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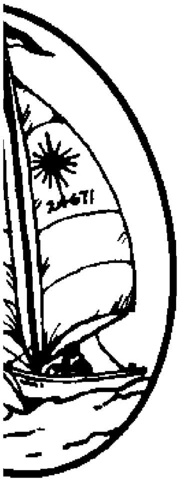
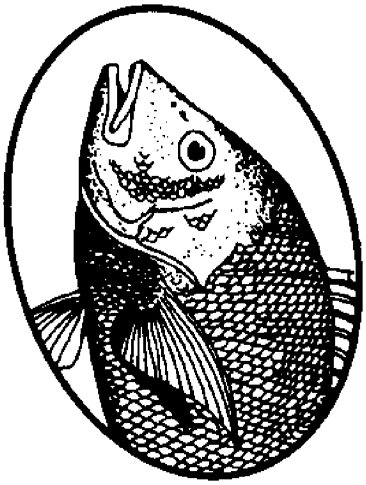
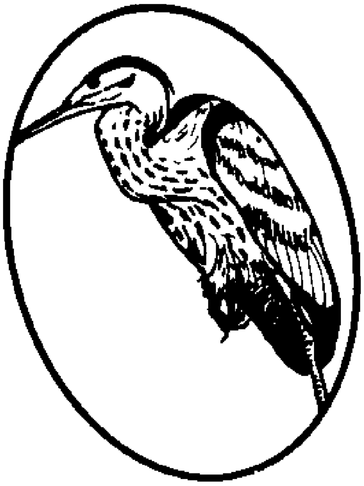
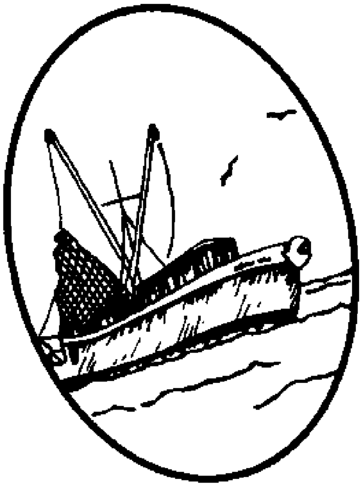
Modeling Estuarine Migration and Abundance Of the Brown Shrimp (Penaeus Aztecus) Of Pamlico Sound, North Carolina

Marc-david Cohen and George S. Fishman









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MODELING ESTUARINE MIGRATION AND ABUNDANCE
OF THE BROWN SHRIMP (PENAEUS AZTECUS)
OF PAMLICO SOUND, NORTH CAROLINA

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ABSTRACT

An important aspect of fishery management concerns the relationship between samples of the fish population, the true state of the population, and management prerogatives. This relationship is particularly important to the brown shrimp (Penaeus aztecus) fishery of Pamlico Sound, North Carolina. These shrimp have an annual life cycle that begins in the early spring with migrations as postlarvae from the ocean to the upper reaches of the Pamlico Sound estuaries. In these estuarine nursery grounds they grow to the juvenile stage and with growth migrate toward the ocean. In an effort to assess the size of the commercial population and to help with management decisions, marine biologists sample the length and abundance of the juvenile estuarine population. One important management decision that is based on these data is the time at which to open the commercial shrimping season.

This technical report describes a component of a methodology, based on computer simulation sampling, for the analysis of fishery management decisions of this type. It is the third in a series of five reports that describe the methodology. It discusses modeling the early part of the shrimp life cycle and in particular presents (1) a model of postlarvae shrimp recruitment to the estuaries of Pamlico Sound, (2) a model of juvenile migration from the estuaries, and (3) a model of estuarine shrimp abundance. Each of these models accounts for mean behavior and for random variation that is a significant element of the biological processes. The models also account for autocorrelation in estuarine abundance levels and for interestuarine correlation, which is shown to contribute significantly to variation in recruitment times and abundance levels.

The techniques for estimation of model parameters are given and are demonstrated with data from sampling of 54 nurseries over four years in Pamlico Sound. Furthermore, to demonstrate the simulation methodology, the report describes algorithms that model the spring sampling of the juvenile population. The report also shows statistics on length and abundance evaluated from computer sampling according to those algorithms.

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Introduction

Commercial fisheries often take advantage of the seasonal migration of a fish population. The brown shrimp fishery of Pamlico Sound offers a good example. As part of their life cycle, these shrimp migrate to the upper estuaries in the early spring (recruit to the primary nursery areas) and migrate from the nurseries to the transport or fishable areas during the late spring and summer (recruit to the secondary nurseries and transport areas). The local fishing industry adjusts its effort to coincide with migrations to the fishable areas. Consequently, effective management of the fish stock, typically involving some form of constraint on fishing effort, requires a comprehensive understanding of the relationship between these migration patterns and harvesting effort. As part of a larger study to develop methodologies for evaluating fishery management policies, this report addresses modeling in-migration to the nursery areas and out-migration from those areas to the fishable areas.

For each estuary in-migration is characterized by an arrival time and an abundance level. Each exhibits seasonal variation with an annual period and also shows within-year random behavior. Moreover, between-year random variation in arrival times and abundance levels can be substantial. These random components account for environmental and meteorological variation which affect the population and therefore the harvest. Consequently, they are important when modeling in-migration for management policy analysis. The dependence between estuaries is also an important consideration. For example, early (late) arrivals in one estuary increases the likelihood of early (late) arrivals in other nearby estuaries. This association and a similar interestuarine relationship for abundance levels are essential parts of the in-migration process and consequently are essential to a comprehensive characterization.

Out-migration also exhibits seasonal variation and random fluctuations associated with within-year and between-year variation. We characterize it using a variant of diffusion migration models discussed by Beverton and Holt (1957), Seber (1973), and Ricker (1975). However, our approach differs from theirs in emphasis and application. Rather than focusing on a descriptive analysis of out-migration we concentrate on a stochastic representation that as part of a simulation fishery model accounts for autocorrelation in estuarine abundance levels.

We use the stochastic approach in modeling both in-migration and out-migration. It enables one to sample the yearly arrival times and abundance levels in the estuaries and then to simulate the fishery under those nursery conditions. We call such a sample population an initial population profile. By including the stochastic elements of migration in a larger fishery model one can assess alternative management policies for naturally occurring environmental conditions. For example, with this approach one may learn that a specific management strategy performs particularly well when abundance levels are low or when arrival times are early. With this knowledge management would be alerted to the preferred strategy in a low abundance or early arrival year.

In particular, the report:

(1) Presents a model of in-migration to the primary nurseries.

(2) Presents a model of out-migration to the fishable (transport) areas.

(3) Presents a model of population abundance levels.

(4) Estimates the parameters of these models using data collected on the Pamlico Sound brown shrimp from 1974 to 1977 by the Division of Marine Fisheries of the Department of Natural Resources and Community Development, the State of North Carolina.

(5) Presents a procedure for sampling yearly initial population profiles from a computer representation of the migration and abundance models.

(6) Presents a procedure for computer sampling that models the marine biologists' sampling of the juvenile estuarine population. The report also shows graphic results from such a simulation.

This report is the third in a series of five reports, describing the submodels which are the building blocks of a methodology for management policy evaluation. The first report, Cohen and Fishman (1980), concentrates on modeling growth and the weight-length relationship. The second report, Cohen and Fishman (1982), addresses the biomass-revenue and the mesh-weight relationships. The next report will focus on the relationship between catch biomass and fishing effort. Each of these papers supplies the details of an additional component of the fishery simulation model. The final report will demonstrate the usefulness and versatility of the methodology by exhibiting the results of a sequence of experiments performed with the model for the goal of evaluating a set of management strategies.

1. Biology of the Brown Shrimp of Pamlico Sound

The brown shrimp has an annual life cycle. The adults spawn offshore in open water in early spring and the young (postlarvae) migrate through the inlets into Pamlico Sound in February, March and April (Williams 1964). The postlarvae seek the low salinity, soft muddy regions of the upper estuaries (called primary nurseries) as growing grounds (Williams 1958). The mechanisms by which the postlarvae enter the inlets and traverse the sound are unknown. However, there is evidence that copious migrations occur during flood tides (Williams 1964, 1969) and on new moons (Williams 1964, Williams and Deubler 1968). This suggests that in addition to the annual periodic component a monthly or biweekly component may contribute to the time of peak recruitment.

Within the nursery environment the postlarvae grow to the juvenile stage in four to six weeks (Perez Farfante 1969). As the juveniles mature they migrate downstream to the lower estuaries (secondary nurseries and transport areas) toward the higher salinity deeper water of the Pamlico Sound (Williams 1955). Migrations continue through summer until the population has migrated from the upper estuaries, to the sound, and finally to the open ocean to spawn, thus repeating the cycle (McCoy 1968). There is no evidence to correlate the between-year abundance levels or between-year harvest sizes (Hunt 1979). However, tagging studies have shown that shrimp within the sound migrate to the south and out of the nearest inlet, and once in the open ocean they continue their southern migration (McCoy 1968, McCoy 1972, McCoy and Brown 1967).

Many of the nurseries in the western Pamlico Sound, from Adams Creek in the south to Stumpy Point Bay in the north, produce shrimp in commercial quantities. To protect the young shrimp and other maturing species, policy has been to prohibit fishing in the primary nursery grounds throughout the year (Shrimp Management Policy, 1978). Hence, brown shrimp are harvested outside the primary nursery areas, principally within the secondary nurseries, the transport areas, and the Pamlico Sound. Fishing commences with the opening of the shrimping season by the Division of Marine Fisheries. The commercial fishermen are eager to harvest in the secondary nurseries and transport areas to ensure that commercial sized shrimp neither escape to the ocean nor are captured by competing fishermen.

2. Description of Data

In an effort to associate the arrival times and abundance levels in the nursery areas with an opening date for the fishing season the Division of Marine Fisheries of the North Carolina Department of Natural Resources and Community Development regularly samples the nursery grounds of Pamlico Sound. The marine biologists of the division have fixed sampling locations, choosing several sites in each major estuarine system. Typically, samples are taken with a ¼ inch mesh trawl net under tow from 30 seconds to several minutes. The specific times depend upon the observed abundance level; long tows in low abundance situations.

For each tow, data consist of the frequency of observations in each of 14 length categories listed in Table 1. Each category is identified by the midpoint of the 10 mm interval defining its limits. Data so organized are easily representable in histogram form as well as suiting other needs of the marine biologists.

The data made available to us by the division were sampled once a month in the late spring to early summer from 1974 to 1977 in two regions, one north of the Pamlico River (northern estuaries), extending from Abel Bay to Stumpy Point Bay, and the other south of the Pamlico River (southern estuaries), extending from Adams Creek to Spring Creek. The 54 estuaries in the data were all identified as primary nurseries with the help of the marine biologists of the Division of Marine Fisheries. Table 2 names each one and lists the code assigned to it by the division. The table also lists an index i which identifies each of the sample sites in the development that follows.

The reader should note that data were collected to provide information about the pre-season state of the commercial shrimp population, not to model the migration processes. Accordingly, the models we consider have been restricted in scope to accommodate the limitations of the data.

3. Migration Models

This section presents models of in-migration and out-migration separately, concentrating on time of migration and parameter estimation.

3.1 In-migration to the Primary Nurseries

Three factors dominate the time of in-migration: 1) seasonal variation, 2) random variation, and 3) interestuarine dependence. Although the annual period of in-migration is its most apparent cyclical feature another structural component, observed by biologists, is discernible in the data. This component associates peak recruitment time (time of in-migration or arrival time to the primary nurseries) with flood tides. Compounding the effects of this component are random within-year and between-year variations whose numerous causes include estuarine salinity levels, river currents, wind currents and water temperature. However, no unequivocal structural model relating these causes and variational effects has been developed up to now. In fact, the mechanisms involved are largely a matter for conjecture.

Because of this gap in our understanding and the absence of field experiments for quantifying the relationship between these environmental factors and recruitment time we propose a simple stochastic characterization that results partly from a preliminary analysis of the data and partly from known qualitative features of in-migration.

If one could observe recruitment times directly, patterns in the data would suggest appropriate models of in-migrations. Unfortunately, these data are not available. However, one alternative is to estimate a recruitment time for each sampled shrimp then peruse these estimates for patterns. This is a feasible approach since the approximate length of each sampled shrimp is recorded in the data, and a model of shrimp growth can be used to estimate shrimp age. If one assumes that substantive growth begins after recruitment, this estimate and the known time of sampling yield an estimate of recruitment time. Section 3.2 describes the development of this estimation procedure.

The resulting analysis suggests the existence of at least two major recruitment periods or waves. For the northern sampling sites Table 3 shows estimates of the probability of recruitment in each spring week for the four year data. Since recruitment time is measured in weeks the probabilities define a probability mass function. This function assigns to each week the probability that a shrimp arrives in that week. For each site the modes or peaks of the mass function are identified by outlined rectangles. One sees that many of the estuaries have two modes whose peaks have considerable magnitude, with their sum often exceeding .70. This supports the contention that in-migration is composed of two recruitment waves. Furthermore, for the northern nurseries the time between recruitment waves (inter-recruitment wave period) is less than five weeks, a period shorter than that over which the data were collected.

This observed multi-wave recruitment is evidence of the relationship between recruitment and tides which has been noted in the past (Williams 1964, 1969). In particular, inter-recruitment wave periods on the order of a month parallel the long period components of the tidal forces. These components have fortnightly, monthly and semi-monthly periods, which result in tide height differences of over a foot in some estuaries (Giese, Wilder and Parker 1979, pp. 7-9).

The data also show a year in which recruitment occurs predominantly in one wave. Considerable recruitment in week 21 of 1976 indicates the favorable conditions during that period, and the consistency across estuaries shows the associative behavior among them.

The reader may also observe that these recruitment times are later than those noted by biologists (Williams 1955a, Williams 1955b, Williams 1964). This results from using in estimation a model of shrimp growth that does not account for slow growth in cold estuarine water in the spring. Consequently, the estimation process consistently underestimates fish age. This however, is no drawback when modeling the fishery for management policy analysis, since by also using this growth model in the fishery simulation model one obtains an accurate profile of the nursery population in mid-spring. This is important since mid-spring is when marine biologists sample the nursery for fishery management purposes. We demonstrate the ability of the simulation model to replicate nursery conditions in a nursery sampling experiment discussed in Section 5.3.

In view of these observations we characterize recruitment to the primary nurseries by at most two waves. Let τ_i^1 denote the time of the first wave and τ_i^2 the time of the second wave for site i. These quantities are integer valued random variables taking values between 1 and 52. They are related by

$$\tau_i^2 \geq \tau_i^1$$

with equality holding in the case of a single recruitment wave. We denote the time between recruitment waves by $\Delta\tau_i$ so that $\Delta\tau_i = \tau_i^2 - \tau_i^1$. Consequently, knowing τ_i^1 and $\Delta\tau_i$ is equivalent to knowing τ_i^1 and τ_i^2.

