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NONUNIFORM GROUNDWATER–CONDUIT DISCHARGE AND HEAD LOSS

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By Stanley S. Butler[1] and David L. Gundlach[2]

INTRODUCTION

The subject of groundwater[3] flow is important in coastal studies because of the effect of outflowing fresh water on the ecology along ocean beaches, the loss of valuable fresh water to the ocean, and the intrusion of sea water into coastal aquifers. Using rational approaches verified by laboratory testing, it is possible to predict groundwater discharge and water levels for nonuniform confined groundwater conduits.

BASIS FOR COMPUTATIONS

The rate of discharge for laminar flow through saturated porous materials can be determined by means of the Darcy equation[4]

$$Q = PIA \quad \ldots\ldots (1)$$

in which P is the coefficient of permeability, I is the hydraulic gradient, and A is the cross-sectional area normal to the direction of flow. For a

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[3]See Appendix IV for definition of terms.

[4]See Appendix III for definition of symbols.

uniform confined conduit Eq. 1 can be written

$$Q = P \frac{H}{L} A \quad \ldots \quad (2)$$

The Darcy equation is valid for laminar flow in which viscous forces predominate. Departure from Darcy's law occurs when inertia forces are significant, and/or when the flow is turbulent. Also, Darcy's law may be inapplicable if clay and colloidal material are abundant in the medium [1].[5]

Whereas uniform groundwater-conduit discharge is calculated by means of Eq. 2, Butler [2] has derived equations for nonuniform conduits, expressed as $Q = \frac{H}{L} (PA)_e$, as follows:

For a truncated-wedge conduit,[6, 7]

$$Q = \frac{H}{L} \left(\frac{(PA)_1 - (PA)_2}{2.3 \log \dfrac{(PA)_1}{(PA)_2}} \right) \quad \ldots \quad (3)$$

For a truncated-pyramid conduit,[8]

$$Q = \frac{H}{L} \sqrt{(PA)_1 \, (PA)_2} \quad \ldots \quad (4)$$

Subscripts 1 and 2 denote the two ends of a length of conduit. The subscripts are interchangeable such that subscript 1 may denote either end.

METHOD OF SOLUTION

The procedure for computing the discharge rate and water levels for a particular conduit is as follows:

1. Plot the product of the coefficient of permeability and the

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[5] Numerals in brackets refer to corresponding items in Appendix II.

[6, 8] See Glossary (Appendix IV) for description of conduits.

[7] Logarithms are to the base 10.

cross-sectional area (PA) versus distance in the direction of flow, as in Fig. 1.

2. Divide the distance into sub-lengths depending on the variation in the quantity (PA). Note that (PA) should vary linearly with distance for use of the truncated-wedge equation, whereas (PA) should vary with the square of the distance for application of the truncated-pyramid equation.

3. For each nonuniform sub-length, determine $(PA)_e$:

If a truncated wedge,

$$(PA)_e = \left(\frac{(PA)_1 - (PA)_2}{2.3 \log \frac{(PA)_1}{(PA)_2}} \right) \quad \ldots \ldots \ldots \quad (5)$$

If a truncated pyramid,

$$(PA)_e = \sqrt{(PA)_1 \, (PA)_2} \quad \ldots \ldots \ldots \quad (6)$$

In Eq. 5 it is helpful to let $(PA)_1$ be the larger of the two (PA)-quantities, so that the ratio $(PA)_1/(PA)_2$ is greater than 1.0.

4. Determine the discharge rate,

$$Q = \frac{H}{L} \left(\frac{L}{\sum \left(\frac{L_s}{(PA)_e} \right)} \right) \quad \ldots \ldots \ldots \quad (7)$$

or

$$Q = \frac{H}{\sum \left(\frac{L_s}{(PA)_e} \right)} \quad \ldots \ldots \ldots \quad (8)$$

See Harr [3] for the basis of Eq. 7.

5. Determine the elevation of the piezometric surface (the level at which water stands in observation wells) by solving Eq. 8 for values of H corresponding to various positions along the conduit.

