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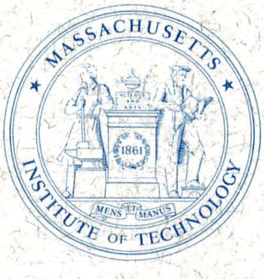
A REVIEW OF EFFECTIVE PLATING TO BE USED IN THE ANALYSIS OF STIFFENED PLATING IN BENDING AND COMPRESSION

BY

D. FAULKNER, RCNC



Massachusetts Institute of Technology

Cambridge, Massachusetts 02139

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This research was carried out during tenure of a British Government Defense Fellowship in the Department of Ocean Engineering, Massachusetts Institute of Technology

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MASSACHUSETTS INSTITUTE OF TECHNOLOGY
CAMBRIDGE, MASS. 02139

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Administrative Statement

Douglas Faulkner presents here a thorough survey of the theoretical and experimental work done on the extent of plating effective in cooperating with structural stiffening members loaded either in bending or compression. Imperfections and residual welding stresses are considered. Naval architecture, aeronautical and civil engineering sources are covered and both the elastic and plastic regimes. Post-buckling behavior of the plating is also taken into account. During design, reliable estimation of the effective breadth of plating can be crucial in predicting the anticipated performance of stiffened plate structures.

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Alfred H. Keil
Director

June, 1973

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ABSTRACT

The review is divided into three main parts. The first is very brief and covers shear lag effects associated with stiffener plate bending. Part II concerns the behavior of unstiffened plate elements in compression, which are referred to as effective "width" effects. The concepts considered are maximum plate strength, and how this is affected by initial distortion, normal pressure and boundary conditions; stress distribution in plate elements before failure; and the "reduced effective width" concept for defining plate element stiffness, as required for use in stiffened-plate collapse theories. Final appraisal and recommendations are made. Part III concerns welding stress effects, and a critical strain theory is advanced for describing welded plate behavior.

The review has of course assessed appropriate test data, including that from three full-scale destroyer tests. Some emphasis has been placed upon the statistical characteristics of the data, in order that this may help establish structural strength distributions for probabilistic approaches to ductile structural reliability.

NOTATION

a, t	Plate element length, thickness
b	Plate element width over which uniform compression is applied
I, Z	Moment of inertia and section modulus of stiffener-plate combination
σ_o, E, ν	Material yield stress, Young's modulus, Poisson's ratio
σ_r	Average longitudinal compression residual welding stress in middle region of plating
E_t	Tangent modulus of plate element in compression (with residual stresses)
m	Mode number of half-wave distortions along the length of the plate
σ_{PE}, σ_P	Elastic and inelastic plate buckling stress
R_r	Plate strength reduction factor due to welding residual stresses
σ_e, σ_a	Edge and average plate stresses
σ_m	Maximum average plate stress
b_e, b_{em}	Effective width of plate, and minimum value
b_e', b_{em}'	Reduced effective width of plate, and minimum value
β	$\frac{b}{t} \sqrt{\frac{\sigma_o}{E}}$ plate width (slenderness) factor

A REVIEW OF EFFECTIVE PLATING TO BE USED IN THE ANALYSIS OF STIFFENED PLATING IN BENDING AND COMPRESSION

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INTRODUCTION

In view of the increasing availability of computer methods of structural analysis, and in view of the dependence of many of these methods on effective breadth and effective width assumptions, it seems that some priority should be given to establishing reliable formulae or data curves which could be incorporated in computer programs. Finite element techniques would probably go a long way towards providing theoretical solutions, but as effort is not readily available for this, a review has been made of the existing state of the art.

Loss of effectiveness of plating in ship grillages can arise from:

(i) Shear lag associated with stiffener bending;

(ii) Reduced stiffness of the plating under axial compression arising from local buckling, in some cases precipitated by residual stress action, and from lack of flatness of the plating caused by initial distortion and/or lateral load. Although the word "stiffness" has been introduced above, it is more usual to associate effectiveness with the load-carrying characteristics of the plate. These two concepts are not, of course, identical and this will be considered.

(iii) More general shear lag arising from ship bending or shear diffusion in way of large discontinuities. This is not considered any further in this note; readers are referred to the second paper by Schade [1].*

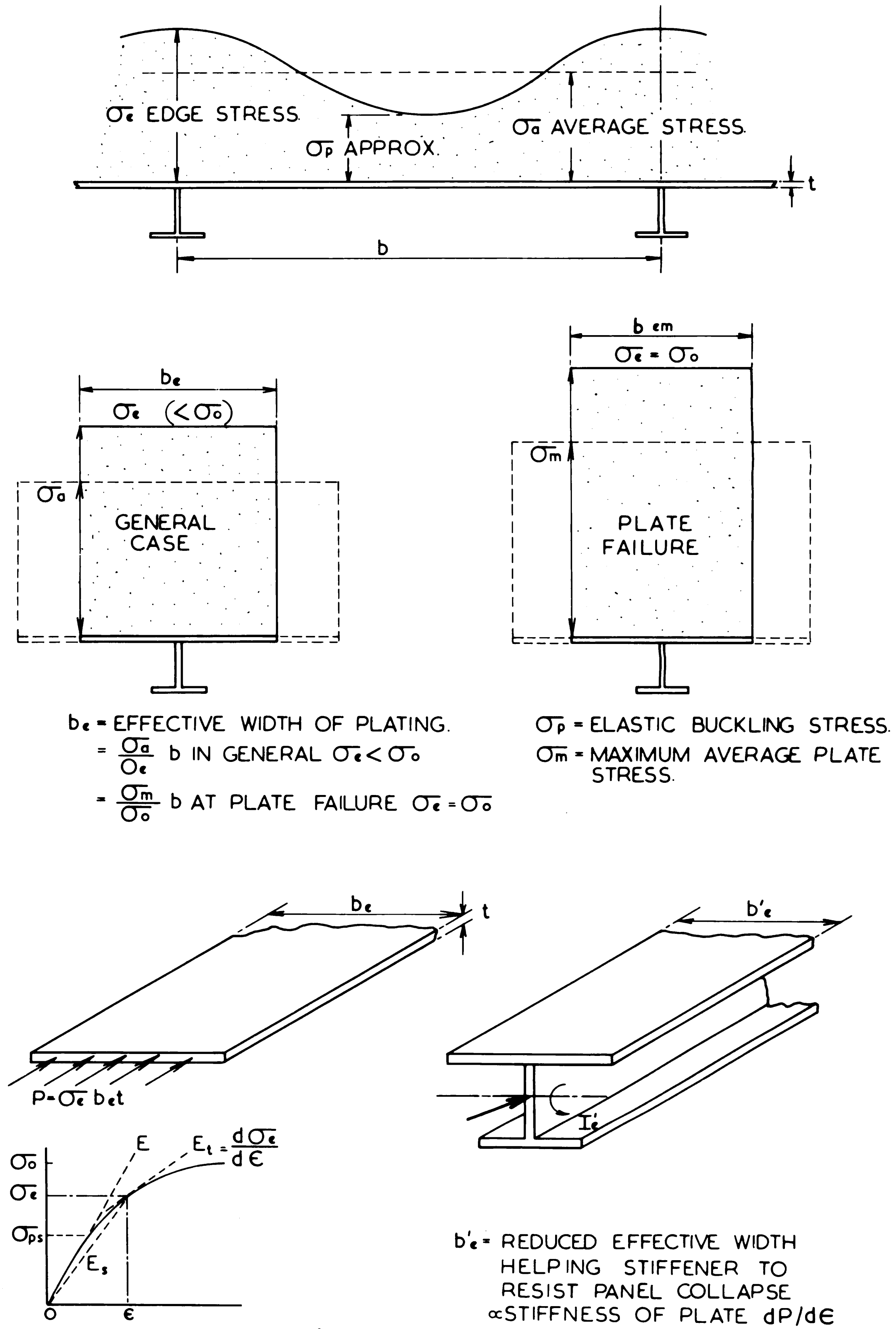
The first two are considered when assessing section properties of stiffeners and effective plating. Professor Schade [1] culled the phrases effective "breadth" to cater for bending shear lag, and "width" for compression effects, but in many practical structures the two effects occur together and they are not separable or additive. For example, there will often by in-plane compression in the plating arising from purely bending loads in the grillage. As that very shrewd American, Admiral Cochrane, said in the discussion to reference 1, "One isn't always clear that the plate will be well informed as to which of these (two) situations it is working in!"

Referring to Figure 1, it will be seen that both phenomena cause a reduction in plate stresses between stiffeners and the effective span of plating required for computing section properties is given by:

$$b_e = \frac{\sigma_a}{\sigma_e} b$$

where σ_e is the edge stress in the plating and σ_a the average stress. For convenience, the designer assumes b_e is constant throughout the lengths of the stiffeners, although theoretically and practically this is not strictly so. Where the effective plating has a greater cross section than have the stiffeners,

*References listed at back.



σe EDGE STRESS
σp APPROX.
σa AVERAGE STRESS
t
b
b em
σe = σo
be
σe (<σo)
σm
σa
GENERAL CASE
PLATE FAILURE
be = EFFECTIVE WIDTH OF PLATING.
= σa/σe b IN GENERAL σe < σo
= σm/σo b AT PLATE FAILURE σe = σo
σp = ELASTIC BUCKLING STRESS.
σm = MAXIMUM AVERAGE PLATE STRESS.
be
t
b'e
P = σe be t
I'e
σo
σe
σps
E
Et = dσe/dε
Es
O
ε
b'e = REDUCED EFFECTIVE WIDTH HELPING STIFFENER TO RESIST PANEL COLLAPSE ∝STIFFNESS OF PLATE dP/dε

Figure 1

it is well known that for single skin structures the curves of I and Z (particularly Z) "flatten out" with increasing b_e; and so errors in selecting b_e will have fairly minor effects on buckling loads and deflections which depend upon I, and much smaller effects on bending stresses which depend upon Z. For sandwich structures, however, such as double bottom, this is not so; section properties vary approximately linearly with b_e and so a wise choice is of the utmost importance.

For single skins, effective width effects are far more important than effective breadth, and so receive greater attention in this review. To maintain balance, most of the historical review of effective width is confined to the appendix.

PART I

EFFECTIVE BREADTH

A simple description of the shear lag phenomenon is given in reference 2. Professor Schade [1] more than anyone has applied plane-stress theory to a variety of ship structures. These analyses show that effective breadth depends upon:

(a) The span of the stiffeners and increases with the span;

(b) The nature of the load, but not its magnitude, and is lowest where high shear exists, e.g., at boundaries and in way of concentrated loads;

(c) The boundary conditions, particularly at the plate sides.

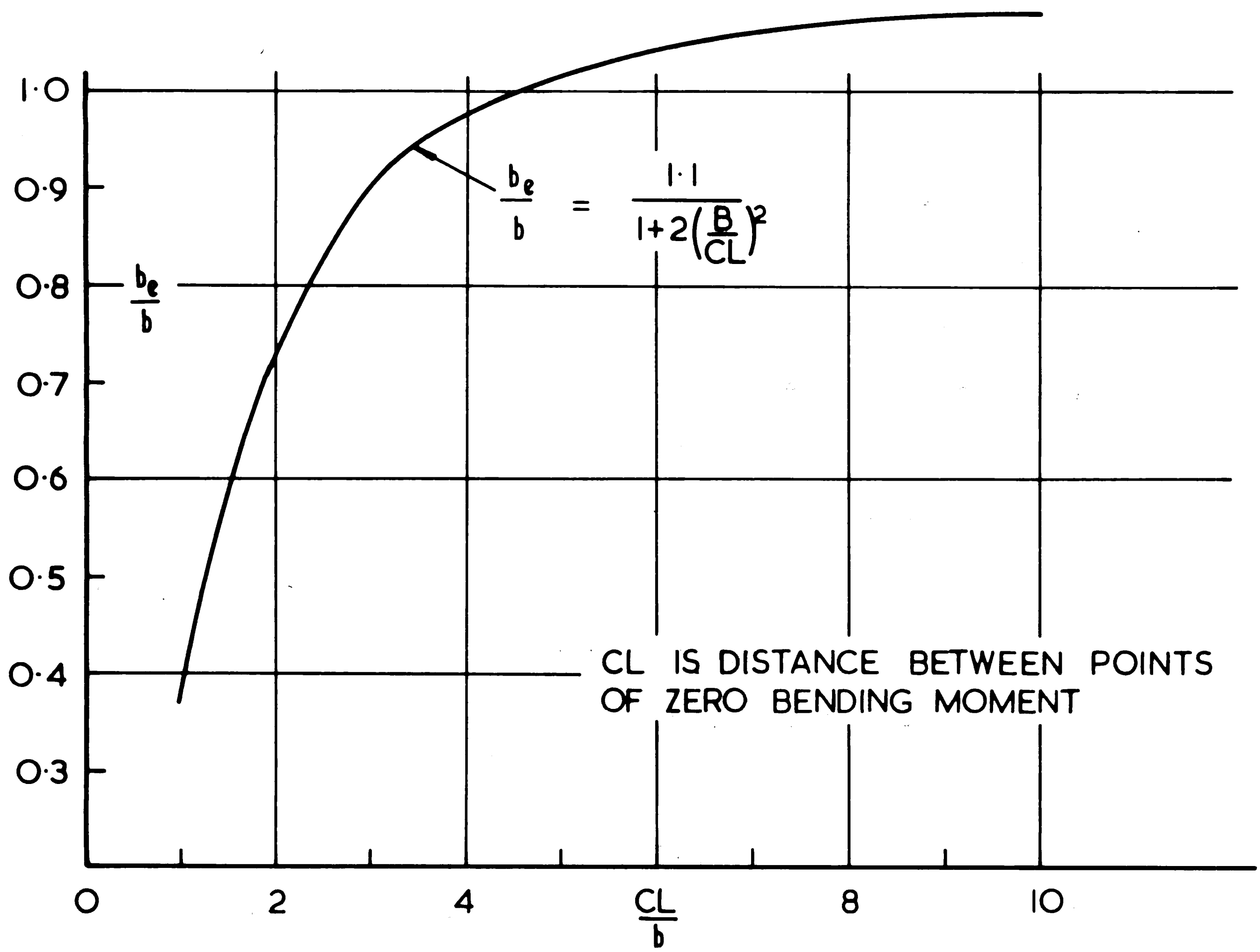
It does not depend upon thickness.*

Typically, for a multiple stiffener configuration the effective breadth ratio b_e/b with a uniformly distributed load would be not less than 0.9 for span/spacing ratios greater than about 3 or 4, depending upon the distance between points of zero bending moment. And so it will be appreciated that very little shear lag may be expected in many ship structures. An algebraic approximation representing uniform loading on multiple stiffeners is given by:

