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THE HYDRODYNAMIC INTERACTION BETWEEN TWO CYLINDRICAL BODIES FLOATING IN BEAM SEAS

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by C. H. Kim

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ABSTRACT

An analysis is given of both wave- and motion-induced forces and moments on the individual bodies of rigidly connected twin cylinders performing three modes of motion in beam seas. The diffracted and radiated waves are evaluated and by the Haskind method the wave-exciting forces are determined.

Furthermore, an anlysis of the hydrodynamic interactions between two different cylindrical bodies floating freely in beam seas is also given and their relative heaving motions are evaluated. A description is given of the application of the strip method in evaluating the hydrodynamic forces and moments as well as the response motions of a twin-hull ocean platform in beam seas.

KEYWORDS

Hydrodynamic Interaction

Diffraction

Radiation

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NOMENCLATURE

A_w	waterplane area
$A_{\pm}^{(m)}$	complex radiated wave amplitude ratio
$A_{\pm}^{(o+e)}$	diffracted wave amplitude ratio
a	incident wave amplitude
B	beam or restoring force (moment) coefficient
C	two-dimensional added mass coefficient
C_{aa}, etc.	added mass coefficient defined in Eq.(29)
C_{SS}, etc.	added mass coefficient defined in Table 1
c	section contour
D	diameter
$D_{\pm}$	constant defined in Eq.(14)
d	depth
F	force or moment
F_{Si}, etc.	hydrodynamic force or moment defined in Table 1
$F^{(m)}$, etc.	wave-exciting force (moment) derived from the radiation of mode m, defined in Eq.(A-17)
$F_S^{(\beta)}$, etc.	wave-exciting force defined in Table 4
F_{ζ}, etc.	wave-exciting force defined in Eq.(24)
$f_S^{(\beta)}$, etc.	non-dimensional exciting force defined in Table 4
$f_{S\pm}$, etc.	non-dimensional exciting force defined in Eq.(18)
f_{Ha}, etc.	non-dimensional heave-exciting force defined in Eq.(29)
G	center of gravity
G	Green's function (source potential)

g	gravitational constant
h	wave elevation
I	moment of inertia
ℓ	hydrodynamic moment arm
ℓ_{SS_r}, etc.	hydrodynamic moment arm defined in Table 2
K	mean wave force or constant defined in Eq.(14)
L	constant defined in Eq.(14)
M	integer, moment or inertial mass
$M_{\eta\eta}$, etc.	inertial mass, moment or moment of inertia defined in Eq.(22)
m	number of mode of motion or sectional mass
m''	two-dimensional added mass
m''_{aa}, etc.	two-dimensional added mass defined in Eq.(27)
m''_{SS}, etc.	two-dimensional added mass defined in Table 1
N	integer or two- or three-dimensional damping coefficient
N_{aa}, etc.	damping coefficient defined in Eq.(27)
N_{SS}, etc.	damping coefficient defined in Table 1
$N_{\eta\eta}$, etc.	damping coefficient defined in Eq.(22)
O	origin of the coordinate system
p	hydrodynamic pressure
Q	source intensity
r	field point
S	amplitude of displacement, surface or spacing
$S_{-\infty}$, etc.	surface at $y \to -\infty$, etc.
s	length of contour, segment or spacing
T	draft of hull or period
t	time

x,y,z	Cartesian coordinate system
X,Y,Z	components of force

SUBSCRIPTS

a	indicating body a
b	indicating body b
D	indicating diffraction
f	indicating force
H	indicating heave or heaving force
h	indicating wave
I	indicating incident wave
i	$\sqrt{-1}$, or indicating the imaginary or hydrodynamic damping part
j	indicating j^{th} segment
k	indicating k^{th} segment
o	indicating origin
r	indicating the real or the hydrodynamic inertial part or relative motion
S	indicating swaying
R	indicating rolling
W	indicating waterplane or waterline
±	indicating $y \to \pm\infty$

SUPERSCRIPTS

e	indicating even function
o	indicating odd function
m	indicating mode of motion

GREEK LETTERS

α	slope of a segment
β	suffix standing for e, o or (e+o)
γ	suffix standing for m for radiation and β for diffraction
∇	volume of hull displacement
δ	non-dimensional damping coefficient
ϵ	phase angle
φ	velocity potential, or rolling motion or suffix indicating roll
λ	wave length
ν	wave number
ρ	water density
χ	yawing motion or suffix denoting yawing
ψ	pitching motion or suffix denoting pitching
η	the y-coordinate of source distribution, swaying motion or suffix denoting swaying
ζ	the z-coordinate of source distribution, heaving motion or suffix denoting heaving
ω	circular frequency

INTRODUCTION

An investigation has been conducted at Stevens Institute of Technology into some problems of the hydrodynamic interaction between two bodies floating in beam seas: 1) the hydrodynamic loadings on individual members of rigidly connected twin cylinders performing three modes of motion in beam seas, and 2) the hydrodynamic interaction and relative heaving motions of two different cylindrical hulls floating freely in close proximity in beam waves. Both problems have not been treated before.

Ohkusu[1,2] has evaluated theoretically the hydrodynamic forces and moments on two or more cylinders heaving, swaying and rolling on a calm water surface. Ohkusu and Takaki[3] have applied these analyses to evaluate the motions of multi-hull ships in waves. Wang and Wahab[4] have also studied the hydrodynamic forces on twin heaving cylinders on a calm water surface.

These investigators used the method of multi-pole expansion to determine the unknown velocity potential. In these analyses, the individual section must be symmetrical about its own vertical midplane and the two cylinders must be identical.

Lee, Jones and Bedel[5] have reported theoretical and experimental evaluations of the hydrodynamic forces on twin heaving cylinders on a calm water surface. Their theoretical analysis followed the method of source distribution over the immersed contours[6] of the cylinders. Consequently, cylinders of arbitrary cross-section not necessarily symmetric about their vertical midplanes can be dealt with. This investigation also assumed the two cylinders to be symmetrically disposed with respect to each other.

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[1]Superior numbers in text matter refer to similarly numbered references listed at the end of this report.

All these investigations[1-5] of multi-hull cylinders assumed that the cylinders are rigidly connected.

The present study also applies the close-fit source distribution method pioneered by Frank.[6] We assume two arbitrarily shaped bodies, which do not have to be either symmetrical about their vertical midplanes or to be symmetrically disposed with respect to each other. Furthermore, the two bodies may be unconnected or connected either rigidly or elastically.

This investigation, based on two-dimensional linearized theory, considers both the radiation and diffraction problems for two arbitrarily shaped cylindrical bodies floating in a train of beam waves; the hydrodynamic inertial and damping forces and moments due to swaying, heaving and rolling of the cylinders on a calm water surface and the forces and moments induced by beam waves on the fixed cylinders are evaluated.

Brief descriptions will be given of the methods used to evaluate the unknown velocity potentials for the radiation and diffraction problems. Since the fundamental velocity potential of a two-dimensional pulsating source of unit intensity located below the free surface and satisfying the required hydrodynamic conditions inside and on the boundaries of the entire deep-water domain is a well-known solution stated by Wehausen and Laitone,[7] it is only necessary to discuss here the kinematical boundary conditions on the body contours.

Two applications of the theory will be considered in detail: 1) the analysis of the hydrodynamic forces on two rigidly connected cylinders, and 2) the relative heaving motions of two unconnected bodies in close proximity. Results of calculations for these cases will be presented and discussed.

The present investigation was partially supported by the National Oceanic and Atmospheric Administration, Office of Sea Grant; Stevens Institute of Technology Project 3841/214.

KINEMATIC BOUNDARY CONDITIONS

THE RADIATION PROBLEM

Consider two arbitrarily shaped parallel cylinders oscillating in prescribed (arbitrary) modes of motion, on or below the calm water surface, with given amplitudes and phases in the form

$$[s]_a = s_a^{(m_a)}\, e^{-i\omega t}$$
$$[s]_b = s_b^{(m_b)}\, e^{-i(\epsilon_{s_a s_b} + \omega t)} \qquad (1)$$

where

$s_a^{(m_a)}$, $s_b^{(m_b)}$ = amplitudes of displacement in the mode of motion m_a and m_b, respectively (m_a or m_b = 2,3,4 corresponding to sway, heave, roll)

$\epsilon_{s_a s_b}$ = phase difference between the motions of bodies a and b

For a certain kind of problem, such as the relative heaving motions between adjacent bodies, it may be convenient to refer the phases of the motions to the wave-exciting force.

The space coordinate system is defined in Figure 1a: the y-axis lies on a calm water surface, the z-axis points vertically upward, and the origin 0 is taken at the midpoint between the two walls of a and b .

The body contours of a and b are approximately represented by polygons with a finite number of segments S_j . A pulsating source of unknown strength is uniformly distributed on each segment to represent the flow induced by the motions of the two bodies. The velocity potential for the source of unit strength at (η,ζ) may be written as:[7]

$$G(y,z;\eta,\zeta)\,e^{-i\omega t} \qquad (2)$$

where (y,z) is the coordinate of a field point. The resultant velocity potential is represented as a sum of all of the discrete source segments of the polygonal approximation to the wetted contours of a and b,

$$\varphi(y,z)^{(m_a,m_b,\epsilon_{S_aS_b})} = \sum_{j=1}^{N} Q_j \int_{S_{j_a}} G(y,z;\eta,\zeta)\,dS + \sum_{k=1}^{M} Q_k \int_{S_{k_b}} G(y,z;\eta,\zeta)\,dS \qquad (3)$$

where S_{j_a}, S_{k_b} = j^{th} and k^{th} polygonal segments of a and b, respectively, where the orientation of the segment is in accordance with Reference 6

Q_j, Q_k = (uniform) complex source intensity of the j^{th} and k^{th} polygonal segments of a and b, respectively

and the time dependent factor $e^{-i\omega t}$ is omitted from hereon.

The unknown source intensities Q are determined numerically, satisfying the kinematic boundary conditions,

$$\frac{\partial\varphi}{\partial n}(y_a,z_a)^{(m_a,m_b,\ \epsilon_{S_aS_b})} = -i\omega u_n^{(m_a)}(y_a,z_a)$$

$$\frac{\partial\varphi}{\partial n}(y_b,z_b)^{(m_a,m_b,\ \epsilon_{S_aS_b})} = -i\omega u_n^{(m_b)}(y_a,z_a)e^{-i\epsilon_{S_aS_b}}$$

with

$$u_n^{(m_a)}(y_a,z_a) = \sin\alpha_k \qquad \text{for } m_a = 2$$
$$= -\cos\alpha_k \qquad 3$$
$$= -(y_a\cos\alpha_k + z_a\sin\alpha_k) \qquad 4$$

α_k = angle of the k^{th} segment S_k with respect to the y-axis for body a, and similarly for body b. The normal velocity $\frac{\partial\varphi}{\partial n}$ in Eq.(4) is taken to be the velocity induced on the k^{th} segment by all of the other segments.

Three special cases of interest may be mentioned. If the bodies a and $\underline{b}$ are rigidly connected twin cylinders, Eq.(4) takes the form

$$\frac{\partial\varphi^{(m)}}{\partial n}(y_a,z_a) = -i\omega u_n^{(m)}(y_a,z_a)$$
$$\frac{\partial\varphi^{(m)}}{\partial n}(y_b,z_b) = -i\omega u_n^{(m)}(y_b,z_b) \tag{4a}$$

while if body $\underline{a}$ is oscillated while body $\underline{b}$ is fixed, Eq.(4) reduces to

$$\frac{\partial\varphi^{(m)}(y_a,z_a)}{\partial n} = -i\omega u_n^{(m)}(y_a,z_a)$$
$$\frac{\partial\varphi^{(m)}(y_b,z_b)}{\partial n} = 0 \tag{4b}$$

and if body $\underline{a}$ is fixed while $\underline{b}$ is oscillated, Eq.(4) reduces to

$$\frac{\partial\varphi^{(m)}}{\partial n}(y_a,z_a) = 0$$
$$\frac{\partial\varphi^{(m)}}{\partial n}(y_b,z_b) = -i\omega u_n^{(m)}(y_b,z_b) \tag{4c}$$

THE DIFFRACTION PROBLEM

Consider an incident wave

$$h = ae^{i\nu y} \tag{5}$$

where

a = wave amplitude

ν = wave number

and the time factor $e^{-i\omega t}$ is omitted in this and subsequent sections. This wave encounters the fixed arbitrarily shaped bodies and is diffracted.

The velocity potential corresponding to the incident wave is

$$\varphi_I^{(o+e)} = -\frac{iga}{\omega} e^{\nu z} e^{i\nu y} \tag{6}$$

which can be expressed as odd and even functions of y ,

$$\varphi_I^{(o)} = \frac{ga}{\omega} e^{\nu z} \sin\nu y$$
$$\varphi_I^{(e)} = -\frac{iga}{\omega} e^{\nu z} \cos\nu y \tag{7}$$

The odd function $\varphi_I^{(o)}$ corresponds to the part of the flow which is asymmetric about the z-axis, while the even function $\varphi_I^{(e)}$ corresponds to the symmetric flow.

The flows represented by the potentials $\varphi_I^{(o)}$ and $\varphi_I^{(e)}$ are disturbed in encountering the bodies. The disturbed flows corresponding to $\varphi_I^{(o)}$ and $\varphi_I^{(e)}$ are described by diffraction potentials, denoted $\varphi_D^{(o)}$ and $\varphi_D^{(e)}$, respectively. These potentials are represented in the same form as the radiation potentials [Eq.(3)], but with different source intensities, Q .

The unknown source intensities are determined by satisfying the boundary conditions

$$\frac{\partial\varphi_D^{(\beta)}}{\partial n}(y_a,z_a) = -\frac{\partial\varphi_I^{(\beta)}}{\partial n}(y_a,z_a)$$
$$\frac{\partial\varphi_D^{(\beta)}}{\partial n}(y_b,z_b) = -\frac{\partial\varphi_I^{(\beta)}}{\partial n}(y_b,z_b) \tag{8}$$

where β = o or e, on the straight-line segments representing the contours of $\underline{a}$ and $\underline{b}$. Thus there are two separate boundary conditions for the asymmetric and symmetric flows.

HYDRODYNAMIC FORCES AND MOMENTS

RELATIONSHIP TO POTENTIALS

The linearized hydrodynamic pressure is given by

$$p^{(\gamma)} = i\rho\omega\varphi^{(\gamma)} \tag{9}$$

where, for the radiation problem $\varphi^{(\gamma)} = \varphi^{(m_a, m_b, \epsilon_{S_a S_b})}$, and for the diffraction problem $\varphi^{(\gamma)} = \varphi_I^{(o+e)} + \varphi_D^{(o+e)}$. Using the single superscript notation for the radiation or diffraction problem, the hydrodynamic forces and moments are given by

$$\begin{aligned} F_S^{(\gamma)} &= -\int_C p^{(\gamma)}\, dz \\ F_H^{(\gamma)} &= \int_C p^{(\gamma)}\, dy \\ F_R^{(\gamma)} &= \int_C p^{(\gamma)}(y\,dy + z\,dz) \end{aligned} \tag{10}$$

where C = contours of bodies a and b.

Non-dimensional hydrodynamic moment arms for the sway and heave components of force, taking into account that the pressure is complex, i.e., $p^{(\gamma)} = p_r^{(\gamma)} + ip_i^{(\gamma)}$, can be written as

$$\ell_{S_r}^{(\gamma)} = -\frac{1}{T}\,\frac{\int_C p_r^{(\gamma)} z\,dz}{\int_C p_r^{(\gamma)}\, dz}$$

$$\ell_{S_i}^{(\gamma)} = -\frac{1}{T}\,\frac{\int_C p_i^{(\gamma)} z\,dz}{\int_C p_i^{(\gamma)}\, dz} \tag{11}$$

[Cont'd]

$$\ell_{H_r}^{(\gamma)} = \frac{1}{T} \frac{\int_C p_r^{(\gamma)} y dy}{\int_C p_r^{(\gamma)} dy}$$

$$\ell_{H_i}^{(\gamma)} = \frac{1}{T} \frac{\int_C p_i^{(\gamma)} y dy}{\int_C p_i^{(\gamma)} dy} \qquad (11)$$

where T = the maximum draft of the two bodies and subscripts S,H,R indicate swaying and heaving forces and rolling moment, respectively.

The hydrodynamic force (moment) has real and imaginary parts, for instance

$$F_S^{(\gamma)} = F_{S_r}^{(\gamma)} + i\ F_{S_i}^{(\gamma)} \qquad (12)$$

In the radiation problem, the real part is called the inertia force (moment) while the imaginary part corresponds to a hydrodynamic damping force (moment).[8] In the diffraction problem, the complex force defines the amplitude and phase of the wave-exciting force relative to the passage of the crest of the incident wave by the origin 0 . The hydrodynamic moment arms also consist of inertial and damping parts in the radiation problem. In the diffraction problem, however, the moment arms of the real and imaginary components are identical because the force and moment maxima occur at the same instant.

FORCE AND MOMENT COEFFICIENTS

Hydrodynamic interactions affect the pressure distributions and forces acting on the individual bodies. Certain components of these resultant forces and moments may be considered as internal forces for the evaluation of dynamic structural loading on rigidly connected cylinders. The hydrodynamic forces and moments exerted on individual bodies are useful for the evaluations of the motions of the individual bodies when they are either coupled or unconnected.

Table 1 defines the various force and moment components, in accordance with Eq.(10), and their corresponding dimensionless coefficients.

TABLE 1. ADDED MASS AND DAMPING COEFFICIENTS

Mode	ADDED MASS COEFFICIENTS		DAMPING COEFFICIENTS	
2	$m''_{SS} = \frac{F^{(2)}_{S_r}}{\omega^2}$	$C_{SS} = \frac{m''_{SS}}{\frac{1}{2}\rho\pi T^2}$	$N_{SS} = \frac{F^{(2)}_{S_i}}{\omega}$	$\delta_{SS} = \frac{N_{SS}}{\frac{1}{2}\omega\rho\pi T^2}$
	$m''_{SH} = \frac{F^{(2)}_{H_r}}{\omega^2}$	$C_{SH} = \frac{m''_{SH}}{\frac{1}{2}\rho\pi T^2}$	$N_{SH} = \frac{F^{(2)}_{H_i}}{\omega}$	$\delta_{SH} = \frac{N_{SH}}{\frac{1}{2}\omega\rho\pi T^2}$
	$m''_{SR} = \frac{F^{(2)}_{R_r}}{\omega^2}$	$C_{SR} = \frac{m''_{SR}}{\frac{1}{2}\rho\pi T^3}$	$N_{SR} = \frac{F^{(2)}_{R_i}}{\omega}$	$\delta_{SR} = \frac{N_{SR}}{\frac{1}{2}\omega\rho\pi T^3}$
3	$m''_{HS} = \frac{F^{(3)}_{S_r}}{\omega^2}$	$C_{HS} = \frac{m''_{HS}}{\frac{1}{2}\rho\pi T^2}$	$N_{HS} = \frac{F^{(3)}_{S_i}}{\omega}$	$\delta_{HS} = \frac{N_{HS}}{\frac{1}{2}\omega\rho\pi T^2}$
	$m''_{HH} = \frac{F^{(3)}_{H_r}}{\omega^2}$	$C_{HH} = \frac{m''_{HH}}{\frac{1}{2}\rho\pi T^2}$	$N_{HH} = \frac{F^{(3)}_{H_i}}{\omega}$	$\delta_{HH} = \frac{N_{HH}}{\frac{1}{2}\omega\rho\pi T^2}$
	$m''_{HR} = \frac{F^{(3)}_{R_r}}{\omega^2}$	$C_{HR} = \frac{m''_{HR}}{\frac{1}{2}\rho\pi T^3}$	$N_{HR} = \frac{F^{(3)}_{R_i}}{\omega}$	$\delta_{HR} = \frac{N_{HR}}{\frac{1}{2}\omega\rho\pi T^3}$
4	$m''_{RS} = \frac{F^{(4)}_{S_r}}{\omega^2}$	$C_{RS} = \frac{m''_{RS}}{\frac{1}{2}\rho\pi T^3}$	$N_{RS} = \frac{F^{(4)}_{S_i}}{\omega}$	$\delta_{RS} = \frac{N_{RS}}{\frac{1}{2}\omega\rho\pi T^3}$
	$m''_{RH} = \frac{F^{(4)}_{H_r}}{\omega^2}$	$C_{RH} = \frac{m''_{RH}}{\frac{1}{2}\rho\pi T^3}$	$N_{RH} = \frac{F^{(4)}_{H_i}}{\omega}$	$\delta_{RH} = \frac{N_{RH}}{\frac{1}{2}\omega\rho\pi T^3}$
	$m''_{RR} = \frac{F^{(4)}_{R_r}}{\omega^2}$	$C_{RR} = \frac{m''_{RR}}{\frac{1}{2}\rho\pi T^4}$	$N_{RR} = \frac{F^{(4)}_{R_i}}{\omega}$	$\delta_{RR} = \frac{N_{RR}}{\frac{1}{2}\omega\rho\pi T^4}$

The first subscript denotes the mode of motion (in place of the parenthetical superscript used previously; the forces, moments, and coefficients do, however, depend on the details of the modes and phases of the motions), while the second subscript denotes the components of force (moment). For example,

$\omega^2 m''_{SR}$ = the roll hydrodynamic inertial moment induced by swaying motion of unit amplitude of displacement

ωN_{RH} = the heave damping force induced by rolling motion of unit amplitude of displacement

The hydrodynamic moment arms, according to Eqs.(11), are presented in Table 2. The subscript notation is analogous to that used for Table 1, so that for example ℓ_{SS_r} indicates the moment arm for the hydrodynamic inertial sway force induced by sway motion.

