A SUGGESTION FOR ANTICIPATING ALTERATIONS IN WAVE ACTION ON SHORES CONSEQUENT UPON CHANGES IN WATER DEPTHS IN HARBORS AND COASTAL WATERS

B. KINSMAN

J.R. SCHUBEL

G.E. CARROLL

M. GLACKIN-SUNDELL

marine sciences research center
state university of new york

new york sea grant institute
suny and cornell

SPECIAL REPORT 27

REFERENCE 79-10

ERRATA SHEET

Page 10

Figure 7C. Number XIII should read VIII.

Pages 30 and 31

Equation (4) and the sentence immediately following it appear as the last paragraph on page 31. They should be inserted on page 30 immediately preceding equation (5).

MARINE SCIENCES RESEARCH CENTER
STATE UNIVERSITY OF NEW YORK
STONY BROOK, NEW YORK 11763

A SUGGESTION FOR ANTICIPATING ALTERATIONS IN WAVE ACTION ON SHORES CONSEQUENT UPON CHANGES IN WATER DEPTHS IN HARBORS AND COASTAL WATERS

Blair Kinsman, J.R. Schubel, George E. Carroll,
Mary Glackin-Sundell

July 1979

*Sponsored by the New York Sea Grant Institute
through a contract with the New York State
Office of General Services.*

Special Report 27

Reference 79-10

Approved for Distribution

J.R. Schubel

J. R. Schubel, Director

ACKNOWLEDGMENTS

This research was supported by the New York Sea Grant Institute through a contract with the New York State Office of General Services. The figures were drafted by Carol Cassidy and Marie Eisel, and the manuscript typed by Susan Risoli. Preparation of the report for printing was supervised by Ms. Jeri Schoof.

TABLE OF CONTENTS

Page

LIST OF TABLES

LIST OF FIGURES

INTRODUCTION

The most important fact to keep in mind is that ocean surface waves, like any waves, transmit energy. In the end, that energy, whose original source was the energy of the wind, is expended on the shore where it plays an important role in coastal erosion. The direction in which the wave energy is transmitted and which part of the coastline it will attack depends on the wave length and the depth of the water in which it runs.

To visualize the effect of depth on a wave train consider Fig. 1. Offshore in *deep* water the wave crests are parallel and the energy they carry, which is proportional to the square of the wave height, is transmitted perpendicular to the wave crests along the wave rays. The energy between wave rays, whatever it may be, remains constant. The speed of energy propagation depends on the length of the wave, the longer the faster, and is unaffected by the water depth. The bottom is too far away to have an influence.

Now, suppose this offshore wave train approaches an even, greatly sloping beach at an angle as shown in Fig. 1. On the right the wave reaches shoaling water, begins to "feel bottom," and slows down while on the left it continues to run at speed. As a result, the crest becomes bowed and the rays curve since the direction of propagation remains locally perpendicular to the crest. The entire train is slewed around and approaches the beach more nearly head on. In short, the wave is refracted, being bent toward shallower water.

When the coastal bathymetry is more complex the direction of wave energy propagation can be very substantially altered and the energy may be either focused or defocused. Consider a wave rolling in over a submarine ridge as shown in Fig. 2. The center of the train meets shallow water first. On either side of the ridge it is still in deeper water and continues to run faster. The rays are bent inward toward the ridge. Since energy between rays remains constant, the converging rays

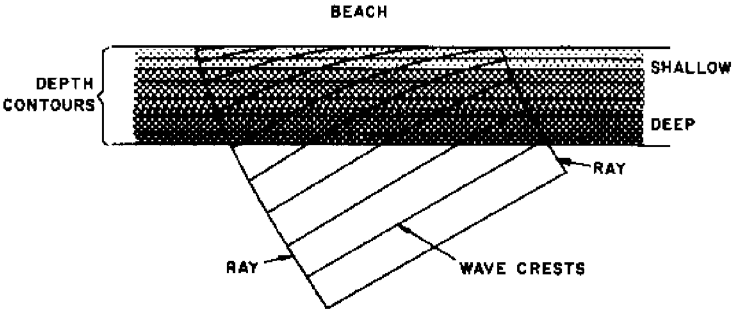

Fig. 1 The refraction of a wave train approaching an even gently sloping beach at an angle [Kinsman, 1965].

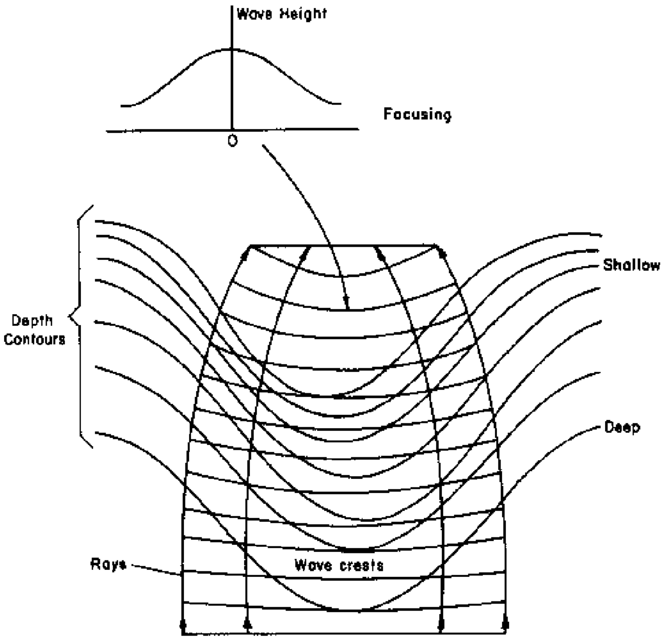

Fig. 2 The refraction of a wave train over a submarine ridge [Kinsman, 1965].

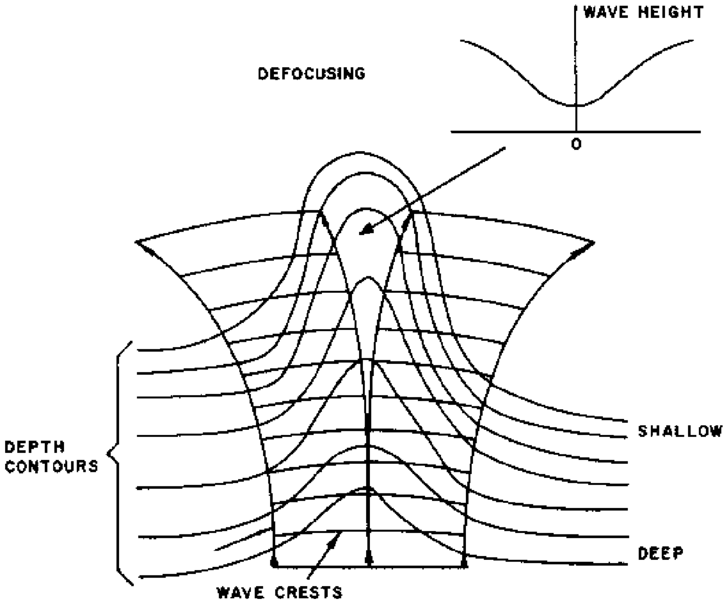

Fig. 3 The refraction of a wave train over a submarine valley [Kinsman, 1965].

confine the energy within a smaller area and the waves respond by becoming higher. Wave energy is focused over a submarine ridge. Coastal headlands are frequently associated with submarine ridges which extend out from them. These ridges focus wave energy on the headlands which are typically sites of rapid erosion.

Exactly the reverse process occurs over submarine valleys, Fig. 3. Over a valley the center continues at the higher speed while the parts to either side in the shallower water slow down. The rays diverge and the wave heights decrease. The wave energy is defocused. This phenomenon is well known to fishermen who anchor over submarine valleys where they can safely ride out even quite severe seas.

The changes in water depth illustrated in Figs. 1-3 are what might be called "gentle". Adjacent rays which start off parallel change direction and converge or diverge but they do not cross. The more abrupt changes in bathymetry often found in nature lead to crossing ray patterns, Fig. 4. On the argument that the energy of the wave train between adjacent rays remains constant and that wave heights increase as the distance between rays decreases one might argue that in the vicinity of the crossing point the wave height must increase indefinitely. Of course, this does not happen. There is a limit to the height of a wave in relaton to its length beyond which the wave becomes unstable and breaks. The Stokes theoretical maximum is H/L = 1/7 = 0.143. Waves this high are seldom found at sea; H/L from 0.1 to 0.008 being typical. When waves become "too high" they break. In breaking,

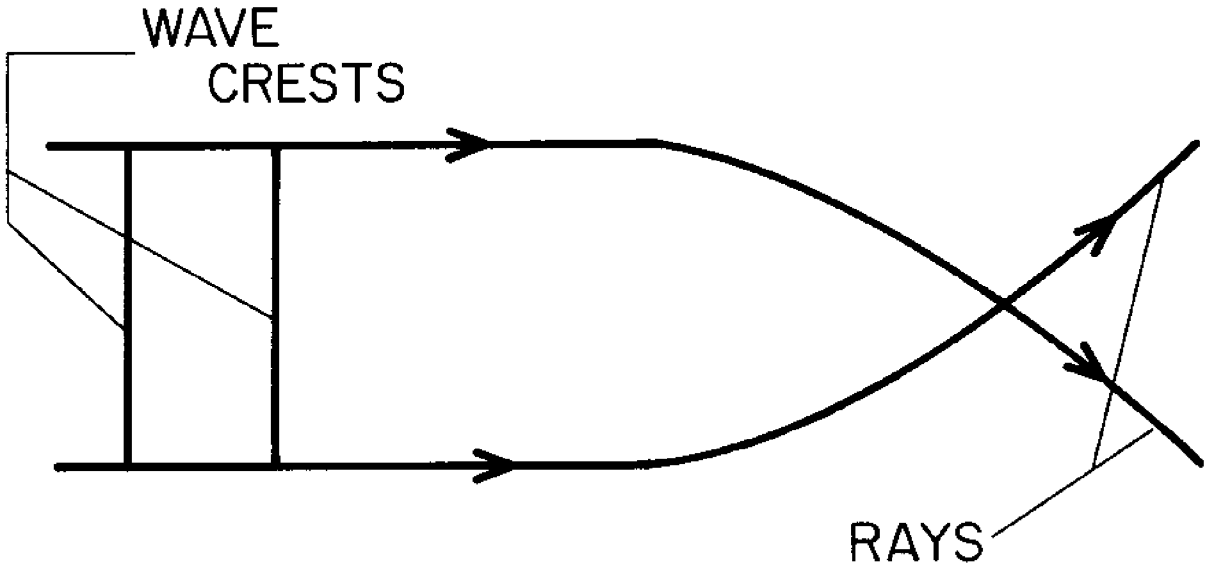

Fig. 4 Crossing rays.

part of the energy carried by the wave is converted to disordered turbulent motion. Increased turbulence is effective in speeding diffusion which, unlike the transmission of energy by waves, is primarily a local process.

The partition between wave energy transmitted through a ray intersection and wave energy converted to turbulent energy is not well established. For practical use it is as well to visualize a ray as the transmission path for a packet of wave energy. If the ray does not cross others, it delivers its entire packet to its coastal target. If it crosses other rays, at each crossing its energy will be fractionally reduced. Areas of multiple ray crossings will be areas of increased turbulence. As a first order estimate of locations of active wave attack and how they will be affected by changes in bathymetry it is useful to ignore energy losses to turbulence.

The concept of rays is very old. It is the foundation of Newton's "Optics" and we have known for many years how to construct the ray patterns associated with ocean surface waves. What is needed is the water depths, which can be had from charts of an area, wave lengths, or equivalently wave periods from which the wave lengths can be calculated, and the initial direction of wave approach. From this information wave rays can be traced from their offshore positions to their final impact on the coast.

Until the advent of the high-speed computer our ability to trace wave rays has been of little practical value to those who must make decisions about coastal protection and the issuance of dredging permits. The process required months of tedious labor at the drafting board for even the simplest situations. Today, wave ray tracing is a practical tool. The existing bathymetry of a region can be stored in the memory of a computer and rays for the characteristic systems of waves which exist offshore rapidly traced in to their points of impact. This identifies those parts of the coast under intense wave attack and permits comparisons of energy expended on the several sections of a coastline. More important, it offers a relatively simple way to determine the effect of any proposed dredging on the distribution of wave energy along the coastline *before* any dredging is done. One simply alters the bathymetry in the computer memory to conform to the depths of the proposed dredging, recomputes the ray patterns, and compares them with the previous results.

It should be stressed that alterations in water depths are neither good nor bad *per se*. Even though dredging may be confined to a small area its effects can be felt over much wider areas. Wave attack can be intensified in already active areas, or it can be reduced, or areas not presently under attack can become areas of active wave erosion. Ideally, if no other object were in view but the minimizing of wave erosion, depth changes would be made that would spread the wave energy uniformly along the coastline. Or perhaps, the structural strength of the coastline, its "erodability", would be considered and the wave energy distributed accordingly. Wave ray tracing can not tell you what decision to make but it can tell you what is likely to happen as a result of any decision you are considering. It will answer the question: "What if I do...?"

Perhaps the best way to see how wave ray tracing can serve you is by way of an illustration.

NEW YORK HARBOR, THE LOWER BAY

The Lower Bay of New York Harbor is the sort of area in which ray tracing can aid decision making. Its bottom shows naturally variable depth complicated by abrupt changes associated with the several dredged channels, Fig. 5. Sand and gravel is also mined there. Currently under consideration are permits for mining

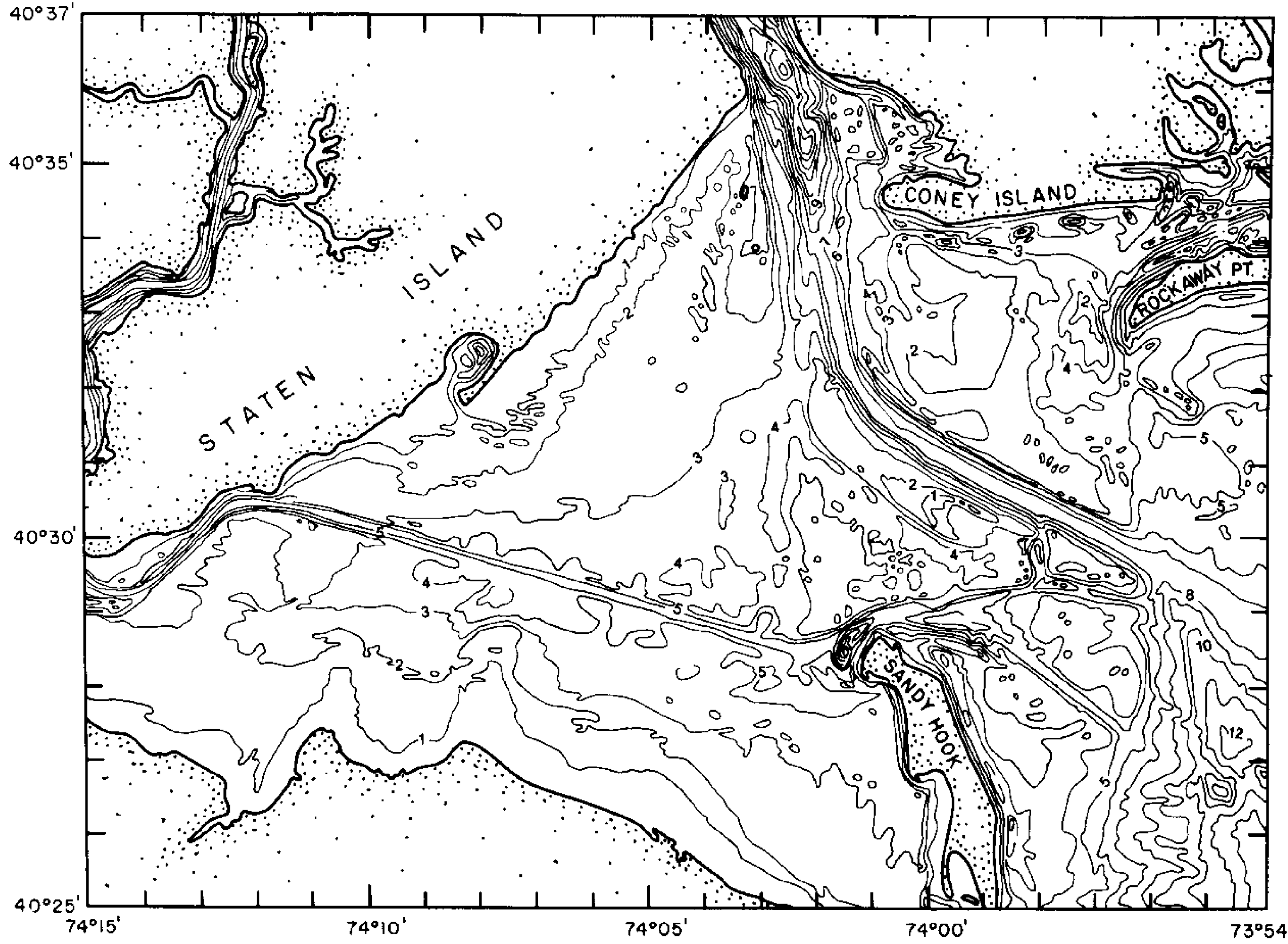

Fig. 5 Map of Lower Bay showing the irregular topography. Contours are in fathoms.

on the East Bank adjacent to Ambrose Channel from buoys 8 to 18, Fig. 6.

The questions are:

(1) What is the present distribution of wave energy along the coast of the Lower Bay?

(2) What will be the present distribution of wave energy along the coast of the Lower Bay if selected areas are dredged to specified depths?

To answer the first question the bathymetry of the Lower Bay must be digitized and stored in the computer memory. This is the most tedious and time-consuming part of the preparatory work. Fortunately, for any area under consideration this task need be done only once and since the information is necessary for many kinds of studies other than ray tracing and may be used repeatedly, the effort is worth while. At this point a number of choices must be made.

Since wave energy propagates, the area to be covered is much larger than the immediate dredging area. It must include all regions from which waves may come and all to which they may go. For the Lower Bay the area selected extends from 74°53'35" West Longitude (the Amboys) to 73°53'35" West Longitude and from 40°23'38" North Latitude (Long Beach) to 40°36'35" North Latitude (Verrazano Narrows).

(A preliminary study in which the area was extended much farther seaward only confirmed what is already well-known: New York has an excellent harbor. Much of the wave energy which offshore is aimed at the harbor mouth between Sandy Hook and Rockaway Point is refracted by the Hudson Canyon and goes ashore on the Jersey coast or on the south shore of Long Island).

For convenience in summarizing the effects of wave refraction, the shores of the Lower Bay have been divided into eleven sections as indicated on Plate O, Fig. 7, and listed in Table 1. This is the second choice to be made.

Bathymetry

The third choice concerns digitization. Since the water depths must be digitized for use in the computer, a grid of points at which the water depths will be recorded must be chosen. A number of considerations, some of them conflicting, govern the choice. Most important, the grid points must be spaced closely enough so that the depths recorded give a good picture of the bathymetry of the area. Abrupt changes in water depth between adjacent points are to be avoided wherever possible. This consideration calls for a small mesh and many grid points. On the other hand, many grid points tax the storage capacity of the computer and increase computing time and costs. A balance must be struck. Then too, there is the question of how well the water depths are actually known. The usual source is navigation charts. These are generally quite good although sometimes in need of up-dating but they may not be precise enough about fine details unless they happen to interfere in some way with ships.

Published charts may be supplemented with unpublished "boat sheets" which report the data on which the charts are based. Boat sheets record many more soundings than do charts.

In extremis a program of measurement can be undertaken but this is both costly and time-consuming and can not usually be justified unless the area in question is of the utmost importance and very sensitive to small changes in water depths as well.

For our example we chose a grid with a cell size of 0.1 nautical mile (185.2 m) on a side. In the north-south direction 130 grid points were used and in the east-west direction 180. This gives us 23,400 water depths to store in the computer. Over most of the Lower bay the digitized data give a good picture of the variations in water depth and are roughly in line with the quality of the information from NOAA Chart 12327 (68th edition, 7 Aug. 1976) from which the depth values were read.

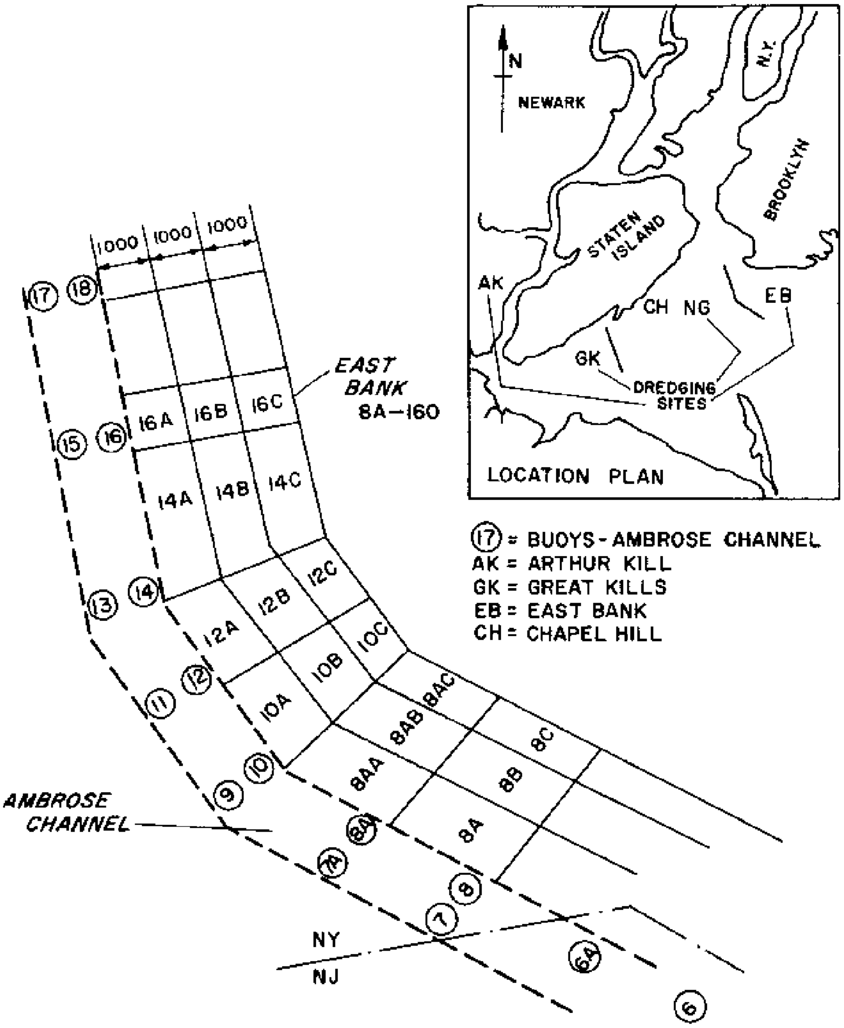

Fig. 6. Segment of Ambrose Channel showing mining sectors on the East Bank.

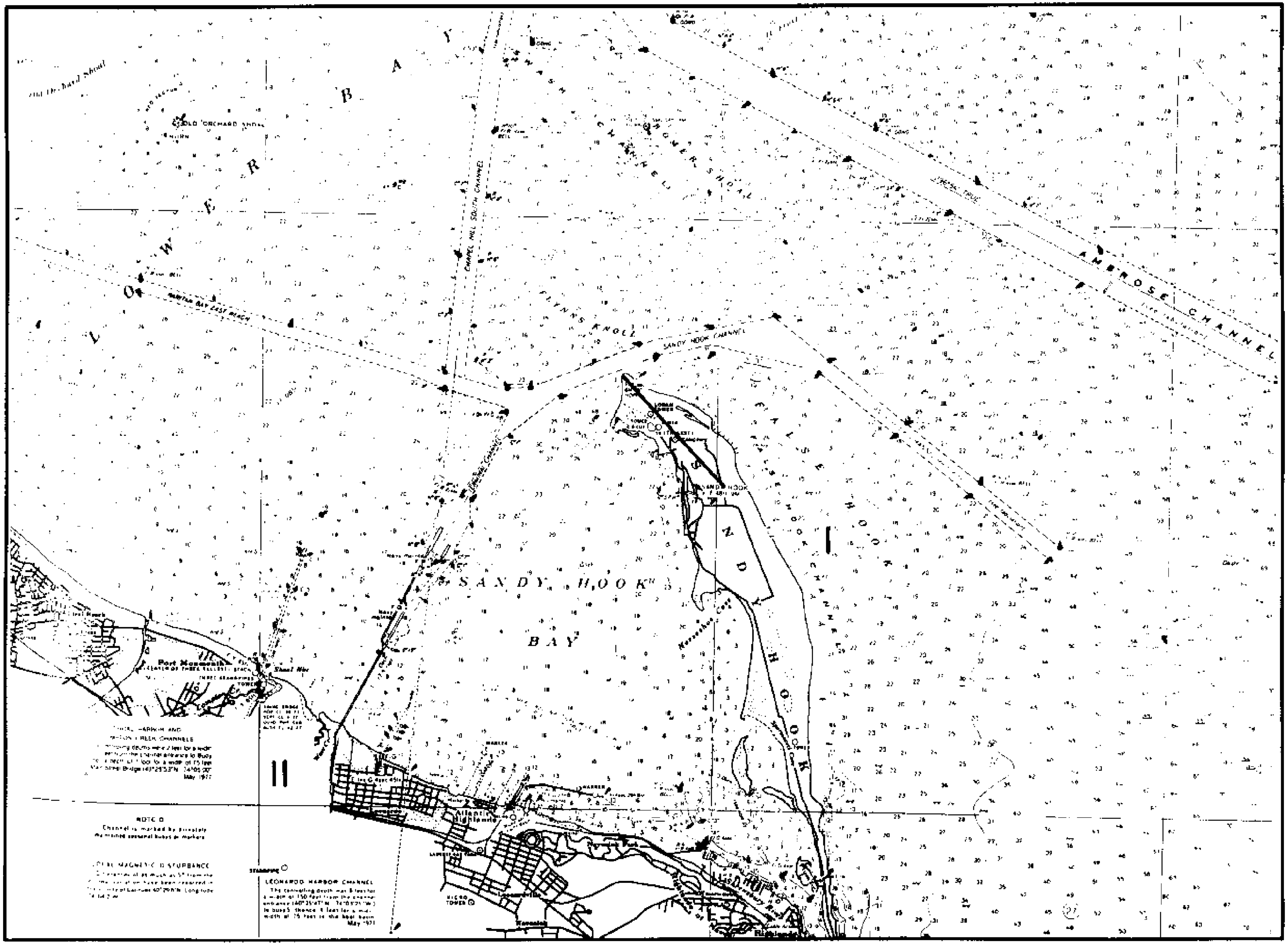

Fig. 7A Map of Lower Bay showing shoreline segments used in summarizing effects of wave refraction.