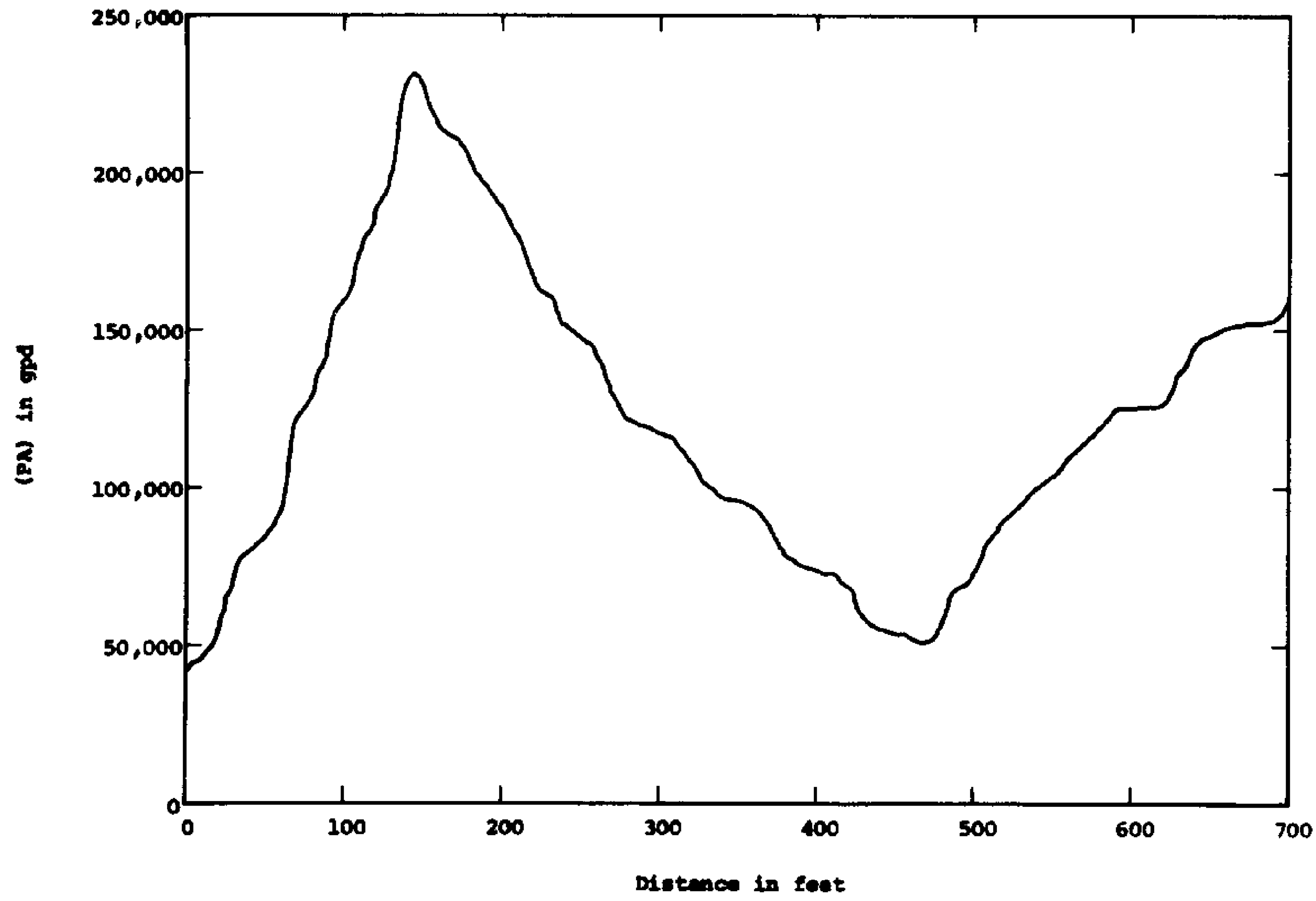

FIG. 1.—VALUES OF (PA) VERSUS DISTANCE

ILLUSTRATIVE EXAMPLE

A nonuniform confined groundwater conduit 700 ft long, of known permeability and cross-sectional area, is to be analyzed for discharge and piezometric-surface characteristics.

Determine the product (PA)[9] at various sections normal to the direction of flow and plot the resulting values with respect to distance as in Fig. 1. Divide the conduit into sub-lengths on the basis of the (PA)-graph. Segments of the (PA)-curve so divided may be treated as either straight lines or sections of parabolic curves of the general form $y = kx^2$. Straight lines represent uniform sections or truncated wedges, whereas parabolic curves represent truncated pyramids.[10]

It is difficult to recognize a parabolic curve representing a truncated pyramid directly from the (PA)-graph. Therefore plot values of $(PA)^{1/2}$ versus distance as shown by Fig. 2. Straight line segments on this graph represent truncated pyramids. Label sub-lengths so identified on the (PA)-graph (Fig. 3).

Some segments may remain undefined. Approximate these as a series of straight lines, designating them truncated wedges, as in Fig. 3.

Table 1 shows the steps in computing the discharge rate. Values for sub-length L_s, for $\sqrt{(PA)_1}$ and $\sqrt{(PA)_2}$, and for $(PA)_1$ and $(PA)_2$ are read directly from the subdivided graphs. The quantity $(PA)_e$ is

---

[9]If permeability varies in the section normal to the direction of flow, use the weighted value

$$(PA) = \sum_{1}^{n} P_n A_n$$

[10]In some cases, it is better to solve in terms of truncated wedges only, thereby simplifying the procedure with negligible loss in accuracy.