TABLE 2. THE HYDRODYNAMIC MOMENT ARMS

MODE	INERTIAL PART	DAMPING PART
2	$\ell_{SS_r} = \ell_{S_r}^{(2)}$	$\ell_{SS_i} = \ell_{S_i}^{(2)}$
	$\ell_{SH_r} = \ell_{H_r}^{(2)}$	$\ell_{SH_i} = \ell_{H_i}^{(2)}$
3	$\ell_{HS_r} = \ell_{S_r}^{(3)}$	$\ell_{HS_i} = \ell_{S_i}^{(3)}$
	$\ell_{HH_r} = \ell_{H_r}^{(3)}$	$\ell_{HH_i} = \ell_{H_i}^{(3)}$
4	$\ell_{RS_r} = \ell_{S_r}^{(4)}$	$\ell_{RS_i} = \ell_{S_i}^{(4)}$
	$\ell_{RH_r} = \ell_{H_r}^{(4)}$	$\ell_{RH_i} = \ell_{H_i}^{(4)}$

The roll moments may be expressed in terms of the heave and sway forces and their respective levers as indicated in Table 3.

TABLE 3. MOMENT COEFFICIENT RELATIONSHIPS

$C_{SR} = C_{SS}\,\ell_{SS_r} + C_{SH}\,\ell_{SH_r}$
$\delta_{SR} = \delta_{SS}\,\ell_{SS_i} + \delta_{SH}\,\ell_{SH_i}$
$C_{HR} = C_{HS}\,\ell_{HS_r} + C_{HH}\,\ell_{HH_r}$
$\delta_{HR} = \delta_{HS}\,\ell_{HS_i} + \delta_{HH}\,\ell_{HH_i}$
$C_{RR} = C_{RS}\,\ell_{RS_r} + C_{RH}\,\ell_{RH_r}$
$\delta_{RR} = \delta_{RS}\,\ell_{RS_i} + \delta_{RH}\,\ell_{RH_i}$

The wave-exciting forces and moments may be expressed in dimensionless form as shown in Table 4.

TABLE 4. THE NON-DIMENSIONAL EXPRESSION OF THE WAVE-EXCITING FORCES AND MOMENTS

$f_S^{(\beta)} = \dfrac{F_S^{(\beta)}}{\rho g a B}$	sway-exciting force
$f_H^{(\beta)} = \dfrac{F_H^{(\beta)}}{\rho g a B}$	heave-exciting force
$f_R^{(\beta)} = \dfrac{F_R^{(\beta)}}{\rho g a BT}$	roll-exciting moment

where

B = beam of the body (a or b)

β = o or e corresponding to the odd or even potential $\varphi_I^{(o)} + \varphi_D^{(o)}$, or $\varphi_I^{(e)} + \varphi_D^{(e)}$

The wave-induced moment can, of course, be represented in terms of the sway and heave forces and corresponding moment arms in accordance with Eq.(11), in a manner similar to the motion-induced moments (Table 3). The separation of wave forces into odd and even parts will be shown to be useful for the case of the catamaran-type ship.

RIGIDLY CONNECTED TWIN CYLINDERS

All of the above formulas may be applied for bodies a, b and a+b, regardless of whether the bodies are similar (some care and consistency must be used in choosing and using relevant dimensions, draft and beam, for use in the dimensionless coefficients if the bodies are not twins). Based on a limited number of numerical calculations for rigidly connected twin cylinders, one obtains the relation between the forces on bodies a and b as shown in Table 5. It is observed that the magnitudes of the forces on a and b are identical in magnitude throughout, while some forces on a and b act in opposite directions.

TABLE 5. THE RELATIONS BETWEEN THE FORCE COEFFICIENTS DUE TO MOTIONS FOR TWIN BODIES a AND b

$$[c_{SH}]_a = -[c_{SH}]_b \;,\; [\delta_{SH}]_a = -[\delta_{SH}]_b$$
$$[c_{SS}]_a = [c_{SS}]_b \;,\; [\delta_{SS}]_a = [\delta_{SS}]_b$$
$$[c_{SR}]_a = [c_{SR}]_b \;,\; [\delta_{SR}]_a = [\delta_{SR}]_b$$
$$[c_{HS}]_a = -[c_{HS}]_b \;,\; [\delta_{HS}]_a = -[\delta_{HS}]_b$$
$$[c_{HH}]_a = [c_{HH}]_b \;,\; [\delta_{HH}]_a = [\delta_{HH}]_b$$
$$[c_{HR}]_a = -[c_{HR}]_b \;,\; [\delta_{HR}]_a = -[\delta_{HR}]_b$$
$$[c_{RH}]_a = -[c_{RH}]_b \;,\; [\delta_{RH}]_a = -[\delta_{RH}]_b$$
$$[c_{RS}]_a = [c_{RS}]_b \;,\; [\delta_{RS}]_a = [\delta_{RS}]_b$$
$$[c_{RR}]_a = [c_{RR}]_b \;,\; [\delta_{RR}]_a = [\delta_{RR}]_b$$

$$[\ell_{SH_r}]_a = -[\ell_{SH_r}]_b \;,\; [\ell_{SH_i}]_a = -[\ell_{SH_i}]_b$$
$$[\ell_{SS_r}]_a = [\ell_{SS_r}]_b \;,\; [\ell_{SS_i}]_a = [\ell_{SS_i}]_b$$
$$[\ell_{HS_r}]_a = [\ell_{HS_r}]_b \;,\; [\ell_{HS_i}]_a = [\ell_{HS_i}]_b$$
$$[\ell_{HH_r}]_a = -[\ell_{HH_r}]_b \;,\; [\ell_{HH_i}]_a = -[\ell_{HH_i}]_b$$
$$[\ell_{RH_r}]_a = -[\ell_{RH_r}]_b \;,\; [\ell_{RH_i}]_a = -[\ell_{RH_i}]_b$$
$$[\ell_{RS_r}]_a = [\ell_{RS_r}]_b \;,\; [\ell_{RS_i}]_a = [\ell_{RS_i}]_b$$
$$[\ell_H^{(o)}]_a = -[\ell_H^{(o)}]_b \;,\; [\ell_H^{(e)}]_a = -[\ell_H^{(e)}]_b$$

Consequently, the resultant hydrodynamic forces and moments may be given as shown in Table 6 where the subscript (a+b) for the rigidly coupled bodies is omitted.

TABLE 6. THE RESULTANT HYDRODYNAMIC FORCES AND MOMENTS DUE TO MOTIONS FOR TWIN CYLINDERS

$c_{SS} = 2[c_{SS}]_a$,	$\delta_{SS} = 2[\delta_{SS}]_a$
$c_{SH} = 0$,	$\delta_{SH} = 0$
$c_{SR} = 2[c_{SR}]_a$,	$\delta_{SR} = 2[\delta_{SR}]_a$
$c_{HS} = 0$,	$\delta_{HS} = 0$
$c_{HH} = 2[c_{HH}]_a$,	$\delta_{HH} = 2[\delta_{HH}]_a$
$c_{HR} = 0$,	$\delta_{HR} = 0$
$c_{RS} = c_{SR}$,	$\delta_{RS} = \delta_{SR}$
$c_{RH} = 0$,	$\delta_{RH} = 0$
$c_{RR} = 2[c_{RR}]_a$,	$\delta_{RR} = 2[\delta_{RR}]_a$

Based on a limited number of numerical calculations of the wave-exciting forces and moments on fixed twin cylinders in beam seas, one finds that the relations between the forces (or moments) on individual bodies a and b are as shown in Table 7. The odd and even components of the wave-exciting forces and moments on bodies a and b are equal in magnitude throughout, whereas some forces on a and b act in opposite directions.

TABLE 7. THE RELATIONS BETWEEN THE WAVE-EXCITING FORCES AND MOMENTS FOR TWIN BODIES a AND b

$[f_S^{(o)}]_a = [f_S^{(o)}]_b$,	$[f_S^{(e)}]_a = -[f_S^{(e)}]_b$
$[f_H^{(o)}]_a = -[f_H^{(o)}]_b$,	$[f_H^{(e)}]_a = [f_H^{(e)}]_b$
$[f_R^{(o)}]_a = [f_R^{(o)}]_b$,	$[f_R^{(e)}]_a = -[f_R^{(e)}]_b$
$[\ell_S^{(o)}]_a = [\ell_S^{(o)}]_b$,	$[\ell_S^{(e)}]_a = [\ell_S^{(e)}]_b$
$[\ell_H^{(o)}]_a = -[\ell_H^{(o)}]_b$,	$[\ell_H^{(e)}]_a = -[\ell_H^{(e)}]_b$

TABLE 8. THE RESULTANT WAVE-EXCITING FORCES AND MOMENTS FOR TWIN CYLINDERS

$$f_S = 2[f_S^{(o)}]_a$$

$$f_H = 2[f_H^{(e)}]_a$$

$$f_R = 2[f_R^{(o)}] = 2[\ell_S^{(o)}\ f_S^{(o)} + \ell_H^{(o)}\ f_H^{(o)}]_a$$

These relations which arise due to symmetry, exhibited graphically in Figures 1a-d, afford considerable simplications for evaluations of catamaran-type configurations.

For convenience, the previously defined force and moment coefficients C,δ (Table 1) will be called the forces and moments. Figure 1a illustrates a typical force system, induced by the heaving motion of the twin bodies. The heaving motion induces both heaving and swaying forces, $[C_{HH}]$, $[\delta_{HH}]$ and $[C_{HS}]$, $[\delta_{HS}]$, respectively, on the individual bodies $\underline{a}$ and $\underline{b}$. The swaying forces $[C_{HS}]$, $[\delta_{HS}]$ on $\underline{a}$ and $\underline{b}$ are equal, opposite and colinear; hence, the resultant forces on the twin bodies (a+b) are only the heaving forces $2[C_{HH}]_a$ and $2[\delta_{HH}]_a$ (see Table 6). Figure 1b represents a typical force system induced by the swaying motion. This motion also induces both heaving and swaying forces on each body. The sway-induced heaving forces $[C_{SH}]$ and $[\delta_{SH}]$ on $\underline{a}$ as well as $\underline{b}$ set up a couple which contributes to the resultant rolling moments $2[C_{SR}]_a$ and $2[\delta_{SR}]_a$ (see Tables 3,6). The sums of the swaying forces on the twin bodies ($\underline{a}$+$\underline{b}$) are equal to $2[C_{SS}]_a$, $2[\delta_{SS}]_a$, which also contribute to the resultant rolling moments $2[C_{SR}]_a$, $2[\delta_{SR}]_a$ (see Tables 3,6).

Another typical force system is that induced by the rolling motion, as illustrated in Figure 1c. The rolling motion induces the heaving forces, $[C_{RH}]$, $[\delta_{RH}]$ and the swaying forces $[C_{RS}]$, $[\delta_{RS}]$ on each body. The heaving forces set up a couple and hence contribute to the resultant rolling moments C_{RR}, δ_{RR} on the twin bodies ($\underline{a}$+$\underline{b}$) [see Tables 3,6]. The swaying forces on a and b are equal. Their resultants on (a+b) are equal to $2[C_{RS}]_a$ and $2[\delta_{RS}]_a$, and also contribute to the rolling moments C_{RR}, δ_{RR} (Tables 3,6).

The non-dimensional expressions of the wave-exciting forces and moments are defined in Table 4. Referring to Figure 1d, first let us observe the typical wave-induced force system. The even and odd wave potentials induce both sway- and heave-exciting forces. The sway-exciting forces on a,b, $[f_S^{(e)}]_a$, $[f_S^{(e)}]_b$, are equal, opposite and colinear, while the heave-exciting forces $[f_H^{(o)}]_a$, $[f_H^{(o)}]_b$ set up a couple.

We see from the figure that the resultant roll-exciting moment and the resultant sway-exciting force are due only to the odd wave potential, whereas the resultant heave-exciting force is due only to the even wave potential. (See Table 8.)

THE RADIATED AND DIFFRACTED WAVES

We consider at first the evaluation of the radiated and diffracted waves generated from rigidly connected twin cylinders floating in a regular beam wave.

The radiated and diffracted waves generated by a monohull cross section floating in a regular beam wave were evaluated in the previous work.[9] The radiated or the diffracted wave is the vector sum of the far field waves induced by the pulsating sources $Q^{(\gamma)}$ distributed on the sectional contours, where $\gamma = m$ (mode number) for the radiation, and $\gamma = \beta$ (0+e) for the diffraction problem.

The asymptotic expression of the velocity potential $\varphi^{(\gamma)}$ at $y \to \pm\infty$ for both radiation (γ=m) and diffraction (γ=β) are identical in their forms (see Ref.9).

$$\varphi_{\pm}^{(\gamma)} = \frac{A_{\pm}'^{(\gamma)}}{\nu} e^{\nu z} e^{i(\pm\nu y \mp \epsilon_{\pm}'^{(\gamma)})} \tag{13}$$

where ± suffix refers to $y \to +\infty$ or $-\infty$. It is to be noted that $\varphi_{\pm}^{(\gamma)}$ denotes both the potential per unit amplitude of displacement in forced oscillation for γ=m and that per unit amplitude of the incident wave for γ=β. The $A_{\pm}'^{(\gamma)}$ and $\epsilon_{\pm}'^{(\gamma)}$ are evaluted in a fashion similar to that given in Ref.9, where the arbitrarily shaped geometry and both symmetric and asymmetric flow conditions will require that the term with $(-1)^m$ in the formula Eq.(25) of the above reference should be taken as zero. Hence,

$$A'^{(Y)}_{\pm} = \sqrt{C^{(Y)2}_{\pm} + D^{(Y)2}_{\pm}}$$

$$\epsilon'^{(Y)}_{\pm} = \tan^{-1}\left[\frac{-D^{(Y)}_{\pm}}{C^{(Y)}_{\pm}}\right]$$

$$C^{(Y)}_{\pm} = \left\{\sum_{j=1}^{N} \pm Q^{(Y)}_j K_j + Q^{(Y)}_{N+j} L_j\right\}_a + \left\{\sum_{j=1}^{N} \pm Q^{(Y)}_j K_j + Q^{(Y)}_{N+j} L_j\right\}_b$$

$$D^{(Y)}_{\pm} = \left\{\sum_{j=1}^{N} \pm Q^{(Y)}_j L_j - Q^{(Y)}_{N+j} K_j\right\}_a + \left\{\sum_{j=1}^{N} \pm Q^{(Y)}_j L_j - Q^{(Y)}_{N+j} K_j\right\}_b \qquad (14)$$

$$K_j = e^{\nu\zeta_{j+1}} \cos(\nu\eta_{j+1} + \alpha_j) - e^{\nu\zeta_j} \cos(\nu\eta_j + \alpha_j)$$

$$L_j = e^{\nu\zeta_{j+1}} \sin(\nu\eta_{j+1} + \alpha_j) - e^{\nu\zeta_j} \sin(\nu\eta_j + \alpha_j)$$

where

a,b = the suffixes indicating the terms of the bodies a and b, respectively

$Q^{(Y)}_j, Q^{(Y)}_{N+j}$ = the real and imaginary parts of the complex source strength, uniformly distributed over the elementary j^{th} segment of bodies a or b

(η_j, ζ_j),etc = the coordinates of the end points of the j^{th} segments

α_j = the slope of the j^{th} segment

The far field wave $h_{\pm}^{(\gamma)}$ is derived from the far field potential $\varphi_{\pm}^{(\gamma)}$ (Eq.13) as

$$h_{\pm}^{(\gamma)} = \frac{iA_{\pm}^{\prime(\gamma)}}{\omega} e^{i(\pm\nu y \mp \epsilon_{\pm}^{\prime(\gamma)})} \tag{15}$$

Let the complex amplitude ratio be

$$A_{\pm}^{(\gamma)} = \frac{A_{\pm}^{\prime(\gamma)}}{\omega} e^{\mp i \epsilon_{\pm}^{\prime(\gamma)}} \; ; \; A_{\pm}^{(\gamma)} = \left|A_{\pm}^{(\gamma)}\right| e^{i\epsilon_{\pm}^{(\gamma)}} \tag{16}$$

The energy conservation law leads to the well-known relation between the hydrodynamic damping coefficient $N^{(m)}$ and the radiated wave amplitude ratio $\left|A_{\pm}^{(m)}\right|$

$$N^{(m)} = \frac{\rho g^2}{\omega^3} \left|A_{\pm}^{(m)}\right|^2 \tag{17}$$

where the $\pm$ suffix indicates the radiated wave at $y \rightarrow +\infty$ or $-\infty$. It is noted that $\left|A_{\pm}^{(4)}\right|$ has the dimension of length; in other words $\left|A_{\pm}^{(4)}\right|$ is not non-dimensional, whereas the $\left|A_{\pm}^{(2)}\right|$, $\left|A_{\pm}^{(3)}\right|$ are.

By the Haskind method,[10,11] one* can also evaluate the wave-exciting forces and moments by the radiated wave amplitudes (Eq.16). For the wave progressing to the positive and negative ends of the y-axis (Eq.5), the wave-exciting forces are given by

$$\begin{aligned} f_{S_{\pm}} &= \frac{A_{\mp}^{(2)}}{\nu B} \\ f_{H_{\pm}} &= \frac{A_{\mp}^{(3)}}{\nu B} \\ f_{R_{\pm}} &= \frac{A_{\mp}^{(4)}}{\nu B T} \end{aligned} \tag{18}$$

*See Appendix A

Referring to Appendix A and The Diffraction Problem, we see that by the Haskind method the exciting forces and moments on individual bodies a or b cannot be evaluated.

Based on the theories of Haskind[12] and Maruo,[13] it can be shown in a fashion similar to what was done in Appendix A, that the mean wave force K on twin cylinders fixed in beam seas is determined by the diffracted wave amplitude (see Ref.9):

$$\frac{K}{\frac{1}{2}\rho g a^2} = \left| A_-^{(o+e)} \right|^2 \tag{19}$$

EQUATIONS OF MOTION OF A TWIN HULL OCEAN PLATFORM IN BEAM SEAS

By employing the strip method, one can approximately evaluate the hydrodynamic forces and moments on a twin hull oscillating in waves. The equations of the coupled heaving and pitching motions and the coupled swaying, rolling and yawing motions of the rigidly connected twin hull platform, being a rigid body, are identical with those for a monohull platform.[14] Since the wave-exciting forces and moments are presently available only for beam seas, we restrict consideration to the beam sea motions. However, the same equations will be applicable to the oblique sea motions when the exciting forces in oblique seas can be predicted. The coupled equations for heaving and pitching are:

$$\begin{bmatrix} \{(B_{\zeta\zeta}-\omega^2 M_{\zeta\zeta})-i\omega N_{\zeta\zeta}\} & -\{B_{\psi\zeta}-\omega^2 M_{\psi\zeta})-i\omega N_{\psi\zeta}\} \\ -\{(B_{\psi\zeta}-\omega^2 M_{\psi\zeta})-i\omega N_{\psi\zeta}\} & \{B_{\psi\psi}-\omega^2 M_{\psi\psi})-i\omega N_{\psi\psi}\} \end{bmatrix} \begin{bmatrix} \frac{\zeta}{a} \\ \frac{\psi}{a} \end{bmatrix} = \begin{bmatrix} \frac{F_\zeta}{a} \\ \frac{F_\psi}{a} \end{bmatrix} \qquad (20)$$

The equations for coupled swaying, rolling and yawing motions are given by:

$$\begin{bmatrix} (-\omega^2 M_{\eta\eta}-i\omega N_{\eta\eta}) & (-\omega^2 M_{\chi\eta}-i\omega N_{\chi\eta}) & (-\omega^2 M_{\varphi\eta}-i\omega N_{\varphi\eta}) \\ (-\omega^2 M_{\eta\chi}-i\omega N_{\eta\chi}) & (-\omega^2 M_{\chi\chi}-i\omega N_{\chi\chi}) & (-\omega^2 M_{\varphi\chi}-i\omega N_{\varphi\chi}) \\ (-\omega^2 M_{\eta\varphi}-i\omega N_{\eta\varphi}) & (-\omega^2 M_{\chi\varphi}-i\omega N_{\chi\varphi}) & (B_{\varphi\varphi}-\omega^2 M_{\varphi\varphi}-i\omega N_{\varphi\varphi}) \end{bmatrix} \begin{bmatrix} \frac{\eta}{a} \\ \frac{\chi}{a} \\ \frac{\varphi}{a} \end{bmatrix} = \begin{bmatrix} \frac{F_\eta}{a} \\ \frac{F_\chi}{a} \\ \frac{F_\varphi}{a} \end{bmatrix} \qquad (21)$$

where the time factor $e^{-i\omega t}$ is omitted in both cases. As we can see in Eqs.(20) and (21), the motions consist of sway η, heave ζ, roll φ, pitch ψ, and yaw χ. It is, as usual, assumed that the motions η, ζ are the translatory motions of the center of gravity G about its midposition of oscillation and that φ, ψ, χ are the motions about the axes through G at its midposition of oscillation.