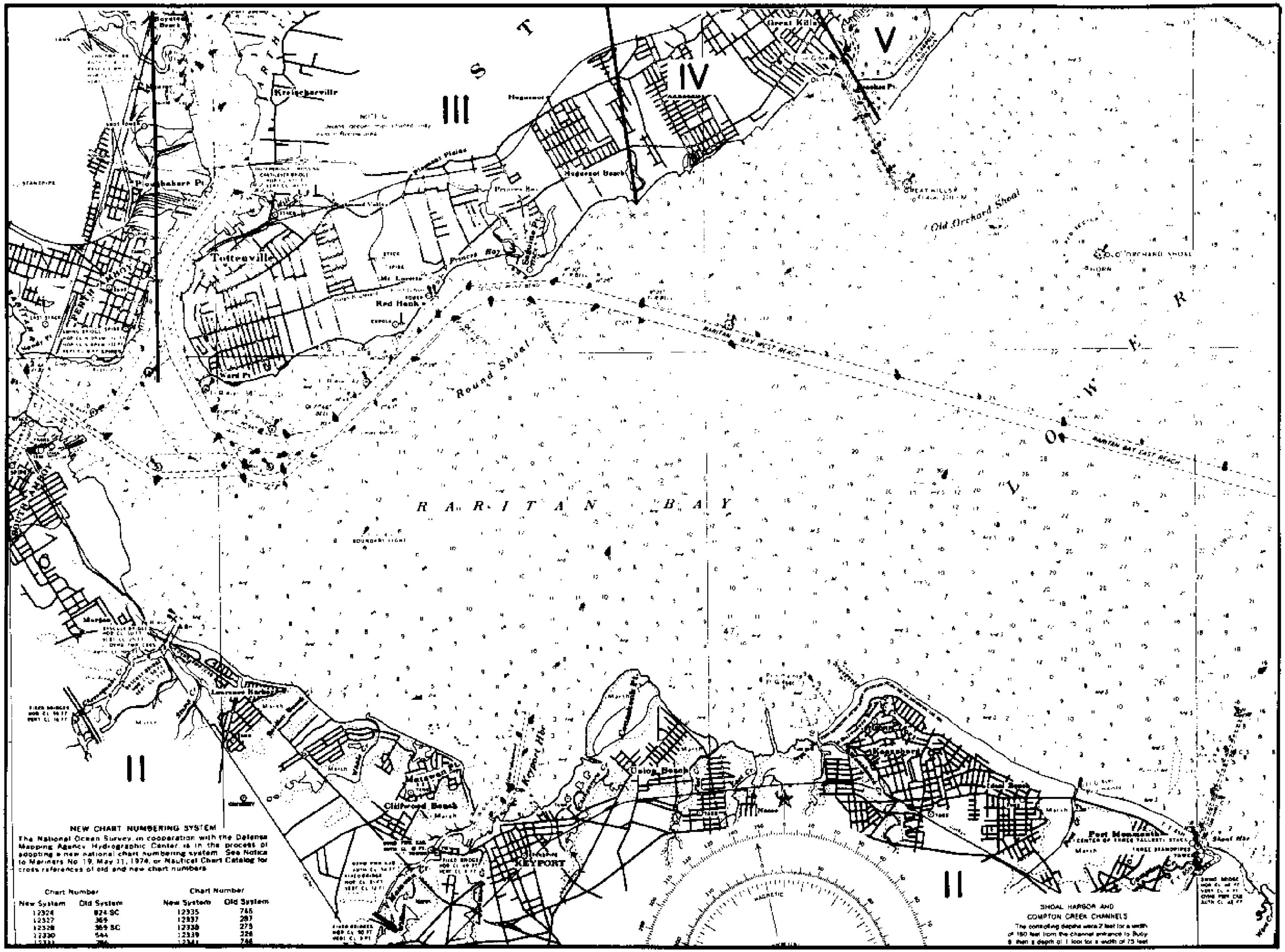

Fig. 7B Map of Lower Bay showing shoreline segments used in summarizing effects of wave refraction.

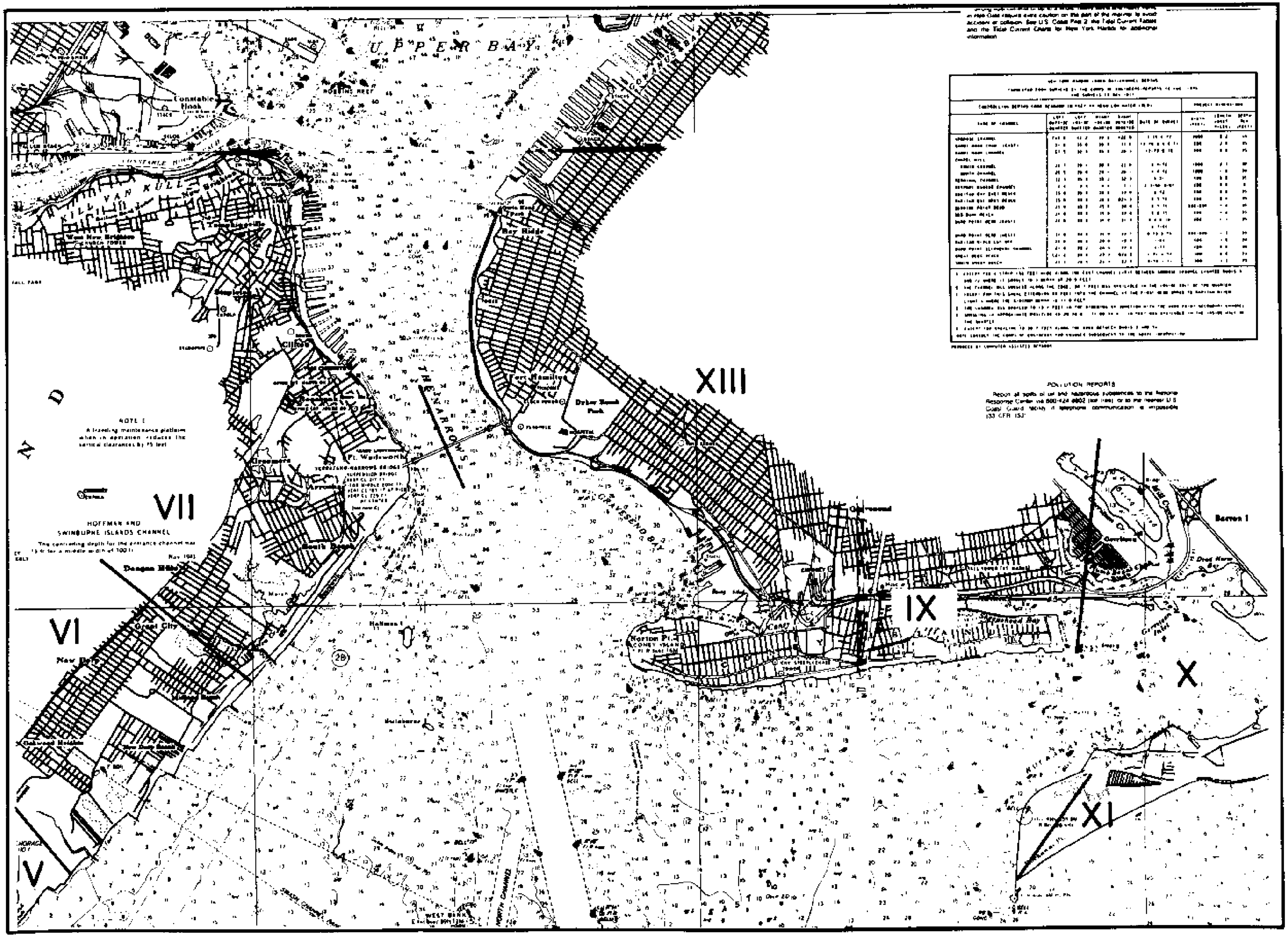

Fig. 7C Map of Lower Bay showing shoreline segments used in summarizing effects of wave refraction.

Table 1. Division of the Shoreline of the Lower Bay

Impact Strip	Description	Strip Length (m)
I	Sandy Hook, seaward face	5,186
II	Sandy Hook to Perth Amboy	62,968
III	Ward Point to Hugenot Beach	6,852
IV	Hugenot Beach to Great Kills Harbor	3,611
V	Crookes Point, NE 2200 yards	2,074
VI	End of V to Midland Beach	4,093
VII	Midland Beach to Verrazano Narrows	5,593
VIII	Gravesend Bay	4,093
IX	Coney Island	6,908
X	Gerritsen Inlet	4,074
XI	Rockaway Point, seaward face	1,926

The chart datum is mean low water. The tidal range in the area is 4.5 to 5.0 feet. Thus, there will be a difference in the ray patterns at high and low water. If the changes in water depth are sufficient to raise questions about where wave energy comes ashore, a small subprogram can be used to correct the mean low water depths to high water and the rays traced at high water for comparision. In fact, it would be simple enough to adjust the water depths by a time varying function representing the tide but such refinement seems unwarranted unless the tidal ranges are extreme.

One feature of the bathymetry of the Lower Bay is not well-represented by the grid size used: the dredged channels; Ambrose, Chapel Hill, Sandy Hook, Terminal, and Raritan. In passing across the channels the water depths characteristically double in a few hundredths of a nautical mile. For proper representation, i.e., to keep changes in depth between adjacent grid points modest, we would have to use a grid mesh of something like 0.01 nautical miles on a side. If we did, we would have to store 2,340,000 water depths instead of 23,400 which is impractical--and the bathymetry of the Lower Bay is not known that well in any case. The difficulty introduced is that from time to time the computer will be faced with such large depth differences that it can not decide in which direction to continue the ray. When that happens the computer simply abandons the ray it is working on and begins again on the next ray. The aborted ray is left hanging and its point of impact on the shoreline remains unknown. Some information loss results and must be borne but it is not large enough to be crippling.

Waves

The fourth choice is: What waves to use? The very best wave information would be a knowledge of the directional wave energy spectra for the wave systems which appear off the area of interest, their frequency of appearance, and their seasons; in short, the wave climatology for the area.

A directional wave energy spectrum is, conceptually, a very simple thing. It sorts the wave energy of a complex wave system, which is proportional to the square of the wave height, by frequency (the reciprocal of the wave period) and direction of travel. Unfortunately, very few such spectra have ever been measured so that it is most unlikely that a clear-cut, well-established wave climatology will be available. To undertake the difficult measurements necessary and to extend them over years is not usually within the scope of decision-making and planning agencies. Such work is perhaps best left to an agency like NOAA.

Some wave information can be gathered from watermen and sailors. These men, who work in an area, can tell you from which directions waves generally come and furnish some estimate of their periods.

In the total absence of wave information one can fall back on an exploration of waves of all periods and directions of approach which could affect the area of interest. This is not entirely satisfying since waves of some periods and with some directions of travel may seldom or never be provided by existing natural conditions and will thus be practically irrelevant. On the other hand, an exploration of the full range of possibilities will guard against surprise after the fact.

In a semi-enclosed area like the Lower Bay locally generated waves are of relatively little importance. There isn't enough room for them to grow to any appreciable extent. It is the offshore waves which reach the area through the mouth between Sandy Hook and Rockaway Point that supply most of the energy.

In the open ocean the nondirectional spectrum, one which sorts energy by frequency only, usually looks something like Fig. 8. There is very little energy in waves with very low frequencies (very long periods) but as one goes to higher frequencies (somewhat shorter periods but still long) there is a very abrupt rise in the energy to a peak after which the energy tails of more or less as the -5 power of the frequency. For our illustrative calculation we have selected six waves to suggest the spectrum. How many you should use depends on how well you know the characteristic spectra for your area, how

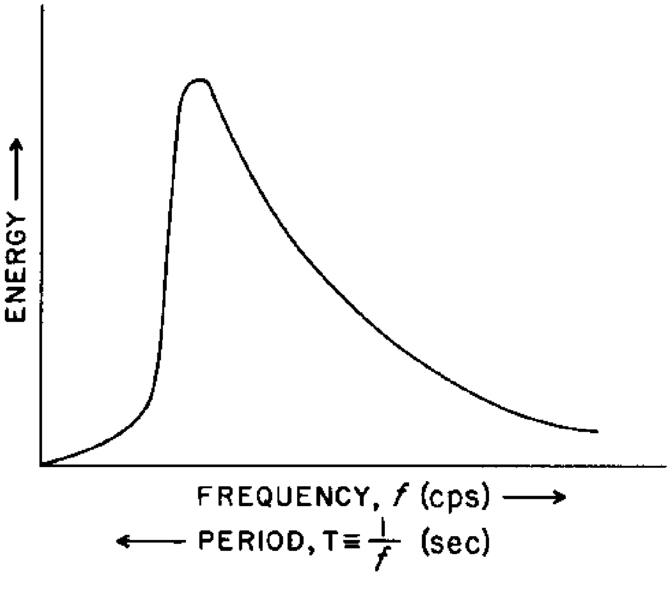

Fig. 8 Non-directional energy spectrum sorted by frequency.

Table 2. Waves Selected for Ray Tracing in the Lower Bay

Period (sec)	Direction (°T)	Frequency (cps)	Length (ft)	Stokes Max Ht. (ft)	Plate Code
2	270	0.5	20	2.86	2/270/-
2	297	0.5	20	2.86	2/297/-
6	270	0.167	180	25.71	6/270/-
6	297	0.167	180	25.71	6/297/-
10	270	0.1	500	71.43	10/270/-
10	297	0.1	500	71.43	10/297/-

comprehensive you want to make your analysis, and how much you have to spend for computing. The selected waves are listed in Table 2.

The three periods were chosen to illustrate three conditions: 2-second waves have speeds dependent only on wave length and are unaffected by refraction over most of the Lower Bay, 6-second waves have speeds which are significantly affected by both the wave length and the water depth over most of the Lower Bay, 10-second waves have speeds determined by the water depths (The wave length no longer plays a part.) over the entire Lower Bay. The two directions chosen for waves of each of the three periods were directly from the East traveling toward 270°T and parallel to Ambrose Channel traveling toward 297°T.

Wave periods are most often used to specify waves since they are their most easily measured feature and do not change during refraction. Their corresponding lengths may be had from the formula $L = 5.12T^2$, T in seconds and L in feet.

There is no unique height associated with waves of a given period. Heights may range from infinitesimal to the Stokes theoretical maximum, one-seventh of the length. Since the energy of a wave is proportional to the square of the wave height, if each wave has grown to its theoretical maximum, the energy ratios would be 1:81:625 corresponding to the period ratios 2:6:10. Waves seldom grow to these heights and experience suggests energy ratios of 1:8:6--values which have been used in our illustrative calculation. If the energy spectra are known, the energy ratios will, of course, be taken from them.

Dredging

An area for which sand and gravel mining permits have been sought is shown in Fig. 6. Its relation to the rest of the Lower Bay appears as the shaded area in Plate O. It is a comparatively small area but, as will be seen, the effects of deepening the area will be widely felt. For the purposes of our calculation we have considered 4 cases, Table 3.

The first case is the pattern of wave refraction as it now exists with no dredging. For the second and third cases 4 subareas were selected: 8C, 8AC, 10A, and 14C. The selected areas are identified by number in Fig. 6 and by heavy shading in Plate O. For the second case the selected areas were dredged to a depth of 45 feet and for the third case to a depth of 90 feet. For the fourth case the entire shaded area was dredged to 90 feet. "Dredged" in this context means simply that the computer was instructed to substitute the specified depths for the actual depths stored in its memory for

Table 3. Dredging Cases Considered

Case	Description	Plate Code
1	No dredging	-/-/0
2	Dredging selected areas to 45 ft.	-/-/45
3	Dredging selected areas to 90 ft.	-/-/90
4	Dredging entire area to 90 ft.	-/-/90A

those grid points falling within the specified areas. Each case is identified on the wave ray plates by the third number of a triple: 0 for case 1, 45 for case 2, 90 for case 3, and 90A for case 4.

THE RAY DIAGRAMS

There are 24 ray diagrams grouped in 4 sets of 6 according to the amount of dredging, Table 4. Each of these ray diagrams, overlaid on Plate O, traces the energy paths for the wave through the Lower Bay to its impact on the coastline. In each case an initial wave crest perpendicular to the direction of propagation was taken off the mouth of the harbor and 25 equally spaced rays were traced. The rays are numbered at their initial and terminal ends to make it easy to see where the energy from any particular ray comes ashore. The numbers of rays whose impact points shift with dredging have been circled. In the two cases where the computer was unable to continue the ray to the shore the initial number is crossed with a slash and no terminal number is shown.

Along the rays, ticks are entered at 1-minute intervals. From these the progress of the wave energy can be judged. They may usefully be thought of as the position of the wave crest from minute to minute--at least so long as the rays do not cross. When the rays become tangled the concept is less useful. The lines representing wave crests, those joining corresponding ticks on the wave rays, could have been entered on the ray diagrams but have not been since they would serve no purpose and the diagrams are already tangled enough. It should be noticed that as a wave crosses shoal water and slows down the time ticks become more closely spaced. This is particularly evident in the diagrams for the 6- and 10-second waves as they come ashore, e.g., Plate A 6/297/0, ray 9 and Plate A 10/297/0, ray 15, since the longer period waves travel faster in deep water but it is equally true of the 2-second waves, e.g., Plate A 2/297/0, ray 6.

There are many interesting things to be seen on these ray diagrams and we will discuss some of them for each plate group.

Plates A

This group of plates shows wave refraction within the Lower Bay as it presently exists to the extent that NOAA Chart 12327 accurately portrays the bathymetry.

For 2-second waves, whether traveling toward 270°T or 297°T, there is little or no refraction until the waves begin their run-up in the shoaling water fronting the shore. In A 2/270/0 with the exception of rays 16, 17, and 22-24 the rays are essentially straight lines. This could have been anticipated since waves are not refracted when the water depths are greater than half the wave length. For 2-second waves the length is 20 feet and the Lower Bay is almost everywhere deeper than 10 feet. Rays 16 and 17 are bent as they pass over Romer Shoal to the west of Ambrose Channel where water depths shallow to as little as 5 feet. Rays 22-24 are bent in

Table 4. Wave Ray Diagrams

Plate Number	Dredging	Wave Period (sec)	Initial Direction of Wave Approach (°T)
A 2/270/0	None	2	270
A 2/297/0		2	297
A 6/270/0		6	270
A 6/297/0		6	297
A 10/270/0		10	270
A 10/297/0		10	297
B 2/270/45	Selected Subareas to 45 ft.	2	270
B 2/297/45		2	297
B 6/270/45		6	270
B 6/297/45		6	297
B 10/270/45		10	270
B 10/297/45		10	297
C 2/270/90	Selected Subareas to 90 ft.	2	270
C 2/297/90		2	297
C 6/270/90		6	270
C 6/297/90		6	297
C 10/270/90		10	270
C 10/297/90		10	297
D 2/270/90A	Entire Area to 90 ft.	2	270
D 2/297/90A		2	297
D 6/270/90A		6	270
D 6/297/90A		6	297
D 10/270/90A		10	270
D 10/297/90A		10	297

passing over the East Bank just east of the mining area where depths as small as 7 feet are found. However, before and after passing the shoals the paths of even these rays are straight lines. In A 2/297/0 the results are similar. Ray 3 is refracted over Flynns Knoll, 10 and 11 over Romer Shoal, and 14, 16, and 17 over East Bank.

Since the 2-second waves are not refracted while passing over the mining area, it is obvious that further deepening of the water will not affect them. Plates B 2/270/45, C 2/270/90, and D 2/270/90A will be identical with Plate A 2/270/0 and Plates B 2/297/45, C 2/297/90, and D 2/297/90A with Plate A 2/297/0.

The ray diagrams for the 6-second waves are more interesting. The rays set off briefly as straight lines in the initial directions and, from time to time, have reasonably straight segments. Their length is 180 feet and, while the Lower Bay is nowhere 90 feet deep, it still has areas deep enough to make refraction, locally, minimal. A glance at Plates A 6/270/0 and A 6/297/0 makes it more than obvious that refracted rays, more often than not hit way wide of the points at which they were initially aimed. For example, in Plate A 6/270/0 the Point Comfort-Port Monmouth stretch of impact strip II, which would seem to be well-sheltered from waves from the east by Sandy Hook, is reached by rays 10, 14, and 20. Ray 15, initially aimed at Red Bank on Staten Island, comes ashore in Gravesend Bay.

Following the paths of individual rays can be informative and sometimes surprising. For example, consider ray 10 on Plate A 10/297/0. It starts out at 297°T toward impact strip VI on Staten Island but by the end of the fifth minute it finds itself crossing Ambrose Channel aimed at Coney Island, impact strip IX. For the next 18 minutes it rolls on toward Coney Island but is then bent sharply around to the southwest and recrosses Ambrose Channel after which it is again sharply bent to the left onto a more or less southerly course and finally comes ashore in impact strip II just behind Sandy Hook in Navesink Park.

If you will spend a few minutes following the individual ray paths, you will find it easy to assent to the proposition that it is very difficult, if not impossible, to say, *a priori*, where wave energy associated with a particular wave period and initial direction will come ashore.

Interesting as tracing single rays may be, considering groups of rays may be even more informative. On Plate A 10/297/0 rays 13, 14,16, and 18 approach Ambrose Channel on its side--ray 13 actually runs along the Channel for the first 4 minutes--but all approach at angles so small that they can not cross the Channel and are bent away to the right. None of these rays is initially aimed at Coney Island, impact strip IX, but all of them wind up there in a tight cluster. Ambrose Channel acts somewhat like the reflection of light from a smooth water surface and is responsible for the heightened wave action focused on Coney Island.

Rays 10, 15, 21, and 22 do manage to cross Ambrose Channel in tis upper reaches where the approach is more nearly perpendicular. For waves of any particular period there is a critical angle of approach. If the angle is less than the critical angle the ray will be "reflected" by the Channel.

The same situation is found on the western side of Ambrose Channel. Rays 6 and 11 approach but at too fine an angle and can not cross. They finally come ashore at Annadale, impact strip IV--another region of heavy wave activity.

It is common knowledge that impact strips IV-VI and impact strip IX are among the regions of the most intense wave erosion in the Lower Bay. It would not be too much to say that they are the direct handiwork of the Ambrose Channel. We will never get rid of Ambrose Channel but it would be interesting to "fill it in" in the computer memory and see whether the concentrations of rays in impact strips IV-VI and IX were not much reduced.

A speculation of more practical potential starts from the notion that, if one deep channel can reflect waves, so can another. Might not the sand and gravel mining area be profitable relocated close off Coney Island instead of next to Ambrose Channel and oriented so that a trough dredged along it would reflect the approaching wave rays? In this way coastal protection for Coney Island might become a cost-free by-product of mining. Ambrose Channel and a trough of the proper depth and orientation could be made to form a sort of "wave guide" that would divert the wave energy. Of course, the redirected energy must come ashore somewhere and its ultimate destination had better be carefully anticipated. Relief for Coney Island that became a concentrated attack on the Verrazano Narrows could be quite unpopular. The best solution would reduce the concentrations and spread the energy more evenly around the whole shoreline.

The position of Ambrose Channel is very clear on Plate A 10/297/0 even

Table 5. Impact Shifts when the Selected Areas are Dredged to 45 Feet

Plate Number	Ray Number	Present Impact Strip	Impact Strip After Dredging
B 2/270/45	none	--	--
B 2/297/45	none	--	--
B 6/270/45	15	VIII	IX
	16	VII	VI
	18	VI	VII
	19	IX	V
	22	IV	V
	23	VI	VI
B 6/297/45	12	X	IX
	14	IX	IX
	16	IX	IX
	17	VI	VII
	18	VII	VI
	19	VI	VI
	22	VI	VI
B 10/270/45	15	V	II
	17	V	II
	18	IX	IX
	20	IX	IX
B 10/297/45	14	IX	II
	16	IX	IX
	17	VII	IX
	18	IX	V
	22	V	V

without bothering to overlay the ray diagram on Plate O. It is equally clear on Plates A 10/270/0 and A 6/297/0. It can also be seen on Plate A 6/270/0 although in this case Plate O helps.