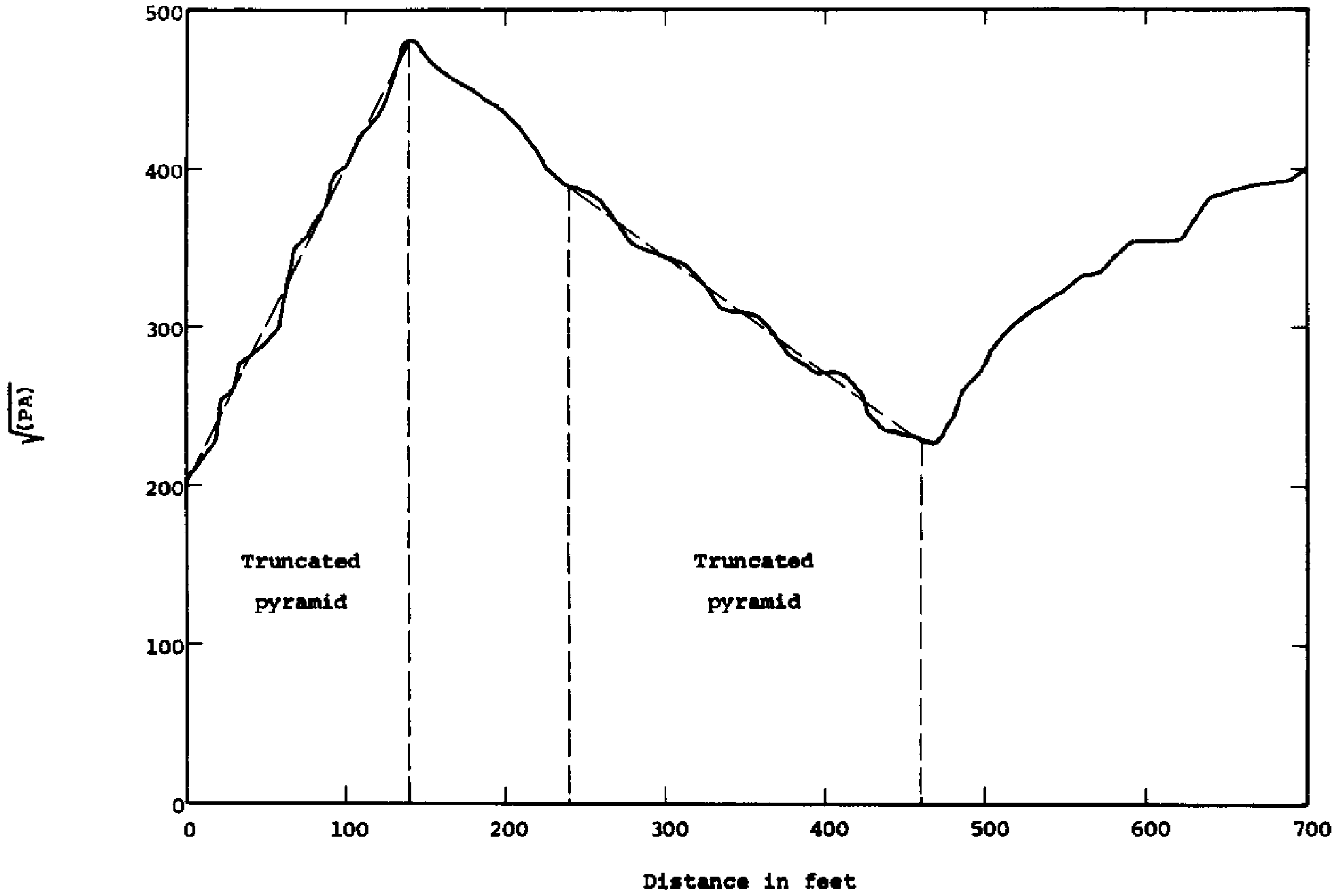

FIG. 2.—SUB-LENGTHS FOR TRUNCATED PYRAMIDS IDENTIFIED ON THE $\sqrt{(PA)}$ -GRAPH

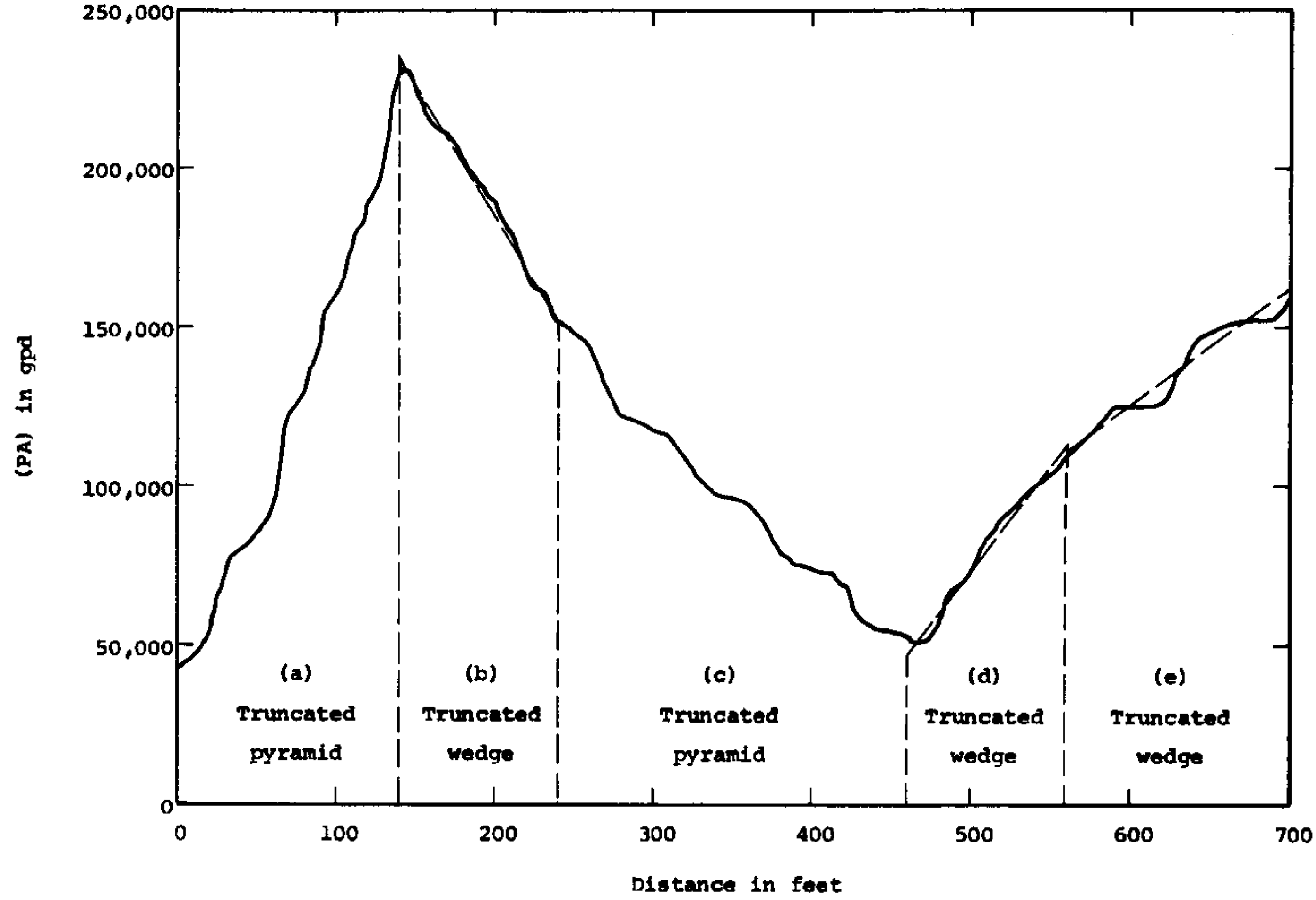

FIG. 3.—SUB-LENGTHS IDENTIFIED ON THE (PA)-GRAPH

TABLE 1.—COMPUTATIONS FOR NONUNIFORM GROUNDWATER-CONDUIT DISCHARGE

Sub-length	L_s feet	$\sqrt{(PA)_1}$	$\sqrt{(PA)_2}$	$(PA)_1$ [11] gpd	$(PA)_2$ [11] gpd	$\frac{(PA)_e}{(PA)_{ave}}$	$(PA)_e$ gpd	$\frac{L_s}{(PA)_e}$ ft/gpd
(a) Truncated pyramid	140	200	481	40,000	231,000	0.709[12]	96,000[12]	0.00146
(b) Truncated wedge	100			237,000	152,000	0.985	192,000	0.00052
(c) Truncated pyramid	220	389	229	151,000	52,000	0.874	89,000	0.00247
(d) Truncated wedge	100			47,000	113,000	0.942[12]	75,000[12]	0.00133
(e) Truncated wedge	140			111,000	162,000	0.989[12]	135,000[12]	0.00104
								0.00682

$$Q = \frac{H}{\sum \left(\frac{L_s}{(PA)_e} \right)} = \frac{H}{0.00682}$$

[11] In the curve fitting, it is not necessary that the end values of (PA) for adjoining sub-lengths match.

[12] $(PA)_1$ and $(PA)_2$ are interchanged in computation of $(PA)_e$.

determined either by Eq. 5 (with the aid of Fig. 4) or Eq. 6, or as follows.

Enter Fig. 5 with the value of the ratio $(PA)_1/(PA)_2$[13] and determine the quantity $(PA)_e/(PA)_{ave}$.[14] Then, since $(PA)_{ave}$ is known:

$$(PA)_{ave} = \frac{(PA)_1 + (PA)_2}{2} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (9)$$

the value for $(PA)_e$ is easily determined.

