evaluate $E\{ (\tilde{L}_{tk} - \mu_L^{\sim}(t,k))^n \}$ with Table 6 from (3.7).
6. $n \leftarrow n + 1$.
7. If $n \leq 6$ go to 5.
8. Evalute $\mu_W(t,k)$ from (3.8a).
9. Evaluate $\sigma_W^2(t,k)$ from (3.8b).
10. $k \leftarrow k + 1$.
11. If $k \leq 2$ go to 2.
12. $pr(S|t) \leftarrow \rho\, pr(S|t,1) + (1-\rho)\, pr(S|t,2)$.
13. $\rho' \leftarrow \rho\, pr(S|t,1)/pr(S|t)$.
14. Approximate $\mu_W(t)$ by (3.11a).
15. Approximate $\sigma_W^2(t)$ by (3.11b).
16. Evaluate $E\, B_j$ from (3.13a).
17. Evaluate $\mathrm{var}\, B_j$ from (3.13b).

18. Approximate μ_{V_j} by (2.6a).

19. Approximate $\sigma^2_{V_j}$ by (2.6c).

20. $q \leftarrow 1$.

21. Independently sample Z_1, Z_2 and Z_3 from a standard normal distribution.

22. $B_j \leftarrow E\,B_j + Z_1 \sqrt{\text{var } B_j}$.

23. Evaluate M_j with $Z = Z_2$ from (2.8).

24. $M_j \leftarrow \lfloor M_j \rfloor$.

25. $V_j \leftarrow M_j\,\mu_{V_j} + Z_3\,\sigma_{V_j} \sqrt{M_j}$.

26. Store the qth sample of V_j in $V(j,q)$.

27. $q \leftarrow q + 1$.

28. If $q \leq Q$ go to 21.

29. Return the Q samples in $V(j,q)$.

4. An Example of the Biomass-Revenue Model as a Management Tool

When coupled with a comprehensive population model the characterization of catch revenue has potential for addressing several management concerns. For example, one way to evaluate alternative management strategies is to compare catch revenues computed from a fishery model operating under the alternative schemes. Since catch revenue is a random quantity one can accomplish the comparison by means of a sampling experiment imbedded in a computer based simulation of the fishery. By sampling catch revenue the simulator observes the effects of the management strategy on revenue for varying environmental and economic conditions. To illustrate the approach, we compare catch revenue obtained from two alternative net meshes, nets I and II, in a population setting that includes growth in fish size. In particular, we focus on the relationship between net mesh preference and the length of the fishing period and consider how fishing costs affect these decisions.

Consider a population of 1000 single age-class shrimp first at risk in the last week in July (j = 30). The initial age of this population is chosen so that it is the youngest group for which the net I selection probability is greater than 0.5 . This corresponds to a four week old population whose members have a 90.6 mm mean length and a 6.5 mm standard deviation of length as determined using our model of shrimp growth (Cohen and Fishman 1980).

To provide a meaningful yet simple example, we assume growth in size, but no out-migration, in-migration or natural mortality; that is, although the individuals in the population are growing in length, and therefore weight, the number of individuals in the population are neither increasing nor decreasing from natural causes. Any shrimp not captured in week j remains in the fishing grounds, grows as described in Cohen and Fishman (1980), and remains a candidate for selection in week j + 1 . Since the selection probability for either net is greater than 0.5 , four weeks of fishing are sufficient for catching virtually the entire 1000 fish.