The distribution of peak recruitment times is also important, and a complete characterization of it includes provisions for the relationship among recruitment waves in the different estuaries, as well as the dependence between the first wave and the length of the period between waves. A characterization of the joint distribution of the vector $(\tau_i^1, \Delta\tau_i : i = 1,\ldots,54)$ accomplishes these goals. Furthermore, this multidimensional distribution fulfills the requirements of the simulation model for management strategy evaluation.

3.2 Estimating the Parameters of In-migration

Ideally, one estimates the joint distribution of the vector $(\tau_i^1, \Delta\tau_i : i = 1,\ldots,54)$ from a sample of the recruitment times. Since four years of data are too limited to support this extensive an endeavor, we restrict the estimation to: 1) the vector of means $(E\tau_i^1 : i = 1,\ldots,54)$, where $E\tau_i^1$ is the expected time of the first recruitment wave in estuary i, 2) the covariance matrix Σ_τ, where the entry in row i and column j is the covariance between the first recruitment wave in estuaries i and j, and 3) the probability mass functions of the time between recruitment waves. In section 5.1 we return to the problem of identifying the joint distribution.

Estimating the Mean Vector and Covariance Matrix of Recruitment Time.

Here we complete the details of the procedure for estimating the probability mass function of recruitment times as outlined in Section 3.1. In particular, we show how the mean vector $(E\tau_i^1 : i = 1,\ldots,54)$ and the covariance matrix Σ_τ are estimated.

As the procedure requires a characterization of shrimp growth we use the growth in length model described in Cohen and Fishman (1980). This model asserts that the length L of a Δt week old fish has probability density function

$$g(\ell|\Delta t) = \sum_{s=1}^{2} \rho_s \phi\left(\frac{\ell-\mu_L(\Delta t,s)}{\sigma_L(\Delta t,s)}\right) \qquad (3.1)$$

where

$$\phi(x) = \frac{1}{\sqrt{2\pi}}e^{-x^2/2}$$

is the standard normal density function, ρ_s is the probability that a shrimp has sex s (s=1 for female and s=2 for male), and $\mu_L(\Delta t,s)$ and $\sigma_L{}^2(\Delta t,s)$ are, respectively the mean and variance of L. Further description of these parameters is in Cohen and Fishman (1980).

Consider a shrimp with length $L=\ell$ at time t, and let T_0 denote the time it arrives at a primary nursery. Since we assume that shrimp growth begins at time of recruitment the shrimp is $\Delta T \equiv t - T_0$ weeks old. Although one cannot observe T_0 directly it is possible to estimate it. One way is to estimate age Δt and then use the relation $T_0 = t - \Delta T$. We follow this approach, and estimate ΔT as the age most likely for a shrimp with length ℓ. Then the estimate $\hat{t}_0$ of T_0 is the quantity that satisfies

$$g(\ell|t-\hat{t}_0) = \max_{0\le x\le t} g(\ell|t-x). \tag{3.2}$$

In practice a $\hat{t}_0$ is found for each sampled shrimp. The collection is then grouped by estuary and year since this partition is needed for estimating $(E\tau_i^1 : i = 1, \ldots,54)$ and Σ_τ.

For sampling site i, in a given year let

J_i = number of samples

n_{ij} = number of shrimp in the jth sample

$t(i,j)$ = time of the jth sample

ℓ_{ijk} = length category of the kth fish in the jth sample

$t_0(i,j,k)$ = recruitment time of the kth fish in the jth sample.

We estimate $t_0(i,j,k)$ by solving (3.2) for $\hat{t}_0$ when $\ell = \ell_{ijk}$ and $t = t(i,j)$ and denote this estimate by $\hat{t}_0(i,j,k)$. To expedite computation, a two step procedure is employed. The first step, described by Algorithm EA in Appendix B, finds an age estimate for a fish having the length of each length category. For category n, λ_n denotes this length. The age estimate for a fish with this length is denoted by $\Delta\hat{t}(\lambda_n)$, and it satisfies

$$g(\lambda_n|\Delta\hat{t}(\lambda_n)) = \max_{0\le x\le 52} g(\lambda_n|x). \tag{3.3}$$

This approach limits computation since for each i, j and k, $\ell_{ijk}=\lambda_n$ for some $1\le n\le 14$, so that

$$\hat{t}_0(i,j,k) = t(i,j) - \Delta\hat{t}(\ell_{ijk}). \tag{3.4}$$

Therefore, regardless of the number of observations one solves only 14 maximization problems. A more direct approach would require the solution of the maximization in (3.2) for each observation. Furthermore, since our intent is to restrict recruitment time to integer values, $\Delta\hat{t}(\lambda_n)$ can be evaluated using a search procedure on an appropriately sized grid. Algorithm EA describes one such procedure. With the estimates $\{\Delta\hat{t}(\lambda_n) : n = 1,\ldots,14\}$ in hand one need only cycle through the data once and evaluate each $\hat{t}_0(i,j,k)$ as in (3.4). Algorithm RT in Appendix B describes this.

After executing these algorithms one has the collection $\{\hat{t}_0(i,j,k) : k = 1,\ldots,n_{ij};\ j = 1,\ldots,J_i\}$ for each estuary (i) and year of data (not denoted). In order to identify the arrival waves we evaluate the discrete probability mass function for each of these sequences and identify their modes. Let $p_i(n)$ be the empirically determined probability that a shrimp in estuary i entered in week n. Then

$$p_i(n) = \frac{1}{n_{ij}J_i} \sum_{k=1}^{n_{ij}} \sum_{j=1}^{J_i} I_{[n,n+1)}(\hat{t}_0(i,j,k))$$

where

$$I_{[a,b)}(x) = \begin{cases} 1 & \text{if } a \le x < b \\ 0 & \text{otherwise} \end{cases}$$

is the indicator function, and $\{p_i(n) : n = 1,\ldots,52\}$ is one such mass function.

When tabled these sample functions provide information on the structure of recruitment. Table 3 illustrates the concept. The yearly character of recruitment and the tendency toward a two wave recruitment are evident there.

We identify the recruitment waves τ_i^1 and τ_i^2 with the modal weeks. These are defined by the two (in some uses only one) maxima in the probability mass functions. For example, the outlined rectangles in Table 3 identify the modes. Since these estimated probability mass functions are subject to sampling error, judgment is used in distinguishing modal weeks. In particular, if the probability mass in a suspected mode is less than .10, the week is not considered the time of a recruitment wave.

To evaluate the mean vector $(E\tau_1^1,\ldots,E\tau_{54}^1)$ and the covariance matrix Σ_τ we let $\hat{\tau}_{im}^1$ denote the week of the first recruitment wave in sampling site i and year m (for 1974 m = 1;...; for 1977 m = 4). Then, an estimate of $E\tau_i^1$ is

$$E\hat{\tau}_i^1 = \frac{1}{4}\sum_{m=1}^{4}\hat{\tau}_{im}^1 .$$

Similarly, the sample covariance matrix is used to estimate Σ_T . The sample variances, on the diagonal of the sample covariance matrix, are

$$\hat{\text{var}}\ \tau_i^1 = \frac{1}{3}\sum_{m=1}^{4}(\hat{\tau}_{im}^1 - E\hat{\tau}_i^1)^2 \qquad i = 1,\ldots,54.$$

Table 4 shows the sample means and variances for the 54 sampling sites.

The off diagonal entries in the sample covariance matrix are used to estimate the correlation coefficients associated with the correlation in recruitment times between nurseries. Examination of these estimates reveal considerable variation in their values. We feel that a large part of this variation results from the small sample size used in estimation rather than representing the true correlation structure. To overcome this limitation we separately average the sample correlation coefficients from three groups of estuaries: among the southern estuaries, among the northern estuaries, and between southern and northern estuaries. Table 5 exhibits these quantities which we denote $\hat{\rho}^{\tau}_{S}$, $\hat{\rho}^{\tau}_{N}$ and $\hat{\rho}^{\tau}_{SN}$ respectively. Analytically we write,

$$\hat{\rho}^{\tau}_{a} = c \sum_{j=n_a}^{n_b} \sum_{i=n_c}^{n_d} \left[\frac{\frac{1}{3} \sum_{m=1}^{4} (\hat{\tau}^{1}_{im} - E\hat{\tau}^{1}_{i})(\hat{\tau}^{1}_{jm} - E\hat{\tau}^{1}_{j})}{(\hat{\mathrm{var}}\ \tau^{1}_{i} \cdot \hat{\mathrm{var}}\ \tau^{1}_{j})^{1/2}} \right]$$

where

a	c	n_a	n_b	n_c	n_d
N	2/(15·14)	1	14	j+1	15
S	2/(39·38)	16	53	j+1	54
SN	1/(39·15)	16	54	1	15

Because of the physical mechanisms of recruitment one expects arrivals in one estuary to be associated with arrivals in the other estuaries. This behavior is evident in Table 3 and is characterized by the positive coefficients $\hat{\rho}^{\tau}_{S}$, $\hat{\rho}^{\tau}_{N}$ and $\hat{\rho}^{\tau}_{SN}$.

In order to sample (on a computer) recruitment times that have correlations $\hat{\rho}^{\tau}_{S}$, $\hat{\rho}^{\tau}_{N}$ and $\hat{\rho}^{\tau}_{SN}$ and variances as in Table 4, requires that we compute a modified covariance matrix. This covariance matrix contains $\hat{\mathrm{var}}\ \tau_i^1$ as the ith diagonal entry and

$$\hat{\rho}_a^{\tau}\ (\hat{\mathrm{var}}\ \tau_i^1\ \hat{\mathrm{var}}\ \tau_j^1)^{1/2} \tag{3.5}$$

where

$$a \equiv \begin{cases} N & \text{if } i,j \le 15 \\ S & \text{if } i,j > 15 \\ SN & \text{otherwise} \end{cases}$$

as the ijth off diagonal entry. We return to this in the discussion of computer sampling of recruitment times in Section 5.2.

An Effect of Correlation between Estuaries on Variation in Recruitment Time.

The correlation between estuaries can have a dramatic effect on the overall variation in recruitment time. Consider for example, the first recruitment times $\tau_1^1,\ldots,\tau_n^1$ for n estuaries. Assume that each has variance σ^2 and that the covariance between any two is $\sigma^2\rho$. Then, the variance of the average recruitment time is

$$\mathrm{var}(\frac{1}{n}\sum_{i=1}^{n}\tau_i^1) = \frac{\sigma^2}{n}(1+(n-1)\rho).$$

To see the effect of inter-estuarine correlation on the average recruitment time we compare this variance with the variance of the average recruitment time when estuaries are independent of one another. Since in the independent case $\rho = 0$ the ratio of the two variances is

$$1 + (n-1)\rho.$$

For ρ as small as $\hat{\rho}_n^{\tau} = .046$ and $n = 54$ the variance of the average recruitment time when accounting for correlation is over 2.4 times the variance in the independent case. This demonstrates that even a small correlation (.046) can have a pronounced effect on the variation in recruitment time.

Estimating the Probability Mass Function of Inter-recruitment Time.

The second recruitment wave is characterized using the time of the first wave and a probability mass function defined on the inter-recruitment times. Recall that $\Delta\tau_i = \tau_i^1 - \tau_i^2$ where $\Delta\tau_i = 0$ if there is no distinct second wave. Let $pr(\Delta\tau_i = m)$ be the probability that the time between recruitment waves in estuary i is m weeks. Then given $\tau_i^1 = t_i^1$ the time of the second wave is

$$\tau_i^2 = t_i^1 + m$$

with probability $pr(\Delta\tau_i = m)$.

To estimate the probability mass $pr(\Delta\tau_i = m)$ we first decompose it into two components and then estimate each component separately. Let p_i be the probability that estuary i has a distinct second wave $(\tau_i^2 \neq \tau_i^1)$ and let p_m^N (p_m^S) be the probability that a northern (southern) estuary has inter-recruitment time m given a distinct second wave. Then

$$pr(\Delta\tau_i = m) = \begin{cases} p_i p_m^N & \text{if } i \text{ is a northern estuary} \\ p_i p_m^S & \text{if } i \text{ is a southern estuary.} \end{cases}$$

One associates p_i with the probability of "success" in a Bernoulli trial, where "success" is a distinct second wave. Consequently, we estimate p_i by the ratio of years with distinct waves to years of data.

Similarly, one associates p_m^N (p_m^S) with the probabilities of a multinomial distribution. Since the largest inter-recruitment wave period is seven weeks there are seven (7) such probabilities. Accordingly, one estimates p_m^N (p_m^S) by

$$\hat{p}_m^N = \frac{\sum_{i=1}^{15} \sum_{n=1}^{4} \delta_{(m)}(\Delta\hat{\tau}_{in})}{4 \cdot 15}$$

$$\hat{p}_m^S = \frac{\sum_{i=16}^{54} \sum_{n=1}^{4} \delta_{(m)}(\Delta\hat{\tau}_{in})}{4 \cdot (54-15)}$$

where the Kronecker delta

$$\delta_{(m)}(x) = \begin{cases} 1 & \text{if } x = m \\ 0 & \text{otherwise} \end{cases} .$$

For each estimate the value of the denominator equals the number of terms in the numerator, which in turn is determined by the number of years of data and the number of estuaries in the northern and southern groups. Table 6 shows estimates of $\{p_m^N, p_m^S : m = 1,\ldots,7\}$ and Table 7 shows estimates of $\{p_i : i = 1,\ldots,54\}$.

3.3 Out-migration to the Transport Areas

During spring and summer the population of growing juvenile and young adult shrimp migrate from the upper estuaries toward the higher saline water in the Pamlico Sound and open ocean. To some extent the shrimp stratify by size from the upper reaches of the estuaries to the estuarine mouths. The larger shrimp are found in the saline water in the estuarine mouth.

By regulation (North Carolina Fisheries Regulations for Coastal Waters 1978) the nurseries are permanently closed to fishing. However, the secondary nurseries, typically in the estuarine mouths, are opened to fishing on a given date which signifies the start of the shrimping season. Marine managers choose this date through an analysis of the estuarine length and abundance samples. Understanding the relationship between early nursery conditions on the one hand, and the opening date and fishery productivity on the other hand, is of crucial importance to both managers and fishermen. One goal of the research, of which this report is a part, is to develop a methodology to study this relationship. Toward this end we present a model of the migrations from nursery to transport area.

This type of migration has traditionally been represented (e.g. see Beverton and Holt 1957, pp. 136-148) by the solution to the differential equation

$$\frac{dN}{dt} = -TN,$$

where N is the number of fish in the nursery at time t and T is the instantaneous rate of out-migration (transport) from the primary nursery. The motivation for this representation is based on the notion that the rate of out-migration depends linearly on population size. Its solution

$$N(t) = N_0 e^{-Tt}, \tag{3.6}$$

describes the population size as it changes in time (t) under the assumption that N_0 were initially (at t = 0) in the population.

A similar model, which we extend, uses (3.6) as a description of mean behavior (Seber 1973, pp. 328-333).

As it stands, (3.6) aggregates migratory behavior over the entire Pamlico nursery system resulting in a gross generalization of population dynamics. The loss in structural detail is more pronounced than one expects from a representation that models mean behavior. To limit this loss, we continue to account for the differences between estuaries as was done in Sections 3.1 and 3.2. In particular, we extend the abundance representation (3.6) in a way that is consistent with the model of nursery recruitment.