$$b_e = \frac{1.1}{1+2\left(\frac{B}{CL}\right)^2} \qquad (1)$$

where CL is the distance between points of zero bending moment. This is plotted in Figure 2. Schade argues that, in practice,

*Naval architects have for many years been led somewhat astray by Pietzker's and Hovgaard's reference to plate thickness, and the old 30t confusion still exists.



1·0
0·9
0·8
0·7
0·6
0·5
0·4
0·3
be/b
be/b = 1·1 / 1+2(B/CL)²
CL IS DISTANCE BETWEEN POINTS OF ZERO BENDING MOMENT
0
2
4
CL/b
8
10

Figure 2

loads cannot exist as sharp concentrations, but instead are somewhat spread or distributed by structure. Therefore, the reduction in b_e expected theoretically at such points will be lessened, and the actual effective breadth will approach that for a uniform load over most of the span. He suggested, therefore, that design estimates of b_e be based on uniform load data. Schade's work has more recently been extended into the plastic range [3], and to cover stiffened plates under axial tension or uniform bending [4].

A limitation of plane-stress solutions is that they take no account of:

(a) Normal deflections such as initial distortion or as caused by lateral load;

(b) Residual stress effects;

(c) Plate buckling effects as mentioned earlier.

Clarkson has demonstrated [5], albeit for a very limited range of structures, that welding distortion can be rather important in reducing plate effectiveness. This gets worse as plate thickness decreases, not only because of the large-deflection behavior of the plating, but also because welding distortions are then relatively larger. Theory is still a long way behind practice, and so the designer has to resort to empirical rules based on experimental data. It would appear from the foregoing that an empirical rule should include:

- stiffener spacing b
- plate thickness t

Clarkson agreed in principle but, as there is as yet insufficient data, he was content to recommend half the stiffener spacing as being effective plating for single skin structures, viz.:

$$b_e = \frac{1}{2} b$$

He argued that in design stress is usually more important than deflection, and this was fairly insensitive to b_e for single skins, as mentioned earlier; also 10-20 percent accuracy is probably the best that can be achieved in practice.

The only exceptions to this advice that the present writer feels are justified are:

(a) In the case of double-bottom structures where intercostal longitudinals are fitted, inter-stiffener deflections seem to cause less serious reduction in plate effectiveness than they do in single skins of the same b and t. This is borne out by full-scale measurements of strain in double bottoms conducted for the Admiralty Ship Welding Committee, and more recent BSRA test on double-bottom structures. As an interim recommendation, the writer advises taking three-quarters the plating effective when considering overall behavior of the double bottom. In critical cases, finite element calculation may be advisable.

(b) Where the stresses in the plating arising from grillage bending are compressive and are greater than three-quarters of the local plate buckling stress, then the effective plating should be governed by the rules discussed in the next section under "effective width" of plating in compression.

It is considered reasonable to assume these breadths are constant throughout the lengths of the beams.

There are two principal ways of determining effective breadth experimentally [6]:

(i) Directly, by strain measurements across the flange and integrating the deduced stresses [8];

(ii) Indirectly, by measuring beam deflections and/or flange stresses at web positions and estimating effective section properties by fitting these measurements to agree with the theoretical response of the structure.

There can be little doubt that the first method is preferable, but it demands a far greater experimental effort and is rarely carried out these days. There appears to be a wealth of such data collected for the Admiralty Ship Welding Committee [7], but very little has been assessed critically and put to good use. The second method is very crude for single skin ship structures, and we must be prepared to adjust our recommendations as further data become available.

PART II

EFFECTIVE WIDTH OF LONG PLATES

Maximum Plate Strength in Compression

A plate is considered long if the length 'a' of its unloaded edges is greater than 'b' the width of the loaded edges. It is well known that the so-called critical stress

$$\sigma_{PE} = \frac{k\pi^2 E}{12(1-\nu^2)} \left(\frac{t}{b}\right)^2 \qquad (2)$$

has relatively little meaning in practical terms. In contrast to columns at high b/t ratios, nothing sensational happens at the theoretical critical load, and final failure can occur at much higher stresses providing the sides of the plate are constrained to remain straight. On the other hand, if the component plates are of relatively low b/t ratio, then collapse will be determined by compressive "squash" at stresses approaching the compressive yield stress of the material. It is assumed that the reader is familiar with the fundamental mechanics of post-buckling behavior of plates (see, for example, page 459 of reference 9).

In predicting plate element strength, it is the maximum stress σ_m that matters; the average applied stress at which the component plates will finally collapse when their edge stresses rreach the yield stress of the material σ_o. Unfortunately, the theoretical determination of σ_m for even the simplest and most idealized case is difficult. Satisfactory solutions making use of finite element analysis are just beginning to emerge, and are being checked against available test data. At present, there are available a proliferation of empirical formulae, each of limited application. Designers, nevertheless, have to design thin-plate assemblies, and in general they manage to do so with fair efficiency. This must certainly be regarded as an area where theory is still a long way behind practice.

Studies of plate compressive strength have been mainly of a semiempirical nature and can be divided into two classes:

(a) Those based on the concept of "effective width" b_e;

(b) Those which seek to establish an empirical formula for maximum stress of the form $\sigma_m = F(\sigma_{PE}, \sigma_o)$.

These two methods represent very roughly the civil engineering (including naval architecture) and aircraft approaches respectively, and both have led to design rules. Unfortunately, the numerous formulae and experimental data give widely varying results, and so a close examination and assessment has been necessary before making firm recommendations can be made for ship purposes.

The effective width method, suggested originally by the typical distribution of stress in a post-buckled plate (see Figure 1) is particularly appropriate for structural steel with its sharp yield-point. The structure is considered as an assembly of plate elements. It assumes that at collapse the load is entirely taken by two yielding strips of material adjacent to the supported edges (or one such strip in the case of a flange), while the remaining central portion is unstressed (see Figure 1). The combined width of the two assessed active strips, called the effective width b_{em}, is assumed to be a constant multiple of the

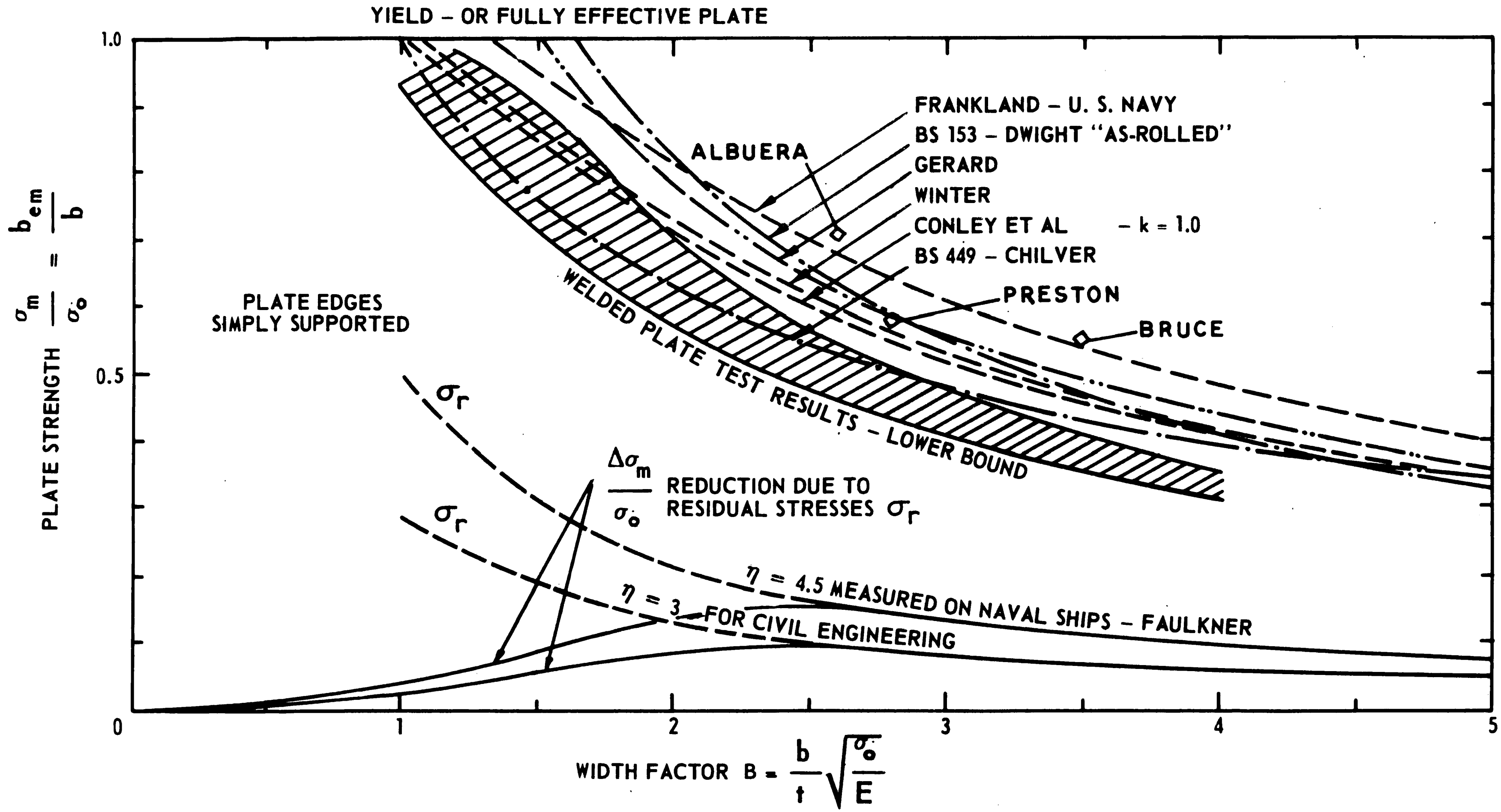

YIELD – OR FULLY EFFECTIVE PLATE
1.0
0.5
0
1
2
3
4
5
PLATE STRENGTH $\frac{\sigma_m}{\sigma_o} = \frac{b_{em}}{b}$
WIDTH FACTOR $B = \frac{b}{t}\sqrt{\frac{\sigma_o}{E}}$
FRANKLAND – U. S. NAVY
BS 153 – DWIGHT "AS-ROLLED"
GERARD
WINTER
CONLEY ET AL – k = 1.0
BS 449 – CHILVER
ALBUERA
PRESTON
BRUCE
PLATE EDGES SIMPLY SUPPORTED
WELDED PLATE TEST RESULTS – LOWER BOUND
σ_r
σ_r
$\frac{\Delta\sigma_m}{\sigma_o}$ REDUCTION DUE TO RESIDUAL STRESSES σ_r
η = 4.5 MEASURED ON NAVAL SHIPS – FAULKNER
η = 3 – FOR CIVIL ENGINEERING

Figure 3

thickness for a given material. The predicted collapse load for a plate wider than $b_{em}t\sigma_o$, the possible effect of any extra width over and above b_{em} being ignored. Narrower plates are assumed to be capable of resisting the full squash load (area x σ_o). Thus the curve relating collapse load with width for a given thickness as predicted by this method has a horizontal cutoff as in Figure 3.