Referring to the formulas of the transfer of forces from the origin 0 of the coordinate system to the center of gravity G of the hull as given in Eqs.(B-14),(B-15),(B-16) of Appendix B, one can readily evaluate the motion-induced sectional forces and moments with respect to G if the resultant forces and moments on the rigidly connected twin cylinders are known with respect to the origin 0 . The force coefficients given in Table 6, such as $C_{HH}, \delta_{HH}, C_{SS}, \delta_{SS}, C_{SR}, \delta_{SR}, C_{RS}, \delta_{RS}, C_{RR}, \delta_{RR}$, are those with respect to the origin 0 .

By summing the sectional forces and moments, the resulting forces and moments are obtained:

$$M_{\eta\eta} = \rho \nabla + \int_{-\ell_1}^{\ell_2} m''_{SS}\, dx$$

$$N_{\eta\eta} = \int_{-\ell_1}^{\ell_2} N_{SS}\, dx$$

$$M_{\chi\eta} = \int_{-\ell_1}^{\ell_2} m''_{SS}\, x dx$$

$$N_{\chi\eta} = \int_{-\ell_1}^{\ell_2} N_{SS}\, x dx$$

$$M_{\varphi\eta} = \int_{-\ell_1}^{\ell_2} (m''_{SR} \pm \overline{OG}\, m''_{SS})\, dx$$

$$N_{\varphi\eta} = \int_{-\ell_1}^{\ell_2} (N_{SR} \pm \overline{OG}\, N_{SS})\, dx$$

$$M_{\chi\chi} = I_{\chi} + \int_{-\ell_1}^{\ell_2} m''_{SS}\, x^2 dx$$

$$N_{\chi\chi} = \int_{-\ell_1}^{\ell_2} N_{SS}\, x^2 dx \qquad (22)$$

[Cont'd]

$$M_{\varphi\chi} = \int_{-\ell_1}^{\ell_2} (m''_{SR} \pm \overline{OG}\, m''_{SS})\, x dx$$

$$N_{\varphi\chi} = \int_{-\ell_1}^{\ell_2} (N_{SR} \pm \overline{OG}\, N_{SS})\, x dx$$

$$M_{\varphi\varphi} = I_\varphi + \int_{-\ell_1}^{\ell_2} \{m''_{RR} \pm \overline{OG}(2m''_{SR} \pm \overline{OG} \cdot m''_{SS})\}\, dx$$

$$N_{\varphi\varphi} = \int_{-\ell_1}^{\ell_2} \{N_{RR} \pm \overline{OG}\,(2N_{SR} \pm \overline{OG} \cdot N_{SS})\}\, dx$$

$$B_{\varphi\varphi} = \rho g \nabla\, \overline{GM}_\varphi$$

$$M_{\zeta\zeta} = \rho \nabla + \int_{-\ell_1}^{\ell_2} m''_{HH}\, dx$$

$$N_{\zeta\zeta} = \int_{-\ell_1}^{\ell_2} N_{HH}\, dx \qquad (22)$$

$$B_{\zeta\zeta} = \rho\, g\, A_w$$

$$M_{\psi\zeta} = M_{\zeta\psi} = \int_{-\ell_1}^{\ell_2} m''_{HH}\, x dx$$

$$N_{\psi\zeta} = N_{\zeta\psi} = \int_{-\ell_1}^{\ell_2} N_{HH}\, x dx$$

$$B_{\psi\zeta} = B_{\zeta\psi} = 2\rho g \int_{-\ell_1}^{\ell_2} y_w\, x dx$$

$$M_{\psi\psi} = I_\psi + \int_{-\ell_1}^{\ell_2} m''_{HH}\, x^2 dx$$

$$N_{\psi\psi} = \int_{-\ell_1}^{\ell_2} N_H\, x^2 dx$$

$$B_{\psi\psi} = \rho g \nabla\, \overline{GM}_\psi = 2\rho g \int_{-\ell_1}^{\ell_2} y_w x^2 dx + \overline{KB} - \overline{KG}$$

where

∇ = total displacement volume of the hull

y_w = beam of one section at waterline

$\pm\overline{OG}$ = distance between the origin and the center of gravity G. The + sign is taken for G above O , and the - sign is taken for G below O .

$\overline{GM}_\varphi, \overline{GM}_\psi$ = metacentric height for roll and pitch, respectively

$I_\varphi, I_\psi, I_\chi$ = moment of inertia of the hull about rolling, pitching and yawing axes, respectively

$\overline{KB}, \overline{KG}$ = distance of center of buoyancy B and center of gravity G from the keel of the hull, respectively

Note that the added mass m'' and damping coefficient for each section are as shown in Table 6. In the above formulas M and N , such as $M_{\eta\eta}, N_{\eta\eta}$, denote the virtual mass and the damping coefficient of the hull per unit amplitude of acceleration and velocity of oscillation, respectively. The first subscript designates the motion and the second subscript designates the component force (or moment).

The formulas for sectional wave-exciting forces and moments are given in Table 8. According to the force transfer, we write them in the following form:

$$f_S = 2\left[f_S^{(o)}\right]_a$$

$$f_H = 2\left[f_H^{(e)}\right]_a \tag{23}$$

$$f_R = 2\left[\left(\ell_S^{(o)} \pm \frac{\overline{OG}}{T}\right) f_S^{(o)} + \left(\ell_H^{(o)} \pm \frac{\overline{OG}}{T}\right) f_H^{(o)}\right]_a$$

where subscript a denotes the force (moment) on body a only. The rolling moment f_R is now with respect to the transverse axis through G .

The wave-exciting force and moment per unit amplitude of the incident wave are

$$\frac{F_\eta}{a} = \rho \; g \int_{-\ell_1}^{\ell_2} f_S \; y_w \; dx$$

$$\frac{F_\zeta}{a} = \rho \; g \int_{-\ell_1}^{\ell_2} f_H \; y_w \; dx \qquad (24)$$

$$\frac{F_R}{a} = \rho \; g \int_{-\ell_1}^{\ell_2} f_R \cdot T \cdot y_w \; dx$$

where

T = draft of a section

y_w = beam of a cross section of a single hull on the waterline

RELATIVE HEAVING MOTIONS OF TWO CYLINDERS FLOATING FREELY IN BEAM SEAS

The most general kinematical boundary condition which can be imposed on two different floating cylinders is given by Eq.(4). By applying this boundary condition, one can determine the motion-induced hydrodynamic forces and moments exerted on each body when both are oscillating with different modes and phases.

The simplified kinematical boundary conditions given by Eq.(4b) and (4c) will be utilized in the prediction of the relative motions of the bodies.

Consider, as one of the simplest cases, the relative heaving motion of two different cylinders in beam seas. It is assumed that the motions of the bodies are restrained in the lateral and rotational directions and furthermore the heaving motions induce only heaving forces. The heave-induced heaving forces on a and b will then be functions of the heaving motion (displacement) ζ_a, ζ_b of the bodies a and b. Hence the heaving forces will be represented by

$$F_a = F_a(\zeta_a, \zeta_b) \quad \text{and} \quad F_b = F_b(\zeta_a, \zeta_b) \tag{25}$$

Since we adopt the condition of small motions, the variation of corresponding forces with the displacement will be restricted to the first order force derivatives in the Taylor expansion given by

$$\begin{array}{cc} \dfrac{\partial F_a(o,o)}{\partial \zeta_a} , & \dfrac{\partial F_a(o,o)}{\partial \zeta_b} \\[2ex] \dfrac{\partial F_b(o,o)}{\partial \zeta_a} , & \dfrac{\partial F_b(o,o)}{\partial \zeta_b} \end{array} \tag{26}$$

all evaluated at the calm free surface.

The force derivatives, or forces induced by unit displacement of the bodies in the presence of each other, are determined by satisfying the linearized kinematical boundary conditions, Eq.(4b) and Eq.(4c). The radiation problem with boundary conditions given by Eq.(4b) determines the force derivatives $\frac{\partial F_a}{\partial \zeta_a}$, $\frac{\partial F_b}{\partial \zeta_a}$, whereas the one with the boundary condition expressed by Eq.(4c) evaluates the force derivatives $\frac{\partial F_a}{\partial \zeta_b}$, $\frac{\partial F_b}{\partial \zeta_b}$. They are represented by the added mass and damping coefficients. With omission of the time factor $e^{-i\omega t}$, they are

$$\frac{\partial F_a}{\partial \zeta_a} = \omega^2 m''_{aa} + i\omega N_{aa}$$

$$\frac{\partial F_b}{\partial \zeta_a} = \omega^2 m''_{ab} + i\omega N_{ab}$$

$$\frac{\partial F_a}{\partial \zeta_b} = \omega^2 m''_{ba} + i\omega N_{ba} \tag{27}$$

$$\frac{\partial F_b}{\partial \zeta_b} = \omega^2 m''_{bb} + i\omega N_{bb}$$

The first subscript indicates the moving body, and the second indicates the body on which the hydrodynamic force is exerted.

The heave-exciting forces F_{H_a}, F_{H_b} acting on bodies a and b are evaluated by the procedure previously described in the diffraction problem.

Having obtained all the hydrodynamic force derivatives, one can readily write the equations of the coupled heaving motion of two cylinders, incorporating the hydrostatic (buoyancy) and inertia forces, for the unknown motions ζ_a and ζ_b :

$$\{-\omega^2(m_a+m''_{aa}) - i\omega N_{aa} + B_a\} \frac{\zeta_a}{a} + (-\omega^2 m''_{ba} - i\omega N_{ba}) \frac{\zeta_b}{a} = \frac{F_{H_a}}{a}$$

$$\{-\omega^2(m''_{ab} - i\omega N_{ab}) \frac{\zeta_a}{a} + \{ -\omega^2(m_b+m''_{bb}) - i\omega N_{bb} + B_b\} \frac{\zeta_b}{a} = \frac{F_{H_b}}{a} \tag{28}$$

where

m_a, m_b = mass of bodies $\underline{a}$ and $\underline{b}$, respectively

B_a, B_b = restoring force on bodies $\underline{a}$ and $\underline{b}$, respectively

For convenience the non-dimensional coefficients for the hydrodynamic forces in Eq.(28) are introduced:

$$
\begin{aligned}
C_{aa} &= \frac{m''_{aa}}{\rho \frac{\pi}{2} T_a^2} & \delta_{aa} &= \frac{N_{aa}}{\omega\rho \frac{\pi}{2} T_a^2} \\
C_{ba} &= \frac{m''_{ba}}{\rho \frac{\pi}{2} T_a^2} & \delta_{ba} &= \frac{N_{ba}}{\omega\rho \frac{\pi}{2} T_a^2} \\
C_{ab} &= \frac{m''_{ab}}{\rho \frac{\pi}{2} T_b^2} & \delta_{ab} &= \frac{N_{ab}}{\omega\rho \frac{\pi}{2} T_b^2} \\
C_{bb} &= \frac{m''_{bb}}{\rho \frac{\pi}{2} T_b^2} & \delta_{bb} &= \frac{N_{bb}}{\omega\rho \frac{\pi}{2} T_b^2} \\
f_{H_a} &= \frac{F_{H_a}}{\rho g a B_a} & f_{H_b} &= \frac{F_{H_b}}{\rho g a B_b}
\end{aligned}
\qquad (29)
$$

NUMERICAL CALCULATIONS

A series of numerical calculations were carried out for the hydrodynamic characteristics of the rigidly connected twin cylinders as well as for two different cylinders floating freely in beam seas.

The results of the numerical calculations are presented in figures which are discussed in the following.

ADDED MASS AND DAMPING COEFFICIENTS FOR TWIN CYLINDERS

Figures 2,3 and 4 exhibit the various hydrodynamic forces and moments on one of a pair of half-immersed circular cylindrical bodies as functions of the frequency parameter $\frac{\nu B}{2}$. The definitions of the various forces on this body $\underline{a}$ are given in Tables 1, 2 and 3. The hydrodynamic forces on body $\underline{b}$ are evaluated by employing the formulas in Table 5 with the aid of the information given in Figures 2, 3 and 4. The resultant forces and moments on the twin bodies ($\underline{a}$+$\underline{b}$) due to the swaying, heaving and rolling motions are evaluated by employing the formulas in Table 6 with the aid of the data in Figures 2, 3 and 4.

WAVE-EXCITING FORCES AND MOMENTS FOR TWIN CYLINDERS

Figure 5 illustrates the behavior of the sway- and heave-exciting forces on body $\underline{a}$ due to odd and even wave potentials as functions of the wave frequency. The vectorial sums of the forces such as $f^{(o+e)}=f^{(o)}+f^{(e)}$ on $\underline{a}$, $\underline{b}$, and ($\underline{a}$+$\underline{b}$) are shown in Figures 6 and 7. In this connection, refer to Tables 7 and 8.

It is interesting to observe the sway-exciting forces on $\underline{a}$ and $\underline{b}$. The same behavior has been exhibited in a recent study by Ohkusu[15] on the hydrodynamic interaction of three vertical cylinders (see Fig.13, Ref.15).

The Haskind method was applied to the evaluations of the resultant wave-exciting forces on the twin bodies and the results confirm exactly the corresponding values obtained by the present method. (See Figures 6, 7 and 8.) A relevant discussion of this matter will be given in the following section.

RADIATED AND DIFFRACTED WAVES

For the twin bodies, the radiated waves $A_-^{(m)}$ and the diffracted waves $A_-^{(o+e)}$ were evaluated according to the formulas in Eqs.(14) and (16) [see Figure 8]. The radiated wave amplitudes $A_-^{(m)}$ are very useful in evaluating the damping force coefficients $N^{(m)}$ or $\delta^{(m)}$ and the wave-exciting forces $f^{(o+e)}$ (see Eqs.17,18). By comparing Figures 2, 6, 7 and 8, one can readily confirm the validity of the formulas in Eqs.(17),(18). For instance, the sway-induced damping force coefficient δ_{SS} in Figure 2 can be evaluated by the sway-induced radiated wave amplitude ratio $|A_-^{(2)}|$ in Figure 8 according to Eq.(17). Similarly, the sway-exciting force in Figure 6 can be evaluated by the amplitude ratio $A_-^{(2)}$ according to Eq.(18), which furnishes both the force amplitude and the phase angle.

The diffracted wave amplitude ratio $A_-^{(o+e)}$ is applied to estimating the mean wave force on the fixed twin bodies in the given incident wave, Eq.(5). The sudden drop of the value of $A_-^{(o+e)}$ at frequency around 0.45 may be ascribed to the effect of the hydrodynamic interaction between the bodies in that close proximity.

WAVE-EXCITING FORCES ON SOME SEMI-SUBMERSIBLE CROSS SECTIONS

We chose two simple cross sections: one submerged and the other surface-piercing. Figure 9a and 9b represent the sway- and heave-exciting forces on the submerged circular cross section. It is seen from these figures that the interaction effects are negligibly small for the given frequency range. The wave-exciting forces on surface-piercing twin bodies are plotted against the wave frequency in Figures 10a, 10b. It is readily seen that the influence of the interaction effect on the sway-exciting forces is remarkable while the influence on the heave-exciting forces is relatively small. Similar behavior was pointed out by Ohkusu,[15] as described previously.

SOME HYDRODYNAMIC CHARACTERISTICS OF TWO DIFFERENT CYLINDRICAL BODIES FLOATING IN BEAM SEAS

Some aspects of the hydrodynamic interaction between two arbitrarily shaped cylindrical floats were investigated numerically (see Figures 11a, 11b, 11c). First we evaluated the heaving added mass and damping coefficients C, δ on bodies a and b for heaving motions with unit amplitude and different phases. The results are plotted against the relative phase angle ϵ_b at two different frequencies $\nu(B_a+B_b)/2 = 0.45$ and 0.26 (Figure 11a. The results show significant differences in the hydrodynamic characteristics of two freely floating cylinders on account of the different phases.

As a special case of the above, we calculated the hydrodynamic forces on the swaying and heaving body a in the presence of the fixed body b (Figure 11b). Two draft ratios $T_b/T_a = 2$ and 15 were taken in order to observe the partial false wall effect on the hydrodynamic forces. The figure also exhibits the hydrodynamic inertial forces C_{HH}, C_{SS} on an isolated swaying and heaving body a. It is seen that the increment of the swaying added mass C_{SS} due to the increase of the depth of the false wall is nearly independent of the frequency, whereas the heaving added masses C_{HH} are remarkably dependent on the depth of the body b, T_b/T_a, and the frequency.

It should be noted that the wall effect on the swaying and rolling motions of a cylinder can be accurately evaluated by utilizing the present method. The problem of considering two bodies performing swaying or rolling motions with 180-deg phase differences will be equivalent to that of a body executing the same mode of motions in the presence of a wall (i.e., taking image effects). The wave-exciting forces, however, can only be approximately determined by means of the present approach by increasing the depth parameter of the fixed body b.

The heave-exciting forces $F_H^{(o+e)}$ on bodies a and b were calculated and plotted in Figure 11c. For comparison, the heave-exciting forces on isolated a and b are also plotted in the same figure, thereby demonstrating the interaction effects.

RELATIVE HEAVING MOTIONS OF TWO ARBITRARILY SHAPED CYLINDERS IN BEAM SEAS

Figures 12a and 12b illustrate the behavior of the heave hydrodynamic forces exerted on bodies a and b which are induced by the heaving motion of body a while body b is fixed, and vice versa. [See Eq. (27).] Note that the first subscript refers to the moving body and the second subscript refers to the body on which the force is exerted.

It is seen from the figures that 1) the influence of the heaving motion of the larger body b on the fixed smaller body a is significantly large, while the influence of the heaving motion of the smaller body a on the fixed larged body b is negligibly small, and 2) the nondimensional forces on a induced by a are nearly the same as the corresponding nondimensional forces on b induced by b .

Using the data given in Figures 11c, 12a and 12b in the formula of Eq. (28), we evaluated the heave responses of bodies a and b as well as the motion of body a relative to b . The results are exhibited in Figure 12c.

The range of wave frequency (or of wave length), for which the calculations were carried out, unfortunately does not cover the entire practical range. Due to limited funds, the calculations were not extended to cover the range of wave length $\lambda/(B_a+B_b) < 7$ so that no information could be obtained in that range.

An attempt is made, however, to illustrate the qualitative behavior of the responses in that frequency range by a smooth extrapolation of the calculated response curves together with a notion of the expected responses at infinity as well as in the resonant frequency range.

The heaving response ζ_a/a of body a relative to body b is evaluated by the vector difference of the individual responses ζ_a/a and ζ_b/a .

CONCLUSIONS

A general method has been developed for the study of the hydrodynamic interaction of two cylindrical bodies of arbitrary shape executing any mode of motion in beam seas. In fact, two types of interaction problem have been considered: 1) rigidly connected twin cylinders, and 2) freely floating cylindrical bodies. The hydrodynamic pressure distributions and the resulting forces and moments have been determined for each individual body undergoing arbitrary motion, with varying relative phases.