In our discussion of migration and later in our discussion of abundance we focus on catch per unit of fishing effort (CPUE) as a measure of abundance. The CPUE is the number of shrimp captured in the standard nursery sample discussed in Section 2. Note that catch per unit effort is a random variable. When using this quantity as a measure of abundance it is important to know the conditions under which it is evaluated. For short fishing periods, CPUE is a reliable measure of current fish density and consequently abundance. On the other hand, if CPUE is determined from the catch of a commercial fishery over a long period, where vessels can compete with one another and fish can migrate considerable distances, then the CPUE is a more difficult quantity to interpret. Beverton and Holt (p. 27, 1957) identify this issue and refine the definition of CPUE to resolve these difficulties. In order to maintain this distinction, and emphasize the relationship to abundance we refer to sample CPUE in this study as shrimp density.

There is a direct proportionality between abundance and density.

In particular, if shrimp density is uniform throughout the nursery then abundance is the product of density and nursery volume. Since shrimp density is measured in terms of a standard tow, nursery volume must also be measured in terms of a standard tow, so that one unit of volume is the volume of water that is sampled in a standard tow.

For estuary i and recruitment wave k let

$N_i^k(t)$ = a random variable that denotes the contribution of the shrimp in recruitment wave k to shrimp density at time t,

α_i^k = a model parameter.

Then as a description of mean shrimp density, (3.6) becomes

$$E\{N_i^k(t)\,|\,N_i^k(\tau_i^k)=n\}=\begin{cases} 0 & \text{if } t<\tau_i^k \\ n\exp\{-\alpha_i^k(t-\tau_i^k)\} & \text{if } \tau_i^k\le t \end{cases} \qquad (3.7)$$

This is the expected contribution of recruitment wave k to shrimp density at site i and time t given that recruitment wave k has density n at τ_i^k. Comparing (3.7) to (3.6) one sees that n takes the role of N_o, $t-\tau_i^k$ the role of t, and α_i^k the role of T. Now, α_i^k is the parameter that reflects the rate of out-migration and natural mortality, and consequently is the focus of estimation. For reading ease, in the remainder of this report we write N_i^k for $N_i^k(\tau_i^k)$.

In this setting, as in similar examples in Seber (1973), it is natural to assume that $N_i^k(t)$ given $N_i^k=n$ is a binomial variate with parameters n and $\exp\{-\alpha_i^k(t-\tau_i^k)\}$. One arrives at this characterization by considering the nursery sojourn time for a shrimp recruited at τ_i^k as an exponential random variable with rate parameter α_i^k and location parameter τ_i^k . Its density function is

$$\frac{1}{\alpha_i^k} \exp\{-\alpha_i^k (t-\tau_i^k)\},$$

so that the probability that the shrimp remains in the nursery at time $t>\tau_i^k$ is

$$\exp\{-\alpha_i^k (t-\tau_i^k)\}.$$

If one assumes that shrimp migrate independently of each other and have identical sojourn time distributions then $N_i^k(t)$ given $N_i^k=n$ has the aforementioned binomial distribution. The exponential model also yields $1/\alpha_i^k$ as the mean nursery sojourn time for shrimp arriving in estuary i at τ_i^k.

The reader should note that if N_i^k and τ_i^k are not given, then the distribution of $N_i^k(t)$ depends on the distributions of N_i^k and τ_i^k, and in general is difficult to characterize. In simulation one approach is to sample N_i^k and τ_i^k, and then sample $N_i^k(t)$ given N_i^k from the appropriate binomial distribution. We return to this topic in Section 5.

3.4 Estimating the Parameters of Out-Migration

Recall that for the estuarine length and abundance samples discussed in Section 3.2, J_i denoted the number of sample tows taken in estuary i and t(i,j) denoted the time of the jth sample. For estimation we identify sample time t(i,j) with time t of model (3.7). Since this model specifies that all shrimp recruit to the nursery either at τ_i^1 or τ_i^2 it agrees with the model of recruitment in Sections 3.1 and 3.2. However, under this assumption, a difficulty arises when estimating $\{ \alpha_i^k : i=1,\ldots 54; k=1,2\}$, since estimation techniques require that each shrimp sample be identified with either τ_i^1 or τ_i^2 . For any sample j satisfying $\tau_i^1 \le t(i,j) \le \tau_i^2$ this presents no problem, these shrimp having been recruited at τ_i^1 . However, for $t(i,j)<\tau_i^1$ an additional assumption is necessary. We assume that these shrimp are recruited at τ_i^1 . Similarly, for $t(i,j)>\tau_i^2$ we assume that the shrimp in sample j are recruited at τ_i^2 . Examination of Table 3 shows that

$$\sum_{t(i,j)<\tau_i^1} p_i(t(i,j))$$

is typically (for alternative sampling sites and years) less than .10. Hence, the first additional assumption is rarely needed. This adjustment is realized via the transformation

$$t'(i,j) = \begin{cases} t(i,j) & \text{for} \quad t(i,j) \ge \hat{\tau}_i^1 \\ \hat{\tau}_i^1 & \text{for} \quad t(i,j) > \hat{\tau}_i^1 \end{cases}$$

From (3.6) one can solve for α_i^k in terms of $E\{ N_i^k(t'(i,j)) \mid N_i^k=n \}$ and $E\{ N_i^k(t'(i,j+1)) \mid N_i^k=n \}$. Then, using the estimate n_{ij} of $E\{ N_i^k(t'(i,j)) \mid N_i^k=n \}$ one obtains the estimate of α_i^k given by

$$\hat{\alpha}_i^k = \frac{1}{\sum_{j=1}^{J_i} I_{(k)}(j)} \sum_{j=1}^{J_i} \frac{\ln (n_{ij}/ n_{ij+1})}{t'(i,j+1)-t'(i,j)} I_{(k)}(j) \qquad (3.8)$$

for i=1,..., 54 and k=1,2. Here,

$$I_{(k)}(j) = \begin{cases} 1 & \text{if } k=1 \text{ and } t'(i,j+1) \le \hat{\tau}_i^2 \\ 1 & \text{if } k=2 \text{ and } t'(i,j) > \hat{\tau}_i^2 \\ 0 & \text{otherwise.} \end{cases}$$

The derivation of (3.8) is presented in Appendix A.

Estimating the out-migration rate in this way yields the sequence of estimates $\{\hat{\alpha}_i^k : k=1,2;\ i=1,\ldots,54\}$ for each year of data. Analogous to the aggregation in Section 3.2, we aggregate the estimates $\{\hat{\alpha}_i^k : i=1,\ldots,14\}$ k=1,2 and $\{\hat{\alpha}_i^k : i=15,\ldots,54\}$ k=1,2. Then $\hat{\alpha}_N^k$ k=1,2 and $\hat{\alpha}_S^k$ k=1,2 are the weighted averages for the northern and southern sites, respectively. Where the weight for element $\hat{\alpha}_i^k$ is the ratio of the cumulative number of sampled shrimp used when estimating α_i^k to the total number sampled. For example, we have

$$\hat{\alpha}_N^k = \frac{\sum_{\text{years of data}} \sum_{i=1}^{15} \hat{\alpha}_i^k \sum_{j=1}^{J_i} n_{ij} I_{(k)}(j)}{\sum_{\text{years of data}} \sum_{i=1}^{15} \sum_{j=1}^{J_i} n_{ij} I_{(k)}(j)}$$

Table 8 shows the estimates of the brown shrimp out-migration rates. Note that the rates are higher for the southern estuaries than the northern estuaries. If this is not merely a result of sampling variation, then it indicates that the shrimp in the southern nurseries on the average have a shorter nursery sojourn time than those in the northern nurseries. As a consequence one expects shrimp in the southern transport areas to be on the average smaller than those in the northern transport areas. No attempt has been made to examine this issue.

4. Abundance Model

Although shrimp size is an important factor in determining catch revenue (Cohen and Fishman 1982), overall population abundance plays an even more important role. Consequently, the ability to predict abundance levels early in the season would help alert commercial fishermen of the year's harvest potential. With this knowledge and the guidance of fishery managers, fishermen could make more informed fishing decisions. For example, in years of limited abundance it might be more profitable for larger vessels to choose an alternative, such as fishing in a region with a more promising outlook, or rigging their vessels for an alternative species.

Since all brown shrimp recruit to the nurseries at roughly the same time, the abundance expected by the commercial fishery can be estimated from nursery samples of juvenile shrimp. In fact, marine biologists evaluate early nursery abundance as an aid in determining fishery opening date and season outlook.

As with the time of nursery recruitment, the level of abundance exhibits yearly variations attributable to various environmental factors. It is generally acknowledged that because of the extreme fecundity of shrimp, nursery environmental conditions contribute more than spawning population size in determining the yearly abundance level. This is supported by the Hunt et al (1979) Study which reports on the relationship between several environmental factors, including temperature and salinity, and yearly harvest size.

Here, we take a different approach. Although we account for random environmental factors, we characterize abundance at nursery recruitment time in a way that focuses on reconstruction of an arbitrary year with a computer coded version of the model. Then, as will be described in a future report, by analyzing a sequence of carefully designed experiments with the computer model, we derive a relationship between nursery conditions, that includes abundance, management decisions and harvest level.

In this section we concentrate on characterizing abundance, and we continue developing model (3.7) of Section 3.3. Recall that N_i^k is shrimp density at sampling site i and recruitment time τ_i^k. Assuming that shrimp density is uniform throughout nursery i, one takes abundance to be the product of N_i^k and nursery volume. Since (3.7) explicitly accounts for shrimp density at recruitment and then describes migrations from the primary nursery, it is sufficient to characterize the joint distribution of $\{N_i^k : k=1,2; i=1,\ldots,54\}$

to complete the description of population abundance and its dynamics within the nursery.

4.1 Estimating the Parameters of the Abundance Model

If one observed abundance at the time of nursery recruitment then characterizing the joint distribution of $\{N_i^k: k=1,2; i=1,\ldots 54\}$ would be straightforward. However, these quantities have not been sampled. To overcome this, we use the estimates $\hat{\alpha}_N^k$ and $\hat{\alpha}_S^k$ in representation (3.7) to estimate N_i^k for k=1,2 and i=1, ...,54 for each year of data. Let n_i^k be this estimate. Then from (3.7) we have

$$n_i^k = \frac{\sum_{j=1}^{J_i} n_{ij}\exp[\hat{\alpha}_E(t'(i,j)-\hat{\tau}_i^k)]I_{[a(k),b(k))}(t'(i,j))}{\sum_{j=1}^{J_i} I_{[a(k),b(k))}(t'(i,j))} \qquad (4.1)$$

where

$$a(1) = 0$$

$$a(2) = b(1) = \hat{\tau}_i^2$$

$$b(2) = 52,$$

$$I_{[a,b)}(x) = \begin{cases} 1 & \text{if } a \le x < b \\ 0 & \text{otherwise} \end{cases}$$

is an indicator function, and

$$\hat{\alpha}_E = \begin{cases} \hat{\alpha}_N^k & \text{if } 1 \le i \le 15 \\ \hat{\alpha}_S^k & \text{if } 16 \le i \le 54. \end{cases}$$

In this way, sampling site i recruitment density is estimated for each of the four years data at each recruitment time. Table 9 shows the sum of n_i^1 and n_i^2 obtained with (4.1) for the brown shrimp data. Using these quantities we evaluate the sample mean vector and the sample covariance matrix of the sequence $\{ N_i : i=1,\dots, 54\}$, where $N_i = N_i^1 + N_i^2$. Table 10 shows the sample mean $\hat{E}N_i$ and sample variance $\widehat{\text{var}}\, N_i$ calculated using the four years data on $n_i^1 + n_i^2$ for each nursery $i = 1,\dots 54$.

It is also important to characterize the inter-estuarine correlation in recruitment shrimp density. In the next section we show how this correlation has a pronounced effect on the variation in yearly abundance. As in Section 3.2, where we examined the correlation in recruitment time between estuaries, we calculate three correlation coefficients, each an average of sample correlation coefficients. We separately average the correlation coefficients of the northern estuaries, of the southern estuaries, and those coefficients involving both northern and southern estuaries. We denote these by $\hat{\rho}_N^N$, $\hat{\rho}_S^N$ and $\hat{\rho}_{SN}^N$, respectively. For example, $\hat{\rho}_N^N$ is

$$\frac{2}{15\cdot 14}\sum_{i=1}^{14}\sum_{k=i+1}^{15}\hat{\rho}_{ik}$$

where

$$\hat{\rho}_{ik} = (\widehat{\text{var}}\, N_i\ \widehat{\text{var}}\, N_k)^{-1/2} \sum_{\substack{\text{years of}\\ \text{data}}} (n_i-\hat{E}N_i)(n_k-\hat{E}N_k).$$

The other coefficients are similarly defined.

Table 11 shows these quantities. Note that they are positive, in agreement with our understanding of the physical mechanisms of recruitment. That is, when density is high (low) in one estuary it is likely to be high (low) in other estuaries.

In Section 5.2 we show that for simulation purposes it is useful to have estimates of the mean and variance of the transformed random vector $(\ln N_1, \ldots, \ln N_{54})$, as well as the correlation coefficients for the three groups of estuaries. To obtain these quantities we follow the procedure described above on the transformed estimates $\{ \ln N_i^k : k=1, 2 ; i=1, \ldots, 54 \}$ for each year of data. Tables 12 and 13 exhibit the estimated quantities.

The quantities $\{ n_i^k : k=1,2 ; i=1, \ldots, 54 \}$ can also be used to estimate the probability that an arbitrary shrimp entering estuary i enters at τ_i^k. We denote this probability by p_i^k. Here, the minimum variance unbiased estimator of p_i^1 is

$$\hat{p}_i^1 = \sum_{\substack{\text{years of}\\ \text{data}}} (n_i^1) \Big/ \sum_{\substack{\text{years of}\\ \text{data}}} (n_i^1+n_i^2),$$

and of $\hat{p}_i^2$ is $\hat{p}_i^2 \equiv 1 - \hat{p}_i^1$ (Johnson and Kotz pp.56-57, 1969). Table 14 shows $\hat{p}_i^1$ for $1 \leq i \leq 54$ obtained from the brown shrimp data.

Now, we turn to a discussion of the effects of inter-estuarine correlation on the variance in abundance. This parallels the discussion in Section 3.2.

The Effects of Correlation Between Estuaries on Abundance Variation

In Section 3.2 we showed how the inclusion of inter-estuarine recruitment correlation can have a significant effect on the variation in the average yearly recruitment times. An analogous situation exists with regard to variation in overall yearly abundance levels. In this case positive correlation between abundance levels in various nursery areas increases the variation in total system wide abundance over what it would be if the nurseries were treated as independent.