The effective width approach is coupled with the name of von Kàrmàn [10], although not originated by him (see appendix for a brief history). He suggested that b_{em} should be taken as the width of plate in the material concerned for which $\sigma_b = \sigma_o$. Thus, for a simply supported web (k = 4) equation (2) leads to the minimum effective width in which the edge stress σ_e is equal to the plate yield stress σ_o:

$$b_{em} = \pi t \sqrt{\frac{E}{3(1-\nu^2)\sigma_o}} = 1.9t\sqrt{\frac{E}{\sigma_o}} \qquad (3)$$

when $\nu = 0.3$. For mild steel having σ_o = 16 tsi, b_{em} would be 55 t. Tests have suggested that this is too high and current figures in use are 40-50 t in many ship codes, and 45 t for non-welded webs in steel girder bridges [11]. von Kàrmàn also suggested generalizing the result for determining the stress distribution prior to collapse by replacing σ_o by $\sigma_e \leq \sigma_o$.

Typical of the empirical formula approach for predicting plate strength, where the parameters are suggested by simple dimensional analysis, is:

$$\frac{\sigma_m}{\sigma_o} = C\left(\frac{\sigma_{PE}}{\sigma_o}\right)^n$$

$$0 < n \leq 1/2; \quad 0 < C \leq 1 \qquad (4)$$

where the constants n and C can be adjusted to suit test results for the material and class of member under study. For a material such as aluminum, it is necessary to state what is meant by σ_o, and it is customary to take this as the 0.2 percent proof stress (σ_2). The value assigned to n has a pronounced effect on the shape of the curve relating collapse load with width for a plate of given thickness. n = 0.5, C = 1 produces a curve with a sharp knee at $\sigma_{PE} = \sigma_o$, which corresponds exactly with von Kàrmàn's effective width concept. These are the upper limits for n and C. Chilver [12,13] in work on light gauge channels proposed n = 1.3, with C = 0.66 for cold formed steel (Marguerre many years earlier used n = 1/3, as discussed in the appendix, but with C = 1 his results are far too optimistic for steel plates, especially at large b/t values). Gerard [14], studying individual simply-supported long flat plates and square tubes of various aluminum and magnesium alloys and steel, found relatively good correlation among these various materials even though they differ significantly in E and σ_o. He proposed using

$$\frac{m}{\sigma_o} = \frac{1.42}{\beta^{0.45}} \tag{5}$$

and claimed (somewhat optimistically, it is felt) this fits the data within ± 10 percent. Equation (5) is equivalent to equation (4) when C = 0.824 and n = 0.425. β is the plate slenderness ratio

$$\beta = \frac{b}{t}\sqrt{\frac{\sigma_o}{E}}$$

These two methods of effective width and the empirical formula approach to σ_m are claimed by Dwight [15] to be in conflict in two important respects:

(a) In the first place, whereas for all $n < 1/2$ the empirical formula (4) gives a steady increase in strength with plate width (a 26 percent increase for double the width, if $n = 1/3$), the von Kàrmàn approach suggests a constant collapse load $b_{em}t\sigma_o$ independent of the width, provided this is greater than b_{em}, of course.

(b) Secondly, the empirical formula method takes account of the conditions along the unloaded edges of a plate since these affect σ_b, whereas the effective width method does not. Thus, using equation (4) with $n = 1/3$, a clamped plate ($k = 7$) would be indicated as 20 percent stronger than a simply-supported one ($k = 4$). The effective width method as presented so far allows no increase for the clamped case.

Previous experiments have shown that for simply-supported plates the compressive strength does tend to rise with increasing plate width, but only rather slowly. In tests on welded box-columns [59] it was found that plates containing locked-in stresses due to welding can be appreciably weaker than stress-free ones (this will be discussed later), and in this case the strength rises even more steeply with plate width.

Little experimental data is available on the effect of edge rotational restraints on plate strength. Clamping the side edges produces a 75 percent increase in σ_b, but is unlikely to have anything like so much effect on σ_m. Indeed, recent carefully conducted experiments [17,18] appear to show that the difference in strength between clamped and simply-supported long steel plates is hardly worth considering. Over the range of practical interest (β = 3, say) it is likely to be appreciably less than the 20 percent indicated by taking n = 1/3 in equation (4), and it does not appear enough, and it perhaps does not warrant higher design stresses. This will be discussed again.

The effect of restraints at the edges of the plate against movement in the plane of the plate has been considered by Marguerre [19] and more recently and exhaustively by Cox [20]. He concluded that the buckling stress is determined chiefly by the degree of restraint applied to its edge against rotation; but restraint against in-plane movement may also have considerable influence. In their effects on the behavior of the plate after buckling, lateral in-plate restraint assumes greater importance and may occasionally cause the plate to behave in an apparently anomalous manner. With complete restraint the stabilizing membrane stresses exert their maximum effect in enabling the plate to regain its stability in the distorted shape. However, these studies are largely theoretical and entirely elastic; and there appears to be no experimental evidence to suggest that complete lateral restraint significantly raises plate strength. In most practical ship structures this in-plane restraint is in any case far from complete because it is provided largely by the transverse stiffeners which form a relatively small proportion of the total cross section. The effect is therefore ignored henceforth.

As applied in more sophisticated circles, the shortcomings just mentioned in the effective width approach are, in fact, more imaginary than real. For example, there are many dual-term effective-width formulae of the form:

$$\frac{b_{em}}{b} = \frac{C_1}{\beta} - \frac{C_2}{\beta^2} \qquad (6)$$

It will be seen that the second term, which was introduced to overcome the experimentally found optimism of the single-term von Kàrmàn equation (3) at low β values, does in fact yield an increase in effective width as b increases. This overcomes the first supposed limitation. Moreover, the slenderness factor β is related to the buckling stresses by the well-known equation

$$\beta^2 = \frac{k\pi^2}{12(1-\nu^2)} \left(\frac{\sigma_o}{\sigma_{PE}}\right) \qquad (7)$$

and so the second limitation is easily overcome by inserting the appropriate buckling coefficient k, depending on the boundary constraints. It follows that formulae expressed in terms of the plate geometry and material properties can also be expressed in terms of their buckling stress and yield stress, and so the difference in the two approaches are perhaps more imaginary than real and can easily be overcome.

So far, the discussion has been rather general and idealized in nature; and it is time we considered the practically important effects of initial distortion, normal pressure, residual stresses, and failure occurring before the edge stress reaches yield due perhaps to stiffener collapse.

<u>Effect of Initial Distortion</u>

In a practical structure, distortions are usually caused by the manufacturing process, and, in particular, in modern ship structures they are associated with residual welding stresses.

For the moment, we will ignore the effect of these stresses and consider only the stiffness and strength of a stress-free, but distorted, plate.

Engineers are usually well aware of the fact that an initially deformed plate loses stiffness immediately load is applied. Thus the efficiency of the plate, defined as the ratio of the total contraction of a perfectly plane plate to that of a deflected plate, is continually reducing as load increases, even before buckling occurs. It can be demonstrated that this definition of plate efficiency is identical with the effective width ratio b_e/b used in this review. It is commonly recognized that this loss in efficiency is small until buckling loads are approached, particularly in plates which are longer than they are wide. Even for wide plates, Murray [21,22] has shown, using an elastic analysis, that initial deflections do not in general lower the efficiency of plating significantly unless the plating is very thin. He suggested this initial deflection should not exceed 0.3t if great loss in efficiency is to be avoided.
Horne extended this wide plate analysis into the elasto-plastic zone [23], thereby throwing light on the phenomenon of the inevitable progress of buckling and deflection growth under certain conditions. H. H. Bleich [24] has refined Horne's theory to render it more in keeping with actual ship conditions. Unfortunately, these conditions are often present in the bottoms of transversely framed ships of all-welded construction, and the best remedy is to alter the system of framing to longitudinal framing. Therefore, wide-plate analysis is considered no further in this review.

An investigation for simply supported square plates in aircraft structures [25] showed that:

(a) As expected, the effects of initial deflection upon buckle growth and effective width are most marked near the theoretical flat-plate critical stress.

(b) At stresses well below the critical stress, the behavior of the plate is very much the same as for an initially flat plate.

(c) The effective width is at all values of stress less than that of an initially flat plate.

If we assume, pessimistically, that for a long simply supported plate, the initial deflections are in asymmetrical waves whose half lengths are equal to the plate width (i.e., they are in the lowest buckling mode), then the square plate conclusions above will apply also to the long plate. Loss of effectiveness of plating is approximately proportional to the square of the initial deflection. With the random ripples which usually occur in welded ships, or with one single lobe, the loss in effectiveness is appreciably reduced and is small in longitudinally stiffened ships. The effect of initial deflection on the maximum end load the plate can carry has been examined recently at Cambridge [15,26]. In spite of several simplifying assumptions (a completely rigorous analysis is very difficult mathematically), both authors' experimental results show good agreement with the theories evolved and demonstrate an appreciable reduction in maximum load capacity, even for very small initial deflections.

For example, an initial deflection of only t/20 in a mild steel plate having b/t = 50 appears to lower σ_m by about 15%. With random ripples, of course, a larger out-of-flatness would be needed to cause the same drop in σ_m. There will inevitably be a harmonic of the initial deflection which is in sympathy with the buckled form, and with other nearby buckling harmonics, and these components will be the most damaging in lowering σ_m. Reference 26 also accounts for residual stress effects, but this will be discussed later.

Since drafting this report (1970), several useful theoretical references have appeared which warrant further study. Moxham [27] has produced an elasto-plastic program which is able to allow for residual stresses, and for three kinds of initial out-of-flatness. Mansour [28] has extended his linear orthotropic plate theory to include the effects of initial curvature and combined loads, and has now extended the theory to the nonlinear range [16]. Dawson and Walker have produced a semiempirical approach which allows explicitly for generally defined imperfections [50], and which shows good agreement with test results. Becker is entering the field [29] and the quality of his small-scale experimental work shows considerable promise.

Measured Initial Distortion

Nearly three hundred plate distortion measurements were taken for the author by Naval Constructor students on typical areas of frigate bottom plating in dry docks in 1965. They show, for example, that the average deflection was 0.30t or 0.005b in the least fair frigate, and 0.11t and 0.0024b in the best. Maximum deflections were generally about three times these values, with the very occasional large local depression of about 1.5t in the worst frigate near a welded seam. It will be observed, therefore, that the fairness of construction varies appreciably between the building yards, and in the worst cases plate unfairness will without doubt appreciably lower plate stiffness and strength in compression.

The results have been grouped statistically and mean central values plotted in Figure 4 [30]. The distortions arise mainly from the side welds of the stiffeners, and the deformation has a marked m = n = 1 mode. Dimensional analysis suggests that, when the stiffener webs (t_w) are thinner than the plate (t), the fillet weld leg length is governed by t_w and we may expect

$$\frac{\delta_P}{t} = K\ \beta^2\ \left(\frac{t_w}{t}\right)^2 \tag{7}$$

In fact, a regression analysis for the warship measurements referred to yielded a nearly linear dependence with t_w/t, viz.,