So much for things as they are. Now let us go on to interfere with them.

Plates B

Plates B show the wave rays after the four selected areas (heavy shading, Plate O) have been dredged to a depth of 45 feet. Only the rays whose impact points are changed by the depth change need immediately concern us. Their ray numbers have been circled and the changes are given in Table 5.

Plates C

Plates C show the wave rays when the four selected areas (heavy shading, Plate O) have been dredged to 90 feet. The ray numbers whose impact points have been changed are circled. The changes are given in Table 6. Ray 19 aborts on Plate C 6/270/90.

Plates D

Plates D show the wave rays when the entire mining area (shading, Plate O) has been dredged to 90 feet. The ray numbers whose impact points have been changed are circled. The changes are given in Table 7. Ray 16 aborts on Plate D 10/297/90A.

Table 6. Impact Shifts when the Selected Areas are Dredged to 90 Feet

Plate Number	Ray Number	Present Impact Strip	Impact Strip After Dredging
C 0/270/90	none	--	--
C 0/297/90	none	--	--
C 6/270/90	15	VIII	IX
	16	VII	VI
	18	VI	II
	22	IV	V
	23	VI	VI
C 6/297/90	12	X	IX
	14	IX	IX
	16	IX	VI
	17	VI	VII
	18	VII	IX
	19	VI	VI
	22	VI	V
C 10/270/90	15	V	V
	17	V	V
	18	IX	X
	20	IX	IX
C 10/297/90	14	IX	
	16	IX	X
	17	VII	IX
	18	IX	V
	22	V	V

ENERGY DISTRIBUTION ALONG THE SHORES OF THE LOWER BAY

The contemplation of wave ray diagrams may be satisfying to those interested in waves and their behavior but for managers and planners some more succinct presentation is needed. They are not so much concerned with the details of the transit of wave energy through the Lower Bay as they are with where it is finally expended.

We have noted that waves of the several periods need not all carry the same energy and have suggested that an energy ratio of 1:8:6 corresponding to the periods 2, 6, and 10 seconds would be reasonable to use. We will, therefore, assign to each ray of the 2-second waves 1 arbitrary unit of energy, to each ray of the 6-second waves 8 units of energy, and to each ray of the 10-second waves 6 units. Thus, in each of the 4 cases we are considering that $2 \times 25 \times 1 + 2 \times 25 \times 8 + 2 \times 25 \times 6 = 750$ arbitrary units of energy start off initially and, barring aborted rays, come ashore somewhere. It is only necessary to cumulate these energy packets by coastal impact strips to form an idea of the distribution of wave energy around the shores of the Lower Bay.

If there were no refraction at all, the wave rays would move in straight lines until they met the shore. In that hypothetical case we would have the distribution shown in Table 8.

Table 7. Impact Shifts when the Entire Area Has been Dredged to 90 Feet

Plate Number	Ray Number	Present Impact Strip	Impact Strip After Dredging
D 2/270/90A	none	--	--
D 2/297/90A	none	--	--
D 6/270/90A	15	VIII	IX
	16	VII	VII
	18	VI	VIII
	19	IX	VII
	22	IV	V
	23	VI	VI
D 6/297/90A	12	X	IX
	14	IX	IX
	16	IX	II
	17	VI	VIII
	18	VII	IX
	19	VI	IX
D 10/270/90A	15	V	V
	17	V	X
	18	IX	X
	20	IX	IX
D 10/297/90A	14	IX	II
	15	IX	VII
	17	VII	IX
	18	IX	V
	21	V	VI
	22	V	IV

From Plates A we can estimate the wave energy distribution for the Lower Bay as it exists according to the depths shown on NOAA 12327. The results are shown in Table 9.

In a similar way we get the wave energy distribution for the Lower Bay when the selected areas are dredged to 45 feet from Plates B with the results shown in Table 10.

From Plates C when the selected areas have been dredged to 90 feet we have Table 11.

From Plates D when the entire mining area has been dredged to 90 feet we have Table 12.

For easy comparison Table 13 displays the energy per meter of shoreline for the hypothetical case of not refraction, Table 8, and for each of the 4 dredging situations under consideration, Tables 9-12.

The first and most obvious thing that strikes the eye is that impact strips I and XI, the seaward faces of Sandy Hook and Rockaway Point, are subjected to the heaviest wave attacks which is only what we would expect. What seems a bit startling is that dredging in the mining area could increase the weight of the attack on these seaward faces. When the selected areas are dredged to 90 feet the energy per meter $\times 10^{-4}$ on the seaward face of Sandy Hook increases by 31.2 units (8.3%). To see why this is so look at

Table 8. Energy Distribution in the Absence of Wave Refraction

A. Approaching toward 270°T

Impact Strip	Ray Number	Number of Rays	Energy Associated with Waves of Periods 2	6	10	Total Energy toward 270°T
I	1 - 9	9	9	72	54	135
II	10 - 13	4	4	32	24	60
III	14 - 18	5	5	40	30	75
IV	19,20	2	2	16	12	30
V	21 - 24	4	4	32	24	60
VI - X	--	0	0	0	0	0
XI	25	1	1	8	6	15
		25				375

B. Approaching toward 297°T

Impact Strip	Ray Number	Number of Rays	Energy Associated with Waves of Periods 2	6	10	Total Energy toward 297°T
I	1,2	2	2	16	12	30
II - IV	--	0	0	0	0	0
V	3 - 7	5	5	40	30	75
VI	8 - 15	8	8	64	48	120
VII	16 - 22	7	7	56	42	105
VIII - X	--	0	0	0	0	0
XI	23 - 25	3	3	24	18	45
		25				375

C. Total Energy Distribution

Impact Strip	Total Energy	Total Energy per Meter of Shoreline (× 10^{-4})
I	165	318.2
II	60	9.5
III	75	109.5
IV	30	83.1
V	135	650.9
VI	120	293.2
VII	105	187.7
VIII	0	0.0
IX	0	0.0
X	0	0.0
XI	60	311.5
	750	

Table 9. Wave Energy Distribution for the
Lower Bay as it is Now

Impact Strip	Wave 2/270		Wave 2/297		Wave 6/270		Wave 6/297		Wave 10/270		Wave 10/297		Total Energy (ΣE)	Total Energy per Meter of Shoreline ($\times 10^{-4}$)
	# of Rays	E	# of Rays	E	# of Rays	E	# of Rays	E	# of Rays	E	# of Rays	E		
I	9	9	2	2	9	72	3	24	7	42	5	30	179	345.2
II	3	3	0	0	3	24	0	0	0	0	1	6	33	5.2
III	5	5	0	0	0	0	0	0	0	0	0	0	5	7.3
IV	3	3	1	1	1	8	3	24	2	12	4	24	72	199.4
V	4	4	4	4	0	0	0	0	5	30	3	18	56	270.0
VI	0	0	8	8	5	40	8	64	1	6	2	12	130	317.6
VII	0	0	7	7	2	16	2	16	1	6	1	6	51	91.2
VIII	0	0	0	0	1	8	0	0	0	0	0	0	8	19.6
IX	0	0	0	0	3	24	5	40	6	36	6	36	136	196.9
X	0	0	0	0	0	0	1	8	0	0	0	0	8	19.6
XI	1	1	3	3	1	8	3	24	3	18	3	18	72	373.8
	25	25	25	25	25	200	25	200	25	150	25	150	750	

Table 10. Wave Energy Distribution for the
Lower Bay when the Selected Areas
Have been Dredged to 45 Feet

Impact Strip	Wave 2/270		Wave 2/297		Wave 6/270		Wave 6/297		Wave 10/270		Wave 10/297		Total Energy (ΣE)	Total Energy per Meter of Shoreline ($\times 10^{-4}$)
	# of Rays	E	# of Rays	E	# of Rays	E	# of Rays	E	# of Rays	E	# of Rays	E		
I	9	9	2	2	9	72	3	24	7	42	5	30	179	345.2
II	3	3	0	0	3	24	0	0	2	12	2	12	51	8.1
III	5	5	0	0	0	0	0	0	0	0	0	0	5	7.3
IV	3	3	1	1	0	0	3	24	2	12	4	24	64	177.2
V	4	4	4	4	2	16	0	0	3	18	4	24	66	318.2
VI	0	0	8	8	5	40	8	64	1	6	2	12	130	317.6
VII	0	0	7	7	2	16	2	16	1	6	0	0	45	80.5
VIII	0	0	0	0	0	0	0	0	0	0	0	0	0	0.0
IX	0	0	0	0	3	24	6	48	6	36	5	30	138	199.8
X	0	0	0	0	0	0	0	0	0	0	0	0	0	0.0
XI	1	1	3	3	1	8	3	24	3	18	3	18	72	373.8
	25	25	25	25	25	200	25	200	25	150	25	150	750	

Table 11. Wave Energy Distribution for the Lower Bay when the Selected Areas Have been Dredged to 90 Feet

Impact Strip	Wave 2/270		Wave 2/297		Wave 6/270		Wave 6/297		Wave 10/270		Wave 10/297		Total Energy (ΣE)	Total Energy per Meter of Shoreline ($\times 10^{-4}$)
	# of Rays	E	# of Rays	E	# of Rays	E	# of Rays	E	# of Rays	E	# of Rays	E		
I	9	9	2	2	9	72	3	24	7	42	6	36	185	356.7
II	3	3	0	0	4	32	0	0	0	0	1	6	41	6.5
III	5	5	0	0	0	0	0	0	0	0	0	0	5	7.3
IV	3	3	1	1	0	0	3	24	2	12	4	24	64	177.2
V	4	4	4	4	1	8	1	8	5	30	4	24	78	376.1
VI	0	0	8	8	5	40	7	56	1	6	2	12	122	298.1
VII	0	0	7	7	1	8	2	16	1	6	0	0	37	66.2
VIII	0	0	0	0	0	0	0	0	0	0	0	0	0	0.0
IX	0	0	0	0	3	24	6	48	5	30	4	24	126	182.4
X	0	0	0	0	0	0	0	0	0	0	1	6	6	14.7
XI	1	1	3	3	1	8	3	24	4	24	3	18	78	405.0
	25	25	25	25	24*	192	25	200	25	150	25	150	742*	

*Ray 19 for wave 6/270 aborts; 8 energy units unaccounted for.

Table 12. Wave Energy Distribution for the Lower Bay when the Entire Mining Area Has been Dredged to 90 Feet

Impact Strip	Wave 2/270		Wave 2/297		Wave 6/270		Wave 6/297		Wave 10/270		Wave 10/297		Total Energy (ΣE)	Total Energy per Meter of Shoreline ($\times 10^{-4}$)
	# of Rays	E	# of Rays	E	# of Rays	E	# of Rays	E	# of Rays	E	# of Rays	E		
I	9	9	2	2	9	72	3	24	7	42	5	30	179	345.2
II	3	3	0	0	3	24	1	8	0	0	2	12	47	7.5
III	5	5	0	0	0	0	0	0	0	0	0	0	5	7.3
IV	3	3	1	1	0	0	3	24	2	12	5	30	70	193.9
V	4	4	4	4	1	8	0	0	4	24	1	6	46	221.8
VI	0	0	8	8	4	32	5	40	1	6	3	18	104	254.1
VII	0	0	7	7	3	24	1	8	1	6	1	6	51	91.2
VIII	0	0	0	0	1	8	1	8	0	0	0	0	16	39.1
IX	0	0	0	0	3	24	8	64	5	30	4	24	142	205.6
X	0	0	0	0	0	0	0	0	2	12	0	0	12	29.5
XI	1	1	3	3	1	8	3	24	3	18	3	18	72	373.8
	25	25	25	25	25	200	25	200	25	150	24*	144	744*	

*Ray 16 for wave 10/297 aborts; 6 energy units unaccounted for.

Table 13. Wave Energy Distributions
Along the Shoreline of the Lower Bay
In Arbitrary Energy Units per Meter of Shoreline × 10^{-4}
For the Hypothetical Case of No Wave Refraction
And for Four Dredging Situations

	Impact Strips										
	I	II	III	IV	V	VI	VII	VIII	IX	X	XI
No Refraction	318.2	9.5	109.5	83.1	650.9	293.2	187.7	0.0	0.0	0.0	311.5
No Dredging	345.2	5.2	7.3	199.4	270.0	317.6	91.2	19.6	196.9	19.6	373.8
Selected Areas Dredged to 45 ft.	345.2	8.1	7.3	177.2	318.2	317.6	80.5	0.0	199.8	0.0	373.8
Selected Areas Dredged to 90 ft.	356.7	6.5	7.3	177.2	376.1	298.1	66.2	0.0	182.4	14.7	405.0
Entire Mining Area Dredged to 90 ft.	345.2	7.5	7.3	193.9	221.8	254.1	92.1	39.1	205.6	29.5	373.8

Plate C 10/297/90. Ray 14, which carries 6 arbitrary energy units and starts parallel to Ambrose Channel and to the east of it, has its direction completely reversed, passes back out to sea, and is refracted to come ashore on impact strip I. This instance is very striking but Table 13, as a whole, confirms that the effects of dredging in the comparatively small mining area are felt to a greater or lesser degree in the wave energy distribution around the entire periphery of the Lower Bay.

Within the Lower Bay there are two regions of heavy wave attack: the region composed of impact strips IV, V, and VI (Hugenot Beach to Midland Beach on Staten Island) and impact strip IX (Coney Island). This is so now and it is apparent that it will remain so whichever of the three alternative dredging schemes is permitted. That these are now regions of heavy wave activity in the Lower Bay is common knowledge and it is reassuring that our analysis for "No Dredging" confirms what we already know. It gives us more confidence that the analysis for situations which could only be verified by irremediably tearing up the Lower Bay are also valid.

While the two regions of high wave activity remain so, there are shifts in the intensity of the attack on points within them under different dredging conditions. For example, when the selected areas are dredged to 45 feet (13.7 m) the attack on IV is abated, the attack on V (the spit enclosing Great Kills Harbor) is intensified by 48.2×10^{-4} units(17.9%), and the attack on VI remains the same. If the selected areas are dredged to 90 feet (27.4 m), the attack on IV is not further changed, the attack on V again increases by 57.9×10^{-4} units--a total increase over the present heavy attack of 39.3%--, the attack on VI actually decreases from its current value by 19.5×10^{-4} units, a drop of 6.1%. If either of these dredging schemes is permitted, the spit protecting Great Kills Harbor may be in a bad way.

Of the three dredging schemes the proposal to dredge the entire mining area to 90 feet is best for the Great Kills Harbor region impact strips IV-VI. While not exactly turning the region into one of peace and quiet it does reduce the wave intensity in all three impact strips: strip IV by 5.5×10^{-4} units (2.8%), strip V by 48.2×10^{-4} units (17.9%), and strip VI by 63.5×10^{-4} units (20.0%). However, what you gain on the swings you lose on the roundabouts. The improvement in strips IV-VI must be paid for by an increased attack on Coney Island, strip IX, where there is an increase of 8.7×10^{-4} units (4.4%).

The easing of conditions on Staten Island would make them a little harder on Coney Island. Since both islands have serious shore erosion problems, the choice will be difficult. The dredging of the selected areas to 45 feet and to 90 feet intensify and reduce wave energy on Coney Island, respectively, but not very much.

If Coney Island is your only concern, then you will issue permits to bring the selected areas to 90 feet which brings a drop of 14.5×10^{-4} units (7.4%).

It should be pointed out that the decision to be made is not limited to a choice between no mining and mining to some selected depth. There is also the option of mining and then back-filling to a selected depth. While the mining is in progress a transient redirection of wave energy will occur. However, its effects can be minimized by concentrating dredging and back-filling in a brief period selected to coincide with the most favorable wave climate. Once the sand had been removed, the pit could be back-filled with unwanted fine-grained dredged material, thus solving two problems: provision of construction material and disposal of wastes. The final depth of the sand cap would be chosen to give the most desirable distribution of wave energy.

Whether back-filling and capping is a feasible operation and whether it would immobilize deeply buried contaminants to a sufficient extent deserves study. The possibility of solving two problems at once is intriguing.

It should also be pointed out that if changes in bathymetry associated with sand mining are unacceptable, there may be a solution short of prohibiting mining. Mining could be restricted to small areas that would be intensively mined to relatively great depths. Once the sand had been removed, the pit could be back-filled with fine-grained dredged material and capped with sand. If these operations can be successfully combined, it would solve two problems--the need for sand and the need for acceptable disposal sites for contaminated dredged materials. This is a problem worth investigating.

The decision is yours and this analysis can't make it for you. However, it can and does tell you what you are letting yourself--and everybody else--in for if you follow this course of action or that.

WHAT REMAINS TO BE DONE

Obviously, it would be unwise to make any decision about the Lower Bay on the basis of our illustrative calculations. We have considered waves of only 3 periods and 2 directions of travel. When the trends of the New Jersey and Long Island coasts are considered, it is evident that waves traveling anywhere between 260°T and 350°T can enter the harbor mouth. Further, wave periods from 2 seconds to, perhaps, 16 seconds should be considered. If we feel that increments of 5° and 1 second will give a sufficiently detailed estimate of the wave energy distribution around the Lower Bay, 285 wave ray diagrams (15 periods, 19 directions) would have to be calculated, appropriate energy weights assigned to each, and the total energy per unit length of impact strip determined. The final result, the wave energy distribution around the shoreline of the Lower Bay, would be no more difficult to understand or cumbersome to use than the illustrative distributions we have already seen. It is the labor involved in arriving at them when not 6 but 285 wave ray diagrams must be plotted and handled that gives one pause.

It is true that, if it is known that waves of some periods and directions of travel *never* occur off the mouth of the Lower Bay with appreciable energy, they may be omitted and the number of wave ray diagrams somewhat reduced but this doesn't really relieve the difficulty of handling large masses of graphical material. And remember that this unwieldy mass is generated each time some new alteration in water depth is explored. The prospect is about as uninviting as an invitation to read the Manhattan telephone directory in detail from cover to cover.

Another difficulty with the graphical presentation of wave ray diagrams as used in our illustration lies in the incompatibility between the computer and the plotter. Digital computers are fast (very fast). Plotters are, in comparison, slow (as slow as the proverbial molasses in January). It takes slightly longer than forever to get the plots made and runs the cost up. What is needed is some way to get the whole job carried through to the wave energy distributions rapidly and easily so that many alternatives can be explored and compared.

Fortunately, there is a way to meet this demand which is well within the state of the computer art. The trick is to leave it all up to the computer. A program associated with the ray tracing program can persuade the computer to take each ray as it comes ashore, multiply by its energy weight, assign the energy to the proper impact strip, and cumulate the energies as it goes along. The final print-out would be the wave energy distribution; (in our illustration, *one* line of Table 13) for each dredging situation considered no matter how many periods and directions have gone to make it up.

The drawback to this simple solution is that the wave ray diagrams are lost and they can be useful. For example, in our illustration, without the wave ray diagrams we would not have noticed the reflecting properties of Ambrose Channel nor would we have been led to speculate on the possibility of creating "wave guides" by dredging as a means of coastal protection. Of course, one could have the computer store the ray calculations and print them out later--much later--for visual inspection but this would put us right back to square one with a mass of graphic material on our hands. Even worse, most of it would be of no interest. What we need is a way to pick out from the mass those few diagrams which will further study.

The solution is a visual output on line with the computer which will show the rays and ray patterns as they develop. For purposes like this a cathode ray tube (television tube) has already been used. For example, the screen could be made to display an identification, say 4/275 for a 4-second wave traveling toward 275°T, and the ray pattern traced on the screen as it developed. If the operator saw anything worth further study, he would press a button instructing the computer to store that particular ray diagram for use in the plotter. If the ray pattern were of no interest, the operator would simply let it pass. In this way only those ray diagrams necessary for further consideration would be preserved in permanent graphic form.

The final step to make this a practical working tool is to provide for easy and rapid alteration of the input.

One needs digitized bathymetry for the area of interest. In the course of preparing this paper we have digitized the bathymetry of the Lower Bay. It should be stored so that it can be used for subsequent studies of the area. As additional areas such as sections of the North Shore of Long Island and Fire Island come under study their digitized bathymetries should be added to the library. Ideally, in the end the library will cover all the coastal areas of a political unit like New York State and any manager who wants to explore the effects of a proposed alteration in water depths will have the digitized bathymetry he needs at hand.

The operator needs a way to alter the bathymetry either at selected clusters of grid points to simulate local dredging or over the entire grid to simulate changes in the tide level.

An easy way to divide the shoreline into impact strips is needed. There is no one all-purpose division suitable for every problem that comes to hand and the operator should be able to change the division at will.

The periods, directions, and energy weights of the waves it is desired to use must be entered. Again flexibility and ease are the prime considerations.

All of these additions to programs and hardware can be made. They will take time and ingenuity and they will have a price. However, if the tool appeals as worth having, the tool can be bought.

Let it be stressed again: This wave ray-computer method will not make the decisions. Wave erosion is only one of the processes that enter. There are others which can be quantified such as long-shore currents (whose study also needs the bathymetry) and shore erodability (whose study does not). In addition, there are nonquantifiable, intangible factors such as political advisability which must affect the decision. In the end, only an acute intelligence supported by judgement and wisdom is qualified to decide; not a computer, not science. As Dr. Girardo Hiriart LeB. once gracefully expressed it,

"Lo que la natura no lo da, Salamanca no lo presta."

The GI's of World War II put it more brutally,

"There ain't no issue number for brains."

Kinsman, Blair. 1965. WIND WAVES their generation and propagation on the ocean surface, Prentice-Hall, Inc., Englewood Cliffs, N.J. 676pp.

APPENDIX

DESCRIPTION OF COMPUTER PROGRAM

The computer program for the calculation and plotting of surface wave rays consists of a main program and ten subroutines which are listed here plus three standard Calcomp plotter subroutines. A generalized flow chart of the program (not including the plotter subroutines) appears in Fig. A-1.

MAIN PROGRAM

MAIN consists of two major sections which control the operation of the computer program. The first is the input section where the five major card groups depicted in Fig. A-2 are read in. More detailed information on these parameters may be found in the section DESCRIPTION OF INPUT PARAMETERS.

The second section of MAIN is the computation section consisting of three major control loops. The first loop sequences through each of the INA ray angles defined in the input. The second loop sequences through each of the INP ray periods defined in the input.

Once an angle and a period have been set, TITLE is called to initialize the plotting area. The final loop is then executed to sequence through the N0R wave rays defined in the input, a call being made to RAYN for each of them in turn. An individual plot therefore consists of N0R wave ray tracks each of which has the same initial ray angle and ray period but a unique ray origin.

When the three loops are exhausted, the number of plots drawn, which should equal INA times INP, is printed and the job terminates.

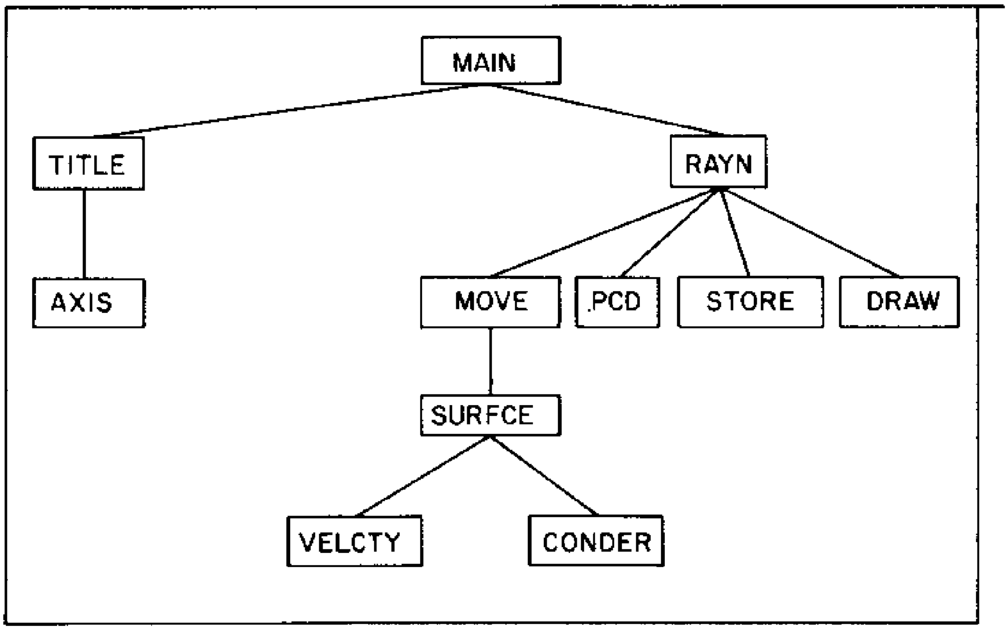

Fig. A1 Schematic flow chart of computer program, excluding plotter subroutines.