A numerical procedure and corresponding computer program adaptable to the CDC-6600 have been developed for evaluating the response motions of a Catamaran-type ocean platform in beam seas. Results of calculations are not presented in this paper since systematic calculations were not performed which would show trends for comparison with those of experimental studies.[16]

A technique has been developed for the evaluation of relative motions of two bodies floating in beam seas. This has been applied to the evaluation of the relative heaving motions of two different cylindrical hulls.

It should be mentioned that this technique is applicable to the evaluation of wall effect on the hydrodynamic forces and moments on a body undergoing a prescribed motion in the presence of a vertical wall, as well as to the approximate determination of the wave-exciting forces and moments (by increasing the depth of "false" wall).

It is recommended that the present "strip method" be applied to systematic calculations for a Catamaran hull in oblique seas, to determine the hydrodynamic forces and moments and response motions as well as the structural loadings, and that the presently developed method for evaluation of relative heaving motion of two freely floating bodies in beam seas be extended to cases of other modes of motion in oblique seas. In the latter case, the hydrodynamic forces, moments and structural loadings on each individual hull will also be furnished.

ACKNOWLEDGMENTS

The author wishes to thank Dr. J. P. Breslin, Dr. S. Tsakonas, Mr. E. Numata and Mr. J. Mercier for their expressions of interest and advice during the period of research.

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APPENDIX A

APPLICATION OF THE HASKIND METHOD TO TWIN CYLINDERS

Haskind's method[10] states that the wave-exciting forces and moments depend on the asymptotic behavior of the radiation potential per unit velocity amplitude. Newman[11] extended the theory and established the formulas relating the exciting forces (moments) to the radiated amplitude ratio $A_{\pm}^{(m)}$. The abovementioned method is applied to the evaluation of the wave-exciting forces (moments) on a rigidly connected twin body fixed in beam seas.

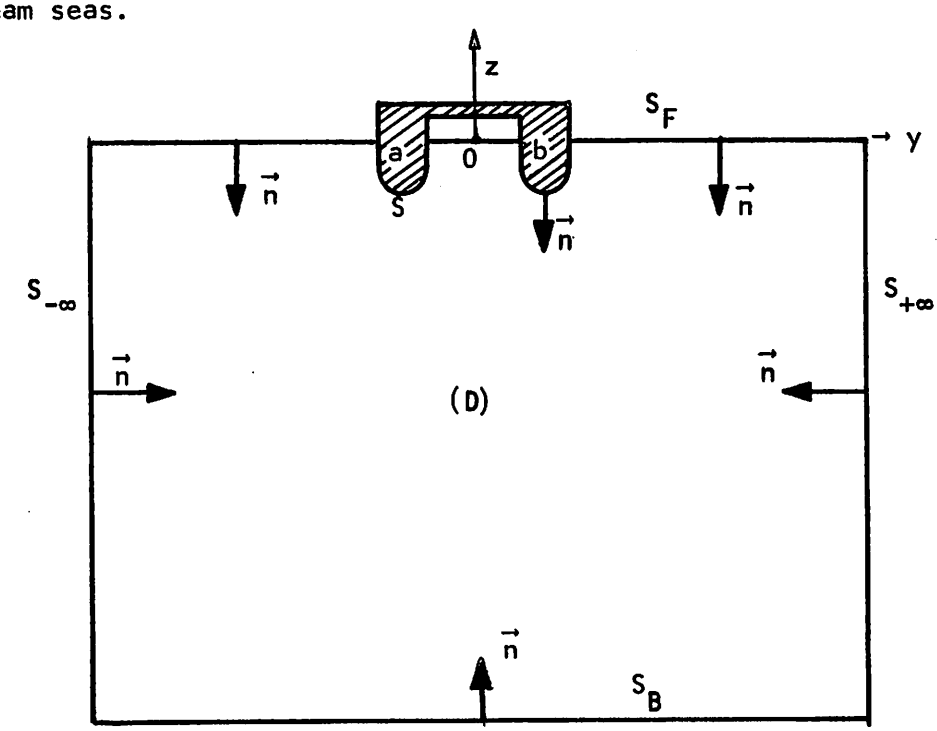


FIGURE A-1

Suppose the rigidly connected twin bodies $\underline{a}$ and $\underline{b}$ are floated in the incident wave

$$h = ae^{i(\pm\nu y-\omega t)} \tag{A-1}$$

where $\pm$ indicates the direction of incident wave oncoming from negative and positive ends of the y-axis, respectively, and define the three modes of the wave-induced motion as

$$S = S^{(m)}\ e^{-i\omega t} \tag{A-2}$$

Then, the velocity potential consists of φ_I , φ_D , $\varphi^{(m)}$ which are

$$\varphi_I = -i\,\frac{ga}{\omega}\,e^{\nu z}\,e^{\pm i\nu y} \tag{A-3}$$

$$\varphi_D = \varphi_{D_a} + \varphi_{D_b} \tag{A-4}$$

$$\varphi^{(m)} = \varphi_a^{(m)} + \varphi_b^{(m)} \tag{A-5}$$

where

φ_I = incident wave potential

φ_D = diffracted wave potential

$\varphi^{(m)}$ = forced wave potential (or radiation potential) per unit displacement amplitude

m = number of mode of motion

The component potentials satisfy:

1. The condition of continuity of the liquid in the water domain.
2. The linearized free surface condition.
3. The deep water condition.
4. The radiation condition.
5. The kinematical boundary conditions on the surfaces of bodies <u>a</u> and <u>b</u>.

The above conditions are formulated in the following forms:

1. $$\nabla^2\varphi = 0 \quad \text{in water domain} \tag{A-6}$$

2. $$\left(\frac{\partial}{\partial z} - \nu\right)\varphi = 0 \quad \text{on} \quad z = 0 \tag{A-7}$$

3. $$\frac{\partial\varphi}{\partial z} = 0 \quad \text{at } z = -\infty \tag{A-8}$$

4. $$\lim_{y\to\pm\infty}\left(\frac{\partial}{\partial y} \mp i\nu\right)\varphi = 0, \qquad \varphi = \varphi_D \ \text{ or } \ \varphi^{(m)} \tag{A-9}$$

5. $$\frac{\partial\varphi^{(m)}(r_a)}{\partial n} = -i\omega\, u_n^{(m)}(r_a)\ ,\quad \frac{\partial\varphi^{(m)}(r_b)}{\partial n} = -i\omega\, u_n^{(m)}(r_b) \qquad \text{(A-10)}$$

6. $$\frac{\partial\varphi_D(r_a)}{\partial n} = -\frac{\partial\varphi_I(r_a)}{\partial n}\ ,\quad \frac{\partial\varphi_D(r_b)}{\partial n} = -\frac{\partial\varphi_I}{\partial n}(r_b) \qquad \text{(A-11)}$$

where r_a, r_b — denotes points on the surfaces of bodies a and b, respectively

$u_n^{(m)}$ — normal component per unit displacement amplitude of motion

(m) — number indicating the mode of motion

The normal displacement components are:

$$\begin{aligned} u^{(2)} &= \sin\alpha \\ u^{(3)} &= -\cos\alpha \\ u^{(4)} &= -(y\cos\alpha + z\sin\alpha) \end{aligned} \qquad \text{(A-12)}$$

The α is the angle of a segment with respect to the y-axis.

Now consider the potentials φ_D and $\varphi^{(m)}$ together in the entire water domain (D) as illustrated in Figure A-1, and write Green's formula:

$$\int_{S_F+S_B+S_{+\infty}+S_{-\infty}+S_a+S_b} \left(\varphi_D \frac{\partial\varphi^{(m)}}{\partial n} - \varphi^{(m)} \frac{\partial\varphi_D}{\partial n}\right) dS = 0 \qquad \text{(A-13)}$$

The integration is performed piecewise. By making use of the boundary conditions, 1) the free surface condition on S_F, 2) the bottom condition on S_B, and 3) the identity of the form of φ_D and $\varphi^{(m)}$ at $y \to +\infty$ or $y \to -\infty$ on the surface $S_{+\infty}$ or $S_{-\infty}$, one obtains the formula

$$\int_{S_a+S_b} \left(\varphi_D \frac{\partial\varphi^{(m)}}{\partial n} - \varphi^{(m)} \frac{\partial\varphi_D}{\partial n}\right) dS = 0 \qquad \text{(A-14)}$$

or

$$\int_{S_a+S_b} \varphi_D \frac{\partial\varphi^{(m)}}{\partial n}\, dS = \int_{S_a+S_b} \varphi^{(m)} \frac{\partial\varphi_D}{\partial n}\, dS \qquad \text{(A-15)}$$

Let $F^{(m)}$ be the sway- and heave-exciting forces F_S, F_H and roll-exciting moment F_R for m=2,3,4 and consider the integration of the linearized hydrodynamic pressure p

$$p = i\rho\omega(\varphi_I + \varphi_D) \tag{A-16}$$

on the body surfaces S_a+S_b:

$$F^{(m)} = -\int_{S_a+S_b} p\, u_n^{(m)}\, dS \tag{A-17}$$

Using Eqs. (A-10),(A-12) and (A-16), the above expressions are transformed into:

$$F^{(m)} = \rho \int_{S_a+S_b} (\varphi_I+\varphi_D) \frac{\partial\varphi^{(m)}}{\partial n}\, dS \tag{A-18}$$

Furthermore, according to Eqs. (A-11) and (A-15)

$$\varphi_D \frac{\partial\varphi^{(m)}}{\partial n} = \varphi^{(m)} \frac{\partial\varphi_D}{\partial n} = -\varphi^{(m)} \frac{\partial\varphi_I}{\partial n} \qquad \text{on } S_a \text{ and } S_b$$

Equation (A-18) therefore takes the following form

$$F^{(m)} = \rho \int_{S_a+S_b} \left(\varphi_I \frac{\partial\varphi^{(m)}}{\partial n} - \varphi^{(m)} \frac{\partial\varphi_I}{\partial n}\right) dS \tag{A-19}$$

By applying Green's formula to the potentials $\varphi^{(m)}$ and φ_I in domain (D), one writes

$$\int_{S_a+S_b+S_F+S_B+S_{+\infty}+S_{-\infty}} \left(\varphi_I \frac{\partial\varphi^{(m)}}{\partial n} - \varphi^{(m)} \frac{\partial\varphi_I}{\partial n}\right) dS = 0 \tag{A-20}$$

As in the case of the integration in Eq.(A-13), piecewise integration is carried out and since the integration on S_F and S_B vanish, the wave-exciting force $F^{(m)}$ in Eq.(A-19) reduces to

$$F^{(m)} = -\rho \int_{S_{+\infty}+S_{-\infty}} (\varphi_I \frac{\partial \varphi^{(m)}}{\partial n} - \varphi^{(m)} \frac{\partial \varphi_I}{\partial n}) \, dS \tag{A-21}$$

The asymptotic express of $\varphi^{(m)}$ was given in Eq.(13) as

$$\varphi_{\pm}^{(m)} = \frac{A_{\pm}^{'(m)}}{\nu} e^{\mp i \epsilon_{\pm}^{'(m)}} e^{\nu z} e^{\pm i \nu y}$$

Employing the definition of the complex amplitude ratio as given in Eq.(16)

$$A_{\pm}^{(m)} = \frac{A_{\pm}^{'(m)}}{\omega} e^{\mp i \epsilon_{\pm}^{'(m)}}$$

one writes the radiation potential $\varphi_{\pm}^{(m)}$ in the form

$$\varphi_{\pm}^{(m)} = \frac{\omega A_{\pm}^{(m)}}{\nu} e^{\nu z} e^{\pm i \nu y} \tag{A-22}$$

Taking the incident wave as $\varphi_I = -\frac{iga}{\omega} e^{\nu z} e^{i\nu y}$, the integrand in Eq.(A-21) becomes

$$\left. \begin{aligned} \varphi_I \frac{\partial \varphi^{(m)}}{\partial n} - \varphi^{(m)} \frac{\partial \varphi_I}{\partial n} &= 0 && \text{on } S_{+\infty} \\ &= -2ga A_{-}^{(m)} e^{2\nu z} && \text{on } S_{-\infty} \end{aligned} \right\} \tag{A-23}$$

whereas for the incident wave $\varphi_I = -\frac{iga}{\omega} e^{\nu z} e^{-i\nu y}$, then

$$\left. \begin{aligned} \varphi_I \frac{\partial \varphi^{(m)}}{\partial n} - \varphi^{(m)} \frac{\partial \varphi_I}{\partial n} &= -2ga A_{+}^{(m)} e^{2\nu z} && \text{on } S_{+\infty} \\ &= 0 && \text{on } S_{-\infty} \end{aligned} \right\} \tag{A-24}$$

Setting Eqs.(A-23) and (A-24) in Eq.(A-21), and executing the integral $\int_{-\infty}^{0} e^{2\nu z} dz = \frac{1}{2\nu}$, one determines the wave-exciting force $F^{(m)}$ in the form

$$F_{\pm}^{(m)} = \frac{\rho g a}{\nu} A_{\mp}^{(m)} \tag{A-25}$$

where ± of $F_{\pm}^{(m)}$ indicates the direction of the incident waves oncoming from the negative and positive ends of the y-axis and ∓ of $A_{\mp}^{(m)}$ indicates the negative and positive ends of the y-axis, respectively.

When one compares the two equations, Eq.(A-19) and Eq.(A-21), and attempts to evaluate the wave forces on an individual body such as <u>a</u> or <u>b</u> , one realizes that the individual forces are not obtained because the integration in Eq.(A-21) requires the simultaneous presence of both bodies.

The failure of Haskind's method to determine the wave-induced forces on the individual bodies has been confirmed numerically. The amplitude ratios $A_{-}^{(m)}$ induced by the sources distributed on each body were evaluated and then the corresponding wave-exciting forces $F_{+}^{(m)}$ were determined according to the formula of Eq.(A-25). When these values were compared with the values determined by Eq.(10), after the complete solution of the diffraction problem, the limitations of the Haskind method became obvious.

APPENDIX B

TRANSFER OF THE FORCE DERIVATIVES

We consider the force derivatives induced by unit amplitude displacements η, ζ, φ of sway, heave, and roll, respectively, of rigidly connected twin cylinders — all defined with respect to a coordinate system through the origin 0 (see Fig.1a).

Since the swaying and heaving forces Y, Z and the rolling moment M are functions of the small displacements η, ζ, φ, i.e.,

$$Y(\eta,\zeta,\varphi) \quad , \quad Z(\eta,\zeta,\varphi) \quad \text{and} \quad M(\eta,\zeta,\varphi) \tag{B-1}$$

the linear terms of the force and moment derivatives

$$\begin{matrix} \frac{\partial Y}{\partial \eta} & \frac{\partial Y}{\partial \zeta} & \frac{\partial Y}{\partial \varphi} \\ \\ \frac{\partial Z}{\partial \eta} & \frac{\partial Z}{\partial \zeta} & \frac{\partial Z}{\partial \varphi} \\ \\ \frac{\partial M}{\partial \eta} & \frac{\partial M}{\partial \zeta} & \frac{\partial M}{\partial \varphi} \end{matrix} \tag{B-2}$$

representing the force and moment per unit amplitude of displacement can be expressed in terms of added mass and damping coefficients (see Table 1) as

$$\begin{aligned} \frac{\partial Y}{\partial \eta} &= \omega^2 m''_{SS} + i\omega N_{SS} \\ \frac{\partial Z}{\partial \eta} &= \omega^2 m''_{SH} + i\omega N_{SH} \\ \frac{\partial M}{\partial \eta} &= \omega^2 m''_{SR} + i\omega N_{SR} \end{aligned} \tag{B-3}$$

$$\frac{\partial Y}{\partial \zeta} = \omega^2 m''_{HS} + i\omega N_{HS}$$

$$\frac{\partial Z}{\partial \zeta} = \omega^2 m''_{HH} + i\omega N_{HH} \qquad \text{(B-4)}$$

$$\frac{\partial M}{\partial \zeta} = \omega^2 m''_{HR} + i\omega N_{HR}$$

$$\frac{\partial Y}{\partial \varphi} = \omega^2 m''_{RS} + i\omega N_{RS}$$

$$\frac{\partial Z}{\partial \varphi} = \omega^2 m''_{RH} + i\omega N_{RH} \qquad \text{(B-5)}$$

$$\frac{\partial M}{\partial \varphi} = \omega^2 m''_{RR} + i\omega N_{RR}$$

The transfer of the force derivatives (Eq.B-2) defined with respect to a coordinate system with origin 0 , to the coordinate system with origin at "G" (center of gravity above the waterline along the z-axis) is to be considered.

Let the motions about G be η', ζ', φ' and let the forces Y',Z',M' be the functions of η',ζ',φ':

$$Y'(\eta',\zeta',\varphi') \quad , \quad Z'(\eta',\zeta',\varphi') \quad , \quad M'(\eta',\zeta',\varphi') \qquad \text{(B-6)}$$

Since the motions about 0 and G are small,

$$\begin{aligned} \eta &= \eta' + \overline{OG}\ \varphi' \\ \zeta &= \zeta' \\ \varphi &= \varphi' \end{aligned} \qquad \text{(B-7)}$$

The two force systems (Y,Z,M) , (Y',Z',M') are related as follows:

$$\begin{aligned} Y'(\eta',\zeta',\varphi') &= Y(\eta,\zeta,\varphi) \\ Z'(\eta',\zeta',\varphi') &= Z(\eta,\zeta,\varphi) \\ M'(\eta',\zeta',\varphi') &= M(\eta,\zeta,\varphi) + \overline{OG} \cdot Y(\eta,\zeta,\varphi) \end{aligned} \qquad \text{(B-8)}$$

The differentials of the forces and displacements are written as

$$dY = \frac{\partial Y}{\partial \eta} d\eta + \frac{\partial Y}{\partial \zeta} d\zeta + \frac{\partial Y}{\partial \varphi} d\varphi$$

$$dZ = \frac{\partial Z}{\partial \eta} d\eta + \frac{\partial Z}{\partial \zeta} d\zeta + \frac{\partial Z}{\partial \varphi} d\varphi \qquad \text{(B-9)}$$

$$dM = \frac{\partial M}{\partial \eta} d\eta + \frac{\partial M}{\partial \zeta} d\zeta + \frac{\partial M}{\partial \varphi} d\varphi$$

$$dY' = \frac{\partial Y'}{\partial \eta'} d\eta' + \frac{\partial Y'}{\partial \zeta'} d\zeta' + \frac{\partial Y'}{\partial \varphi'} d\varphi'$$

$$dZ' = \frac{\partial Z'}{\partial \eta'} d\eta' + \frac{\partial Z'}{\partial \zeta'} d\zeta' + \frac{\partial Z'}{\partial \varphi'} d\varphi' \qquad \text{(B-10)}$$

$$dM' = \frac{\partial M'}{\partial \eta'} d\eta' + \frac{\partial M'}{\partial \zeta'} d\zeta' + \frac{\partial M'}{\partial \varphi'} d\varphi'$$

$$d\eta = d\eta' + \overline{OG}\ d\varphi'$$

$$d\zeta = d\zeta' \qquad \text{(B-11)}$$

$$d\varphi = d\varphi'$$

$$dY' = dY$$

$$dZ' = dZ \qquad \text{(B-12)}$$

$$dM' = dM + \overline{OG}\ dY$$

Hence,

$$\frac{\partial Y'}{\partial \eta'} d\eta' + \frac{\partial Y'}{\partial \zeta'} d\zeta' + \frac{\partial Y'}{\partial \varphi'} d\varphi' = \frac{\partial Y}{\partial \eta} (d\eta' + \overline{OG}\ d\varphi') + \frac{\partial Y}{\partial \zeta} d\zeta' + \frac{\partial Y}{\partial \varphi} d\varphi'$$

$$\frac{\partial Z'}{\partial \eta'} d\eta' + \frac{\partial Z'}{\partial \zeta'} d\zeta' + \frac{\partial Z'}{\partial \varphi'} d\varphi' = \frac{\partial Z}{\partial \eta} (d\eta' + \overline{OG}\ d\varphi') + \frac{\partial Z}{\partial \zeta} d\zeta' + \frac{\partial Z}{\partial \varphi} d\varphi' \qquad \text{(B-13)}$$

[Cont'd]

$$\frac{\partial M'}{\partial \eta'} d\eta' + \frac{\partial M'}{\partial \zeta'} d\zeta' + \frac{\partial M'}{\partial \varphi'} d\varphi' = \frac{\partial M}{\partial \eta} (d\eta' + \overline{OG}\, d\varphi') + \frac{\partial M}{\partial \zeta} d\zeta' + \frac{\partial M}{\partial \varphi} d\varphi' + \overline{OG} \left\{ \frac{\partial Y}{\partial \eta}(d\eta' + \overline{OG}\, d\varphi') + \frac{\partial Y}{\partial \zeta} d\zeta' + \frac{\partial Y}{\partial \varphi} d\varphi' \right\} \quad \text{(B-13)}$$

The above are reduced to

$$\left. \begin{aligned} \frac{\partial Y'}{\partial \eta'} &= \frac{\partial Y}{\partial \eta} \\ \frac{\partial Y'}{\partial \zeta'} &= \frac{\partial Y}{\partial \zeta} \\ \frac{\partial Y'}{\partial \varphi'} &= \frac{\partial Y}{\partial \varphi} + \overline{OG} \cdot \frac{\partial Y}{\partial \eta} \end{aligned} \right\} \quad \text{(B-14)}$$

$$\left. \begin{aligned} \frac{\partial Z'}{\partial \eta'} &= \frac{\partial Z}{\partial \eta} \\ \frac{\partial Z'}{\partial \zeta'} &= \frac{\partial Z}{\partial \zeta} \\ \frac{\partial Z'}{\partial \varphi'} &= \frac{\partial Z}{\partial \varphi} + \overline{OG} \cdot \frac{\partial Z}{\partial \eta} \end{aligned} \right\} \quad \text{(B-15)}$$

$$\left. \begin{aligned} \frac{\partial M'}{\partial \eta'} &= \frac{\partial M}{\partial \eta} + \overline{OG} \cdot \frac{\partial Y}{\partial \eta} \\ \frac{\partial M'}{\partial \zeta'} &= \frac{\partial M}{\partial \zeta} + \overline{OG} \cdot \frac{\partial Y}{\partial \zeta} \\ \frac{\partial M'}{\partial \varphi'} &= \frac{\partial M}{\partial \varphi} + \overline{OG} \left(\frac{\partial M}{\partial \eta} + \overline{OG} \frac{\partial Y}{\partial \eta} + \frac{\partial Y}{\partial \varphi} \right) \end{aligned} \right\} \quad \text{(B-16)}$$

The transferred force derivatives in Eqs.(B-14,15,16) are readily presented in terms of $\overline{OG}$ and the added mass and damping coefficients, previously determined with respect to the origin 0 . It is to be noted that all the above formulas are valid for each single body as well as for the twin bodies.