In this example we compare eight management strategies. Each strategy restricts mesh size to one of two possible sizes and specifies one of four alternative fishing periods. Table 7 enumerates the

Table 7

Alternative Management Strategies

Strategy	Mesh Size Restriction	Fishing Period Limits
1	Net I	Week 30
2	Net I	Weeks 30 through 31
3	Net I	Weeks 30 through 32
4	Net I	Weeks 30 through 33
5	Net II	Week 30
6	Net II	Weeks 30 through 31
7	Net II	Weeks 30 through 32
8	Net II	Weeks 30 through 33

eight options. In a stochastic setting the simulator has available several measures for comparing the performance of these alternative strategies. We consider four: the mean cumulative catch revenue and the three quartiles of cumulative catch revenue. Although means are the most frequently considered comparative measures, the quartiles can be used to obtain another perspective. A quartile is the value of catch revenue that an arbitrary observation does not exceed with probability .25, .50 or .75 (depending on which of the three quartiles is being considered). Thus, we say that 25% (50% or 75%) of the time the catch revenue will be less than the lower quartile (median or upper quartile). Since each of these gives additional information about the distribution of catch revenue, each provides an alternative measure of comparison. For example, suppose a manager is particularly sensitive to an occasional low catch revenue and wants to choose a management alternative to minimize this possibility. In this case he could use the lower quartile for comparing alternatives.

Regardless of the comparison measure, the underlying issue in this example involves the tradeoff between growth and fishing costs. This can be seen by considering the alternatives at the end of week j : fish week $j + 1$ or cease fishing. In week $j + 1$ the size of the fish tend to be larger than in week j , and hence they command a higher price. However, since there are fewer fish in week $j + 1$ a fisherman must expend additional effort to catch them. Therefore, the management strategy falls into one of two types: 1) use net I, capture the bulk of the population early in the four week period and then cease fishing or 2) use net II, take a longer period to capture the bulk of the population, but gain higher revenue due to growth at a greater expense in fishing costs. Although one may argue that in a scenario without migration it is preferable to initiate fishing late in the fishing period, here we restrict fishing to commence in week 30. This behavior is characteristic of a competitive fishery such as a shrimp fishery where on the opening date the non-cooperative fishermen vie for the largest share of the population.

Let $R_n^{(k)}$ denote the cumulative catch revenue in dollars obtained from fishing the population in weeks 30 through n. The superscript (k) denotes net I for $k = 1$, and net II for $k = 2$. Then,

$$R_n^{(k)} = \begin{cases} R_{n-1}^{(k)} + V_n^{(k)} & \text{for } n = 30,\ldots,33 \\ 0 & \text{for } n = 29 . \end{cases}$$

Furthermore, if in a simulation experiment we denote the ith observation of $R_n^{(k)}$ by $r^{(k)}(n,i)$, then a sample of 1000 such observations is obtained by sampling $V_j^{(k)}$ and $M_j^{(k)}$ using Algorithm R2 and the calling sequence:[5]

Algorithm R3

Given: k

1. $Q \leftarrow 1$.
2. $i \leftarrow 1$.
3. $t \leftarrow 4$.
4. $j \leftarrow 30$.
5. $r^{(k)}(j-1,i) \leftarrow 0$.
6. $\tilde{M}_j^{(k)} \leftarrow 1000$.
7. Sample $M_j^{(k)}$ and $V_j^{(k)}$ using Algorithm R2.
8. $r^{(k)}(j,i) = r^{(k)}(j-1,i) + V_j^{(k)}$.
9. $\tilde{M}_{j+1}^{(k)} \leftarrow (\tilde{M}_j^{(k)} - M_j^{(k)})$.

(continued)

[5]Although more computationally efficient algorithms can be found for sampling $R^{(k)}$ the added discussion required to describe such a procedure is not central to the example, and would lead us away from the purpose of this illustration.

10. $j \leftarrow j+1$.

11. $t \leftarrow t+1$.

12. If $j \le 33$ go to 7.

13. $i \leftarrow i+1$.