Consider the sequence $\{ N_1^1, \ldots, N_m^1 \}$ of shrimp density at recruitment time τ_i^1 for estuaries $1 \leq i \leq m$. If we denote the volume, assumed non-random, in nursery i by V_i, then $V_i N_i^1$ is the abundance at τ_i^1 in estuary i. Rewrite $V_i N_i^1$ as X_i, and for expository convenience assume that $\text{var}(N_i^1) = \text{var}(N_j^1) = \sigma^2$ and $\text{cor}(N_i^1, N_j^1) = \rho$ for $1 \leq i < j \leq 54$. Then the variance of total abundance is

$$\text{var}\left(\sum_{i=1}^{m} X_i \right) = \left(\sum_{i=1}^{m} V_i^2 \right) n\sigma^2 (1 + (m-1)\rho).$$

Treating the nurseries independently implies that $\rho = 0$, so that the ratio of variance of the dependent case to the independent case is

$$(1 + (m-1) \rho).$$

For demonstration, consider

$$\begin{aligned} \rho &= \min \{ \hat{\rho}_S^N , \hat{\rho}_N^N , \hat{\rho}_{SN}^N \} \\ &= \hat{\rho}_S^N \\ &= .125 \end{aligned}$$

For $m=54$, $(1+(m-1) \rho) = 7.625$, showing more than a 7-fold increase in total variation in abundance when accounting for inter-estuarine correlation over not accounting for this dependence. This example demonstrates the importance of accounting for inter-estuarine dependence in a model of population dynamics.

5. The Migration and Abundance Models in a Fishery Simulation for Management Policy Analysis

So far we have focused on the development of models that describe the early spring behavior of the brown shrimp population. The analysis has included: (1) a characterization of primary nursery recruitment, (2) a characterization of out-migration from these nurseries, and (3) a characterization of primary nursery shrimp density. This has been accomplished with the goal in mind of linking these models with models of fishing and economic evaluation of catch (Cohen and Fishman 1982) to form a useful tool for management policy analysis. Toward this end this section describes the distribution of length and abundance of both the shrimp in the primary nurseries and of the outmigrating shrimp.

An example in Section 5.3 illustrates the methodology, where we discuss simulating the process of sampling the primary nurseries in the spring and show graphic results of such a simulation.

5.1 The Distribution of Length and Abundance of Weekly Out-migrants

Recall in Section 3.3, N_i^k denoted shrimp density in nursery i at τ_i^k, the time the kth recruitment wave arrived. Shrimp abundance at τ_i^k equals $V_iN_i^k$, where V_i is the volume of nursery i. We also observed that a shrimp in the nursery at τ_i^k is still in the nursery at $n>\tau_i^k$ with probability $\exp(-\alpha_i^k(n-\tau_i^k))$. Consequently, as in traditional models (Seber 1973), we represented nursery abundance at time n by a binomial variate with parameters $V_iN_i^k$ and $\exp(-\alpha_i^k(n-\tau_i^k))$. Here, we represent time by n to emphasize the weekly basis of sampling discussed in Section 5.3.

In this section we modify this characterization in order (1) to include the probabilistic dependence in observations of abundance at different times, and (2) to describe the number of out-migrants from each nursery. Consider the cohort defined by the kth recruitment wave, and for nursery i let

$A_i^k(n)$ = abundance at time n,
$R_i^k(n)$ = number of out-migrants during week n, and
$M_i^k(n)$ = number of mortalities during week n.

Note that these quantities are random variables. Furthermore, since $A_i^k(n)$ is the abundance of the kth wave, for week $n=\tau_i^k$, $A_i^k(n)=V_iN_i^k$ and for $n<\tau_i^k$, $A_i^k(n)=0$.

Observe that any shrimp in the nursery at time n must either remain in the nursery until time n+1, out-migrate during week n or die during week n. The mass balance equation

$$A_i^k(n)=A_i^k(n+1)+R_i^k(n)+M_i^k(n)$$

formalizes this relationship. Furthermore, it leads to a characterization of the random variables $A_i^k(n+1)$, $R_i^k(n)$ and $M_i^k(n)$ that enhances the traditional model discussed in Section 3.3. In particular, given $a_i^k(n)$ shrimp in the nursery at time n we assume that the vector $(A_i^k(n+1), R_i^k(n), M_i^k(n))$ has a multinomial distribution with parameters $a_i^k(n)$,

$\exp(-\alpha_i^k)$ = probability that a shrimp in the nursery remains there for the week,
$(1-\exp(-\alpha_i^k))(1-p_\mu)$ = probability that a shrimp in the nursery out-migrates during the week, and

$(1-\exp(-\alpha_i^k))p_\mu$ = probability that a shrimp in the nursery dies during outmigration.

One can see that the parameter p_μ is used to distinguish the shrimp that migrate from those that die. The estimation of p_μ is discussed in later technical report.

The multinomial characterization implies that the random variables $A_i^k(n+1)$ and $R_i^k(n)$ have marginal binomial distributions, and that the distribution of $A_i^k(n+1)$ derived from this model is consistent with that discussed in Section 3.3. In particular,

$$A_i^k(n+1)\sim \mathrm{Bin}(a_i^k(n),\exp(-\alpha_i^k)) \tag{5.1}$$

$$R_i^k(n)\sim \mathrm{Bin}(a_i^k(n),\exp(-\alpha_i^k)(1-p_\mu)), \tag{5.2}$$

where the notation $X\sim \mathrm{Bin}(N,p)$ means that X has a binomial distribution with parameters N and p.

Since $A_i^k(n)$ is customarily large, the normal approximation to the binomial distribution is applicable (Johnson and Kotz 1969, pp. 61-68). In particular, the approximation yields for the random variable $A_i^k(n+1)$

$$\begin{aligned}\mathrm{pr}(a<A_i^k(n+1)<b)=&\Phi((b-a_i^k(n)p+1/2)/(a_i^k(n)pq)^{1/2})\\&-\Phi((a-a_i^k(n)p+1/2)/(a_i^k(n)pq)^{1/2})\end{aligned}$$

where $p = \exp(-\alpha_i^k)$

$q = 1-p$

and $\Phi(.)$ is the distribution function of a normal random variable with zero mean and unit variance. In a similar way we approximate the distribution of $R_i^k(n)$ with a normal distribution.

This approach for characterizing abundance preserves an important property of the biological process. In particular, consider the weekly sequence of abundances $A_i^k(\tau_i^k)$, $A_i^k(\tau_i^k+1), \ldots, A_i^k(52)$. Since these represent the abundance of a single cohort at successive times, one would expect that

$$A_i^k(\tau_i^k) \geq A_i^k(\tau_i^k+1) \geq \ldots \geq A_i^k(52).$$

The characterization (5.1) guarantees that this condition holds with probability 1, and thus it induces autocorrelation in the sequence of abundances.

The characterization also preserves an important property of the out-migration process. In the field, once the shrimp have left the primary nursery, they do not return. This means that the number of out-migrants is non-negative. Since $R_i^k(n)$ being a binomial variate implies that $R_i^k(n) \geq 0$ with probability 1, the description of out-migration satisfies this condition.

To remove the dependence on cohort which has been carried along in the characterization of abundance and out-migration, consider abundance and the number of out-migrants as the sum of each cohort's contribution. In particular, abundance in nursery i during week n is

$$A_i(n) = A_i^1(n) + A_i^2(n),$$

and the number of out-migrants during week n is

$$R_i(n) = R_i^1(n)+R_i^2(n).$$

Algorithm SA in Appendix B describes a procedure for sampling $\{(A_1^k(n),\ldots,A_{54}^k(n)):k=1,2;n=1,\ldots,52\}$ and $\{(R_1(n),\ldots R_{54}(n)): n=1,\ldots,52\}$ in a computer simulation that follows our development.

The length of out-migrating shrimp and shrimp in the nursery is also of interest to fishery managers. Let $S_i(n)$ denote the length of an arbitrary shrimp in nursery i at time n. Then $S_i(n)$ is written as

$$S_i(n)=\begin{cases} 0 & \text{for } n\le\tau_i^1 \\ L(n-\tau_i^1) & \text{for } \tau_i^1<n\le\tau^2 \\ L(n-\tau_i^1)I_{[0,p_i^1)}(U)+L(n-\tau_i^2)I_{[p_i^1,1)}(U) & \text{for } \tau_i^2<n. \end{cases} \tag{5.3}$$

Here,

$L(n-\tau_i^k)$ is a random variable with density function (3.1) (with $\Delta t=n-\tau_i^k$) that represents the length of an arbitrary shrimp of age $n-\tau_i^k$,

U is a uniform (0,1) random variable,

p_i^1 is the probability that a shrimp in nursery i arrived in wave 1, and

$$I_{[a,b)}(x) = \begin{cases} 1 & \text{if } a\le x<b \\ 0 & \text{otherwise} \end{cases}$$

is the indicator function.

The mean and variance of $S_i(n)$ will be of use in Section 5.3. The mean is

$$ES_i(n) = \begin{cases} 0 & n \leq \tau_i^1 \\ \sum_{s=1}^{2} \rho_s \mu_L(n-\tau_i^1, s) & \tau_i^1 < n \leq \tau_i^2 \\ \sum_{s=1}^{2} \sum_{k=1}^{2} \rho_s p_i^k \mu_L(n-\tau_i^k, s) & n > \tau_i^2 \end{cases} \tag{5.4a}$$

and its variance is obtained from the second moment,

$$ES_i(n)^2 = \begin{cases} 0 & n \leq \tau_i^1 \\ \sum_{s=1}^{2} \rho_s \{\sigma_L^2(n-\tau_i^1, s) + \mu_L^2(n-\tau_i^1, s)\} & \tau_i^1 < n \leq \tau_i^2 \\ \sum_{s=1}^{2} \sum_{k=1}^{2} \rho_s p_i^k \{\sigma_L^2(n-\tau_i^k, s) + \mu_L^2(n-\tau_i^k, s)\} & \tau_i^2 < n \end{cases} \tag{5.4b}$$

and the identity

$$\text{var } S_i(n) = E\left[S_i(n)^2\right] - \left[ES_i(n)\right]^2. \tag{5.4c}$$

5.2 Simulation Sampling of Abundance and Recruitment Times

To sample the number of migrants in a weekly period using Algorithm SA requires observations of the recruitment wave arrival times $\{(\tau_1^k, \ldots, \tau_{54}^k) ; k=1,2\}$ as well as observations of the shrimp densities $\{(N_1^k, \ldots, N_{54}^k) ; k=1,2\}$ at the recruitment times. This section concerns the distribution of these quantities. For conciseness denote $(\tau_1^k, \ldots, \tau_{54}^k)$ by $\bar{\tau}^k$ and $(N_1^k, \ldots, N_{54}^k)$ by $\bar{N}^k$.

Recall that Sections 3.2 and 4.1 discussed estimation of the mean vector and covariance matrices of $\bar{\tau}^1$ and $\bar{N}$ where $\bar{N}$ was defined to be $\bar{N}^1 + \bar{N}^2$. Also, the relationships between recruitment times τ_i^1 and τ_i^2, and between shrimp densities N_i^1 and

N_i^2 were characterized in a way that the distribution of the conditional random vectors $\bar{\tau}^2$ given $\bar{\tau}^1$, $\bar{N}^1$ given $\bar{N}$, and $\bar{N}^2$ given $\bar{N}$ were easily described. Algorithms ST1 and SN1 in Appendix B enumerate a procedure for computer sampling of these quantities that follows the earlier discussion. Because we have characterized these quantities as conditional random variables we now need to focus on the distribution of $\bar{\tau}^1$ and of $\bar{N}$.

Sampling $\bar{\tau}^1$ and $\bar{N}$

As mentioned in earlier sections the data are not sufficient to support estimation of the multidimensional distributions of recruitment time $\bar{\tau}^1$ and shrimp density $\bar{N}$. In the absence of estimation we assume that $\bar{\tau}^1$ has a multinormal distribution with mean $E\bar{\tau}^1 = (E\tau_1^1, E\tau_2^1, \ldots, E\tau_{54}^1)$ and covariance $\Sigma_{\bar{\tau}}$. This covariance matrix has $(\text{var } \tau_1^1, \ldots, \text{var } \tau_{54}^1)$ on the main diagonal and covariances evaluated with (3.5) on the off-diagonals. The estimation of these parameters is discussed in Section 3.2. The multinormal distribution is convenient to use in this application because the mean and covariance completely specify the distribution, no additional parameters are needed. In general this is not the case, additional parameters would be needed to assure that $\bar{\tau}^1$ would have covariance $\Sigma_{\bar{\tau}}$.

This would require estimation of additional parameters from the scant data.

Recall that the model of recruitment discussed in Section 3.2 aggregated all recruits within weekly period n to time n, the beginning of the week. Consequently, τ_i^k only assumed integer values. This is inconsistent with the normal characterization. To reconcile this difference in a computer simulation, we sample a vector from the multinormal distribution with mean $E\bar{\tau}^1$ and covariance matrix $\Sigma_{\bar{\tau}}$ then take the largest integer less than or equal to each element as the recruitment time.

Now, we turn to characterizing the distribution of shrimp density $\bar{N}$. There is empirical evidence supporting the assumption that $\bar{N}$ has a multilognormal distribution. In a ten year study in North Carolina, Williams (1969) observed that the abundance of Penaeus Aztecus in postlarvae samples followed a lognormal distribution. This finding supports the model of recruitment abundance that assumes that environmental factors affect abundance multiplicatively (Ricker 1975). By considering environmental factors as multiplicative weights of a base recruitment population, the actual yearly recruitment can be represented by the product of each factor and the base population. Under the assumption that the environmental factors are independent, identically distributed random variables (whose logarithms have finite first and second movements), the actual recruitment population has an approximate lognormal distribution. In view of the empirical evidence we approximate $(N_1, \ldots, N_{54})$ by a multilognormal random variable with mean $(\hat{E}N_1, \ldots, \hat{E}N_{54})$ and covariances evaluated from the variances

($\hat{\text{var}}\ N_1$, ..., $\hat{\text{var}}\ N_{54}$) and correlation coefficients $\hat{\rho}_N^N$, $\hat{\rho}_S^N$, and $\hat{\rho}_{SN}^N$ as discussed in section 4.1.

Algorithms ST1 and SN1 in Appendix B describe the procedure for sampling $(\tau_1^1,\ldots,\tau_{54}^1)$ and $(N_1,\ldots,N_{54})$, respectively. Both of these procedures call subroutines (auxiliary algorithms) COV and LTM which are also in Appendix B. COV constructs a covariance matrix from a vector of variances and correlation coefficients as shown in (3.5), while LTM is needed for sampling a multinormal random variable on the computer. LTM performs the Cholesky decomposition of a covariance matrix using an algorithm taken from Fishman (1980).

One final note concerns sampling $(N_1,\ldots,N_{54})$ from a multilognormal distribution. This distribution can be characterized in at least two ways; one in terms of the mean and covariance of $(N_1,\ldots,N_{54})$ and the other in terms of the mean and covariance of the transformed vector $(\ln N_1, \ldots, \ln N_{54})$. For simulation purposes it is practical to generate an observation from the distribution of the transformed random variable which has a multinormal distribution and then to apply the inverse transformation by exponentiating each element of the generated observation of $(\ln N_1, \ldots, \ln N_{54})$. Recall that in Section 4.1 we discussed estimating the mean vector and covariance matrix of $(\ln N_1,\ldots,\ln N_{54})$. We use these estimates in algorithm SN1.

