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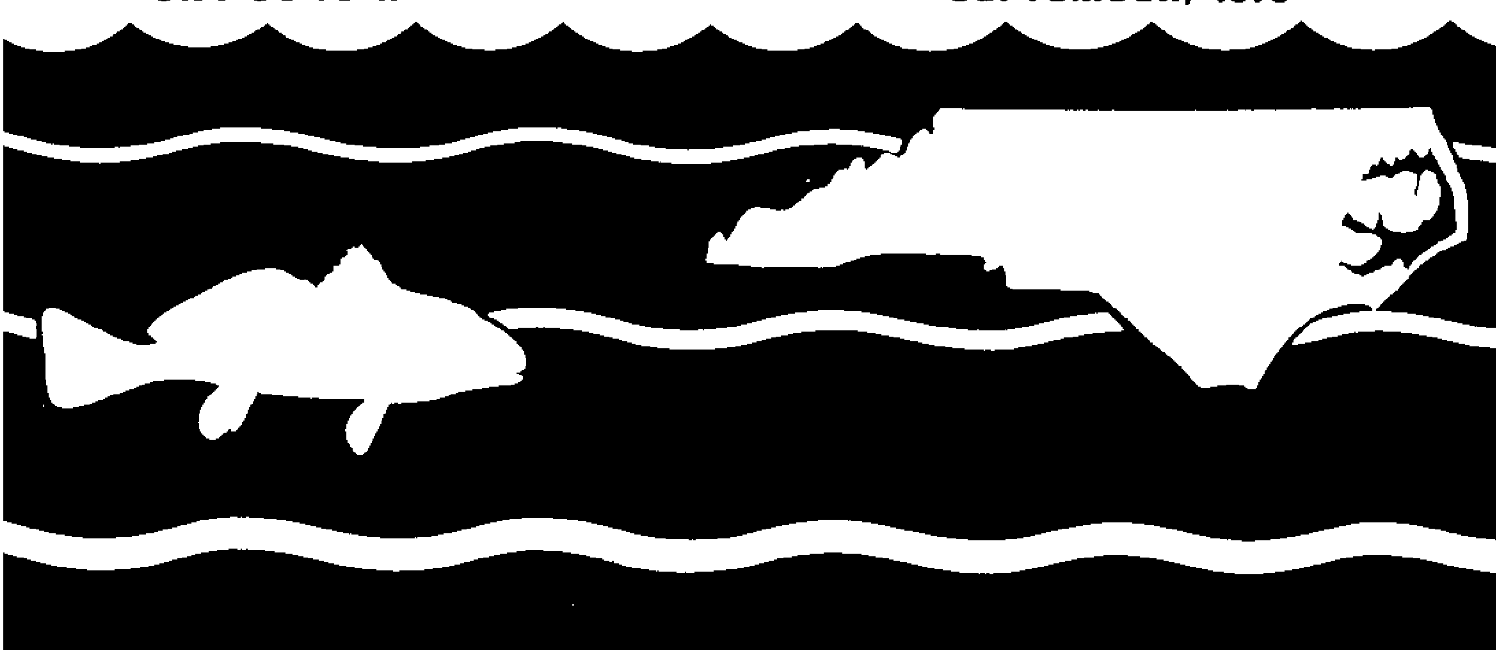

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by

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INTRODUCTION

Past studies (4, 2) indicate that when waves encounter current, the characteristics of waves undergo changes due to the interactions between the waves and the current. Similarly, the forces exerted on an object immersed in water are also affected by the presence of current (8, 9). It was noted (8), however, that the effects of current on waves and hence on fluid forces on objects diminish with increasing wind speed.

Safety analysis of marine structures subjected to oscillatory fluid forces requires consideration of possible fatigue failure (7). The presence of current is expected to play an important role in the evaluation of fatigue damage of the structures. This is because most of the fatigue damages inflicted in the structures take place under moderate wind conditions which prevail over much of the life of the structures and under which the effects of current on waves and fluid forces are most pronounced (8).

Under the action of random forcing functions, the quantity often used in connection with fatigue damage evaluation is the probability density function of the peaks of the forces induced in the structures (3, 7). Most marine structures are relatively rigid and under normal wave conditions operate within the linear range; the forces in these structures are linearly proportional to the externally applied forces. With these considerations, this study undertakes to examine the influence of current and wave-current interactions on the probability density function of the peaks of the fluid force on a rigid cylinder.

For simplicity, only deep water, zero mean, stationary, Gaussian random waves are considered. The current is assumed to be steady, uniformly distributed in depth. The fluid force is evaluated at an element of the cylinder of unit length situated immediately beneath the mean water level.

INFLUENCE OF CURRENT ON WAVE FIELD KINEMATICS

In evaluating fluid force, the Morrison's formula is used in this study. That is, fluid force is considered to consist of two parts, the inertia component, linearly proportional to fluid particle acceleration, and the drag component, nonlinearly related to fluid particle velocity. In subsequent computation of the probability density function of the peaks of fluid forces, the quantities σ_v, the standard deviation of fluid particle velocity, and σ_a, $\sigma_{\dot{a}}$, those of fluid particle acceleration and its derivative, are required. These quantities in turn are determined from their respective spectrum. Thus, the influence of current on wave frequency spectrum and on spectra of fluid particle velocity, acceleration and its derivative are briefly discussed first.

It was shown (2) that under the action of a steady current, the frequency spectrum of surface waves of a stationary gravity wave field in deep water is given by

$$\phi(n) = \frac{4\phi^*(n)}{[1 + (1 + \frac{4Un}{g})^{1/2}]\ [(1 + \frac{4Un}{g})^{1/2} + (1 + \frac{4Un}{g})]} \qquad (1)$$

in which, n is total frequency, U is current velocity, g is gravitational acceleration and $\phi(n)$ and $\phi^*(n)$ are respectively the frequency spectra of surface waves with and without the influence of current. In this study $\phi^*(n)$ is taken to be

$$\phi^*(n) = \frac{\alpha g^2}{n^5} \exp\ (-\beta (\frac{n_o}{n})^4) \qquad (2)$$

the Kitaigorodskii-Pierson-Moskowitz spectrum, in which α and β are non-dimensional constants equal to 0.8×10^{-2} and 0.74 respectively and $n_o = g/W$ with W the mean wind speed. The spectra $\phi(n)$ for various values of current speed U and for wind speed W = 40 mph are plotted in Fig. 1. It is seen that when the current is in the direction of the waves, that is, when U is positive, the surface spectrum is lowered. On the other hand, under adverse current, the surface wave spectrum increases in magnitude. When the current is negative, there is a cut-off frequency in the surface wave spectrum determined by the condition $1 + \frac{4Un}{g} = 0$, beyond which no wave can exist.

