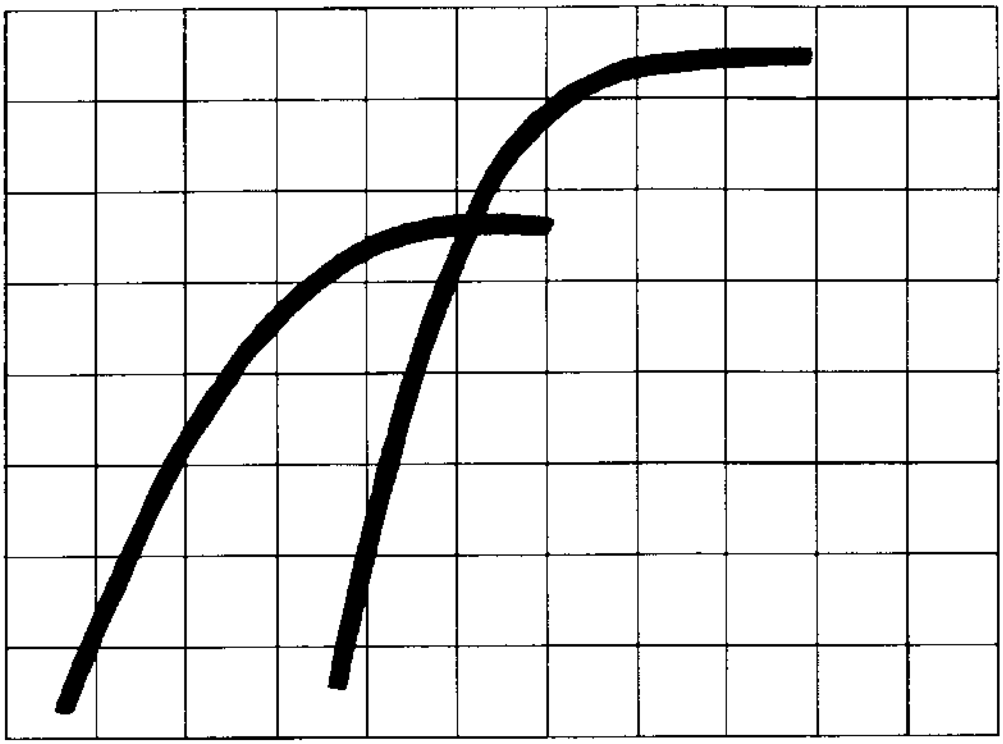

Application of a Seasonal Harvesting Model to Two North Carolina Shrimp Fisheries

Robert Kellogg, J.E. Easley Jr. and Thomas Johnson

UNC Sea Grant Publication UNC-SG-86-03

APPLICATION OF A SEASONAL HARVESTING MODEL TO TWO NORTH CAROLINA SHRIMP FISHERIES

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This work was sponsored by the Office of Sea Grant, NOAA, U.S. Department of Commerce, under Grant No. NA83AA-D-00012 and the North Carolina Department of Administration. The U.S. Government is authorized to produce and distribute reprints for governmental purposes notwithstanding any copyright that may appear hereon.

UNC Sea Grant College Publication UNC-SG-86-03

March 1986

TABLE OF CONTENTS

INTRODUCTION

The timing of the opening for a seasonal fishery such as the inshore shrimp fishery can potentially influence the total revenue obtainable from the fishery. The traditional approach in North Carolina has been to open the shrimp fishery when the shrimp have reached a marketable size. However, shrimp increase in value as they grow, both from weight gain and higher market prices for the larger size groups. Thus it is possible that a later season opening may increase total revenue.

The purpose of this study is to determine the <u>optimal</u> time to open the harvest season for two North Carolina shrimp fisheries--the New River fishery and the Pamlico Sound fishery (exclusive of tributaries and bays). The general seasonal harvesting model developed by Kellogg (1985) is adapted for use in the analysis. Optimal solutions were generated for several different scenarios by varying natural mortality, initial population size and price. Solutions were then compared to the "unregulated" case to determine if the season opening should be delayed beyond the time when shrimp reach marketable size.

OPTIMAL TIMING OF HARVEST FOR THE NEW RIVER SHRIMP FISHERY

Brief Description of the New River Shrimp Fishery

The New River estuary is located in Onslow County, North Carolina. Whereas it is an important fishery locally, the New River share of the total value of the shrimp catch in North Carolina has generally been below 5 percent. The estuary is unique in that most of the tidal portion of the river basin is surrounded by Camp LeJeune, a military base. As a result, there has been little or no commercial development within the estuary.

Three species of shrimp are harvested commercially in the New River: brown shrimp (<u>Penaeus aztecus</u>), pink shrimp (<u>Penaeus duorarum</u>), and white shrimp (<u>Penaeus setiferus</u>). Brown shrimp is by far the most important species, typically constituting 70 to 90 percent of the annual shrimp catch (Table 1). Brown shrimp are harvested from about mid-June through September. White shrimp represent only a very small proportion of the total catch (0.4-2.8 percent), even in a year that is favorable for the growth of the species (such as 1980). White shrimp (when present) are harvested late in the year, from September through November. Pink shrimp are harvested during two periods. Overwintering adults ("early" pink shrimp) are harvested in the spring from May until the end of June, after which time most of the population has migrated out of the estuary. A second pink shrimp harvest ("late" pink shrimp), with young shrimp that migrated into the estuary in the early spring, occurs from mid-August until November. The significance of pink shrimp in the commercial catch varies from about 10 to 30 percent.

Table 1. Summary of annual catch statistics for the New River shrimp fishery. Data from the Division of Marine Fisheries.

Year	Harvest period (month/day)	Number of weeks in harvest period	Annual catch (pounds heads-off)	% of annual catch	Total number[a] of shrimp harvested
			Early Pink Shrimp		
1979	3/29-7/6	15	24735	8.8	1,502,362
1980	5/1-6/20	8	16732	5.8	885,031
1981	5/22-6/30	7	6951	19.6	373,012
1982	5/1-7/9	10	10485	4.6	641,197
			Brown Shrimp		
1979	6/29-10/29	21	242083	86.6	11,753,993
1980	6/5-10/17	21	256383	89.1	14,398,666
1981	6/30-10/1	14	25472	71.8	985,906
1982	6/11-10/9	19	184700	80.8	9,144,577
			Late Pink Shrimp		
1979	8/14-12/19	19	10706	3.8	779,964
1980	9/5-11/17	11	6543	2.3	465,555
1981	8/14-11/6	13	2538	7.2	167,195
1982	8/14-11/19	15	32660	14.3	2,049,315
			White Shrimp		
1979	10/19-12/16	7	1973	0.7	159,285
1980	8/15-11/25	16	8110	2.8	332,511
1981	9/15-10/1	3	512	1.4	22,016
1982	8/28	1	840	0.4	36,120

[a]Calculated by multiplying the catch in pounds times the median of the size range reported in units of number per pound and then summed over all records.

The life histories of the three species are similar. Eggs are spawned and hatch in the ocean. The young migrate to estuarine nursery areas where they grow rapidly into juveniles and subadults. They then return to the ocean where they become sexually mature, spawn, and die.

The commercial catch statistics reported by the Division of Marine Fisheries (see Kellogg 1985 for a compilation of this dataset) identifies three geartypes for the New River: 1) vessel (craft weighing 5 tons or more and registered as a merchant vessel of the United States) using shrimp trawls, 2) boat (any craft not identified as a vessel) using shrimp trawls, and 3) boat using channel nets. Channel nets are a passive form of collection gear, capturing shrimp that are migrating toward the ocean. Approximately 55 to 65 percent of the catch is taken by boats using shrimp trawls, 20 to 35 percent is taken by vessels, and 10 to 20 percent is taken by channel nets. For the 1979-1982 period, the yearly average catch per effort (i.e., average catch per daily fishing trip) was 30 to 120 pounds per boat for shrimp trawls, 80 to 1000 pounds per vessel, and 100 to 300 pounds per boat for channel nets.

A prominent characteristic of the shrimp market is the dependence of price on shrimp size. Larger shrimp bring a higher price than smaller shrimp (Table 2). (Exceptions occur in Table 2, but probably result from averaging over different time periods within the year.) In 1980, for example, New River brown shrimp in the 26 to 30 per-pound category (heads off) sold for 4.35 dollars per pound (ex-vessel price). However, brown shrimp in the 61 to 70 per-pound category sold for only 1.89 dollars, a price difference of nearly 2.50 dollars per pound. This increase in price with increasing shrimp size and the rapid growth rate exhibited by shrimp during the potential harvest season are two reasons why the timing of harvest is important.

The North Carolina Department of Natural Resources, Division of Marine Fisheries (DMF) is responsible for the promulgation of rules and regulations governing the harvest of shrimp in North Carolina. The principal regulations involve opening and closing the harvest season in secondary nursery areas and designation of primary nursery areas. Primary nursery areas--such as small creeks and bays--are defined by regulation and are closed at all times. Secondary nursery areas are opened and closed by proclamation. The DMF traditionally has opened the season when shrimp were large enough to have commercial value.

The overlap between the brown shrimp harvest season and the pink shrimp harvest seasons results in a potential discard problem. Small brown shrimp are sometimes collected as bycatch during the harvest for early pink shrimp, and small late pink shrimp are often collected as bycatch during the harvest for brown shrimp. When these shrimp are too small to have any commercial value, they are discarded, and are thus lost to the fishery (see Waters et al. 1980).

A description of the economic characteristics of the South Atlantic shrimp fishery is presented in South Atlantic Fishery Management Council (1981), including the ex-vessel market, the domestic wholesale market, the export market and a description of businesses, markets and organizations associated with the shrimp fishery.

Table 2. Prices and annual catch statistics for New River shrimp by species and size class, all geartypes combined. Data from the North Carolina Division of Marine Fisheries. Weight is in pounds (heads off), and price is in dollars per pound.

		Species							
		Early Pink		Brown		Late Pink		White	
Year	Shrimp per pound (heads off)	Weight	Avg. price	Weight	Avg. price	Weight	Avg. price	Weight	Avg. price
1979	26-30	--	--	--	--	--	--	--	--
	31-36	--	--	1152	4.37	--	--	--	--
	36-40	--	--	5041	3.98	--	--	--	--
	41-45	46	2.65	15069	3.46	--	--	--	--
	46-50	--	--	192087	2.71	--	--	--	--
	51-55	6197	2.75	12959	2.90	2557	2.76	--	--
	56-60	7891	2.96	9418	2.95	1426	2.59	--	--
	61-70	9341	2.75	5857	2.65	486	2.16	421	2.60
	>70	1260	2.41	500	2.19	6237	2.53	1552	2.17
	Total	24735	2.80	242083	2.81	10706	2.58	1973	2.26
1980	26-30	--	--	878	4.35	--	--	--	--
	31-36	487	3.28	--	--	--	--	2925	3.09
	36-40	1186	3.11	14552	3.31	--	--	1628	3.01
	41-45	189	3.15	19745	3.18	--	--	1440	2.94
	46-50	4145	2.76	63353	2.92	376	2.65	1523	2.86
	51-55	1750	2.61	28451	2.80	1354	2.59	71	2.90
	56-60	8780	2.48	54108	2.07	--	--	--	--
	61-70	88	2.40	55760	1.89	1668	1.86	456	1.75
	>70	107	2.40	19536	1.90	3145	1.64	67	1.75
	Total	16732	2.64	256383	2.47	6543	1.95	8110	2.92
1981	26-30	--	--	1920	3.71	--	--	--	--
	31-36	38	4.00	5475	3.52	--	--	--	--
	36-40	--	--	7626	3.39	--	--	--	--
	41-45	1728	3.68	8573	3.42	--	--	512	3.32
	46-50	--	--	1298	3.35	--	--	--	--
	51-55	1855	2.99	580	4.00	--	--	--	--
	56-60	2473	2.55	--	--	225	2.60	--	--
	61-70	857	2.36	--	--	2123	3.03	--	--
	>70	--	--	--	--	190	2.50	--	--
	Total	6951	2.93	25472	3.46	2538	2.95	512	3.32

Table 2. (continued)

Year	Shrimp per pound (heads off)	Early Pink		Brown		Late Pink		White	
		Weight	Avg. price	Weight	Avg. price	Weight	Avg. price	Weight	Avg. price
1982	26–30	--	--	--	--	--	--	--	--
	31–36	--	--	3249	3.82	--	--	--	--
	36–40	215	5.00	3765	3.71	--	--	--	--
	41–45	881	4.64	51603	3.56	--	--	840	3.50
	46–50	346	4.81	48168	3.44	--	--	--	--
	51–55	803	4.00	43717	3.31	10218	3.45	--	--
	56–60	1289	3.51	28422	3.27	7087	3.30	--	--
	61–70	6481	3.32	4657	3.21	10423	3.25	--	--
	>70	470	3.24	1119	3.22	4932	3.12	--	--
	Total	10485	3.59	184700	3.42	32660	3.30	840	3.50

The Model

The general seasonal harvesting model presented in Kellogg (1985) was adapted for application to the New River shrimp fishery. The problem contains three state variables, one each for the population dynamics of brown shrimp and the two year classes of pink shrimp. White shrimp were excluded because they constitute a negligible portion of the fishery.

At the time of this study, the channel net fishery was unregulated, and thus was not subject to the opening/closing schedule. (Channel nets were included under proclamation authority in October, 1985.) Moreover, channel netters use passive gear and collect only shrimp that are migrating to the open ocean. The channel net catch would be modelled as natural mortality whether or not the shrimp were captured by channel netters or migrated to the ocean. (For the present study, no distinction is made between emigration and mortality.) Consequently, the activities of the channel net fishermen have no bearing on the optimal timing of harvest, and are excluded from the analysis.

The New River shrimp harvesting problem can be formally stated as follows:

maximize with respect to $\Phi(t)$

$$PV = \int_0^T \{[P(g_1,t,w)g_1(z,t)x_1(t)+P(g_2,t,w)g_2(z,t)x_2(t) + P(g_3,t,w)g_3(z,t)x_3(t)]qE(R,t) - c_1E(R,t)\}e^{-\delta t}\,\Phi(t)\,dt$$

$$\dot{x}_1 = F_1(z,t)x_1(s_1) - M_1(x_1,z,t) - qE(R,t)x_1(t)\,\Phi(t)$$

$$x_1(s_1) \text{ given and } x_1(t) \geq 0,$$

$$\dot{x}_2 = F_2(z,t)x_2(s_2) - M_2(x_2,z,t) - qE(R,t)x_2(t)\,\Phi(t)$$

$$x_2(s_2) \text{ given and } x_2(t) \geq 0,$$

$$\dot{x}_3 = F_3(z,t)x_3(s_3) - M_3(x_3,z,t) - qE(R,t)x_3(t)\,\Phi(t)$$

$$x_3(s_3) \text{ given and } x_3(t) \geq 0,$$

where

$P(g_i,t,w)$ = the market price equation in dollars per pound,

$g_i(z,t)$ = the shrimp size equation for species i in pounds per shrimp,

w = vector of exogenous demand variables,

z = vector of environmental variables,

$x_i(t)$ = population size in numbers for species i,

$E(R,t)$ = the fishing effort equation,

R = the return per standard unit of fishing effort,

q = the catchability coefficient for the standard unit of fishing effort,

c_1 = cost per unit of effort,

$F_i(z,t)$ = the recruitment function for species i,

$M_i(x_i,z,t)$ = the natural mortality function (including emigration) for species i,

i=1 ==> brown shrimp,

i=2 ==> early pink shrimp,

i=3 ==> late pink shrimp,

t = time in units of weeks starting from April 1, and

s_i = the time when species/cohort i first becomes vulnerable to capture.

$\Phi(t)$ = the decision variable ($\Phi(t)=0$ implies a closed season and $\Phi(t)=1$ implies an open season).

The purpose of this model is to determine the season opening/closing schedule, $\Phi(t)$, that maximizes the discounted present value of net returns to the harvesting sector. The potential harvest season spans April 1 (t=0) to December 31 (t=T). The weekly discount rate, δ, was set equal to 0.001827 for this study, which is equivalent to an annual discount rate of 10 percent. This is a real rate, which is required because all prices and costs are in units of uninflated dollars (1967 dollars).

Note that the market price equation, $P(g_i,t,w)$, is not a function of the quantity of New River shrimp harvested. Waters (1983) demonstrated that the shrimp industry in North Carolina is a "price-taker." Consequently, ex-vessel price is determined on the basis of national supply and demand. However, since ex-vessel price depends on the size of the shrimp harvested, the regulating agency can influence the price per pound by adjusting the timing of harvest. And since effort depends on price (through R, the return per unit of effort), the regulatory agency indirectly influences fishing effort as well. Because other factors are also involved, however, the regulatory agency cannot control prices and effort, only influence them.

Components of the Model

Market Price Equation

The market price (ex-vessel) for New River shrimp was modelled as a linear function of shrimp size and the national shrimp price, as follows:

$$P_{New\ River} = a_0 + a_1(Size) + a_2(Size)^2 + a_3(P_{national}).$$

The ex-vessel demand for shrimp harvested in North Carolina has been shown to be perfectly elastic (Waters 1983)--that is, North Carolina prices are not affected by the quantity of shrimp harvested in North Carolina. Consequently, the predominant determinant of the local shrimp price is the national price. The measure of the national shrimp price used in this study was the shrimp price index (PSHRIMP) reported by the Department of Commerce in Current Fishery Statistics. This monthly index is calculated as follows:

$$\text{Index} = \frac{\text{Current price times 1967 quantity}}{\text{1967 average monthly value}}.$$

It represents a relative price of shrimp. For example, if the index is 185.0, the price of shrimp that sold for 1.00 dollars per pound in 1967 has increased to 1.85 dollars per pound. Since the index includes price changes resulting from general price inflation, the index was divided by the consumer price index to convert it to constant 1967 dollars.

As noted previously, shrimp size is also an important determinant of the market price. Shrimp size was included in the model as a quadratic to account for any non-linearity that might exist. Size was measured in units of number per pound, heads off. Thus, the larger the shrimp, the smaller the ''size,'' and so the sign of the derivative of price with respect to size is expected to be negative (that is, $a_1+2a_2(Size)<0$). Shrimp size for the New River was reported by the Division of Marine Fisheries in units of number per pound, heads off, for nine size classes. For estimation purposes, the midpoint of the range for each size class was used as the size estimate.

Weekly values for catch in weight and total revenue were obtained from the Division of Marine Fisheries as part of the weekly catch statistics collected on shrimp. Weekly ex-vessel price was determined as the ratio of revenue to total weight (pounds, heads-off), and thus represents a weighted average price. Ex-vessel price was calculated separately for each size class, but data for different species of the same size were combined. These prices were then adjusted by dividing them by the appropriate monthly consumer price index so that all prices were in constant 1967 dollars.

The weekly ex-vessel shrimp prices for the New River and monthly values of the shrimp price index were used to estimate the above model. The model had an R^2 of 0.557 (Table 3), and all independent variables were highly

Table 3. Parameter estimates and statistics associated with the market price prediction equation for New River shrimp[a].

$$P_{New\ River} = a_0 + a_1(Size) + a_2(Size)^2 + a_3(PSERIMP).$$

Source	d.f.	Sum of Squares	Mean Square
Model	3	9.85085608	3.28361869
Error	345	7.83400476	0.02270726
Corrected total	348	17.68486084	

Model F = 144.61   Pr > F = 0.0001 R^2 = 0.557022

Parameter	Estimate	t for H_0: Parameter=0	Pr > |t|	Std Error of Estimate
a_0	1.55598933	14.26	0.0001	0.10908382
a_1	-0.02502662	-6.64	0.0001	0.00376797
a_2	0.00012549	4.04	0.0001	0.00003106
a_3	0.34175075	12.13	0.0001	0.02816784

[a] Size is measured in number of shrimp per pound. Thus a larger size measure corresponds to smaller shrimp.

significant. In addition, the partial derivative with respect to the size variable was negative, as expected.

In applications, values for the shrimp price index would need to be forecasted before the equation could be used to predict prices for New River shrimp. Several investigators have experimented with models of the shrimp fishery on a national scale. The most successful models were simultaneous equation models including a supply equation (Hopkins et al. 1983; Thompson and Roberts 1983; Blomo et al. 1982). Imports, inventories and supply were important variables (or equations) in these studies. Studies incorporating the shrimp price index are not currently available from the published literature.

For solving the optimization problem in the present study, the average values of PSHRIMP for 1979 and 1982 (2.10 and 1.66 in 1967 dollars, respectively) were used. The relationship between market price and shrimp size with PSHRIMP equal to the 1982 average is illustrated in Figure 1. In addition, price was set equal to zero if size was smaller than 85 per pound.

The final input required is shrimp size in units of number per pound, heads off. A size prediction equation for each species is developed in the next subsection. The market price equations, $P(g_i,w,t)$, were obtained by incorporating the results of the size prediction equations into the above equation.