The discharge rate for the nonuniform conduit is

$$Q = \frac{H}{\sum \left(\frac{L_s}{(PA)_e} \right)} = \frac{H}{0.00682}$$

where H is the total head loss in the direction of flow for the entire 700-ft conduit. In this example, if H = 65.0 ft, then

$$Q = \frac{65.0}{0.00682} = 9530 \text{ gpd}$$

In addition, the head loss for any portion of the conduit can be computed by Eq. 8 in the form

$$H = Q \sum \left(\frac{L_s}{(PA)_e} \right)$$

The resulting values are used to determine the elevation of the piezometric surface, as shown in Table 2 and Fig. 6.

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[13] Interchange subscripts 1 and 2, if necessary, so that the value of the ratio $(PA)_1/(PA)_2$ is greater than 1.0.

[14] See Appendix I for the derivation of Fig. 5.

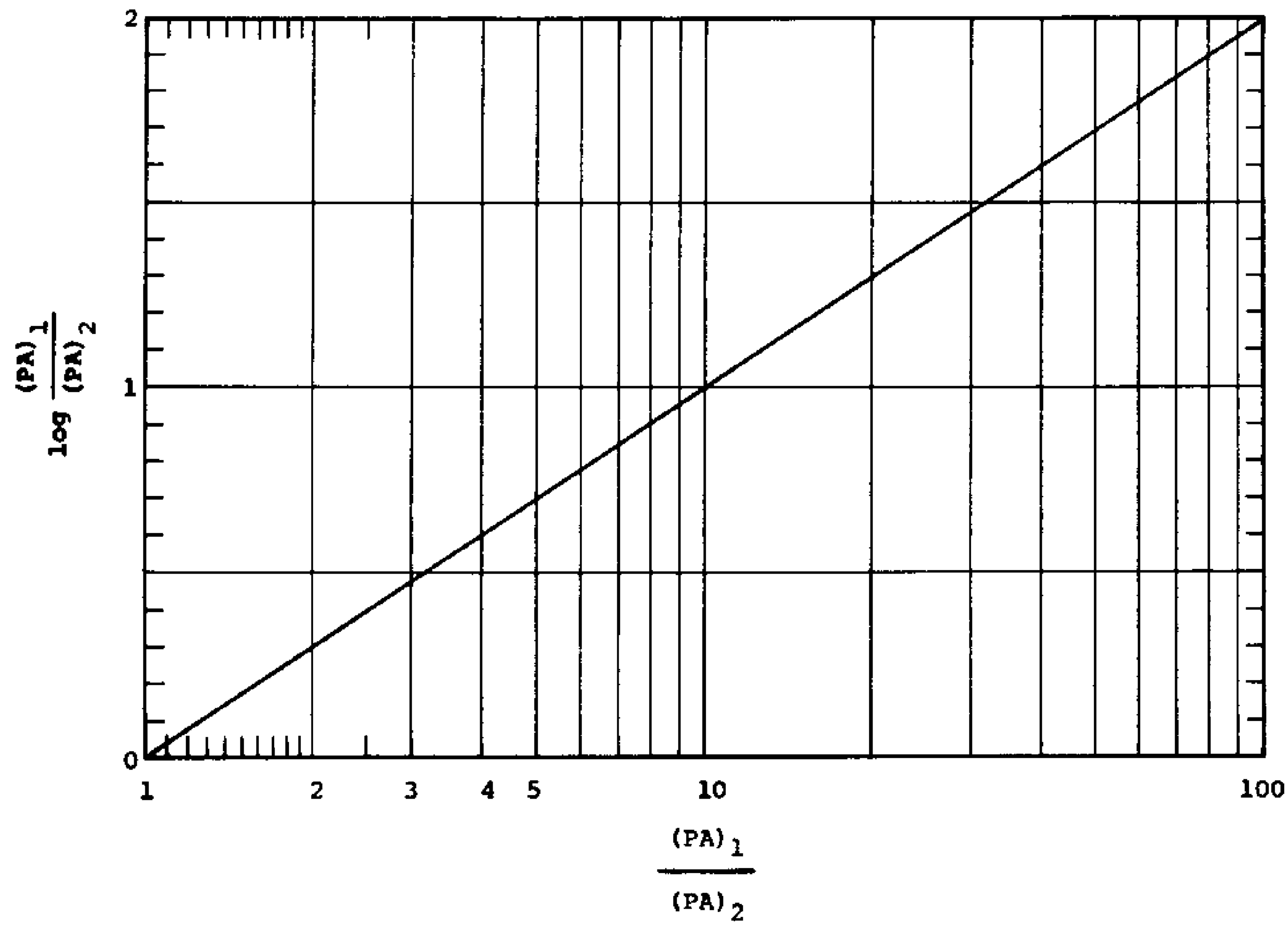

FIG. 4.—LOG VALUES OF $(PA)_1/(PA)_2$