The spectra of fluid particle velocity V(t), acceleration a(t), and its derivative $\dot{a}(t)$ at the mean water level are, within the accuracy of linear wave theory, respectively given by

$$\phi_{VV}(n) = n^2\phi(n) \tag{3}$$

$$\phi_{aa}(n) = n^4\phi(n) \tag{4}$$

and

$$\phi_{\dot{a}\dot{a}}(n) = n^6\phi(n) \ . \tag{5}$$

The standard deviations σ_v of fluid particle velocity and σ_a, $\sigma_{\dot{a}}$ of fluid particle accelerations and its derivative are respectively obtained from

$$\sigma_v = [\int_n \phi_{VV}(n)dn]^{1/2} \tag{6}$$

$$\sigma_a = [\int_n \phi_{aa}(n)dn]^{1/2} \tag{7}$$

and

$$\sigma_{\dot{a}} = [\int_n \phi_{\dot{a}\dot{a}}(n)dn]^{1/2} \tag{8}$$

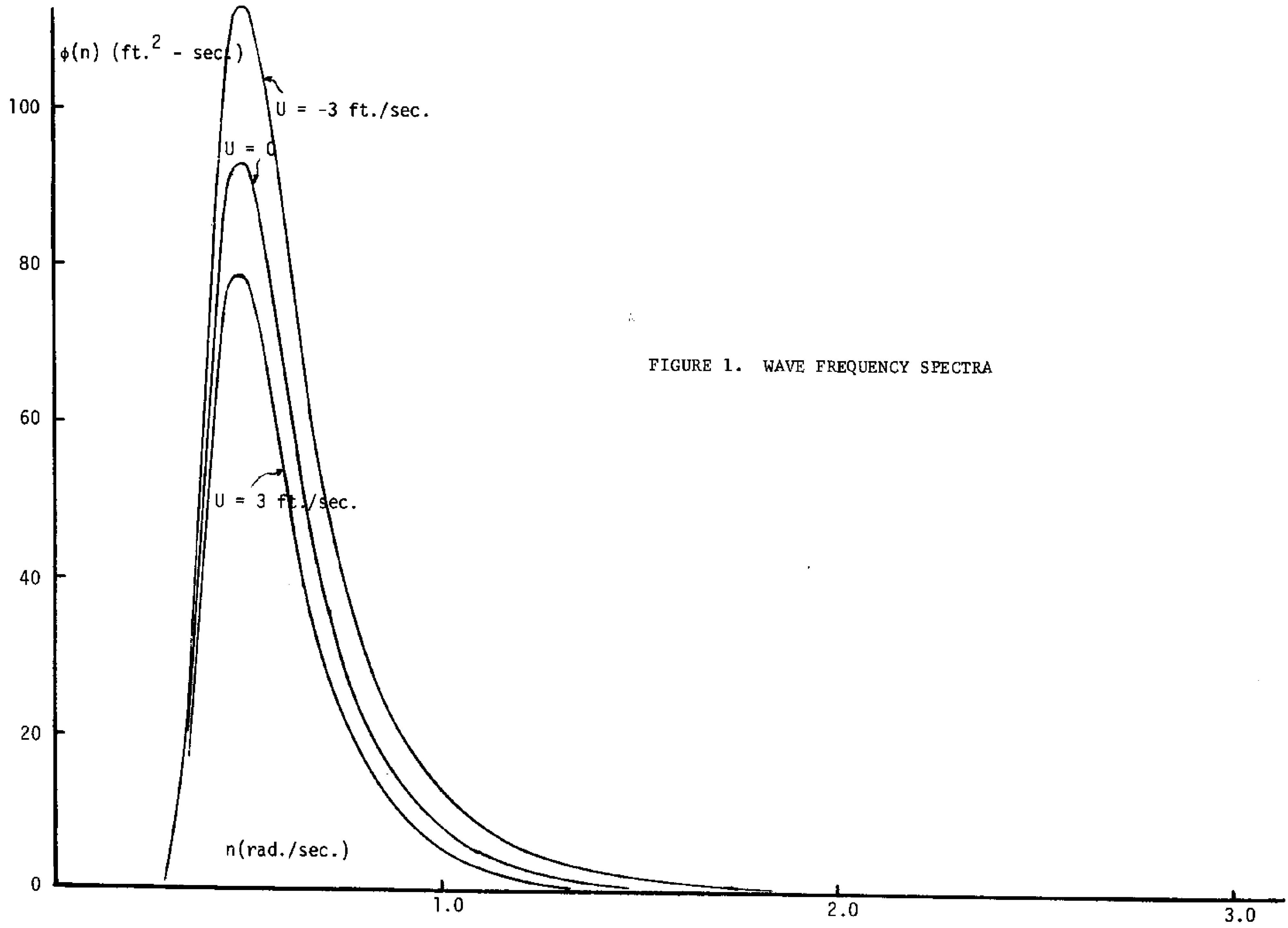

FIGURE 1. WAVE FREQUENCY SPECTRA

in which the integrations extend over the entire frequency range for gravity waves. Fig. 2 shows the effect of wave-current interactions on fluid particle velocity spectrum under various current conditions. Spectra of fluid particle acceleration and its derivative exhibit similar characteristics and are therefore not shown.

PEAK DISTRIBUTION OF FLUID FORCE

The probability density function $f_p(x;t)$ of the peaks of random process X(t) is (3)

$$f_p(x;t) = \frac{-1}{E[M_T(t)]} \int_{-\infty}^{o} z f_{XYZ}(x,o,z;t)dz \quad (9)$$

In Eq. 9, $f_p(x;t)dxdt$ is the probability of the occurrence of a peak of the random process X(t) with magnitude between x and x + dx from time t to t + dt. The quantity $f_{XYZ}(\cdot,\cdot,\cdot;t)$ is the joint probability density function of the random processes X(t), $Y(t) = \dot{X}(t)$ and $Z(t) = \ddot{X}(t)$. $E[M_T(t)]$ is the expected value of the total number of peaks of the process X(t), per unit time, regardless of their magnitudes. Here and hereafter "over-dot" denotes differentiation and $E[\cdot]$ denotes the expected value of the random quantity enclosed in the bracket.