14. If $i \le 1000$ go to 3 else stop.

We estimate $E\,R_n^{(k)}$, the expected value of $R_n^{(k)}$, and $\operatorname{var} R_n^{(k)}$, the variance of $R_n^{(k)}$ for $k = 1,2$ and $n = 30,\ldots,33$ by

$$E\,R_n^{(k)} = \frac{1}{1000} \sum_{i=1}^{1000} r^{(k)}(n,i)$$

and

$$\operatorname{var} R_n^{(k)} = \frac{1}{999} \sum_{i=1}^{1000} \left(r^{(k)}(n,i) - \hat{E}\,R_n^{(k)}\right)^2$$

respectively. Furthermore, we estimate the lower quartile, the median, and the upper quartile of $R_n^{(k)}$ by

$$Q_n^{(k)}(p) = \min\left\{x: \sum_{i=1}^{1000} I_{[0,r^{(k)}(n,i))}(x) > 1000(1-p)\right\},$$

where $p = .25, .50$, and $.75$ respectively.[6]

[6]Since these estimators are based on 1000 replications of a simulation their variance will be small. For this reason, and to avoid additional notation we do not distinguish the estimators from the parameters they estimate.

4.1 Discussion

Optimal Management Strategy Based on Revenue

For each mesh size, Table 8 shows estimates of the aforementioned parameters of the distribution of cumulative catch revenue as well as the probability of selecting an arbitrary population member for $n = 30, \ldots, 33$. All of these quantities were obtained from a sampling experiment performed using Algorithm R3. The table shows that if fishing is permitted for only one week, $n = 30$, net I is preferred to net II. This can be seen by comparing the sample mean and quartiles in the row $n = 30$ for net I, with the corresponding quantities for net II.

If fishing is permitted for more than one week, then a large mesh net is preferable to a small mesh net. In particular, for $n = 33$, the sample mean and quartiles for net II are larger than those for net I. These observations agree with intuition; for in a nonmigrating population with individuals growing and increasing in value, it is preferable to select the larger, higher priced fish. In this case the optimal management strategy is to permit fishing for the entire four week period ($n = 33$) and to allow only the use of net II, yielding a sample mean revenue of 33.30.

Table 8

Comparing Revenue for Alternative Management Policies
(revenue in dollars)

Net I

Strategy	n	pr(S|t)	$E\ R_n^{(1)}$	var $R_n^{(1)}$	$Q_n^{(1)}(.25)$	$Q_n^{(1)}(.50)$	$Q_n^{(1)}(.75)$
1	30	.83	18.5	34.5	14.4	18.4	22.6
2	31	.90	24.1	44.0	19.6	24.0	28.4
3	32	.94	24.9	45.2	20.2	24.9	29.2
4	33	.96	25.0	45.6	20.5	25.0	29.3

Net II

Strategy	n	pr(S|t)	$E\ R_n^{(2)}$	var $R_n^{(2)}$	$Q_n^{(2)}(.25)$	$Q_n^{(2)}(.50)$	$Q_n^{(2)}(.75)$
5	30	.56	13.1	23.0	9.8	13.0	16.3
6	31	.70	25.4	44.0	20.8	25.3	29.9
7	32	.78	31.4	55.4	26.4	31.4	36.1
8	33	.83	33.3	58.4	28.1	33.1	38.4

Optimal Management Strategy Based on Profit

Although a helpful example for illustrating Algorithm R3, this scenario neglects the important consideration of fishing costs. Let C denote the weekly cost in dollars of fishing the population in the example. Then $(n - 29)C$ is the cost of fishing weeks 30 through n inclusive. For expository purposes we define cumulative profit in dollars as

$$P_n^{(k)} = R_n^{(k)} - (n - 29)C \qquad \text{For } n = 30,\ldots,33 .$$

With $C = 5$ and 10 chosen for demonstrative purposes, Table 9 shows sample means, variances and quartiles of $P_n^{(k)}$ for $n = 30,\ldots,33$ and $k = 1$ and 2. Although $P_n^{(k)}$ is a random variable $(n - 29)C$ is not; thus, except for the sample variances, the estimates in Table 9 are obtained from those in Table 8 by subtracting $(n - 29)C$ from each entry in row n.