Shrimp Size Equations

The model used to predict shrimp size is similar to the model used by Kellogg and Spitsbergen (1983) to predict the size of bay scallop meats. The bay scallop model was a modified Brody-Bertalanffy growth model while the model used here is the logistic growth model with a temperature dependent growth coefficient. The logistic model can be written as:

$$S_t^{-1} = S_\infty^{-1}(1-e^{-kt}) + S_0^{-1}e^{-kt},$$

where S_t is size at time t, S_∞ is the maximum size, S_0 is the size at the beginning of the time period, and k is a growth coefficient. A temperature dependent growth coefficient was incorporated into the model. Because shrimp are cold-blooded, temperature is the dominant factor affecting growth. When the temperature changes, the growth rate changes as well. Thus, kt was replaced by the following:

$$B(C,t) = b_1t + b_2C,$$

where C is cumulative water temperature in degree-weeks (Centigrade degrees) from April 1. The resulting model for shrimp size is as follows:

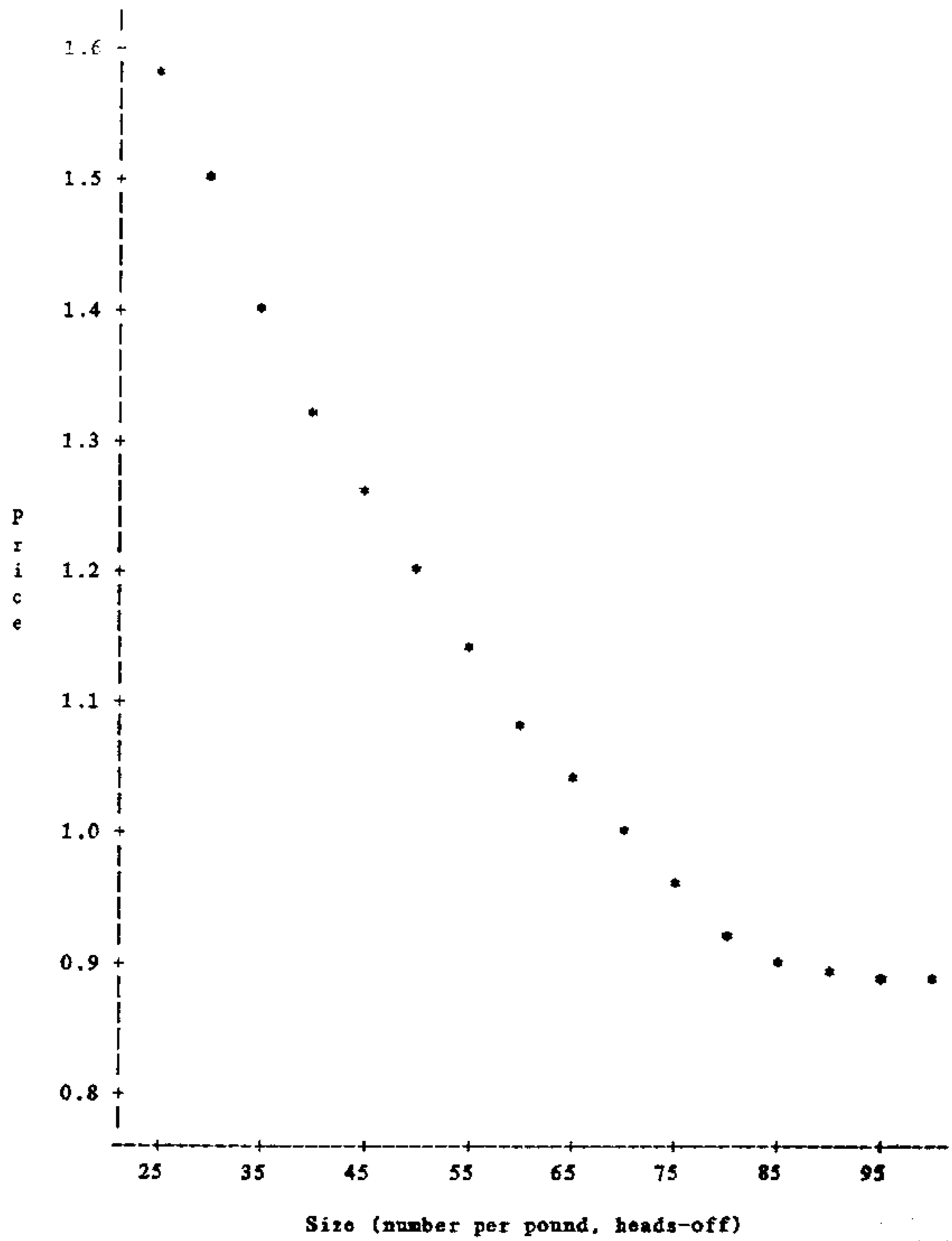

Figure 1. Relationship between market ex-vessel price (1967 dollars) per pound and shrimp size for the New River shrimp fishery with PSHRIMP equal to 1.66 dollars.

$$S_t^{-1} = S_\infty^{-1}(1-e^{-B(C,t)}) + S_0^{-1}e^{-B(C,t)},$$

where S_t = size in pounds per shrimp at time t,

S_∞ = the average "maximum" size attainable before migration to the ocean,

S_0 = size at t=0,

t = time in weeks from April 1, and

C = cumulative water temperature in degree-weeks (Centigrade degrees) from April 1.

The reciprocal of S_t, which is in units of number per pound, was used to estimate the model. This is the basic unit of measure used in the shrimp industry. (It is also the unit of measure required by the price prediction equation developed in the last subsection.) Since the size at the beginning of the time period is determined by the mesh size and so is known, S_0^{-1} was treated as a variable and assigned a value corresponding to the smallest size class vulnerable to harvest. A reasonable value for S_0^{-1} is 85 shrimp per pound (heads off). The final model that was estimated is thus:

$$S_t' = S_\infty'[1-e^{-(b_1t+b_2C)}] + 85e^{-(b_1t+b_2C)},$$

where $S_t'=1/S_t$, and b_1, b_2 and S_∞' are parameters.

With this model, shrimp growth is a function of the size of shrimp at time t and the temperature at time t. This is shown explicitly below by taking the deriviative of S_t' with respect to time and simplifying (T(t) is water temperature as a function of time):

$$d(S_t')/dt = [b_1t+b_2T(t)][S_t'-(S_t'^2/S_\infty')].$$

The commercial catch dataset reported by the Division of Marine Fisheries was used to estimate this model. Size was reported according to nine size classes for shrimp large enough to have commercial value; data on discarded shrimp were not available. Development of a predictive equation using this data will result in an equation for <u>expected shrimp size</u>, and does not constitute a biological growth model. This is because of the continual migration of shrimp into and out of the harvest area and because of the link between migration and shrimp size. Small shrimp stay in the creeks and bays until they reach a certain size (or stage of maturity) and then enter the harvest area. This recruitment does not take place instantly, but rather is protracted over several weeks. Shrimp begin to migrate toward the ocean as they mature, removing the larger shrimp from the sampled population. Additionally, fishermen discard shrimp too small for commercial value (less than about 90-95 mm); thus, the smaller shrimp in the distribution are not sampled. Consequently, size data from commercial catch statistics represent--roughly--the size of shrimp that a commercial fisherman might encounter at a particular time. (Laney and Copeland (1981) discuss how concurrent processes of recruitment and emigration bias shrimp growth models that are estimated using field data.)

The size measure used for estimation (number per pound, heads off) was determined as the midpoint of the range defining each size class (see Kellogg 1985). The smallest size class was set equal to 85 per pound because most shrimp that measure more than about 100 per pound (90-95 mm) and smaller are discarded by the commercial fisherman. With 100 as the upper limit, 85 is the midpoint of the range (70 to 100) for the smallest size class. Weekly averages were calculated by weighting each observation by the proportion of the catch that it represented. Size data for channel netters were excluded because they fish exclusively near the mouth of the estuary using passive gear and are more likely to capture larger shrimp that are migrating to the ocean (Richard Carpenter, Division of Marine Fisheries, personal communication). The resulting dataset (in units of number per pound) is presented in Table 4. Cumulative water temperature in degree-weeks from April 1 was calculated for each week using the cumulative temperature predictive equation developed by Kellogg (1985).

A non-linear least squares procedure (Marquardt iteration) was used to fit the above model. The R^2 for brown shrimp was 0.972 for the complete model and 0.519 after subtracting the contribution of the mean (Table 5). All three parameters were (asymptotically) statistically significant. The model and parameters presented in Table 5 were used in subsequent analyses to predict the size of brown shrimp. (Because of the very high correlation (>0.99) between the b_i's, the parameter values are intended only for predictive purposes and no interpretation of individual coefficients is attempted.) The average maximum size attainable before migration to the ocean (S_∞') was estimated to be 44.8 per pound. This value corresponds closely to the "equilibrium" modal length of 125 mm reported by McCoy (1972) for brown shrimp in the New River during the period of migration.

For early pink shrimp, the non-linear least squares fit resulted in a R^2 of 0.990 for the complete model and 0.546 after subtracting the contribution of the mean (Table 6). S_∞ was statistically significant at α=0.05, whereas the b_i's were borderline significant at α=0.10. This model was used in subsequent analyses to predict the size of early pink shrimp. (This equation performs well for t<16, which covers the period when early shrimp are present, but it predicts continued growth at a slow rate for t>16 because of the warm water temperature in the summer. Consequently, the estimate of S_∞ does not represent the average maximum size prior to migration, as it did for brown shrimp.)

The modified logistic model did not explain a significant portion of the variability in the data for late pink shrimp; all of the parameters were non-significant (Table 7). The failure of the model for this group was probably due to continual recruitment of small shrimp, especially in 1982 (1982 was the best of the four years for late pink shrimp catches). This had the effect of negating any trend of progressively increasing shrimp size.

Table 4. Weekly average size (number per pound, heads-off) of shrimp collected from the New River by commercial fishermen. Data for channel netters were excluded. (Week=0 is April 1.)

	Brown shrimp				Early pink shrimp				Late pink shrimp			
Week	1979	1980	1981	1982	1979	1980	1981	1982	1979	1980	1981	1982
0	--	--	--	--	85.0	--	--	--	--	--	--	--
1	--	--	--	--	85.0	--	--	--	--	--	--	--
2	--	--	--	--	67.6	--	--	--	--	--	--	--
3	--	--	--	--	63.7	--	--	--	--	--	--	--
4	--	--	--	--	58.4	--	--	--	--	--	--	--
5	--	--	--	--	--	58.0	--	--	--	--	--	--
6	--	--	--	--	65.0	--	--	60.8	--	--	--	--
7	--	--	--	--	64.5	55.4	59.1	65.0	--	--	--	--
8	--	--	--	--	54.9	60.0	54.0	--	--	--	--	--
9	--	--	--	--	65.0	58.0	59.9	65.0	--	--	--	--
10	--	85.0	--	--	58.0	58.0	56.6	67.3	--	--	--	--
11	--	85.0	--	85.0	58.0	55.5	--	65.0	--	--	--	--
12	--	85.0	--	85.0	--	40.2	--	63.7	--	--	--	--
13	52.2	--	53.0	65.0	58.0	--	43.0	62.9	--	--	--	--
14	50.8	64.2	42.1	--	58.0	--	--	--	--	--	--	--
15	48.2	70.0	37.8	55.1	--	--	--	43.0	--	--	--	--
16	48.0	62.3	--	47.0	--	--	--	--	--	--	--	--
17	50.1	58.9	43.0	49.8	--	--	--	--	--	--	--	--
18	48.0	--	38.0	53.9	--	--	--	--	--	--	--	--
19	53.0	46.9	33.6	50.9	--	--	--	--	--	--	65.0	--
20	54.9	50.3	--	50.2	--	--	--	--	85.0	--	--	53.0
21	50.9	46.7	--	54.3	--	--	--	--	--	--	--	--
22	45.4	--	34.5	53.9	--	--	--	--	--	--	65.0	58.0
23	--	44.1	--	--	--	--	--	--	--	85.0	--	--
24	42.5	40.5	46.7	40.3	--	--	--	--	85.0	85.0	65.0	59.9
25	46.5	--	--	42.1	--	--	--	--	85.0	--	--	57.7
26	--	--	35.7	42.3	--	--	--	--	--	--	65.0	57.7
26	--	46.6	--	42.5	--	--	--	--	--	70.7	--	59.1
28	--	47.1	--	44.2	--	--	--	--	--	66.5	--	75.0
29	42.5	51.9	--	--	--	--	--	--	85.0	72.7	--	70.3
30	44.2	--	--	--	--	--	--	--	85.0	--	--	85.0
31	47.5	--	--	--	--	--	--	--	85.0	51.9	70.3	85.0
32	--	--	--	--	--	--	--	--	--	--	--	75.8
33	--	--	--	--	--	--	--	--	54.6	65.0	--	76.7
34	--	--	--	--	--	--	--	--	53.9	--	--	75.0
35	--	--	--	--	--	--	--	--	55.7	--	--	--
36	--	--	--	--	--	--	--	--	--	--	--	--
37	--	--	--	--	--	--	--	--	71.1	--	--	--
38	--	--	--	--	--	--	--	--	68.5	--	--	--

Note: Dashes indicate no catch reported for that time period.

Table 5. Parameter estimates and statistics for the prediction equation for brown shrimp. Time is in weeks from April 1 and C is cumulative water temperature (Centigrade degrees) in day-degrees from April 1.

MODEL: $S'_t = S'_\infty[1-e^{-(b_1t+b_2C)}] + 85e^{-(b_1t+b_2C)}$.

Non-linear least squares summary statistics

SOURCE	DF	SUM OF SQUARES	MEAN SQUARE
REGRESSION	3	151021.23987263	50340.41329088
RESIDUAL	52	4314.96587604	82.98011300
UNCORRECTED TOTAL	55	155336.20574866	
(CORRECTED TOTAL)	54	8969.54567304	

PARAMETER	ESTIMATE	ASYMPTOTIC STD. ERROR	ASYMPTOTIC 95 % CONFIDENCE INTERVAL LOWER	UPPER
S'_∞	44.85600674	2.23973662	40.36165308	49.35036041
b_1	-0.83807329	0.28654704	-1.41307114	-0.26307544
b_2	0.00582483	0.00190434	0.00200350	0.00964616

ASYMPTOTIC CORRELATION MATRIX OF THE PARAMETERS

	S'_∞	b_1	b_2
S'_∞	1.000000	-0.675153	0.688285
b_1	-0.675153	1.000000	-0.999345
b_2	0.688285	-0.999345	1.000000

Table 6. Parameter estimates and statistics for the prediction equation for early pink shrimp. Time is in weeks from April 1 and C is cumulative water temperature (Centigrade degrees) in day-degrees from April 1.

MODEL: $S'_t = S'_\infty[1-e^{-(b_1t+b_2C)}] + 85e^{-(b_1t+b_2C)}$,

Non-linear least squares summary statistics

SOURCE	DF	SUM OF SQUARES	MEAN SQUARE
REGRESSION	3	121751.10632467	40583.70210822
RESIDUAL	30	1202.75399587	40.09179986
UNCORRECTED TOTAL	33	122953.86032054	
(CORRECTED TOTAL)	32	2651.77325800	

PARAMETER	ESTIMATE	ASYMPTOTIC STD. ERROR	ASYMPTOTIC 95 % CONFIDENCE INTERVAL LOWER	UPPER
S'_∞	48.40370982	14.89263926	17.98910168	78.81831796
b_1	0.47164785	0.28297359	-0.10625715	1.04955285
b_2	-0.00226640	0.00137344	-0.00507133	0.00053853

ASYMPTOTIC CORRELATION MATRIX OF THE PARAMETERS

	S'_∞	b_1	b_2
S'_∞	1.000000	0.798061	-0.517600
b_1	0.798061	1.000000	-0.927115
b_2	-0.517600	-0.927115	1.000000

Table 7. Parameter estimates and statistics for the prediction equation for late pink shrimp. Time is in weeks from April 1 and C is cumulative water temperature (Centigrade degrees) in day-degrees from April 1.

MODEL: $S'_t = S'_\infty[1-e^{-(b_1t+b_2C)}] + 85e^{-(b_1t+b_2C)}$,

Non-linear least squares summary statistics

SOURCE	DF	SUM OF SQUARES	MEAN SQUARE
REGRESSION	3	177796.24272947	59265.41424316
RESIDUAL	33	4502.42176794	136.43702327
UNCORRECTED TOTAL	36	182298.66449741	
(CORRECTED TOTAL)	35	4502.88791752	

PARAMETER	ESTIMATE	ASYMPTOTIC STD. ERROR	ASYMPTOTIC 95 % CONFIDENCE INTERVAL LOWER	ASYMPTOTIC 95 % CONFIDENCE INTERVAL UPPER
S'_∞	70.22784665	2.96809478	64.18925146	76.26644184
b_1	-0.51610106	29.43768871	-60.40714184	59.37493972
b_2	0.00437119	0.17997110	-0.36178040	0.37052277

ASYMPTOTIC CORRELATION MATRIX OF THE PARAMETERS

	S'_∞	b_1	b_2
S'_∞	1.000000	-0.157969	0.201928
b_1	-0.157969	1.000000	-0.998193
b_2	0.201928	-0.998193	1.000000

Therefore, the mean value was selected for use in predicting late pink shrimp size in subsequent analyses. The mean size of late pink shrimp in the commercial catches for 1979 to 1982 was 70.3 shrimp per pound (0.0144 pounds per shrimp).

In applications, a forecast of cumulative water temperature--C--is needed. For the present study, the cumulative temperature prediction equation for 1982 was used. The resulting size curves are shown in Figure 2.

Use of these equations to predict shrimp size implies that each species consists of identically sized shrimp regardless of the time of recruitment. It is also important to note that the size equations are applicable only over the time intervals represented by the dataset used to estimate the equations (weeks 10 to 31 for brown shrimp, weeks 0 to 15 for early pink shrimp, and weeks 19 to 38 for late pink shrimp). The prediction equations do not perform well outside of these time intervals. This does not limit the usefulness of the prediction equations, however, because shrimp are either not vulnerable to capture or have migrated from the estuary at times outside of these intervals.

The size prediction equations were also used as inputs to the market price prediction equation. The variable "Size" in that model is equivalent to S_t'. By substituting S_t' into the price equation, it can be written as

$$P(g_i, w, t) = a_0 + a_1 S_t' + a_2 (S_t')^2 + a_3 \text{PSHRIMP},$$

where $S_t' = 1/g_i$, S_t' is in units of number per pound and g_i is in units of pounds per shrimp. The price per shrimp, $P(g_i, w, t) g_i(z, t)$ (with PSHRIMP equal to the 1982 average), is plotted in Figure 3.

Fishing Effort Equation

Anderson (1977) characterized fishermen (and their fishing vessels) as "producers of effort rather than of fish." With this concept of fishing effort it is clear that fishing effort is not only an input into the fishery production function, but is also a "product" produced according to a separate production function. The demand for the "product" is a derived demand based on the demand for fish. The supply curve for the "product" can, in principle, be determined from the cost curves associated with the production function for fishing effort. Assuming fishermen maximize profits, the amount of fishing effort supplied to a particular fishery is determined by the equilibrium conditions in the finished product (fish) and factor product (fishing effort) markets.