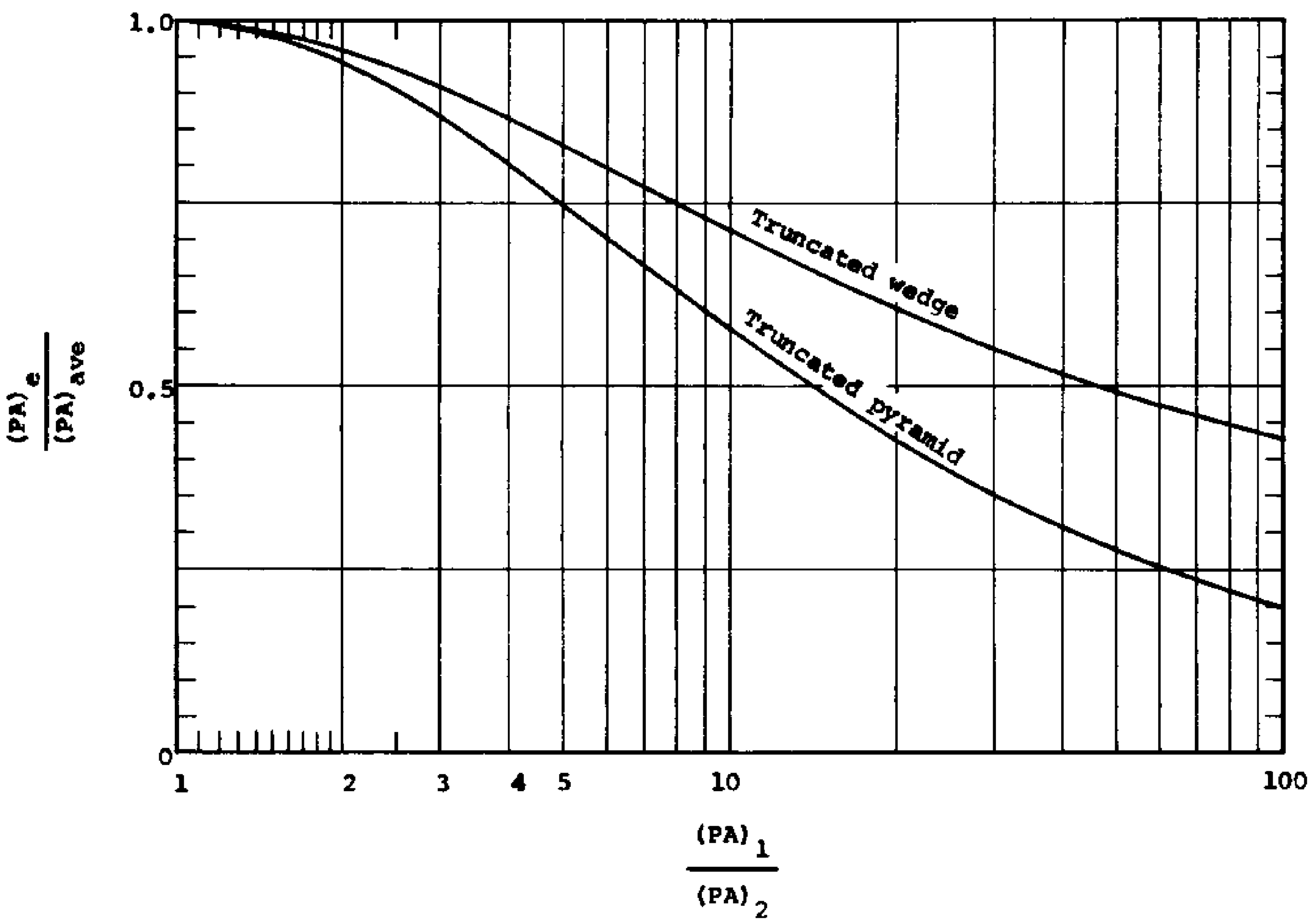

FIG. 5.—$(PA)_e/(PA)_{ave}$ VERSUS $(PA)_1/(PA)_2$

TABLE 2.—COMPUTATIONS FOR PIEZOMETRIC ELEVATION

Sub-length	L_s feet	L feet	$\sum\left(\frac{L_s}{(PA)_e}\right)$[15] ft/gpd	H feet	Piezometric elevation feet
		0	0	0	65.0
(a)	140				
		140	0.00146	13.9	51.1
(b)	100				
		240	0.00198	18.9	46.1
(c)	220				
		460	0.00445	42.4	22.6
(d)	100				
		560	0.00578	55.1	9.9
(e)	140				
		700	0.00682	65.0	0

EXPERIMENTAL ANALYSIS

A model simulating various types of nonuniform conduits was constructed at the University of Southern California Hydraulic Research Laboratory. The test results verified Eq. 5 and 6 for the truncated wedge and for the truncated pyramid. Further testing is anticipated in this area in the future.

CONCLUSIONS

The method described herein for computing the discharge rate and

[15] Data taken from last column of Table 1.