An approximate and conservative estimate of the probability density function of peaks is (3)

$$f_p(x;t) = \frac{-1}{E[M_T(t)]} \frac{dE[N_+(x;t)]}{dx} \quad (10)$$

in which

$$E[N_+(x;t)] = \int_{o}^{\infty} y f_{XY}(x,y;t)dy \quad (11)$$

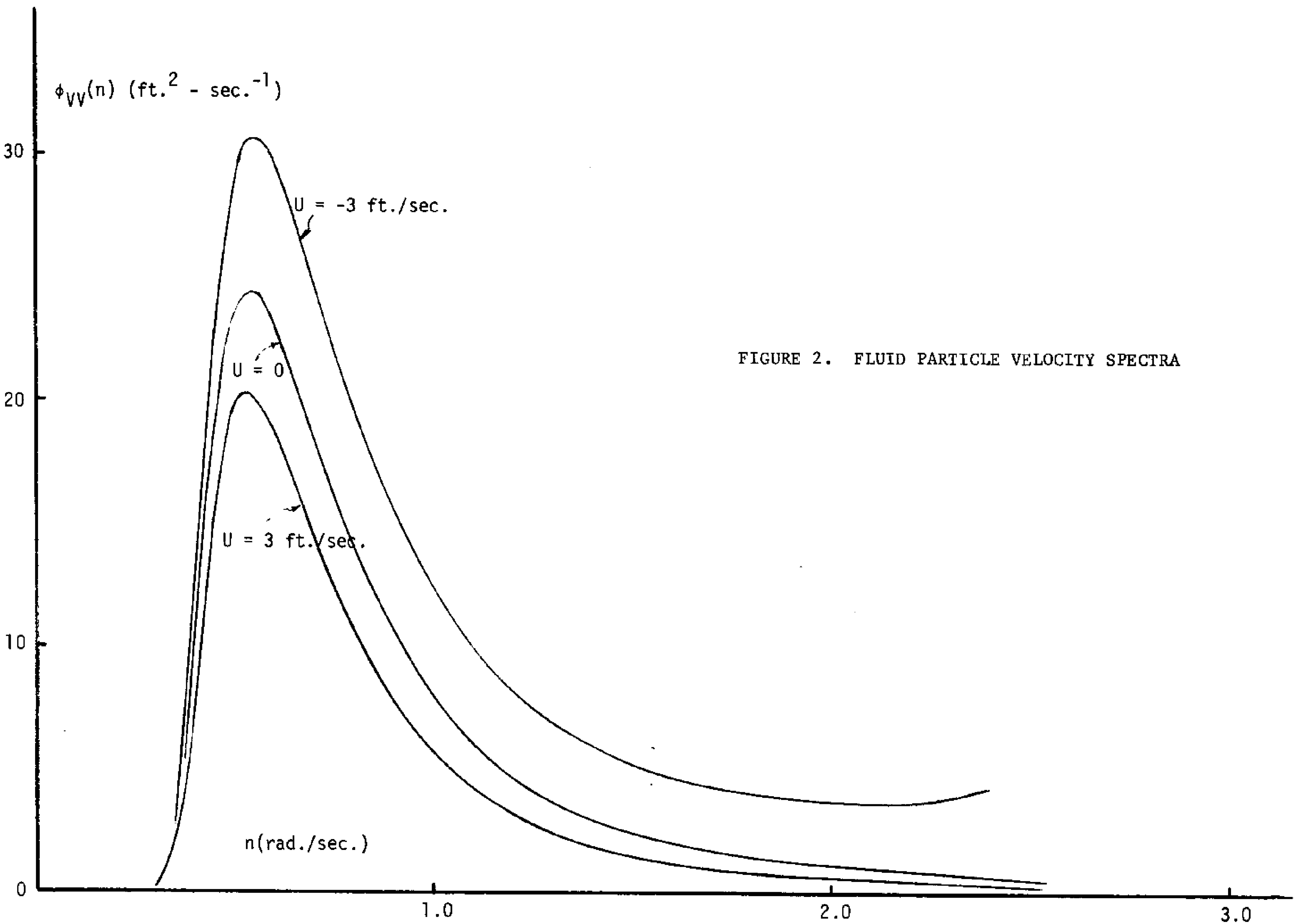

FIGURE 2. FLUID PARTICLE VELOCITY SPECTRA

is the expected value of the rate of threshold crossings, from below, at the threshold level x, of the process X(t). In Eq. 11, $f_{XY}(.,.;t)$ is the joint probability density function of the processes X(t) and $Y(t) = \dot{X}(t)$.

Associated with the process X(t), the expected value of damage, per unit time, denoted by E[D(t)], is given by (3)

$$E[D(t)] = c^{-1}E[M_T(t)] \int_{-\infty}^{\infty} x^b f_p(x;t)dx \tag{12}$$

in which b and c are constants representing the characteristics of the fatigue properties of the material. It is seen from Eq. 12 that in evaluating fatigue damage, only the product of $E[M_T(t)]$ and $f_p(x;t)$ is required. That is, it suffices to examine the quantity $\frac{dE[N_+(x;t)]}{dx}$ of Eq. 10.

It is mentioned here that the time parameter t that appears in all the quantities in Eqs. 9 through 12 can be dropped when X(t) is a stationary process.

Consider now the fluid force exerted on an element of unit length of a cylinder situated at mean water level of a random, stationary, zero mean Gaussian sea under the influence of a steady uniform current. According to Morrison's formula, the fluid force F(t) is

$$F(t) = C_D V(t)|V(t)| + C_M a(t) \tag{13}$$

in which a(t) and V(t) are respectively the fluid particle acceleration and velocity. Here V(t) = U + v(t), the sum of the steady current velocity U and the oscillatory particle velocity v(t) corresponding to wave motion. In Eq. 13, $C_D = \rho K_D d$ and $C_M = \rho K_M \frac{\pi d^2}{4}$, with ρ = 2 lbs. $\sec^2$/ft. ft.^3, density of water, d, diameter of cylinder, K_D = 0.5 to 0.7 and K_M = 1.4 to 2.0, the drag and inertia coefficients (5). In this study, the values of K_D and K_M are chosen to be 0.5 and 1.4 respectively.