Table 9

Comparing Profit for Alternative Management Policies

(profit in dollars)

$C = 5.00$

Net I

Strategy	n	$E\,P_n^{(1)}$	$\mathrm{var}\,P_n^{(1)}$	$Q_n^{(1)}(.25)$	$Q_n^{(1)}(.50)$	$Q_n^{(1)}(.75)$
1	30	13.5	34.5	9.4	13.4	17.6
2	31	14.1	44.0	9.6	14.0	18.[illegible]
3	32	9.9	45.2	5.2	9.9	14.2
4	33	5.0	45.6	0.5	5.0	9.3

Net II

Strategy	n	$E\,P_n^{(2)}$	$\mathrm{var}\,P_n^{(2)}$	$Q_n^{(2)}(.25)$	$Q_n^{(2)}(.50)$	$Q_n^{(2)}(.75)$
5	30	8.1	23.0	4.8	8.0	11.3
6	31	15.4	44.0	10.8	15.3	19.9
7	32	16.4	55.4	11.4	16.4	21.1
8	33	13.3	58.4	8.1	13.1	18.6

$C = 10.00$

Net I

Strategy	n	$E\,P_n^{(1)}$	$\mathrm{var}\,P_n^{(1)}$	$Q_n^{(1)}(.25)$	$Q_n^{(1)}(.50)$	$Q_n^{(1)}(.75)$
1	30	8.5	34.5	4.4	8.4	12.5
2	31	4.1	44.0	-0.4	4.0	8.4
3	32	-5.1	45.2	-9.8	-5.1	-0.8
4	33	-5.0	45.6	-19.5	-15.0	-10.7

Net II

Strategy	n	$E\,P_n^{(2)}$	$\mathrm{var}\,P_n^{(2)}$	$Q_n^{(2)}(.25)$	$Q_n^{(2)}(.50)$	$Q_n^{(2)}(.75)$
5	30	3.1	23.0	-0.2	3.0	6.3
6	31	4.6	44.0	0.8	5.3	9.9
7	32	1.4	55.4	-3.6	1.4	6.1
8	33	-6.7	58.4	-11.9	-6.9	-1.6

Table 9 shows that for $C = 5$, regardless of the comparison measure, the optimal strategy is to permit fishing in weeks 30 through 32 with net II. As the table shows, this strategy results in a mean profit of 16.40. However, for $C = 10$ the table reveals a preference for permitting fishing only one week with net I. This action, which yields sample mean profit of 8.50, contrasts with the optimal strategy in the lower cost scenario. Here fishing costs are high enough to offset benefits from growth and the use of a large mesh net. This example demonstrates how the optimal strategy responds to changes in fishing costs.

Although the optimal strategies are independent of comparison measure, one should not conclude that it is unnecessary to consider alternative measures, and simply compare sample means. In a more comprehensive fishery scenario, involving a multi age-class population and complex management alternatives, it is unlikely that each measure will result in identical optimal policies. In this case a manager needs to articulate his criterion of optimality clearly.

The Value of Fishing Cost Information

An additional observation concerns the manager's decision when he is uncertain of the true fishing cost. By comparing the expected profit from following an optimal strategy, based on correctly known cost, with the expected profit obtained from following a suboptimal policy, based on unknown cost, one can quantify the value of information about fishing cost. We illustrate this analysis by considering several scenarios where the manager has limited information about cost but must choose to follow a policy that assumes either $C = 5$ or $C = 10$. For example, if $C = 5$ but the manager acts as if $C = 10$, then his decision, as shown in Table 9, is to require use of net I and to permit fishing only for week 30. Since in fact $C = 5$, the sample mean profit from this decision is 13.50. On the other hand, if C was truly 10, his decision result in a sample mean profit of 8.5, the optimal expected profit when $C = 10$. If one assumes that each cost value is equally likely, the expected value of acting as if $C = 10$ is 11. In a similar manner, if $C = 5$ and $C = 10$ are equally likely an estimated expected value of profit of 8.9 results when the manager acts as if $C = 5$. Comparing 8.9 to 11 shows that it is preferable on the basis of estimated expected profit to act as if $C = 10$.