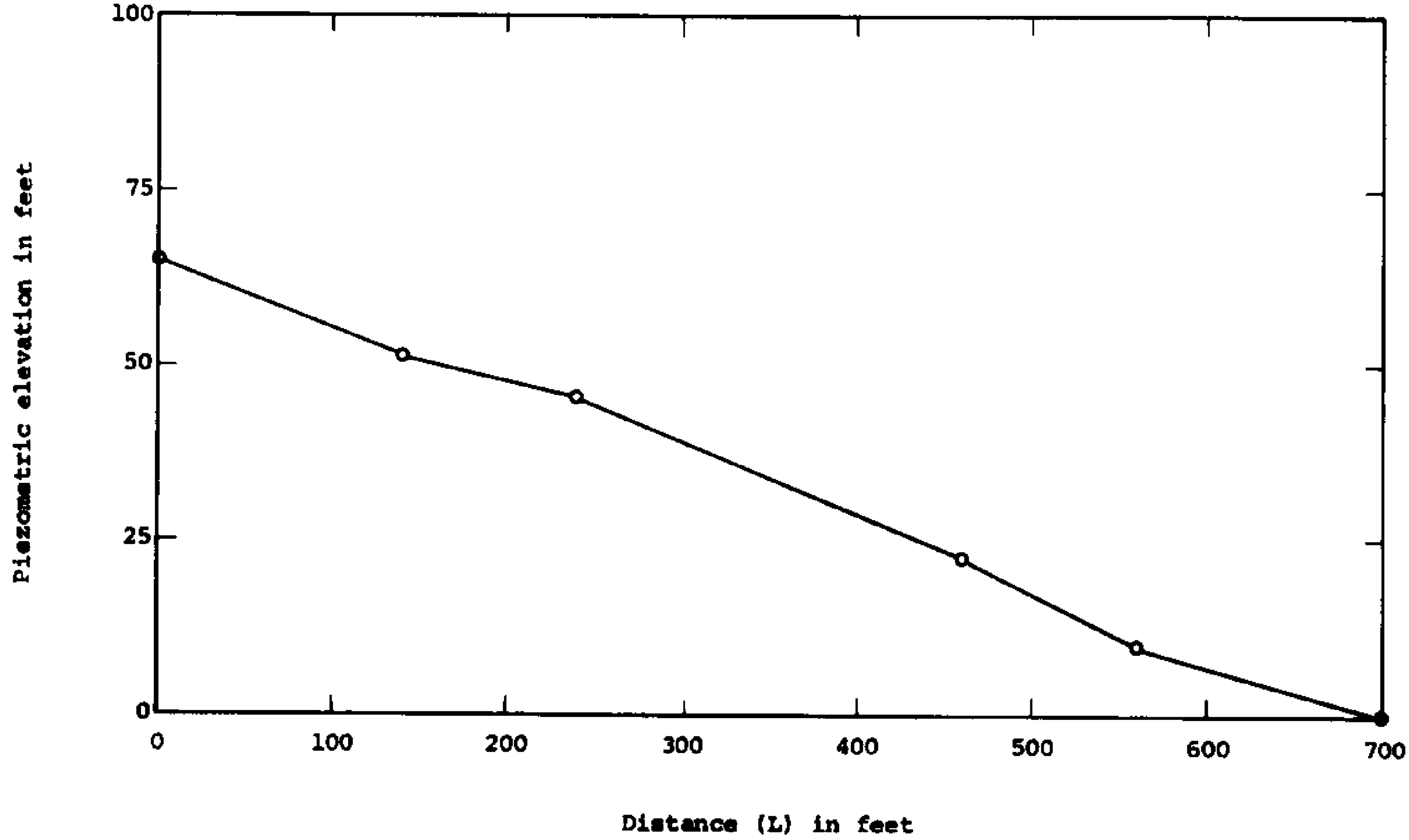

FIG. 6.—PIEZOMETRIC ELEVATION VERSUS DISTANCE[16]

[16]Any number of intermediate points can be determined by the method described herein.

piezometric water levels for a nonuniform confined groundwater conduit is relatively simple, and can be applied to any conduit configuration. Although designed as an approximate method, it is fairly accurate, provided reliable information is available to permit division of the conduit into sub-lengths on the basis of permeability and cross-sectional area.