To use Eq. 11 to evaluate $\frac{dE[N_+(x)]}{dx}$ of the stationary, random process $X(t) = F(t)$, it is necessary to find the joint probability density function $f_{XY}(.,.)$ of $X(t) = F(t)$ and the derivative $Y(t) = \dot{F}(t)$ of the fluid force $X(t) = F(t)$ which, from Eq. 13, is

$$\dot{F}(t) = 2C_D|V(t)|\ a(t) + C_M\dot{a}(t)\ . \tag{14}$$

This may be achieved by the standard method of transformation of random variables (6). Thus, introducing an auxiliary random quantity $Z(t) = V(t)$ and first determine the joint probability density function $f_{XYZ}(.,.,.)$. That is,

$$f_{XYZ}(x,y,z) = f_{Va\dot{a}}(V,a,\dot{a})/|J| \tag{15}$$

in which $|J| = C_M^2$ is the Jacobian of transformation, the function $f_{Va\dot{a}}(V, a,\dot{a})$ is the Gaussian joint probability density function of $V(t)$, $a(t)$ and $\dot{a}(t)$,

$$f_{Va\dot{a}}(V,a,\dot{a}) = \frac{1}{(2\pi)^{3/2}|S|^{1/2}}$$

$$\exp[\frac{1}{2|S|}(S_{VV}(V-U)^2 + 2S_{V\dot{a}}(V-U)\dot{a} + S_{aa}a^2 + S_{\dot{a}\dot{a}}\dot{a}^2)] \tag{16}$$

in which

$$\begin{aligned}
|S| &= \sigma_v^2\,\sigma_a^2\,\sigma_{\dot{a}}^2 - \sigma_a^6 \\
S_{VV} &= \sigma_a^2\,\sigma_{\dot{a}}^2 \\
S_{V\dot{a}} &= \sigma_a^4 \\
S_{aa} &= |S|\ /\sigma_a^2
\end{aligned} \tag{17}$$

and

$$S_{\dot{a}\dot{a}} = \sigma_v^2\,\sigma_a^2 \quad .$$

In Eq. 15, the arguments V, a and $\dot{a}$ of $f_{Va\dot{a}}(V,a,\dot{a})$ are to be replaced by

$$V = z$$

$$a = (x - C_D|z|z)/C_M$$

and

$$\dot{a} = [y - \frac{2C_D|z|}{C_M}(x - C_D|z|z)]/C_M \quad . \tag{18}$$

The joint probability density function $f_{XY}(.,.)$ of $X(t) = F(t)$ and $Y(t) = \dot{F}(t)$ is the marginal density to the joint probability density function $f_{XYZ}(.,.,.)$

$$f_{XY}(x,y) = \int_{-\infty}^{\infty} f_{XYZ}(x,y,z)\, dz \quad . \tag{19}$$

The expected value of the rate of threshold crossings, from below, of the process $X(t) = F(t)$, at threshold level x, is, from Eq. 11,

$$E[N_+(x)] = \int_{-\infty}^{\infty} y f_{XY}(x,y)\,dy = \int_{-\infty}^{\infty} dz \int_{-\infty}^{\infty} y f_{XYZ}(x,y,z)\,dy\,. \tag{20}$$

The integration in Eq.(20) with respect to y can be achieved explicitly. After taking the derivative of the resulting expression of $E[N_+(x)]$ from Eq. 20 with respect to x,

$$\frac{dE[N_+(x)]}{dx} = \frac{1}{C_M^2(2\pi)^{3/2}|S|^{1/2}}$$

$$\Bigg\{ \frac{-1}{2C}\int_{-\infty}^{\infty}\Big[\frac{S_{VV}B}{C_D|z|} + \frac{4CC_D|z|}{C_M}\big(B - \frac{A}{2C}\big)\Big]\exp\Big[\frac{-1}{2|S|}\Big(\frac{S_{VV}A^2C_M^2}{4S_{V\dot{a}}^{\,2}} + \frac{S_{aa}B^2}{4C_D^2z^2} - CB\big(B-\frac{A}{C}\big)\Big]dz$$

$$+ \sqrt{\frac{|S|}{C}}\int_{-\infty}^{\infty}\Big[\frac{2C_D|z|G}{C_M} + \big(B - \frac{A}{2C}\big)\Big(\sqrt{\frac{C}{|S|}}\;\frac{2C_D|z|}{C_M}\, e^{-E^2/2} - \frac{BG}{2C_M\sigma_a^2C_D|z|}\Big)\Big]$$

$$\exp\left[- \frac{A^2 C_M{}^2}{8\sigma_v^2 \ \sigma_a^8} - \frac{B^2}{8\sigma_a^2 C_D^2 \ z^2} \right] dz \Bigg\} \qquad (21)$$

in which

$$A = \frac{2S_{VV}(z - U)}{C_M}$$

$$B = \frac{2C_C|z|}{C_M} \ (x - C_D z|z|)$$

$$C = \frac{S_{a\dot{a}}}{C_M^2} \qquad (22)$$

$$E = \sqrt{\frac{C}{|S|}} \ \left(\frac{A}{2C} - B \right)$$

$$G = \sqrt{2\pi} \ (0.5 - P(E) \ )$$

$$P(E) = \int_o^E T(q) dq$$

$$T(q) = \frac{1}{\sqrt{2\pi}} \ e^{-q^2/2} \quad .$$

It is recognized that $P(\cdot)$ is the error function (6).

Thus for given wind and current conditions, once the quantities σ_v, σ_a and $\sigma_{\dot{a}}$ are computed from Eqs. 1 through 8, the quantity $\frac{dE[N_+(x)]}{dx}$ can be obtained by integrating Eq. 21 with respect to z numerically.