We can represent this symbolically by letting $k^*(C)$ and $n^*(C)$ be the optimal k and n given cost C. Then these quantities satisfy

$$E[P_{n^*(C)}^{k^*(C)} | C] = \max_{\substack{1 \le k \le 2 \\ 30 \le n \le 33}} E[P_n^{(k)} | C] .$$

With this notation we represent the expected profit when acting as if $C = 10$ when in fact $C = 5$ by $E[P_{n^*(10)}^{k^*(10)} | 5]$. As we have observed this equals 13.5. If we denote the probability that $C = 5$ by $pr(C = 5)$ and the probability that $C = 10$ by $pr(C = 10)$, then

$$E[P_{n^*(10)}^{k^*(10)}] = E[P_{n^*(10)}^{k^*(10)} | 5] \; pr(C = 5) + E[P_{n^*(10)}^{k^*(10)} | 10] \; pr(C = 10)$$

is the expected profit when acting as if $C = 10$ and $E[P_{n^*(5)}^{k^*(5)}]$, similarly defined, is the expected profit when acting as if $C = 5$.

The reader should note that there are at least two interpretations of the probabilities, $pr(C = 5)$ and $pr(C = 10)$. One holds that the cost C is not a random quantity but a constant with one of two values, 5 or 10 , unknown to the manager. The probabilities then represent the manager's expert subjective estimate of the probability that each cost is correct. The other interpretation holds that cost is a random quantity, and that these probabilities define the probability mass function of this random variable. In either case our analysis holds.

Recall we observed that if

$$pr(C = 5) = pr(C = 10) = .5$$

then

$$E[P_{n^*(10)}^{k^*(10)}] = 11$$

and

$$E[P_{n^*(5)}^{k^*(5)}] = 8.9 \quad .$$

Since $11 > 8.9$ the manager acts as if $C = 10$. On the other hand, if the manager had perfect information of the current cost he would choose the decision $k^*(5)$, $n^*(5)$ when $C = 5$, and $k^*(10)$, $n^*(10)$ when $C = 10$. Thus, with perfect information the estimated expected profit is $E[P^{k*(5)}_{n*(5)}|5] = 16.4$ when $C = 5$ and $E[P^{k*(10)}_{n*(10)}|10] = 8.5$ when $C = 10$. Then, under the assumption that $pr(C = 5) = pr(C = 10) = .5$ the expected value of profit is

$$.5E[P^{k*(5)}_{n*(5)}|5] + .5E[P^{k*(10)}_{n*(10)}|10] = 12.45 .$$

Hence, an estimate of the expected value of fishing cost information, namely the difference between the expected value of profit under perfect information and the expected value of profit when acting as if $C = 10$ (the best choice when $pr(C = 5) = pr(C = 10) = .5$) , is the difference between 12.45 and 11 or 1.45 (Schlaifer 1969).

This example serves to illustrate three aspects of the techniques presented in this report: 1) an incorporation of the stochastic revenue characterization into a biological model, 2) a technique for sampling revenue based on the biomass-revenue model, and 3) a use of the data obtained from a sampling experiment to address management concerns with decision analysis techniques.

5. Summary

We have accomplished several tasks in this paper. First, we presented a description of the relationship between the weight of a captured shrimp and the regional price of that shrimp that accounts for structural and random variation. Second, we have estimated parameters for the model from data of the North Carolina brown shrimp fishery of Pamlico Sound. Third, we have extended this representation to relate regionally captured biomass to regional revenue. Fourth, we have extended the model of mesh selectivity proposed by Beverton and Holt in a way that is compatible with the model of growth described by Cohen and Fishman (1980). Finally, we have shown how the model of mesh selectivity is incorporated into the biomass-revenue relationship, and in an example we have demonstrated how the characterization can be used for decision making. Furthermore the methodology presented in this paper is applicable to a multiple age-class population. We address this issue in another report.