$$\delta_P/t = 0.12\ \beta^2 (t_w/t) \tag{8}$$

$$\text{for } t_w \leq t\ ,\ \beta \leq 3$$

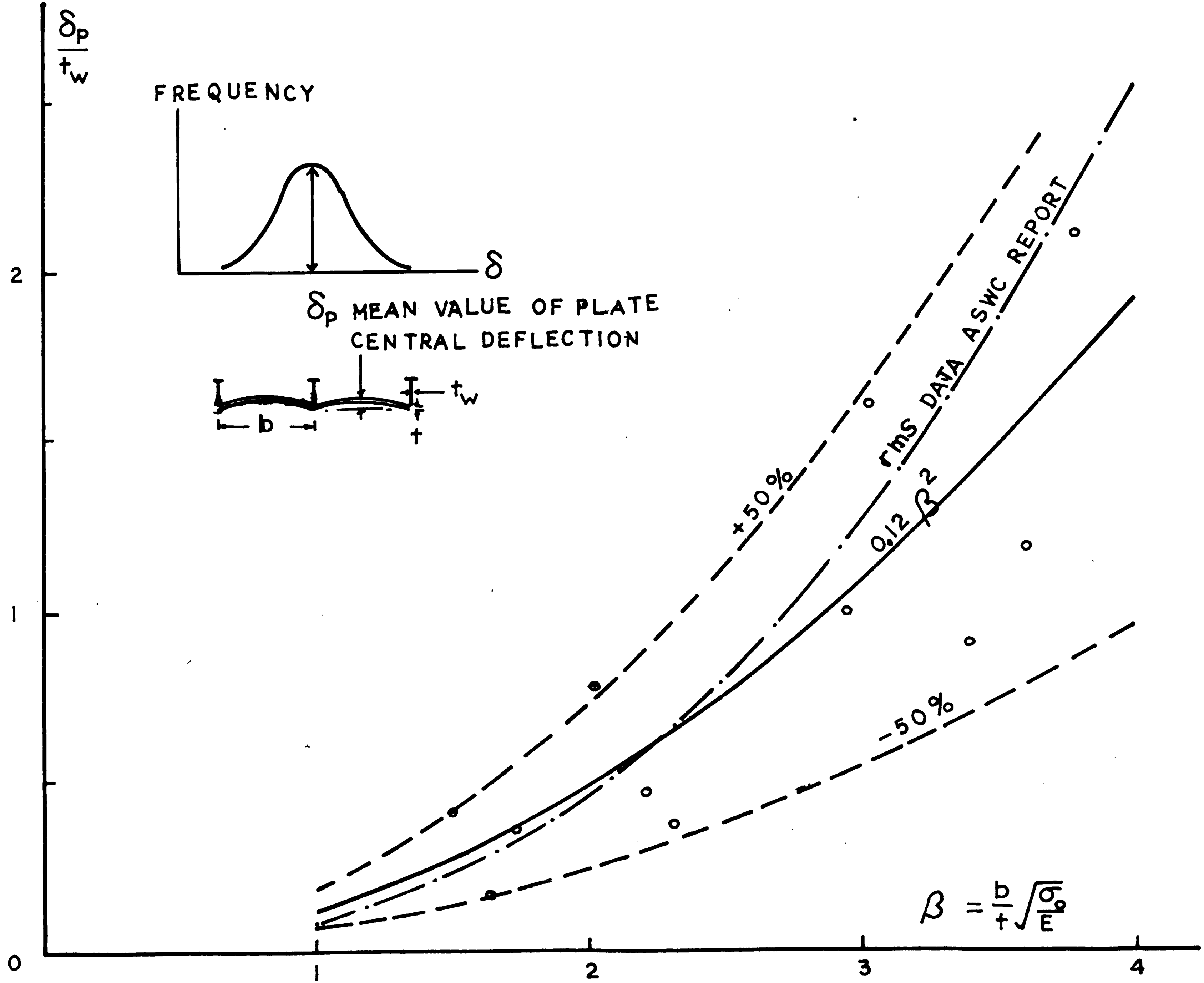
For slenderer plates ($\beta > 3$), a coefficient of 0.15 is more accurate. When $t_w > t$, then the fillet weld leg length will normally be determined by plate thickness t. In this case

$$\delta_P/t = K\ \beta^2 \tag{9}$$

Since for much of the warship data $t_w \simeq t$, equation (9) was plotted with K = 0.12 in Figure 4, which also includes some data of merchant ships [7]. Noting that rms data is a little more than mean values, we can conclude that a coefficient of 0.15 is probably more appropriate for merchant ship structures.

In civil engineering structures coefficients less than 0.1

Figure 4


$\frac{\delta_p}{t_w}$
FREQUENCY
δ
δ_p MEAN VALUE OF PLATE CENTRAL DEFLECTION
t_w
b
t
+50%
rms DATA ASWC REPORT
$0.12\,\beta^2$
-50%
$\beta = \frac{b}{t}\sqrt{\frac{\sigma_o}{E}}$
0
1
2
1
2
3
4

would usually be more appropriate. With intermittent welding this would be reduced still further.

Effect of Normal Pressure

There are many parts of a ship's structure which experience normal as well as in-plane loading. However, it is usually only in the bottom structure that these two occur together at their maximum values. The question arises as to whether the normal loads are sufficiently large to produce a noticeable effect on the compressive buckling and failure loads or not. The increase in buckling strength is essentially due to the tensile membrane stresses which are caused by the deflection of the plate under normal load. As these membrane stresses are usually very small (for deflections $< 0.5t$), it can be concluded that normal load in ships only slightly raises the critical load, and will not prevent the usual buckled pattern (see page 497, reference 9). The effect on ultimate load is less clearly understood and is being examined at the Naval Construction Research Establishment, Dunfermline, and for the Ship Structure Committee [29].

The author [30,31] has considered the effects of normal pressure on plate strength and stiffness, and has proposed formulations which make extensive use of interaction equations. When the lateral pressure is greater than $\sigma_o^2/E\beta^2$, then a transfer to effective width equations for clamped plate edges is proposed (see later).

Other Boundary and Load Conditions

The discussion has been general, but the buckling and effective width equations quoted so far have assumed edges held straight but free to rotate. The effects of initial deflection and normal load have been mentioned, but other factors which could be relevant to the behavior of plate elements in a complete structure are:

(a) Edges may be rotationally restrained--in particular, constraints on the long unloaded edges (e.g., by heavy roller or Vee groove supports during plate tests) may affect strength and stiffness.

(b) In-plane working of the unloaded edges is difficult to prevent in single plate experiments, but would be inhibited in a stiffened-plate structure.

(c) Inelastic material behavior has not been properly considered.

(d) The effect of plate element length has not been adequately considered, other than to assume plates are "long" if $a > b$. This can be shown to be a reasonable approximation for plate buckling, and indeed for plate strength; but there is no doubt that the post-failure stiffness of plate elements varies appreciably with length, as explained by Ractliffe [17].

(e) Poisson effects, and other in-plane loading effects have not been discussed.

With all these shortcomings it is perhaps small wonder there are very significant differences between the various available formulations. The review revealed these could amount to 100%, though; for the unwary engineer seeking guidance these differences themselves would not necessarily be immediately apparent.

Even if one confines one's interest to one material like steel and refuses to look at research reports, the discrepancy between the various "accepted" codes is itself quite remarkable. A glance at Figure 3 shows this to be so, even for the preferred expressions remaining after a weeding-out process.

The boundary conditions at the unloaded edges are the most important as far as long plate compression is concerned. The position is complicated by the largely indeterminate conditions at plate elements in ship grillages, as mentioned earlier. For rotation simple supports are commonly assumed since open-section longitudinals which are torsionally weak are supposed to provide very little rotational constraint. This argument has been challenged [34] on the grounds that, since the buckled wave forms are short compared with the span between stiffener supports for long plates, and since these waves alternate in sign, the twisting moments they induce in the stiffeners alternate rapidly in direction and are largely self-equilibrating. This causes only very minor twisting along the stiffener edges, and so, perhaps, clamped assumptions are more valid. These produce a 75% increase in elastic buckling stress, and very probably would noticeably increase load-shortening stiffness for most of the range prior to collapse. However, clamping the side edges is unlikely to have anything like so much effect on σ_m. Carefully conducted experiments at Cambridge [17] indicate that the difference in strength between clamped and simply supported long steel plates is perhaps not worth considering, though more recent work by Moxham [27] suggests distinct differences with slender plates ($\beta < 2.5$, say). Certainly it would be too optimistic to substitute the clamped value of σ_b in any of the effective width formulae in Appendix 1.

In a ship in-plane warping of the plate is strongly resisted by virtue of the restraint from adjacent panels. This is not simulated in single plate tests. The effect of this restraint was first considered theoretically by Marguerre [19], and more recently and exhaustively by Cox [20] and by Mayers and Budiansky [37]. Cox demonstrated that, while their effect was not as great on elastic buckling as were rotational restraints, in-plane restraints were important, and may occasionally cause the plate to behave in an apparently anomalous manner. With complete restraint the stabilizing membrane stresses exert their maximum effect in enabling the plate to regain its stability in the distorted shape. However, his studies are largely theoretical and entirely elastic. The NACA report carried the theory into the post-buckling plastic region and indicated that the load-carrying capacity of a plate with edges constrained to remain straight should be markedly superior to that for a plate with edges free to warp. Experimental data on this subject is a little confusing and far from satisfactory, especially again for the effect on stiffness prior to failure. Besseling [38] carried out tests on Alclad and Dural sheets sandwiched between a pair of rigid grillages. The results for the single-bay specimens apparently differed significantly from those for the three-and five-bay ones. However, the tests as a whole were not entirely satisfactory owing to the difficulties of preserving intimate contact between the plate and the stiffeners and the eccentricity of the applied load. In contrast, recent tests at NSRDC [39] on threebay panels showed the same load-carrying behavior as for single plates, though massive stiffeners were required to force the plates to achieve maximum load. Reference has already been made to Hoff [33] who in sumarizing previous experimental data concluded that the effect of warping on effective width is immaterial. It seems, therefore, that the best, and again possibly conservative, conclusion we can draw is that warping has little if any effect on load capacity for ductile materials, but could appreciably affect plate stiffness for which further experimental data is required.

The above conclusion is quite similar to that found when considering rotational constraint, and it is interesting to note that in both cases the high ductility of most structural metals appears to obviate the effects of edge restraints as far as load capacity is concerned. This is a welcome help to hard-pressed engineers, but at the same time a hindrance to fundamental understanding, which could be especially important when contemplating the strength of nonductile materials such as glass-reinforced plastics.

Davidson [40] has made an excellent review of work prior to 1965, especially theoretical aspects, and Dwight [41] continues to keep us informed of the excellent work he has been directing at Cambridge. Future theoretical developments must surely lie with finite element methods of sufficient generality to account for real shapes and boundary conditions. A start has been made in that direction [125,131]. Smith [126] has discussed the need for establishing reliable formulae or data curves which could be incorporated in computer programs.

Three Destroyer Tests

Figure 3 also includes long plate strength data deduced from three pre-World War II destroyers tested to destruction in dry dock, and collectively reported by Vasta [57] and referred to by Lewis and Gerard [58]. They were quoted at the time to support the U.S. Navy plate strength equation attributed to Frankland (see Appendix), viz:

$$\frac{b_{em}}{b} = \frac{2.25}{\beta} - \frac{1.25}{\beta^2}$$

It will be seen from Figure 3 that the results for ALBUERA (British destroyer longitudinally framed) and BRUCE (U.S. destroyer mixed stiffening) lie just above the Frankland equation, which itself provides an upper bound over most of the β range to the various design codes. PRESTON was a U.S. destroyer mainly stiffened transversely.

However, the author in discussing reference 51 suggested that not too much importance be attached to the perhaps surprising optimism of these results. The main reasons for this are seen to be:

(a) The plates were riveted to stiff flanges which suggest that their edge constraints may have been appreciably nearer to the clamped condition (for which it is better suited).

(b) The average stress in the plate elements will be somewhat less than the overall average because the longitudinals and heavy longitudinal structure will generally sustain higher stresses at failure than will the plating; while some attempt was made to allow for this, it is believed the errors this division of load entails are more likely to lead to plate strength overestimates than otherwise.

(c) In particular, there must remain considerable uncertainty about the yield stress of these longitudinal stiffeners which quite commonly are known to have higher values than for the plating.

(d) Using simple bending theory to calculate overall average stresses at failure can lead to several small errors such as

- uncertainty in the elasto-plastic neutral axis at collapse
- the plate σ_u/σ_y values quoted in the original papers are really based on M_u/M_y values, where M_u is the ultimate moment measured and M_y is the yield moment assumed from material properties

and so errors arising from ignoring plastic redistribution within the cross section have been ignored. In the case of WOLF, shear buckling of the side plating certainly occurred before failure.

Lastly, the results quoted (with the exception of the transversely framed PRESTON) are appreciably optimistic compared with recently collected test data, discussed later.