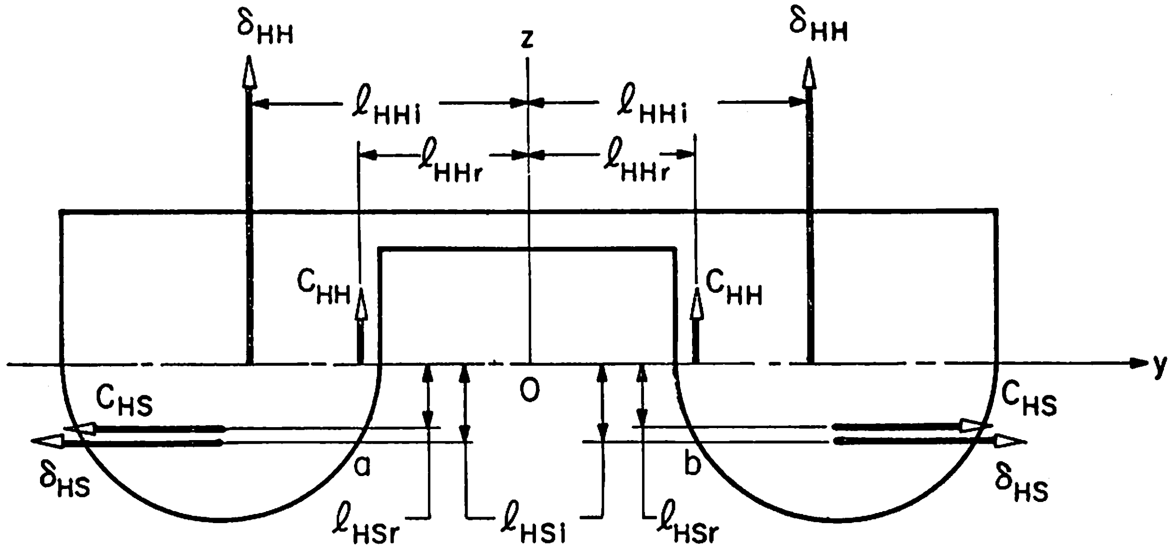


FIG. 1a. A TYPICAL SYSTEM OF FORCES INDUCED BY HEAVING MOTION

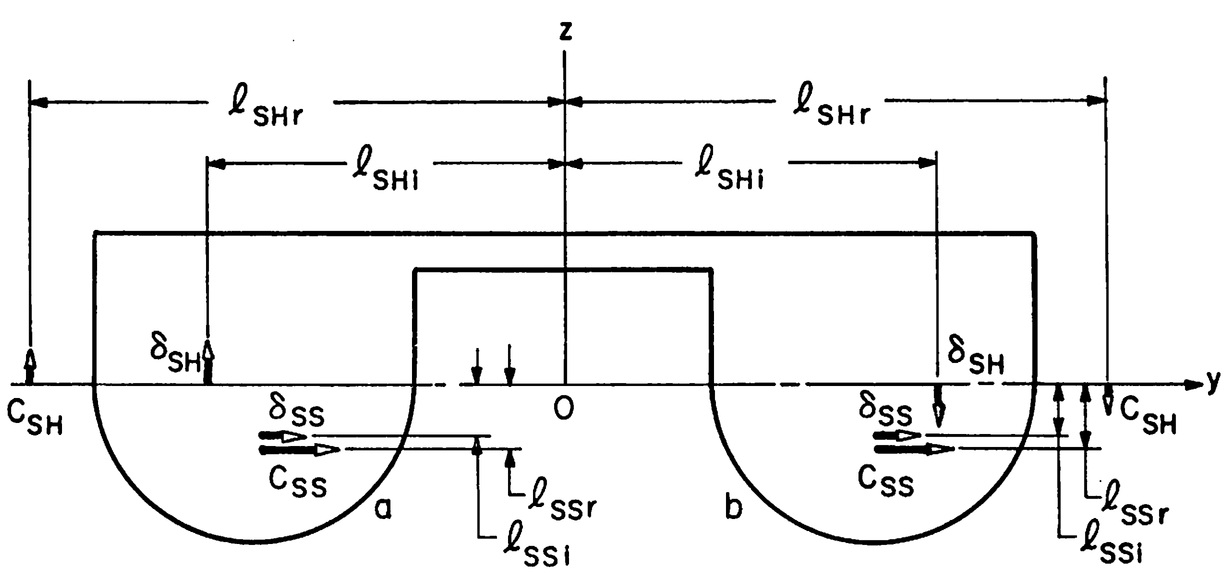


FIG. 1b. A TYPICAL SYSTEM OF FORCES INDUCED BY SWAYING MOTION

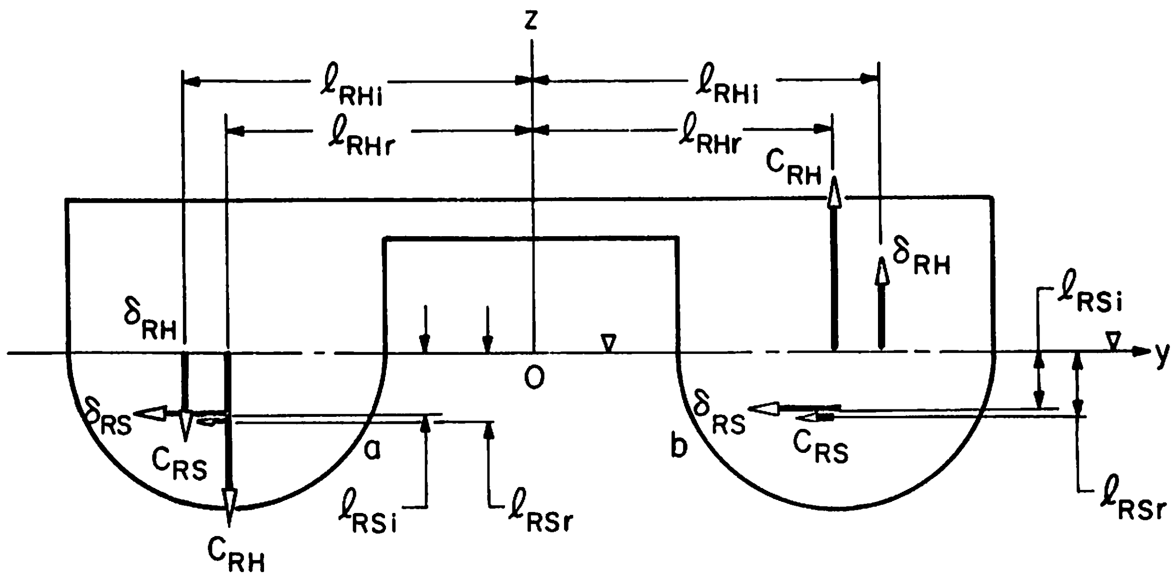


FIG. Ic. A TYPICAL SYSTEM OF FORCES INDUCED BY ROLLING MOTION

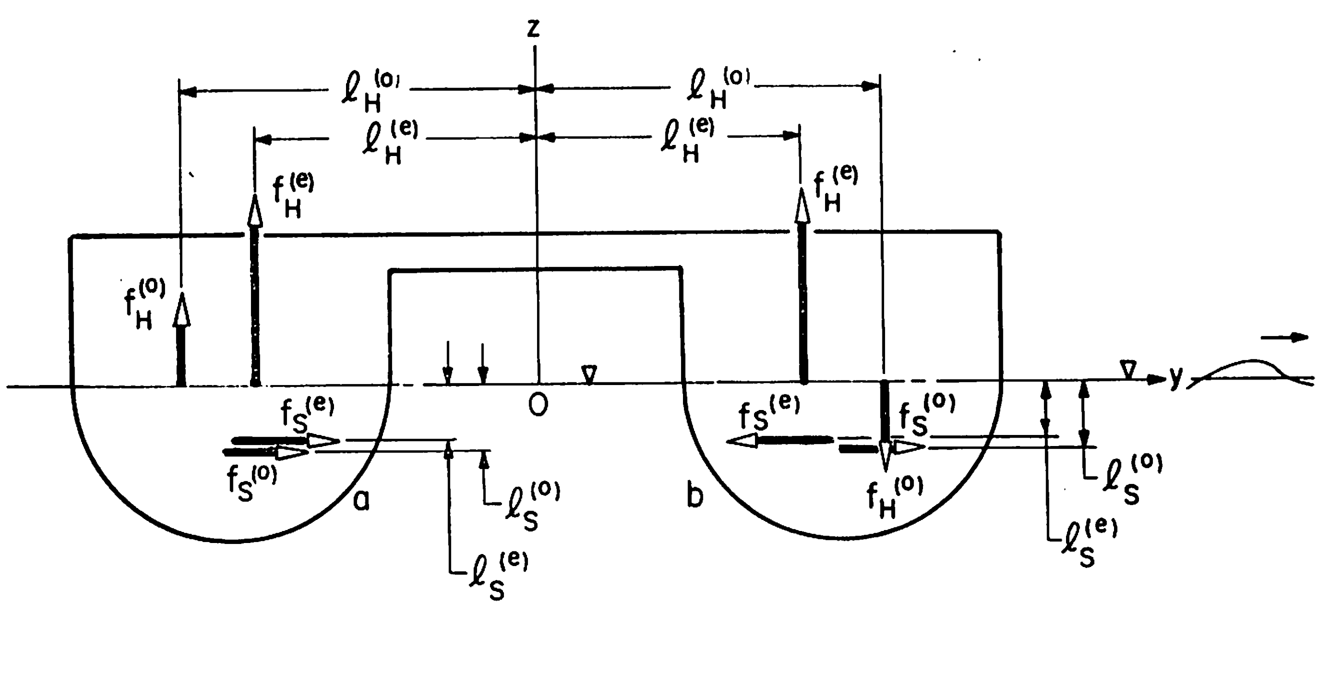


FIG. Id. A TYPICAL SYSTEM OF FORCES INDUCED BY WAVES

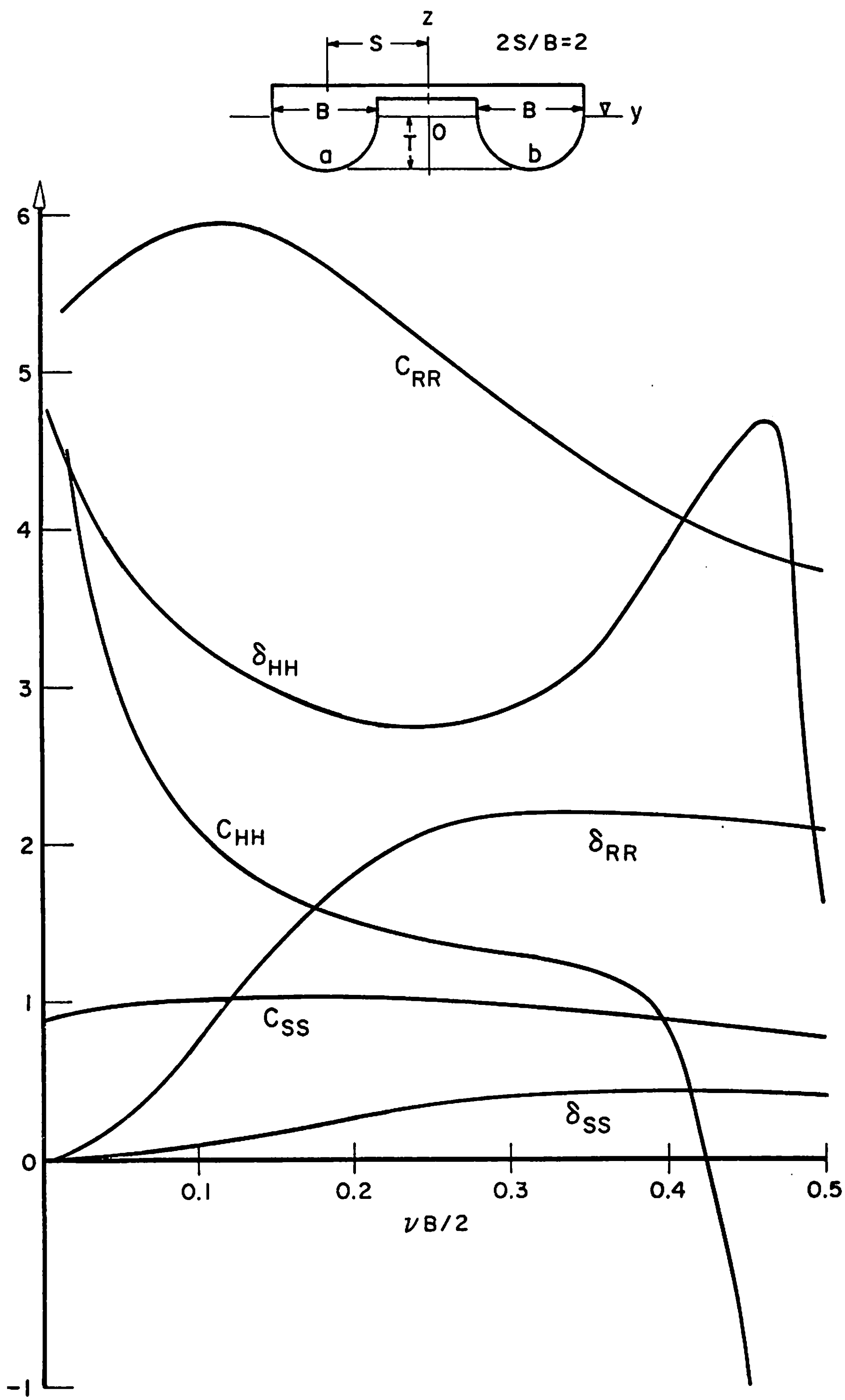


FIG. 2. HEAVING, SWAYING AND ROLLING ADDED MASS AND DAMPING COEFFICIENTS FOR CYLINDER a