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Appendix A

In this appendix we justify the approximations (2.6a) and (2.6b) of μ_{v_j} and $Ev_{j\ell}^2$ where

$$v_{j\ell} = W_{j\ell} \{\gamma_j - \beta_j e^{-\delta_j W} K(W_{j\ell})\eta_{j\ell} \} \quad .$$

For reading convenience we suppress the subscripts j and ℓ and write this expression as

$$v = W(\gamma - \beta e^{-\delta W} K(W)\eta) \quad ,$$

where $K(W)$ is defined as in (2.1). Since v contains two sources of variation, η and W, we denote expectation with respect to each random variable by subscripting the expectation operator E by either η or W. Hence

$$\mu_v = E_W E_\eta v \tag{A.1a}$$

and

$$Ev^2 = E_W E_\eta v^2 \tag{A.1b}$$

Since the right hand side of (A.1a) is equivalent to

$$E_W \{W(\gamma - \beta e^{-\delta W} K(W)\mu_\eta)\}$$

we write

$$\mu_v = \mu_W \gamma - \beta \mu_\eta E_W(We^{-\delta W}K(W)) \;. \tag{A.2a}$$

Similarly from (A.1b),

$$Ev^2 = (\mu_W^2 + \sigma_W^2)\gamma^2 - 2\beta\gamma \mu_\eta E_W(W^2 e^{-\delta W}K(W)) +$$

$$\beta^2 (\mu_\eta^2 + \sigma_\eta^2) E_W (W^2 e^{-2\delta W}K(W)) \;. \tag{A.2b}$$

Examination of (A.2a) and (A.2b) shows that to complete the evaluation of μ_v and Ev^2 the quantities

$$E_W (W e^{-\delta W}K(W)) \tag{A.3a}$$

and

$$E_W (W^2 e^{-\xi W}K(W)) \quad \text{for } \xi = 2\delta \text{ and } \delta \tag{A.3b}$$

require explicit representation. Recall that the distribution function of W is $F_W(\cdot)$. By definition one obtains

$$E_W(W e^{-\delta w_{K(W)}}) = \int_0^{\infty} x e^{-\delta w_{K(x)}} \, dF_w(x) = \sum_{k=0}^{11} \int_{\omega_{k+1}}^{\omega_k} x e^{-\delta w_k} \, dF_w(x) \; .$$

This expression is approximately equal to

$$\sum_{k=0}^{11} \int_{\omega_{k+1}}^{\omega_k} x e^{-\delta x} \, dF_w(x) \quad , \tag{A.4}$$

with the error of approximation dependent both on the distribution function $F_w(\cdot)$ and on the partition $\{\omega_k : k=0,\ldots,12\}$. In practice this is a good approximation because the partition has been determined in the marketplace as a solution to a minimization problem. In theory, the marketplace is a setting where both the buyer and seller will only agree on the partition for which the error of this approximation is "closest to zero". We leave this notion vague to avoid issues not central to the discussion, and we write (A.4) as

$$\int_{\omega_{12}}^{\omega_0} x e^{-\delta x} \, dF_w(x)$$

which is equivalent to

$$E_W(W e^{-\delta W}) - \int_0^{\omega_{12}} x e^{-\delta x} dF_w(x) - \int_{\omega_0}^{\infty} x e^{-\delta x} dF_w(x) \quad .$$

Because shrimp whose weights are less than ω_{12} are not commercially valuable, and shrimp whose weights are greater than ω_0 are extremely rare, the quantity

$$\int_0^{\omega_{12}} x e^{-\delta x} \, dF_w(x) + \int_{\omega_{12}}^{\infty} x e^{-\delta x} \, dF_w(x)$$

is small relative to $E_W(We^{-\delta W})$. Thus we approximate (A.3a) by

$$E_W\,(W\,e^{-\delta\,W})\;. \tag{A.5a}$$

Similarly, we approximate (A.3b) by

$$E_W\,(W^2\,e^{-\xi\,W})\;. \tag{A.5b}$$

These expressions, in contrast to (A.3a) and (A.3b), can be evaluated by an application of the methods of statistical differentials (Johnson and Kotz 1969). Let $\Delta = W - \mu_W$ and substitute $\Delta + \mu_W$ for W in (A.5a). Thus

$$E_W\,(W\,e^{-\delta\,W}) = E_W\,[(\Delta + \mu_W)\,e^{-\delta\,\Delta}e^{-\delta\,\mu_W}]\;.$$