Under the action of negative current, the relevant quantity is not the peak but the trough distribution of fluid force. That is, quantity $\frac{dE(N_-(x)]}{dx}$ is used for this evaluation of fatigue damage in which $E[N_-(x)]$ is the expected value of the rate of threshold crossings, per unit time, of $X(t) = F(t)$, from above, at threshold level x. The expression of $\frac{dE[N_-(x)]}{dx}$ is, of course, the the same as that of $\frac{dE[N_+(x)]}{dx}$ except the signs of U and x are changed accordingly

It should be mentioned here that the assumptions underlying Eq. 10 implies that $\frac{dE[N(x)]}{dx}$ is to be evaluated only for absolute values of the threshold level x greater than that of the expected value of the process $X(t) = F(t)$ (3).

GAUSSIAN APPROXIMATION OF FLUID FORCE

In dealing with random phenomena, Gaussian assumption is often invoked for reasons of mathematical expediency. It is therefore of interest to examine the influence that Gaussian assumption of fluid force has on the quantity $\frac{dE[N_+(x)]}{dx}$ of the fluid force under consideration.

Referring back to Eq. 20, the joint probability density function $f_{XY}(x,y)$ of $X(t) = F(t)$ and $Y(t) = \dot{F}(t)$ is required. If it is assumed that the processes $X(t) = F(t)$ and $Y(t) = \dot{F}(t)$ are jointly Gaussian, then by stationary assumption of $X(t) = F(t)$, they are uncorrelated and therefore statistically independent. Thus

$$f_{XY}(x,y) = f_X(x) f_Y(y) \tag{23}$$

in which $f_X(x)$ and $f_Y(y)$ are Gaussian with respective parameters $E[F]$, σ_F and $E[\dot{F}]$, $\sigma_{\dot{F}}$, the expected value and standard deviation of $X(t) = F(t)$ and $Y(t) = \dot{F}(t)$. Under the assumption of Gaussian stationary sea state, within the accuracy of linear wave theory, it was shown (1) that

$$E[F] = 2C_D\, \sigma_v^2\, [\gamma T(\gamma) + (1 + \gamma^2)\, P(\gamma)] \tag{24}$$

in which $\gamma = U/\sigma_v$ is a parameter measuring the relative strength of current. The standard deviation σ_F of $X = F(t)$ is, by definition

$$\sigma_F = \{ E[F^2] - E^2[F]\}^{1/2} \tag{25}$$

in which $E[F^2]$ is the second statistical moment of $X = F(t)$ which, following Borgman (1), is

$$E[F^2] = C_D^2 \sigma_v^4(\gamma^4 + 6\gamma^2 + 3) + C_M^2 \sigma_a^2 . \tag{26}$$

It can be similarly shown that

$$E[\dot{F}] = 0 \tag{27}$$

and

$$\sigma_{\dot{F}} = [4C_D^2 \sigma_a^2 \sigma_v^2 (1 + \gamma^2) + C_M^{\,2} \sigma_{\dot{a}}^2]^{1/2} . \tag{28}$$

The expected value of the rate of threshold crossings, from below, of $X(t) = F(t)$, in this case is, from Eq. 20

$$E[N_+(x)] = \int_{-\infty}^{\infty} y f_{XY}(x,y)\,dy = f_X(x) \int_{-\infty}^{\infty} y f_Y(y)\,dy = \frac{\sigma_{\dot{F}}}{\sqrt{2\pi}} f_X(x) \tag{29}$$

and its derivative is

$$\frac{dE[N_+(x)]}{dx} = \frac{-\sigma_{\dot{F}}}{2\pi\sigma_F^2} \left(\frac{x - E[F]}{\sigma_F} \right) \exp \left[- \frac{1}{2} \left(\frac{x - E[F]}{\sigma_F} \right)^2 \right]. \tag{30}$$

In adverse current, the quantity $\frac{dE[N_-(x)]}{dx}$ is used which is simply $\frac{dE[N_+(x)]}{dx}$ with the signs of x and E[F] changed accordingly.

NUMERICAL RESULTS

The absolute value of the quantity $\frac{dE[N(x)]}{dx}$ of the fluid force on an element of a cylinder of unit length and diameter is now computed and presented graphically for wind speed W = 40 mph and current speeds U = 0, = ±3 ft./sec. For each current speed used, the value $\frac{dE[N(x)]}{dx}$ is computed from both Eq. 21

for the non-Gaussian case and Eq. 30, for the case where fluid force is assumed to be Gaussian. Except when U = 0, results are obtained for the cases where wave-current interactions are considered and ignored. In the former case, Eqs. 1 through 8 are used to obtain the basic quantities σ_v, σ_a and $\sigma_{\dot{a}}$. In the latter case, values of σ_v, σ_a, $\sigma_{\dot{a}}$ are obtained by replacing $\phi(n)$ by $\phi^*(n)$ in Eqs. 3, 4, and 5. For all cases considered, the value of x ranges from $x = E[F]$ to $x = E[F] \pm 3\sigma_F$ depending on whether peak or trough is considered.

In Fig. 3, the values of $\left| \frac{dE[N_+(x)]}{dx} \right|$ for the case U = 0 are presented. Curve 1 represents the non-Gaussian case and Curve 2, the Gaussian case. It is immediately apparent that the Gaussian case deviates appreciably from the non-Gaussian case and for larger values of x to which most fatigue damage is due, the Gaussian case underestimates the quantity $\frac{dE[N_+(x)]}{dx}$.

That the Gaussian case departs from the non-Gaussian case is due largely to the fact that while the processes $X(t) = F(t)$ and $Y(t) = \dot{F}(t)$ are truly uncorrelated, they are not actually statistically independent, a fact which the assumption that the processes are jointly Gaussian fails to reflect.

Figure 4 gives values of $\left| \frac{dE[N_+(x)]}{dx} \right|$ for the case when U = 3 ft./sec. Curves 1 and 2 represent the non-Gaussian case and Curves 3 and 4 are for the Gaussian case. Comparing curves 1 and 2, it is seen that wave-current interactions give rise to less peaks than when interactions are ignored. This is because when current is in the direction of wave propagation the sea becomes calmer due to wave-current interactions. Examination of Curves 3 and 4 indicates that the same holds true for the Gaussian case. Finally, comparison of Curve 1 with Curve 3 and Curve 2 with Curve 4 shows that Gaussian approximation of fluid force tends to underestimate the quantity $\left| \frac{dE[N_+(x)]}{dx} \right|$ for larger values of x.