In most fishery applications, fishing effort is taken as exogenous. But with fishing effort defined as a "product," it is obvious that it is endogenous to the harvesting problem, depending on the cost of producing effort and the return to effort. The return to a unit of fishing effort is

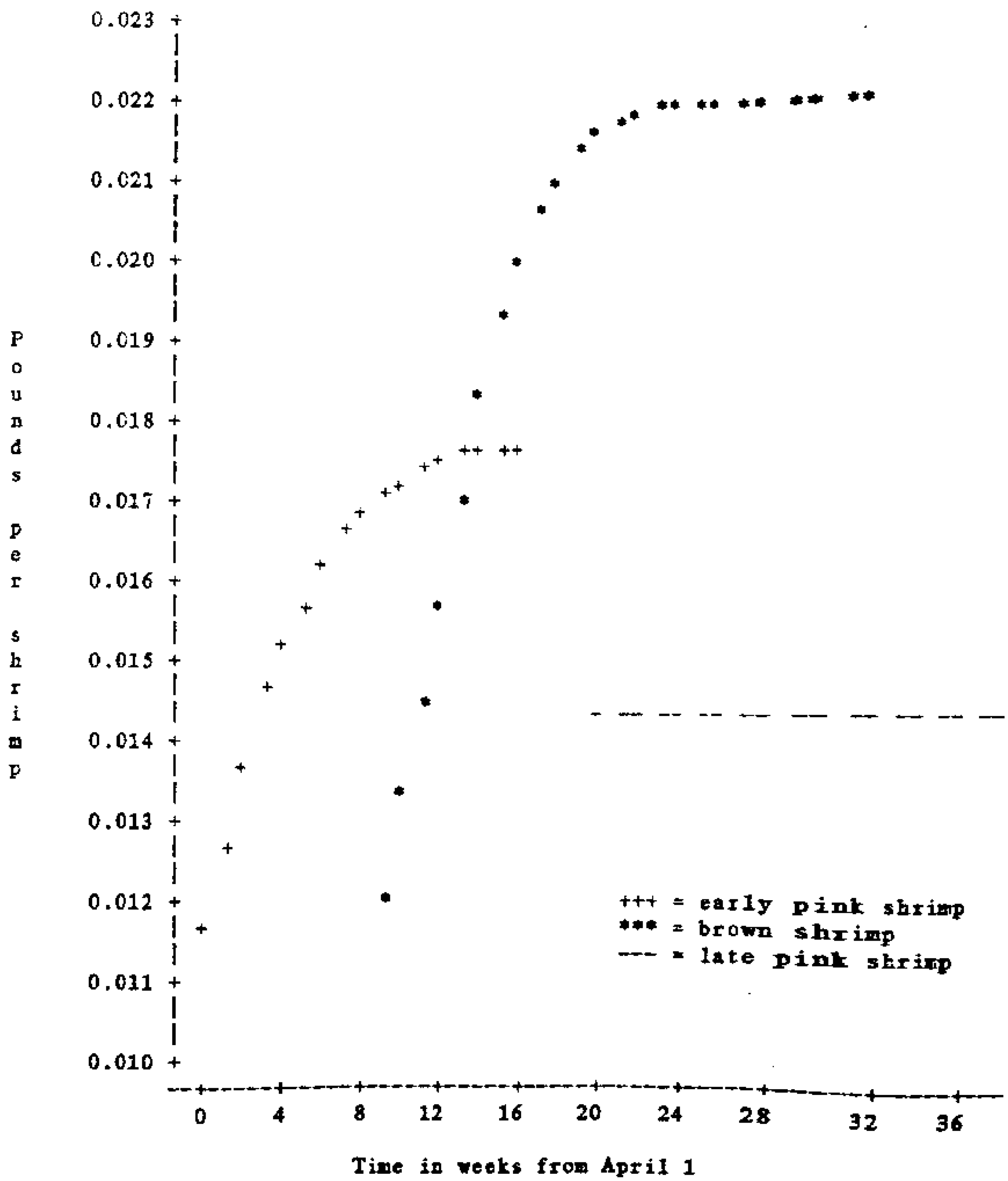

Figure 2. Size equations used in the shrimp harvesting problem.

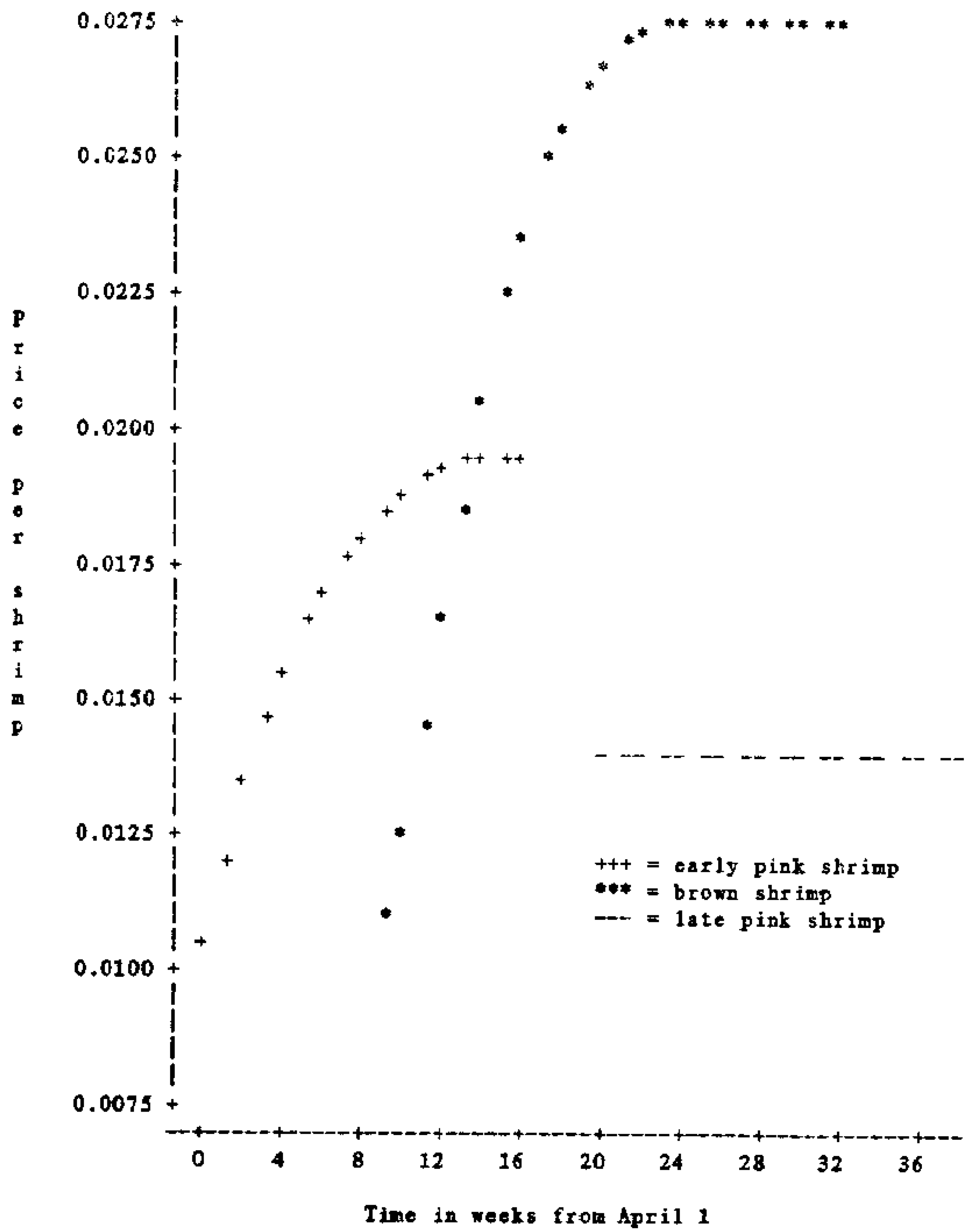

Figure 3. Price per shrimp, $P(g_i,w,t)g_i(z,t)$, used to obtain solutions to the New River shrimp harvesting problem. Price is in 1967 dollars.

the price of fish multiplied by the catch per unit of effort, which in turn is determined by the season opening/closing schedule. McGaw (1981) successfully modelled fishing effort for the Georges Bank scallop industry as an endogenous function depending on catch. In the present study, an endogenous prediction equation for the supply of fishing effort was developed.

Prior to determining the prediction equation for effort, it is necessary to define a standard unit of effort and to combine the two effort levels designated in the catch statistics (boats and vessels) into a single measure. The standard unit of effort was taken to be a vessel-day, corresponding to the empirical measure of vessel landings in the catch statistics dataset. Boat landings were converted to vessel-days on the basis of the relative catch per unit of effort. The conversion factor was determined as follows:

$$\text{Conversion factor} = \left[\sum_{i=1}^{n} \frac{\text{weekly catch per effort for boats}}{\text{weekly catch per effort for vessels}} \right] / n,$$

where i=1,2,...n weekly observations over the four-year study period. Results are shown below:

conversion factor	=	0.3127
standard deviation	=	0.1478
sample size	=	53
minimum value	=	0.0790
maximum value	=	0.7748

Weekly values for vessel-days were then determined by multiplying boat-days by 0.3127 and adding that number to weekly values for vessel-days. The return to a standard unit of effort was determined by adding weekly revenue for boats to weekly revenue for vessels and dividing by the number of vessel-days.

The supply of fishing effort for the New River shrimp fishery was initially modelled as follows:

$$\text{Effort} = E(\text{return, fuel cost, interest rate, wage rate, time}),$$

where effort is in vessel-days; fuel cost, interest rate and wage rate are factor costs; and time is a supply shifter intended to capture the seasonality of catch expectations and the effects of seasonal changes in opportunity costs of factors of production. For estimation, the model was formulated as a linear model:

$$\text{Vessel-days per week} = e_0 + e_1R + e_2FUEL + e_3INT + e_4WAGE + e_5t + e_6t^2,$$

where R is revenue per vessel-day in 1967 dollars, FUEL is the annual consumer price index for motor fuel (1967 dollars), INT is the monthly interest rate on 3-month U.S. Treasury bills, WAGE is the weekly average wage in 1967 dollars for insured employment in Onslow County (quarterly data), and t is time in weeks from April 1. The data used to estimate the above equation are presented in Kellogg (1985).

The relationship between effort and revenue per unit of effort is shown in Figure 4 for 1979 to 1982. It is apparent from Figure 4 that the data for 1979 differs substantially from that for 1980 to 1982. The difference may be due to the radically different regulatory policy in 1979. In 1979, a large portion of the estuary was opened to shrimping at one time, resulting in the participation of a record number of fishing craft. No such "grand opening" occurred in 1980 to 1982. For these reasons, only the 1980 to 1982 data were used to estimate the effort function.

Results of estimating the effort equation are presented in Table 8. The three variables representing factor costs were not significant, with a joint F statistic of 0.81. Since there was some suspicion prior to the analysis that these factor costs might not affect supply, the variables were dropped (mean square error test--Wallace (1977).) The R^2 for the reduced model (Model 2, Table 8) was 0.693 and the three explanatory variables--R, t and t^2—were all significant (P>.001). This reduced model was used in subsequent analyses to predict the supply of effort.

In the context of the shrimp harvesting model, effort becomes endogenous because the return per vessel-day is determined at each point in time by the ex-vessel price per shrimp, the population size, and the catchability coefficient for the standard unit of effort:

$$\text{return per vessel-day} = \sum_i P(g_i, w, t) g_i(z, t) q x_i,$$

where i=1,2,3 represents the three shrimp species or cohorts in the fishery.

Cost Equation

Variable costs were modelled as proportional to fishing effort:

$$\text{cost per week} = c_1 E(R, t),$$

where E(R,t) is the function used to predict the number of vessel-days per week and c_1 is the minimum variable cost per vessel-day. Only the minimum variable cost is needed in this problem because the effort function serves as a marginal cost curve for the industry above the minimum cost. As the return increases, more and more fishermen are drawn into the fishery instead of other occupations or alternative fisheries. The minimum is needed here to

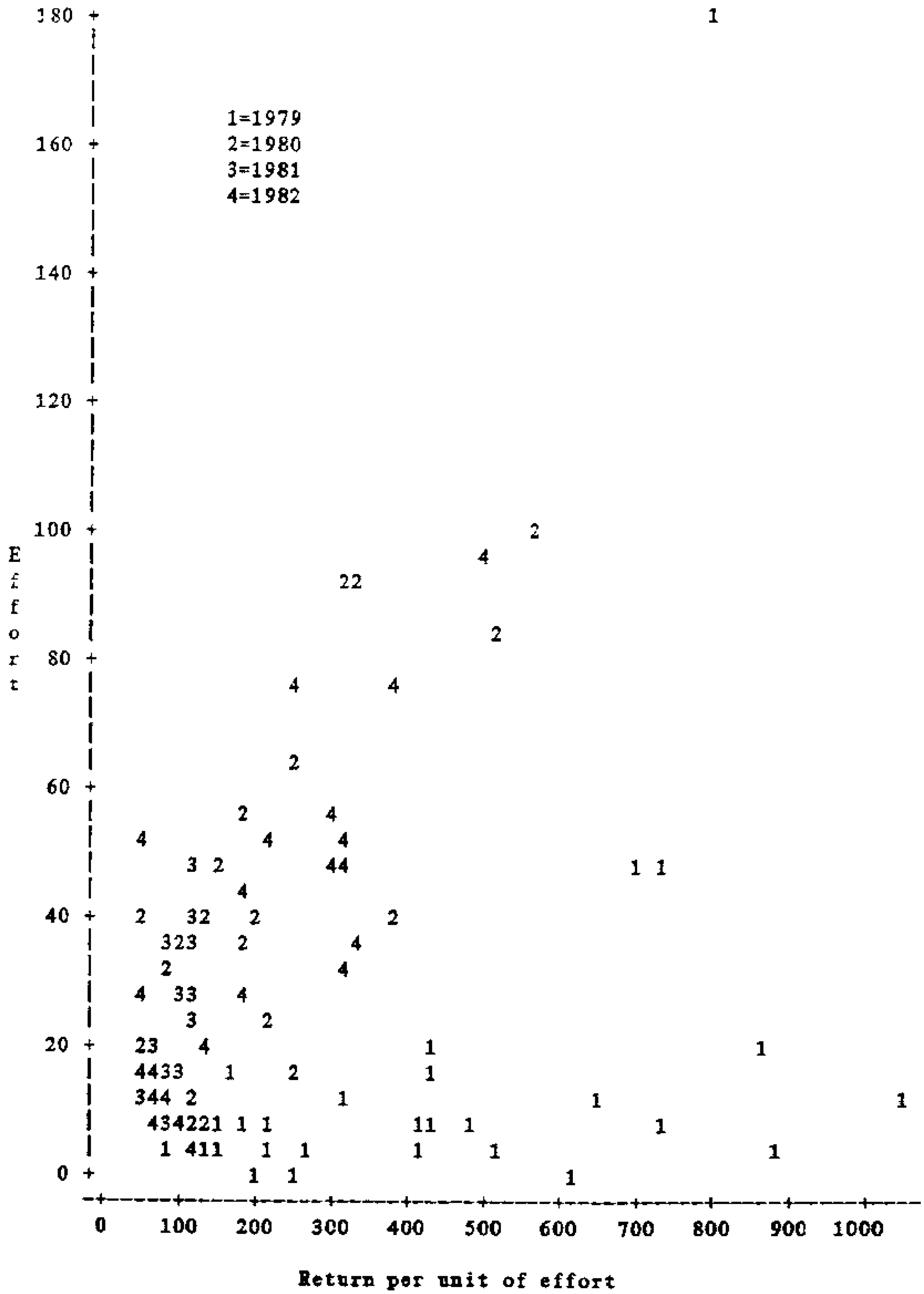

Figure 4. Relationship between effort (vessel-days) and return per vessel-day (1967 dollars) for the New River shrimp fishery, 1979-1982.

Table 8. Parameter estimates and associated statistics for the effort prediction equation for the New River shrimp fishery.

MODEL 1: Vessel-days = $e_0 + e_1R + e_2FUEL + e_3INT + e_4WAGE + e_5t + e_6t^2$

SOURCE	DF	SUM OF SQUARES	MEAN SQUARE	F VALUE
MODEL	6	27516.52164972	4586.08694162	22.38
ERROR	56	11476.31824060	204.93425430	
CORRECTED TOTAL	62	38992.83989032		PR>F = 0.0001

R^2 = 0.705681

VARIABLE	PARAMETER ESTIMATE	T FOR H0: PARAMETER=0	PR > |T|	STD ERROR OF ESTIMATE
INTERCEPT	48.64159134	0.37	0.7098	130.05201648
R	0.10966215	5.19	0.0001	0.02112656
FUEL	-0.19822798	-1.15	0.2534	0.17178343
INT	-0.04102206	-0.05	0.9589	0.79326033
WAGE	0.03076155	0.03	0.9786	1.14198995
t	5.53064084	3.82	0.0003	1.44871680
t^2	-0.14897860	-4.14	0.0001	0.03598905

MODEL 2: Vessel-days = $e_0 + e_1R + e_5t + e_6t^2$

SOURCE	DF	SUM OF SQUARES	MEAN SQUARE	F VALUE
MODEL	3	27018.83032459	9006.27677486	44.38
ERROR	59	11974.00956572	202.94931467	
CORRECTED TOTAL	62	38992.83989032		PR>F = 0.0001

R^2 = 0.692918

VARIABLE	PARAMETER ESTIMATE	T FOR H0: PARAMETER=0	PR > |T|	STD ERROR OF ESTIMATE
INTERCEPT	-22.36829804	-2.14	0.0366	10.46080638
R	0.12404577	6.99	0.0001	0.01774015
t	4.72846896	3.54	0.0008	1.33644307
t^2	-0.12822183	-3.90	0.0003	0.03288982

prevent the linear nature of the effort function from predicting fishing effort during periods when the return is below that which has historically been associated with fishing activity.

Liao (1979) collected information on variable costs for the South Atlantic shrimp fishery, including data on North Carolina shrimp fishermen. From the results of Liao's survey, Waters (1983) derived an estimate of variable cost per vessel day for the vessel size class most common in North Carolina. Waters (1983) estimated that variable costs were 113 dollars (1978 dollars) per day, exclusive of wages, which is equivalent to 57.83 dollars in 1967 dollars. This estimate corresponds closely to the minimum cut-off of the return per vessel-day observed in the catch statistics dataset for the New River. Consequently, the minimum cost per unit of effort, c_1, was set equal to 50 dollars for the present study. This value will keep fishing effort at zero when the return per effort is less than 50 dollars, even though the effort equation will predict low levels of fishing.

Equations of Motion

Equations of motion describe how the resource stock changes over time. A generalized equation of motion for the shrimp harvesting problem is:

$$\dot{x}_i = F_i(z,t) - M_i(x_i,z,t) - qE(R,t)x_i(t)\,\Phi(t).$$

In this form, both the recruitment function ($F_i(z,t)$) and the mortality function ($M_i(x_i,z,t)$) contain environmental variables (z). Although environmental factors are extremely important for both of these functions, sufficient data do not exist at the present time to include them. Additionally, the mortality function will include emigration as well as natural mortality. For the New River fishery, emigration is equivalent to mortality, since the shrimp are lost to the fishery in both cases.

The recruitment function was developed as a probability function (that is, $\int F(t)dt=1$). Recruitment in numbers of shrimp at time t is obtained by multiplying F(t) by an estimate of the population size when shrimp first become vulnerable to capture ($t=s_i$). Changes in population that are due to natural mortality will be assumed to be proportional to the population size. It will further be assumed that the proportion is constant throughout the potential harvest season. Modelling mortality in this way is a common practice in fisheries. (The natural mortality coefficient is also called the instantaneous natural mortality rate in the fisheries literature.) Fishing mortality was estimated by the catch-per-unit-effort production function ($qE(R,t)x_i(t)$). Recreational fishing mortality is ignored in the present study since data on the recreation catch is not available; however, the recreation catch may be significant. Incorporating these assumptions and features into the generalized equation of motion results in the following form which was used in the present study.

$$\dot{x}_i = F_i(t)x_i(s_i) - M_i x_i(t) - qE(R,t)x_i(t)\,\Phi(t).$$

Four parameters or functions must be estimated for each species or cohort: 1) the natural mortality coefficient (M_i), 2) the "initial" population size ($x_i(s_i)$), 3) the recruitment function ($F_i(t)$), and 4) the catchability coefficient (q). Methods used to obtain values for these four items are discussed in detail in Kellogg (1985).

The best estimate for the natural mortality coefficient for the New River shrimp fishery is about 0.35. Solutions to the shrimp harvesting problem were also determined for M equal to 0.15 and 0.25 to evaluate how the harvesting strategy would change if natural mortality were lower. For simplicity, the same mortality coefficient was used for all three species/cohorts. The catchability coefficient estimate used in the problem was 0.0008. Estimates of initial population size for use in the optimization problem were determined by Kellogg (1985) as 215 million for brown shrimp, 5.9 million for early pink shrimp, and 17.0 million for late pink shrimp. (In actual applications, initial population sizes should be estimated by biological sampling early in the season.)