APPENDIX I. —DERIVATION OF FIG. 5

For a truncated wedge,

$$(PA)_e = \frac{(PA)_1 - (PA)_2}{2.3 \log \dfrac{(PA)_1}{(PA)_2}}$$

Let $(PA)_1 = c(PA)_2$, c being greater than 1.0; then

$$(PA)_e = \frac{(PA)_2 \, (c-1)}{2.3 \log c}$$

Divide both sides of the equation by $(PA)_{ave}$.

$$\frac{(PA)_e}{(PA)_{ave}} = \frac{(PA)_2 \, (c-1)}{2.3 \log c \, (PA)_{ave}}$$

Since $(PA)_{ave} = \dfrac{(PA)_1 + (PA)_2}{2}$

$$\frac{(PA)_e}{(PA)_{ave}} = \frac{2(c-1)}{2.3 \log c \, (c+1)} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (10)$$

For a truncated pyramid,

$$(PA)_e = \sqrt{(PA)_1 \, (PA)_2}$$

Let $(PA)_1 = d(PA)_2$, d being equal to or greater than 1.0; then

$$(PA)_e = \sqrt{d} \;\; (PA)_2$$

Divide both sides of the equation by $(PA)_{ave}$.

$$\frac{(PA)_e}{(PA)_{ave}} = \sqrt{d} \;\; \frac{(PA)_2}{(PA)_{ave}}$$

$$\frac{(PA)_e}{(PA)_{ave}} = \frac{2\sqrt{d}}{d+1} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (11)$$

Following are tabulated values computed from Eq. 10 and 11:

Truncated wedge		Truncated pyramid	
$c = \frac{(PA)_1}{(PA)_2}$	$\frac{(PA)_e}{(PA)_{ave}}$	$d = \frac{(PA)_1}{(PA)_2}$	$\frac{(PA)_e}{(PA)_{ave}}$
1.0[17]	1.000	1.0	1.000
2.0	0.963	2.0	0.943
3.0	0.911	3.0	0.866
4.0	0.867	4.0	0.800
5.0	0.829	5.0	0.745
6.0	0.798	6.0	0.700
7.0	0.772	7.0	0.661
8.0	0.749	8.0	0.629
9.0	0.729	9.0	0.600
10.0	0.711	10.0	0.575
20.0	0.605	20.0	0.426
30.0	0.551	30.0	0.353
40.0	0.516	40.0	0.309
50.0	0.492	50.0	0.277
60.0	0.473	60.0	0.254
70.0	0.458	70.0	0.236
80.0	0.446	80.0	0.221
90.0	0.435	90.0	0.209
100.0	0.426	100.0	0.198

[17] As c approaches 1.0, $(PA)_e/(PA)_{ave}$ approaches the limit 1.000.

APPENDIX II. —REFERENCES

1. De Wiest, R. J. M., Geohydrology, pp. 178-179, Wiley, 1965.
2. Butler, S. S., Engineering Hydrology, pp. 78-79, Prentice-Hall, 1957.
3. Harr, M. E., Groundwater and Seepage, p. 27, McGraw-Hill, 1962.

APPENDIX III. —NOTATION

The following symbols have been adopted for use in this paper:

A = cross-sectional area of porous media (such as sand) normal to the direction of flow, in square feet;

A_n = subdivision of cross-sectional area, in square feet;

c = dimensionless constant greater than 1.0;

d = dimensionless constant equal to or greater than 1.0;

gpd = gallons per day;

H = head loss, in feet;

I = hydraulic gradient, equal to head loss per unit length in the direction of flow;

k = dimensionless constant;

L = length, in feet;

L_s = sub-length, in feet;

log = logarithm to the base 10;

n = number of subdivisions of the cross-sectional area;

P = coefficient of permeability, in gallons per day per square foot;

P_n = coefficient of permeability for subdivision of cross-sectional area, in gallons per day per square foot;

PA = product of the coefficient of permeability and of the cross-sectional area, in gallons per day;

$(PA)_e$ = equivalent uniform value, in gallons per day;

Q = rate of discharge, in gallons per day;

x = independent variable, equivalent to distance;

y = dependent variable, equivalent to the quantity (PA).

APPENDIX IV.—GLOSSARY

Aquifer: A body of earth material capable of supplying water at a rate sufficient for economic extraction by wells.

Colloidal material: Soil particles less than two microns in diameter.

Confined groundwater: Groundwater confined by overlying, relatively impermeable material.

Groundwater: Subsurface water occupying the zone of saturation.

Hydraulic gradient: Slope representing the rate of change in head per unit distance in the direction of flow.

Laminar flow: Streamline fluid flow in which successive flow particles follow similar path lines and head loss varies with velocity to the first power.

Nonuniform: Characteristic whereby permeability and cross-sectional area vary with respect to position in the direction of flow.

Permeability: The ability of a material to transmit fluid through its pores when subjected to a difference in head.

Piezometric surface: For a confined aquifer, the imaginary surface representing the height to which water would rise in observation wells penetrating the body of groundwater at various points; that is, the surface representing the sum of the pressure head plus elevation head with respect to the body of water.

Truncated-pyramid conduit: Groundwater conduit in which the product (PA) varies with the square of the distance. Examples: (1) P is uniform but A varies; (2) A is uniform but P varies; (3) both P and A vary.

Truncated-wedge conduit: Groundwater conduit in which the product (PA) varies linearly with distance.

Turbulent flow: Fluid flow in which successive flow particles follow independent path lines and head loss varies approximately with velocity to the second power.

Uniform: Characteristic whereby permeability and cross-sectional area remain constant with respect to position in the direction of flow.