Expanding $e^{-\delta\Delta}$ results in

$$E_W(W\,e^{-\delta\,W}) = E_W\,[(\Delta + \mu_W)e^{-\delta\,\mu_W}\sum_{i=0}^{\infty}\frac{(-\delta\Delta)^i}{i!}]$$

$$= E_W(e^{-\delta\mu_W}[\mu_W\sum_{i=0}^{\infty}\frac{(-\delta\Delta)^i}{i!} + \sum_{i=0}^{\infty}\frac{\Delta(-\delta\,\Delta)^i}{i!}])\;,$$

Then dropping the terms of degree three or greater yields the approximation

$$E_W(W\,e^{-\delta\,W}) \doteq e^{-\delta\,\mu_W}\,[\mu_W(1 + \frac{\delta^2\sigma^2}{2}) - \delta\,\sigma_W^2\,],$$

where the error of approximation depends on the size of the high order (degree three and greater) central moments of W. A similar argument yields that

$$E_W(W^2\,e^{-\xi\,W}) \doteq e^{-\xi\mu_W}\,[\sigma_W^2\,(1 - 2\mu_W\xi + \mu_W^2\,\xi^2/2) + \mu_W^2].$$

(See Johnson and Kotz (1969) for other examples of the method of statistical differentials). Combining these expressions with (A.2a) and (A.2b) results in approximations for μ_v and Ev^2 in terms of μ_w and σ_w^2. In particular,

$$Ev = \mu_v + \mu_w[\gamma - \beta e^{-\delta \mu_w} \mu_\eta (1 + \frac{\delta^2 \sigma^2_w}{2} - \frac{\delta \sigma_w^2}{\mu_w})] \qquad (A.6a)$$

and

$$Ev^2 = \sigma_v^2 + \mu_v^2 + (\mu_w^2 + \sigma_w^2)\gamma^2 - 2\gamma\beta\mu_\eta e^{-\delta\mu_w}[\sigma_w^2(1 - 2\mu_w\delta + \frac{\mu_w^2\delta^2}{2}) + \mu_w^2] +$$

$$\beta^2(\sigma_\eta^2 + \mu_\eta^2) e^{-2\delta\mu_w}[\sigma_w^2(1 - 4\mu_w\delta + 2\mu_w^2\delta^2) + \mu_w^2] \qquad (A.6b)$$

This completes the representation of μ_v and Ev^2, in terms of μ_w and σ_w^2.

Appendix B

In this appendix we show that expressions (3.5), (3.6) and (3.7) are equivalent to $pr(S|t,k)$, $\mu_{\tilde{L}}(t,k)$ and $\{E\ (\tilde{L}_{tk} - \mu_{\tilde{L}}(t,k))^n\}$ respectively. For reading ease we suppress the subscripts and functional notation relating to sex and the age of the single age-class population, namely k and t respectively. From a binomial expansion the selection probability

$$pr(S|L = \ell) = \left(1 - \frac{1}{h^{\ell/\ell^*}}\right)^h$$

$$= \left(1 - e^{-\ell/\ell^*}\ \ln h\right)^h$$

is equivalent to

$$\sum_{m=0}^{h} \binom{h}{m}\ (-1)^m\ e^{-\lambda(m)\ell}$$

where $\lambda(m) \equiv \frac{m}{\ell^*}\ \ln h$. Since L is normally distributed with mean μ_L and variance σ_L^2 ,

$$pr(S) = \int_{-\infty}^{\infty} pr(S|L = \ell)\ \phi\left(\frac{\ell - \mu_L}{\sigma_L}\right)\ d\ell$$

$$= \int_{-\infty}^{\infty} \sum_{m=0}^{h} \binom{h}{m}(-1)^m\ e^{-\lambda(m)\ell}\ \phi\left(\frac{\ell - \mu_L}{\sigma_L}\right)\ d\ell$$

$$= \sum_{m=0}^{h} \binom{h}{m}(-1)^m \int_{-\infty}^{\infty} e^{-\lambda(m)\ell}\ \phi\left(\frac{\ell - \mu_L}{\sigma_L}\right)\ d\ell\ .$$