Recruitment functions ($F_i(t)$) were constructed for each species/ cohort. Ideally, these functions should be modelled as a function of temperature and salinity. However, sufficient information is not available to incorporate these factors or to estimate the function directly from data. Instead, simple probability distributions were constructed to be consistent with the biology of the species and with catch-per-effort data on the smallest size class. For brown shrimp, a triangular recruitment function was selected. Recruitment starts at t=10 and continues through t=16, peaking at t=13. The function is expressed as follows:

$$
\begin{aligned}
&\text{for } t<10 \text{ or } t>16, && F_1 = 0,\\
&\text{for } t=10 \text{ or } t=16, && F_1 = 1/28,\\
&\text{for } t=11 \text{ or } t=15, && F_1 = 3/28,\\
&\text{for } t=12 \text{ or } t=14, && F_1 = 6/28,\\
&\text{for } t=13, && F_1 = 8/28.
\end{aligned}
$$

Early pink shrimp recruitment begins gradually and then increases rapidly to a sharp peak. For the present study, this function was modelled as follows:

$$
\begin{aligned}
&\text{for } t<3, && F_2 = 1/20,\\
&\text{for } t=3, && F_2 = 2/20,\\
&\text{for } t=4, && F_2 = 15/20,\\
&\text{for } t>4, && F_2 = 0.
\end{aligned}
$$

Recruitment of late pink shrimp is a gradual--and more or less steady--process from about week 19 until about the end of the season. The catch-per-effort data supports this, generally. So a rectangular distribution was constructed, spanning from week 19 to week 34, as follows:

$$
\begin{aligned}
&\text{for } t<19 \text{ or } t>34, && F_3 = 0,\\
&\text{for } 18<t<35, && F_3 = 1/16.
\end{aligned}
$$

Calculation of the Optimal Harvest Period

Statement of the Problem

Incorporating these results into the general shrimp harvesting model, the problem can be restated as follows:

maximize with respect to $\Phi(t)$

$$PV = \int_0^T [R(x_1,x_2,x_3)E(R,t)-c_1E(R,t)]e^{-\delta t}\,\Phi(t)\,dt$$

such that

$$\dot{x}_1 = F_1(t)x_1(10) - M_1x_1(t) - qE(R,t)x_1(t)\,\Phi(t)$$
$$x_1=0 \text{ for } t<10,\ x_1(10) \text{ given and } x_1(t)\geq 0,$$

$$\dot{x}_2 = F_2(t)x_2(0) - M_2x_2(t) - qE(R,t)x_2(t)\,\Phi(t)$$
$$x_2(0) \text{ given and } x_2(t)\geq 0,$$

$$\dot{x}_3 = F_3(t)x_3(19) - M_3x_3(t) - qE(R,t)x_3(t)\,\Phi(t)$$
$$x_3=0 \text{ for } t<19,\ x_3(19) \text{ given and } x_3(t)\geq 0,$$

where

$$R(x_1,x_2,x_3) = P(g_1,t,w)g_1(z,t)qx_1(t) + P(g_2,t,w)g_2(z,t)qx_2(t) + P(g_3,t,w)g_3(z,t)qx_3(t),$$

$$P(g_i,t,w) = a_0 + a_1(1/g_i) + a_2(1/g_i)^2 + a_3\text{PSHRIMP}$$

$$1/g_1(z,t) = S'_{1\infty}[1-e^{-(b_{11}t+b_{12}C)}] + 85e^{-(b_{11}t+b_{12}C)},$$

$$1/g_2(z,t) = S'_{2\infty}[1-e^{-(b_{21}t+b_{22}C)}] + 85e^{-(b_{21}t+b_{22}C)},$$

$$1/g_3(z,t) = 70.3,$$

$$E(R,t) = e_0 + e_1R(x_1,x_2,x_3) + e_5t + e_6t^2,$$

$i=1$ ==> brown shrimp,

$i=2$ ==> early pink shrimp,

$i=3$ ==> late pink shrimp,

t = time in units of weeks starting from April 1,

T = natural end of fishing season (December 31), and

$\Phi(t)$ = the decision variable ($\Phi(t)=0$ implies a closed season and $\Phi(t)=1$ implies an open season).

The starting time, $t=s_i$, was different for each of the three species/cohorts. It was selected to correspond to the earliest commercial catch record for 1979-1982. Starting times were: s=0 for early pink shrimp, s=10 for brown shrimp, and s=19 for late pink shrimp. In the model, population size was set equal to zero prior to these starting times. (This has the same effect as setting the equation of motion equal to zero until $t=s_i$.)

The F_i functions (recruitment functions) and the C function (for cumulative temperature in day-degrees from April 1) are discrete functions defined as follows:

t	F_1	F_2	F_3	C	t	F_1	F_2	F_3	C
0	0	1/20	0	0	20	0	0	1/16	3475
1	0	1/20	0	117	21	0	0	1/16	3670
2	0	1/20	0	241	22	0	0	1/16	3860
3	0	2/20	0	372	23	0	0	1/16	4045
4	0	15/20	0	511	24	0	0	1/16	4225
5	0	0	0	657	25	0	0	1/16	4397
6	0	0	0	810	26	0	0	1/16	4561
7	0	0	0	971	27	0	0	1/16	4718
8	0	0	0	1139	28	0	0	1/16	4866
9	0	0	0	1314	29	0	0	1/16	5004
10	1/28	0	0	1494	30	0	0	1/16	5134
11	3/28	0	0	1681	31	0	0	1/16	5254
12	6/28	0	0	1872	32	0	0	1/16	5364
13	8/28	0	0	2067	33	0	0	1/16	5465
14	6/28	0	0	2266	34	0	0	1/16	5557
15	3/28	0	0	2467	35	0	0	0	5639
16	1/28	0	0	2670	36	0	0	0	5714
17	0	0	0	2872	37	0	0	0	5780
18	0	0	0	3075	38	0	0	0	5838
19	0	0	1/16	3276	39	0	0	0	5890

Coefficients were estimated as follows:

$e_0 = -22.37$	$S'_{1\infty} = 44.86$	$S'_{2\infty} = 48.40$
$e_1 = 0.1240$	$b_{11} = -0.8381$	$b_{21} = 0.4716$
$e_3 = 4.728$	$b_{12} = 0.005825$	$b_{22} = -0.002266$
$e_4 = -0.1282$	$a_0 = 1.556$	$a_2 = 0.0001255$
	$a_1 = -0.02503$	$a_3 = 0.34175$

Exogenous variables were assigned the following values:

$M_1 = M_2 = M_3 = 0.15,\ 0.25,\ \text{or}\ 0.35$

$q = .0008,$

$c_1 = 50$

$x_1(10) = 215$ million

$x_2(0) = 5.9$ million

$x_3(19) = 17$ million

PSHRIMP = 1.66 or 2.10

The optimization problem was solved for six sets of exogenous variables--high and low price levels at each of three natural mortality rates. In actual practice, each of the above exogenous variables and the recruitment functions and the cumulative temperature function will need to be estimated or forecasted before solving the harvesting problem. The above problem represents only six hypothetical situations with the same population size and recruitment pattern, and so the solutions obtained will not apply to all harvesting years.

Solution Procedure

The maximum principle is used to solve for the optimal opening/closing schedule (Johnson 1985; Kamien and Schwartz 1981; Kellogg 1985). The maximum principle says that the optimal control can be obtained by maximizing a function called the "Hamiltonian" at each moment over the time horizon of the problem. Here, the Hamiltonian function is

$$\begin{aligned} H(t) = {} & [R(x_1,x_2,x_3)-c_1]E(R,t)e^{-\delta t}\ \phi(t) \\ & + \lambda_1[F_1(t)x_1(10)-Mx_1(t)-qE(R,t)x_1(t)\ \phi(t)] \\ & + \lambda_2[F_2(t)x_2(0)-Mx_2(t)-qE(R,t)x_2(t)\ \phi(t)] \\ & + \lambda_3[F_3(t)x_3(19)-Mx_3(t)-qE(R,t)x_3(t)\ \phi(t)], \end{aligned}$$

where the λ_is are the adjoint, or co-state variables. The Hamiltonian represents the net revenue plus the value of the changes in the resource at each point in time. The adjoint variables represent the value of an additional unit of the stock, also called the marginal user cost or "shadow" price. Since the Hamiltonian is linear in the control variable, the necessary condition for a maximum is expressed using the following switching function:

$$\phi = \begin{cases} 1 & \text{if } [R(x_1,x_2,x_3)-c_1]E(R,t)e^{-\delta t} - \lambda_1 qE(R,t)x_1(t) \\ & \quad - \lambda_2 qE(R,t)x_2(t) - \lambda_3 qE(R,t)x_3(t) \geq 0 \\ 0 & \text{if } [R(x_1,x_2,x_3)-c_1]E(R,t)e^{-\delta t} - \lambda_1 qE(R,t)x_1(t) \\ & \quad - \lambda_2 qE(R,t)x_2(t) - \lambda_3 qE(R,t)x_3(t) < 0 \end{cases}$$

The system of differential equations is

$$\dot{x}_1 = F_1(t)x_1(10) - Mx_1(t) - qE(R,t)x_1(t)\ \phi(t)$$

$$\dot{x}_2 = F_2(t)x_2(0) - Mx_2(t) - qE(R,t)x_2(t)\ \phi(t)$$

$$\dot{x}_3 = F_3(t)x_3(19) - Mx_3(t) - qE(R,t)x_3(t)\ \phi(t)$$

$$\begin{aligned} \dot{\lambda}_1 = {} & \lambda_1 M + \lambda_1 q[E(R,t)+e_1 qP(g_1,t,w)g_1(z,t)x_1(t)]\ \phi(t) \\ & - P(g_1,t,w)g_1(z,t)q[E(R,t)+e_1(R(x_1,x_2,x_3)-c_1)]e^{-\delta t}\ \phi(t) \end{aligned}$$

$$\dot{\lambda}_2 = \lambda_2 M + \lambda_2 q[E(R,t)+e_1 qP(g_2,t,w)g_2(z,t)x_2(t)]\ \phi(t) \\ - P(g_2,t,w)g_2(z,t)q[E(R,t)+e_1(R(x_1,x_2,x_3)-c_1)]e^{-\delta t}\ \phi(t)$$

$$\dot{\lambda}_3 = \lambda_3 M + \lambda_3 q[E(R,t)+e_1 qP(g_3,t,w)g_3(z,t)x_3(t)]\ \phi(t) \\ - P(g_3,t,w)g_3(z,t)q[E(R,t)+e_1(R(x_1,x_2,x_3)-c_1)]e^{-\delta t}\ \phi(t)$$

Sufficient conditions for a solution cannot be derived because initial conditions for the adjoint equations are not specified. The optimal solution is obtained by searching over positive values of $\lambda_1(0)$, $\lambda_2(0)$ and $\lambda_3(0)$ to find the $\phi(t)$ corresponding to the maximum net present value of the season harvest. (See Kellogg (1985) or Kellogg et al. (1985) for a detailed account of the solution procedure.)

The optimal opening/closing schedule was determined to the nearest week. The algorithm used to solve for the optimal $\phi(t)$ is presented in Appendix A. It is similar to the algorithm used by Kellogg et al. (1985) to solve the bay scallop problem, except that three initial λ's are determined. The switching function is solved at the beginning of each week to see whether or not the season should be opened. If the switching function is positive, the harvest and net present value for the period are calculated. The process continues until t=T (December 31).

The "unregulated" case was determined by setting all λ_i's equal to zero for all time periods. This represents the situation in the fishery where the marginal user cost is disregarded (i.e., the unregulated open access situation). The time of fishing for the unregulated case represents the time when it is <u>profitable</u> to harvest shrimp <u>under the assumptions of the model</u>. This condition is required to contrast the optimal solution to the "unregulated" case properly. (The "unregulated" case here applies only to setting the season opening. Other regulations on gear and area restrictions are assumed to remain in force.)

Results

The optimum season opening/closing schedule for each of the six solutions is contrasted with that for the unregulated case in Table 9. Delaying the season improved the net present value of the harvest only at the lowest natural mortality rates. At a natural mortality rate of 0.35, which is approximately equal to the natural mortality rate estimated by McCoy (1972) for the New River, the unregulated case was optimal. At this mortality rate, delaying the harvest decreased the net present value. At a natural mortality rate of 0.15, the net present value was increased only slightly (0.04 percent) over that for the unregulated case by delaying the early pink shrimp fishery. The higher market price produced higher net present values at all three natural mortality rates, but the optimal harvest season was nearly the same as that at the lower market price.

In this example, the dynamics of the late pink shrimp population did not influence the season opening/closing schedule. In each case, the optimal

Table 9. Summary of harvesting solutions for the New River shrimp fishery for six combinations of inputs. Present value and λ_i are in 1967 dollars.

M	PSHRIMP	Optimal $\lambda_i(0)$			Net present value	Optimal season[a]	% harvested		
		$\lambda_1(0)$	$\lambda_2(0)$	$\lambda_3(0)$			x_1	x_2	x_3
0.15	1.66	0.0020	0.0024	0.00	2,210,114	t=11-34	49	15	11
		unregulated (all λ_i=0)			2,209,298	t=4-6,10-34	49	16	11
0.15	2.10	0.0022	0.0030	0.00	2,616,165	t=4,11-34	52	16	11
		unregulated (all λ_i=0)			2,615,232	t=4-7,10-34	52	17	11
0.25	1.66	0.0000	0.0006	0.00	1,538,690	t=4,10-33	36	6	5
		unregulated (all λ_i=0)			1,538,644	t=4-5,10-33	36	6	5
0.25	2.10	0.0000	0.0000	0.00	1,853,466	t=4-5,10-33	38	6	6
0.35	1.66	0.0000	0.0000	0.00	1,139,996	t=4,10-27	27	2	3
0.35	2.10	0.0000	0.0000	0.00	1,390,297	t=4-5,10-30	29	2	3

[a]There are two optimal seasons in most cases. The first is in the spring when early pink shrimp are present, and the second is in the summer when brown shrimp and late pink shrimp are present.

$\lambda_3(0)$ was zero. This occurred because the size and price (and thus value) of late pink shrimp were modelled as constants, which the data indicated. However, if growth, recruitment, natural mortality and emigration could be modelled more carefully, the optimal season opening/closing schedule might be affected by tradeoffs between brown shrimp catches and growth of late pink shrimp during the late summer and fall months.

These results suggest that there is little or no gain from regulating the New River shrimp fishery beyond current practice. In general, the high natural mortality rates result in more value lost when the season opening is delayed than is gained through growth and increased prices. However, with different recruitment functions and population sizes than used here, the optimization model might produce different results. For example, if the brown shrimp population was small and the early pink shrimp population was large, then there would be more tradeoff possibilities early in the season. The same would occur if recruitment patterns were more prolonged and overlapping. Whereas high mortality rates indicate little benefit for delaying the season opening beyond the unregulated harvesting scenario in general, it is possible that gain from regulation might be realized under specific conditions that may occur in some years.

OPTIMAL TIMING OF HARVEST FOR THE PAMLICO SOUND SHRIMP FISHERY

Brief Description of the Pamlico Sound Shrimp Fishery

The Pamlico Sound fishery is located in northeastern North Carolina and produces most of the shrimp catch in the state. For purposes of this study, the Pamlico Sound fishery is restricted to the catch in Pamlico Sound proper (area code 6354), and thus excludes the catch in tributaries to Pamlico Sound and some bays. In this area, shrimp are harvested from early July through November. Brown shrimp is the predominant shrimp species, but pink shrimp are also commercially important in the late summer and early fall. Since 1977, overwintering pink shrimp that emerge in the spring have not contributed significantly to the fishery (Dennis Spitsbergen, Division of Marine Fisheries, personal communication). White shrimp are also not abundant. The North Carolina Division of Marine Fisheries regulates the fishery, traditionally opening the season when shrimp become large enough to have commercial value.

In Pamlico Sound, shrimp are harvested primarily by vessels using shrimp trawls. Catch statistics indicate that time spent fishing by boats was less than 5 percent of the total fishing time in the area, and the catch (in pounds) by boats represented only 2.4 percent of the total catch. (Boats play a larger role in harvesting shrimp in tributaries to Pamlico Sound.)

Catch statistics for 1979 through 1982 are summarized below for the Pamlico Sound fishery.

Shrimp harvest in Pamlico Sound

Year	Total revenue	Weight in pounds			Number	
		Brown shrimp	Pink shrimp	Total	Brown shrimp	Pink shrimp
1979	1,676,631	357,633	96,099	453,732	12,213,047	6,205,293
1980	7,014,667	1,916,865	300,750	2,217,615	66,565,555	17,543,600
1981	1,733,966	432,668	27,707	460,375	11,065,854	1,626,986
1982	6,795,726	1,352,324	203,401	1,555,725	41,843,261	12,469,010

Data on the Pamlico Sound shrimp fishery were obtained from the North Carolina Division of Marine Fisheries and analyzed according to procedures used for the New River shrimp fishery. The resulting dataset is presented in Appendix B of this report. On the basis of these data, functions for shrimp growth, price, and fishing effort were estimated. Estimates of population dynamics, such as population size, catchability coefficient, and natural mortality, were taken from an earlier analysis of the Pamlico Sound fishery by Waters (1983).

The Model

The seasonal harvesting model used for New River shrimp was adapted for application to the Pamlico Sound shrimp fishery. The main differences between the two problems are that the Pamlico Sound fishery is modelled using only two populations ("early" pink shrimp are negligible in the Pamlico Sound area as defined in this study), and the size equations do not incorporate water temperature.

Components of the Model

Shrimp Size Equations

The model used to predict the size of brown shrimp is similar to the basic model used to predict the size of New River shrimp. However, weekly temperature data were not available for the Pamlico Sound fishery, so the model was modified to

$$S_t^{-1} = S_\infty^{-1}(1-e^{-kt}) + 85e^{-kt},$$

where S_t = size in pounds per shrimp at time t,

S_∞ = the average "maximum" size attainable before migration to the ocean,

t = time in weeks from June 1, and

k = a constant growth coefficient.

The reciprocal of S_t, which is in units of number per pound, was used to estimate the model. Average shrimp size for each week was calculated by dividing the weight of shrimp harvested in each week into the number of shrimp harvested in each week, resulting in the weekly average size in number-per-pound, heads off. The number of shrimp harvested per week was determined by multiplying the catch in pounds by the size class (in units of number per pound, heads off) reported by the port sampler (the midpoint of the range was used), and then summed over all size classes that were harvested in each week.

A non-linear least squares procedure (Marquardt iteration) was used to fit the above model. The results for brown shrimp are presented in Table 10. The R^2 was 0.455 after subtracting the contribution of the mean. Both parameters were (asymptotically) statistically significant. The average maximum size attainable before migration to the ocean (S_∞^{-1}) was estimated to be 30.4 shrimp per pound. This value corresponds roughly to a length of 140 millimeters, which is a reasonable value for migrating brown shrimp in Pamlico Sound.

The above model could not be used for pink shrimp--convergence could not be obtained. A second order function of time was used instead. The model and associated statistics are presented in Table 11. As was done for brown shrimp, the reciprocal of S_t was used to estimate the model. The first derivative was negative, as required, for all but the last four time periods. (Using size measured in number-per-pound, a negative derivative means the animals are increasing in weight as the season progresses.) The positive derivative in the last four time periods poses no problem, however, because the change in size that occurs is very slight (see Figures 5 and 6).

Table 10. Parameter estimates and statistics for the size prediction equation for brown shrimp. Time is in weeks from June 1.

$$\text{MODEL:} \quad S_t^{-1} = S_\infty^{-1}[1-e^{-(kt)}] + 85e^{-(kt)},$$

Non-linear least squares summary statistics

SOURCE	DF	SUM OF SQUARES	MEAN SQUARE
REGRESSION	2	100411	50205
RESIDUAL	83	3454	41.623
UNCORRECTED TOTAL	85	103866	
(CORRECTED TOTAL)	84	6333	

PARAMETER	ESTIMATE	ASYMPTOTIC STD. ERROR	ASYMPTOTIC 95 % CONFIDENCE INTERVAL LOWER	UPPER
S_∞^{-1}	30.43599440	1.00059432	28.44584448	32.42614433
k	0.26891204	0.02790746	0.21340499	0.32441908

Table 11. Parameter estimates and statistics for the size prediction equation for pink shrimp. Time is in weeks from June 1.

$$\text{MODEL:} \quad S_t^{-1} = b_0 + b_1 t + b_2(t^2)$$

Source	d.f.	Sum of Squares	Mean Square
Model	2	4923.0363	2461.51817
Error	75	4261.5925	56.82123
Corrected total	77	9184.6288	

Model F = 43.32   Pr > F = 0.0001 R^2 = 0.536

Parameter	Estimate	t for H_0: Parameter=0	Std Error of Estimate
b_0	116.1157	13.9955	8.2966
b_1	-4.7997	-5.0341	0.9534
b_2	0.0965	3.7760	0.0256

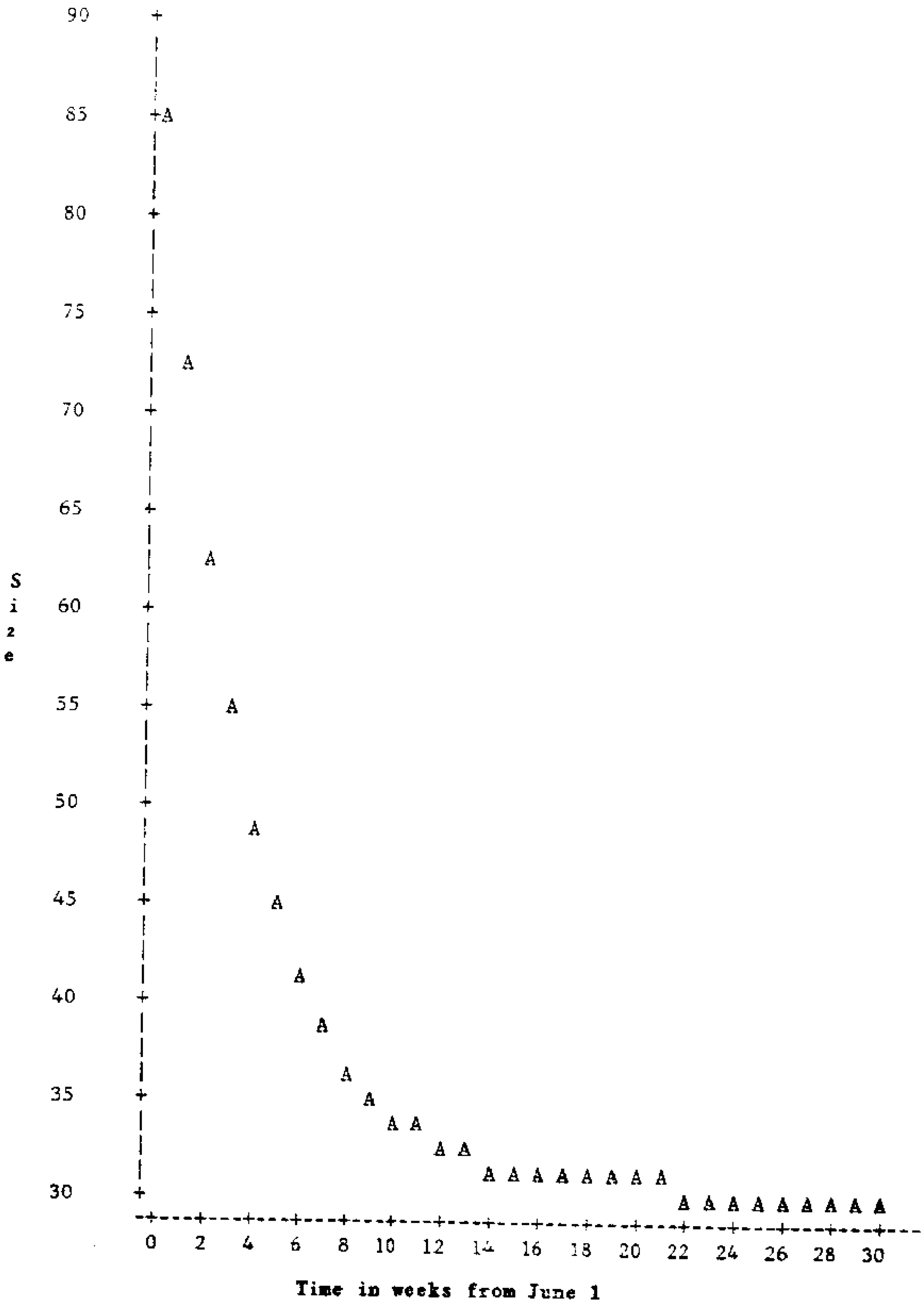

Figure 5. Size function (number per pound, heads off) for brown shrimp used in the Pamlico Sound shrimp harvesting problem.