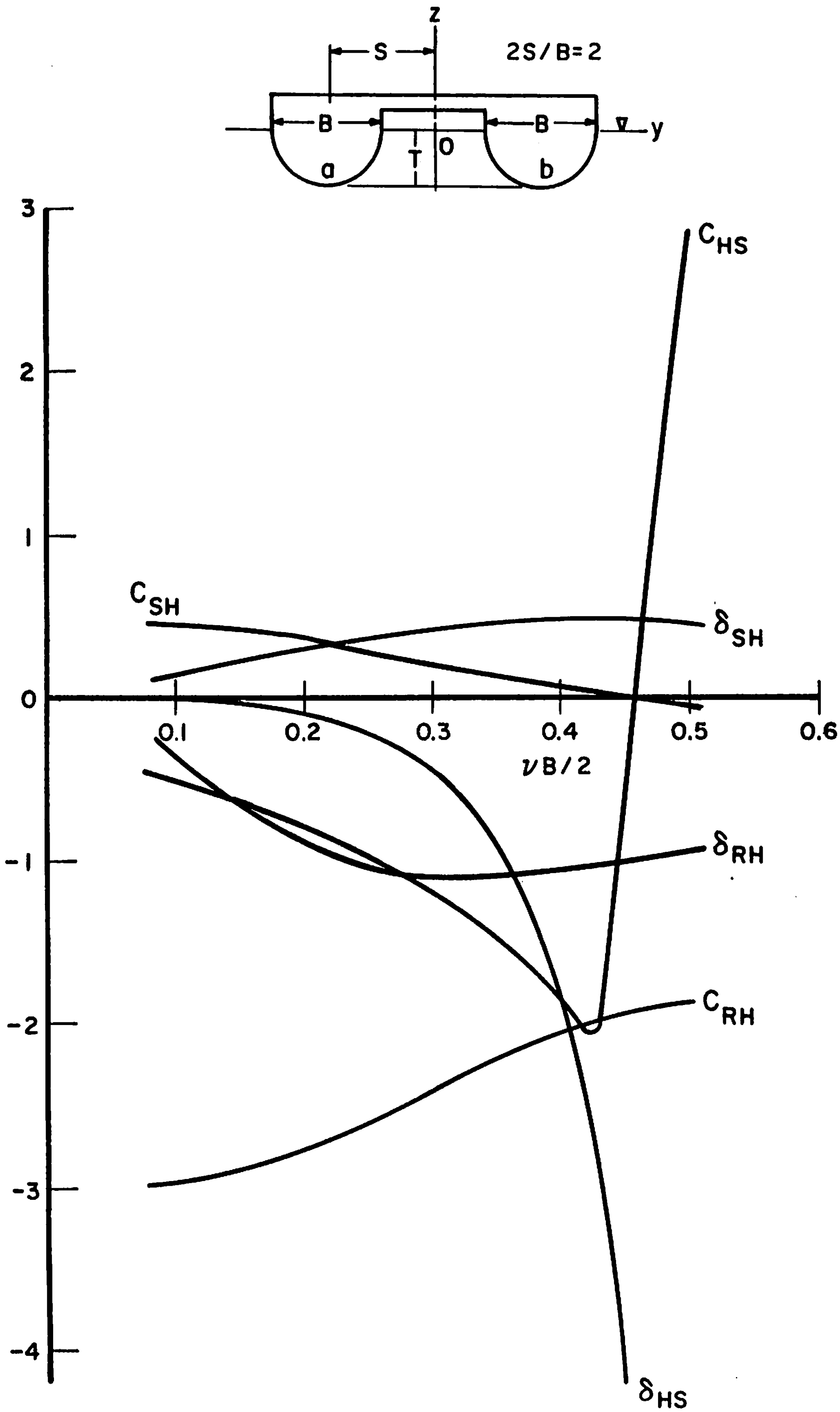


FIG. 3. HEAVE-INDUCED SWAYING, SWAY-INDUCED HEAVING, AND ROLL-INDUCED HEAVING AND SWAYING FORCES ON CYLINDER a

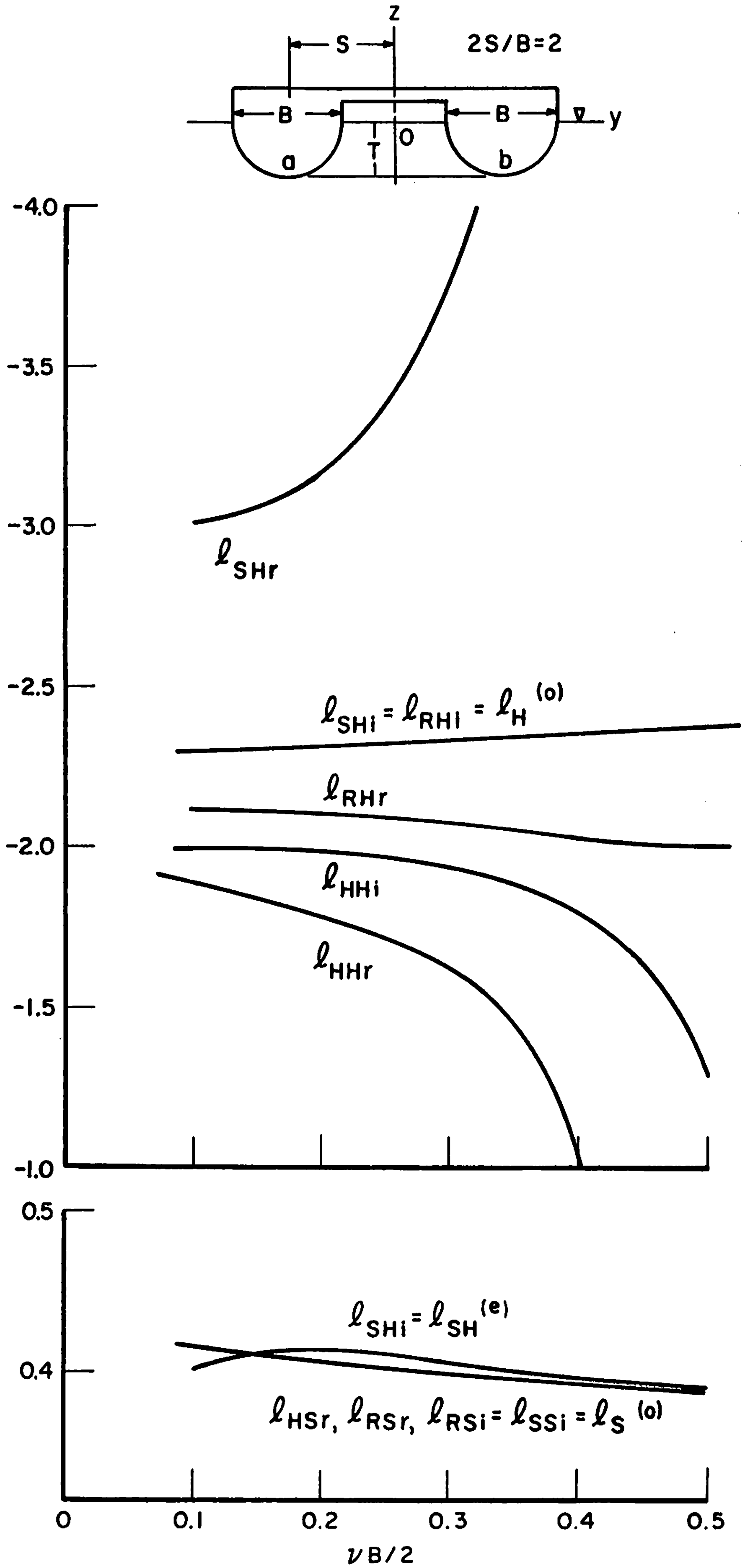


FIG. 4. HYDRODYNAMIC MOMENT ARMS ON CYLINDER a

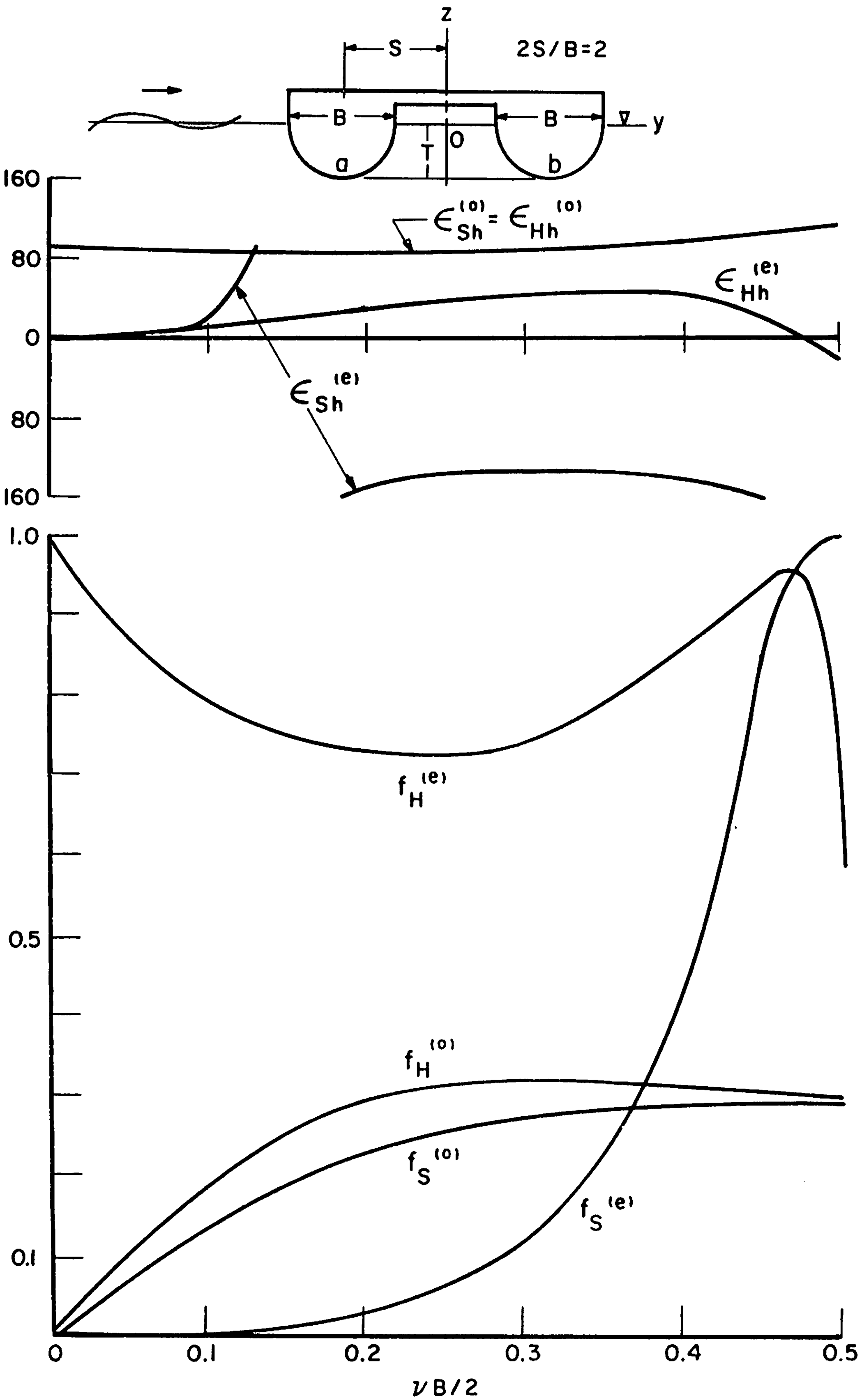


FIG. 5. THE SWAY-AND HEAVE-EXCITING FORCES ON CYLINDER a INDUCED BY EVEN AND ODD WAVES

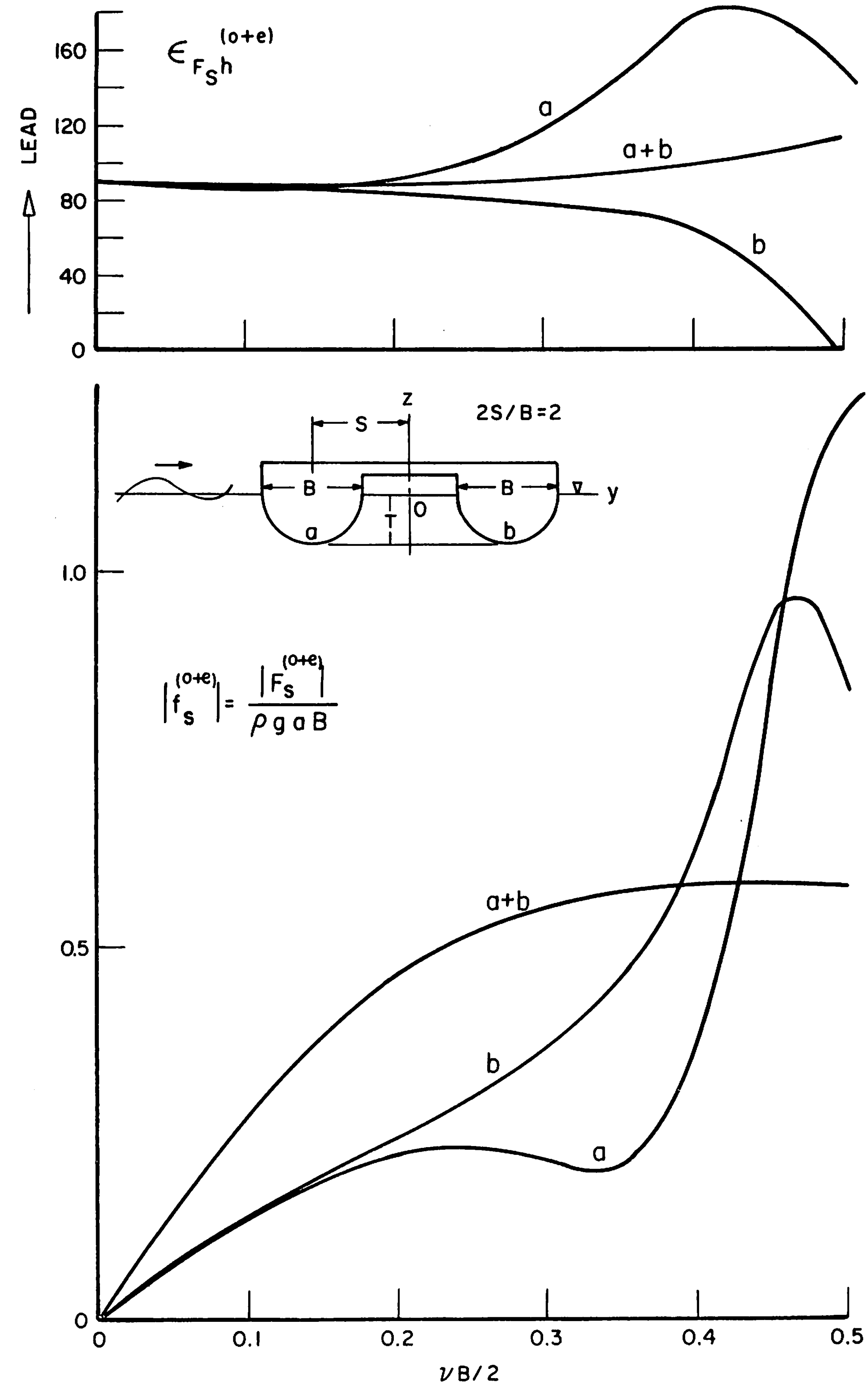


FIG. 6. THE SWAY-EXCITING FORCES ON CYLINDERS a,b AND a+b

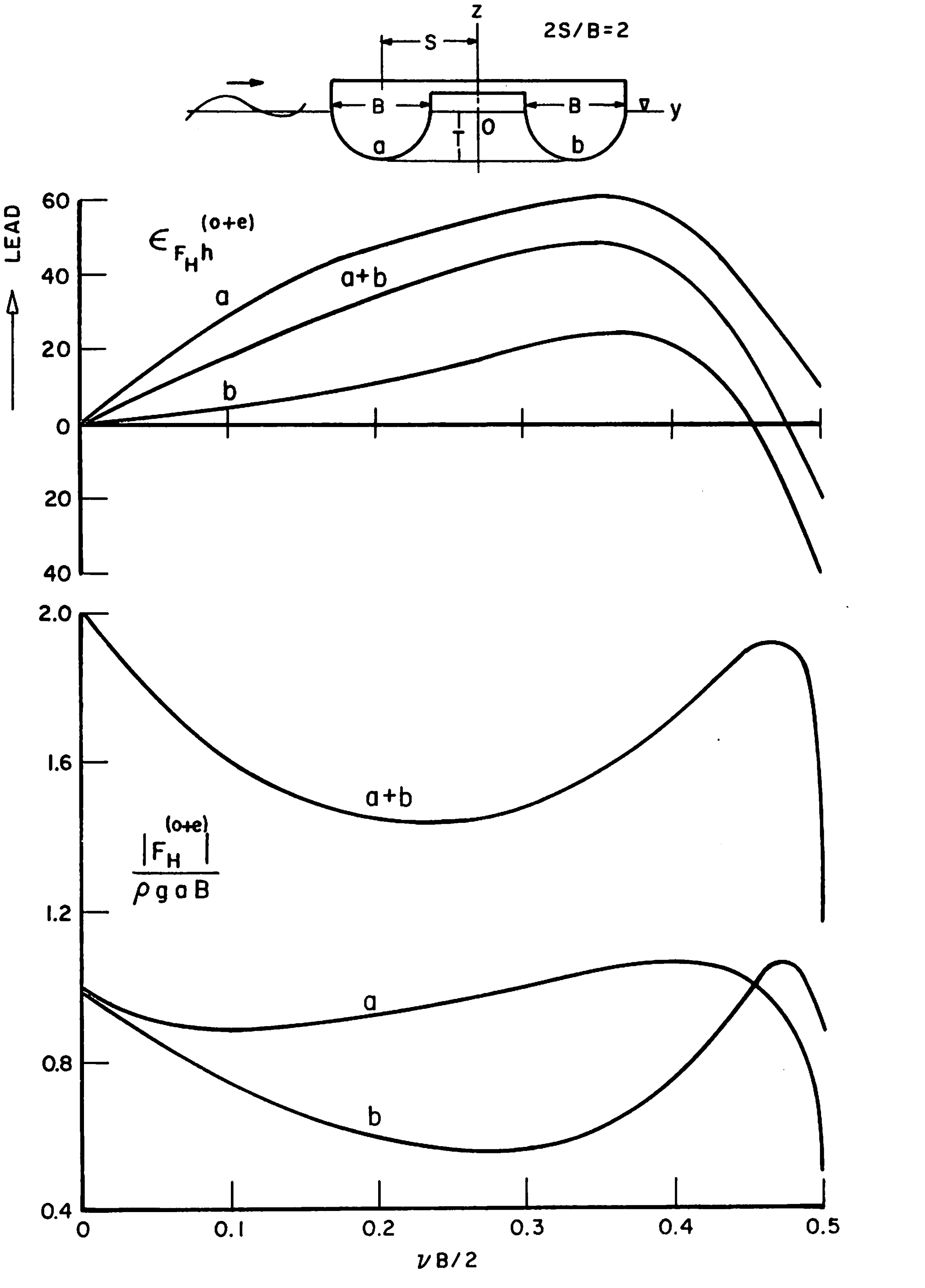


FIG. 7. THE HEAVE-EXCITING FORCES ON CYLINDERS a, b AND (a+b)