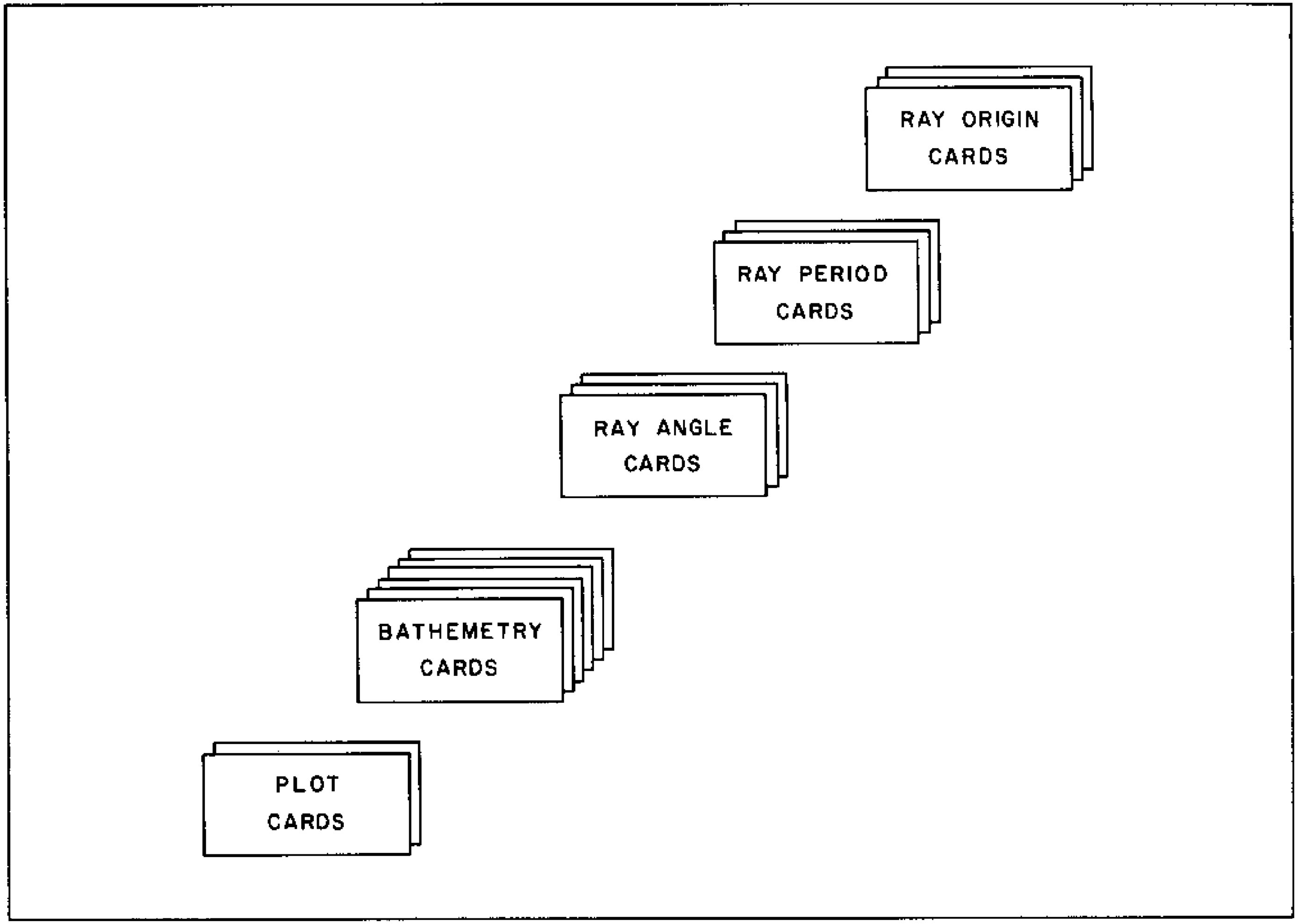

Fig. A-2 Schematic diagram of input card deck.

SUBROUTINE TITLE

TITLE is the first subroutine called by MAIN and it must be called to initialize the plotter for each new plot. The routine begins by centering the new plot vertically (Y-direction) and by establishing a new plot origin at the lower left corner of the plot border. A label is then plotted to the left of the border consisting of the project number, date, plot number, plot scale, wave period and time between crest marks. A straight line rectangular border is then drawn and depending on the value of NAX, an optional call to AXIS may be made to add calibrated X- and Y-axes.

SUBROUTINE AXIS

AXIS, if optionally called by TITLE, labels the X- and Y-axes and draws calibration marks at integral grid locations along each axis. For proper operation of this routine, both the X- and the Y-axis should be an integral number of grid units in length.

SUBROUTINE RAYN

RAYN is called by MAIN for each new wave ray to determine its path from origin to termination point. RAYN calls MØVE to determine the coordinates of each new point along a ray. The goodness of fit of this point to the least squares plane computed by SURFCE is then made by a call to PCD. RAYN then calls STØRE to save the coordinates of this new point for subsequent plotting. A wave ray is considered terminated if it reaches the grid boundary, if it reaches shore, or if the curvature iteration in MØVE fails to converge. Once the ray has reached its termination point, RAYN calls DRAW to plot its track.

Depending on the value of the print flag (NPT), RAYN outputs information in either of two formats. If NPT equals zero, then wave ray characteristics are output only for the rays origin and termination points. If NPT is not equal to zero, then wave ray characteristics are output for all points along the ray.

SUBROUTINE MOVE

MØVE is called by RAYN to calculate the coordinates of each new point along a wave ray. The incremental step (D) is computed and together with the curvature used to obtain the present point, the approximate location of the next point is computed. MØVE then calls SURFCE to obtain the curvature of this new point. Curvatures of the current and new points are then averaged with this new curvature being used to obtain a new approximation of the next point. This procedure continues until one of three termination conditions is reached.

If two successive curvature averages differ by a factor less than 0.00009/D then convergence is assumed and the new point is accepted. If the average curvatures used on the 18th and 20th iterations differ by a factor less than 0.00009/D, then the curvature used to obtain the new point will be the average of the curvatures computed on the 19th and 20th iterations. This is done due to the fact that the curvature approximations have converged to two values and the iteration is oscillating between the two.

If neither of the prior two conditions are met, then the iteration has failed to converge and no new point is accepted. Finally, MØVE checks to see if the new point has reached the grid boundary and control is returned to RAYN.

SUBROUTINE SURFCE

SURFCE is called by MØVE to calculate ray curvature (K) for a specific point along a wave ray. For this point a plane is fitted by least squares to the four closest depth values in the bathemetry matrix. The water depth (h) is then obtained by interpolating on this plane. If the water depth is less than or equal to zero, then the ray has reached shore and control is returned to MOVE. If water depth is greater than zero, the partial of water depth with respect to the direction normal to the ray (n) is computed using:

$$\frac{\partial h}{\partial n} = - \frac{\partial h}{\partial x} \sin A + \frac{\partial h}{\partial y} \cos A \qquad (1)$$

where A is the direction of travel.

VELCTY and CØNDER are then called to compute wave speed (C) and the partial of wave speed with respect to the normal. If water depth divided by deep water wavelength (W_L) is less than or equal to 0.5, then ray curvature is computed using:

$$K = \frac{1}{C} \left(\frac{-\partial C}{\partial n} \right) \qquad (2)$$

Otherwise, ray curvature is set equal to zero. Deep water wavelength is defined by:

$$W_L = \frac{gT^2}{2\pi} \qquad (3)$$

where g is the acceleration due to gravity in feet per second per second and T is the wave period in seconds.

SUBROUTINE VELCTY

VELCTY is called by SURFCE each time a wave speed is to be calculated. If the water depth divided by the deep water wavelength is greater than 0.5, then the wave speed is set equal to the deep water wave speed (Ws). The deep water wave speed is given by:

$$C = \frac{gT}{2\pi} \tanh \left(\frac{2\pi h}{CT} \right) \qquad (5)$$

This equation is iterated until successive values for C differ by a factor less than 0.00005.

SUBROUTINE CONDER

CØNDER is called by SURFCE to convert the partial of water depth with respect to the direction normal to the ray into the partial of wave speed with respect to the normal. This is given by:

$$\frac{\partial C}{\partial n} = \frac{\partial h}{\partial n} . W \qquad (6)$$

where

$$W = \frac{1}{k'} \cdot \left[\frac{1}{\frac{Ck''}{1+k''C} + \frac{Ck''}{1-k''C} + \ln(1+k''C) - \ln(1-k''C)} \right]$$

$$k' = \frac{T}{4\pi}$$

$$k'' = \frac{2\pi}{gT}$$

SUBROUTINE PCD

PCD is called by RAYN after each point along a ray has been calculated in order to get a measure of the goodness of fit of the linear interpolation plane computed by SURFCE. For each of the four depth values closest to the point, PCD computes the percentage difference between the actual value at that point and the corresponding point on the computed plane. The maximum of these is then taken as a measure of the goodness of fit.

SUBROUTINE STORE

After each point along a ray has been calculated, RAYN calls STØRE. The X- and Y- coordinates of the point are stored in two arrays so that they will be available later for plotting purposes. Additionally, if the input parameter CIN is greater than zero, the X- and Y- coordinates of the wave crest marks will be calculated and stored. If CIN equals zero, no crest mark positions will be calculated or stored.

SUBROUTINE DRAW

When all points have been calculated for a given ray, RAYN calls DRAW to plot the entire ray track from its origin to its termination point or vice versa whichever is optimal in terms of pen movement. Depending on the value of the variable FAN, defined in the BLØCK DATA section of the program, the rays may be numbered on the plot. If FAN is less than zero, rays will be numbered at their origins. If FAN is greater than zero, rays will be numbered at their termination points. If FAN equals zero, ray numbering will be suppressed. Additionally, if the input parameter CIN is greater than zero, marks will be placed along the rays, normal to the direction of travel, to designate crest positions. If CIN equals zero, no crest marks will be drawn.

DESCRIPTION OF INPUT PARAMETERS

The input data for this program consists of five major card groups: Plot cards, Bathemetry cards, Ray angle cards, Ray period cards, and Ray origin cards. Each of the input parameters on these cards will be described in a four column format. The first is the variable name used for the parameter in the program. The second column contains the starting and ending columns on the input card for that parameter. The third column contains the FØRTRAN input format used to read the parameter, The fourth and final column gives a description of the parameter.

$$W_s = \frac{gT}{2\pi} \tag{4}$$

If the water depth divided by the deep water wavelength is less than or equal to 0.5, the wave speed is computed using:

REFERENCE

Wilson, W. Stanley, 1966. A Method for Calculating and Plotting Surface Wave Rays, Technical Memorandum No. 17, U.S. Army Coastal Engineering Research Center.

PLOT CARDS

Card one:

PROJCT	1- 6	A6	Project number.
DATE1, DATE2	7-14	2A4	Date field (MM/DD/YY).

Card two:

	1- 5	5X	Blank.
NOR	6- 9	I4	Number of rays to be calculated for this set of plots.
MM	10-13	I4	Number of rows in the bathemetry matrix.
NN	14-17	I4	Number of columns in the bathemetry matrix.
GRID	18-24	F7.0	Number of feet per grid unit in the bathemetry matrix.
DCON	25-31	F7.0	Multiplicative factor to convert input bathemetry matrix to feet.
	32-43	12X	Blank.
NPT	44-47	I4	Print flag. Zero indicates abbreviated printout, non-zero indicates unabbreviated printout.
NAX	48-51	I4	Calibration flag. Zero indicates plain borders, non-zero indicates calibration marks are to be drawn at each grid unit on the border.
CIN	52-58	F7.0	Time between crest marks along a ray expressed in seconds. Zero indicates no crest marks should be drawn.
HT	59-67	F9.3	Height for a specific plot expressed in inches.

BATHEMETRY CARDS

CMAT(I,J)	1-60	10F6.1	Bathemetry matrix. Read by column, the first depth value of each column beginning on a new card. Each column

will take (MM+9)/10 input cards and there will be NN such groups.

RAY ANGLE CARDS

Card one:

INA	1- 3	I3	Number of initial wave ray angles.

Remaining cards:

ANG(I)	1-64	8F8.3	Initial wave ray angles, expressed in degrees, measured counterclockwise from the positive x-axis. There will be (INA+7)/8 such cards.

RAY PERIOD CARDS

Card one:

INP	1- 3	I3	Number of wave ray periods.

Remaining cards:

PER(I)	1-64	8F8.2	Wave ray periods, expressed in seconds. There will be (INP+7)/8 such cards.