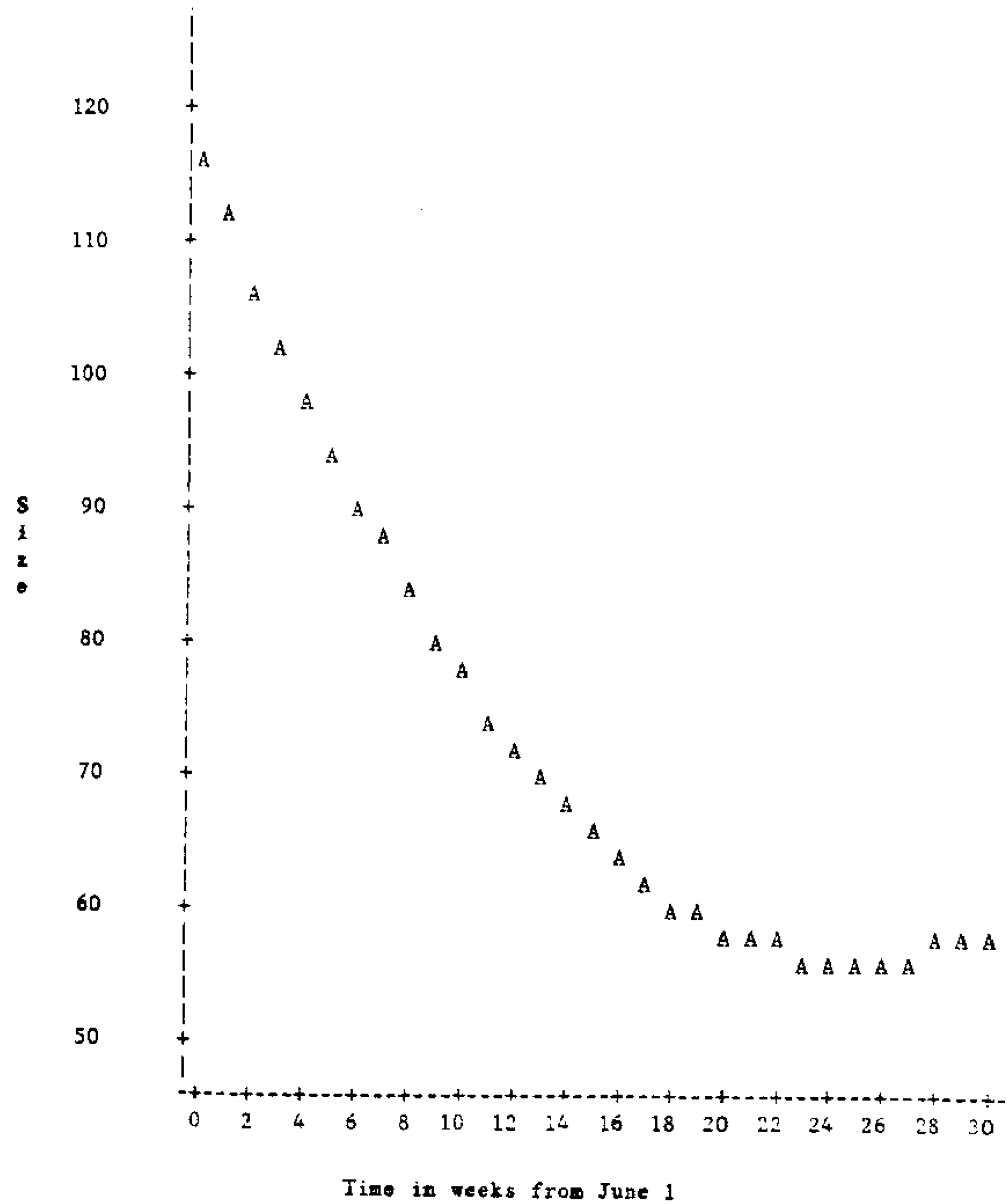

Figure 6. Size function (number per pound, heads off) for pink shrimp used in the Pamlico Sound shrimp harvesting problem.

Market Price Equation

The market price (ex-vessel) was modelled in the same manner as for the New River shrimp fishery:

$$P_{Pamlico} = a_0 + a_1(Size) + a_2(Size)^2 + a_3(PSHRIMP).$$

Weekly ex-vessel prices were determined as the ratio of total revenue to total weight, and thus represent a weighted average price. Prices were then adjusted by dividing them by the appropriate monthly consumer price index so that all prices were in constant 1967 dollars. (Note that prices for each size class, rather than weekly average prices, were used to fit the New River shrimp price equation.)

The results of estimating the above model are shown in Table 12. The model had an R^2 of 0.891 and all independent variables were highly significant. In addition, the partial derivative with respect to the size variable was negative, as expected, and the coefficients compared favorably with those determined for New River shrimp.

After incorporating the size prediction equation into the market price prediction equation, price is predicted as a function of time and PSHRIMP. In addition, the model was adjusted so that price would equal zero if size was smaller than 85 per pound. This did not affect brown shrimp, which were larger than 85 per pound throughout the potential harvest season, but resulted in a zero price for pink shrimp prior to t=8. The resulting price curves (with PSHRIMP equal to 2.10 dollars) are shown in Figures 7 and 8.

Fishing Effort Equation

The standard unit of effort was taken to be a vessel-hour. (Note that the standard unit of effort used for the New River fishery was a vessel-day.) Boat-hours were converted to vessel-hours on the basis of the relative catch per unit of effort using the same procedure described for New River shrimp. Statistics for the relative catch per unit of effort are shown below:

range	=	0.10689-1.428
mean	=	0.44323
median	=	0.37673
standard deviation	=	0.27488
sample size	=	58

Since the distribution was skewed (non-normal), the median measure was used to convert boat-hours to vessel-hours. Total hours fished per week was determined by multiplying boat-hours by 0.37673 and adding that number to vessel-hours. The return to a standard unit of effort (revenue per hour

Table 12. Parameter estimates and statistics associated with the market price prediction equation for Pamlico Sound shrimp.

Model: $P_{Pamlico} = a_0 + a_1(Size) + a_2(Size)^2 + a_3(PSHRIMP)$.

Source	d.f.	Sum of Squares	Mean Square
Model	3	20.31201165	6.77067055
Error	162	2.49066546	0.01537448
Corrected total	165	22.80267711	

Model F = 440.38 Pr > F = 0.0001 R^2 = 0.890773

Parameter	Estimate	t for H_0: Parameter=0	Pr > |t|	Std Error of Estimate
a_0	1.42073903	13.89	0.0001	0.10228085
a_1	-0.03525977	-10.46	0.0001	0.00337076
a_2	0.00016368	5.03	0.0001	0.00003252
a_3	0.65890638	17.81	0.0001	0.03698644

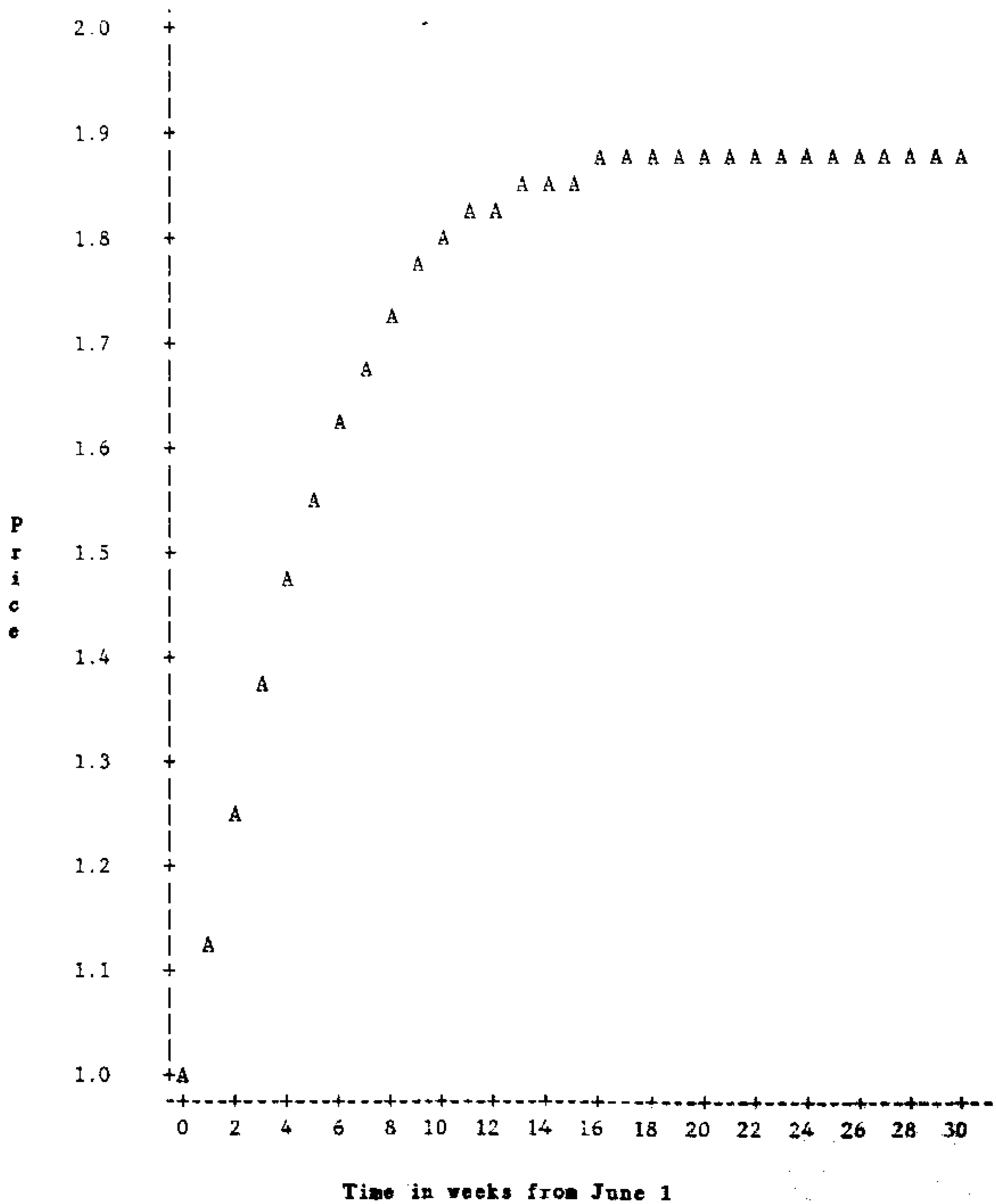

Figure 7. Price function (dollars per shrimp, 1967 dollars) for brown shrimp used in the Pamlico Sound shrimp harvesting problem. (PSHRIMP-2.10)

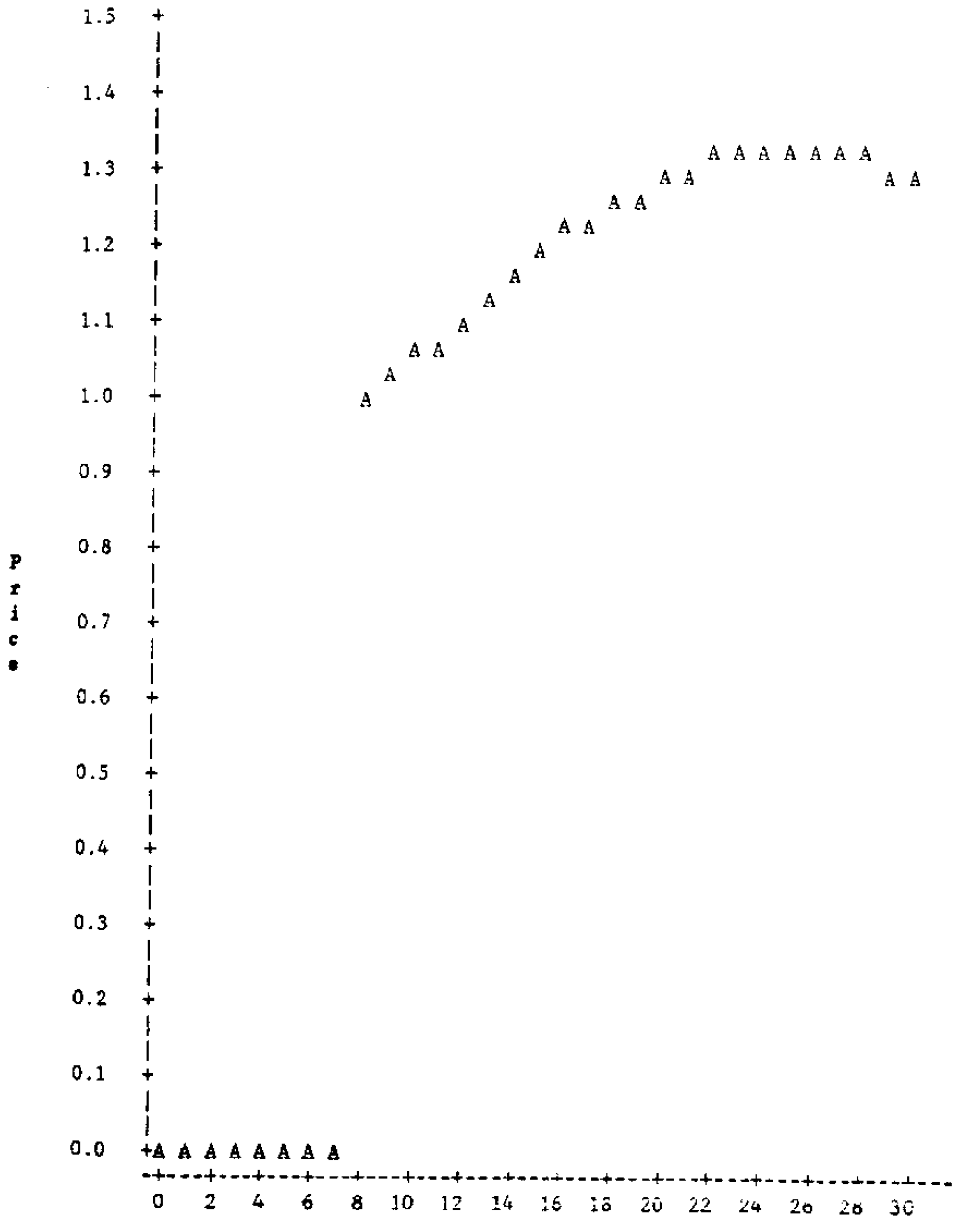

Figure 8. Price function (dollars per shrimp, 1967 dollars) for pink shrimp used in the Pamlico Sound shrimp harvesting problem. (PSHRIMP=2.10)

fished) was determined by adding weekly revenue for boats to weekly revenue for vessels and dividing by the number of vessel-hours.

The effort supply model estimated was as follows:

$$\text{Vessel-hours per week} = e_1R + e_2t + e_3t^2$$

where R is revenue per vessel-hour in 1967 dollars and t is time in weeks from June 1. The approximately linear relationship between effort (vessel-hours per week) and revenue per unit effort is shown in Figure 9. The relationship between effort and time is shown in Figure 10.

Prior to estimating the model, one observation was labelled an outlier and deleted from the dataset. The outlier is shown graphically in Figure 11. The return per effort for this observation was unusually high even though the amount of effort expended during that week was very low (12 hours). Variation from year to year among returns per unit of effort are also apparent from Figure 11.

Results of estimating the effort equation are presented in Table 13. The R^2 was 0.5538 and the three explanatory variables--R, t and t^2--were all statistically significant ($P<0.05$). (Note: Estimation of the model with an intercept term resulted in no significant reduction in the error term.)

Cost Equation

Variable costs were again modelled as proportional to fishing effort:

$$\text{cost per week} = cE(R,t),$$

where E(R,t) is the function used to predict the number of vessel-hours per week and c is the minimum variable cost per vessel-hour. From Figure 9 it can be seen that few observations occurred when the return fell below about 7 dollars per hour. Assuming an 8-hour day, this is roughly equivalent to Waters' estimated cost of 57.83 dollars per day (1967 dollars). The minimum cost per unit of effort, c, was therefore set equal to 7 dollars for the present study. This value keeps fishing effort at zero when the return per effort is less than 7 dollars (even though the linear effort equation will predict low levels of fishing).

Equations of Motion

Equations of motion are the same as those used for the New River fishery except for the parameter values. Parameter values for all but the recruitment function for brown shrimp were taken from Waters (1983).