Since the integral is equivalent to the moment generating function of a normal random variable evaluated at $\lambda(m)$ one has

$$\int_{-\infty}^{\infty} e^{-\lambda(m)\ell} \phi\left(\frac{\ell-\mu_L}{\sigma_L}\right) d\ell = e^{-\lambda(m)\mu_L + \lambda(m)^2 \sigma_L^2/2} ,$$

and thus pr(S), the unconditional selection probability, equals

$$pr(S) = \sum_{m=0}^{h} \binom{h}{m} (-1)^m e^{-\lambda(m)\mu_L + \lambda(m)^2 \sigma_L^2/2} .$$

This expression agrees with (3.5). Furthermore, because the joint probability that L is in the interval (a,b) and the fish is selected is

$$\sum_{m=0}^{h} \binom{h}{m} (-1)^m \int_a^b e^{-\lambda(m)\ell} \phi\left(\frac{\ell-\mu_L}{\sigma_L}\right) d\ell , \quad (B.1)$$

the expected length of a selected shrimp is

$$\mu_L^{\sim} = \frac{1}{pr(S)} \sum_{m=0}^{h} \binom{h}{m} (-1)^m \int_{-\infty}^{\infty} \ell e^{-\lambda(m)\ell} \phi\left(\frac{\ell-\mu_L}{\sigma_L}\right) d\ell . \quad (B.2)$$

Moreover, one can show that

$$\int_{-\infty}^{\infty} \ell e^{-\lambda(m)\ell} \phi\left(\frac{\ell-\mu_L}{\sigma_L}\right) d\ell = \Lambda(m)\,\Theta(m)$$

where

$$\Lambda(m) = \mu_L - \lambda(m)\,\sigma_L^2$$
$$\Theta(m) = \exp[-\lambda(m)\,\mu_L + \lambda(m)^2 \sigma_L^2/2] .$$

Hence the expression for the mean selected length

$$\mu_L^{\sim} = \sum_{m=0}^{h} \binom{h}{m} (-1)^m \Lambda(m)\,\Theta(m)$$

agrees with (3.6) .

To evaluate the nth central moment of $\tilde{L}$ we expand $(\tilde{L} - \mu_{\tilde{L}})^n$ using the binomial expansion and then apply the expectation operator so that

$$E\left((\tilde{L} - \mu_{\tilde{L}})^n\right) = \sum_{s=0}^{n} \binom{n}{s} E(\tilde{L}^s)(-\mu_{\tilde{L}})^{n-s} \quad . \tag{B.3}$$

But as in (B.2) ,

$$E(\tilde{L}^s) = \frac{1}{pr(S)} \sum_{m=0}^{h} \binom{h}{m}(-1)^m \int_{-\infty}^{\infty} \ell^s e^{-\lambda(m)\ell} \phi\left(\frac{\ell-\mu_L}{\sigma_L}\right) d\ell$$

$$= \frac{1}{pr(S)} \sum_{m=0}^{h} \binom{h}{m}(-1)^m \Theta(m) \int_{-\infty}^{\infty} \ell^s \phi\left(\frac{\ell-\Lambda(m)}{\sigma_L}\right) d\ell \; . \tag{B.4}$$

Substituting expression (B.4) for $E(\tilde{L}^s)$ in (B.3) yields (3.7) for $E\left((\tilde{L}-\mu_{\tilde{L}})^n\right)$.