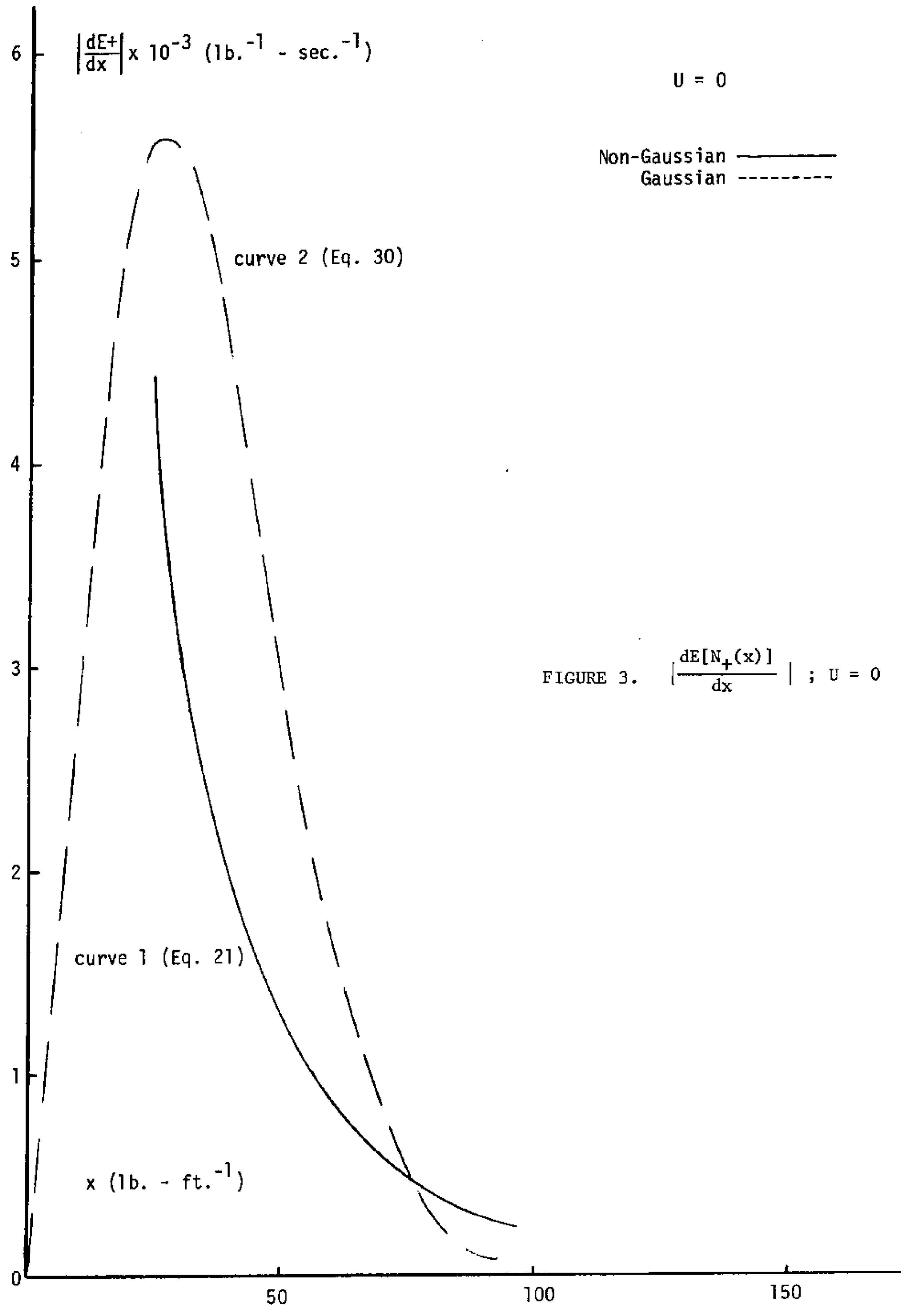

FIGURE 3. $\left|\frac{dE[N_+(x)]}{dx}\right|$; U = 0

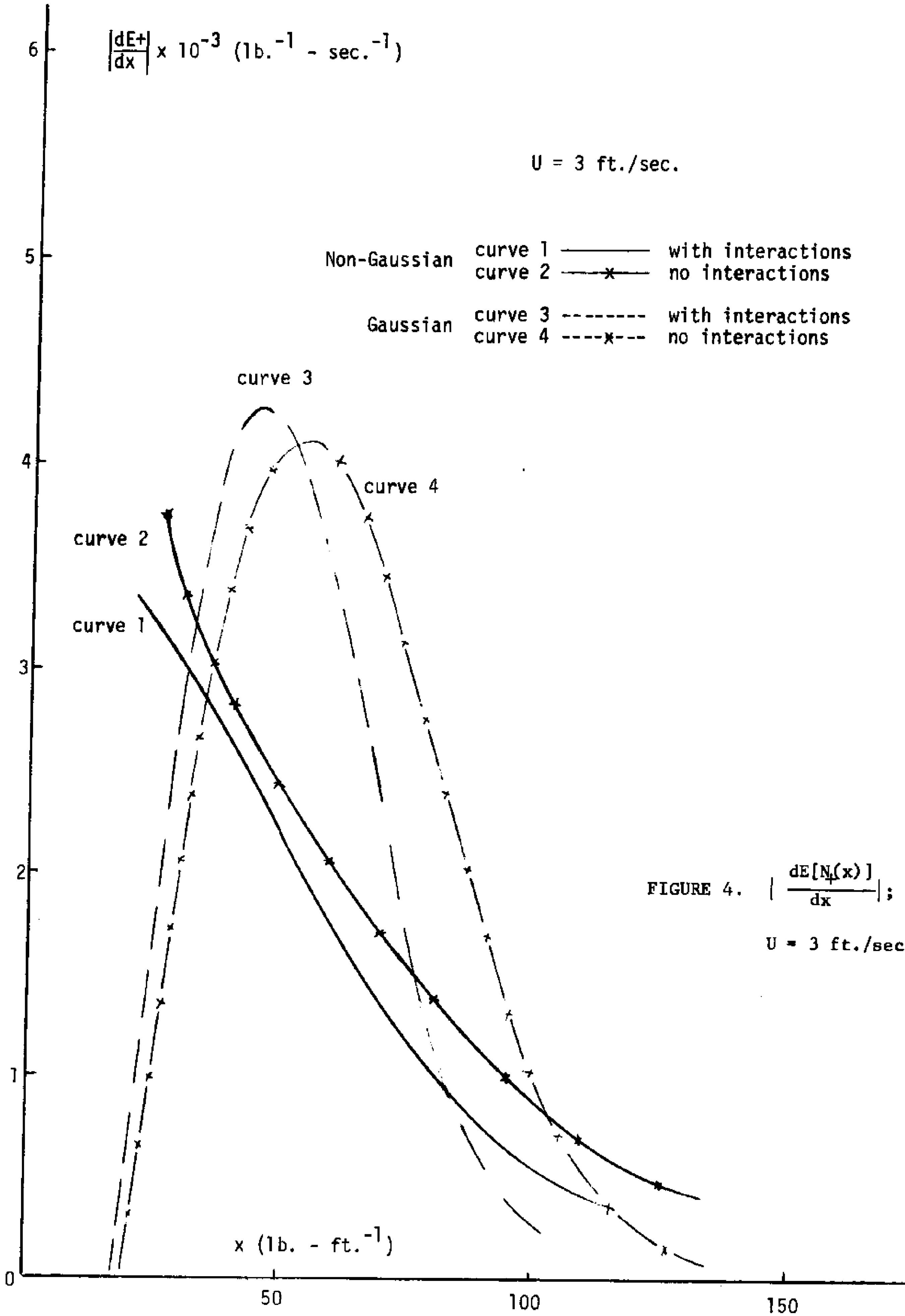

FIGURE 4. $\left|\frac{dE[N_+(x)]}{dx}\right|$;

U = 3 ft./sec

Figure 5 gives values of $\left|\frac{dE[N_{-}(x)]}{dx}\right|$ for U = -3 ft./sec. Comparison of curves 1 and 2 shows that wave-current interactions increase the probability of occurrence of troughs for larger magnitudes of fluid force. This is due to the fact that in adverse current, while small waves break, large waves steepen with attending energy pile-up. Examination of curves 3 and 4 indicates that the same holds true for the Gaussian case. It is noted that curves 2 and 4, which represent the case when no wave-current interactions are considered, are merely mirror images of the corresponding curves in Fig. 4. Figure 5 also shows, as in the case of Fig. 4, that