RAY ORIGIN CARDS

X	1- 6	F6.2	X-coordinate of ray origin expressed in grid units.
Y	7-12	F6.2	Y-coordinate of ray origin expressed in grid units. There will be N0R such cards.

```
C
C     PROGRAM FOR THE CALCULATION AND PLOTTING OF SURFACE WAVE RAYS
C
C     WRITTEN IN FORTRAN IV FOR THE UNIVAC 1110 COMPUTER AND THE
C     CALCOMP 910/563 PLOTTING SYSTEM. LOGICAL UNIT 2 WAS USED FOR
C     THE PLOTTER TAPE OUTPUT. IN ADDITION TO THE SUBROUTINES LISTED,
C     THIS PROGRAM NEEDS THE CALCOMP SUBROUTINES "PLOT" , "SYMBOL",
C     AND "NUMBER". FOR INFORMATION ON THESE ROUTINES REFER TO
C     "PROGRAMMING CALCOMP PEN PLOTTERS", SEPTEMBER 1969. CALIFORNIA
C     COMPUTER PRODUCTS,INC.
C
C     THIS PROGRAM WAS PREPARED BY GEORGE F. CARROLL, MARINE SCIENCES
C     RESEARCH CENTER, SUNY AT STONY BROOK AND REPRESENTS A MODIFICATION
C     OF A PROGRAM ORIGINALLY PREPARED BY W. STANLEY WILSON, DEPARTMENT
C     OF OCEANOGRAPHY, JOHNS HOPKINS UNIVERSITY.
C
      CHARACTER*6 PROJCT
      DIMENSION S(3,3),EM(4,3),E(3),YVW(3),CMAT(100,100),C(4),
     1 AX(5000),AY(5000),CTOUR(5),X(50),Y(50),ANG(100),PER(100)
      COMMON S,EM,E,YVW,CMAT,C,AX,AY,CTOUR,PROJCT,D,TT,CXY,MAX,
     1 GRID,DCON,DEP,WL,AMM,ANN,DY,FAN,DATE1,DATE2,CIN
C
      CALL PLOTS(0,0,2)
C
C     INPUT PLOT CARDS
C
      READ(5,499)PROJCT,DATE1,DATE2
  499 FORMAT(A6,2A4)
      READ(5,500)NOR,MM,NN,GRID,DCON,NPT,NAX,CIN,HT
  500 FORMAT(5X,3I4,2F7.0,12X,2I4,F7.0,F9.3)
C
C     INPUT BATHEMETRY CARDS
C
      DO 10 J=1,NN
      READ(5,501)(CMAT(I,J),I=1,MM)
  501 FORMAT(10F6.1)
   10 CONTINUE
C
C     INPUT RAY ANGLE CARDS
C
      READ(5,503)INA,(ANG(I),I=1,INA)
  503 FORMAT(I3/(8F8.3))
C
C     INPUT RAY PERIOD CARDS
C
      READ(5,504)INP,(PER(I),I=1,INP)
  504 FORMAT(I3/(8F8.2))
C
C     INPUT RAY ORIGIN CARDS
C
      READ(5,502)(X(I),Y(I),I=1,NOR)
  502 FORMAT(2F6.2)
C
C
      MMAX=5000
      LI=53
      LII=(LI-4)/3
      CIN=CIN/3600.0
      WL=32.2*TT*TT/6.2831854
      AMM=FLOAT(MM-1)
```

```
      ANN=FLOAT(NN-1)
      DY=ANN/HT
      SCLI=GRID*DY*12.0
      YT=15.0-HT/2.0
      RT=AMM/DY
      NPLOT=0
C
C
      DO 40 IT=1,INA
      A=ANG(IT)*0.0174532925
      DO 30 IP=1,INP
      TT=PER(IT)
      NPLOT=NPLOT+1
      CALL TITLE(NPLOT,NAX,SCLI,HT)
      DO 20 N=1,NOR
      MAX=1
      XX=X(N)
      YY=Y(N)
      AA=A
      CALL RAYN(XX,YY,AA,NPLOT,N,MMAX,LI,NPT,LII)
   20 CONTINUE
      CALL PLOT(RT+6.0,-YT,-3)
   30 CONTINUE
   40 CONTINUE
C
C
      CALL PLOT(0.0,0.0,999)
      WRITE(6,9999)NPLOT
 9999 FORMAT('1JOB ENDED NORMALLY , ',I3,' PLOTS DRAWN')
      STOP
      END
```

```
C
C
C
      BLOCK DATA
C
      CHARACTER*6 PROJCT
      DIMENSION S(3,3),EM(4,3),E(3),YVW(3),CMAT(180,130),C(4),
     1 AX(5000),AY(5000),CTOUR(5)
      COMMON S,EM,E,YVW,CMAT,C,AX,AY,CTOUR,PROJCT,D,TT,CXY,MAX,
     1 GRID,DCON,DEP,WL,AMM,ANN,DY,FAN,DATE1,DATE2,CIN
C
      DATA ((S(I,J),J=1,3),I=1,3)/0.75,3*-0.5,1.0,0.0,-0.5,0.0,1.0/,
     1 ((EM(L,I),L=1,4),I=1,3)/4*1.0,0.0,2*1.0,3*0.0,2*1.0/,
     2 FAN/0.0/
      END
C
C
      SUBROUTINE TITLE(NPLOT,NAX,SCL1,HT)
C
      CHARACTER*6 PROJCT,IMAGE*77
      DIMENSION S(3,3),EM(4,3),E(3),YVW(3),CMAT(180,130),C(4),
     1 AX(5000),AY(5000),CTOUR(5)
      COMMON S,EM,E,YVW,CMAT,C,AX,AY,CTOUR,PROJCT,D,TT,CXY,MAX,
     1 GRID,DCON,DEP,WL,AMM,ANN,DY,FAN,DATE1,DATE2,CIN
C
      CALL PLOT(0.0,15.0-HT/2.0,-3)
  701 RT=AMM/DY
      XNPLOT=NPLOT
      ENCODE(200,IMAGE)PROJCT,DATE1,DATE2,IFIX(XNPLOT+.5),IFIX(SCL1+.5),
     1 TT,IFIX(CIN*3600.+.5)
  200 FORMAT('PROJ.NO.  ',A6,',  ',2A4,',  PLOT NO. ',I3,',  SCL = 1/',
     1 I6,',  TT = ',F4.1,',  CIN = ',I3)
      CALL SYMBOL(-1.5,0.0,0.21,IMAGE,90.,76)
      IF(NAX)705,704,705
  704 CALL PLOT(0.0,0.0,3)
      CALL PLOT(0.0,HT,2)
      GOTO 706
  705 CALL AXIS(0.,0.,1HY,1,HT ,90.,0.,DY)
      CALL AXIS(0.,0.,1HX,-1,RT,0.,0.,DY)
      CALL PLOT(0.0,HT,3)
  706 CALL PLOT(RT,HT,2)
      CALL PLOT(RT,0.0,2)
      IF(NAX)707,708,707
  708 CALL PLOT(0.0,0.0,2)
  707 YHT=HT
      RETURN
      END
C
C
      SUBROUTINE AXIS(X,Y,BCD,NC,SIZE,THETA,YMIN,DY)
C
      SIGN=1.0
      IF(NC)1,2,2
    1 SIGN=-1.0
    2 NAC=IABS(NC)
      TH=THETA*0.017453294
      N=DY*SIZE+0.5
      CTH=COS(TH)
      STH=SIN(TH)
      TN=N
      XB=X
      YB=Y
      XA=X-0.1*SIGN*STH
      YA=Y+0.1*SIGN*CTH
      CALL PLOT(XA,YA,3)
      DO 20 I=1,N
      CALL PLOT(XB,YB,2)
      XC=XB+CTH/DY
      YC=YB+STH/DY
      CALL PLOT(XC,YC,2)
      XA=XA+CTH/DY
      YA=YA+STH/DY
      CALL PLOT(XA,YA,2)
      XB=XC
   20 YB=YC
      ABSV=YMIN+TN
      XA=XB-(.20*SIGN-.05)*STH-.02857*CTH
      YA=YB+(.20*SIGN-.05)*CTH-.02857*STH
      N=N+1
      DO 30 I=1,N
      IF(AMOD(ABSV,5.))100,101,100
  101 CALL NUMBER(XA,YA,0.1,ABSV,THETA,-1)
  100 ABSV=ABSV-1.
      XA=XA-CTH/DY
   30 YA=YA-STH/DY
      TNC=NAC+7
      XA=X+(SIZE/2.0-.06*TNC)*CTH-(-.07+SIGN*.36)*STH
      YA=Y+(SIZE/2.0-.06*TNC)*STH+(-.07+SIGN*.36)*CTH
      CALL SYMBOL(XA,YA,0.14,BCD,THETA,NAC)
```

```
      RETURN
      END
C
C
      SUBROUTINE RAYN(X,Y,A,NPLOT,N,MMAX,LI,NPT,LII)
C
      CHARACTER*6 PROJCT
      DIMENSION S(3,3),EM(4,3),E(3),YVW(3),CMAT(180,130),C(4),
     1 AX(5000),AY(5000),CTOUR(5)
      COMMON S,EM,E,YVW,CMAT,C,AX,AY,CTOUR,PROJCT,D,TT,CXY,MAX,
     1 GRID,DCON,DEP,WL,AMM,ANN,DY,FAN,DATE1,DATE2,CIN
C
      NDP=1
      NFK=1
      NGO=1
      KREST=0
      KCIN=0
      CALL SURFCE(X,Y,A,FK,NFK,NDP)
      CALL MOVE(X,Y,A,FK,NGO,MIT,NFK,NDP)
      TIME=0.0
      ANGLE=A*57.29577951
      IF(NPT)100,101,100
  100 WRITE(6,7)PROJCT,DATE1,DATE2,NPLOT,TT,N
    7 FORMAT(1H1,11HPROJECT NO.,A6,2H, ,2A4,10H, PLOT NO.,I3,
     1 10H, PERIOD =,F5.1,14H SEC., RAY NO.,I3,1H.//)
      WRITE(6,150)
  150 FORMAT(1H ,3X,3HMAX,6X,1HX,8X,1HY,8X,5HANGLE,6X,4HTIME,4X,
     1 6HPCTDIF,5X,5HDEPTH,6X,1HD//)
      GOTO 19
  101 IF(N-1)800,800,801
  801 IF(MOD(N,LII))803,800,803
  800 WRITE(6,850)PROJCT,DATE1,DATE2,NPLOT,TT
  850 FORMAT(12H1PROJECT NO.,A6,2H, ,2A4,10H, PLOT NO.,I3,
     1 10H, PERIOD =,F5.1,5H SEC.///)
      WRITE(6,851)
  851 FORMAT(8H RAY NO.,4X,3HMAX,6X,1HX,8X,1HY,8X,5HANGLE,6X,4HTIME//)
  803 WRITE(6,853)N,MAX,X,Y,ANGLE,TIME
  853 FORMAT(1H ,I6,1X,I7,2F9.2,F11.2,F10.3)
      GOTO 19
    3 MAX=1+MAX
      IF(MAX+KCIN-MMAX)399,400,400
  400 WRITE(6,401)
  401 FORMAT(80X,36HDIMENSION OF OUTPUT-ARRAYS EXCEEDED.)
      GOTO 15
  399 ZCXY=CXY
      CALL MOVE(X,Y,A,FK,NGO,MIT,NFK,NDP)
      GOTO(396,402),NDP
  402 WRITE(6,403)
  403 FORMAT(80X,18HRAY REACHED SHORE.)
      GOTO 15
  396 IF(D/DY-.005)700,700,702
  700 WRITE(6,701)
  701 FORMAT(80X,26HRAY REACHED SHALLOW WATER.)
      GOTO 15
  702 GOTO(397,397,404),MIT
  404 WRITE(6,405)
  405 FORMAT(80X,40HCURVATURE APPROXIMATIONS NOT CONVERGING.)
      GOTO 15
  397 IF(NPT)180,20,180
  180 IF(MOD(MAX,LI))20,5,20

    5 WRITE(6,7)PROJCT,DATE1,DATE2,NPLOT,TT,N
      WRITE(6,150)
   20 TIME=TIME+(D*GRID/(1800.*(CXY+ZCXY)))
      ANGLE=A*57.29577951
   19 IF(NPT)160,161,160
  160 CALL PCD(C,E,PCTDIF)
      WRITE(6,12)MAX,X,Y,ANGLE,TIME,PCTDIF,DEP,D
   12 FORMAT(I7,2F9.2,F11.2,F10.3,F10.1,F10.2,F10.3)
  161 KMAX=MAX
      PX=X
      PY=Y
      CALL STORE(X,Y,A,KMAX,TIME,KCIN,KREST)
      GOTO(10,11),MIT
   11 IF(NPT)170,169,170
  169 WRITE(6,853)N,MAX,X,Y,ANGLE,TIME
  170 WRITE(6,9)
    9 FORMAT(1H+,80X,19HCURVATURE AVERAGED.)
   10 IF(MAX-1)4,4,13
    4 GOTO(3,402),NDP
   13 GOTO(3,406),NGO
  406 WRITE(6,407)
  407 FORMAT(80X,26HRAY REACHED GRID BOUNDARY.)
   15 IF(NPT)190,191,190
  191 WRITE(6,1233)N,KMAX,PX,PY,ANGLE,TIME
 1233 FORMAT(1H+,I6,1X,I7,2F9.2,F11.2,F10.3,/)
  190 CALL DRAW(N,KMAX,KCIN,KREST)
      RETURN
      END
```

```
C                                                                       MSRC0269
C                                                                       MSRC0270
      SUBROUTINE MOVE(X,Y,A,FK,NGO,MIT,NFK,NDP)                         MSRC0271
C                                                                       MSRC0272
      CHARACTER*6 PROJCT                                                MSRC0273
      DIMENSION S(3,3),EM(4,3),E(3),YVW(3),CMAT(190,130),C(4),          MSRC0274
     1 AX(5000),AY(5000),CTOUR(5)                                       MSRC0275
      COMMON S,EM,E,YVW,CMAT,C,AX,AY,CTOUR,PROJCT,D,TT,CXY,MAX,         MSRC0276
     1 GRID,DCON,DEP,WL,AMM,ANN,DY,FAN,DATE1,DATE2,CIN                  MSRC0277
C                                                                       MSRC0278
      MIT=1                                                             MSRC0279
      GOTO(201,202),NFK                                                 MSRC0280
  201 D=0.5                                                             MSRC0281
      GOTO 203                                                          MSRC0282
  202 D=AMAX1(DEP/WL,0.025*DY)                                          MSRC0283
  203 IF(MAX-2)38,102,104                                               MSRC0284
  102 FKBAR=FK                                                          MSRC0285
  104 DO 20 II=1.20                                                     MSRC0286
   39 DELA=FKBAR*D                                                      MSRC0287
      AA=A+DELA                                                         MSRC0288
      ABAR=A+.5*DELA                                                    MSRC0289
      DELX=D*COS(ABAR)                                                  MSRC0290
      DELY=D*SIN(ABAR)                                                  MSRC0291
      XX=X+DELX                                                         MSRC0292
      YY=Y+DELY                                                         MSRC0293
      CALL SURFCE(XX,YY,AA,FKK,NFK,NDP)                                 MSRC0294
      GOTO(101,6),MIT                                                   MSRC0295
  101 GOTO(10,38),NDP                                                   MSRC0296
   10 FKBAR=0.5*(FK+FKK)                                                MSRC0297
      IF(II-18)5,37,9                                                   MSRC0298
   37 FKKPP=FKBAR                                                       MSRC0299
    5 IF(MAX-2)7,7,9                                                    MSRC0300
```

```
    7 IF(II-1)20,20,9                                                   MSRC0301
    9 IF(ABS(FKKP-FKBAR)-(0.00009/D))6,6,19                             MSRC0302
   19 IF(II.EQ.20)GOTO 21                                               MSRC0303
   20 FKKP=FKBAR                                                        MSRC0304
   21 IF(ABS(FKKPP-FKBAR)-(0.00009/D))18,18,17                          MSRC0305
   17 MIT=3                                                             MSRC0306
      GOTO 38                                                           MSRC0307
   18 FKBAR=0.5*(FKBAR+FKKP)                                            MSRC0308
      FKKP=FKBAR                                                        MSRC0309
      MIT=2                                                             MSRC0310
      GOTO 39                                                           MSRC0311
    6 IF((XX-0.5)*((AMM-0.5)-XX))2,2,3                                  MSRC0312
    3 IF((YY-0.5)*((ANN-0.5)-YY))2,2,8                                  MSRC0313
    2 NGO=2                                                             MSRC0314
    8 X=XX                                                              MSRC0315
      Y=YY                                                              MSRC0316
      A=AA                                                              MSRC0317
      FK=FKK                                                            MSRC0318
   38 RETURN                                                            MSRC0319
      END                                                               MSRC0320
C                                                                       MSRC0321
C                                                                       MSRC0322
      SUBROUTINE SURFCE(X,Y,A,FK,NFK,NDP)                               MSRC0323
C                                                                       MSRC0324
      CHARACTER*6 PROJCT                                                MSRC0325
      DIMENSION S(3,3),EM(4,3),F(3),YVW(3),CMAT(190,130),C(4),          MSRC0326
     1 AX(5000),AY(5000),CTOUR(5)                                       MSRC0327
      COMMON S,EM,E,YVW,CMAT,C,AX,AY,CTOUR,PROJCT,D,TT,CXY,MAX,         MSRC0328
     1 GRID,DCON,DEP,WL,AMM,ANN,DY,FAN,DATE1,DATE2,CIN                  MSRC0329
      I=X+1.                                                            MSRC0330
      J=Y+1.                                                            MSRC0331
      FI=I                                                              MSRC0332
      FJ=J                                                              MSRC0333
      XL=X+1.-FI                                                        MSRC0334
      YL=Y+1.-FJ                                                        MSRC0335
      IF(MAX-1)1,1,4                                                    MSRC0336
    4 IF(ZI-FI)1,2,1                                                    MSRC0337
    2 IF(ZJ-FJ)1,3,1                                                    MSRC0338
    1 ZI=FI                                                             MSRC0339
      ZJ=FJ                                                             MSRC0340
      C(1)=CMAT(I,J)                                                    MSRC0341
      C(2)=CMAT(I+1,J)                                                  MSRC0342
      C(3)=CMAT(I+1,J+1)                                                MSRC0343
      C(4)=CMAT(I,J+1)                                                  MSRC0344
      DO 318 II=1,3                                                     MSRC0345
      YVW(II)=0.                                                        MSRC0346
      DO 318 L=1,4                                                      MSRC0347
  318 YVW(II)=YVW(II)+C(L)*EM(L,II)                                     MSRC0348
      DO 319 II=1,3                                                     MSRC0349
      E(II)=0.                                                          MSRC0350
      DO 319 JJ=1,3                                                     MSRC0351
  319 E(II)=E(II)+S(II,JJ)*YVW(JJ)                                      MSRC0352
    3 DEP=(E(1)+E(2)*XL+E(3)*YL)*DCON                                   MSRC0353
      IF(DEP)320,320,324                                                MSRC0354
  320 NDP=2                                                             MSRC0355
      GOTO 403                                                          MSRC0356
  324 IF((DEP/WL)-0.5)321,321,322                                       MSRC0357
  321 NFK=2                                                             MSRC0358
      GOTO 323                                                          MSRC0359
  322 NFK=1                                                             MSRC0360
```

```
  323 CALL VELCTY(CXY,TT,MAX,DEP,NFK)
      PCX=E(2)*DCON
      PCY=E(3)*DCON
      DN=-PCX*SIN(A)+PCY*COS(A)
      CALL CONDER(DN,TT,CXY,MAX,NFK)
      GOTO(401,402),NFK
  401 FK=0.0
      GOTO 403
  402 FK=-DN/CXY
  403 RETURN
      END
C
C
      SUPROUTINE VELCTY(CXY,TT,MAX,DEP,NFK)
C
      IF(MAX-1)101,101,102
  101 BAR=6.2831854/TT
      CXX0=TT*32.2/6.2831854
      CCC=CXX0
      GOTO 103
  102 CCC=XCXY
  103 GOTO(104,105),NFK
  104 CXY=CXX0
      GOTO 106
  105 DO 1000 M=1,90
      CXY=CXX0*TANH(BAR*DEP/CCC)
      IF(ABS(CXY-CCC)-.00005)106,1000,1000
 1000 CCC=(CXY+CCC)/2.
  106 XCXY=CXY
      RETURN
      END
C
C
      SUBROUTINE CONDER(DN,TT,CXY,MAX,NFK)
C
      IF(MAX-1)101,101,102
  101 C1=TT/12.5663708
      C2=6.2831854/(32.2*TT)
  102 GOTO(105,104),NFK
  104 C3=C2*CXY
      A1=C3/(1.+C3)
      A2=C3/(1.-C3)
      A3=LOG(1.+C3)
      A4=LOG(1.-C3)
      DN=(DN/C1)*(1./(A1+A2+A3-A4))
  105 RETURN
      END
C
C
      SUBROUTINE PCD(C,E,PCTDIF)
C
      DIMENSION C(4),E(3)
C
      IF(C(1)*C(2)*C(3)*C(4))901,900,901
  900 PCTDIF=999.
      GOTO 902
  901 P1=ABS((C(1)-E(1))/C(1))
      P2=ABS((C(2)-E(1)-E(2))/C(2))
      P3=ABS((C(3)-E(1)-E(2)-E(3))/C(3))
      P4=ABS((C(4)-E(1)-E(3))/C(4))
```

```
      PCTDIF=100.*AMAX1(P1,P2,P3,P4)
  902 RETURN
      END
C
C
      SUBROUTINE STORE(X,Y,A,KMAX,TIME,KCIN,KREST)
C
      CHARACTER*6 PROJCT
      DIMENSION S(3,3),EM(4,3),E(3),YVW(3),CMAT(180,130),C(4),
     1 AX(5000),AY(5000),CTOUR(5)
      COMMON S,EM,E,YVW,CMAT,C,AX,AY,CTOUR,PROJCT,D,TT,CXY,MAX,
     1 GRID,DCON,DEP,WL,AMM,ANN,DY,FAN,DATE1,DATE2,CIN
C
      IF(CIN)403,403,410
  410 IF(KMAX-1)400,400,401
  400 AT=0.0
  403 K=KMAX+KCIN
      AX(K)=X
      AY(K)=Y
      IF(CIN)205,205,402
  402 ZA=A
      ZCXY=CXY
      GOTO 205
  401 ET=TIME-AT
      IF(CIN-ET)405,404,403
  404 K=KMAX+KCIN
      AX(K)=-X
      AY(K)= Y
      KREST=KREST+1
```

```
      AT=AT+CIN                                                         MSRC0450
      GOTO 402                                                          MSRC0451
  405 DSC=(ET-CIN)*(CXY+ZCXY)*3600./(GRID*2.)                           MSRC0452
      AA=(A+ZA)/2.                                                      MSRC0453
      XM=DSC*COS(AA)                                                    MSRC0454
      YM=DSC*SIN(AA)                                                    MSRC0455
      K=KMAX+KCIN                                                       MSRC0456
      AX(K)=-X+XM                                                       MSRC0457
      AY(K)= Y-YM                                                       MSRC0458
      KREST=KREST+1                                                     MSRC0459
      KCIN=KCIN+1                                                       MSRC0460
      AT=AT+CIN                                                         MSRC0461
      GOTO 401                                                          MSRC0462
  205 RETURN                                                            MSRC0463
      END                                                               MSRC0464
C                                                                       MSRC0465
C                                                                       MSRC0466
      SUBROUTINE DRAW(N,KMAX,KCIN,KREST)                                MSRC0467
C                                                                       MSRC0468
      CHARACTER*6 PROJCT                                                MSRC0469
      DIMENSION S(3,3),EM(4,3),E(3),YVW(3),CMAT(180,130),C(4),          MSRC0470
     1 AX(5000),AY(5000),CTOUR(5)                                       MSRC0471
      COMMON S,EM,E,YVW,CMAT,C,AX,AY,CTOUR,PROJCT,D,TT,CXY,WAX,         MSRC0472
     1 GRID,DCON,DEP,WL,AMM,ANN,DY,FAN,DATE1,DATE2,CIN                  MSRC0473
C                                                                       MSRC0474
      XN=N                                                              MSRC0475
      KMAX=KMAX+KCIN                                                    MSRC0476
      IF(AX(KMAX))600,601,601                                           MSRC0477
  600 AX(KMAX)=-AX(KMAX)                                                MSRC0478
      KREST=KREST-1                                                     MSRC0479
  601 IF(MOD(N,2))104,103,104                                           MSRC0480
```

```
  103 KTWO=KMAX-1                                                       MSRC0481
      KADD=-1                                                           MSRC0482
      LAST=+1                                                           MSRC0483
      MC=KREST+1                                                        MSRC0484
      IF(FAN)201,201,200                                                MSRC0485
  200 CALL NUMBER(AX(KMAX)/DY,AY(KMAX)/DY,0.175,XN,0.0,-1)              MSRC0486
  201 CALL PLOT(AX(KMAX)/DY,AY(KMAX)/DY,3)                              MSRC0487
      IF(KMAX-1)106,106,105                                             MSRC0488
  104 KTWO=+2                                                           MSRC0489
      KADD=+1                                                           MSRC0490
      LAST=KMAX                                                         MSRC0491
      MC=0                                                              MSRC0492
      IF(FAN)110,111,111                                                MSRC0493
  110 CALL NUMBER(AX(1)/DY,AY(1)/DY,0.175,XN,0.0,-1)                    MSRC0494
  111 CALL PLOT(AX(1)/DY,AY(1)/DY,3)                                    MSRC0495
      IF(KMAX-1)106,106,105                                             MSRC0496
  105 IF(CIN)300,300,301                                                MSRC0497
  301 IF(AX(KTWO))302,300,300                                           MSRC0498
  300 CALL PLOT(AX(KTWO)/DY,AY(KTWO)/DY,2)                              MSRC0499
      GOTO 303                                                          MSRC0500
  302 AX(KTWO)=-AX(KTWO)                                                MSRC0501
      WI=0.05                                                           MSRC0502
      MC=MC+KADD                                                        MSRC0503
      IF(MOD(MC,10))500,501,500                                         MSRC0504
  501 WI=0.10                                                           MSRC0505
  500 XPN=AX(KTWO)/DY                                                   MSRC0506
      YPN=AY(KTWO)/DY                                                   MSRC0507
      K=KTWO-KADD                                                       MSRC0508
      XPL=AX(K)/DY                                                      MSRC0509
      YPL=AY(K)/DY                                                      MSRC0510
      DSC=SQRT((XPN-XPL)**2.+(YPN-YPL)**2.)                             MSRC0511
      CALL PLOT(XPN,YPN,2)                                              MSRC0512
      XB=+WI*(YPN-YPL)/DSC                                              MSRC0513
      YB=-WI*(XPN-XPL)/DSC                                              MSRC0514
      CALL PLOT(XPN+XB,YPN+YB,2)                                        MSRC0515
      CALL PLOT(XPN-XB,YPN-YB,2)                                        MSRC0516
      CALL PLOT(XPN,YPN,2)                                              MSRC0517
  303 IF(KTWO-LAST)109,106,109                                          MSRC0518
  109 KTWO=KTWO+KADD                                                    MSRC0519
      GOTO 105                                                          MSRC0520
  106 IF(KADD)208,108,108                                               MSRC0521
  208 IF(FAN)107,205,205                                                MSRC0522
  107 CALL NUMBER(AX(1)/DY,AY(1)/DY,0.175,XN,0.0,-1)                    MSRC0523
      GOTO 205                                                          MSRC0524
  108 IF(FAN)205,205,207                                                MSRC0525
  207 CALL NUMBER(AX(KMAX)/DY,AY(KMAX)/DY,0.175,XN,0.0,-1)              MSRC0526
  205 RETURN                                                            MSRC0527
      END                                                               MSRC0528
```

FACULTY PRESS, INC., BROOKLYN, N.Y.