Sampling the Initial Population Profile

To summarize, algorithms thus far presented provide a method for generating initial population profiles on a computer. By executing the sequence of algorithms ST1, SN1, ST2, SN2 and SA one generates: (1) the number of weekly out-migrants to each secondary nursery and (2) the weekly abundance within each nursery. Furthermore, with the characterization of shrimp length (5.3) one has a complete description of the nursery population of length and abundance as a function of time. In the next section we demonstrate one application of these models.

5.3 Simulating Primary Nursery Sampling

The value of simulation methodology becomes apparent when one joins the algorithms for sampling population profiles with a model of fishing and the economic evaluation of catch (see Cohen and Fishman 1982). This enables one to sample fishery yield for alternative initial populations so that questions regarding effective fishery management can be addressed. For example, one valuable analysis is to relate early nursery conditions with fishery yield in an effort to determine a preferred fishery opening date. We follow this example in detail in a later technical report. However, to demonstrate the potential of the methodology here, we show length and abundance data obtained from a computer simulation of the marine biologists spring nursery sampling.

In order to simulate this aspect of the fishery we require a model of fishing. Since we have thus far not presented such a model we briefly outline one. Let p(e) denote the probability of catching an arbitrary shrimp given e units of fishing effort.

Here, fishing effort is a measure of the fishery capitalization and human resources that are applied to shrimping in a week. In the present application e measures the effort in 100 foot hours of net used to sample a primary nursery. This means that for $e=1$ a 100 foot net is trawled for 1 hour. Richer (1975) and Seber (1973) show analogous definitions for effort in other applications.

Let $\tilde{N}_i(n)$ denote a sample of the abundance (or more accurately shrimp density) in nursery i at time n. Under the assumption that shrimp are uniformly distributed throughout the nursery and that each is caught independently of the others (Seber 1973) one obtains

$$\tilde{N}_i(n) \sim \mathrm{Bin}(A_i(n), p(e)).$$

However, for large $A_i(n)$ the normal approximation to the binomial is applicable, so that

$$\tilde{N}_i(n) \sim \mathcal{N}(A_i(n)p(e),\ A_i(n)p(e)(1-p(e)))$$

where the notation $X \sim \mathcal{N}(\mu,\sigma^2)$ means that X has a normal distribution with mean μ and variance σ^2. This yields a method for simulating primary nursery abundance sampling.

Furthermore, since managers typically consider the sample average of shrimp length when analyzing nursery sampling data we turn to simulating this aspect of sampling. Expression (5.3) and the result of a central limit theorem (Feller 1968) provide the means for accomplishing this. An arbitrary shrimp in a given sample of $\tilde{N}_i(n)$ shrimp has length $S_i(n)$, with mean and variance $ES_i(n)$ and $\operatorname{var} S_i(n)$ as specified in (5.4). And similarly, for large $\tilde{N}_i(n)$ the sample average length, denoted $\bar{S}_i(n)$, is approximately normally distributed with mean $ES_i(n)$ and variance $\operatorname{var} S_i(n) / \tilde{N}_i(n)$.

Summarizing the Length and Abundance Information

The analysis of the last section completes the information needed to describe a procedure for simulation sampling of each of the 54 primary nurseries. Such a procedure would result in a large number of observations as in fact occurs in practice. In the field in a single week, at least 108 observations are made; 54 of abundance and 54 of mean length. Consequently, when nurseries are sampled for 10 weeks (May into July) a manager must evaluate the results of over 1000 observations to determine policy, a formidable task.

Although in practice a manager may want to determine separate policy for each estuary, or for groups of estuaries, in this example we assume that policy is uniform throughout Pamlico Sound. To simplify data analysis and demonstrate the computer sampling technique we focus on statistics that summarize the information contained in these numerous observations of length and abundance. In each week we consider the average sample abundance $\bar{N}(n)$ and the average sample mean length $\bar{S}(n)$ each computed from observations in the 54 primary nurseries. These two quantities summarize the state of the fishery in a way that avoids pitfalls associated with examining a large set of observations. One pitfall that is avoided, is the tendency to focus on those nurseries with comparatively large abundances. Basing policy decisions on extreme values in this way may lead to high variability in the revenue generated by the fishery. Since typically, a goal of management is to limit, if not minimize, variation this approach is to be avoided.

Algorithm SNUR in Appendix B enumerates the steps for simulation sampling of these quantities.

Discussion of Simulation Results

A simulation of primary nursery sampling using Algorithm SNUR yielded observations of the catch in a 30 second tow $\bar{N}(n)$ and the mean shrimp length in the catch $\bar{S}(n)$ for $n=18$ (May) to 29 (July) for 1000 independent sampling replications. Table 15 contains summary statistics from the data generated by the simulation experiment alongside similar statistics for the data described in Section 2. We refer to the latter data set as the field data. The statistics presented for each of these groups include the lower quartile (Q1), the median (Q2), the upper quartile (Q3), the mean and the standard deviation.

These statistics provide a means of comparing data obtained from the simulation model with the field data. Figure 1 is a plot of the sample median (Q2) of catch $\bar{N}(n)$, on the vertical axis, vs. sample mean length $\bar{S}(n)$, on the horizontal axis, for each node on the graph, and each curve is identified with data from which it is derived.

Although clear distinctions exist between the two curves, both exhibit relatively parallel downward sloping segments from their peak abundance levels. However, greater variability exists in the field data abundance than in the simulated abundance, no doubt attributable in part to the limited number of observations from which the field data are derived. These field data represent at most four years of observations, whereas the simulated data represent 1000 replications of a year. Therefore, the reason for the smoother simulated curve is clear.

Also observe in Figure 1 the more rapid growth in length in the simulated data than in the field data. We can study this discrepancy more closely by examining the series of histograms of length obtained from the simulated data in Figure 2 for weeks 18 through 26. To facilitate comparison of the length data the field data mean length and standard deviation of length are shown at the base of each histogram. The short horizontal line indicates the field data mean length and the vertical line shows one standard deviation from the mean.

The figure shows a tendency for the simulated growth in length to be more rapid than growth in length in the field data. However, the figure also shows that the mean shrimp length from the field data is often close to the mode of the histogram and that one standard deviation from the mean often overlaps a substantial portion of the histogram. This overlap together with the parallel downward slopes in Figure 1 encourage us to regard the mechanism that produced the simulated length data as a reasonable one to use in our analyses.

Figure 3 is a similar graph of histograms of simulated abundance by week. The long tail on the histogram shows the high variability in the simulation generated data as does the long length of the line representing the standard deviation of abundance from the field data. It is also instructive to compare the sample medians of abundance of the two groups of data. Figure 4 shows this comparison. Also included in the figure are observations of sample abundance made by Williams (1955) in 1952-1953. Williams' data were scaled since his measure of sampling effort differs from that used here. Consequently, one should focus on the shape of curves from his data rather than the magnitude. Based on the observed level of agreement, one can regard the mechanism for generating the simulated abundance data to be a reasonable one to employ for present purposes.

Use of Simulation Sampling in a Model for Management Policy Analysis

The sampling technique can be particularly useful for examining the effects of management policy on the fishery. Consider a model of the fishery that enables the sampling of fishery performance measures, such as the market value of catch or the quantity of fish landed, as a function of management policy. If the model is based on sampling the population profile, as we have suggested, it will reflect considerable detail of the biology of the species.

This approach can be used to show how fishery performance changes with respect to alternative policies and various environmental conditions. More importantly, it provides a tool for designing policy that exploits the current environmental conditions. For example, consider sampling on a computer a sequence of weekly estuarine length and abundance representing an arbitrary year. The fishery performance under a host of alternative policies can be evaluated while conditioning on this sequence of samples. Using these data the preferred policy can be identified. In this way we relate management policy to fishery performance. In the example of the shrimp fishery the application of interest is the identification of fishery opening dates that optimize fishery performance as a function of estuarine length and abundance data.

6. Summary

In this technical report we presented 1) a model of post larvae brown shrimp migration into the primary estuaries of the Pamlico Sound, 2) a model of juvenile brown shrimp migrations from the primary estuaries to the secondary estuaries and transport areas, and 3) a model of estuarine abundance. Each of these models accounts for mean behavior and for the random variation that is a significant element of the biological processes. Furthermore, the models of recruitment and abundance also account for the correlation between estuaries. To highlight the importance of inter-estuarine correlation we have shown how such correlation affects the variation in mean recruitment time and in overall abundance.

Techniques for estimation of model parameters are given and are demonstrated with data on length and abundance of juvenile brown shrimp. These data represent four years sampling in 54 estuaries in Pamlico Sound.

The models presented are part of a fishery simulation model for the analysis of management policy. They characterize the initial conditions in the estuarine nurseries in a way that enables computer sampling of a typical years' spring nursery conditions. Furthermore, they provide the capability of simulating the nursery sampling that the marine biologists perform each spring in their effort to assess fishery conditions for management purposes. In Section 5 we developed the simulation model and showed that the simulation of nursery sampling of length and abundance compares favorably to sampling in the field.

TABLE 1

Length Categories Used by the Division of Marine Fisheries in Data Collection

Length Category (mm)	Interval of Lengths Defining Category
15	< 20
25	20 to 30
35	30 to 40
45	40 to 50
55	50 to 60
65	60 to 70
75	70 to 80
85	80 to 90
95	90 to 100
105	100 to 110
115	110 to 120
125	120 to 130
135	130 to 140
145	140 to 150

TABLE 2

Division of Marine Fisheries
Primary Estuary Sampling Sites

NORTH

Primary Estuary	Sampling Site Code	i
Abel Bay	SB1	1
Spensers Bay	SB4	2
Rose Bay	RB2, RB4	3,4
Swan Quarter Bay	SQB1,SQB3,SQB4	5,6,7
Juniper Bay	JB1	8
East Bluff Bay	OC2	9
Wysocking Bay	WB1,WB2	10,11
Far Creek	FC1,FC3	12,13
Long Shoal River	LSR1,LSR3	14,15

SOUTH

Primary Estuary	Sampling Site Code	i
Lower Spring Creek	A-2	16
Goose Creek	A-3	17
Upper Spring Creek	A-16	18
Oyster Creek	A-53	19
Clark Creek	A-58	20
Dog Creek	A-59	21
Long Creek	B-10	22
Porpoise Creek	B-20,B-21	23,24
Middle Bay	B-30,B-32	25,26
Jones Bay	B-40	27
Pitch Creek	B-43,B-44	28,29

TABLE 2 (continued)

Primary Estuary	Sampling Site Code	i
Gales Creek	C-N-5	30
Smith Creek	C-N-10	31
Chapel Creek	C-N-14	32
Ball Creek	C-S-2	33
Bonner Bay	D-4	34
Dipping Vat Creek	C-8	35
Swan Creek	E-1,E-2	36,37
Greens Creek	E-15	38
Orchard Creek	F-1	39
Pierce Creek	F-3	40
Whittaker Creek	F-4	41
Krenshaw Creek	F-12	42
Dawson Creek	F-22	43
Turnagain Bay	G-1,G-3	44,45
Back Creek	G-20	46
Cedar Creek	G-23	47
Issac Creek	G-24	48
Clubfoot Creek	H-2	49
Long Bay	J-2	50
Creek off Thorofare Bay	J-10,J-31	51,52

TABLE 2 (continued)

Primary Estuary	Sampling Site Code	i
North Bay	J-20	53
Beard Creek (off Neuse River)	L-1	54

TABLE 3

Estimated Probability Mass Functions of Recruitment Time to Northern Pamlico Sound Primary Nurseries for 1974 to 1977

(Table Enteries are $p_i(n)$)††

Week† n (1974)

Sampling Site		April			May					June				July		
i	Code	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
1	SB1				.02	$\boxed{.44}$	.05	$\boxed{.36}$	.02	.07	.04	.02	.00			
2	SB4				.03	$\boxed{.31}$	.04	$\boxed{.49}$	.02	.05	.03	.00	.01			
3	RB2				$\boxed{.24}$	.05	.01	.06	$\boxed{.64}$		.00		.00			
4	RB4				$\boxed{.36}$	.01	.01	.19	$\boxed{.31}$	.03	.06	.01	.00			
5	SQB1		.03				$\boxed{.29}$		.03	.06	$\boxed{.32}$	.20		.02	.04	.02
6	SQB3			.00		.03	$\boxed{.48}$	.33	.00	.04	.05	.01	.02	.02	.01	.01
7	SQB4								.03	.13	$\boxed{.46}$	.36				
8	JB1		.01			.05	$\boxed{.73}$			.00	.06	.04			.01	.07
9	OC2			.05		.01	$\boxed{.81}$	.00	.01	.03	.03		.00	.01	.01	.01
10	WB1			$\boxed{.39}$	.11	.00	.11	$\boxed{.32}$	.00	.00	.03	.00		.00	.00	.00
11	WB2						.04	$\boxed{.13}$	.03	.08	.28	$\boxed{.32}$		.02	.04	.02
12	FC1			$\boxed{.12}$	.00		.00	$\boxed{.60}$	.01	.03	.06	.14				
13	FC3			$\boxed{.35}$	.02	.01	.04	$\boxed{.40}$	.01	.02	.09	.04				
14	LSR1				$\boxed{.49}$	.00	.03	$\boxed{.32}$	.00	.01	.10	.00		.03	.01	
15	LSR3				.03	.03	.17	$\boxed{.47}$	.02	.04	$\boxed{.21}$		.00			

TABLE 3 (continued)

Week† n (1975)

Sampling Site		April			May				June				July			
i	Code	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
1	SB1					$\boxed{.46}$	.13	$\boxed{.32}$	.02	.01	.01	.02	.01			
2	SB4				.11	$\boxed{.46}$	.03	$\boxed{.40}$								
3	RB2					.05		.12	$\boxed{.47}$	.24		.12				
4	RB4				$\boxed{.65}$	.03	.05		$\boxed{.22}$	.05						
5	SQB1				.18	.09	$\boxed{.36}$					.09	.09	.19		
6	SQB3				.03	.10	$\boxed{.34}$	$\boxed{.34}$		.02	.03	.03	.07	.04		
7	SQB4				$\boxed{.75}$	.00	.09	.07		.02			.04			
8	JB1						$\boxed{1.0}$									
9	OC2				$\boxed{.19}$				.17	$\boxed{.25}$	.17	.19				
10	WB1				$\boxed{.25}$	.04	.06	$\boxed{.53}$	.04	.01	.01			.01		.01
11	WB2				$\boxed{.26}$	.02	.16	$\boxed{.47}$	.01	.01	.01	.02	.01			
12	FC1				.00		.01	.05	.13	.18	$\boxed{.54}$	.07	.00			
13	FC3			.01	.11	$\boxed{.45}$	.07	$\boxed{.19}$	.01	.04	.04	.02	.04			
14	LSR1				.03		.11	$\boxed{.42}$	.02	.01	.11	.06	.07	.09	.02	.02
15	LSR3				.06	.03	.05	$\boxed{.50}$	.32	.01	.00		.01			