Waters (1983) summarized natural mortality rates estimated by biologists for the Pamlico Sound fishery and calculated an average instantaneous rate of

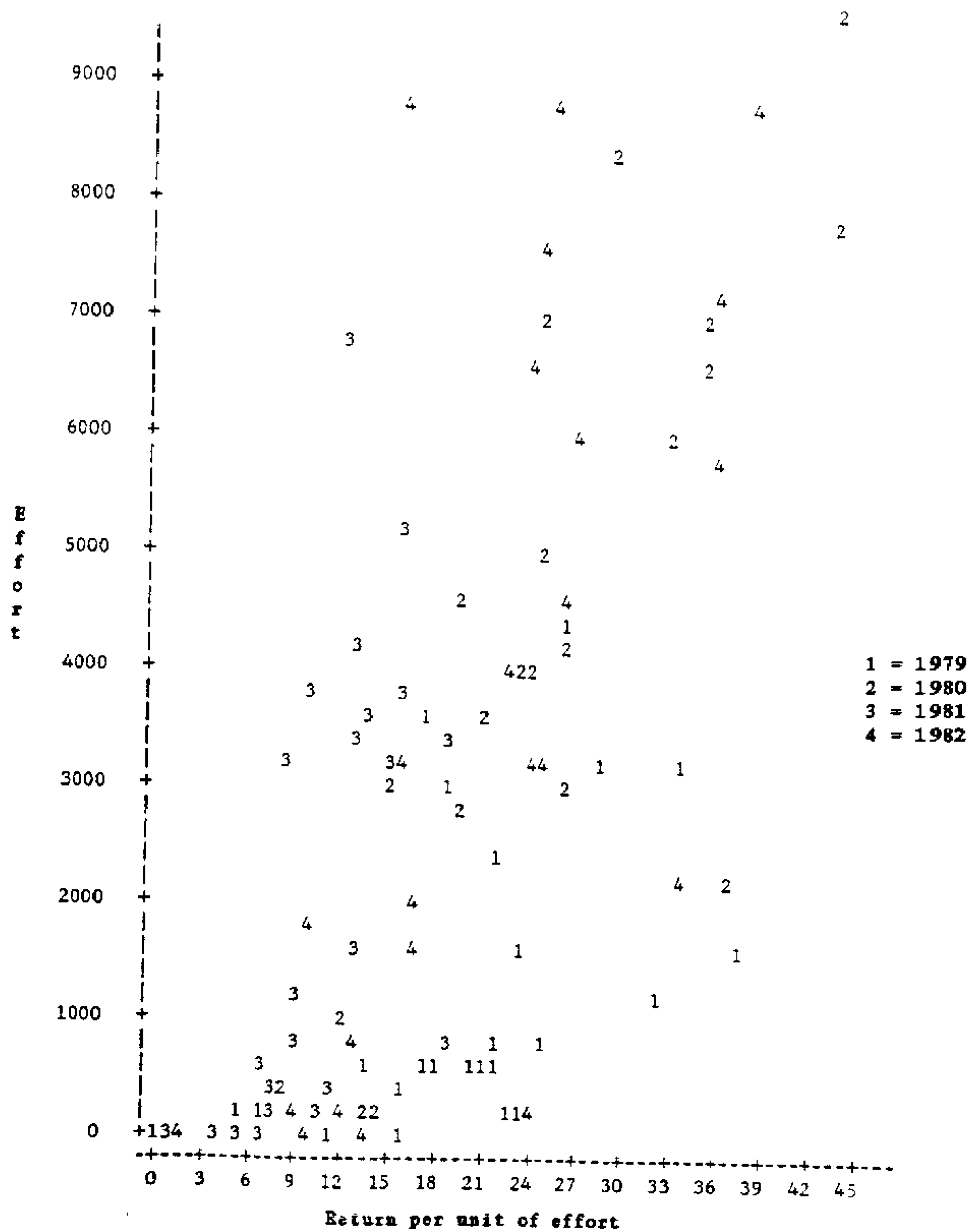

Figure 9. Relationship between effort (vessel-hours) and return per unit of effort (1967 dollars) for the Pamlico Sound shrimp fishery, 1979–1982. (30 observations hidden)

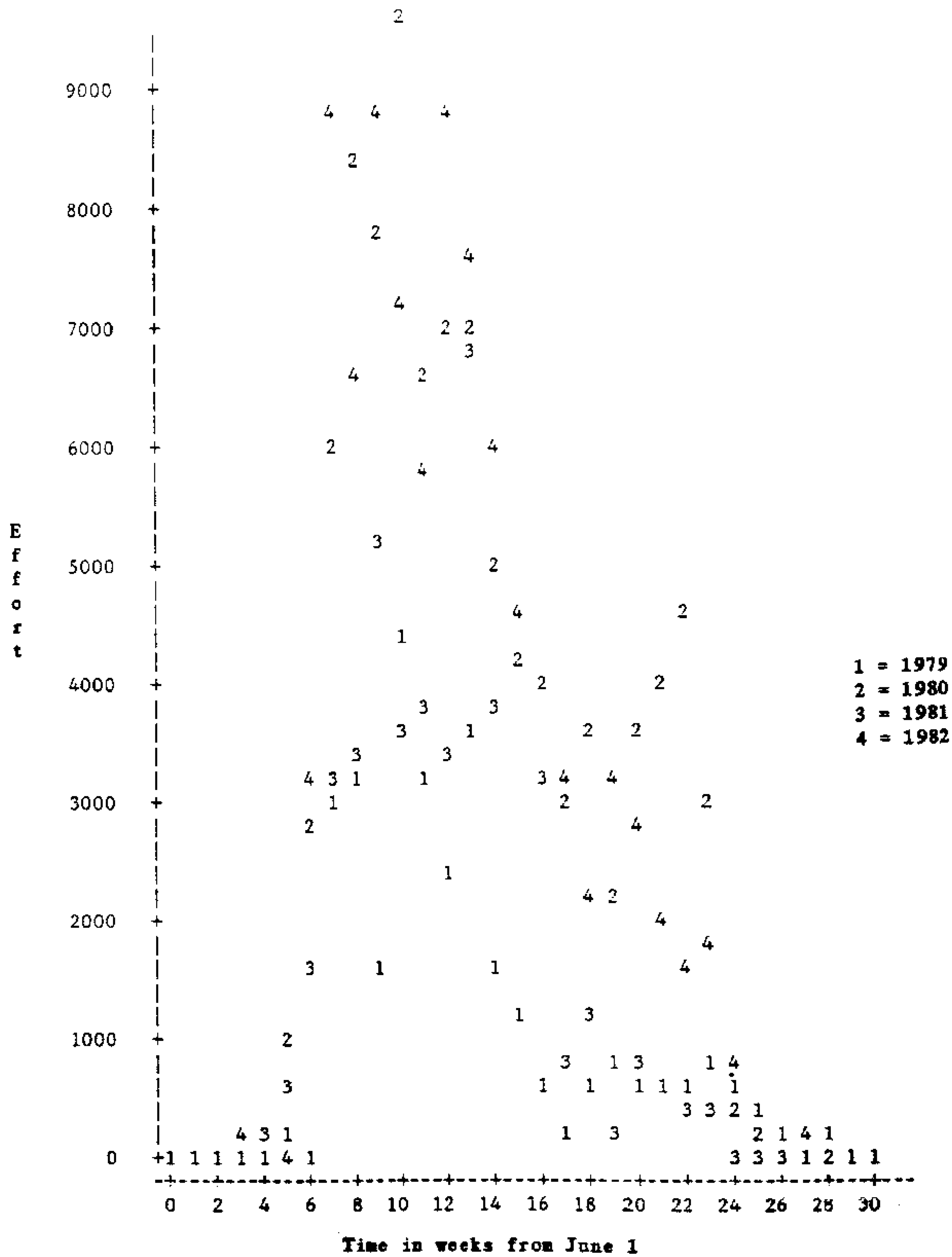

Figure 10. Fishing effort (vessel-hours) versus time in weeks from June 1 for the Pamlico Sound shrimp fishery, 1979-1982. (28 observations hidden)

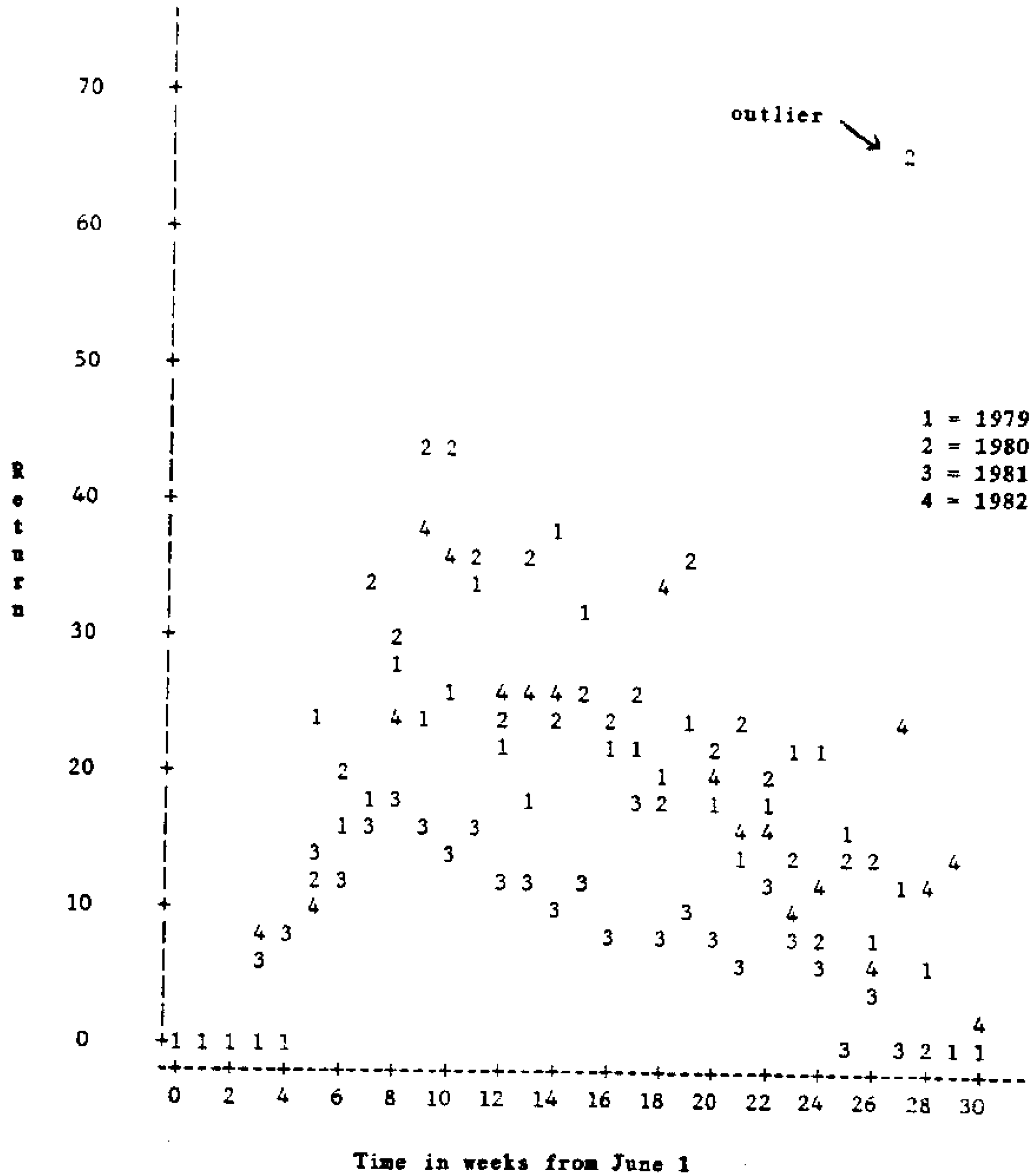

Figure 11. Return per unit of effort (1967 dollars) versus time in weeks from June 1 for the Pamlico Sound shrimp fishery, 1979-1982. (25 observations hidden)

Table 13. Parameter estimates and associated statistics for the effort prediction equation for the Pamlico Sound shrimp fishery.

MODEL: $\text{Vessel-hours} = e_1R + e_1t + e_2t^2$

SOURCE	DF	SUM OF SQUARES	MEAN SQUARE	F VALUE
MODEL	3	1007730672.499	335910224.166	112.88
ERROR	120	357106056.501	2975883.804	
CORRECTED TOTAL	123	1364836729.000		PR>F = 0.0001

VARIABLE	PARAMETER ESTIMATE	T FOR HO: PARAMETER=0	PR > |T|	STD ERROR OF ESTIMATE
R	123.8073169	6.07	0.0001	20.4019520
t	165.7874888	2.17	0.0321	76.4493074
t^2	-7.2548539	-2.62	0.0098	2.7653114

0.30 per week. This value was used in the present study, and additional simulations were done using 0.2 and 0.1 to determine the effects of natural mortality on the optimal harvesting solution. The same mortality rate was applied to both species.

Waters (1983) calculated that an initial population of 240 million brown shrimp (as of approximately July 1) was consistent with the catch statistics for 1978. For pink shrimp, Waters concluded that recruitment of one million shrimp per day was consistent with the same dataset. These values were also used in the present study. Since recruitment for pink shrimp occurred from t=5 to t=24 in the present model, the initial population size used was 140 million (equivalent to recruitment of one million per day for 20 weeks). The analysis was also done using lower initial population sizes of 200 million brown shrimp and 70 million pink shrimp. In actual applications of the model, initial population sizes should be estimated by biological sampling early in the season. It should be noted that these population sizes are for <u>vulnerable</u> shrimp only, and thus exclude shrimp that do not survive to catchable size.

Recruitment functions ($F_i(t)$) were constructed as simple probability distributions so as to be consistent with the biology of the species and with catch-per-effort data on the smallest size class. The recruitment function used for brown shrimp is expressed as follows:

for $t>9$ or $t<4$, $F_1 = 0$,
for $t=4$ or $t=9$, $F_1 = 0.05$,
for $t=5$, $F_1 = 0.15$, and
for $t=6$, 7 or 8, $F_1 = 0.25$.

This is arbitrary, but reasonable after examining the catch-per-effort data for the smallest size class and comparing it to catch-per-effort data for all size classes combined. As mentioned above, the recruitment function for pink shrimp was constant from t=5 through t=24, so F_1=0.05.

In this model, the catchability coefficient, q, represents the proportion of the population removed by a single unit of effort--in this case, by one vessel-hour. Waters (1983) estimated a value of 0.0000109383 per vessel-hour for q using information specific to Pamlico Sound. This value for q was used in the present study.

Calculation of the Optimal Harvest Period

Statement of the Problem

Incorporating the results of the previous section into the shrimp harvesting model, the problem can be re-stated as follows (variables are defined on pages 6 and 7):

maximize with respect to $\Phi(t)$

$$PV = \int_0^T [R(x_1,x_2)E(R,t) - cE(R,t)]e^{-\delta t}\, \phi(t)\, dt$$

such that

$$\dot{x}_1 = F_1(t)x_1(4) - M_1 x_1(t) - qE(R,t)x_1(t)\, \phi(t)$$
$x_1=0$ for $t<4$, $x_1(4)$ given and $x_1(t) \geq 0$,

$$\dot{x}_2 = F_2(t)x_2(5) - M_2 x_2(t) - qE(R,t)x_2(t)\, \Phi(t)$$
$x_2=0$ for $t<5$, $x_2(5)$ given and $x_2(t) \geq 0$,

where

$$R(x_1,x_2) = P(g_1,t,w)g_1(z,t)qx_1(t) + P(g_2,t,w)g_2(z,t)qx_2(t),$$

$$P(g_i,t,w) = a_0 + a_1(1/g_i) + a_2(1/g_i)^2 + a_3 PSHRIMP$$

$$1/g_1(z,t) = S_\infty^{-1}[1-e^{-(kt)}] + 85e^{-(kt)},$$

$$1/g_2(z,t) = S_\infty^{-1}[1-e^{-(kt)}] + 85e^{-(kt)},$$

$$E(R,t) = e_1 R(x_1,x_2) + e_2 t + e_3 t^2,$$

$i=1$ ==> brown shrimp,

$i=2$ ==> pink shrimp,

t = time in units of weeks starting from June 1, and

$\Phi(t)$ = the decision variable ($\phi(t)=0$ implies a closed season and $\phi(t)=1$ implies an open season).

The purpose of this model is to determine the season opening/closing schedule, $\phi(t)$, that maximizes the discounted present value of net returns to the harvesting sector. The potential harvest season spans from June 1 ($t=0$) through December ($t=T=30$). The weekly discount rate, δ, will be set equal to 0.001827 for this study, which is equivalent to an annual discount rate of 10 percent. This is a real rate, which is required because all prices and costs are in units of uninflated dollars (1967 dollars).

The starting time, $t=s_i$, was four for brown shrimp and five for pink shrimp. In the model, population size was set equal to zero prior to this starting time. Since pink shrimp have no commercial value until t=8 in the model, pink shrimp harvested at t=5, t=6 and t=7 represent "bycatch," which is usually discarded by the fishermen (see Waters (1983) for a complete discussion of the bycatch problem).

The F_i functions (recruitment functions) are discrete functions defined as follows:

t	F_1	F_2	t	F_1	F_2	t	F_1	F_2
0	0.00	0.00	10	0.00	0.05	20	0.00	0.05
1	0.00	0.00	11	0.00	0.05	21	0.00	0.05
2	0.00	0.00	12	0.00	0.05	22	0.00	0.05
3	0.00	0.00	13	0.00	0.05	23	0.00	0.05
4	0.05	0.00	14	0.00	0.05	24	0.00	0.05
5	0.15	0.05	15	0.00	0.05	25	0.00	0.00
6	0.25	0.05	16	0.00	0.05	26	0.00	0.00
7	0.25	0.05	17	0.00	0.05	27	0.00	0.00
8	0.25	0.05	18	0.00	0.05	28	0.00	0.00
9	0.05	0.05	19	0.00	0.05	29	0.00	0.00
						30	0.00	0.00

Coefficients were estimated as follows:

$e_1 = 123.807$
$e_2 = 165.787$
$e_3 = -7.2548$

$S_\infty^{-1} = 30.436$
$k = 0.2689$
$b_1 = 116.1157$
$b_2 = -4.7997$
$b_3 = 0.0965$

$a_0 = 1.4207$
$a_1 = -0.03526$
$a_2 = 0.0001637$
$a_3 = 0.658906$

Exogenous variables were assigned the following values:

$M_1 = M_2 = 0.1,\ 0.2,\ \text{or}\ 0.3$
$q = .0000109383$
$c = 7$
$x_1(4) = 240$ or 200 million
$x_2(5) = 140$ or 70 million
PSHRIMP = 1.66 or 2.10

The optimization problem was solved for 12 sets of exogenous variables—high and low price levels at each of three natural mortality rates and two population sizes. In actual practice, each of the above exogenous variables and the recruitment functions will need to be estimated or forecasted before solving the harvesting problem. The above problem represents only 12 hypothetical situations with the same recruitment pattern, and so the solutions obtained will not apply to all harvesting years.

Solution Procedure

The Hamiltonian (see page 29) is

$$
\begin{aligned}
H(t) = {} & [R(x_1,x_2)-c]E(R,t)e^{-\delta t}\,\Phi(t) \\
& + \lambda_1[F_1(t)x_1(4)-Mx_1(t)-qE(R,t)x_1(t)\ \Phi(t)] \\
& + \lambda_2[F_2(t)x_2(5)-Mx_2(t)-qE(R,t)x_2(t)\ \Phi(t)]
\end{aligned}
$$

which leads to the switching function:

$$
\Phi = \begin{cases}
1 & \text{if } [R(x_1,x_2)-c]E(R,t)e^{-\delta t} - \lambda_1 qE(R,t)x_1(t) \\
 & \quad - \lambda_2 qE(R,t)x_2(t) \ \geq 0 \\
0 & \text{if } [R(x_1,x_2)-c]E(R,t)e^{-\delta t} - \lambda_1 qE(R,t)x_1(t) \\
 & \quad - \lambda_2 qE(R,t)x_2(t) \ < 0
\end{cases}
$$

The system of differential equations is

$$\dot{x}_1 = F_1(t)x_1(4) - Mx_1(t) - qE(R,t)x_1(t)\ \Phi(t)$$

$$\dot{x}_2 = F_2(t)x_2(5) - Mx_2(t) - qE(R,t)x_2(t)\ \Phi(t)$$

$$
\begin{aligned}
\dot{\lambda}_1 = {} & \lambda_1 M + \lambda_1 q[E(R,t)+e_1 qP(g_1,t,w)g_1(z,t)x_1(t)]\ \Phi(t) \\
& - P(g_1,t,w)g_1(z,t)q[E(R,t)+e_1(R(x_1,x_2)-c)]e^{-\delta t}\ \Phi(t)
\end{aligned}
$$

$$
\begin{aligned}
\dot{\lambda}_2 = {} & \lambda_2 M + \lambda_2 q[E(R,t)+e_2 qP(g_2,t,w)g_2(z,t)x_2(t)]\ \Phi(t) \\
& - P(g_2,t,w)g_2(z,t)q[E(R,t)+e_2(R(x_1,x_2)-c)]e^{-\delta t}\ \Phi(t)
\end{aligned}
$$

The optimal solution is obtained by searching over positive values of the adjoint variables, $\lambda_1(0)$ and $\lambda_2(0)$, to find the $\Phi(t)$ corresponding to the maximum net present value of the season harvest. The optimal opening/closing schedule was determined to the nearest week. The algorithm used to solve for the optimal $\Phi(t)$ is presented in Appendix C. It is similar to the algorithm used to solve the New River shrimp problem. The unregulated case was determined by setting both $\lambda_i(0)$'s equal to zero.

Results

The optimum season opening/closing schedule for each of the 12 solutions is contrasted with that for the unregulated case in Table 14. Delaying the season improved the net present value of the harvest only at the lowest natural mortality rate. At natural mortality rates of 0.2 and 0.3 the unregulated case was optimal. At these mortality rates it was not possible to find $\lambda_i(0)$'s greater than zero that resulted in a higher present value of the harvest. At a natural mortality rate of 0.1, the net present value was

Table 14. Summary of harvesting solutions for the Pamlico Sound shrimp fishery for 12 combinations of inputs. Present value and λ_i are in 1967 dollars.