Reduced Effective Width

The strength of a stiffened panel finally depends upon the stability of the stiffeners and some associated plating. In determining how much plate to consider effective in resisting, for example, column collapse between transverse frames, it is important to recognize that the support given by the plate to the stiffeners is governed by its stiffness just prior to and at the moment of collapse. Wagner [32] was the first to emphasize the difference between load-carrying capacity of the plate and support given by the sheet to stringers against buckling. For the latter, as Hoff has so well described [33], it is the ability of the plate to take over ADDITIONAL loads which is important for the stability of the combination, rather than the actual capacity of the plate to carry loads. The essential in these considerations is to allow for the decrement with increasing compressive strain of the load-carrying capacity of the waved or buckled plate by reducing the value of the effective width b_e in very similar manner to the allowance made for the deviation from Hooke's law by reducing the modulus of elasticity of the material when the critical stress for a strut exceeds the limit of proportionality. Referring to Figure 3, the compressive load carried by the plate before failure is $P = \sigma_e b_e t = \sigma_a bt$. We require the stiffness of the plate $dP/d\varepsilon$ where ε is the average compression strain. By definition

$$\frac{dP}{dE} = b'_e t \left(\frac{d\sigma_e}{d\varepsilon}\right) \tag{10}$$

where $P = \sigma_e b_e t$

$$\therefore \quad \frac{dP}{dE} = [b_e (\frac{d\sigma_e}{d\varepsilon}) + \sigma_e (\frac{db_e}{d\varepsilon})]$$

$$= t(\frac{d\sigma_e}{d\varepsilon}) [b_e + \sigma_e (\frac{db_e}{d\sigma_e})]$$

Equating with equation (10), this leads to the reduced effective width:

$$b'_e = b_e + \sigma_e\left(\frac{db_e}{d\sigma_e}\right) \qquad (11)$$

Hence, when the compressive load P increases, the plate element behaves in such a manner as if the value of its effective width were b'_e. This is actually less than b_e since the second term in equation (11) is negative. b_e is referred to as the REDUCED EFFECTIVE WIDTH, or sometimes in aircraft engineering as the TANGENT WIDTH [127] since it uses the slope of the load-compression graph. This follows readily from a simple manipulation of equation (11) to give

$$b'_e = b\left(\frac{d\sigma_a}{d\sigma_e}\right) \qquad (12)$$

Equations (11) and (12) are perfectly general and apply in the inelastic range. Generalizing von Kármán's effective width equation (3) for $\sigma_e < \sigma_o$ for use in equation (12), we have:

$$b_e = 1.9t\sqrt{\frac{E}{\sigma_e}}$$

i.e., $$b\,\sigma_a = \sigma_e b_e = 1.9t\sqrt{E\,\sigma_e}$$

Hence $$b'_e = b\,d\,\sigma_a = \frac{1.9t}{2}\sqrt{\frac{E}{\sigma_e}}$$

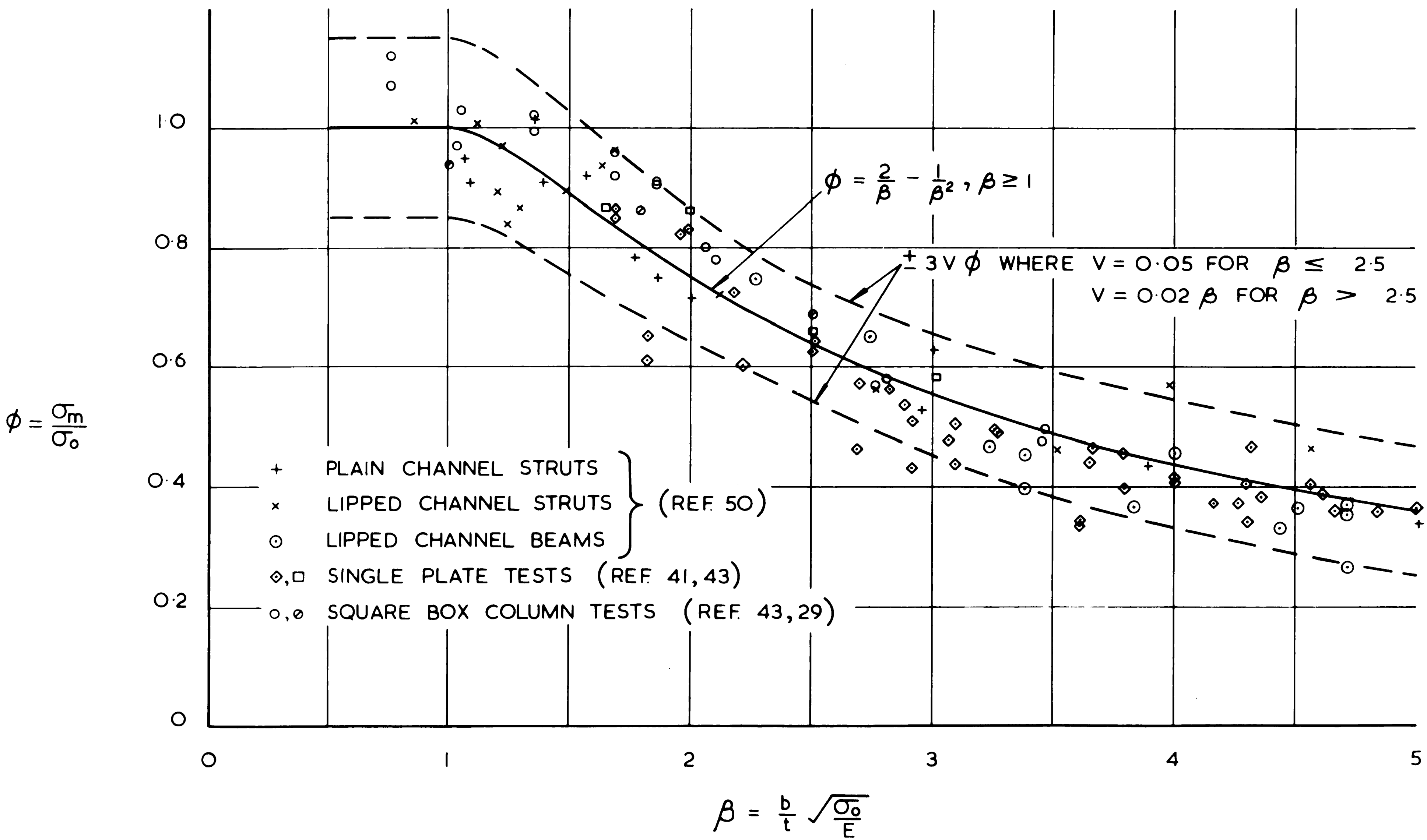
i.e., $$b'_e = \frac{1}{2}b_e$$

For other effective width equations, the reduction is often more than half. One shortcoming of the above analysis is that it takes no account of plate element length, since the same insensitivity to length is assumed as for plate buckling and strength.

Appraisal and Recommendations

The foregoing remarks are probably enough to illustrate why researchers have found it so hard to reconcile theory with practice, which justifies a semiempirical approach to the problem

Figure 5



$\phi = \frac{2}{\beta} - \frac{1}{\beta^2}, \beta \geq 1$
$\pm 3V\phi$ WHERE $V = 0{\cdot}05$ FOR $\beta \leq 2{\cdot}5$
$V = 0{\cdot}02\beta$ FOR $\beta > 2{\cdot}5$
PLAIN CHANNEL STRUTS
LIPPED CHANNEL STRUTS
LIPPED CHANNEL BEAMS
(REF. 50)
SINGLE PLATE TESTS (REF. 41, 43)
SQUARE BOX COLUMN TESTS (REF. 43, 29)
$\phi = \frac{\sigma_m}{\sigma_o}$
$\beta = \frac{b}{t}\sqrt{\frac{\sigma_o}{E}}$
1·0
0·8
0·6
0·4
0·2
0
0
1
2
3
4
5

based on test results from plates having typical initial deflections and also, preferably, typical residual stresses (considered in Part III). A review for wide plates ($a < b$) is given elsewhere [30].

A. Pinned Plates

The review has indicated an abundance of theory and experimental data on the elastic behavior of flat or idealized plates, especially for lightweight low modulus aircraft alloys at high width factors. Steel data are more scarce; the main sources of vetted data being individual plates tested and reviewed at NSRDC [42], individual plates tested at Cambridge [15,17,18,27] which have been compiled together [43]. These provide nearly 50 test points on plate strength over a range of β up to 5, and they are plotted in Figure 5.

In addition, about 20 results are plotted in Figure 5 from about 20 square box-column tests at Cambridge [44,45,46] which again have been compiled by Dwight [43], and from about 20 cold-formed channel strut [47] and beam tests [48,49] which have been compiled by Dawson and Walker [50], and 3 results from small-scale box strut tests by Becker [29] are included.

Superimposed on the diagram is a plate strength equation proposed by the author in 1964 [51], and which has subsequently been found to provide excellent agreement with strut-panel test data:*

$$\frac{\sigma_m}{\sigma_o} = \frac{b_{em}}{b} = \frac{2}{\beta} - \frac{1}{\beta^2} \qquad \text{for } \beta \geq 1 \tag{13a}$$

*To be published in The Structural Engineer, London.

It will be seen that this provides a very reasonable mean curve through the data, with coefficients of variation assessed as

$$\begin{aligned} \nu &= 0.05 \text{ for } 0.5 \leq \beta \leq 2.5 \\ &= 0.02\beta \text{ for } \beta > 2.5 \end{aligned} \qquad (13b)$$

The generalized form of equation (13) for use in the restricted range $0.7\,\sigma_o \leq \sigma_e \leq \sigma_o$ and $\beta \geq 1$ is [31,51]:

$$\frac{\sigma_a}{\sigma_e} = \frac{b_e}{b} = \frac{2}{\beta}\sqrt{\frac{\sigma_o}{\sigma_e}} - \frac{1}{\beta^2}\left(\frac{\sigma_o}{\sigma_e}\right) \qquad (14)$$

Applying equation (11) leads to reduced effective width expressions for $\beta \geq 1$ of:

$$\frac{b'_e}{b} = \frac{1}{\beta}\sqrt{\frac{\sigma_o}{\sigma_e}} \qquad (15)$$

$$\frac{b'_{em}}{b} = \frac{1}{\beta} \qquad (16)$$

Maquoi and Massonnet [52] have very recently provided a critical review of the various effective width formulae, and concluded that equations (13) and (14) were the best available for box-girder use. Equation (13) is a little more optimistic than Winter's well-known equation [48] recently modified [53], but appreciably more pessimistic than Frankland's equation [54] which has been used by naval architects.

A few words should be mentioned concerning the use of equations (13), (14) and (15), which the reviewer finally recommends. First, it is aimed at predicting <u>average</u> values of plate strength and stiffness, whereas equations used in design codes are sometimes more nearly "lower bound" formulations--though it is not by any means always obvious where this is intended. The author has a strong preference for using mean strength predictions, as these are much more valuable for a statistical treatment of the subject [30], as has already been applied to ship structures [56]. It was for this reason that the coefficients of variation derived from test data were quoted.

Secondly, and arising from this, because the statistical scatter from test data is large, researchers and designers are urged not to be too influenced by just a few test points, however well the tests were conducted. There will always remain a high chance that one test result could be appreciably lower or higher than the mean expected, and many results are necessary before confidence is established.

Thirdly, the generalized expressions (14) and (15) were intended for use in the stiffened-plate assembly, where collapse may very well occur by some form of stiffener failure before the edge stress in the plate elements has reached the yield stress of the material, as commonly supposed. The underlying theory for its use in this way is given in reference 30, with a brief presentation for computer-assisted design and analysis purposes outlined in reference 31. These last two references also provide for a treatment of the wide plate problem ($a < b$).

B. Clamped Edges

In cases where the longitudinal stiffeners in a panel are torsionally strong, or where the lateral pressure is sufficiently large, the use of clamped edges may be more appropriate. In the light of recent Cambridge work and NCRE test data briefly referred to, it has been proposed [30,31] that when the lateral pressure is greater than $\sigma_o^2/E\beta^2$ expressions corresponding to plates with clamped boundaries be used in the range $\beta \geq 1.25$ and $0.7\,\sigma_o \leq \sigma_e \leq \sigma_o$:

$$\frac{b_{em}}{b} = \frac{2.5}{\beta} - \frac{1.5625}{\beta^2} \tag{17}$$

$$\frac{b_e}{b} = \frac{2.5}{\beta}\sqrt{\frac{\sigma_o}{\sigma_e}} - \frac{1.5625}{\beta^2}\left(\frac{\sigma_o}{\sigma_e}\right) \tag{18}$$

$$\frac{b'_{em}}{b} = \frac{1.25}{\beta} \tag{19}$$

$$\frac{b'_e}{b} = \frac{1.25}{\beta}\sqrt{\frac{\sigma_o}{\sigma_e}} \tag{20}$$

The curious use of 4 decimals in the coefficient of the second terms is necessary to avoid computational anomalies.

PART III

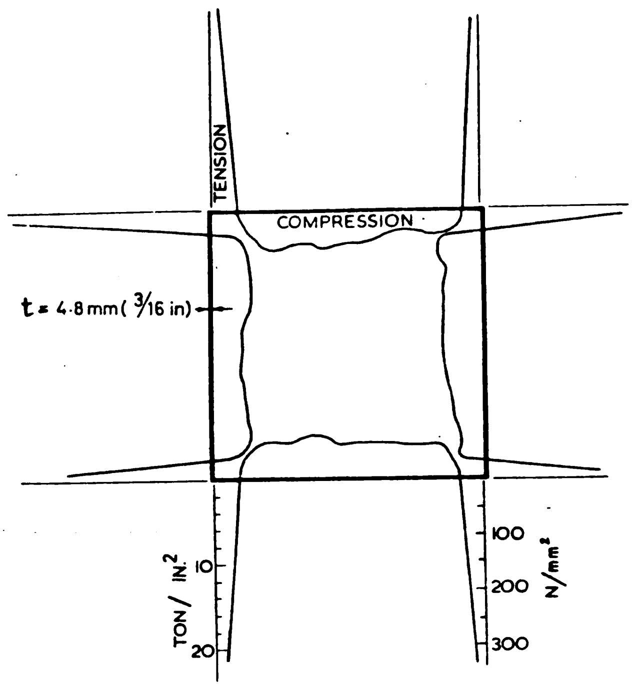
WELDING STRESS EFFECTS

The historical appendix reveals that welding stress effects in plating have been studied in Russia, Japan, Britain and the United States, and the author's own work has been particularly inspired by the work at Cambridge and Lehigh. Space and time prevent a fuller discussion of all these investigations and so the author has based this section of the review mainly around his own work as reported in reference 30 and outlined for design use in reference 31. This preference is supported by good agreement with experiments, as can be seen from Figure 8 and discussed below.