TABLE 3 (continued)

Week† n (1976)

Sampling Site i	Code	April 16	17	18	May 19	20	21	22	June 23	24	25	July 26	27	28	29	30
1	SB1				.01	.19	.57	.11		.02	.02	.02	.04			
2	SB4				.01	.10	.72	.10		.01		.01	.03			
3	RB2				.02	.16	.61	.09		.02	.05	.02	.02			
4	RB4					.09	.49	.19		.04	.09	.03	.01	.02	.02	
5	SQB1					.07	.77	.07					.03	.02	.04	
6	SQB3				.03	.11	.64	.07		.03	.02	.02	.03			
7	SQB4						.91	.03								.06
8	JB1					.01	.68	.05			.01	.02	.08	.13	.02	
9	OC2				.01	.07	.75	.04			.03	.02	.03			
10	WB1				.01	.10	.63	.15			.01	.02	.04			
11	WB2				.00	.02	.39	.46		.02	.02	.01	.02			
12	FC1				.02	.03	.25	.32		.05	.08	.04	.08	.06	.01	.02
13	FC3															1.0
14	LSR1						.07	.85	.03		.01	.02	.02			
15	LSR3							.35					.17			.48

TABLE 3 (continued)

Week† n (1977)

Sampling Site i	Code	April 16	17	18	May 19	20	21	22	June 23	24	25	July 26	27	28	29	30
1	SB1	.24			.06	.55	.07	.02	.03	.01	.01					
2	SB4			.05	.33	.32	.12	.05	.04	.07		.03				
3	RB2			.02	.13	.38	.19	.19	.03	.02	.03					
4	RB4	.21		.05	.11	.36	.02	.12	.13							
5	SQB1	.43		.02	.04	.30	.09	.02	.07	.01	.00	.01	.00			
6	SQB3	.40		.01	.02	.26	.06	.04	.20	.01	.01					
7	SQB4	.00		.02	.05	.50	.14	.05	.09	.05	.05	.05				
8	JB1					.22	.03	.15	.22	.14	.14	.10				
9	OC2	.63				.05	.05	.10	.05	.05						.05
10	WB1	.05		.01	.18	.38	.11	.20	.06	.01	.01					
11	WB2	.05		.00	.04	.42	.05	.26	.07	.05	.02	.03				
12	FC1		.02		.08	.35	.05	.17	.16	.06	.04	.04	.01			
13	FC3	.26		.00	.06	.36	.04	.12	.06	.01	.00	.06	.02	.02		
14	LSR1				.04	.54	.03	.28	.03	.02	.05	.01				
15	LSR3				.01	.05		.01	.01	.08	.59	.25	.01			

† The weeks are numbered sequentially starting with the first week in January.

†† For some estuaries the sum of the recruitment time probability mass function may be less than 1.0. This is the result of roundoff error and/or small values of $P_i(t)$ for $t > 30$ (not shown in the table).

TABLE 4

Estimates of the Mean and Variance of the Time of the First Recruitment Wave

Sampling Site i	$\hat{E}\tau_i^1$	$\widehat{\text{var}}\ \tau_i^1$	Sampling Site i	$E\hat{\tau}_i^1$	$\widehat{\text{var}}\ \tau_i^1$
1	19.25	4.92	28	19.00	1.00
2	20.25	.25	29	22.33	4.33
3	20.75	2.92	30	19.25	0.92
4	18.75	4.25	31	19.50	3.00
5	19.25	5.58	32	20.00	7.00
6	19.75	6.25	33	20.25	3.58
7	21.25	6.92	34	19.00	0.00
8	20.75	0.25	35	20.50	1.67
9	19.25	5.58	36	22.00	2.00
10	19.50	1.67	37	18.53	9.33
11	20.75	2.25	38	20.25	2.92
12	21.25	8.92	39	22.50	40.33
13	21.00	38.67	40	20.50	1.67
14	20.75	2.25	41	19.50	0.33
15	22.75	2.25	42	21.33	4.33
16	18.50	0.50	43	21.33	1.33
17	19.67	0.33	44	19.00	1.00
18	21.00	2.00	45	18.67	0.33
19	19.50	0.50	46	18.75	2.25
20	19.75	2.92	47	19.50	4.05
21	19.25	6.25	48	18.00	0.00
22	20.75	6.92	49	18.00	2.00
23	22.00	11.33	50	19.33	0.33
24	21.75	0.25	51	19.25	0.25
25	21.00	1.33	52	16.00	37.00
26	20.50	3.00	53	19.50	1.67
27	23.00	50.00	54	22.00	1.00

TABLE 5

Estimated First Recruitment Wave Correlation Coefficients

Coefficient	Value
$\hat{\rho}_S^{\tau}$	.227
$\hat{\rho}_N^{\tau}$	.046
$\hat{\rho}_{SN}^{\tau}$	.136

TABLE 6

Estimates of the Probability Mass Function of Inter-recruitment Times given a Distinct Second Wave

m	$\hat{p}_m^N$	$\hat{p}_m^S$
1	.583	.025
2	.125	.300
3	.000	.225
4	.291	.200
5	.000	.150
6	.000	.050
7	.000	.050

TABLE 7

Estimates of the Probability of a Distinct Second Wave

Sampling Site i	$\hat{p}_i$	Sampling Site i	$\hat{p}_i$
1	0.50	28	0.33
2	0.25	29	0.33
3	0.25	30	0.75
4	0.75	31	0.25
5	0.75	32	0.33
6	0.25	33	0.25
7	0.00	34	0.33
8	0.50	35	0.00
9	0.25	36	0.00
10	0.50	37	0.33
11	0.75	38	0.25
12	0.50	39	0.00
13	0.50	40	0.75
14	0.50	41	0.25
15	0.25	42	0.33
16	0.50	43	0.00
17	0.33	44	0.00
18	0.50	45	0.00
19	0.50	46	0.25
20	0.75	47	0.50
21	0.25	48	0.00
22	0.50	49	0.25
23	0.00	50	0.00
24	0.25	51	0.75
25	0.25	52	0.33
26	0.25	53	0.25
27	0.00	54	0.33

TABLE 8

Estimates of the Out-Migration Rates

$\hat{\alpha}_N^1 = .438$ $\alpha_S^1 = .499$

$\hat{\alpha}_N^2 = .338$ $\alpha_S^2 = .579$

TABLE 9

Estimates of Relative Abundance at Nursery Recruitment to Northern Sampling Sites

$$n_i = n_i^1 + n_i^2$$

Sampling Site		Year			
i	Code	1974	1975	1976	1977
1	SB1	532	184	130	318
2	SB4	482	109	150	493
3	RB2	474	32	355	322
4	RB4	279	48	144	197
5	SQB1	66	17	90	312
6	SQB3	199	64	332	209
7	SQB4	31	28	26	161
8	JB1	54	11	84	189
9	OC2	188	30	82	9
10	WB1	433	123	146	74
11	WB2	170	218	219	310
12	FC1	350	228	150	899
13	FC3	425	197	34	560
14	LSR1	284	71	364	227
15	LSR3	378	61	4	229

Table 10

Estimates of the Mean and Variance of Abundance

Sampling Site i	$\widehat{EN}_i$	$\widehat{\text{var}}\ N_i (x10^3)$	Sampling Site i	$\widehat{EN}_i$	$\widehat{\text{var}}\ N_i (x10^3)$
1	291.07	32.02	28	442.33	5.08
2	308.57	43.08	29	1000.67	2337.56
3	295.89	35.28	30	7586.25	193477.30
4	167.25	9.39	31	109.25	3.39
5	121.25	17.03	32	112.67	51.92
6	201.99	11.64	33	136.50	5.22
7	61.69	4.43	34	40.67	.26
8	84.35	5.77	35	1525.00	18.42
9	77.27	6.44	36	671.00	570.31
10	194.32	26.26	37	176.33	20.30
11	229.37	3.39	38	544.25	346.07
12	406.85	114.35	39	160.75	23.08
13	304.05	54.88	40	3851.50	29201.78
14	236.55	15.26	41	1026.75	1416.41
15	186.12	28.67	42	105.00	1.03
16	163.50	35.11	43	103.33	4.23
17	1735.00	7442.63	44	485.00	279.08
18	357.50	1.10	45	90.33	2.16
19	2223.50	65.16	46	47.25	2.23
20	5580.50	115907.70	47	81.50	7.32
21	559.50	373.66	48	126.00	18.82
22	1057.75	932.82	49	190.00	9.61
23	390.50	34.76	50	174.67	44.58
24	416.50	189.65	51	237.75	58.03
25	405.50	141.46	52	473.67	3.29
26	533.75	215.49	53	423.75	367.45
27	136.00	25.09	54	1354.67	4898.87

Table 11

Estimated Abundance Coorelation Coefficients

Coefficient	Value
$\hat{\rho}_S^N$	.125
$\hat{\rho}_N^N$	.359
$\hat{\rho}_{SN}^N$	.242

TABLE 12

Estimates of the Mean and Variance of the Log Transformed Abundance

Sampling Site i	$\hat{E}(\ln N_i)$	$\widehat{var}(\ln N_i)$	Sampling Site i	$\hat{E}(\ln N_i)$	$\widehat{var}(\ln N_i)$
1	5.53	0.29	28	6.08	0.02
2	5.22	0.46	29	5.82	2.24
3	5.32	1.17	30	7.02	4.21
4	4.94	0.43	31	4.58	2.35
5	4.32	1.06	32	4.53	0.47
6	5.16	0.34	33	4.71	0.57
7	3.78	0.57	34	3.64	0.15
8	4.01	1.09	35	4.76	0.50
9	3.80	1.31	36	6.01	1.18
10	5.04	0.41	37	4.69	1.38
11	5.41	0.04	38	5.86	0.88
12	5.77	0.44	39	3.94	0.53
13	5.19	1.04	40	7.32	2.15
14	5.31	0.39	41	6.23	1.87
15	4.23	3.01	42	4.62	0.06
16	4.56	1.27	43	4.45	0.45
17	6.05	3.34	44	5.79	0.72
18	5.88	0.01	45	4.40	0.23
19	7.70	0.01	46	3.09	2.11
20	6.06	5.99	47	4.00	0.91
21	5.65	1.91	48	4.39	1.04
22	6.29	2.20	49	5.14	0.23
23	5.85	0.27	50	4.64	1.02
24	5.33	2.04	51	4.93	1.02
25	5.71	0.54	52	4.25	9.30
26	5.55	2.62	53	5.19	1.87
27	4.34	1.36	54	4.89	7.91

TABLE 13

Estimated Log Transformed Abundance Correlation Coefficients

Coefficient	Value
$\hat{\rho}_S^{\ln N}$	.127
$\hat{\rho}_N^{\ln N}$	.328
$\hat{\rho}_{SN}^{\ln N}$	.228

TABLE 14

Estimates of the Probability that a Shrimp Recruits at τ_i^1 Given it Recruits

Sampling Site i	$\hat{p}_i^1$	Sampling Site i	$\hat{p}_i^1$
1	.45	28	.70
2	.60	29	.72
3	.36	30	.69
4	.55	31	.71
5	.39	32	.78
6	.43	33	.85
7	1.00	34	.38
8	.62	35	1.00
9	.36	36	1.00
10	.48	37	.54
11	.52	38	.76
12	.32	39	1.00
13	.38	40	.68
14	.56	41	.50
15	.52	42	.67
16	.34	43	1.00
17	.66	44	1.00
18	.77	45	1.00
19	.40	46	.72
20	.69	47	.82
21	.50	48	1.00
22	.48	49	.65
23	1.00	50	1.00
24	.40	51	.52
25	.54	52	.33
26	.85	53	.74
27	1.00	54	.41

TABLE 15
Simulation Generated Data and Field Data Summary Statistics

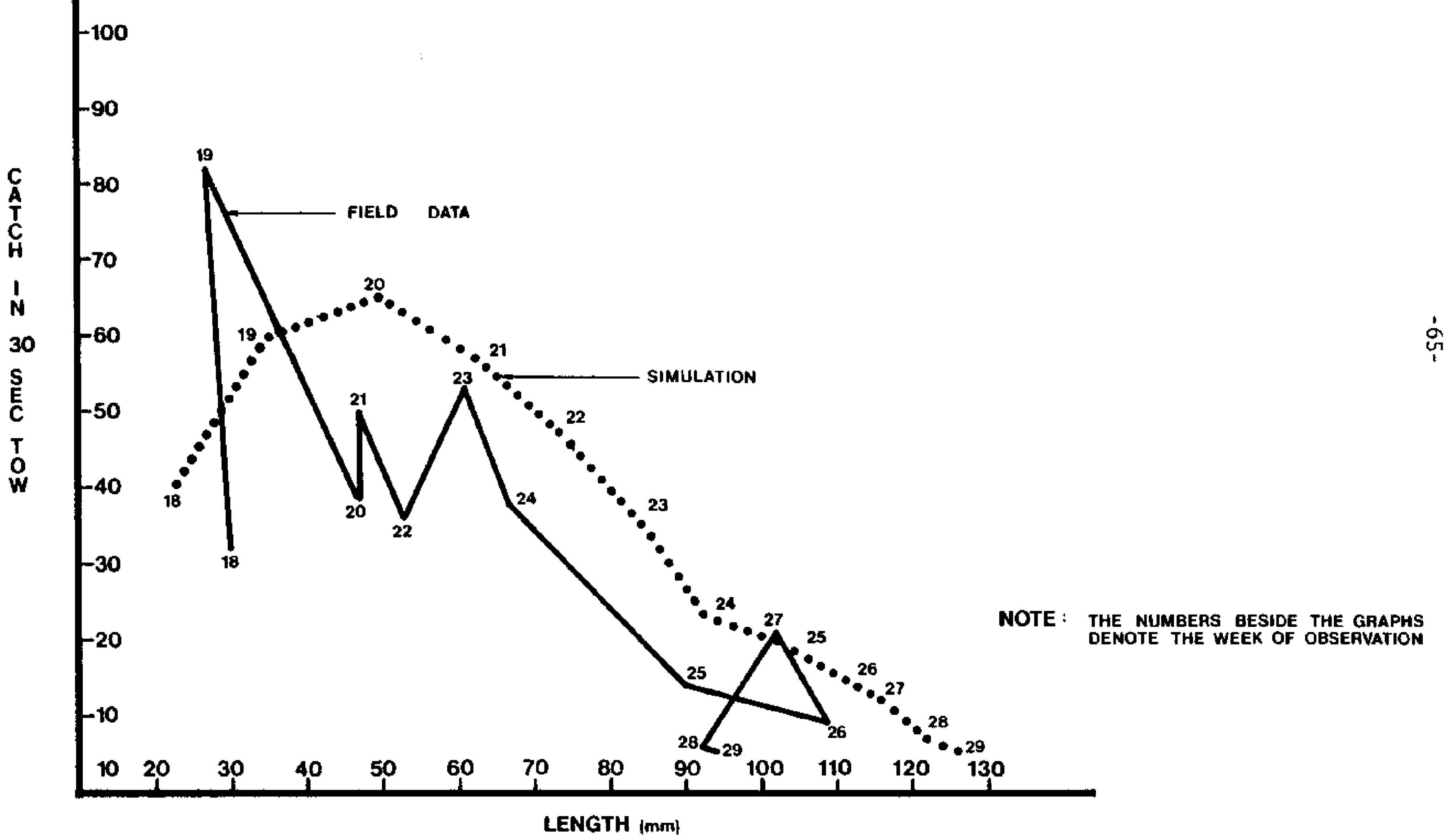
Month	Week	Data	No.	Length					Catch				
				Q1	Q2	Q3	Mean	StDev	Q1	Q2	Q3	Mean	StDev
	18	S	1000	12	23	34	28.3	12.1	19	41	84	94.9	241.3
		F	1	30	30	30	30	-	32	32	32	32	-
May	19	S	1000	26	35	46	37.6	18.1	33	60	112	122.1	241.8
		F	3	20	27	43	30.2	11.7	20	82	168	89.8	74.2
	20	S	1000	40	50	60	50.7	18.5	37	65	113	122.6	221.5
		F	4	31	47	53	43.9	12.0	17	39	79	45.2	33.6
	21	S	1000	52	63	74	63.2	17.8	33	57	103	108.0	183.5
		F	4	44	47	53	47.8	4.7	27	50	150	76.0	72.0
	22	S	1000	63	74	86	74.1	19.2	28	47	82	93.1	192.7
June		F	4	44	53	70	55.7	14.1	14	36	69	39.8	29.3
	23	S	1000	72	84	98	85.0	22.5	22	36	64	84.3	370.7
		F	4	47	61	69	59.2	11.8	21	53	153	75.4	73.1
	24	S	1000	79	93	108	94.4	26.3	16	27	47	69.2	453.0
		F	4	63	67	80	69.9	9.3	28	38	60	42.0	17.7
	25	S	1000	86	103	121	104.8	29.0	12	19	35	55.0	473.5
		F	4	75	90	107	90.5	16.7	6	14	36	19.1	16.3
	26	S	1000	93	112	134	113.6	38.1	8	14	25	39.5	335.8
July		F	3	53	109	110	90.7	32.7	9	9	62	26.8	30.6
	27	S	1000	91	116	142	121.3	57.5	6	10	18	27.2	223.0
		F	4	79	102	113	97.9	17.4	7	21	51	26.1	23.6
	28	S	1000	93	122	151	130.4	146.6	4	7	13	18.7	139.7
		F	4	90	92	106	96.2	9.6	3	6	40	16.4	23.5
	29	S	1000	94	127	164	133.9	76.5	3	5	9	12.9	89.6
		F	4	87	94	112	97.7	13.5	3	5	11	6.5	4.0