M	PSHRIMP	Optimal $\lambda_i(0)$		Net present value	Optimal season	Millions harvested	
		$\lambda_1(0)$	$\lambda_2(0)$			x_1	x_2
		Initial population size of brown shrimp = 240 million Initial population size of pink shrimp = 140 million					
0.10	1.66	0.01170	0.00275	3,753,280	t = 6-29	97.8	34.7
		unregulated (all λ_i=0)		3,744,378	t = 5-29	98.3	34.8
0.10	2.10	0.01505	0.00399	5,058,593	t = 6-29	106.7	38.3
		unregulated (all λ_i=0)		5,048,728	t = 5-29	107.3	38.4
0.20	1.66	0.00000	0.00000	2,015,456	t = 5-26	59.1	17.7
0.20	2.10	0.00000	0.00000	2,826,793	t = 5-27	66.0	20.0
0.30	1.66	0.00000	0.00000	1,263,909	t = 5-18	40.2	9.6
0.30	2.10	0.00000	0.00000	1,810,726	t = 5-25	44.8	9.0
		Initial population size of brown shrimp = 200 million Initial population size of pink shrimp = 70 million					
0.10	1.66	0.01000	0.00230	2,616,990	t = 6-28	73.2	14.8
		unregulated (all λ_i=0)		2,609,653	t = 5-28	73.6	14.8
0.10	2.10	0.01300	0.00330	3,542,338	t = 6-28	80.2	16.3
		unregulated (all λ_i=0)		3,534,565	t = 5-28	80.6	16.3
0.20	1.66	0.00000	0.00000	1,376,341	t = 5-20	43.3	6.9
0.20	2.10	0.00000	0.00000	1,938,299	t = 5-23	48.7	8.2
0.30	1.66	0.00000	0.00000	859,726	t = 5-15	29.1	3.7
0.30	2.10	0.00000	0.00000	1,238,335	t = 5-16	33.1	4.4

increased slightly over that for the unregulated case by delaying the season one period. However, the gain was negligible. The higher market price produced higher net present values at all three natural mortality rates, but the opening date of the optimal harvest season was the same as that at the lower market price. Similarly, the higher population size resulted in a higher present value, but had no effect on the opening date of the harvest season.

As was found for the New River fishery, there appears to be little or no gain from regulating the Pamlico Sound shrimp fishery beyond current practice. In general, the high natural mortality rates result in more value lost when the season opening is delayed than is gained through growth and increased prices. This is consistent with the conclusion by Waters (1983) that protecting juvenile shrimp from harvest in Pamlico Sound did not result in significantly higher gains to fishermen's income.

SUMMARY AND CONCLUSIONS

A bioeconomic optimal control model presented by Kellogg et al. (1985) was used to determine the optimal season opening/closing schedule for the New River and Pamlico Sound shrimp fisheries. Several solutions were obtained for each fishery by varying natural mortality, initial population size and price. In both cases, the analysis showed little or no gain from delaying the season opening beyond the time when shrimp first reach marketable size.

This is not a new result. Kutkuhn (1966) studied the dynamics of a shrimp fishery in the eastern Gulf of Mexico and similarly concluded that the high growth rates were insufficient to offset substantial losses due to expected mortality. He also concluded that postponing the start of fishing beyond when shrimp reach marketable size (larger than the 70 headless-count designation) was not feasible. McCoy (1972) and Purvis and McCoy (1972) studied the New River fishery and pink shrimp in Pamlico Sound using analytical methods similar to Kutkuhn (1966) and concurred with this management strategy. However, Purvis and McCoy (1974) recommended that some gain would be obtained for the brown shrimp fishery in Pamlico Sound if fishing was prohibited until the shrimp reached a count of 46 to 50 per pound, heads-off. This latter management strategy is not supported by the results of the present study.

However, with different recruitment functions and population sizes than used here, the optimization model might produce different results. Although the examples examined here are representative of the two fisheries, they are nonetheless hypothetical cases only. A different balance between pink and brown shrimp recruitment and abundance could result in situations where a delay in the season opening and perhaps even a short mid-season closure would enhance the value of the fishery. The fisheries should be monitored closely during the pre-harvest period and the model applied for each year in order to determine if exceptions to the existing management strategy are desirable.

Bioeconomic optimal control models are not the only input that should be used by the fishery manager in promulgating regulations. Some aspects of a fishery are not easily incorporated into a model, such as income redistribution, political realities, dynamics of ecosystems, and catastrophic weather events. But management models such as the one used here can provide important insights that cannot be obtained in any other way.

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APPENDIX A: SOLUTION ALGORITHM FOR THE NEW RIVER SHRIMP HARVESTING PROBLEM

The following program is written in IBM-PC Basic.

```
10 REM       SHRIMP HARVESTING PROGRAM: SHRIMP3.BAS
20 REM
30 REM EXOGENOUS VARIABLES USED IN THE PROGRAM
40 REM
50 M=.25              'VALUES USED ARE 0.25 AND 0.35
60 Q=.0008
70 C1=50
80 PSHRIMP=2.1   'VALUES USED ARE 1.66 AND 2.10
90 CUMPV=0:CUMHARV1=0:CUMHARV2=0:CUMHARV3=0
100 X1=0:X2=0:X3=0
110 REM
120 REM  FUNCTIONS USED IN THE PROGRAM
130 REM
140 DEF FNSIZE1(C,T)=1/(44.86*(1-EXP(.8381*T-.005825*C))
    +85*EXP(.8381*T-.005825*C))
150 DEF FNSIZE2(C,T)=1/(48.4*(1-EXP(-.4716*T+.002266*C))
    +85*EXP(-.4716*T+.002266*C))
160 SIZE3=1/70.3
170 DEF FNPRICE1(SIZE1)=1.556-.02503*(1/SIZE1)
    +.0001255*(1/(SIZE1^2))+.34175*PSHRIMP
180 DEF FNPRICE2(SIZE2)=1.556-.02503*(1/SIZE2)
    +.0001255*(1/(SIZE2^2))+.34175*PSHRIMP
190 DEF FNPRICE3(SIZE3)=1.556-.02503*(1/SIZE3)
    +.0001255*(1/(SIZE3^2))+.34175*PSHRIMP
200 DEF FNEFFORT(R,T)=-22.37+.124*R+4.728*T-.1282*(T^2)
210 DEF FNDISCOUNT(T)=EXP(-.001827*T)
220 DEF FNX1DOT(E,X1)=-M*X1-Q*E*X1*PHI
230 DEF FNX2DOT(E,X2)=-M*X2-Q*E*X2*PHI
240 DEF FNX3DOT(E,X3)=-M*X3-Q*E*X3*PHI
250 DEF FNL1DOT(E,R,L1)=(-(E+R*.124-C1*.124)*Q*P1*SIZE1*D*PHI)
    +(L1*(M+Q*PHI*(.124*Q*P1*SIZE1*X1+E)))
260 DEF FNL2DOT(E,R,L2)=(-(E+R*.124-C1*.124)*Q*P2*SIZE2*D*PHI)
    +(L2*(M+Q*PHI*(.124*Q*P2*SIZE2*X2+E)))
270 DEF FNL3DOT(E,R,L3)=(-(E+R*.124-C1*.124)*Q*P3*SIZE3*D*PHI)
    +(L3*(M+Q*PHI*(.124*Q*P3*SIZE3*X3+E)))
280 REM
290 REM SET UP FOR PRINT AND INITIAL LAMBDAS
300 INPUT "INITIAL VALUE FOR L1";L1
310 INPUT "INITIAL VALUE FOR L2";L2
320 INPUT "INITIAL VALUE FOR L3";L3
330 LPRINT "M=";M;"  PSHRIMP=";PSHRIMP;"  L1(0)=";L1;" L2(0)=";L2;
    " L3(0)=";L3
340 U$="## #######.# # ########## ####### ####### #.###### #.######
    #.###### ### #######"
```

```
350 LPRINT "T" TAB(6) "SWITCH" TAB(13) "PHI" TAB(22) "X1" TAB(30) "X2"
    TAB(39) "X3" TAB(48) "L1" TAB(57) "L2" TAB(66) "L3" TAB(70) "EFF"
    TAB(76) "PV"
360 REM
370 REM  THE MAIN PROGRAM
380 REM
390 FOR T=0 TO 39
400 REM DEFINING THE DISCRETE FUNCTION FOR CUMULATIVE TEMPERATURE
410 IF T=0 THEN C=0
420 IF T=1 THEN C=117
430 IF T=2 THEN C=241
440 IF T=3 THEN C=372
450 IF T=4 THEN C=511
460 IF T=5 THEN C=657
470 IF T=6 THEN C=810
480 IF T=7 THEN C=971
490 IF T=8 THEN C=1139
500 IF T=9 THEN C=1314
510 IF T=10 THEN C=1494
520 IF T=11 THEN C=1681
530 IF T=12 THEN C=1872
540 IF T=13 THEN C=2067
550 IF T=14 THEN C=2266
560 IF T=15 THEN C=2467
570 IF T=16 THEN C=2670
580 IF T=17 THEN C=2872
590 IF T=18 THEN C=3075
600 IF T=19 THEN C=3276
610 IF T=20 THEN C=3475
620 IF T=21 THEN C=3670
630 IF T=22 THEN C=3860
640 IF T=23 THEN C=4045
650 IF T=24 THEN C=4225
660 IF T=25 THEN C=4397
670 IF T=26 THEN C=4561
680 IF T=27 THEN C=4718
690 IF T=28 THEN C=4866
700 IF T=29 THEN C=5004
710 IF T=30 THEN C=5134
720 IF T=31 THEN C=5254
730 IF T=32 THEN C=5364
740 IF T=33 THEN C=5465
750 IF T=34 THEN C=5557
760 IF T=35 THEN C=5639
770 IF T=36 THEN C=5714
780 IF T=37 THEN C=5780
790 IF T=38 THEN C=5838
800 IF T=39 THEN C=5890
810 REM
820 REM DEFINING THE DISCRETE RECRUITMENT FUNCTIONS
830 IF T<10 THEN F1=0
840 IF T=10 OR T=16 THEN F1=1/28
850 IF T=11 OR T=15 THEN F1=3/28
```

```
860 IF T=12 OR T=14 THEN F1=6/28
870 IF T=13 THEN F1=8/28
880 IF T>16 THEN F1=0
890 IF T=0 THEN F2=.05
900 IF T=1 THEN F2=.05
910 IF T=2 THEN F2=.05
920 IF T=3 THEN F2=.1
930 IF T=4 THEN F2=.75
940 IF T>4 THEN F2=0
950 IF T<19 OR T>34 THEN F3=0 ELSE F3=1/16
960 REM
970 REM DEFINING INITIAL POPULATION SIZES
980 IF 9<T<17 THEN X1START=215000000# ELSE X1START=0
990 IF T<5 THEN X2START=5900000! ELSE X2START=0
1000 IF 18<T<35 THEN X3START=17000000# ELSE X3START=0
1010 REM
1020 REM RECRUITMENT
1030 X1=X1+F1*X1START
1040 X2=X2+F2*X2START
1050 X3=X3+F3*X3START
1060 REM INITIALIZING AND SETTING PRINT VALUES
1070 X1PR=X1
1080 X2PR=X2
1090 X3PR=X3
1100 L1PR=L1
1110 L2PR=L2
1120 L3PR=L3
1130 HARV1=0:HARV2=0:HARV3=0:PV=0
1140 REM
1150 SIZE1=FNSIZE1(C,T)
1160 SIZE2=FNSIZE2(C,T)
1170 P1=FNPRICE1(SIZE1)
1180 P2=FNPRICE2(SIZE2)
1190 P3=FNPRICE3(SIZE3)
1200 R=(P1*SIZE1*Q*X1)+(P2*SIZE2*Q*X2)+(P3*SIZE3*Q*X3)
1210 E=FNEFFORT(R,T)
1220 IF E<0 THEN E=0
1230 D=FNDISCOUNT(T)
1240 SWITCH=(R*E-C1*E)*D-L1*Q*E*X1-L2*Q*E*X2-L3*Q*E*X3
1250 IF SWITCH>0 THEN PHI=1 ELSE PHI=0
1260 REM
1270 REM SOLVING DIFFERENTIAL EQUATIONS
1272 REM PRICE AND SIZE ARE HELD CONSTANT THROUGH THE WEEK
1280 FOR N=1 TO 5
1290 REM
1300 REM CALC OF CUM. HARVESTS AND PRESENT VALUE
1310 H=.2
1320 RT=T+(N-1)*H
1330 E0=FNEFFORT(R,RT)
1340 E1=FNEFFORT(R,RT+.5*H)
1350 E3=FNEFFORT(R,RT+H)
1360 R=(P1*SIZE1*Q*X1)+(P2*SIZE2*Q*X2)+(P3*SIZE3*Q*X3)
1370 SUBHARV1=H*Q*E0*X1*PHI
```

```
1380 SUBHARV2=H*Q*E0*X2*PHI
1390 SUBHARV3=H*Q*E0*X3*PHI
1400 HARV1=HARV1+SUBHARV1
1410 HARV2=HARV2+SUBHARV2
1420 HARV3=HARV3+SUBHARV3
1430 SUBPV=(P1*SIZE1*Q*X1+P2*SIZE2*Q*X2+P3*SIZE3*Q*X3-C1)*E0*D*H*PHI
1440 PV=SUBPV+PV
1450 REM
1460 REM CALCULATION OF NEW LAMBDAS
1470 REM LAMBDA1
1480 K0L1=FNL1DOT(E0,R,L1)
1490 K1L1=FNL1DOT(E1,R,L1+.5*H*K0L1)
1500 K2L1=FNL1DOT(E1,R,L1+.5*H*K1L1)
1510 K3L1=FNL1DOT(E3,R,L1+H*K2L1)
1520 L1=L1+(H/6)*(K0L1+2*K1L1+2*K2L1+K3L1)
1530 IF L1<0 THEN L1=0
1540 REM LAMBDA2
1550 K0L2=FNL2DOT(E0,R,L2)
1560 K1L2=FNL2DOT(E1,R,L2+.5*H*K0L2)
1570 K2L2=FNL2DOT(E1,R,L2+.5*H*K1L2)
1580 K3L2=FNL2DOT(E3,R,L2+H*K2L2)
1590 L2=L2+(H/6)*(K0L2+2*K1L2+2*K2L2+K3L2)
1600 IF L2<0 THEN L2=0
1610 REM LAMBDA3
1620 K0L3=FNL3DOT(E0,R,L3)
1630 K1L3=FNL3DOT(E1,R,L3+.5*H*K0L3)
1640 K2L3=FNL3DOT(E1,R,L3+.5*H*K1L3)
1650 K3L3=FNL3DOT(E3,R,L3+H*K2L3)
1660 L3=L3+(H/6)*(K0L3+2*K1L3+2*K2L3+K3L3)
1670 IF L3<0 THEN L3=0
1680 REM
1690 REM CALCULATION OF NEW X'S
1700 REM X1
1710 K0X1=FNX1DOT(E0,X1)
1720 K1X1=FNX1DOT(E1,X1+.5*H*K0X1)
1730 K2X1=FNX1DOT(E1,X1+.5*H*K1X1)
1740 K3X1=FNX1DOT(E3,X1+H*K2X1)
1750 X1=X1+(H/6)*(K0X1+2*K1X1+2*K2X1+K3X1)
1760 IF X1<0 THEN X1=0
1770 REM X2
1780 K0X2=FNX2DOT(E0,X2)
1790 K1X2=FNX2DOT(E1,X2+.5*H*K0X2)
1800 K2X2=FNX2DOT(E1,X2+.5*H*K1X2)
1810 K3X2=FNX2DOT(E3,X2+H*K2X2)
1820 X2=X2+(H/6)*(K0X2+2*K1X2+2*K2X2+K3X2)
1830 IF X2<0 THEN X2=0
1840 REM X3
1850 K0X3=FNX3DOT(E0,X3)
1860 K1X3=FNX3DOT(E1,X3+.5*H*K0X3)
1870 K2X3=FNX3DOT(E1,X3+.5*H*K1X3)
1880 K3X3=FNX3DOT(E3,X3+H*K2X3)
1890 X3=X3+(H/6)*(K0X3+2*K1X3+2*K2X3+K3X3)
1900 IF X3<0 THEN X3=0
```

```
1910 NEXT N
1920 CUMPV=CUMPV+PV
1930 CUMHARV1=CUMHARV1+HARV1
1940 CUMHARV2=CUMHARV2+HARV2
1950 CUMHARV3=CUMHARV3+HARV3
1960 LPRINT USING U$;T,SWITCH,PHI,X1PR,X2PR,X3PR,L1PR,L2PR,L3PR,E,PV
1970 NEXT T
1980 PROP1=CUMHARV1/215000000#
1990 PROP2=CUMHARV2/5900000!
2000 PROP3=CUMHARV3/17000000#
2010 LPRINT "CUMHARV1=";CUMHARV1;"  PROPORTION=";PROP1
2020 LPRINT "CUMHARV2=";CUMHARV2;"  PROPORTION=";PROP2
2030 LPRINT "CUMHARV3=";CUMHARV3;"  PROPORTION=";PROP3
2040 LPRINT "CUMPV=";CUMPV
2050 END
```

APPENDIX E: SUMMARY OF COMMERCIAL CATCH STATISTICS FOR THE PAMLICO SOUND SHRIMP FISHERY

Commercial catch data were obtained from the North Carolina Division of Marine Fisheries. Beginning in 1979, the Division of Marine Fisheries and the National Marine Fisheries Service collected weekly catch statistics on shrimp and other selected fisheries in the state. Information included date, geartype, fishing area, number of landings, catch in weight (pounds, heads-off), and the average ex-vessel price per pound for each species and size class. (Some additional catch data were collected during a pilot study in 1978, but were excluded from this analysis because it is believed to be incomplete.) Only data designated for Pamlico Sound proper (area 6354) are included in this analysis. Data from surrounding tributaries and some bays are excluded.

Two geartypes were listed for Pamlico Sound: 1) vessels (craft weighing 5 tons or more and registered as a merchant vessel of the United States) using shrimp trawls, and 2) boats (any craft not identified as a vessel) using shrimp trawls. A measure of weekly total fishing effort (hours fishing) was obtained by adjusting the number of boat-hours to vessel-hours and adding the two together. The method used to adjust boat-hours to vessel-hours is presented in the text.

The weekly values reported here were determined using procedures reported by Kellogg (1985). Each week represents a Friday-to-Friday catch. Port samplers usually interviewed all dealers weekly and reported the catch as a weekly aggregate. But prior to 1982, dealers were occasionally missed, and the catch for that week was recorded with the catch of the following week (Katy West, Statistics Coordinator for the Division of Marine Fisheries, personal communication). It was not possible to correct the records; consequently, any errors that are due to this source remain. Another potential source of error involves periods when no catch was reported. If no catch was reported in the dataset for a particular week, the catch was assumed to be zero.

Table B1. Dataset for Pamlico Sound shrimp. Data from the North Carolina Division of Marine Fisheries. Weight is in pounds (heads-off) and revenue and prices are in current dollars (not adjusted for inflation). Time is measured in weeks from June 1.

Week	Total hours fished	Total revenue	Total weight	Revenue per hour fished	Shrimp price index	Consumer price index
			1979			
0	0	0	0	---	523.8	216.6
1	0	0	0	---	523.8	216.6
2	0	0	0	---	523.8	216.6
3	0	0	0	---	523.8	216.6
4	0	0	0	---	498.6	218.9
5	195	9918	3981	50.86	498.6	218.9
6	58	1957	675	33.74	498.6	218.9
7	2971	121452	34874	40.88	498.6	218.9
8	3290	207371	58758	63.03	498.6	218.9
9	1697	88166	24459	51.95	444.7	221.1
10	4303	251097	63889	58.35	444.7	221.1
11	3220	238161	62130	73.96	444.7	221.1
12	2467	120306	29519	48.77	444.7	221.1
13	3553	139139	31974	39.16	438.0	223.4
14	1582	133322	31370	84.27	438.0	223.4
15	1277	92805	20249	72.67	438.0	223.4
16	674	31717	7210	47.06	438.0	223.4
17	230	11587	2940	50.38	450.7	225.4
18	550	25382	7038	46.15	450.7	225.4
19	705	38832	14195	55.08	450.7	225.4
20	653	26182	9352	40.09	450.7	225.4
21	617	18578	7197	30.11	450.7	225.4
22	674	26677	10161	39.58	430.2	227.5
23	888	43520	15596	49.01	430.2	227.5
24	542	26833	9559	49.51	430.2	227.5
25	494	17704	6329	35.84	430.2	227.5
26	144	2346	780	16.29	415.0	229.9
27	74	1882	697	25.43	415.0	229.9
28	139	1697	800	12.21	415.0	229.9
29	0	0	0	---	415.0	229.9
30	0	0	0	---	415.0	229.9
			1980			
0	0	0	0	---	382.9	247.6
1	0	0	0	---	382.9	247.6
2	0	0	0	---	382.9	247.6
3	0	0	0	---	382.9	247.6

Table B1. continued.