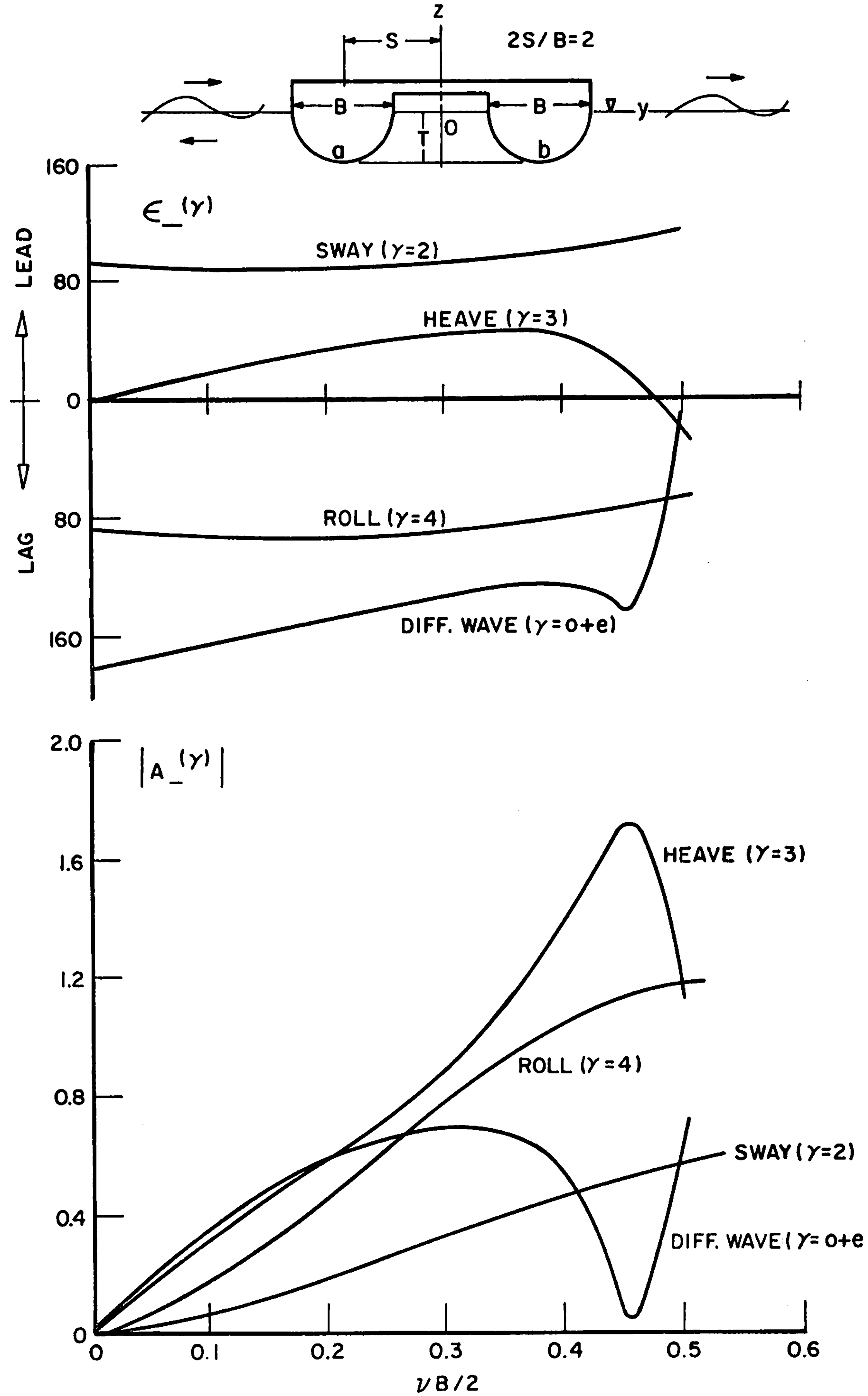


FIG. 8. THE RADIATED AND DIFFRACTED WAVES GENERATED FROM THE TWIN CYLINDERS

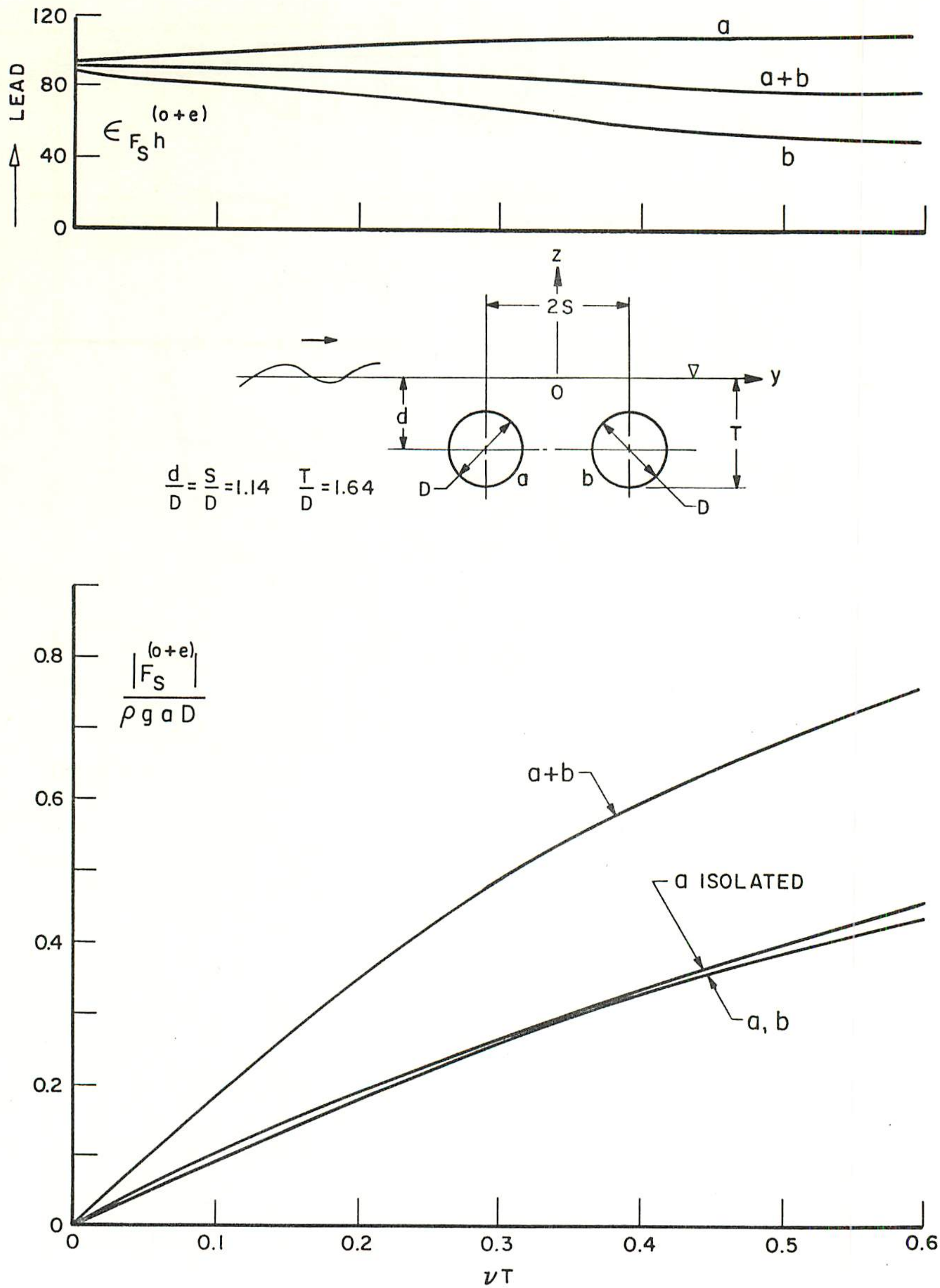


FIG. 9a. SWAY– EXCITING FORCES ON THE SUBMERGED TWIN CIRCULAR CYLINDERS

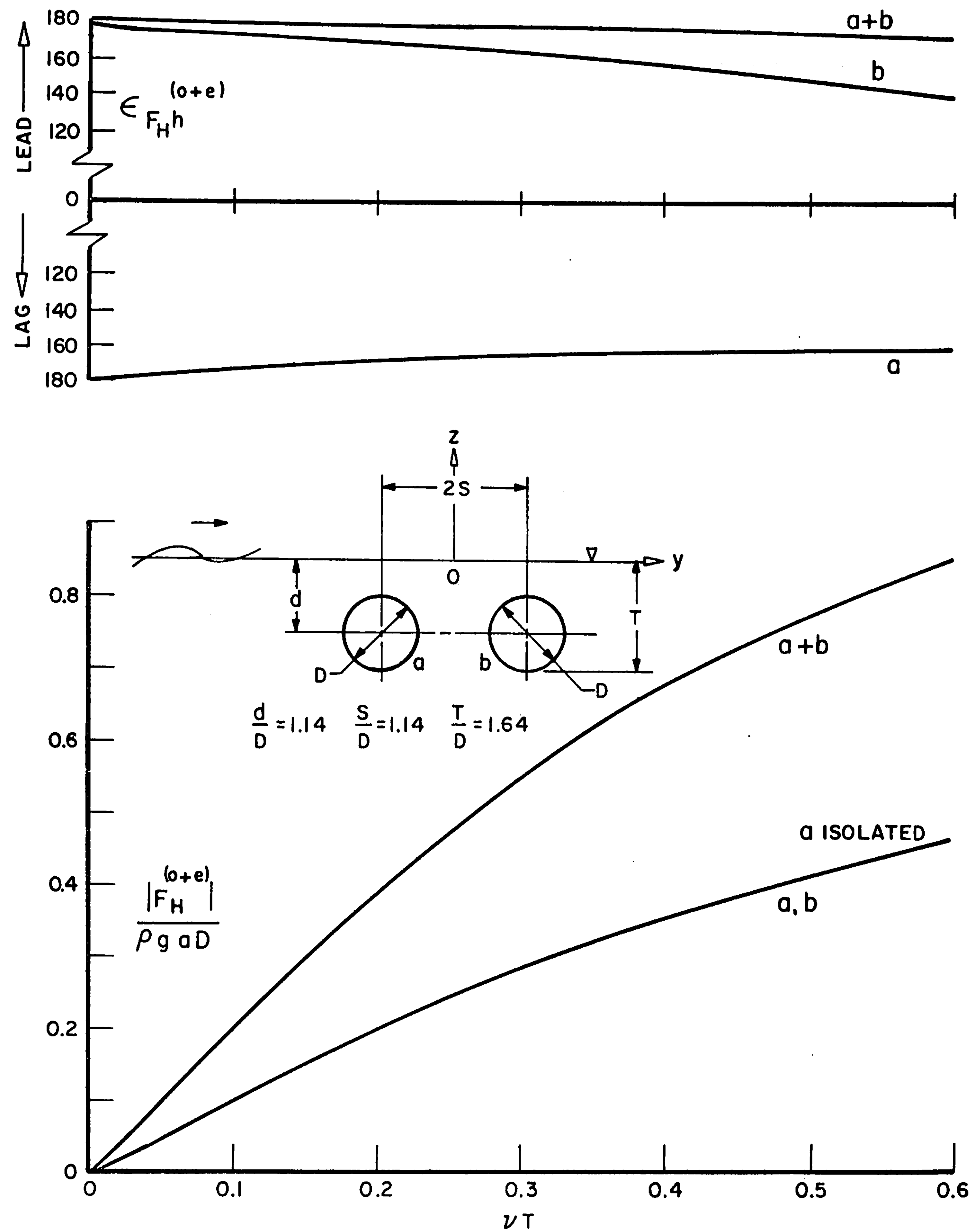


FIG.9b. HEAVE-EXCITING FORCES ON THE SUBMERGED TWIN CIRCULAR CYLINDERS

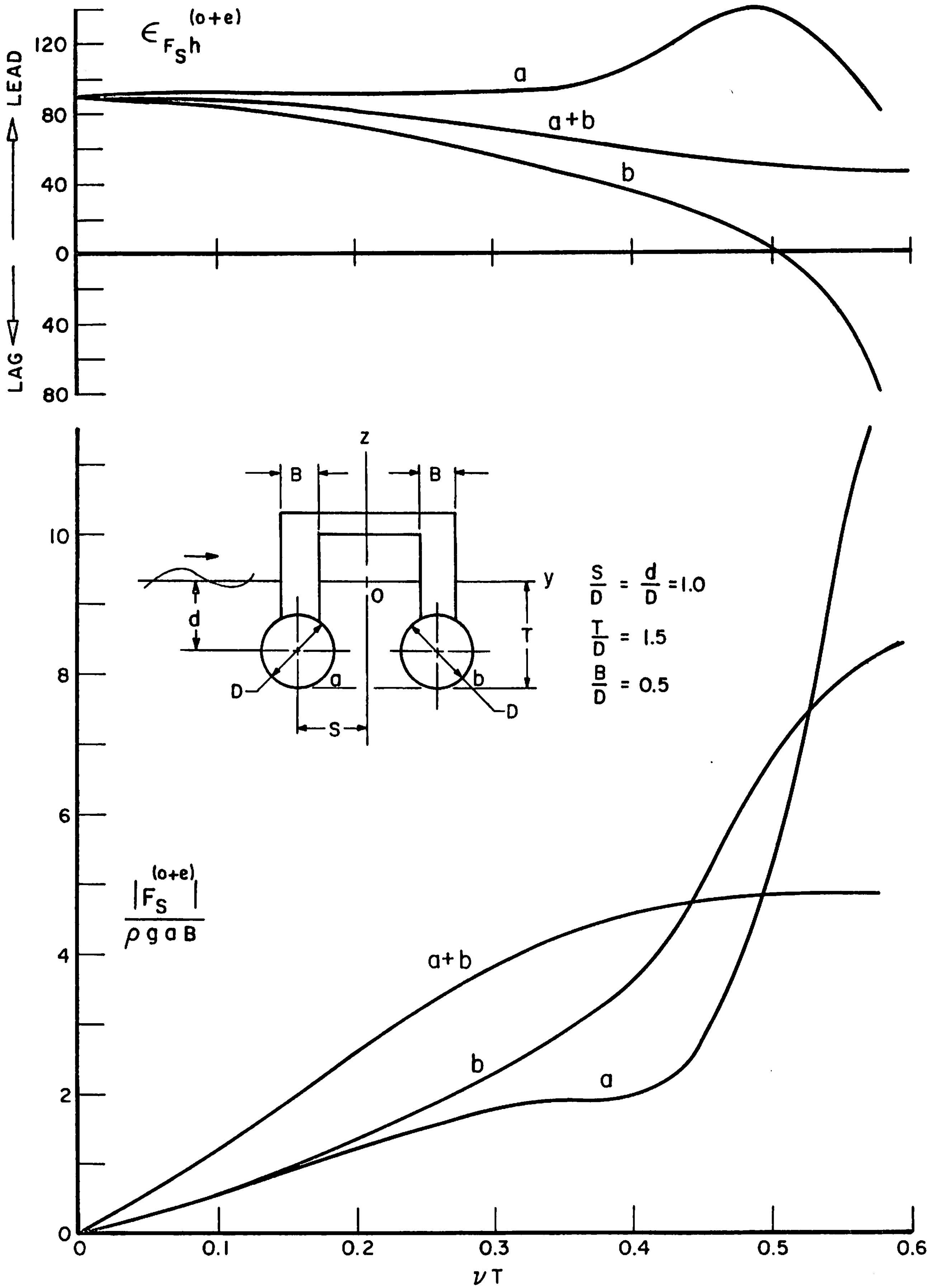


FIG.10a. THE SWAY-EXCITING FORCE ON CYLINDERS a,b AND a+b

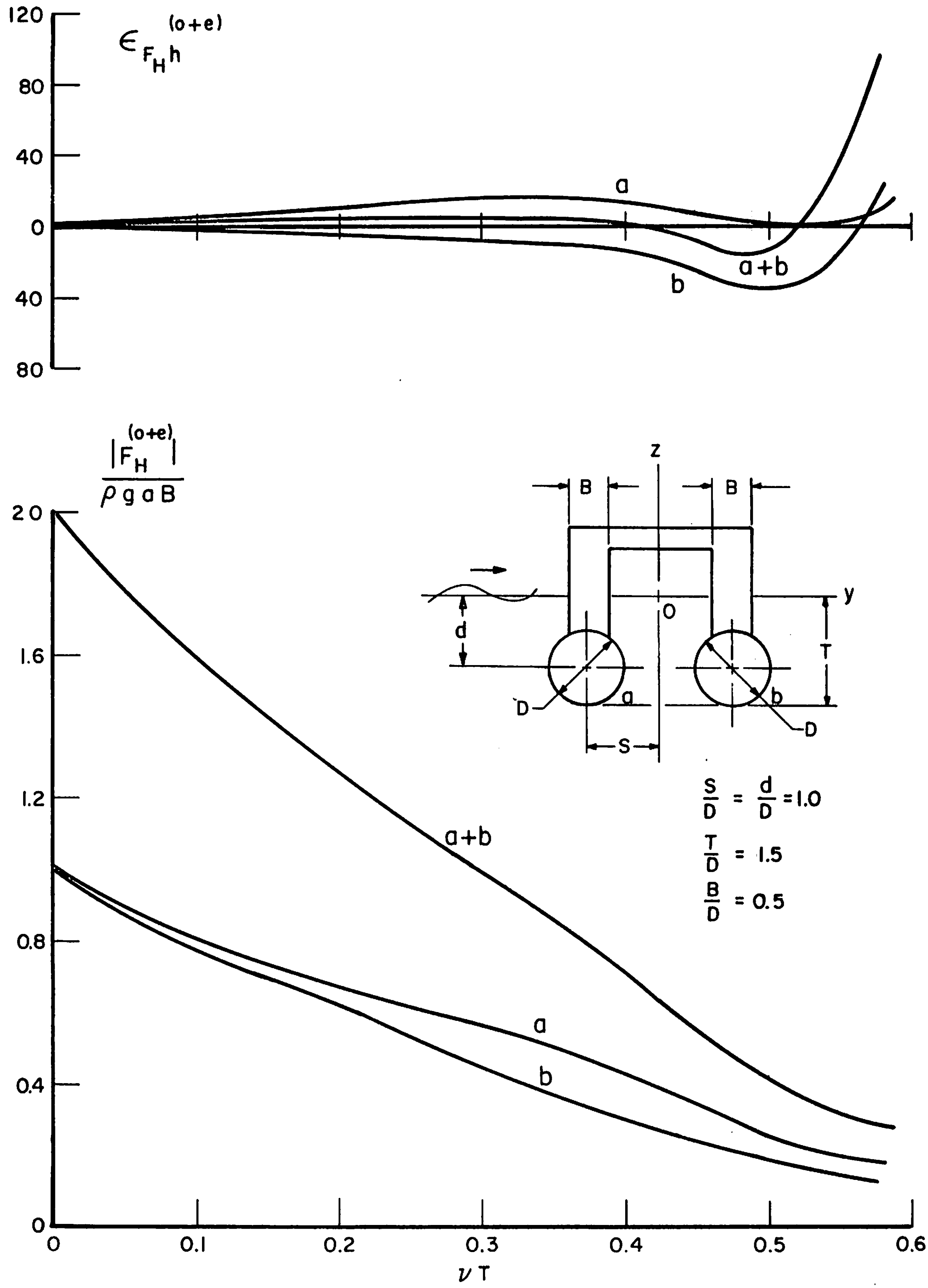
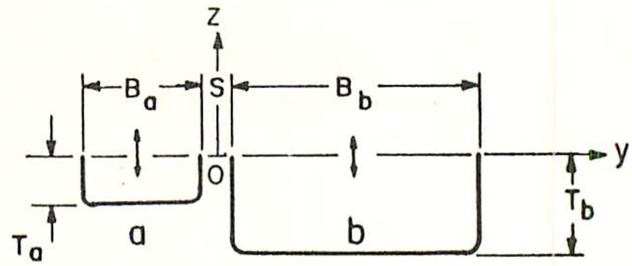


FIG. 10b. HEAVE-EXCITING FORCES ON CYLINDERS a, b AND a+b



$$\zeta_a \cos(\omega t + \epsilon_a)$$
$$\zeta_b \cos(\omega t + \epsilon_b)$$

$$\delta_a = \frac{N_a}{\omega \rho \frac{\pi}{2} T_a^2}, \quad \delta_b = \frac{N_b}{\omega \rho \frac{\pi}{2} T_b^2}$$

$$C_a = \frac{m_a''}{\rho \frac{\pi}{2} T_a^2} \quad C_b = \frac{m_b''}{\rho \frac{\pi}{2} T_b^2}$$