Data: S = Simulated data
F = Field data

Figure 1

A Plot of Sample Median Catch/30 sec tow vs. Sample Median Length

For Simulated Nursery Data and Field Data By Week



NOTE: THE NUMBERS BESIDE THE GRAPHS DENOTE THE WEEK OF OBSERVATION

Figure 2

Histogram of Simulated Shrimp Length in the Nursery and Field Data Mean and Standard Deviation of Length By Week

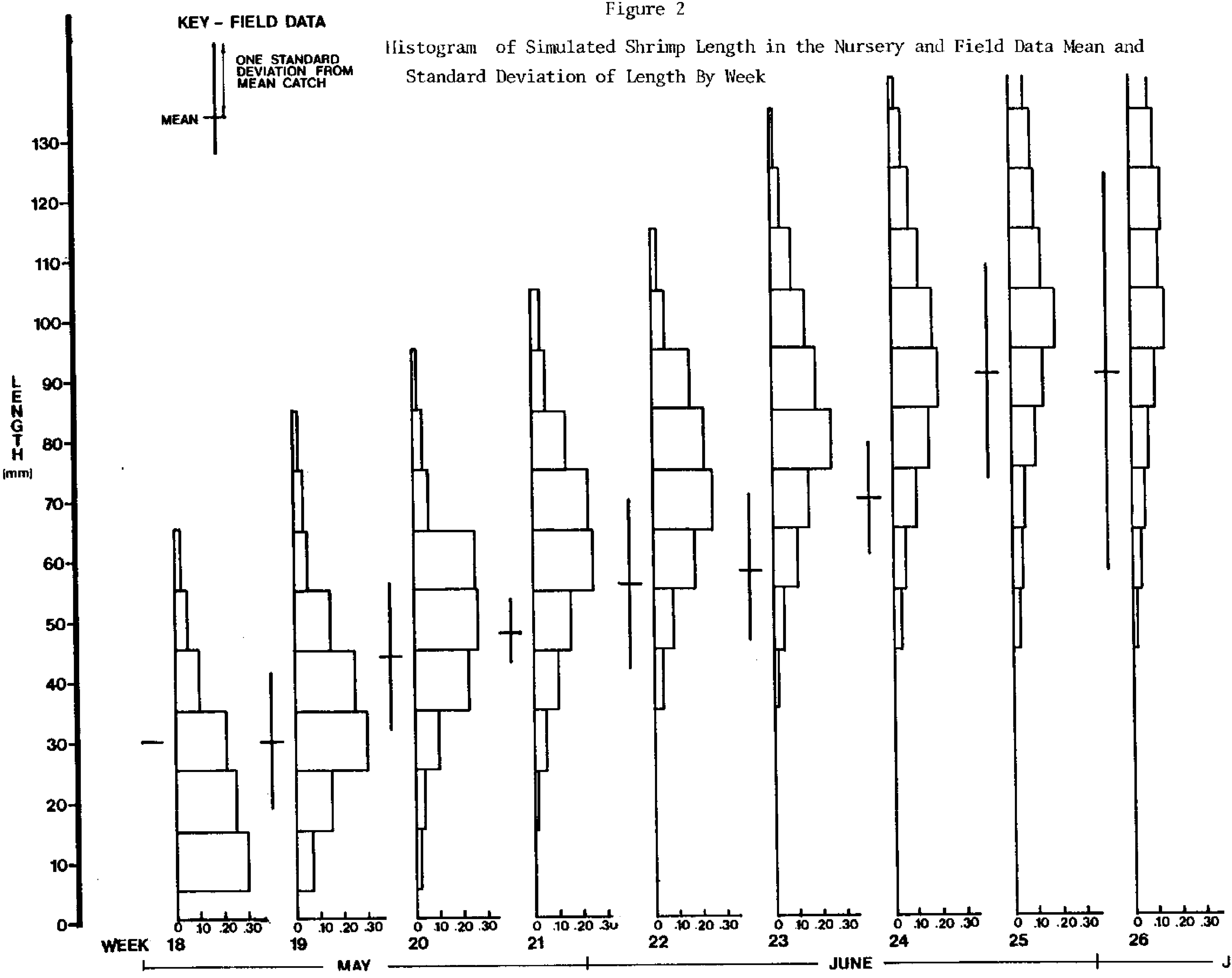


Figure 3
Histogram of Simulated Nursery Catch (30 Sec Tow) and Field Data mean and Standard Deviation of Catch By Week

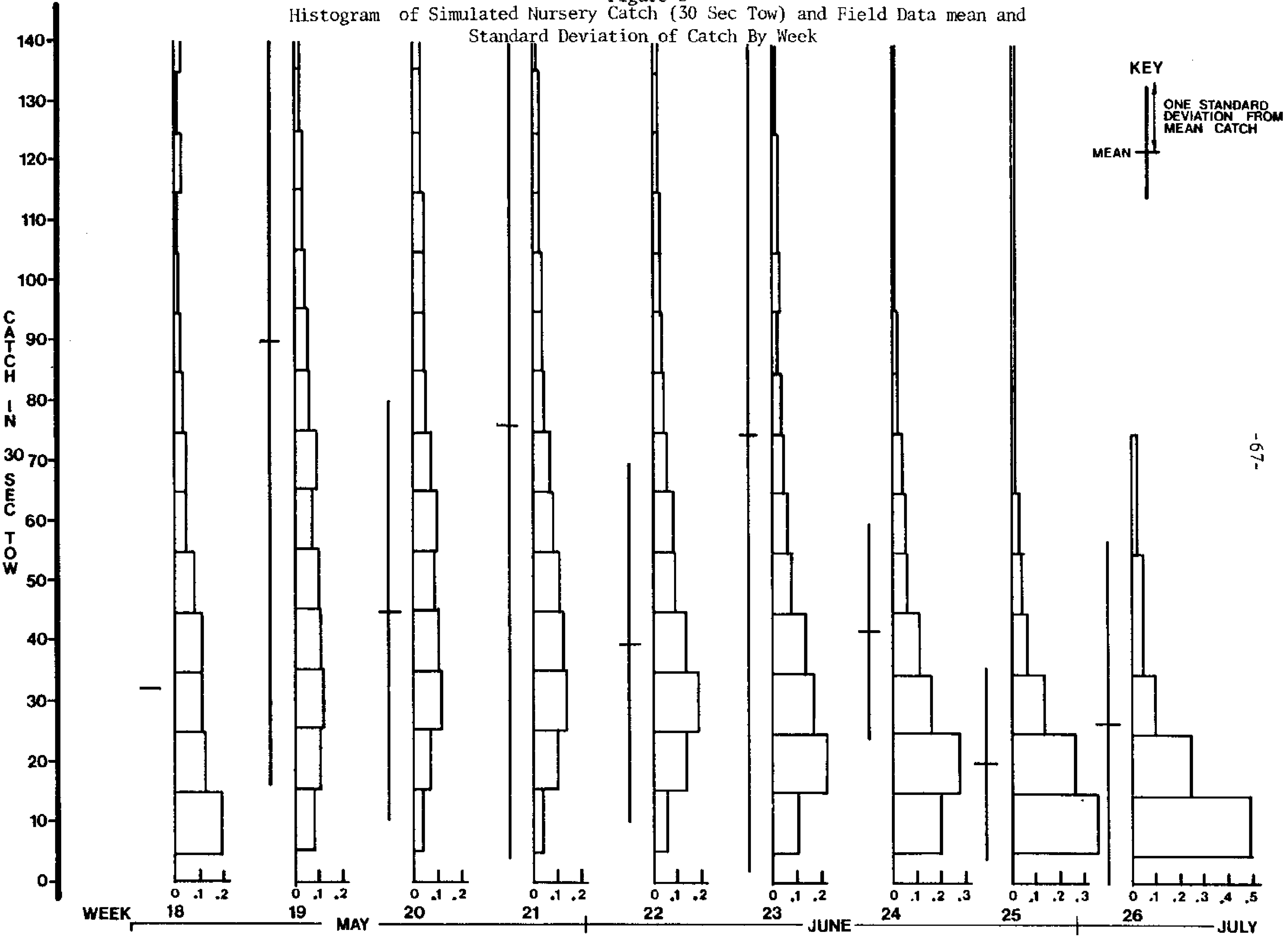
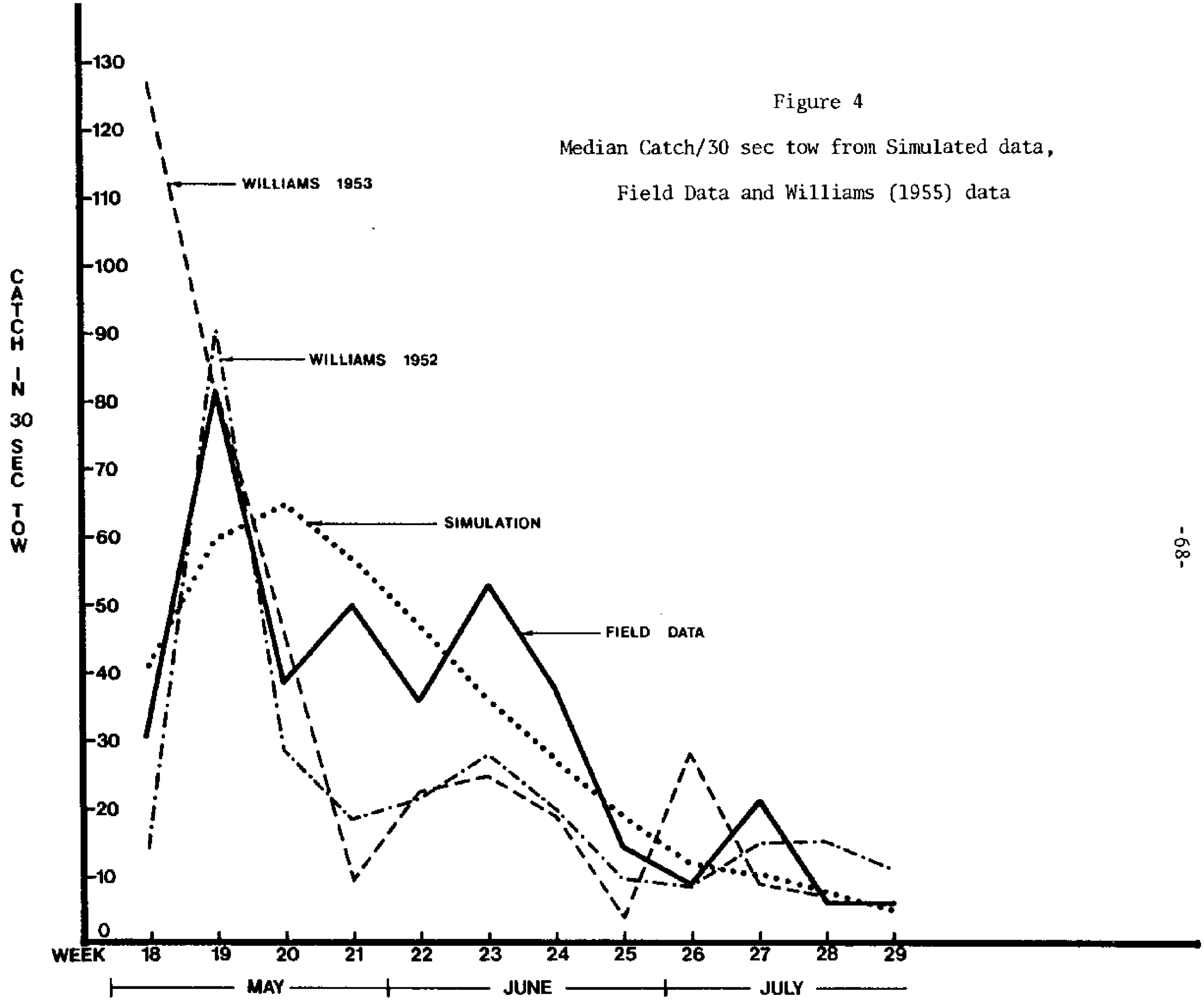


Figure 4

Median Catch/30 sec tow from Simulated data, Field Data and Williams (1955) data



APPENDIX A

In this appendix we show how the estimate $\hat{\alpha}_i^k$ defined in (3.8) is derived from expression (3.7). Recall that n_{ij} is the catch in estuary i in sample j at time t(i,j). Also recall that n_{ij} is an estimate of $E[N_i^k(t'(i,j))|N_i^k=n]$ where

$$E[N_i^k(t'(i,j))|N_i^k=n] = \begin{cases} 0 & \text{if } t'(i,j)<\hat{\tau}_i^k \\ n\exp[-\alpha_i^k(t'(i,j)-\hat{\tau}_i^k)] & \text{otherwise.} \end{cases} \qquad (A.1)$$