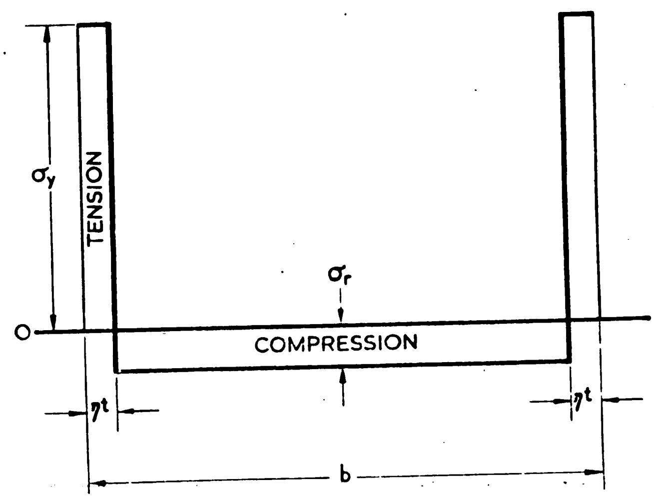
Maximum Pinned Plate Strength in Compression

When stiffening members are welded to the plate, the welding temperatures take on such extreme values that considerable residual stresses result from the process which can seriously degrade plate strength. With the contraction arising from these welds, it is found that the weld metal, together with the web and plate material in the immediate vicinity, are invariably left stressed at about σ_o in tension. This "tension block" typically extends three to six thicknesses out from the weld each side, and depends on the cross-section area of weld deposit and on the welding conditions, sequence, constraint, etc. (see Figures 6 and 7).

As Figure 7 illustrates, the tension block is offset from the stiffener-plate centroid and thus bending occurs as shown. To preserve equilibrium along the direction of the stiffener, the tension must be balanced by residual compression which exists



(a). *Typical residual stress pattern in a welded box member* $b/t = 80$, $\sigma_e = 402$ N/mm² (26 ton/in²).



(b) *Idealized residual stress pattern in a web with edge welds (definition of* η*). Note that the area of the compression stress block must equal combined area of tension blocks.*

Figure 6. Typical Residual Welding Stresses

Square Box Columns

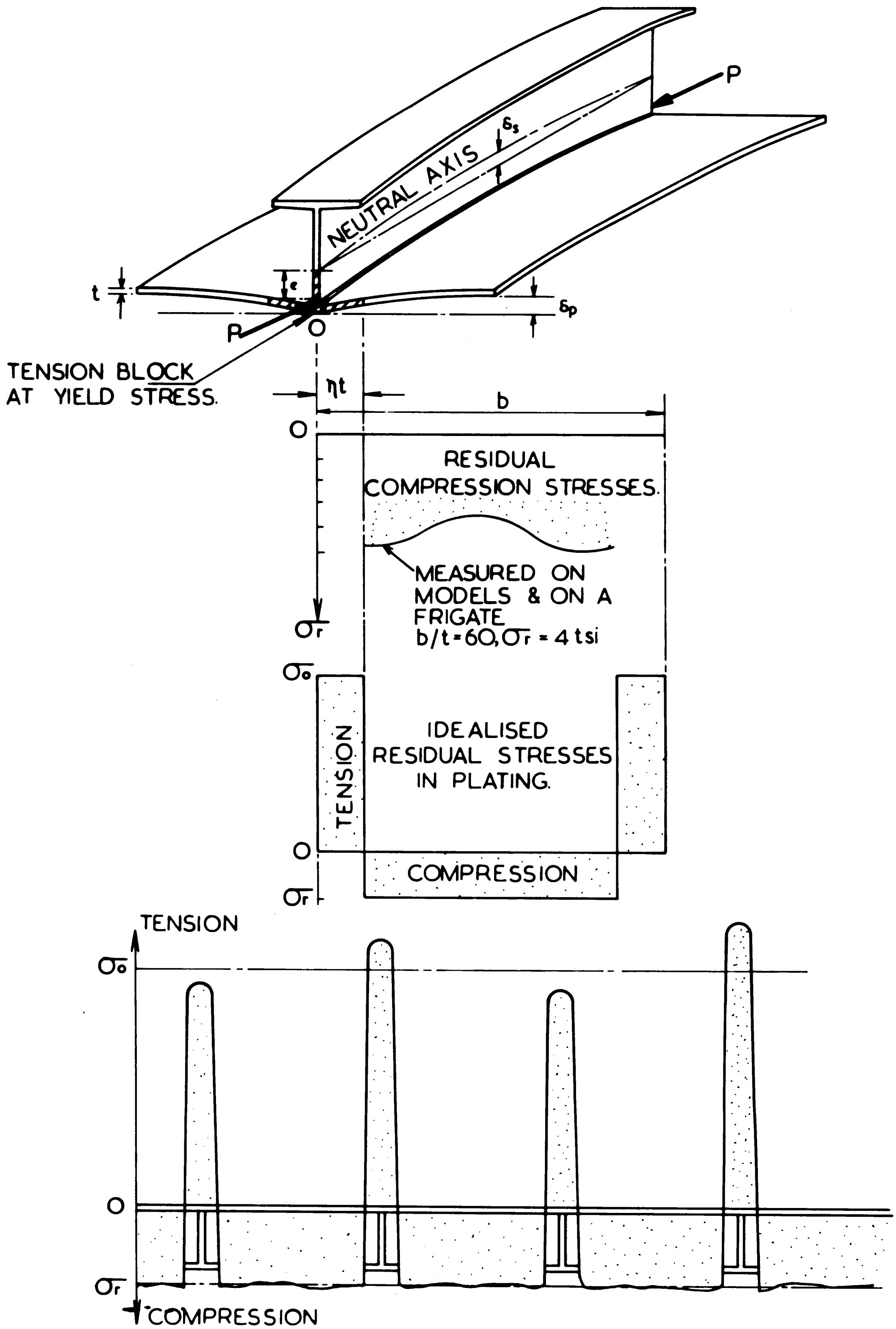
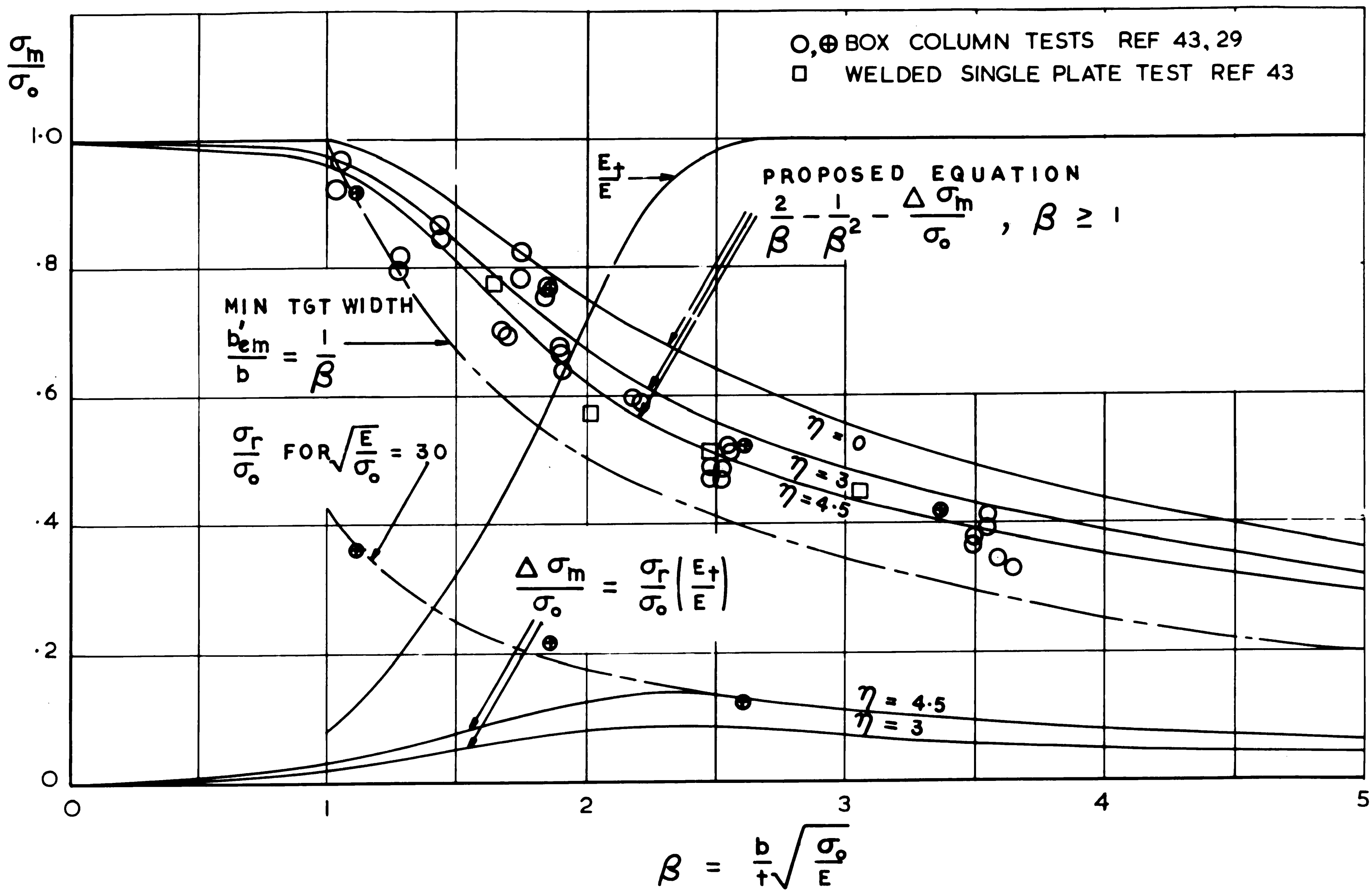

P
NEUTRAL AXIS
δ_s
t
δ_p
P
O
TENSION BLOCK
AT YIELD STRESS.
ηt
b
O
RESIDUAL
COMPRESSION STRESSES.
MEASURED ON
MODELS & ON A
FRIGATE
b/t = 60, σ_r = 4 tsi
σ_r
σ_o
TENSION
IDEALISED
RESIDUAL STRESSES
IN PLATING.
O
COMPRESSION
σ_r
TENSION
σ_o
O
σ_r
COMPRESSION

Figure 7



O, ⊕ BOX COLUMN TESTS REF 43, 29
□ WELDED SINGLE PLATE TEST REF 43
$\frac{\sigma_m}{\sigma_o}$
1·0
·8
·6
·4
·2
O
$\frac{E_t}{E}$
PROPOSED EQUATION
$\frac{2}{\beta} - \frac{1}{\beta^2} - \frac{\Delta \sigma_m}{\sigma_o}$, $\beta \geq 1$
MIN TGT WIDTH
$\frac{b'_{em}}{b} = \frac{1}{\beta}$
$\frac{\sigma_r}{\sigma_o}$ FOR $\sqrt{\frac{E}{\sigma_o}} = 30$
$\frac{\Delta \sigma_m}{\sigma_o} = \frac{\sigma_r}{\sigma_o}\left(\frac{E_t}{E}\right)$
$\eta = 0$
$\eta = 3$
$\eta = 4{\cdot}5$
$\eta = 4{\cdot}5$
$\eta = 3$
O
1
2
3
4
5
$\beta = \frac{b}{t}\sqrt{\frac{\sigma_o}{E}}$

Figure 8

largely in the plate. This equilibrium requirement provides a relationship between the magnitude of the compressive residual stress σ_r in the plating and the widths ηt of the tension zones each side of the weld

$$\frac{\sigma_r}{\sigma_o} = \frac{2\eta}{(b/t) - 2\eta} \qquad (21)$$

Reference 30 contains a method for estimating the design value of η for single pass welding taking stress shakedown into account. Values of $\eta = 4.5$ to 6 are typical for as-welded ships, but values of 3 to 4.5 are more appropriate for ship design after allowing for shakedown. Values of $\eta = 3$ to 4.5 also seem reasonable for most as-welded civil engineering structure, with proportionately lower values if intermittent welding is used.

Equation (21) is reasonably well supported for values of $\beta > 1$, as shown in Figure 8. The form of this expression indicates how rapidly σ_r increases with reducing b/t and, whereas (peak) values as high as 3/4 σ_o have been measured in fabricated test sections, typical ship values generally run around 1/4 σ_o.