$$\frac{B_a}{2T_a} = \frac{B_b}{2T_b} = 1.25$$

$$\frac{B_a}{B_b} = 0.5$$

$$\frac{S}{B_b} = 0.1$$

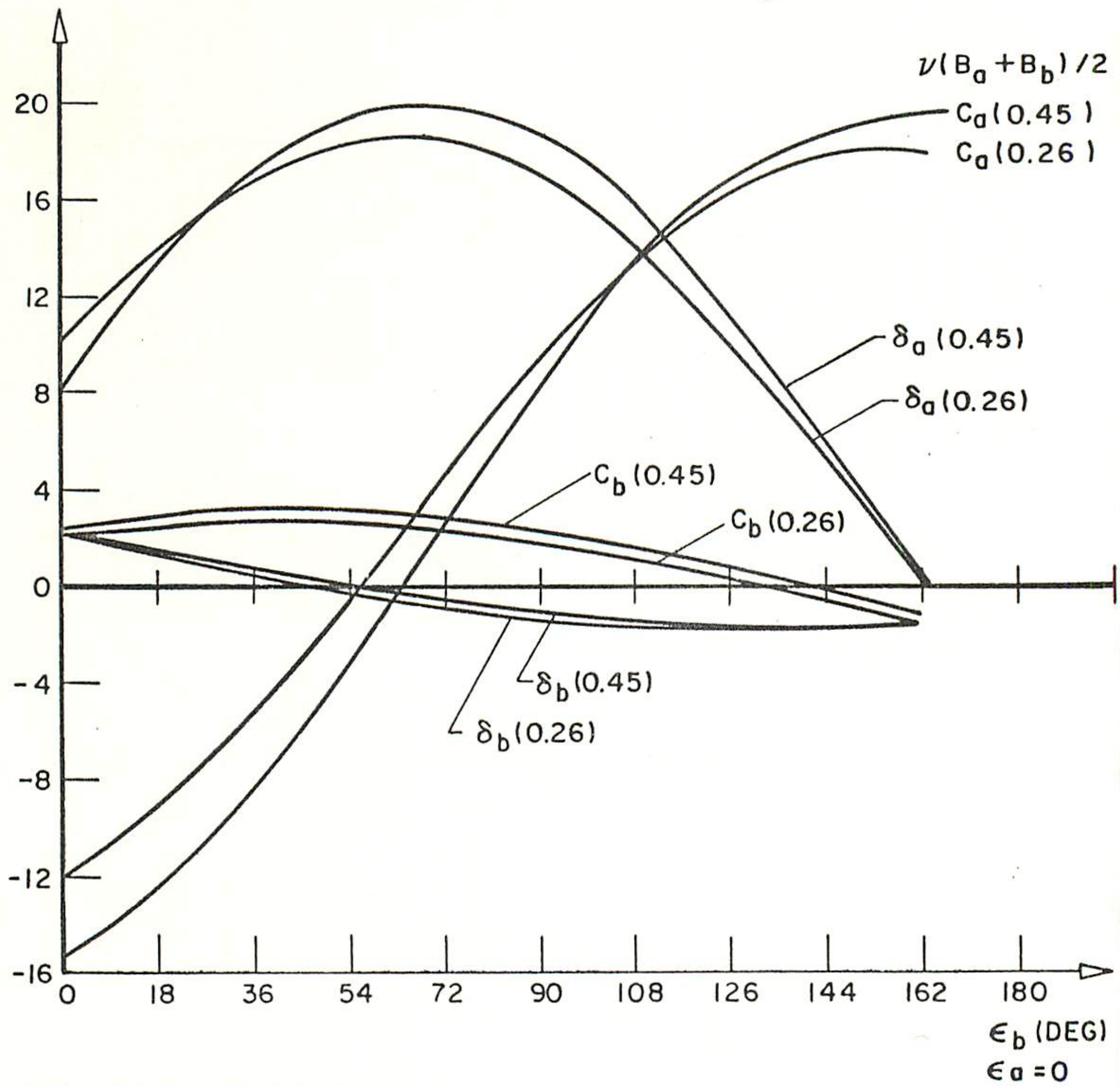


FIG. 11a. HYDRODYNAMIC CHARACTERISTICS OF TWO HEAVING CYLINDERS AS FUNCTION OF PHASE DIFFERENCE

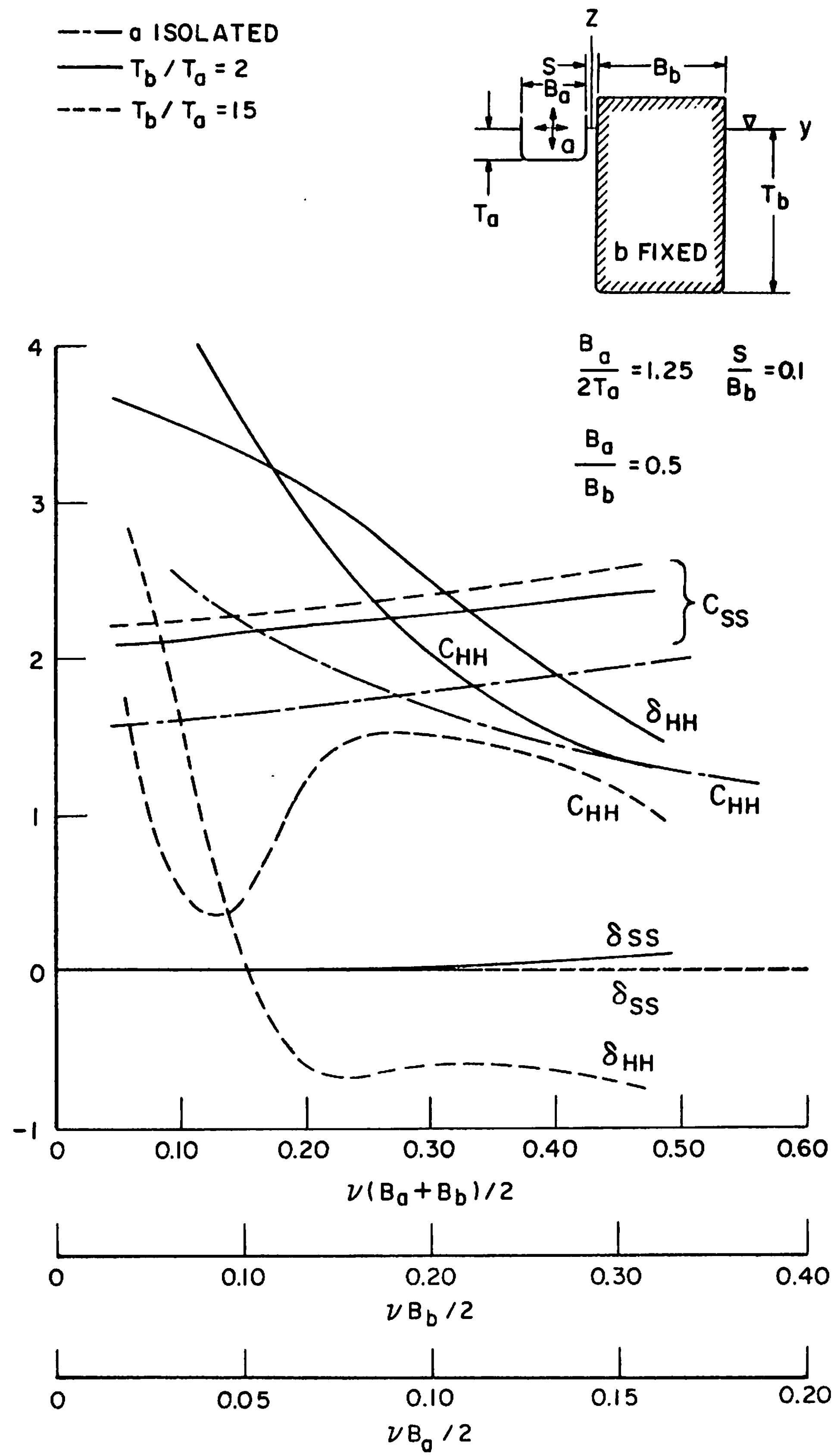


FIG. 11 b. SWAYING AND HEAVING HYDRODYNAMIC FORCES ON CYLINDER a INFLUENCED BY FIXED CYLINDER b

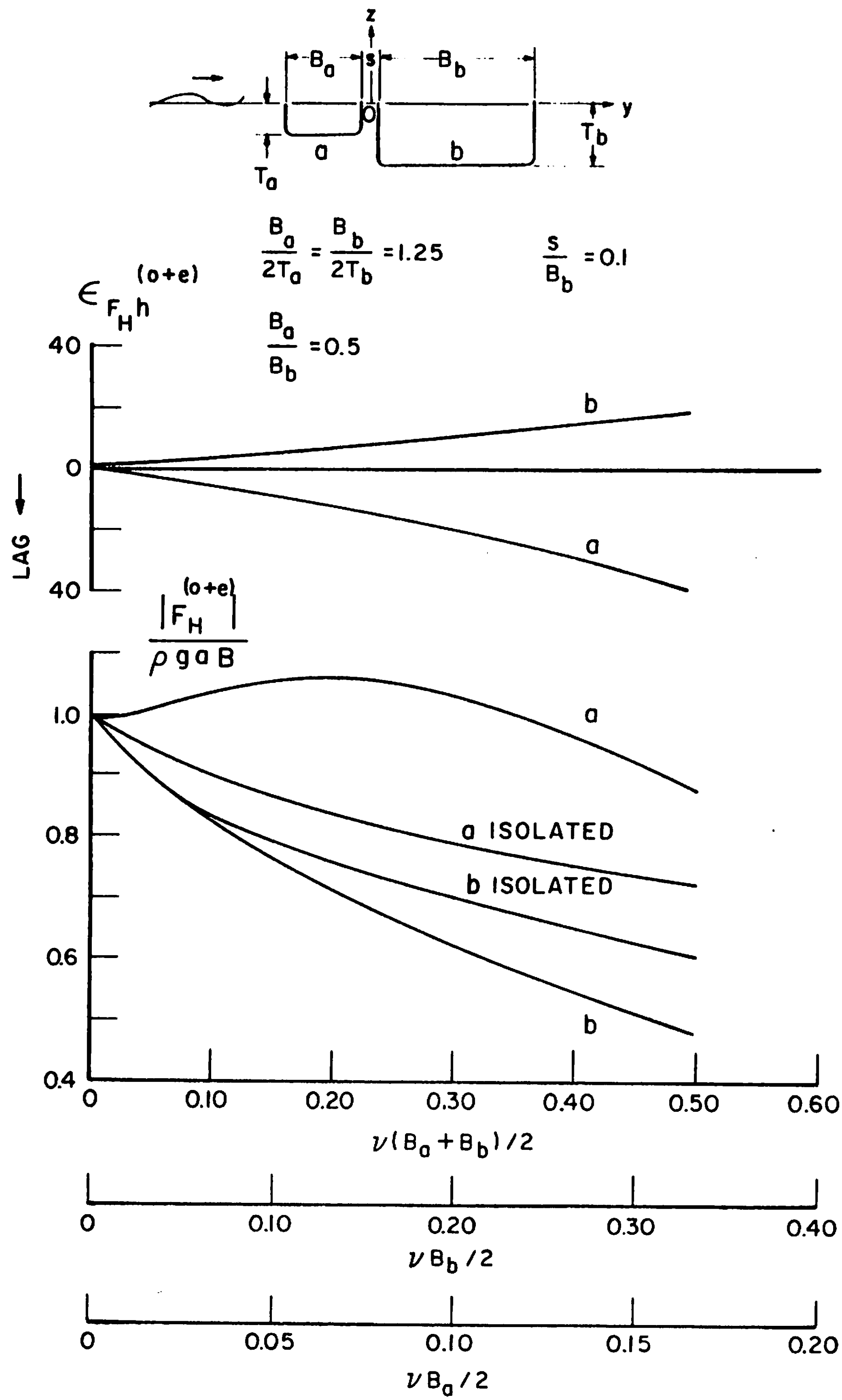
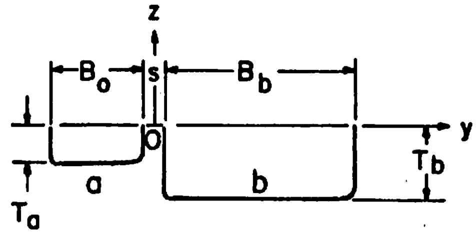


FIG. 11c. HEAVE-EXCITING FORCES ON CYLINDERS a AND b

$$C_{aa} = \frac{m''_{aa}}{\rho \frac{\pi}{2} T_a^2} \qquad C_{ba} = \frac{m''_{ba}}{\rho \frac{\pi}{2} T_a^2}$$

$$C_{ab} = \frac{m''_{ab}}{\rho \frac{\pi}{2} T_b^2} \qquad C_{bb} = \frac{m''_{ba}}{\rho \frac{\pi}{2} T_b^2}$$



$$\frac{B_a}{2T_a} = \frac{B_b}{2T_b} = 1.25 \qquad \frac{s}{B_b} = 0.1$$

$$\frac{B_a}{B_b} = 0.5$$

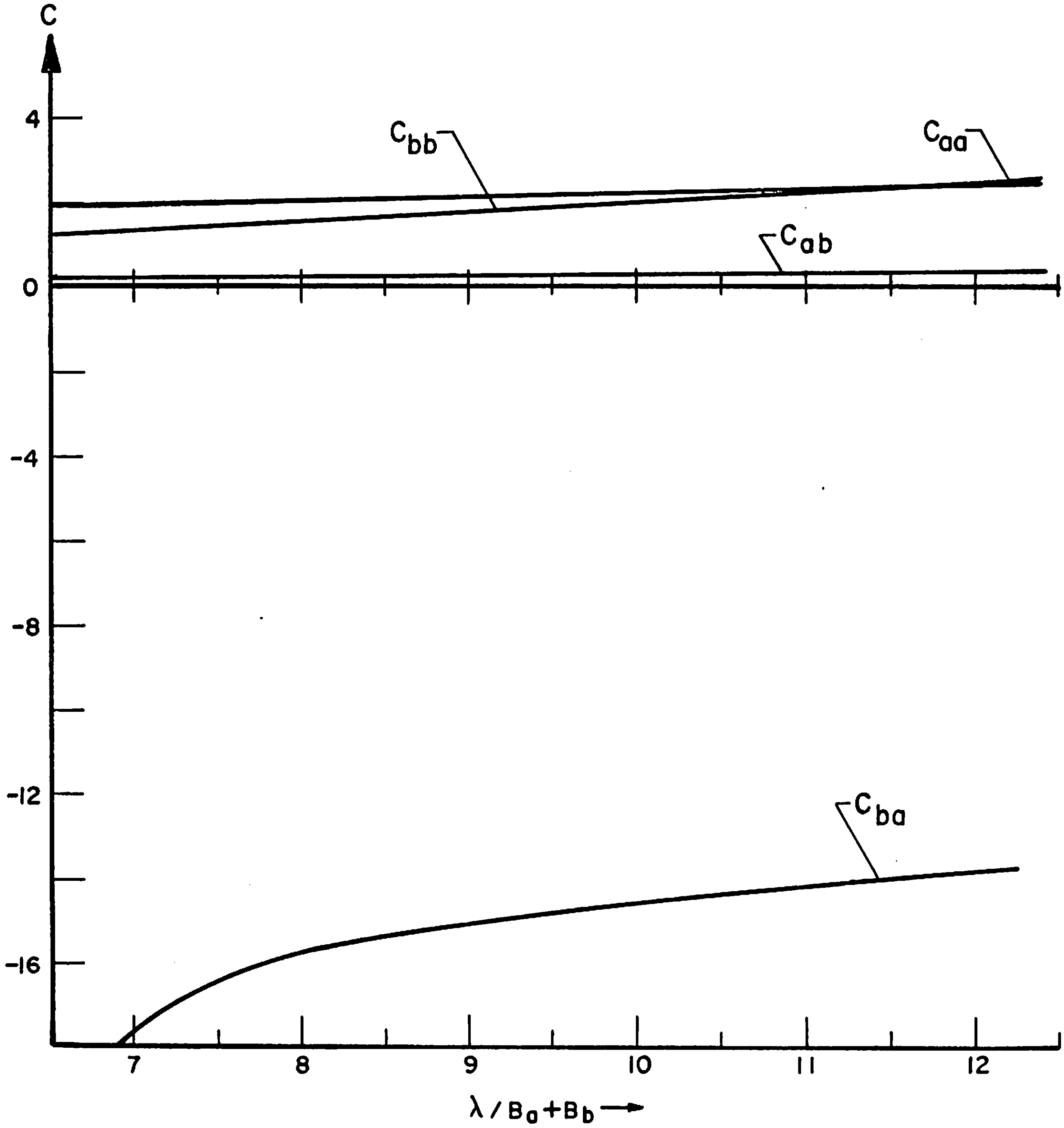
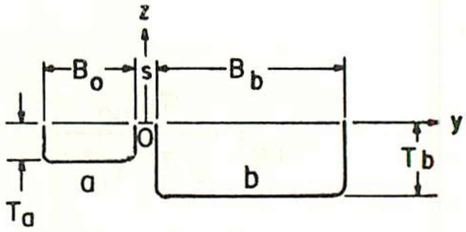


FIG. 12a. HEAVE ADDED MASS VERSUS WAVE LENGTH

$$\delta_{aa} = \frac{N_{aa}}{\omega\rho \frac{\pi}{2} T_a^2} \qquad \delta_{ba} = \frac{N_{ba}}{\omega\rho \frac{\pi}{2} T_a^2}$$

$$\delta_{ab} = \frac{N_{ab}}{\omega\rho \frac{\pi}{2} T_b^2} \qquad \delta_{bb} = \frac{N_{ab}}{\omega\rho \frac{\pi}{2} T_b^2}$$



$$\frac{B_a}{2T_a} = \frac{B_b}{2T_b} = 1.25 \qquad \frac{s}{B_b} = 0.1$$

$$\frac{B_a}{B_b} = 0.5$$

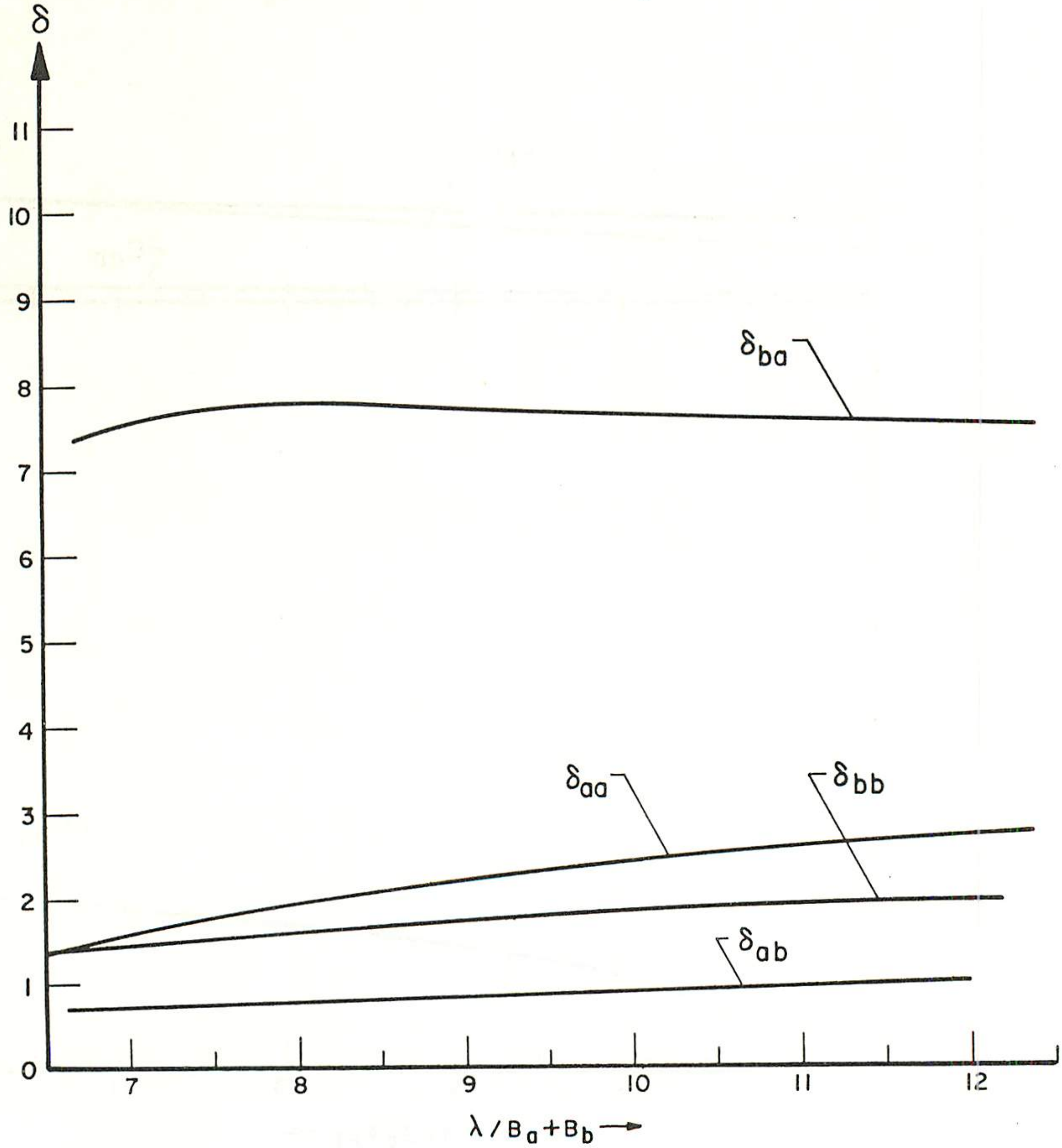


FIG. 12b. HEAVE DAMPING COEFFICIENTS VERSUS WAVE LENGTH

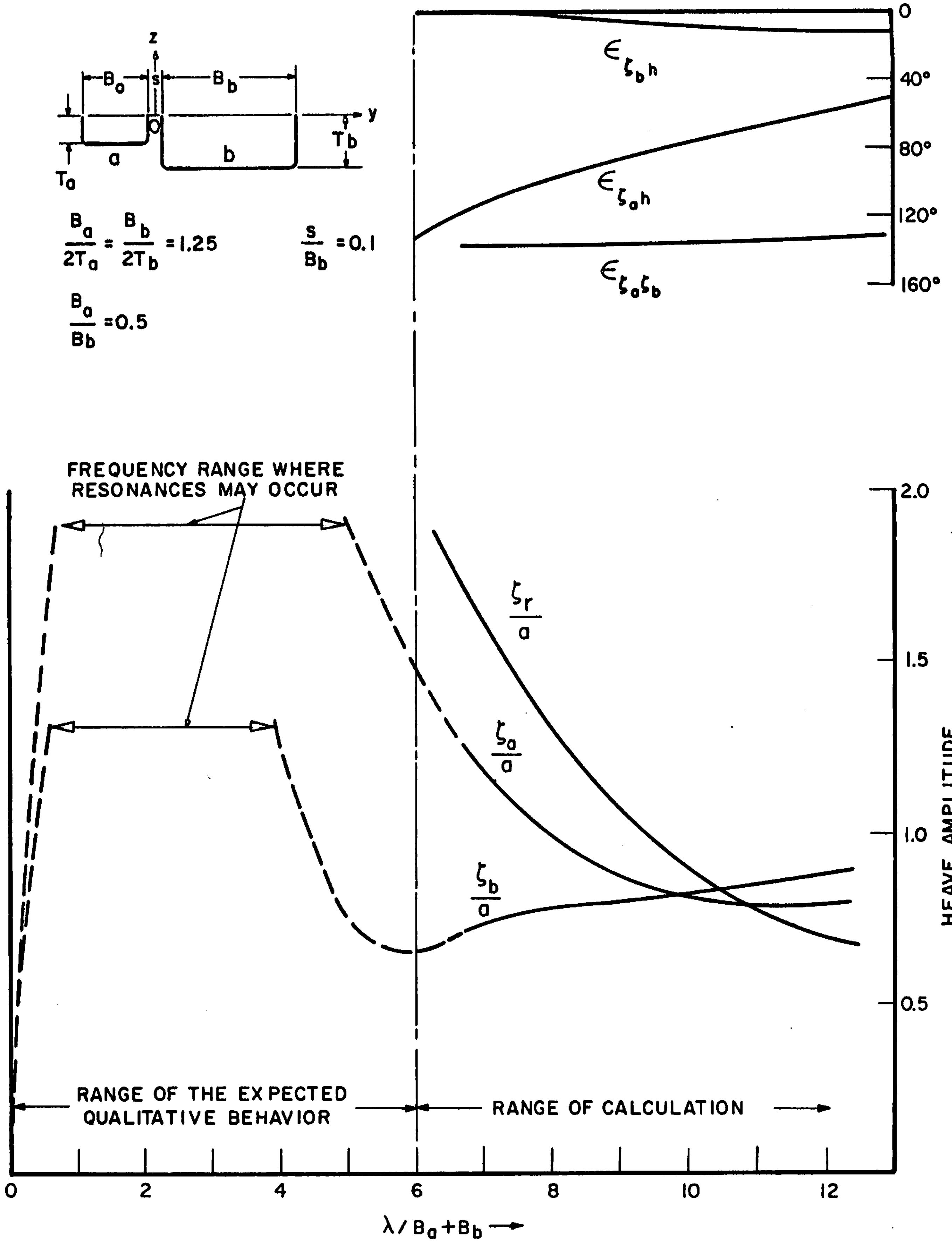


FIG. 12c. HEAVE RESPONSES OF BODIES a AND b VERSUS WAVE LENGTH

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