PLATE A

LEGEND: 2/270/0

PERIOD: 2 sec

LENGTH: 20 ft

BEARING: 270° T

MINING: NONE

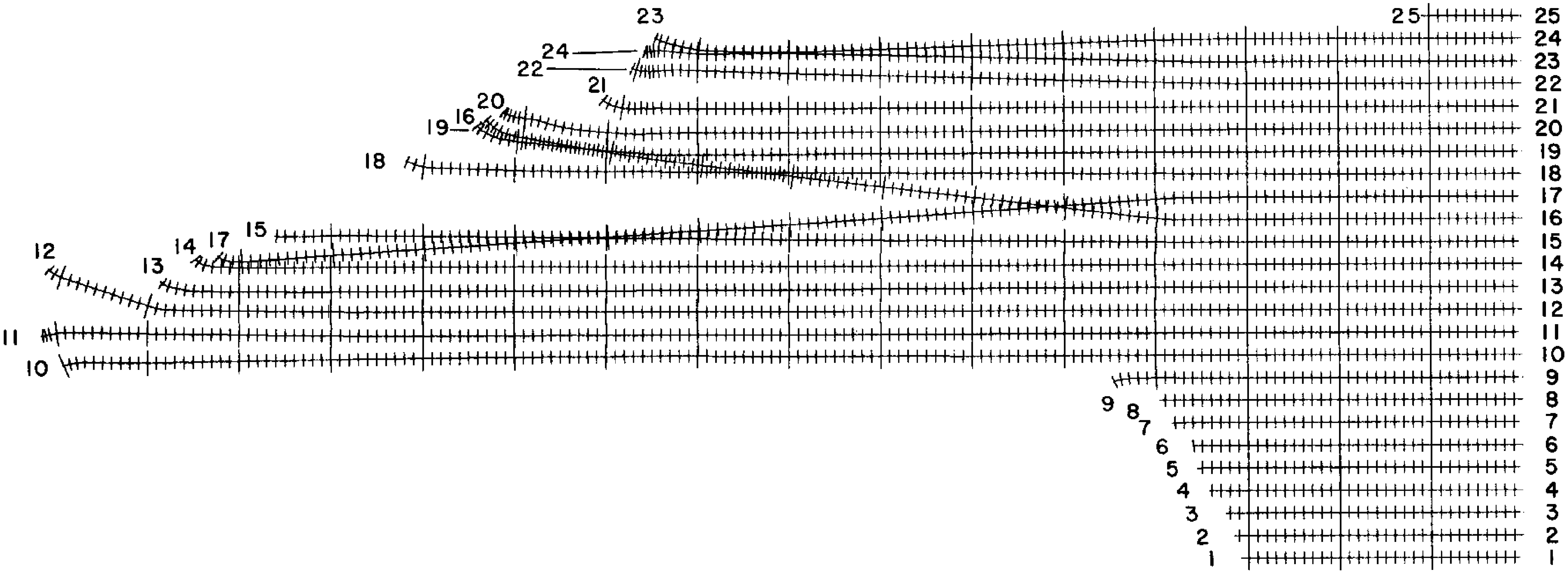

PLATE A

LEGEND: 6/270/0

PERIOD: 6 sec

LENGTH: 184 ft

BEARING: 270° T

MINING: NONE

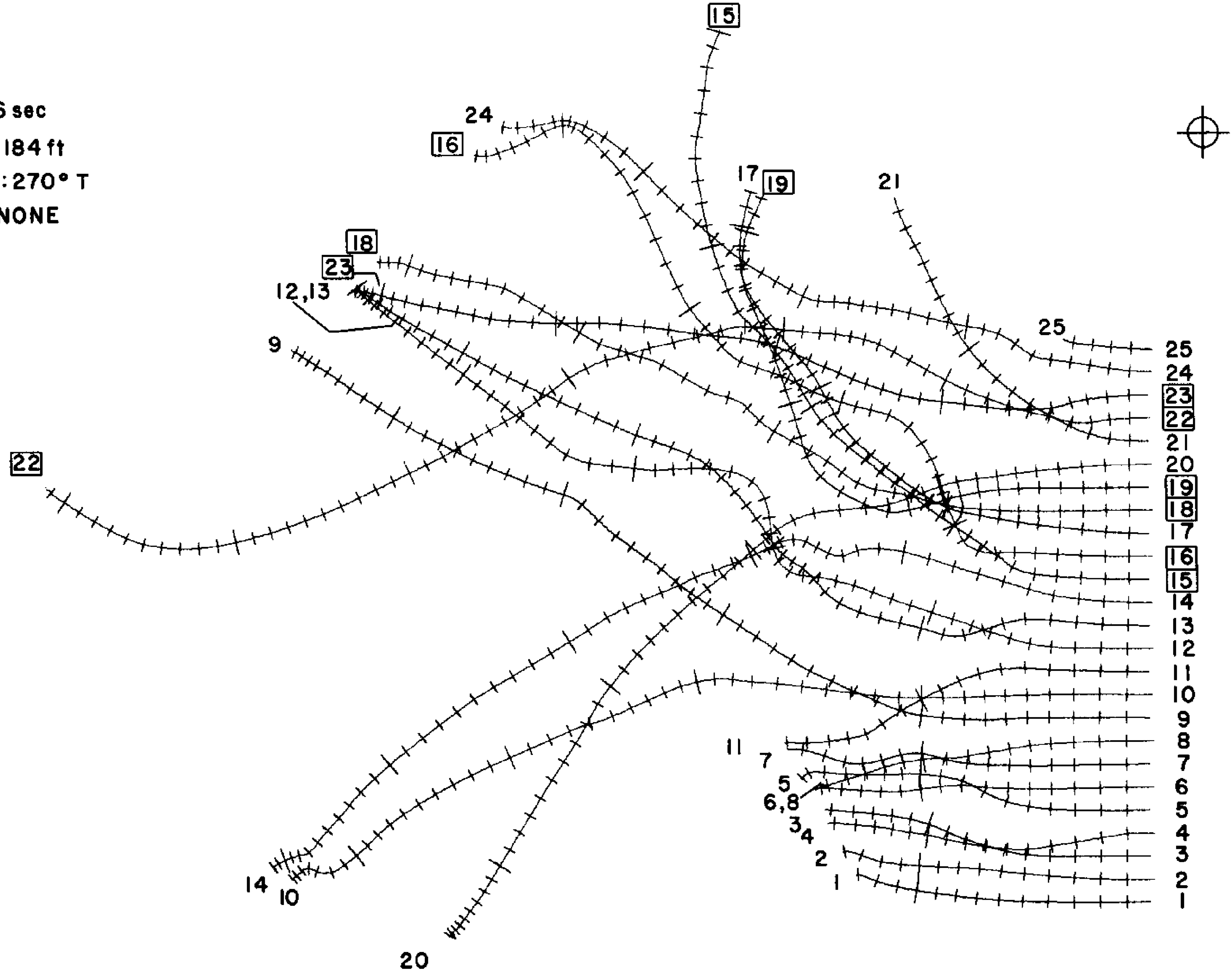

PLATE A

LEGEND: 10/270/0

PERIOD: 10 sec

LENGTH: 512 ft

BEARING: 270° T

MINING: NONE

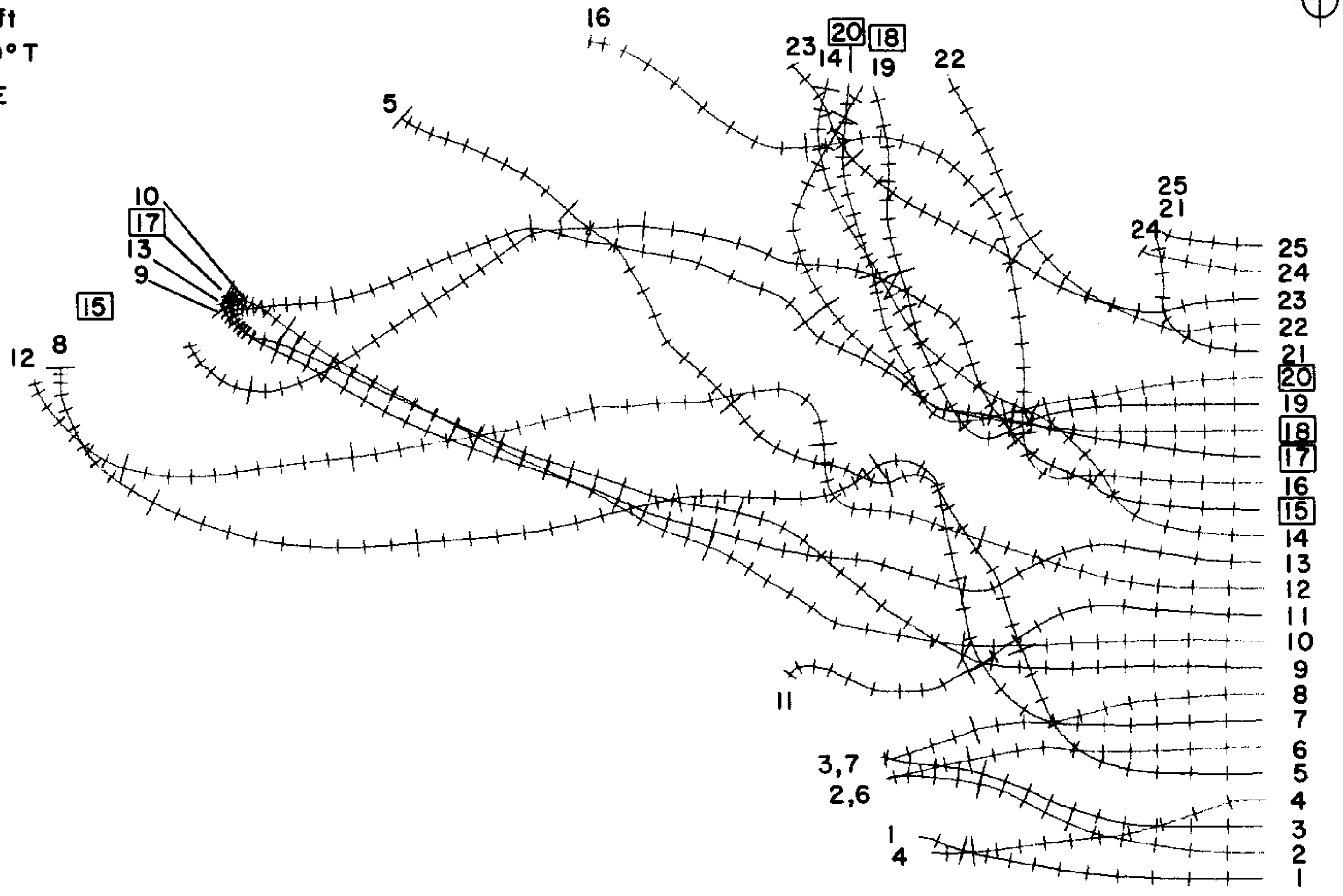

PLATE A

LEGEND: 2/297/0

PERIOD: 2 sec

LENGTH: 20 ft

BEARING: 297° T

MINING: NONE

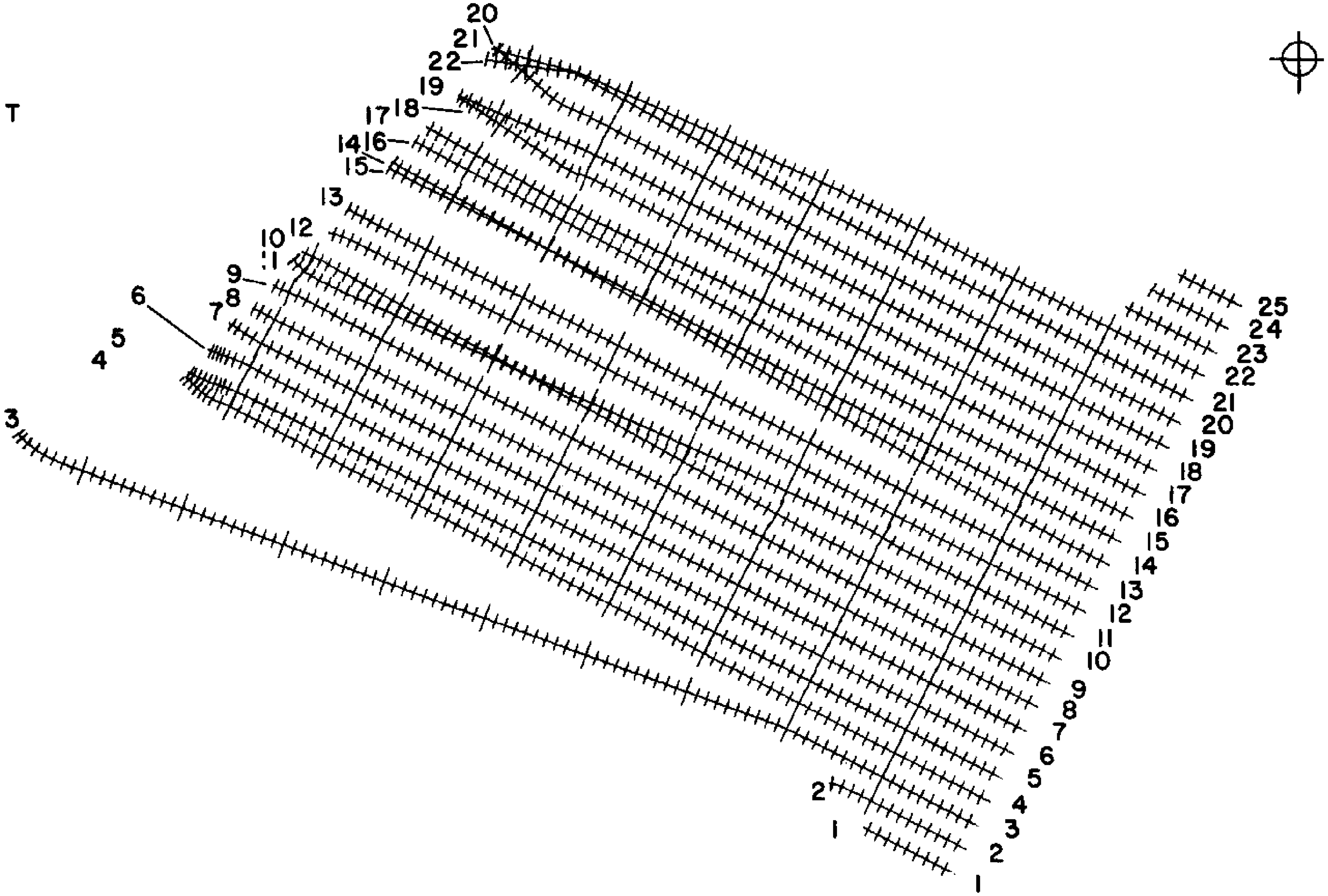

PLATE A

LEGEND: 6/297/0

PERIOD: 6 sec

LENGTH: 184 ft

BEARING: 297° T

MINING: NONE

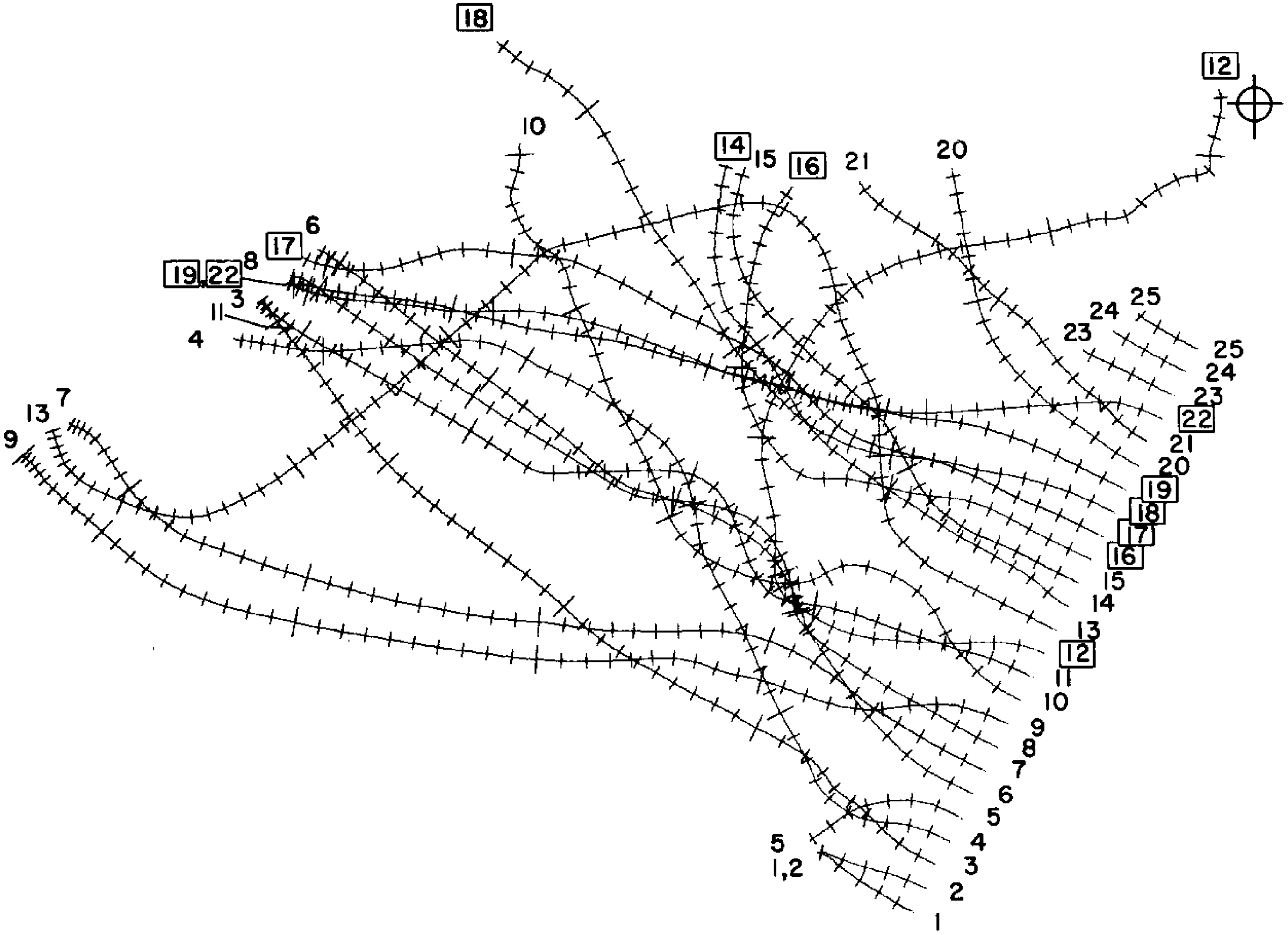

PLATE A

LEGEND: 10/297/0

PERIOD: 10sec

LENGTH: 512ft

BEARING: 297°T

MINING: NONE

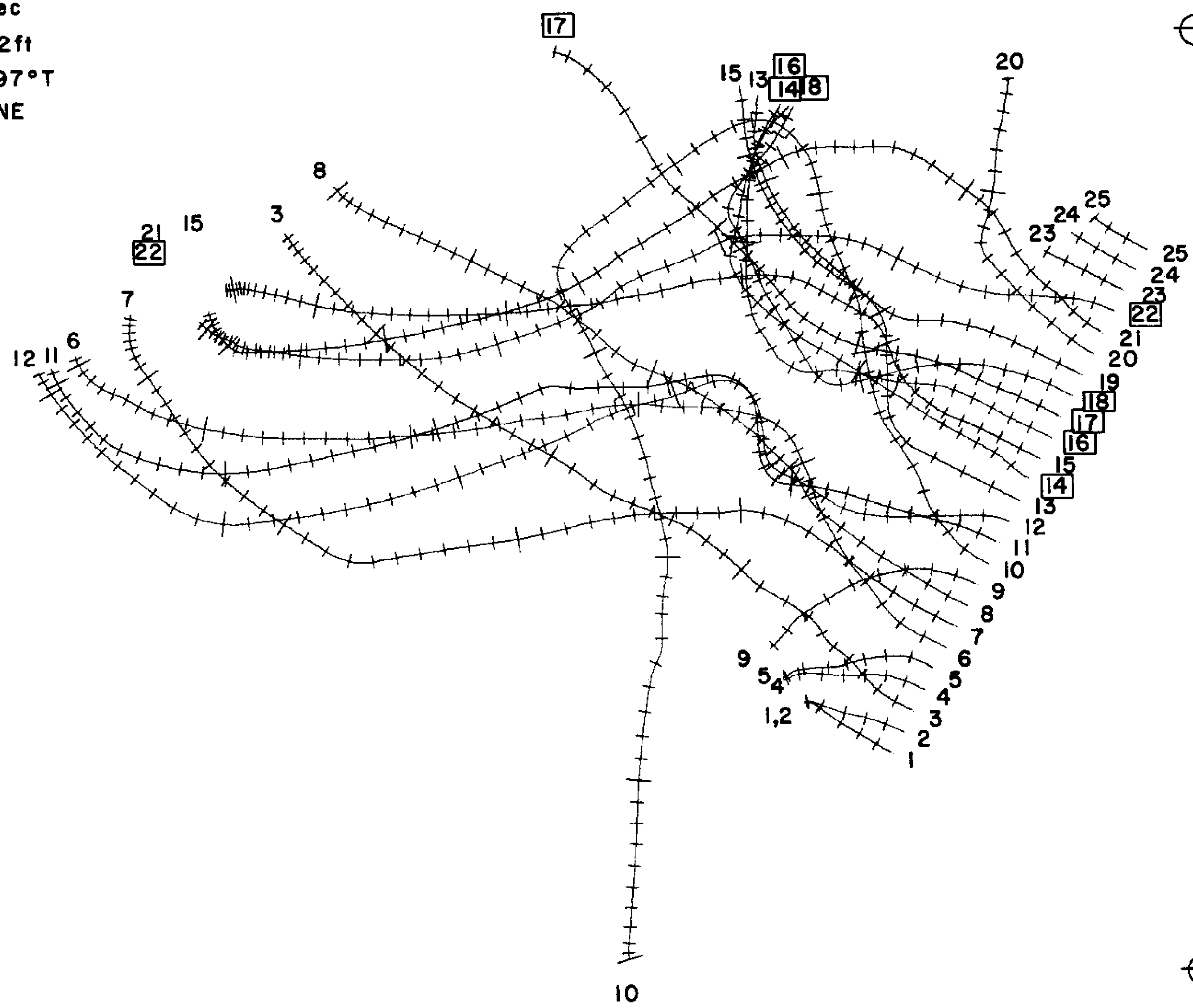

PLATE B

LEGEND: 2/270/45

PERIOD: 2 sec

LENGTH: 20 ft

BEARING: 270° T

MINING: AREAS 8C, 8AC, 10A &
14C to 45 ft

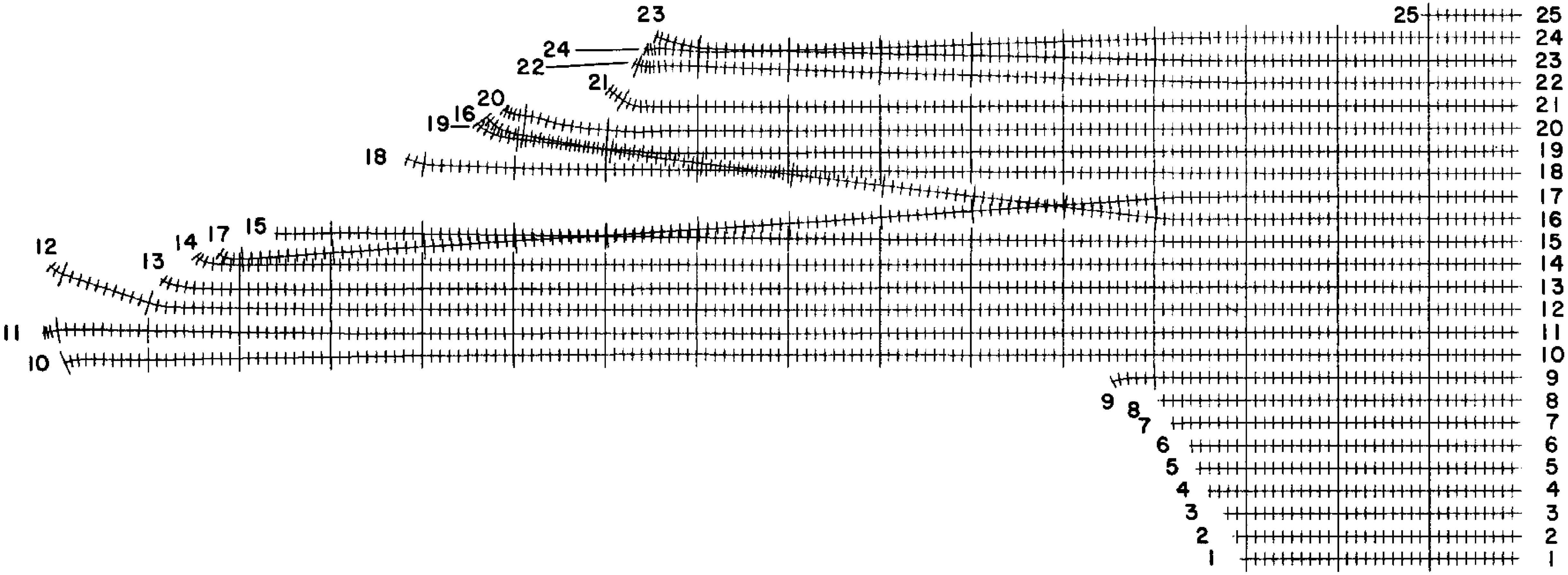

PLATE B

LEGEND: 6/270/45

PERIOD: 6 sec

LENGTH: 184 ft

BEARING: 270° T

MINING: 8C, 8AC, 10A &
14C to 45 ft

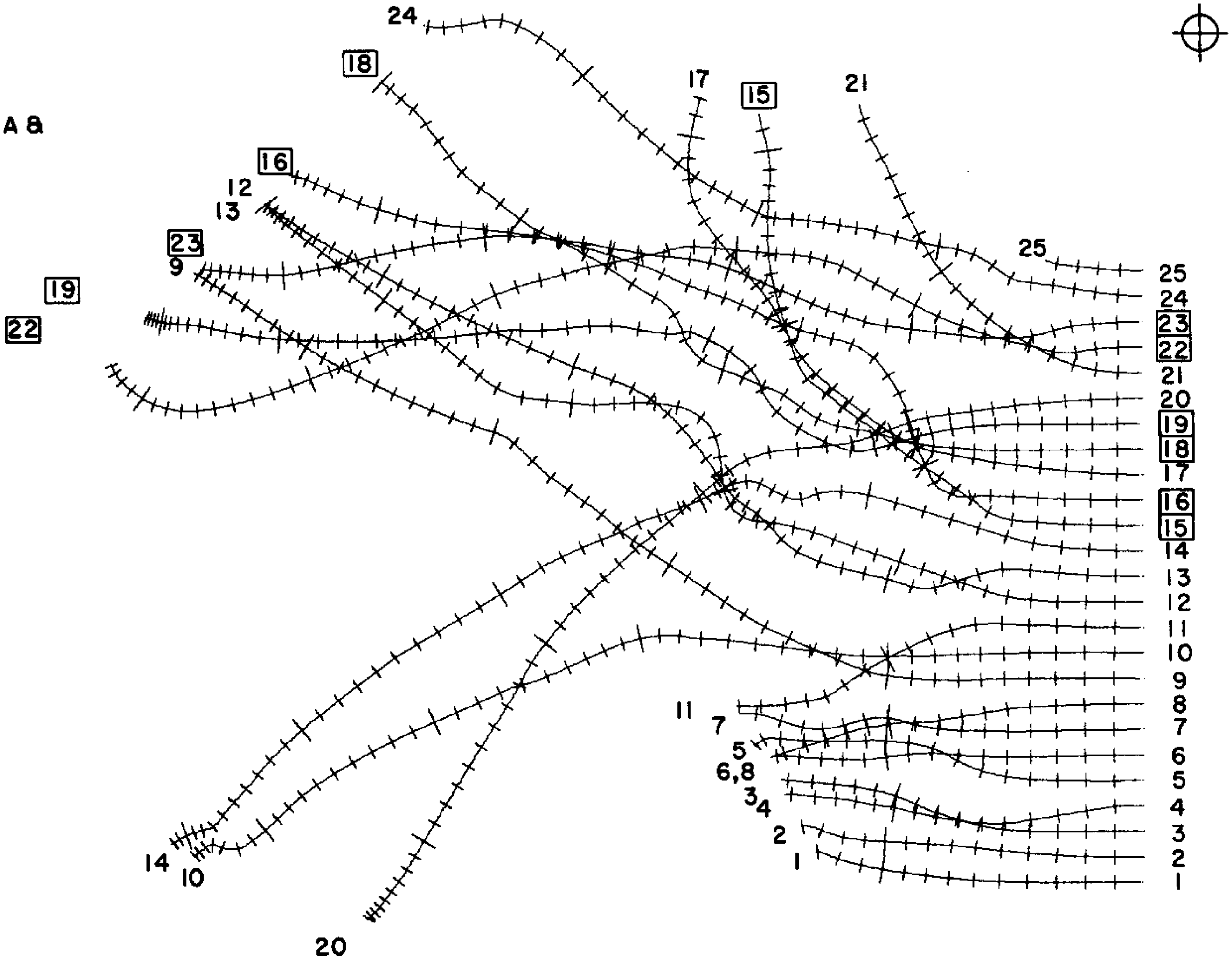

PLATE B

LEGEND: 10/270/45