Welding, besides introducing residual compressive stresses into the plate, also gives rise to initial distortions due to transverse shrinkage. In transversely framed ships the presence of the welding distortions is the much more severe of the two phenomena since such distortions may appreciably affect stiffness and lead to appreciable losses in plate strength. For longitudinally framed ships, however, in which our main interests lie, it is found in general that the main degradation in strength comes from the presence of the residual stresses σ_r rather than the welding distortions. As a consequence, no further discussion of such welding-induced distortions will be included here.

The main concern regarding the residual stress σ_r is the extent to which it reduces plate strength. Flat plate experiments conducted at Lehigh University, in Japan and in England have shown that in the elastic range the simple notion of reducing the long plate elastic buckling stress by σ_r is safe and reasonably accurate for σ_r values up to about 0.2 σ_o. The author has also confirmed that this result is also applicable to plate strength over the same range, namely, the reduction $\Delta\sigma_m = \sigma_r$. The range corresponds approximately to values of $\beta > 2$. At lower values of β, inelastic effects dominate and the following critical strain approach suggested by Becker [29] was proposed [30]. This approach advances two hypotheses:

1. The plate will buckle inelastically when the total compression strain in the middle zone, allowing for the presence of residual stresses, reaches a critical strain level which corresponds to the elastic buckling strain;
2. The reduction in inelastic buckling stress derived from the first hypothesis will also be the reduction in plate strength.

As well as being soundly based, this approach is also attractive because it shows good agreement with experiment. Providing σ_r is not too large, the reduction in plate strength is then conservatively given by

$$\Delta\sigma_m = (E_t/E)\sigma_r \qquad (22)$$

where the correct E_t to be used is the "structural tangent modulus" for the stiffened plate in compression. For flat-yield type materials the ratio E_t/E can be approximated using the Ostenfeld-Bleich quadratic parabolae [9], namely,

$$\frac{E_t}{E} = \frac{\sigma(\sigma_o - \sigma)}{\sigma_{ps}(\sigma_o - \sigma_{ps})} \tag{23}$$

where the structural proportional limit σ_{ps} in compression is approximately related to the material proportional limit σ_p by

$$\sigma_{ps} = \sigma_p - \sigma_r = p_r\sigma_o \tag{24}$$

and for which p_r is typically 0.5 in welded ships. For materials having more rounded stress-strain curves, equation (23) would be unsatisfactory. A formulation based on the Ramberg and Osgood three-parameter description of stress-strain curves [132] has been derived [30] and is preferred for such materials.

By assuming that the inelastic plate buckling stress σ_P is given by

$$\sigma_P = \sqrt{E_t/E}\ \sigma_{PE} \tag{25}$$

where σ_{PE} is given by equation (2) and is the elastic flat plate buckling stress, it can be shown that for a pinned plate, equation (2) with k = 4 combined with equation (23) with $\sigma = \sigma_P$ leads to the following expression for E_t/E.

$$\frac{E_t}{E} = \left[\frac{3.62\beta^2}{13.1 + p_r(1 - p_r)\beta^4}\right]^2 \qquad 0 \le \beta \le \frac{1.9}{\sqrt{p_r}} \tag{26a}$$

$$= \quad 1.0 \qquad \beta \ge \frac{1.9}{\sqrt{p_r}} \tag{26b}$$

This is plotted in Figure 8. Hence, from equation (22) the following plate strength equation was proposed for welded <u>pinned</u> plates:

$$\frac{\sigma_m}{\sigma_o} = \frac{b_{em}}{b} = \begin{cases} \dfrac{2}{\beta} - \dfrac{1}{\beta^2} - \dfrac{\sigma_r}{\sigma_o}\dfrac{E_t}{E} & \beta \geq 1 \quad (27a) \\[2ex] 1 - \dfrac{\sigma_r}{\sigma_o}\dfrac{E_t}{E} & 0 \leq \beta < 1 \quad (27b) \end{cases}$$

where σ_r is defined according to equation (21) and E_t/E is given by equations (26). This is plotted for two values of tension block width η in Figure 8. It will be seen that agreement with test data is good.

By rearranging equations (27) as follows:

$$\frac{\sigma_m}{\sigma_o} = \frac{b_{em}}{b} = \begin{cases} \left[\dfrac{2}{\beta} - \dfrac{1}{\beta^2}\right] R_r & \beta \geq 1 \quad (28a) \\[2ex] R_r & 0 \leq \beta < 1 \quad (28b) \end{cases}$$

the quantity R_r, the strength reduction ratio caused by residual stresses, is defined. For a pinned plate R_r is obviously given by

$$R_r = \begin{cases} 1 - \dfrac{\sigma_r}{\sigma_o}\dfrac{E_t}{E}\dfrac{\beta^2}{2\beta - 1} & \beta \geq 1 \quad (29a) \\[2ex] 1 - \dfrac{\sigma_r}{\sigma_o}\dfrac{E_t}{E} & 0 \leq \beta < 1 \quad (29b) \end{cases}$$

R_r represents directly the reduction plate strength due to welding. For $\eta = 3$ and 4.5 values of $(1 - R_r)$ expressed as percentages are provided in the table:

β	1	1.5	2	2.5	3	4	5
$\eta = 3$	1.8	5.5	10.7	13.3	12.8	12.1	11.7
$\eta = 4.5$	3.2	9.0	16.9	20.9	20.2	18.5	17.8

It will be seen that the percentage degradation in plate strength increases until about $\beta = 2.5$; at this and higher values of β the effects are purely elastic and the percentage degradation is appreciable and approximately constant. At low values of β the effects are small due to yielding effects, as confirmed by test results.

Reduced Effective Widths

For use in predicting stiffener-plate collapse (and in evaluating the average load at failure), it is necessary to consider events before failure when $\sigma_e < \sigma_o$. This has already been considered for the stress-free plate where equation (14) was proposed for b_e and equation (15) derived for b_e' by differentiation. The reduction factor R_r has just been defined for considering the effect of residual stresses on plate strength. While the effects of residual stresses at intermediate loads are complex, a simple but intuitively attractive approach is to assume that the derived residual stress reduction factor R_r can be directly applied to equations (14) and (15) to provide b_e and b_e' when $\sigma_e < \sigma_o$ and residual stresses are present. Thus, using the term $\beta_e = \beta\sqrt{\sigma_o/\sigma_e}$

$$\frac{b_e}{b} = \begin{cases} \left[\dfrac{2}{\beta_e} - \dfrac{1}{\beta_e^2}\right] R_r & \beta_e \geq 1 \quad (30a) \\ R_r & 0 \leq \beta_e < 1 \quad (30b) \end{cases}$$

$$\frac{b_e'}{b} = \begin{cases} \dfrac{1}{\beta_e} R_r & \beta_e \geq 1 \quad (31a) \\ R_r & 0 \leq \beta_e < 1 \quad (31b) \end{cases}$$

The same range for σ_e applies as before, viz., $0.7\sigma_o \leq \sigma_e \leq \sigma_o$.

When $\sigma_e = \sigma_o$ the expressions for b_{em} and b'_{em} are obtained, and are plotted in Figure 8.

Clamped Plates

In the case where the longitudinal stiffeners are torsionally strong or where the lateral pressure is sufficiently large, the use of clamped edges may be more appropriate, as discussed for the stress-free plate. By a similar analysis to that just outlined above for the pinned plate, the following expressions for a clamped plate residual stress reduction factor R_r may be derived.

$$\frac{E_t}{E} = \begin{cases} \left[\dfrac{6.31\beta^2}{39.8 + p_r(1 - p_r)\beta^4}\right]^2 & 0 \leq \beta \leq \dfrac{2.51}{\sqrt{p_r}} \quad (32a) \\ 1.0 & \beta > \dfrac{2.51}{\sqrt{p_r}} \quad (32b) \end{cases}$$

$$R_r = \begin{cases} 1 - \dfrac{\sigma_r}{\sigma_o}\dfrac{E_t}{E}\dfrac{\beta^2}{2.5\beta - 1.5625} & \beta \geq 1.25 \quad (33a) \\ 1 - \dfrac{\sigma_r}{\sigma_o}\dfrac{E_t}{E} & 0 \leq \beta < 1.25 \quad (33b) \end{cases}$$

where σ_r is still defined by equation (21). Equations (33) should be applied directly to equations (17) to (20) presented for the stress-free clamped plate to obtain effective widths and reduced effective widths for the welded clamped plate.

Test Data

The above theory for the strength of welded pinned plates was presented in Figure 8 along with all known reputable test data from about 40 welded-box columns. These data range in scale from tests on large-scale columns based on the design of the Forth Road Bridge [59], intermediate scale columns tested at

Cambridge University [44,45,46], and very small-scale but nevertheless high quality tests carried out by Becker and his associates for the U.S. Ship Structure Committee [29]. Dwight and Moxham compiled all data except Becker's in reference 46.

The author has also tested more than 40 welded Tee-stiffened plate strut-panels in compression [60] and, while these results cannot be directly plotted on Figure 8, it can be said that the support for the underlying theory is good. It will be seen that the difference in strength between welded and stress-free plates is quite marked.

Variance

The quality of the Cambridge test data (the bulk of that presented) is very good, and certainly better than may be expected in as-built production structures. For this reason it is not possible to arrive directly at standard deviations to be used in assessing the strength distributions of welded plate or stiffened plate structures. Nevertheless, the author has applied linear error theory to the derived strength formulations and has found that, despite the high variability in the welding processes themselves, the overall effect of these welding variations in broadening the strength variance is not marked [56]. This is because the welding variations act only on a relatively small "subtraction" term, equation (22), in the overall strength formulations.

Indeed, in one sense, it may be argued that, since the welding of stiffened panels induces a predominantly $m = 1$ initial distortion mode into the plate elements, then for long plate ($a >> b$) these distortions may actually stiffen and strengthen the plate elements by reducing the membrane actions and by reducing the amplitudes of the more harmful m = a/b components of initial distortion.

The final broad conclusion might therefore be that when assessments of overall variance are based on the variances observed in individual plate tests (e.g., equations (13b)), then, provided these plate tests include data from a range of initial distortions expected from the welding conditions, the overall variance expected in welded plate element behavior may only be marginally increased. The initial plate element distortions covered by the plate tests reported in general cover the practical range of welding distortions and so equations (13b) themselves might suffice also for the as-welded plate strength variances. However, to add a little conservatism, the following equations are suggested.

$$v = 0.06 \text{ for } 0.5 \leq \beta \leq 2.5$$

$$v = 0.024\beta \text{ for } \beta > 2.5 \qquad (34)$$

The expected overall variance for the complete structure can then be assessed applying linear error theory to the overall collapse formulations, as illustrated in reference 56.

APPENDIX

BRIEF HISTORICAL REVIEW

Ninety years ago, Box [61] may well have been the first to propose a plate strength equation, even before Bryan's classical paper in 1891 on plate buckling [62]. It is such a reasonable fit for the mild steel plates for which it was intended that, by making assumptions concerning material properties, Box's equation can be generalized to:

$$\frac{\sigma_m}{\sigma_o} \simeq \frac{1}{\sqrt{\beta}}$$

Its form clearly anticipates many later equations (the more important ones are tabled at the end of the appendix), and could indicate extraordinary intuition on Box's part.

Since then the subject has progressed in three overlapping phases representing in turn marine, aeronautical and civil engineering interests in stiffened panels.

Pietzker's early intuition [63] was followed by a dazzling display of mainly German papers in the twenties and thirties. A series of theoretical papers appeared, generally under the title "Die Mittragende Breite" [64,65,66,67,68,19] following the earlier work of the Hungarian, Von Kármán [69,124]. The German naval architect, Schnadel, continued his investigations for nearly a decade, and provided the first post-buckling approximation. Marguerre treated this problem more generally, avoiding many arbitrary assumptions in his one outstanding paper [19]. His expressions are still widely used in aircraft engineering, but have proved to be too optimistic for imperfect steel ship plating. The publications of the sadly neglected pair, Metzer [67] and Miller [68], contain most of the formulae important to the subject. Föppl, Ritz and Chwalla were also working in closely allied fields.

This Teutonic talent was matched in the thirties and early forties only in America by engineers sponsored by NACA [70,71,72, 73,25]. Levy [72] developed the classical "exact" solution of Marguerre's equations, confirming his results. NACA and the USN Experimental Model Basin also rendered a great service in translating the more important German papers. Sweeney, Vasta and Frankland [74,75,54] at the EMB conducting tests on about 100 steel plates laid the foundations for the U.S. Navy's present design codes [76]. This has been followed with more recent work [72,42,39]. Bengston, following the Schnadel-Marguerre trail in a notable paper before the Society just before the war [78], still has a strong following in the United States, though his work contains certain contradictions and is considered doubtful by Bleich [9]. Civil engineering in America seems generally to have been willing to make use of the aeronautical research, and extended it for light-gauge steel use. Winter at Cornell may perhaps be quoted as the leading personality [35,36,48,49]. This has led to design codes [79,80,81,53,93] and was supported by a useful review for the Column Research Council [82]. The best aeronautical reviews in the United States have been made by Hoff [33], Levy et al. [72,73] and Gerard [83,14], while Bleich [9] and Timoshenko [2] serve everyone's interest admirably and were used extensively for this present review. Reference 84 provides probably an older ship review, overtaken by reference 40.