Now, consider samples j and j+1 that satisfy either

$$\hat{\tau}_i^1 \leq t'(i,j) < t'(i,j+1) \leq \hat{\tau}_i^2 \qquad (A.2)$$

or

$$\hat{\tau}_i^2 < t'(i,j) < t'(i,j+1). \qquad (A.3)$$

Samples j and j+1 that satisfy one of these constraints contain shrimp that are identified as having arrived in a single recruitment wave. If j and j+1 satisfy (A.2) then shrimp in both samples arrived in recruitment wave 1 at $\hat{\tau}_i^1$, while if j and j+1 satisfy (A.3) then shrimp in both samples arrived in recruitment wave 2 at $\hat{\tau}_i^2$. For j and j+1 that satisfy either (A.2) or (A.3) we write the ratio of (A.1) for j to (A.1) for j+1 as

$$\frac{n_{ij}}{n_{ij+1}} = \frac{E[N_i^k(t'(i,j))|N_i^k=n]}{E[N_i^k(t'(i,j+1))|N_i^k=n]}$$

$$= \exp[-\alpha_i^k(t'(i,j)-t'(i,j+1))] \qquad (A.4)$$

where k=1 if (A.2) holds and k=2 if (A.3) holds. Note that for some j and j+1 neither (A.2) nor (A.3) may hold. Taking the ratio in this way eliminates the unknown n from equation (A.1). Solving (A.4) for α_i^k yields an expression whose quantities are all known. In particular, we obtain

$$\frac{\ln(n_{ij}/n_{ij+1})}{t'(i,j+1)-t'(i,j)} \tag{A.5}$$

The estimate $\hat{\alpha}_i^k$ as defined by (3.8) is the average of expression (A.5) over all j for which (A.2) holds if k=1, or for which (A.3) holds if k=2. We write the average precisely by defining the function $I_{(k)}(j)$ as

$$I_{(k)}(j) = \begin{cases} 1 & \text{if } k=1 \text{ and } t'(i,j+1) \le \hat{\tau}_i^2 \\ 1 & \text{if } k=2 \text{ and } t'(i,j) > \hat{\tau}_i^2 \\ 0 & \text{otherwise,} \end{cases}$$

then summing the product of $I_{(k)}(j)$ and expression (A.5) over all J_i samples. The estimate $\hat{\alpha}_i^k$ thus obtained is

$$\hat{\alpha}_i^k = \frac{1}{\sum_{j=1}^{J_i} I_{(k)}(j)} \sum_{j=1}^{J_i} \left[\frac{\ln(n_{ij}/n_{ij+1})}{t'(i,j+1)-t'(i,j)}\right] I_{(k)}(j),$$

which agrees with (3.8).

APPENDIX B

This appendix contains algorithms used in (1) estimation of model parameters, for (2) simulation sampling of the initial population profile, and for (3) simulation sampling of spring nursery sampling.

Algorithm EA

```
Given:  $\rho_s$, $\delta^\dagger$, and $\{ \lambda_n : n=1,\dots,14 \}$
1.  $n \leftarrow 1$.
2.  $\max \leftarrow 0$.
3.  $x \leftarrow \delta$.
4.  Evaluate $g(\lambda_n|x)$ using (3.1).
5.  If $\max < g(\lambda_n|x)$ go to 8.
6.  $\max \leftarrow g(\lambda_n|x)$.
7.  $\Delta\hat{t}(\lambda_n) \leftarrow x$.
8.  $x \leftarrow x+\delta$ ; if $x \le 52$ go to 4.
9.  $n \leftarrow n+1$; if $n \le 14$ go to 2.
10. Deliver $\{ \Delta\hat{t}(\lambda_n) : n=1,\dots,14 \}$.
```

† The value of δ determines grid size. We use $\delta = .05$ weeks.

Algorithm RT

```
Given: $\{\Delta\hat{t}(\lambda_n):n=1,\dots,14\}$, $i$, $n_{ij}$, $J_i$, $\{\ell_{ijk}:1\le k\le n_{ij}:1\le j\le J_i\}$.
1.  $j \leftarrow 1$.
2.  $k \leftarrow 1$.
3.  $\hat{t}_0(i,j,k) \leftarrow t(i,j) - \Delta\hat{t}(\ell_{ijk})$.
4.  $k \leftarrow k+1$;  if $k\le n_{ij}$ go to 3.
5.  $j \leftarrow j+1$;  if $j\le J_i$ go to 2.
6.  Return $\{\hat{t}_0(i,j,k):1\le k\le n_{ij};1\le j\le J_i\}$.
```

Algorithm ST1

```
Given: $\hat{\rho}_N^\tau$, $\hat{\rho}_S^\tau$, $\hat{\rho}_{SN}^\tau$, $(E\hat{\tau}_1^1,\ldots,E\hat{\tau}_{54}^1)$, $(\hat{var}\ \tau_1^1,\ldots,\hat{var}\ \tau_{54}^1)$.
1. Call COV given: $(\hat{\rho}_N^\tau, \hat{\rho}_S^\tau, \hat{\rho}_{SN}^\tau, 16, (\hat{var}\ \tau_1^1,\ldots,\hat{var}\ \tau_{54}^1))$ return $\Sigma$.
2. Call LTM given $\Sigma$ return $\underset{\sim}{C}$.
3. For $i=1,\ldots,54$ sample $z_i$ from $N(0,1)$.
4. $i \leftarrow 1$.
5. $\tau_i^1 \leftarrow E\tau_i^1 + \sum_{j=1}^{i} C_{ij}\ z_j$.
6. $\tau_i^1 \leftarrow \lfloor \tau_i^1 \rfloor$.
7. If $\tau_i^1 < 0$ go to 3.
8. $i \leftarrow i+1$. If $i \leq 54$ go to 5.
9. Return $(\tau_1^1,\ldots,\tau_{54}^1)$.
```

Algorithm SN1

```
Given: $\hat{\rho}_N^{\ln N}, \hat{\rho}_S^{\ln N}, \hat{\rho}_{SN}^{\ln N}, (\hat{E}(\ln N_1),\ldots,\hat{E}(\ln N_{54})), (\hat{var}(\ln N_1),\ldots,\hat{var}(\ln N_{54}))$.
1. Call COV given $(\hat{\rho}_N^{\ln N}, \hat{\rho}_S^{\ln N}, \hat{\rho}_{SN}^{\ln N}, 16, (\hat{var}(\ln N_1),\ldots,\hat{var}(\ln N_{54}))$
   return $\Sigma$.
2. Call LTM given $\Sigma$ return $\underset{\sim}{C}$.
3. For $i=1,\ldots,54$ sample $z_i$ from $N(0,1)$.
4. $i \leftarrow 1$.
5. $N_i \leftarrow \hat{E}N_i + \sum_{j=1}^{i} C_{ij} z_j$.
6. If $N_i < 0$ go to 3.
7. $i \leftarrow i+1$. If $i \leq 54$ go to 5.
8. For $i=1,\ldots,54$; $N_i \leftarrow \exp(N_i)$.
9. Return $(N_1,\ldots,N_{54})$.
```

Algorithm ST2

```
Given: $(\tau_1^1,\ldots,\tau_{54}^1)$, $(\hat{p}_1,\ldots,\hat{p}_{54})$, $\{\hat{p}_m^N,\hat{p}_m^S : m=1,\ldots,7\}$.
1.  For i=1,...,54 sample $U_i$ from $U(0,1)$.
2.  $i \leftarrow 1$.
3.  If $U_i > \hat{p}_i$ go to 10.
4.  Sample W from $U(0,1)$.
5.  $j \leftarrow 0$.
6.  $j \leftarrow j+1$.
7.  If $i \leq 15$ and $W > \sum_{m=1}^{j} \hat{p}_m^N$ then go to 6.
8.  If $i > 15$ and $W > \sum_{m=1}^{j} \hat{p}_m^S$ then go to 6.
9.  $\tau_i^2 \leftarrow \tau_i^1 + m+1$.
10. $i \leftarrow i+1$.  If $i \leq 54$ go to 3.
11. Return $(\tau_1^2,\ldots,\tau_{54}^2)$.
```

Algorithm SN2

```
Given: $(N_1,\ldots,N_{54})$, $(\hat{p}_1^1,\ldots,\hat{p}_{54}^1)$.
1.  $i \leftarrow 1$.
2.  Sample Z from $\mathcal{N}(0,1)$.
3.  $N_i^1 \leftarrow N_i \hat{p}_i^1 + Z\sqrt{N_i \hat{p}_i^1 (1-p_i^1)}$.
4.  If $N_i^1 < 0$ then go to 2.
5.  $N_i^2 \leftarrow N_i - N_i^1$.
6.  If $N_i^2 < 0$ then go to 2.
7.  $i \leftarrow i+1$.  If $i \leq 54$ go to 2.
8.  Return $(N_1^1,\ldots,N_{54}^1)$ and $(N_1^2,\ldots,N_{54}^2)$.
```

ALGORITHM SA

```
Given: ${(\tau_1^k,\ldots,\tau_{54}^k), (N_1^k,\ldots, N_{54}^k) : k=1,2}$ , $(V_1\ldots,V_{54})^{\dagger}$,
       and ${\alpha_i^k : k=1,2 ; i=1,\ldots, 54}$, $p_\mu$.
1.  $i \leftarrow 1$.
2.  $k \leftarrow 1$.
3.  $A_i^k(\tau_i^k) \leftarrow V_i N_i$.
4.  $X(i,k) \leftarrow \exp\{-\alpha_i^k\}$.
5.  $k \leftarrow k+1$.  If $k=2$ go to 3.
6.  $i \leftarrow i+1$.  If $i<55$ go to 2.
7.  $i \leftarrow 1$.
8.  $j \leftarrow 1$.
9.  $R_i(j) \leftarrow 0$.
10. $j \leftarrow j+1$.  If $j \le \tau_i^1+1$ go to 9.
11. $n \leftarrow \tau_i^1+1$.
12. $k \leftarrow 1$.
13. Sample $Z_1$ and $Z_2$ from a standard normal distribution.
14. $p \leftarrow (1-X(1,k))(1-p_\mu)$.
15. $q \leftarrow 1-p$.
16. $R_i(n) \leftarrow A_i^k(n)\, p+Z_1 \sqrt{A_i^k(n)pq+R_i(n)}$.
17. $A_i^k(n+1) \leftarrow A_i^k(n)\, X(i,k)+Z_2 \sqrt{A_i^k(n)\, X(i,k)(1-X(i,k))}$.
18. $k \leftarrow k+1$.  If $k \le 2$ go to 13.
19. $n \leftarrow n+1$.  If $n \le 52$ go to 12.
20. $i \leftarrow i+1$.  If $i \le 54$ go to 11.
21. Return ${(R_1(n),\ldots,R_{54}(n)) : n=1,\ldots,52}$ and
           ${(A_1^k(n),\ldots,A_{54}^k(n)) : k=1,2; n=1,\ldots,52}$.
```

† For demonstrative purposes, in the example in Section 5.3

nursery volume is assumed equal at each sampling site i, with total nursery volume equal to 100000, hence V_i=1851.85 for i=1, ...,54. Total nursery volume is chosen so that the simulated mean commercial catch biomass agrees in order of magnitude with observed commercial catch biomass.

Algorithm COV

```
Given: $\rho_1$, $\rho_2$, $\rho_3$, k, and $(V_1,\ldots,V_{54})$.
1. $i \leftarrow 1$.
2. $\sigma_{ii} \leftarrow V_i$.
3. $j \leftarrow i+1$.
4. $\rho \leftarrow \rho_3$.
5. If $i > k$ and $j > k$ then $\rho \leftarrow \rho_2$.
6. If $i < k$ and $j < k$ then $\rho \leftarrow \rho_1$.
7. $\sigma_{ij} \leftarrow \rho \sqrt{V_i V_j}$.
8. $\sigma_{ij} \leftarrow \sigma_{ji}$.
9. $j \leftarrow j+1$. If $j \le 54$ go to 4.
10. $i \leftarrow i+1$. If $i \le 54$ go to 2.
11. Return $\Sigma \equiv \{\sigma_{ij} : i=1,\ldots,54;\ j=1,\ldots,54\}$.
```

Algorithm LTM

```
Given: $\Sigma \equiv \{\sigma_{ij} : i=1,\ldots,54;\ j=1,\ldots,54\}$.
1. $a \leftarrow \sqrt{\sigma_{11}}$.
2. For $i=1,\ldots,54$; $C_{i1} \leftarrow \sigma_{i1}/a$.
3. $i \leftarrow 2$.
4. $C_{ii} \leftarrow (\sigma_{ii} - \sum_{j=1}^{i-1} C_{ij}^2)^{1/2}$.
5. If $i=54$ then return $\underset{\sim}{C} \equiv \{C_{ij} : i=1,\ldots,54;\ j=1,\ldots,54\}$.
6. $i \leftarrow i+1$.
7. For $j=2,\ldots,i-1$; $C_{ij} \leftarrow (\sigma_{ij} - \sum_{l=1}^{j-1} C_{il} C_{jl})/C_{jj}$.
8. Go to 4.
```

Algorithm SNUR

```
Given:  $p(e)$†, $\{(A_1^k(n),\ldots,A_{54}^k(n))$ ; $k=1,2$; $n=1,\ldots,52$ $\}$.
1.  For $n=1,\ldots,52$; $\bar{N}(n) \leftarrow 0$; $\bar{S}(n) \leftarrow 0$.
2.  $i \leftarrow 1$.
3.  $n \leftarrow \hat{\tau}_i^{(1)}$.
4.  Sample $X_1$ from Bin $(A_i^1(n), p(e))$.
5.  Sample $X_2$ from Bin $(A_i^2(n), p(e))$.
6.  $\bar{N}(n) \leftarrow \bar{N}(n) + X_1 + X_2$.
7.  If $\bar{N}(n)=0$ then go to 13.
8.  Evaluate $ES_i(n)$ and var $S_i(n)/(X_1+X_2)$ using (5.3).
9.  Sample $Z$ from $N(0,1)$.
10. $Y_1 \leftarrow ES_i(n) + Z\sqrt{\text{var } S_i(n)/(X_1+X_2)}$.
11. If $Y_1 < 0$ then go to 9.
12. $\bar{S}(n) \leftarrow \bar{S}(n) + Y_1$.
13. $n \leftarrow n+1$.  If $n \leq 52$ go to 4.
14. $i \leftarrow i+1$.  If $i \leq 54$ go to 3.
15. $\bar{S}(n) \leftarrow \bar{S}(n)/54$, $\bar{N}(n) \leftarrow \bar{N}(n)/54$, return.
```

†For a 30 second tow with a 10 foot net $p(e)=1.4288 \times 10^{-7}$.

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