PERIOD: 10 sec

LENGTH: 512 ft

BEARING: 270° T

MINING: 8C, 8AC, 10A &
14C to 45 ft

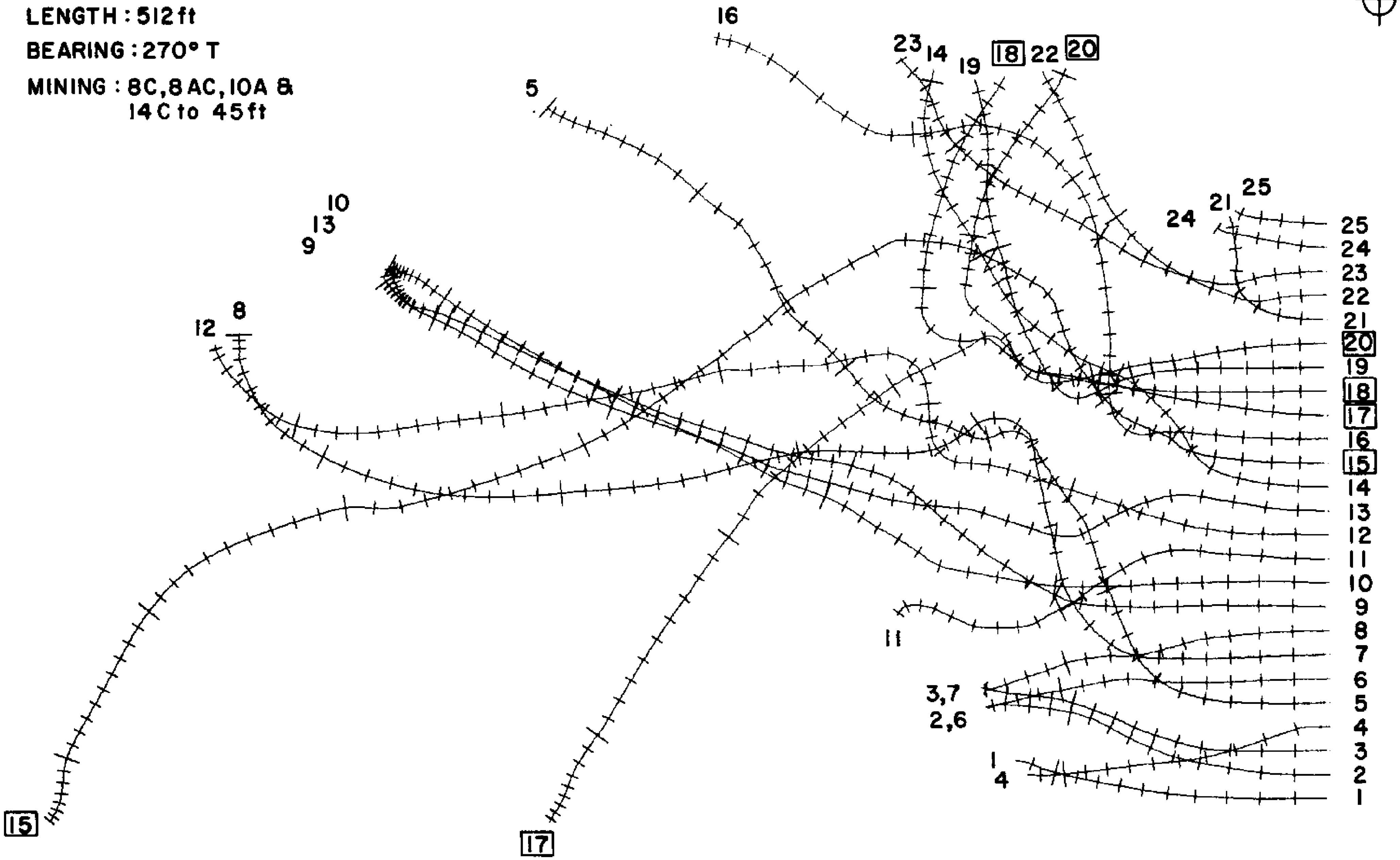

PLATE B

LEGEND: 2/297/45

PERIOD: 2 sec

LENGTH: 20 ft

BEARING: 297° T

MINING: AREAS 8C, 8AC, 10A &
14C to 45 ft

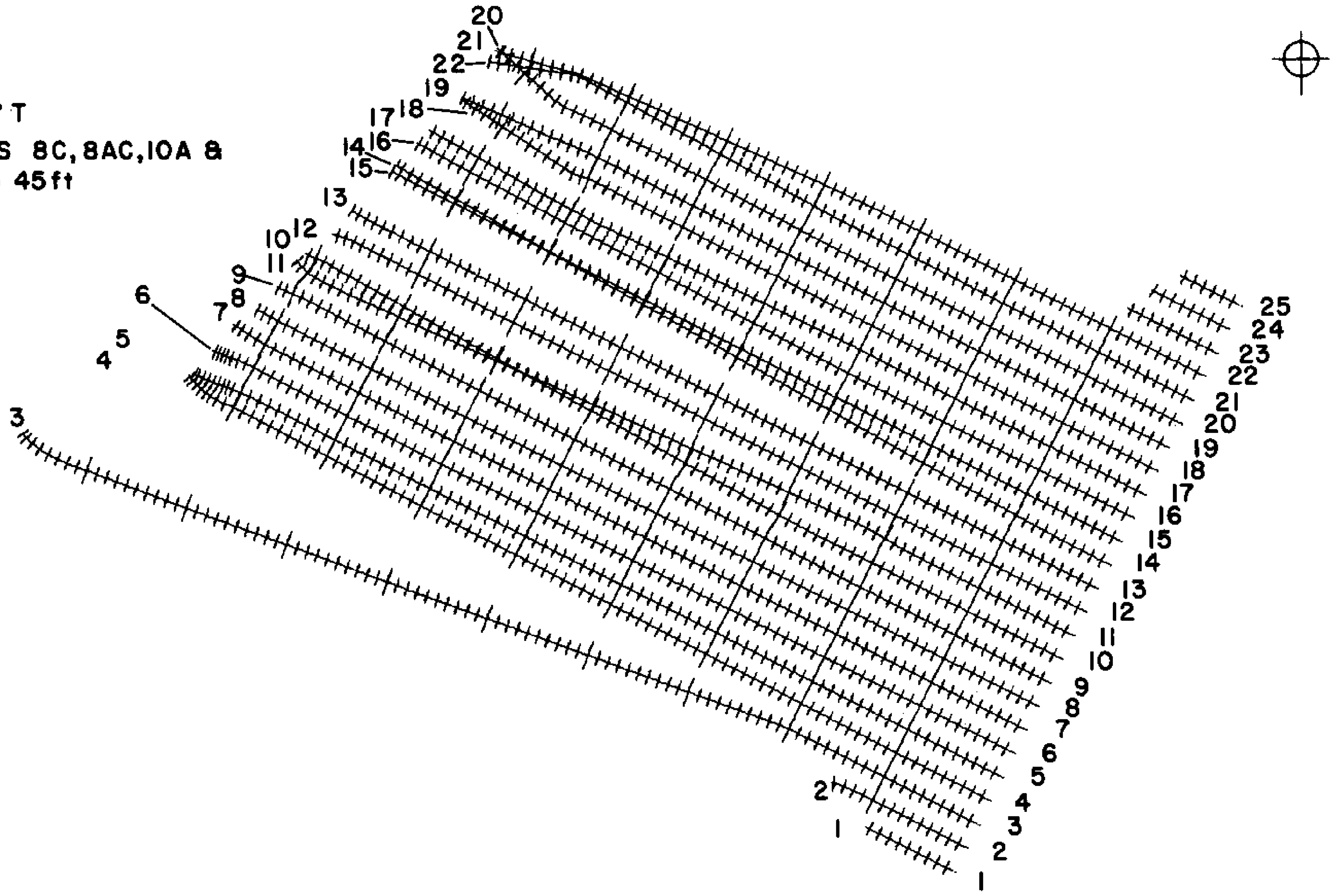

PLATE B

LEGEND: 6/297/45

PERIOD: 6 sec

LENGTH: 184 ft

BEARING: 297° T

MINING: 8C, 8AC, 10A &
14C to 45ft

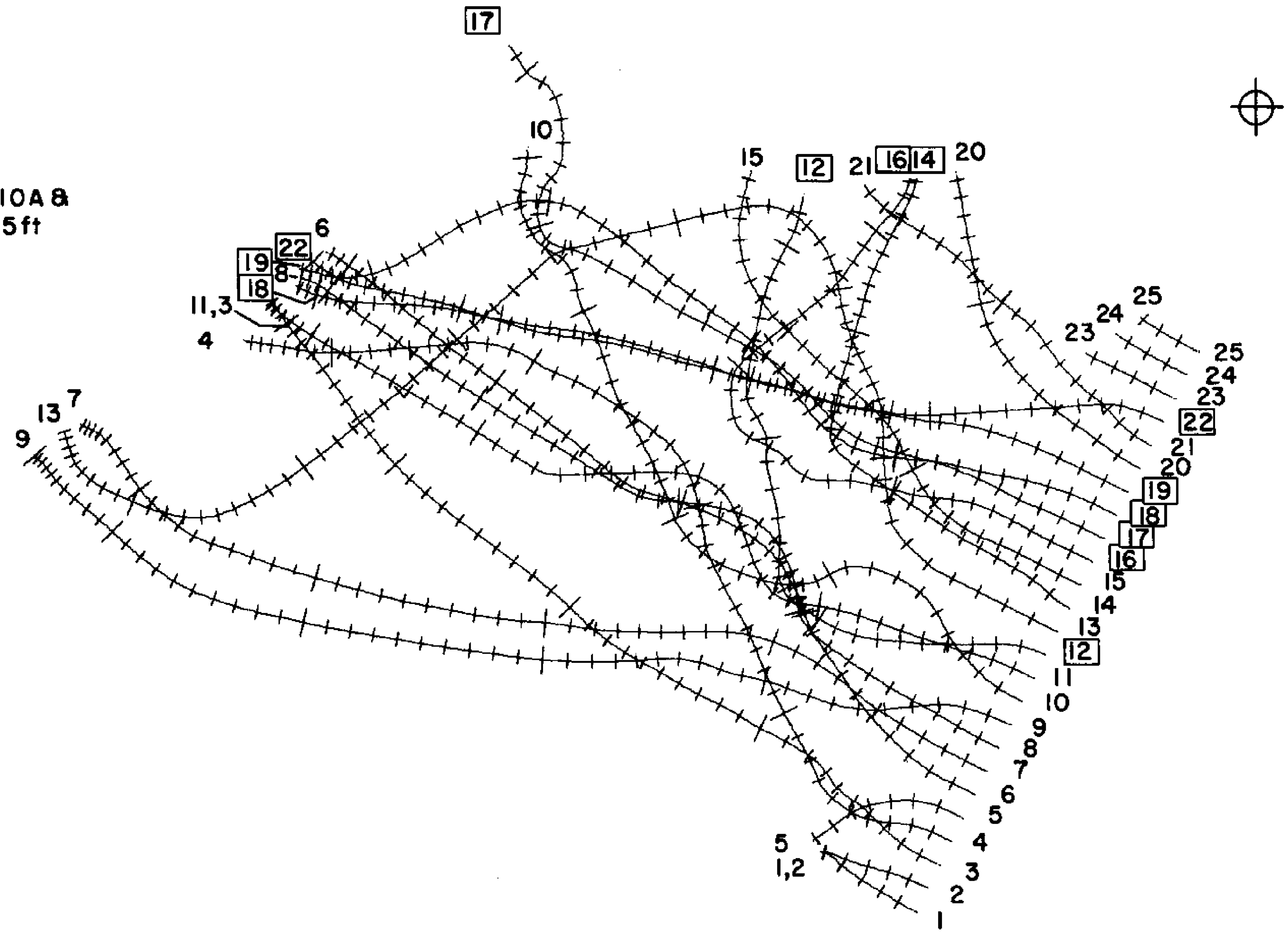

PLATE B

LEGEND: 10/297/45
PERIOD: 10 sec
LENGTH: 184 ft
BEARING: 297° T
MINING: 8C, 8AC, 10A &
14C to 45 ft

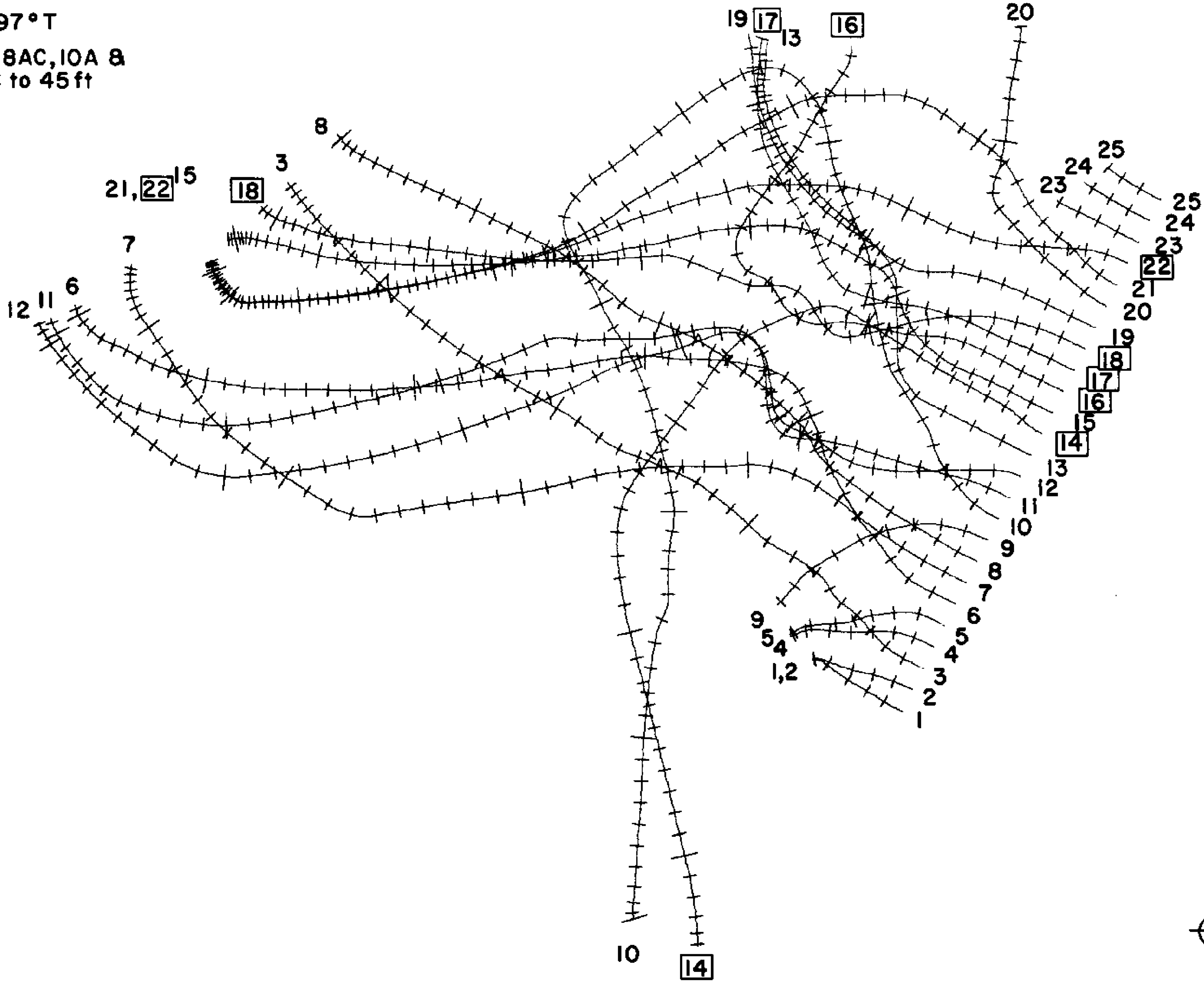

PLATE C

LEGEND: 2/270/90

PERIOD: 2 sec

LENGTH: 20 ft.

BEARING: 270° T

MINING: 8C, 8AC, 10A, 14C

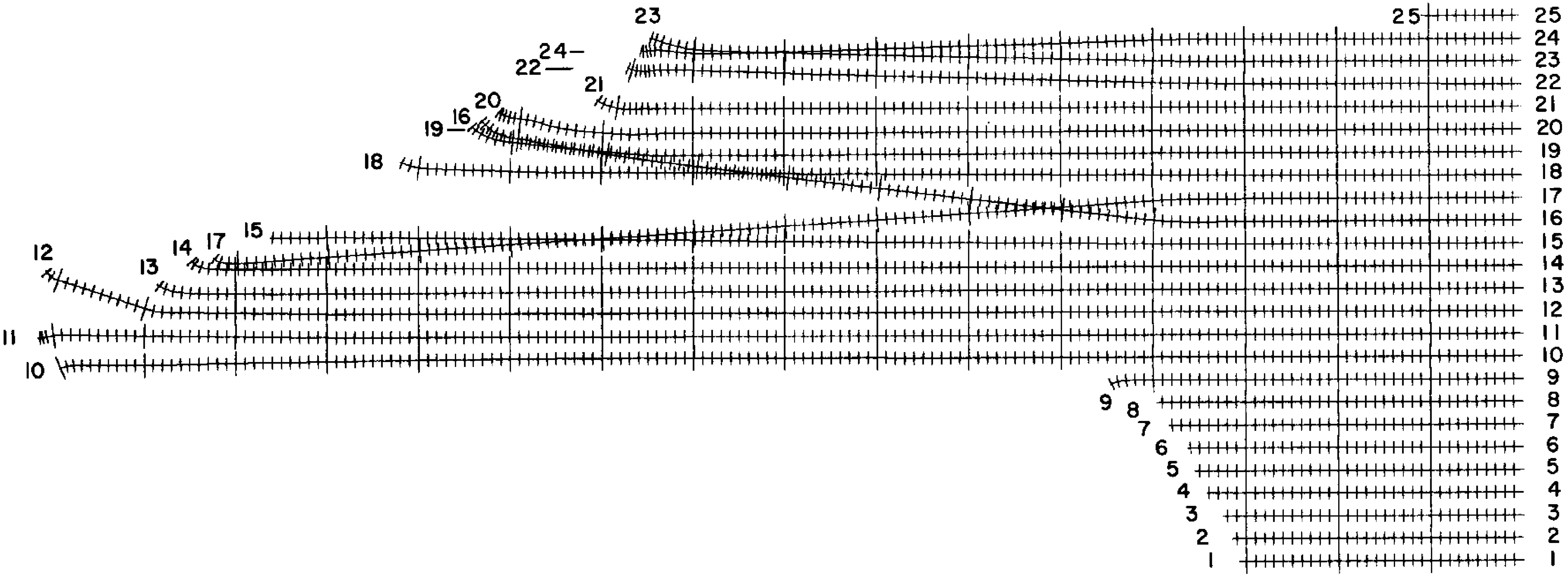

PLATE C

LEGEND: 6/270/90

PERIOD: 6 sec

LENGTH: 184 ft

BEARING: 270° T

MINING: 8C, 8AC, 10A & 14C to 90 ft

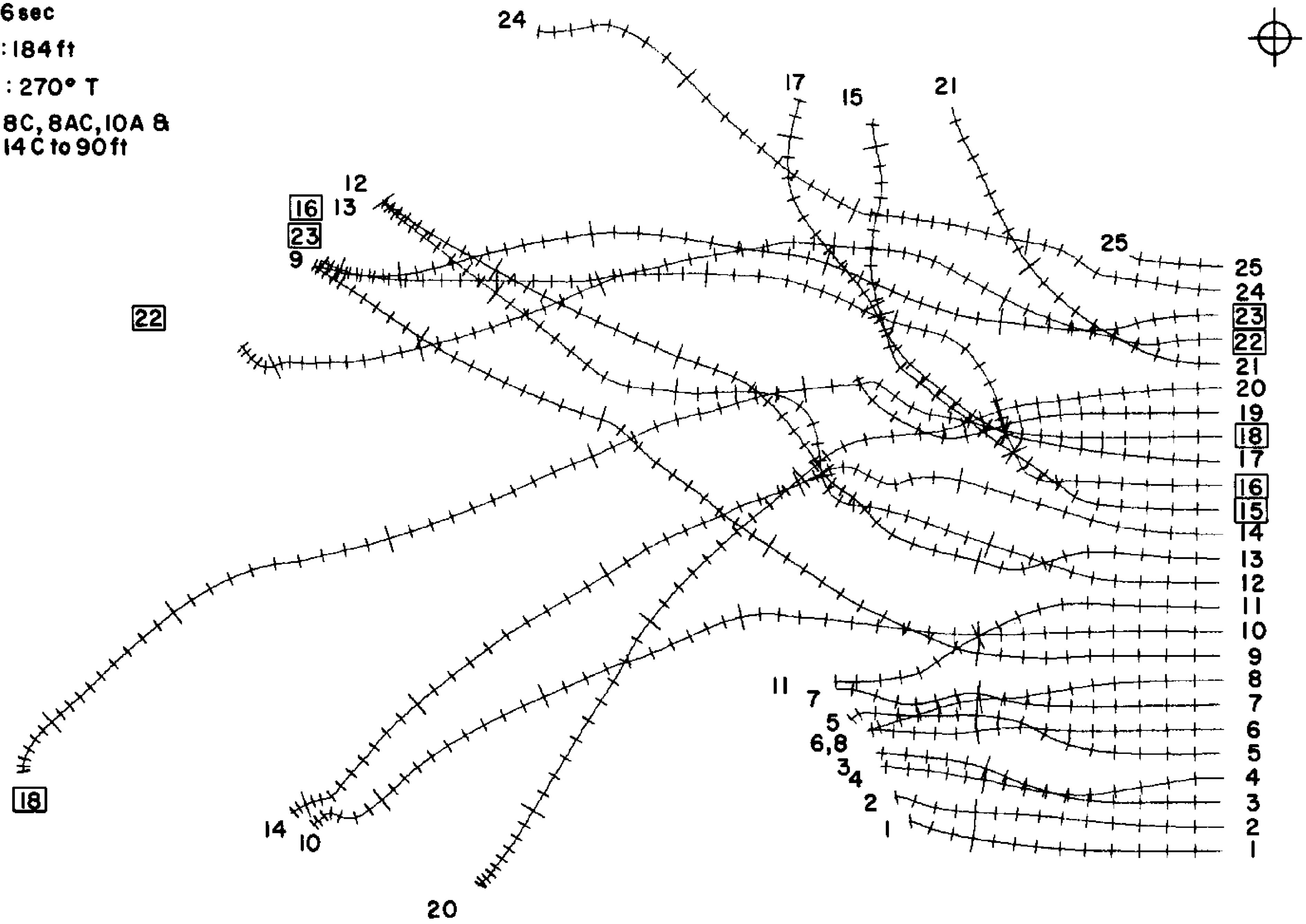

PLATE C

LEGEND: 10/270/90

PERIOD: 10 sec

LENGTH: 512 ft

BEARING: 270° T

MINING: 8C, 8AC, 10A &
14C to 90 ft

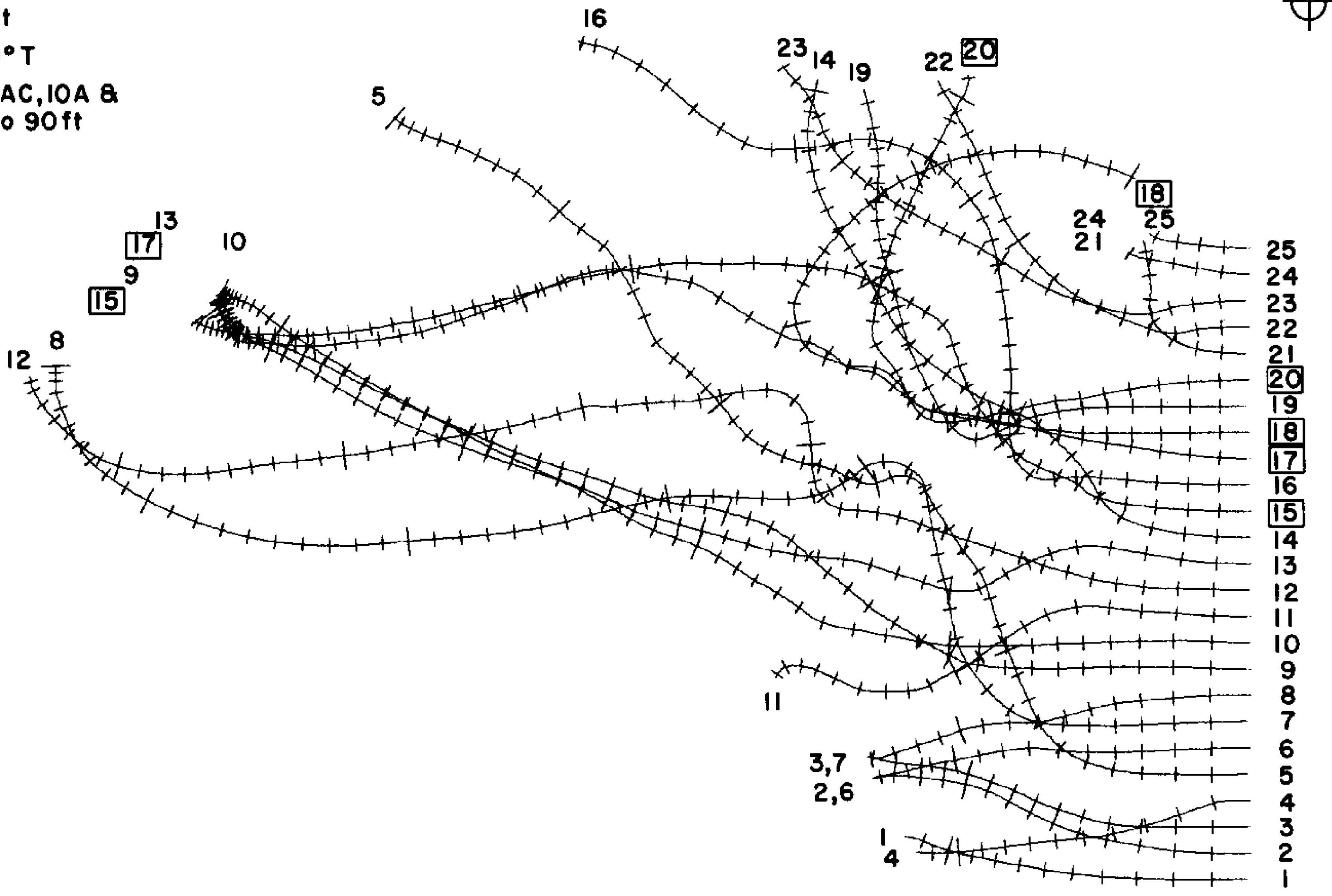

PLATE C

LEGEND: 2/297/90

PERIOD: 2 sec

LENGTH: 20ft.