Gaussian assumption of fluid force tends to underestimate the quantity $\left|\frac{dE[N_{-}(x)]}{dx}\right|$ for larger values of x.

CONCLUDING REMARKS

Presented in this study is the influence of current and wave-current interactions on the quantity $\left|\frac{dE[N(x)]}{dx}\right|$ of fluid force which can be used directly to estimate the expected damage of materials of marine structures. The effect of Gaussian assumption of fluid force on the quantity is also investigated. From the results of this study, it is shown that

1. The phenomenon of wave-current interactions affects the quantity $\frac{dE[N(x)]}{dx}$ considerably and

2. Gaussian assumption of fluid force, while convenient to use, is decisively inadequate for the evaluation of the quantity $\frac{dE[N(x)]}{dx}$.

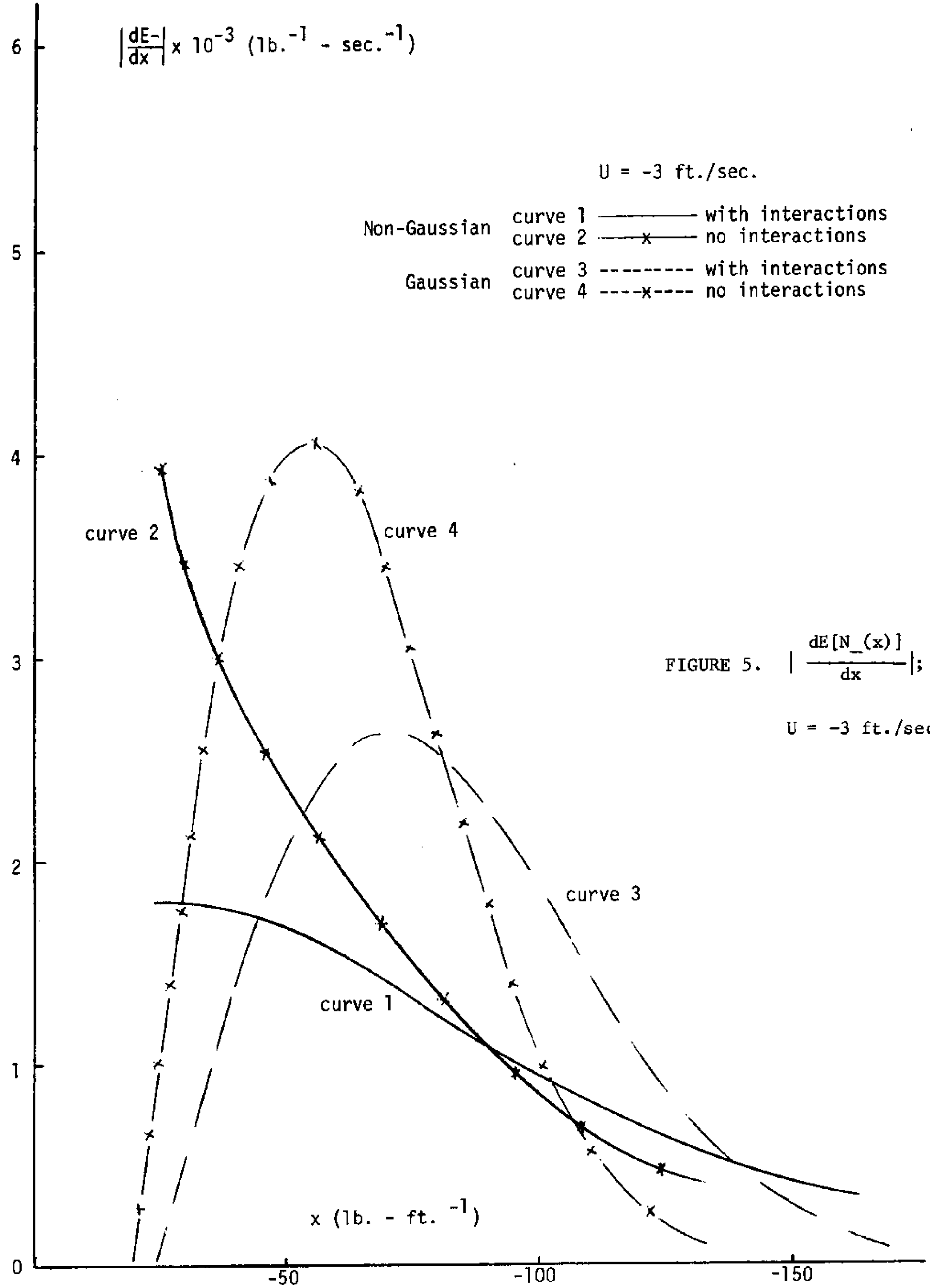

FIGURE 5. $\left|\frac{dE[N\_(x)]}{dx}\right|$;