Week	Total hours fished	Total revenue	Total weight	Revenue per hour fished	Shrimp price index	Consumer price index
4	0	0	0	---	381.6	247.8
5	1072	31583	13469	29.46	381.6	247.8
6	2830	134536	53481	47.54	381.6	247.8
7	6077	498956	177117	82.11	381.6	247.8
8	8382	603066	190396	71.95	381.6	247.8
9	7778	845005	262425	108.64	388.2	249.4
10	9548	1032608	304375	108.15	388.2	249.4
11	6676	590046	169041	88.38	388.2	249.4
12	7002	430177	117295	61.44	388.2	249.4
13	7074	627669	161071	88.73	373.6	251.7
14	5052	313871	91124	62.13	373.6	251.7
15	4186	275648	84072	65.85	373.6	251.7
16	4087	237819	77904	58.19	373.6	251.7
17	3048	204587	64440	67.12	344.4	253.9
18	3675	158961	59706	43.25	344.4	253.9
19	2281	211786	78736	92.85	344.4	253.9
20	3681	199432	73073	54.18	344.4	253.9
21	3901	237325	91730	60.84	344.4	253.9
22	4657	236142	90502	50.71	340.3	256.2
23	2957	112442	48518	38.03	340.3	256.2
24	499	10696	4429	21.43	340.3	256.2
25	264	9289	4120	35.19	340.3	256.2
26	293	10982	4401	37.48	341.3	258.4
27	12	2041	907	170.08*	341.3	258.4
28	0	0	0	---	341.3	258.4
29	0	0	0	---	341.3	258.4
30	0	0	0	---	341.3	258.4
			1981			
0	0	0	0	---	421.6	271.3
1	19	22	7	1.16	421.6	271.3
2	0	0	0	---	421.6	271.3
3	19	278	87	14.63	421.6	271.3
4	137	2960	1155	21.61	382.4	274.4
5	644	24516	8318	38.07	382.4	274.4
6	1568	53803	16524	34.31	382.4	274.4
7	3116	130392	36010	41.85	382.4	274.4
8	3485	176891	46642	50.76	382.4	274.4
9	5222	230135	60642	44.07	329.9	276.5
10	3697	141213	37376	38.20	329.9	276.5
11	3718	163039	42002	43.85	329.9	276.5
12	3397	117130	29126	34.48	329.9	276.5

Table B1. continued.

Week	Total hours fished	Total revenue	Total weight	Revenue per hour fished	Shrimp price index	Consumer price index
13	6792	231082	61261	34.02	373.7	279.3
14	3872	106916	26821	27.61	373.7	279.3
15	4217	150297	36184	35.64	373.7	279.3
16	3216	73843	20148	22.96	373.7	279.3
17	826	42483	11731	51.43	426.1	279.9
18	1267	31659	8875	24.99	426.1	279.9
19	185	5541	1786	29.95	426.1	279.9
20	710	17429	5293	24.55	426.1	279.9
21	574	10855	3335	18.91	426.1	279.9
22	377	11755	3474	31.18	405.1	280.7
23	442	9298	2884	21.04	405.1	280.7
24	89	1674	490	18.81	405.1	280.7
25	0	0	0	---	405.1	280.7
26	67	755	229	11.27	401.9	281.5
27	0	0	0	---	401.9	281.5
28	0	0	0	---	401.9	281.5
29	0	0	0	---	401.9	281.5
30	0	0	0	---	401.9	281.5
			1982			
0	0	0	0	---	453.6	290.6
1	0	0	0	---	453.6	290.6
2	0	0	0	---	453.6	290.6
3	120	3081	1121	25.68	453.6	290.6
4	0	0	0	---	439.5	292.2
5	43	1232	355	28.65	439.5	292.2
6	3170	145249	44090	45.82	439.5	292.2
7	8703	404031	111773	46.42	439.5	292.2
8	6514	458428	116396	70.38	439.5	292.2
9	8846	988188	239323	111.71	492.1	292.8
10	7238	756656	173643	104.54	492.1	292.8
11	5729	598435	136972	104.46	492.1	292.8
12	8830	663688	138670	75.16	492.1	292.8
13	7558	556610	111092	73.65	510.8	293.3
14	6072	475520	90807	78.31	510.8	293.3
15	4519	350545	67242	77.57	510.8	293.3
16	4003	265701	51531	66.38	510.8	293.3
17	3142	231485	47704	73.67	498.0	294.1
18	2258	226156	52138	100.16	498.0	294.1
19	3107	219010	54000	70.49	498.0	294.1
20	2848	161158	42075	56.59	498.0	294.1
21	1997	94897	24541	47.52	498.0	294.1

Table B1. continued.

Week	Total hours fished	Total revenue	Total weight	Revenue per hour fished	Shrimp price index	Consumer price index
22	1630	77723	21251	47.68	517.6	293.6
23	1843	54525	14798	29.58	517.6	293.6
24	871	32681	9125	37.52	517.6	293.6
25	269	10708	3050	39.81	517.6	293.6
26	36	706	215	19.61	525.0	292.4
27	106	7527	2023	71.01	525.0	292.4
28	245	8403	2230	34.30	525.0	292.4
29	79	3120	880	39.49	525.0	292.4
30	50	265	74	5.30	525.0	292.4

e B1. continued.

Brown Shrimp				Pink Shrimp			
Weight	Number	Average size	Average price	Weight	Number	Average size	Average price
			1979				
0	0	---	--	0	0	---	--
0	0	---	--	0	0	---	--
0	0	---	--	0	0	---	--
0	0	---	--	0	0	---	--
0	0	---	--	0	0	---	--
3981	249739	62.73	2.49	0	0	---	--
675	36445	53.99	2.90	0	0	---	--
34870	1447618	41.51	3.48	4	340	85.00	2.00
58732	2301238	39.18	3.53	26	2210	85.00	2.00
23988	868744	36.22	3.63	471	39995	84.92	2.16
62593	2155105	34.43	3.97	1296	108660	83.84	1.83
55277	1773886	32.09	4.08	6853	568945	83.02	1.84
26703	800401	29.97	4.29	2816	230632	81.90	2.08
30105	923505	30.68	4.49	1869	147133	78.72	2.18
23451	565288	24.11	4.93	7919	525575	66.37	2.23
17618	478113	27.14	4.97	2631	176515	67.09	2.00
5798	145874	25.16	5.02	1412	92960	65.84	1.87
2009	55387	27.57	4.59	931	63675	68.39	2.53
3918	118644	30.28	4.47	3120	195386	62.62	2.52
3126	114728	36.70	3.30	11069	713867	64.49	2.58
2504	95972	38.33	3.36	6848	441378	64.45	2.59
1235	51240	41.49	3.81	5962	406236	68.14	2.33
823	24114	29.30	4.67	9338	591964	63.39	2.45
97	2716	28.00	4.49	15499	870439	56.16	2.78
0	0	---	--	9559	536290	56.10	2.81
0	0	---	--	6329	344582	54.44	2.80
130	4290	33.00	4.45	650	37550	57.77	2.72
0	0	---	--	697	59245	85.00	2.70
0	0	---	--	800	51716	64.65	2.12
0	0	---	--	0	0	---	--
0	0	---	--	0	0	---	--
			1980				
0	0	---	--	0	0	---	--
0	0	---	--	0	0	---	--
0	0	---	--	0	0	---	--
0	0	---	--	0	0	---	--
0	0	---	--	0	0	---	--
13469	728345	54.08	2.34	0	0	---	--

Table B1. continued.

	Brown Shrimp				Pink Shrimp			
Week	Weight	Number	Average size	Average price	Weight	Number	Average size	Average price
6	53481	2478540	46.34	2.52	0	0	—	—
7	177117	7408221	41.83	2.82	0	0	—	—
8	190396	6886450	36.17	3.17	0	0	—	—
9	262425	9245101	35.23	3.22	0	0	—	—
10	304305	10135540	33.31	3.39	20	1700	85.00	1.40
11	168955	5584074	33.05	3.49	20	1700	85.00	1.40
12	116827	3621585	31.00	3.68	400	34000	85.00	—
13	157353	4344376	27.61	3.95	3693	260917	70.65	1.65
14	80322	2441553	30.40	3.60	10802	594900	55.07	2.26
15	69988	2221163	31.74	3.52	14084	875350	62.15	2.06
16	58461	1914811	32.75	3.40	19443	1181584	60.77	2.01
17	46115	136422	29.59	3.57	18325	967329	52.79	2.19
18	35813	1303174	36.39	3.12	23057	1518223	65.85	1.95
19	41197	1532806	37.21	3.16	37164	2170816	58.41	2.17
20	42293	1588264	37.55	2.97	30230	1620471	53.60	2.39
21	40758	1590104	39.01	2.93	50377	2831895	56.21	2.30
22	49207	1900756	38.63	2.95	40443	2413079	59.67	2.19
23	7308	240364	32.89	3.23	39910	2347414	58.82	2.13
24	425	14460	34.02	3.07	4004	215744	53.88	2.35
25	0	0	—	—	4120	236175	57.32	2.25
26	650	21450	33.00	3.00	3751	203308	54.20	2.41
27	0	0	—	—	907	68995	76.07	2.25
28	0	0	—	—	0	0	—	—
29	0	0	—	—	0	0	—	—
30	0	0	—	—	0	0	—	—
				1981				
0	0	0	—	—	0	0	—	—
1	0	0	—	—	7	336	48.00*	3.14*
2	0	0	—	—	0	0	—	—
3	0	0	—	—	87	3961	45.53*	3.20*
4	1155	57090	49.43	2.56	0	0	—	—
5	8318	340394	40.92	2.95	0	0	—	—
6	16524	600500	36.34	3.26	0	0	—	—
7	36010	1103677	30.65	3.62	0	0	—	—
8	46642	1259359	27.00	3.79	0	0	—	—
9	60225	1463172	24.30	3.81	392	25687	65.53	2.28
10	36968	893139	24.16	3.80	408	29480	72.25	2.01
11	40831	952353	23.32	3.93	1171	80816	69.01	2.12
12	27981	587853	21.01	4.10	1145	75025	65.52	2.13
13	58809	1395052	23.72	3.84	2452	166094	67.74	2.10

Table B1. continued.

Week	Brown Shrimp Weight	Brown Shrimp Number	Brown Shrimp Average size	Brown Shrimp Average price	Pink Shrimp Weight	Pink Shrimp Number	Pink Shrimp Average size	Pink Shrimp Average price
14	24777	546841	22.07	4.14	2044	134340	65.72	2.10
15	32184	619627	19.25	4.41	4000	254848	63.71	2.11
16	17247	444366	25.76	3.93	2901	191451	65.99	2.09
17	10819	344662	31.86	3.75	912	62608	68.65	2.09
18	5770	170415	29.53	3.85	3105	159073	51.23	3.04
19	1676	57918	34.56	3.17	110	7150	65.00	2.00
20	2871	96793	33.71	3.64	2422	131962	54.48	2.88
21	2240	79745	35.60	3.34	1095	58623	53.54	3.08
22	1621	52898	32.63	3.89	1853	95214	51.38	2.94
23	0	0	---	--	2884	119401	41.40	3.22
24	0	0	---	--	490	23360	47.67	3.42
25	0	0	---	--	0	0	---	--
26	0	0	---	--	229	7557	33.00	3.30
27	0	0	---	--	0	0	---	--
28	0	0	---	--	0	0	---	--
29	0	0	---	--	0	0	---	--
30	0	0	---	--	0	0	---	--
				1982				
0	0	0	---	--	0	0	---	--
1	0	0	---	--	0	0	---	--
2	0	0	---	--	0	0	---	--
3	493	34233	69.44	2.40	628	25314	40.31*	3.02*
4	0	0	---	--	0	0	---	--
5	355	15265	43.00	3.47	0	0	---	--
6	44090	2034964	46.15	3.29	0	0	---	--
7	111773	4314441	38.60	3.61	0	0	---	--
8	116396	4055387	34.84	3.94	0	0	---	--
9	239323	7727266	32.29	4.13	0	0	---	--
10	171493	5375330	31.34	4.39	1950	130658	67.00	2.17
11	129283	3936394	30.45	4.49	7434	528843	71.14	2.28
12	130533	3667446	28.10	4.92	7860	530498	67.49	2.75
13	102814	2769757	26.94	5.19	7923	577371	72.87	2.78
14	82435	1998535	24.24	5.46	8352	615055	73.64	3.07
15	59454	1391807	23.41	5.57	7765	557537	71.80	2.45
16	43289	1036667	23.95	5.62	8206	531836	64.81	2.73
17	35982	913166	25.38	5.46	11575	701752	60.63	2.97
18	34415	1018435	29.59	4.99	17642	1129092	64.00	3.06
19	20713	597499	28.85	5.30	33287	2027964	60.92	3.28
20	11196	345623	30.87	5.09	30879	1786654	57.86	3.37
21	6686	203743	30.47	5.12	17855	969523	54.30	3.40

Table B1. continued.

Week	Brown Shrimp Weight	Number	Average size	Average price	Pink Shrimp Weight	Number	Average size	Average price
22	4306	136068	31.60	5.04	16945	977734	57.70	3.31
23	4446	156493	35.20	4.64	10352	569603	55.02	3.27
24	992	40071	40.39	4.16	8133	433919	53.35	3.51
25	438	14814	33.82	4.76	2612	148297	56.78	3.30
26	0	0	---	--	215	11620	54.05	3.28
27	520	17325	33.32	5.07	1503	92277	61.40	3.25
28	217	8091	37.29	4.71	2013	109153	54.22	3.67
29	682	34441	50.50	3.45	198	10380	52.42	3.87
30	0	0	---	--	74	3930	53.10	3.58

Note: Data indicated by an asterisk (*) were excluded from the analysis.

APPENDIX C: SOLUTION ALGORITHM FOR THE PAMLICO SOUND SHRIMP HARVESTING PROBLEM

The following program is written in IBM-PC Basic.

```
10 REM       SHRIMP HARVESTING PROGRAM FOR PAMLICO SOUND
20 REM
30 REM EXOGENOUS VARIABLES USED IN THE PROGRAM
40 REM
50 M=.30                'VALUES USED ARE 0.1, .2, AND .3
60 Q=.000010938
70 C1=7
80 PSHRIMP=2.1    'VALUES USED ARE 1.66 AND 2.10
90 CUMPV=0:CUMHARV1=0:CUMHARV2=0
100 X1=0:X2=0
110 REM
120 REM  FUNCTIONS USED IN THE PROGRAM
130 REM
140 DEF FNSIZE1(T)=1/(30.436+(85-30.436)*EXP(-.268*T))
150 DEF FNSIZE2(T)=1/(116.1157-4.7997*T+.0965*(T^2))
170 DEF FNPRICE1(SIZE1)=1.4027-.03526*(1/SIZE1)
        +.0001637*(1/(SIZE1^2))+.65891*PSHRIMP
180 DEF FNPRICE2(SIZE2)=1.4027-.03526*(1/SIZE2)
        +.0001637*(1/(SIZE2^2))+.65891*PSHRIMP
200 DEF FNEFFORT(R,T)=123.807*R+165.787*T-7.25485*(T^2)
210 DEF FNDISCOUNT(T)=EXP(-.001827*T)
220 DEF FNX1DOT(E,X1)=-M*X1-Q*E*X1*PHI
230 DEF FNX2DOT(E,X2)=-M*X2-Q*E*X2*PHI
250 DEF FNL1DOT(E,R,L1)=(-(E+R*123.8-C1*123.8)*Q*P1*SIZE1*D*PHI)
        +(L1*(M+Q*PHI*(123.8*Q*P1*SIZE1*X1+E)))
260 DEF FNL2DOT(E,R,L2)=(-(E+R*123.8-C1*123.8)*Q*P2*SIZE2*D*PHI)
        +(L2*(M+Q*PHI*(123.8*Q*P2*SIZE2*X2+E)))
280 REM
290 REM SET UP FOR PRINT AND INITIAL LAMBDAS
300 INPUT "INITIAL VALUE FOR L1";L1
310 INPUT "INITIAL VALUE FOR L2";L2
330 PRINT "M=";M;"  PSHRIMP=";PSHRIMP;"  L1(0)=";L1;" L2(0)=";L2
340 U$="## ####### # ########## ######## #.###### #.###### #####
     ####### ###.## "
350 PRINT "T" TAB(4) "SWITCH" TAB(11) "PHI" TAB(18) "X1" TAB(28)
      "X2" TAB(35) "L1" TAB(45) "L2" TAB(51) "EFF" TAB(58) "PV"
      TAB(66) "R"
360 REM
370 REM  THE MAIN PROGRAM
380 REM
390 FOR T=0 TO 30
810 REM
820 REM DEFINING THE DISCRETE RECRUITMENT FUNCTIONS
830 IF T<4 THEN F1=0
840 IF T=4 OR T=9  THEN F1=.05
```

```
850 IF T=5 THEN F1=.15
860 IF T=6 OR T=7 OR T=8 THEN F1=.25
880 IF T>9  THEN F1=0
900 F2=1/20
905 if T<5 THEN F2=0
910 IF T>24 THEN F2=0
960 REM
970 REM DEFINING INITIAL POPULATION SIZES
980 IF T<11 THEN X1START=240000000#
990 ELSE X1START=0
1000 IF T<25 THEN X2START=(7*20*1000000)
1005 ELSE X2START=0
1010 REM
1020 REM RECRUITMENT
1030 X1=X1+F1*X1START
1040 X2=X2+F2*X2START
1060 REM INITIALIZING AND SETTING PRINT VALUES
1070 X1PR=X1:X2PR=X2:L1PR=L1:L2PR=L2
1130 HARV1=0:HARV2=0:PV=0
1140 REM
1150 SIZE1=FNSIZE1(T)
1160 SIZE2=FNSIZE2(T)
1170 P1=FNPRICE1(SIZE1)
1180 P2=FNPRICE2(SIZE2)
1190 IF SIZE1<(1/85) THEN P1=0
1199 IF SIZE2<(1/85) THEN P2=0
1200 R=(P1*SIZE1*Q*X1)+(P2*SIZE2*Q*X2)
1205 RPRNT=R
1210 E=FNEFFORT(R,T)
1220 IF E<0 THEN E=0
1230 D=FNDISCOUNT(T)
1240 SWITCH=(R*E-C1*E)*D-L1*Q*E*X1-L2*Q*E*X2
1250 IF SWITCH>0 THEN PHI=1 ELSE PHI=0
1260 REM
1270 REM SOLVING DIFFERENTIAL EQUATIONS
1280 FOR N=1 TO 5
1290 REM
1300 REM CALC OF CUM. HARVESTS AND PRESENT VALUE
1310 H=.2
1320 RT=T+(N-1)*H
1330 E0=FNEFFORT(R,RT)
1340 E1=FNEFFORT(R,RT+.5*H)
1350 E3=FNEFFORT(R,RT+H)
1360 R=(P1*SIZE1*Q*X1)+(P2*SIZE2*Q*X2)
1370 SUBHARV1=H*Q*E0*X1*PHI
1380 SUBHARV2=H*Q*E0*X2*PHI
1400 HARV1=HARV1+SUBHARV1
1410 HARV2=HARV2+SUBHARV2
1430 SUBPV=(P1*SIZE1*Q*X1+P2*SIZE2*Q*X2-C1)*E0*D*H*PHI
1440 PV=SUBPV+PV
1450 REM
```

```
REM CALCULATION OF NEW LAMBDAS
REM LAMBDA1
K0L1=FNL1DOT(E0,R,L1)
K1L1=FNL1DOT(E1,R,L1+.5*H*K0L1)
K2L1=FNL1DOT(E1,R,L1+.5*H*K1L1)
K3L1=FNL1DOT(E3,R,L1+H*K2L1)
L1=L1+(H/6)*(K0L1+2*K1L1+2*K2L1+K3L1)
IF L1<0 THEN L1=0
REM LAMBDA2
K0L2=FNL2DOT(E0,R,L2)
K1L2=FNL2DOT(E1,R,L2+.5*H*K0L2)
K2L2=FNL2DOT(E1,R,L2+.5*H*K1L2)
K3L2=FNL2DOT(E3,R,L2+H*K2L2)
L2=L2+(H/6)*(K0L2+2*K1L2+2*K2L2+K3L2)
IF L2<0 THEN L2=0
REM
REM CALCULATION OF NEW X'S
REM X1
K0X1=FNX1DOT(E0,X1)
K1X1=FNX1DOT(E1,X1+.5*H*K0X1)
K2X1=FNX1DOT(E1,X1+.5*H*K1X1)
K3X1=FNX1DOT(E3,X1+H*K2X1)
X1=X1+(H/6)*(K0X1+2*K1X1+2*K2X1+K3X1)
IF X1<0 THEN X1=0
REM X2
K0X2=FNX2DOT(E0,X2)
K1X2=FNX2DOT(E1,X2+.5*H*K0X2)
K2X2=FNX2DOT(E1,X2+.5*H*K1X2)
K3X2=FNX2DOT(E3,X2+H*K2X2)
X2=X2+(H/6)*(K0X2+2*K1X2+2*K2X2+K3X2)
IF X2<0 THEN X2=0
NEXT N
CUMPV=CUMPV+PV
CUMHARV1=CUMHARV1+HARV1
CUMHARV2=CUMHARV2+HARV2
PRINT USING U$;T,SWITCH,PHI,X1PR,X2PR,L1PR,L2PR,E,PV,RPRNT
NEXT T
PRINT "CUMHARV1=";CUMHARV1
PRINT "CUMHARV2=";CUMHARV2
PRINT "CUMPV=";CUMPV
END
```