Britain's contribution has always been relatively small. After the WOLF experiment [85,86] the lead was taken by the

Aeronautical Research Council [20,87,88] and the Royal Aeronautical Society [89,90]. H. L. Cox's 1946 paper [20] must now be regarded as a classic, and he has done more than anyone in the United Kingdom to make the fruits of aeronautical research available to naval architects [91]. Civil engineering interests have recently been served mainly by the universities, notably University College, London [12,47,50,92], Strathclyde [13], and more recently some excellent work, including the effect of welding residual stresses at Cambridge [15,17,18,41,43,44,45]. This has led directly to design codes [11,94]. Mention must also be made of an early paper by the inimitable G. I. Taylor [95]. Except for Murray [21,22,84,96], British naval architects have on the whole been less active [7,97,129,130], especially with experimental work where they have been content to watch activities in America. This relative inactivity has perhaps led to confusion in some quarters to the extent that 30t is still used for bending--almost certainly a legacy from Hovgaard [98] which even reference 1 has not completely dispelled.

The Russians were also very active in this field, including Timoshenko during his early work at Kiev [99,100]. He was the first to derive the expressions for the strain energy of plates with large deflection, which formed the basis of much of the later work elsewhere. This last decade has seen notable Russian research in residual stress effects [101,102,103]. The National Aeronautical Laboratory, Amsterdam, has made notable contributions in the theoretical [104] and experimental [38,105] treatment of the elastic post-buckling load-shortening behavior of long plates. It was on the strength of this Dutch work, and more or less corroborated by ASCE work [105], that the Fritz Engineering Laboratory, Lehigh, concluded [40] that the Koiter equation can be used with confidence, even in the inelastic range, for long plates having a b/t ratio less than 120. This forms the basis of much of FEL's work since then [107,108] which includes the effect of residual welding stresses. Japan has been well represented at FEL [109,110,111] where her main effort as at home [112,113,114] has been on the effects of welding stresses on elastic buckling and on inelastic buckling and collapse. Japanese naval architects have not only been the quickest to recognize the importance of residual stresses, but are now able to apply their work in design.

Gerard has already been mentioned for his work following Stowell [128] on the stability of plates beyond the elastic limit. The main burst of inelastic aeronautical activity was in the late forties and early fifties [115,116,117,118,119] using modern theories of plasticity-deformation theory and Prandtl-Reuss' incremental theory. A little later, a plastic buckling theory was proposed for steel plates [120,121].

Since the WOLF experiment [85] three other destroyers had their backs broken in drydock, and the results have been conveniently summarized [57], as discussed in the main body of this review. The Admiralty Ship Welding Committee reports should be mentioned [7] as containing a wealth of statistical data on plate deflections and effectiveness, but, alas, most of it has not been analyzed or assessed. Other reports which can be recommended for their experimental skill with edge supports and in measuring plate stiffness are references 15, 17, 38 and 105. The

Cambridge work was particularly successful in arresting the collapse and in allowing unhindered shortening of the plate at the unloaded edges. There is a dearth of data on load-shortening behavior. The experimental techniques available for measuring effective plating are outlined in reference 8 and the experimenter may find references 73, 122 and 123 helpful.

For convenience, a table of the more important effective width formulae for stress-free plate elements follows this appendix.

TABLE OF MORE IMPORTANT EFFECTIVE WIDTH FORMULAE

Listed in approximate chronological order

DERIVATION	$\frac{b_{em}}{b} = \frac{\sigma_m}{\sigma_o}$ FORMULAE		REMARKS
	Based on $\beta = \frac{b}{t}\sqrt{\frac{\sigma_o}{E}}$ slenderness ratio	Based on $\frac{\sigma_{PE}}{\sigma_o}$ $(= \frac{3.62}{\beta^2}$ for pinned plates)	
Box [61]	$\sigma_m = 80/\sqrt{b/t}$ tsi $= \sigma_o/\sqrt{\beta}$ (see note)	$.725\sqrt[4]{\frac{\sigma_{PE}}{\sigma_o}}$	Probably the earliest recorded 1883. Note: As generalised in the Appendix.
Pietzker [63]	$40\,\frac{t}{b}$		Steel plates only
Schnadel [66]	$0.5 + \frac{1.81}{\beta^2}$	$0.5(1 + \frac{\sigma_{PE}}{\sigma_o})$	Long pinned plates
		$\frac{1}{3}(1 + \frac{2\sigma_{PE}}{\sigma_o})$	Wide pinned plates
Von Karman* [10]	$\frac{1.9}{\beta}$	$\sqrt{\frac{\sigma_{PE}}{\sigma_o}}$	Optimistic at low β
Sechler* [93]	$\frac{0.77}{\beta^{0.74}} + \frac{2.92}{\beta^{2.74}}$	$0.5(\frac{\sigma_{PE}}{\sigma_o})^{0.37}(1 + \frac{\sigma_{PE}}{\sigma_o})$	
Timoshenko [99,100]	$0.434 + \frac{2.05}{\beta^2}$	$0.434 + 0.566(\frac{\sigma_{PE}}{\sigma_o})$	Square pinned plates

DERIVATION	$\frac{b_{em}}{b} = \frac{\sigma_m}{\sigma_o}$ FORMULAE		REMARKS
	Based on $\beta = \frac{b}{t}\sqrt{\frac{\sigma_o}{E}}$ slenderness ratio	Based on $\frac{\sigma_{PE}}{\sigma_o}$ ($= \frac{3.62}{\beta^2}$ for pinned plates)	
Cox* [87]	$0.19 + \frac{1.54}{\beta}$	$0.19 + 0.81\sqrt{\frac{\sigma_{PE}}{\sigma_o}}$	Thin flat alloy plates, RAS Data Sheets 02.01.17 and 02.01.03
Marguerre* [19]	$\frac{1.535}{\beta^{2/3}}$	$\sqrt[3]{\frac{\sigma_{PE}}{\sigma_o}}$	Fits aeronautical data at high β values
	$0.5 + \frac{1.81}{\beta^2}$	$0.5(1 + \frac{\sigma_{PE}}{\sigma_o})$ Sides free to move transversely.	Square simply supported plates — often used for long plates. It is optimistic for ship plates.
		$\frac{1}{3+\mu}(2 + \frac{\sigma_{PE}}{\sigma_o})$ Sides do not move.	
Bengston [78]		For pinned plates: $0.483 + 0.517\frac{\sigma_{PE}}{\sigma_o}$ for 'long' plate. $0.4 + 0.6\frac{\sigma_{PE}}{\sigma_o}$ for 'square' plate. $0.318 + 0.682\frac{\sigma_{PE}}{\sigma_o}$ for 'wide' plate $\frac{a}{b} = \frac{1}{2}$	Similar to Schnadel and Marguerre, but slightly less optimistic at high β values. Analysis is considered doubtful by Bleich, and certainly too optimistic for ship use.

DERIVATION	$\frac{b_{em}}{b} = \frac{\sigma_m}{\sigma_o}$ FORMULAE		REMARKS
	Based on $\beta = \frac{b}{t}\sqrt{\frac{\sigma_o}{E}}$ slenderness ratio	Based on $\frac{\sigma_{PE}}{\sigma_o}$ (= $\frac{3.62}{\beta^2}$ for pinned plates)	
Bengston [78] (Cont'd)		For clamped plates: the coeffs are 0.6 and 0.4 'square' plates, and 0.5 and 0.5 'wide' plates.	
Frankland [54] (Sweeney and Vasta contributed significantly refs. 74 and 75)	$\frac{2.25}{\beta} - \frac{1.25}{\beta^2}$	$1.19 \frac{\sigma_{PE}}{\sigma_o} - 0.35 \frac{\sigma_{PE}}{\sigma_o}$	Used by U. S. Navy
Koiter [104]	$1.2\ S^{2/5} - 0.65\ S^{4/5} + 0.45\ S^{6/5}$ where $S = \varepsilon_{PE}/\varepsilon_e$		For all edge conditions
Winter* [35,48]	$\frac{1.9}{\beta} - \frac{0.9}{\beta^2}$	$\sqrt{\frac{\sigma_{PE}}{\sigma_o}}\left(1 - \frac{1}{4}\sqrt{\frac{\sigma_{PE}}{\sigma_o}}\right)$	Follows lines pioneered by Sechler [93] and based on tests with thin steel. Very reasonable for stress free plates.
Conley et al [42]	$\frac{1.82\sqrt{k}}{\beta} - \frac{0.82}{\beta^2}$ where $k = \frac{\sigma_{oc}}{\sigma_{ot}}$ ratio of compression and tension yield stresses	$0.96\sqrt{\frac{\sigma_{PE}}{\sigma_{oc}}} - 0.23\frac{\sigma_{PE}}{\sigma_{oc}}$	Good fit for steel and aluminum alloy plates. Not yet adopted by U. S. Navy. More conservative than Frankland equation.

DERIVATION	$\frac{b_{em}}{b} = \frac{\sigma_m}{\sigma_o}$ FORMULAE Based on $\beta = \frac{b}{t}\sqrt{\frac{\sigma_o}{E}}$ slenderness ratio	Based on $\frac{\sigma_{PE}}{\sigma_o}$ $(= \frac{3.62}{\beta^2}$ for pinned plates)	REMARKS
Bleich [9]		$\frac{1+\xi}{1+3\xi^4} + \frac{2\xi^4}{1+3\xi^4}\frac{\sigma_{PE}}{\sigma_o}$ $= \frac{1}{3}(1+2\frac{\sigma_{PE}}{\sigma_o})$ when ξ is large ('wide' plate) $\xi = \frac{\lambda}{b}$ λ = wave length	Extension of Marguerres theory for square plates to cover wide plates. Sides assumed free transversely. Agrees with Schnadels formula for 'wide' plates.
Chilver et al [12,13]	$\frac{1.13}{\beta^{2/3}}$	$0.736\ \sqrt[3]{\frac{\sigma_{PE}}{\sigma_o}}$	Light gauge steel channels. Suitable for stress free plates.
BS 449 [94]	$\frac{1}{\beta^{2/3}}$	$0.65\ \sqrt[3]{\frac{\sigma_{PE}}{\sigma_o}}$	Lower scatter boundry. Found suitable for welded plates by Chilver et al.[13].
AISC (1961) [79]	$\frac{1.69}{\beta}$	$0.89\sqrt{\frac{\sigma_{PE}}{\sigma_o}}$	
Gerard [14,83]	$\frac{c}{\beta^{0.85}}$ c = 1.42 pinned c = 1.80 clamped	$0.824(\frac{\sigma_{PE}}{\sigma_o})^{0.425}$	Claimed to fit all the relevant aeronautical data within ± 10% for many materials.

DERIVATION	$\frac{b_{em}}{b} = \frac{\sigma_m}{\sigma_o}$ FORMULAE Based on $\beta = \frac{b}{t}\sqrt{\frac{\sigma_o}{E}}$ slenderness ratio	Based on $\frac{\sigma_{PE}}{\sigma_o}$ $\left(= \frac{3.62}{\beta^2}\right.$ for pinned plates)	REMARKS
Winter* [53]	$\frac{1.9}{\beta} - \frac{0.79}{\beta^2}$	$\sqrt{\frac{\sigma_{PE}}{\sigma_o}}\left(1 - 0.22\sqrt{\frac{\sigma_{PE}}{\sigma_o}}\right)$	This appears to be a little more optimistic than previous expression used for cold-formed steel structural members, but only because this expression is now recommended for use at the working load and not at failure.
Dwight [43] & BS 153 [11]	$\frac{1.65}{\beta}$	$0.87\sqrt{\frac{\sigma_{PE}}{\sigma_o}}$	Tentative suggestion for new UK design rules for "as rolled" steel plates. For "welded" plates σ_m is reduced and Dwight's treatment is preferred.
Becker [29]			
Faulkner* [51,30]	$\frac{2}{\beta} - \frac{1}{\beta^2}$	$1.05\sqrt{\frac{\sigma_{PE}}{\sigma_o}} - 0.28\,\frac{\sigma_{PE}}{\sigma_o}$	Proposed in 1965 [51] for ship use, confirmed in 1973 as the best mean curve derived from existing reliable data.
	mean strength with coefficient of variation $v = 0.05$ for $\beta \leq 2.5$, and $v = 0.02\beta$ for $\beta > 2.5$		

*These authors proposed the use of their formulae when the edge stress was less than the yield stress by replacing σ_o by σ_e. The use of the edge stress is important in strain and hence stiffness considerations when estimating the collapse of the stiffeners which support the edges of the plate elements.

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