U = -3 ft./sec

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APPENDIX I. - NOTATION

The following symbols are used in this report:

A = quantity defined in Eq. 22;

$a, \dot{a}$ = dummy variable used in Eqs. 15 and 16;

$a(t)$, $\dot{a}(t)$ = fluid particle acceleration and its derivative;

B = quantity defined in Eq. 22;

b = material constant used in Eq. 12;

C = quantity defined in Eq. 22;

C_D, C_M = coefficients used in Eq. 13;

c = material constant used in Eq. 12;

$D(t)$ = rate of damage at time t (Eq. 12);

d = diameter of cylinder;

E = quantity defined in Eq. 22;

$E[\cdot]$ = expected value of the random quantity enclosed in the bracket;

$E[D(t)]$ = expected value of damage per unit time; (Eq. 12);

$E[F]$, $E[\dot{F}]$ = expected values of fluid force $F(t)$ and its derivative $\dot{F}(t)$ respectively (Eqs. 24, 27);

$E[F^2]$ = second statistical moment of fluid force $F(t)$ (Eq. 26);

$E[M_T(t)]$ = expected total number of peaks of random process $X(t)$, regardless of their magnitudes, per unit time (Eqs. 9,10,12);

$E[N_+(x;t)]$, $E[N_+(x)]$ = expected number of crossings of random process $X(t)$, at threshold level x, from below, per unit time (Eqs. 11, 20);

$E[N_-(x;t)]$, $E[N_-(x)]$ = expected number of crossings of random process $X(t)$, at threshold level x, from above, per unit time;

$F(t)$, $\dot{F}(t)$ = fluid force and its derivative on element of a cylinder of unit length (Eqs. 13, 14);

$f_p(x;t)$ = probability density function of peaks of random process $X(t)$ (Eqs. 9, 10);

$f_{Va\dot{a}}(.,.,.)$ = joint probability density function of V(t), a(t) and $\dot{a}(t)$ (Eq. 16);

$f_{XY}(.,.;t)$ = joint probability density function of X(t) and Y(t) = $\dot{X}(t)$ (Eq. 11);

$f_{XY}(.,.)$ = joint probability density function of X(t) = F(t) and Y(t) = $\dot{F}(t)$ (Eqs. 19, 23);

$f_{XYZ}(.,.,.;t)$ = joint probability density function of X(t), Y(t) = $\dot{X}(t)$ and Z(t) = $\ddot{X}(t)$ (Eq. 9);

$f_{XYZ}(.,.,.)$ = joint probability density function of X(t) = F(t), Y(t) = $\dot{F}(t)$ and Z(t) = V(t) (Eq. 15);

$f_X(.)$ = Gaussian probability density function of X(t) = F(t) with parameters E[F] and σ_F (Eq. 23);

$f_Y(.)$ = Gaussian probability density function of Y(t) = $\dot{F}(t)$ with parameters E[$\dot{F}$] and $\sigma_{\dot{F}}$ (Eq. 23);

G = quantity defined in Eq. 22;

g = gravitational acceleration;

J = Jacobian of transformation (Eq. 15);

K_D, K_M = drag and inertia coefficients respectively;

$M_T(t)$ = total number of peaks of random process X(t), regardless of their magnitudes, per unit time;

$N_+(x;t)$, $N_+(x)$ = number of crossings of random process X(t), at threshold level x, from below, per unit time;

$N_-(x;t)$, $N_-(x)$ = number of crossings of random process X(t), at threshold level x, from above, per unit time;

n = frequency;

n_o = g/W (Eq. 2);

P(.) = error function (Eqs. 22, 24);

q = dummy variable;

$|S|$, S_{aa}, $S_{\dot{a}\dot{a}}$, $S_{V\dot{a}}$, S_{VV} = quantities defined in Eq. 17;

$T(q) = \frac{1}{\sqrt{2\pi}} e^{-q^2/2}$ (Eqs. 22, 24);

t = time;

U = current speed;

V = dummy variable;

$V(t)$ = $v(t) + U$, fluid particle velocity;

$v(t)$ = fluid particle velocity corresponding to wave motion;

W = mean wind speed (Eq. 2);

$X(t)$ = random process or $X(t) = F(t)$;

$\dot{X}(t)$, $\ddot{X}(t)$ = first and second derivative of random process $X(t)$;

x = dummy variable;

$Y(t)$ = random process $Y(t) = \dot{X}(t)$ or $Y(t) = \dot{F}(t)$;

y = dummy variable;

$Z(t)$ = random process $Z(t) = \ddot{X}(t)$ or $Z(t) = V(t)$;

α, β = non-dimensional constants (Eq. 2);

γ = U/σ_v;

ρ = density of water;

σ_a, $\sigma_{\dot{a}}$ = standard deviations of fluid particle acceleration $a(t)$ and its derivative $\dot{a}(t)$ respectively (Eqs. 7, 8);

σ_F, $\sigma_{\dot{F}}$ = standard deviations of fluid force $F(t)$ and its derivative $\dot{F}(t)$ respectively (Eqs. 25, 28);

σ_v = standard deviation of fluid particle velocity $V(t)$ (Eq. 6);

$\phi(n)$, $\phi^*(n)$ = wave frequency spectra with and without the influence of current respectively (Eqs. 1, 2);

$\phi_{aa}(n)$, $\phi_{\dot{a}\dot{a}}(n)$ = frequency spectra of fluid particle acceleration $a(t)$ and its derivative $\dot{a}(t)$ respectively (Eqs. 4, 5) and

$\phi_{VV}(n)$ = frequency spectrum of fluid particle velocity $V(t)$ (Eq. 3).

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