BEARING: 297° T

MINING: 8C,8AC,10A,14C

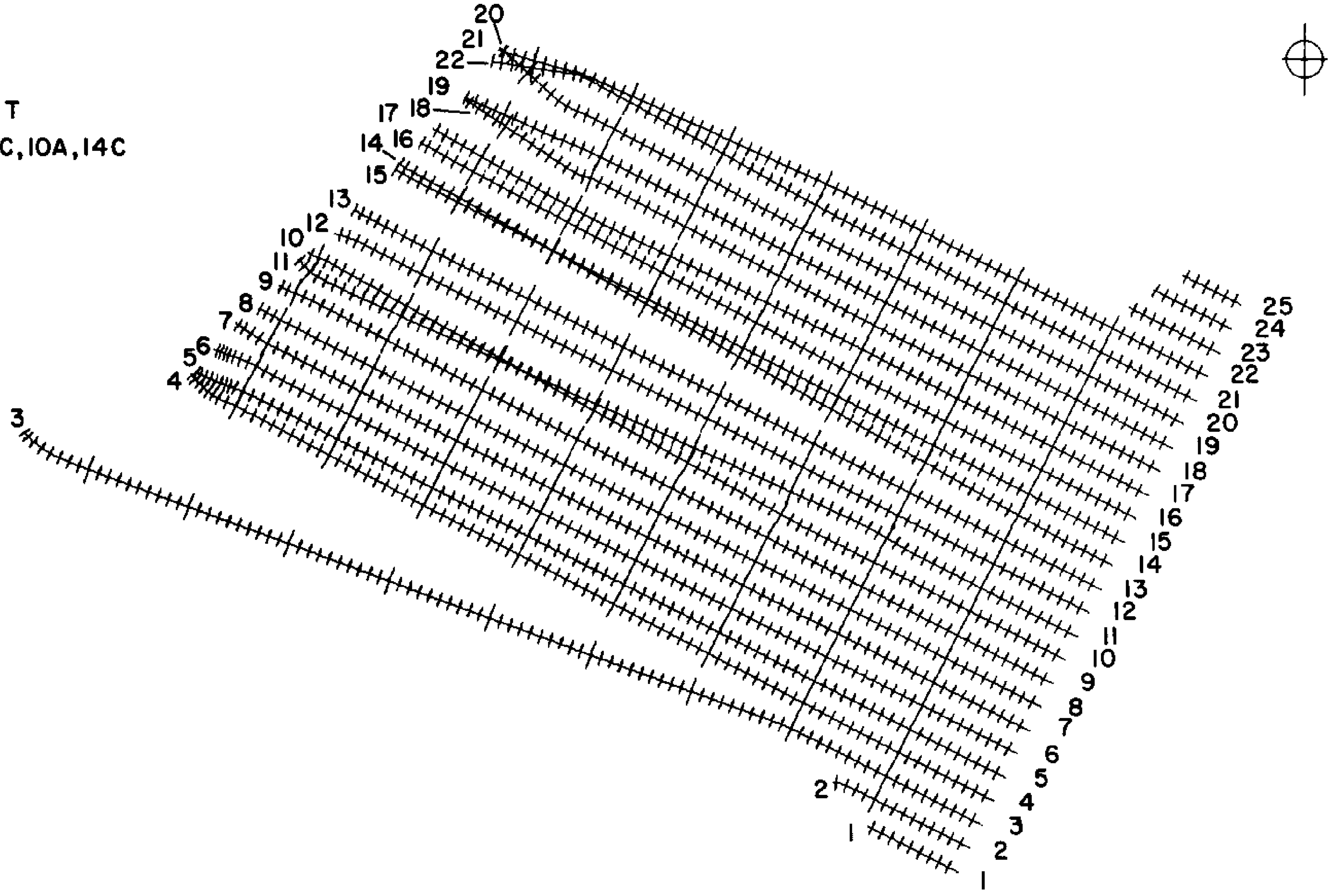

PLATE C

LEGEND: 6/297/90

PERIOD: 6 sec

LENGTH: 184 ft

BEARING: 297° T

MINING: 8C, 8AC, 10A &
14C to 90ft

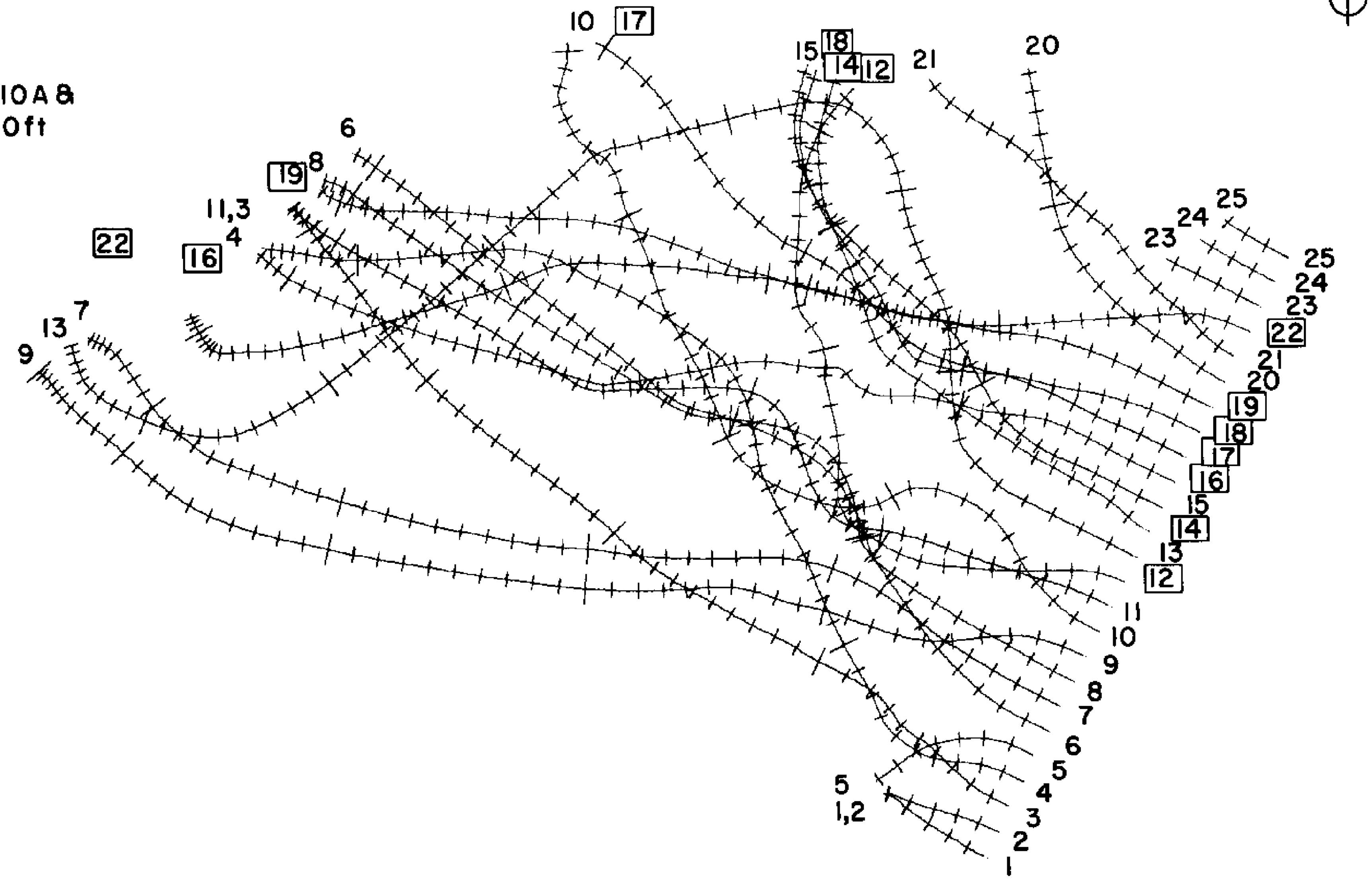

PLATE C

LEGEND: 10/297/90

PERIOD: 10sec

LENGTH: 512ft

BEARING: 297°T

MINING: 8C, 8AC, 10A &
14C to 90ft

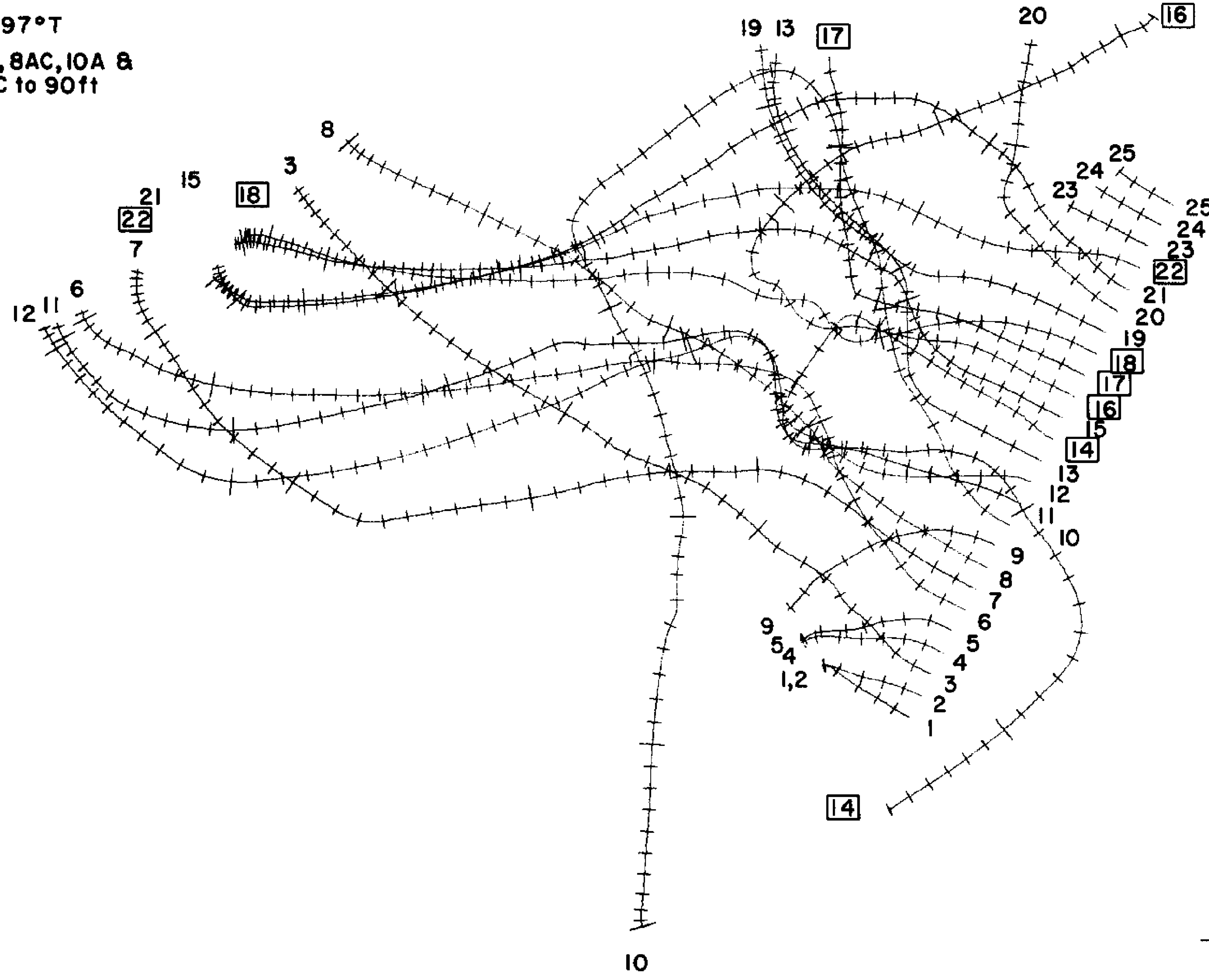

PLATE D

LEGEND: 2/270/90A

PERIOD: 2 sec

LENGTH: 20ft.

BEARING: 270° T

MINING: 90 ft.

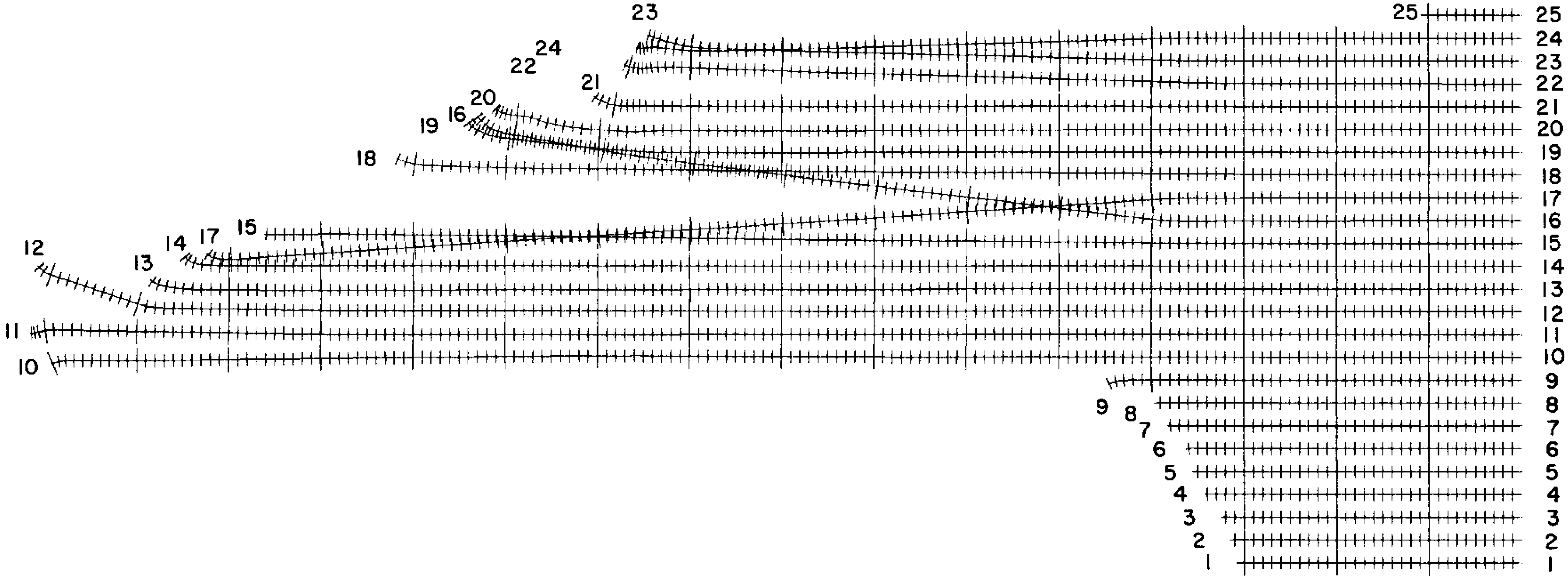

PLATE D

LEGEND: 6/270/90A

PERIOD: 6 sec

LENGTH: 184 ft

BEARING: 270° T

MINING: ENTIRE AREA to 90 ft

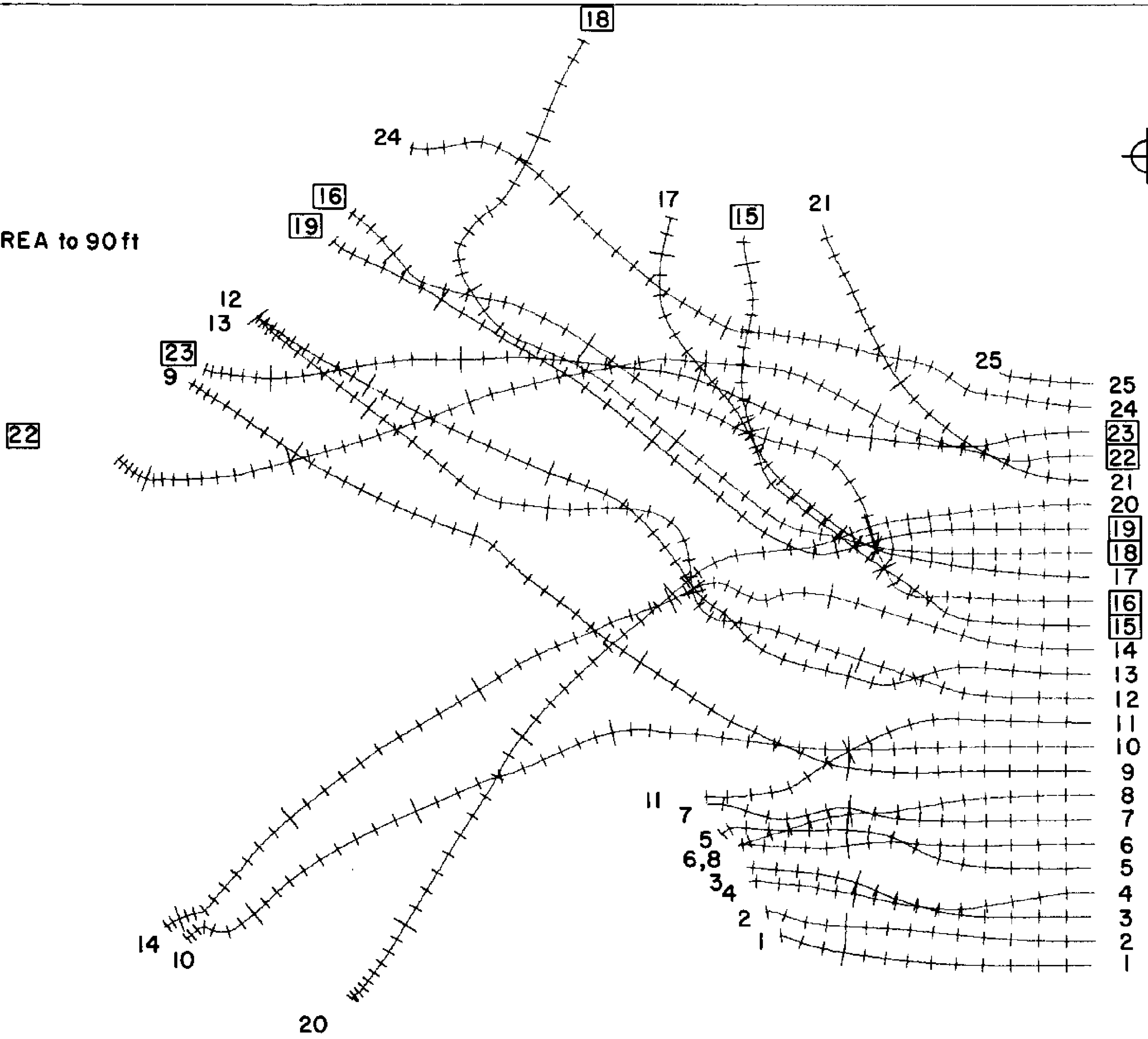

PLATE D

LEGEND : 10/270/90A

PERIOD : 10sec

LENGTH : 512 ft

BEARING : 270° T

MINING : ENTIRE AREA to 90ft

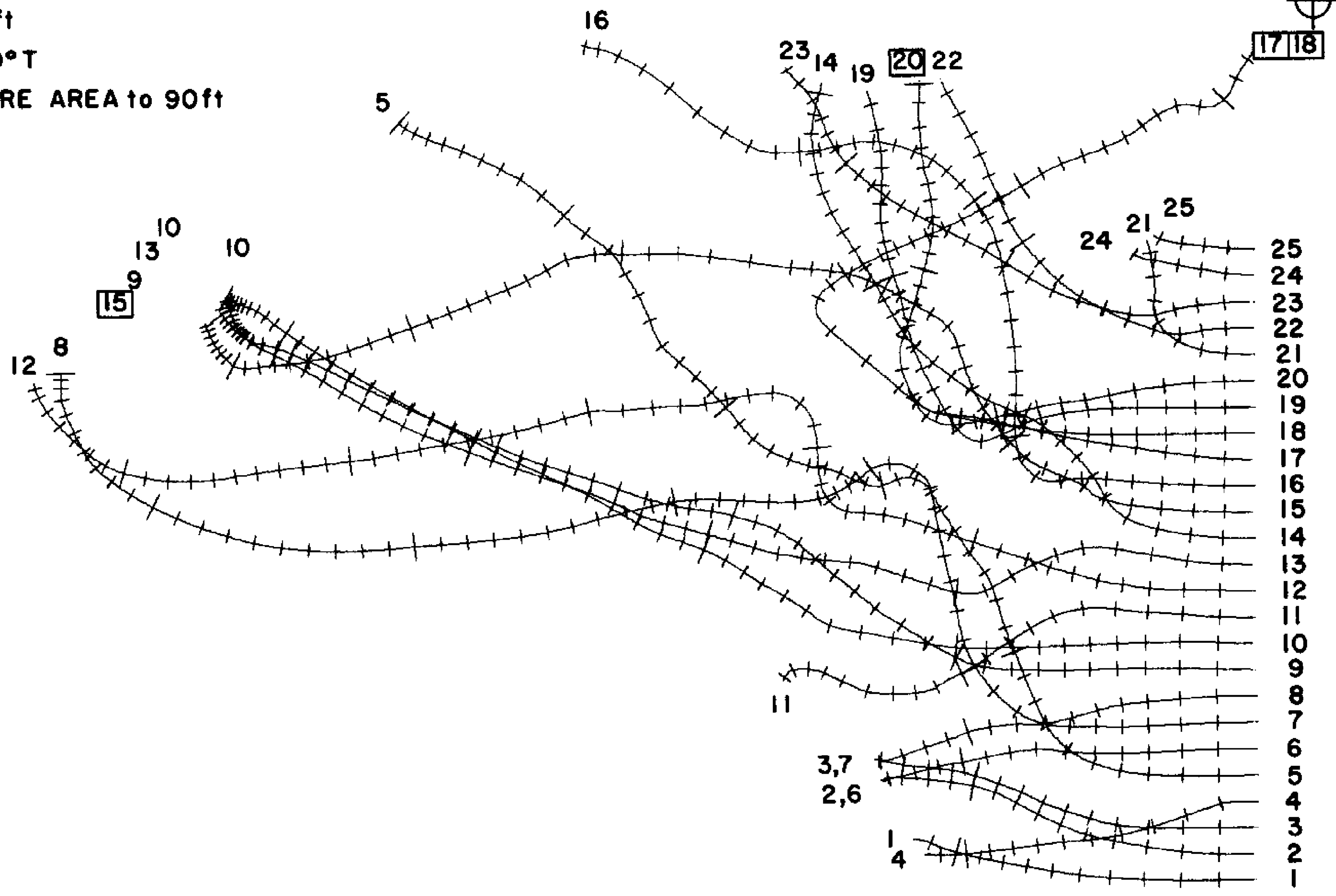

PLATE D

LEGEND: 2/297/90A

PERIOD: 2 sec

LENGTH: 20 ft.

BEARING: 297° T

MINING: 90 ft.

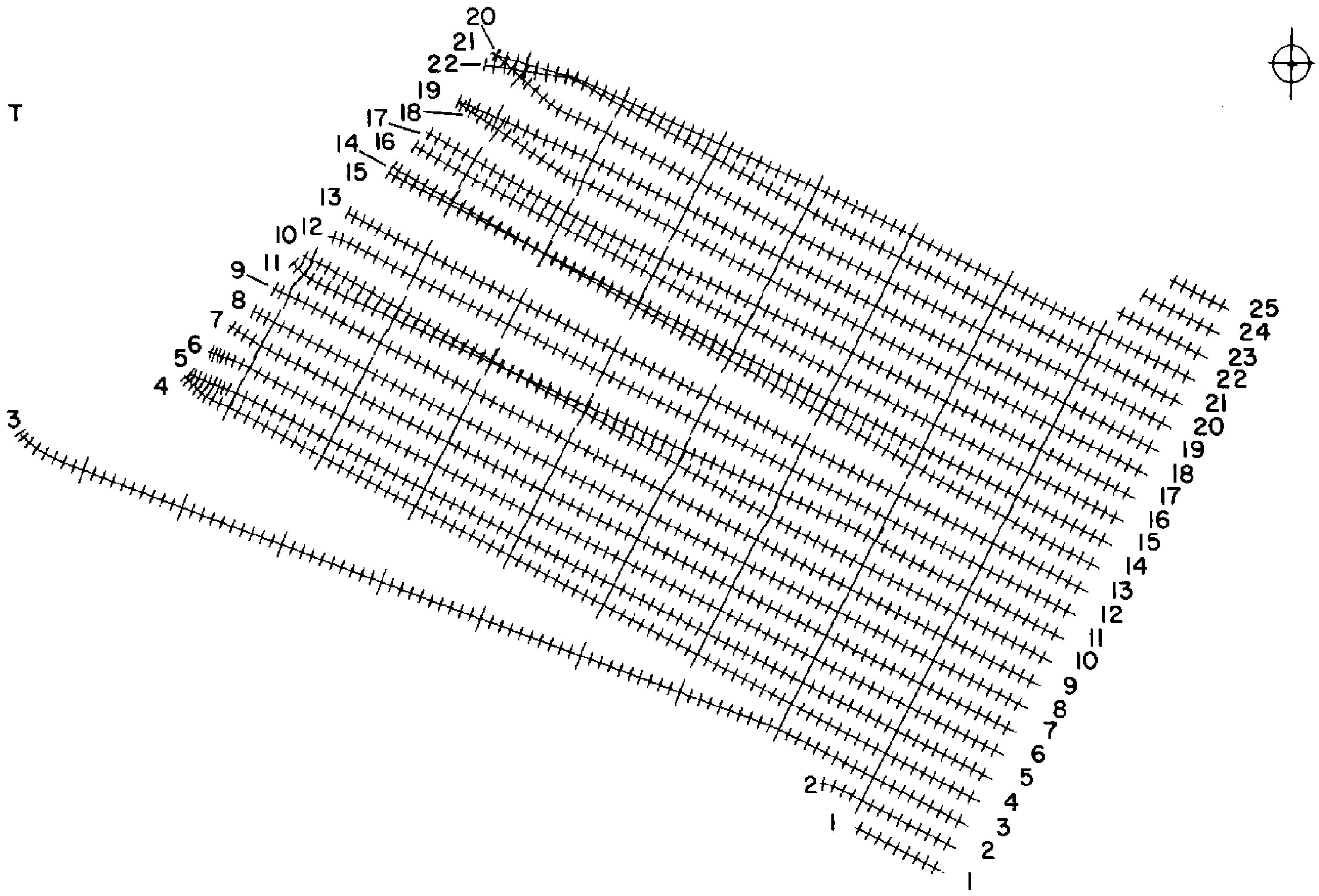

PLATE D

LEGEND: 6/297/90A

PERIOD: 6 sec

LENGTH: 184 ft

BEARING: 297° T

MINING: ENTIRE AREA to 90 ft

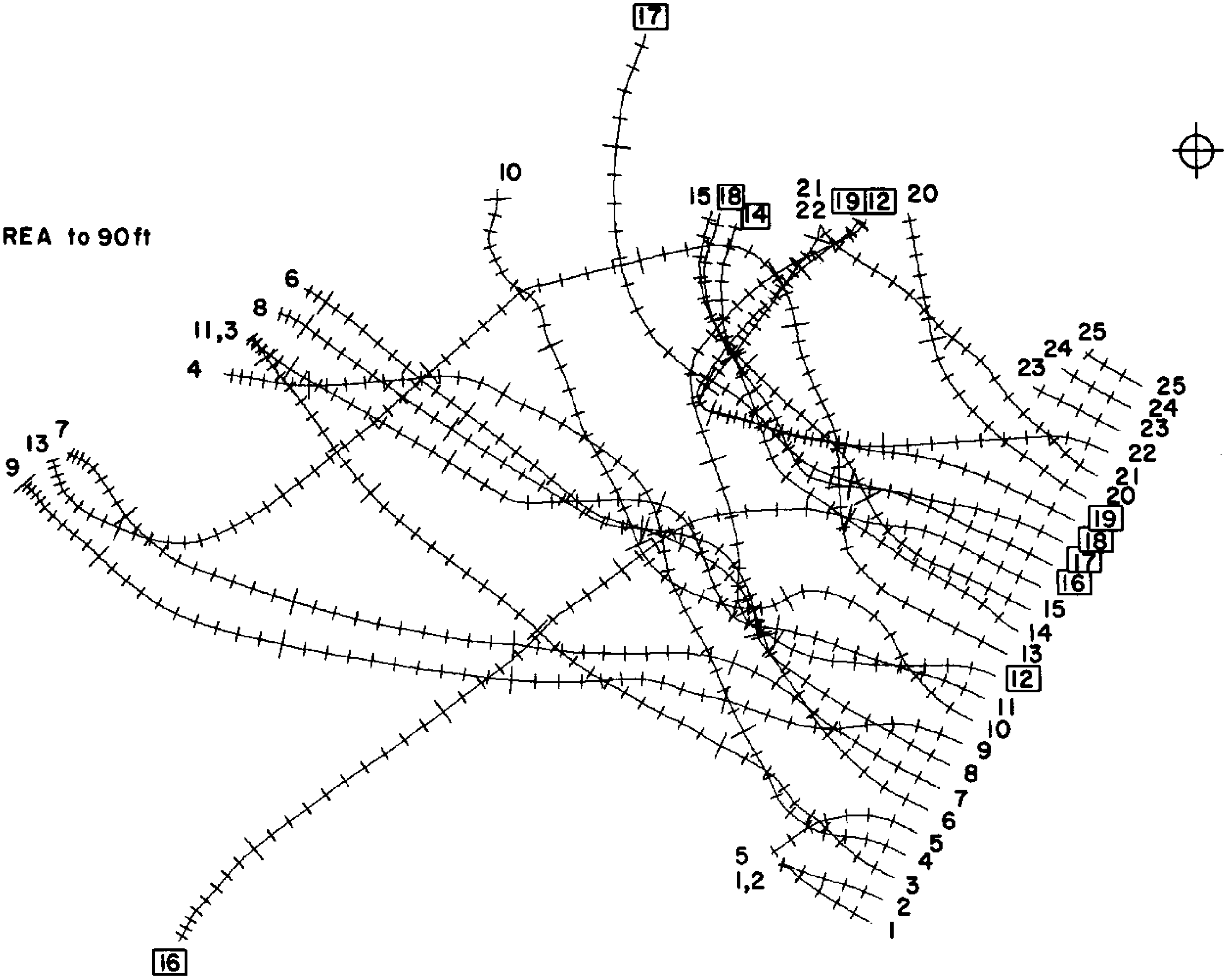

PLATE D

LEGEND: 10/297/90A
PERIOD: 10sec
LENGTH: 512 ft
BEARING: 297° T
MINING: ENTIRE AREA to 90ft