Working Paper 78-4

Submarine Wastewater Outfall Near Field Flow Dynamics And Mixing

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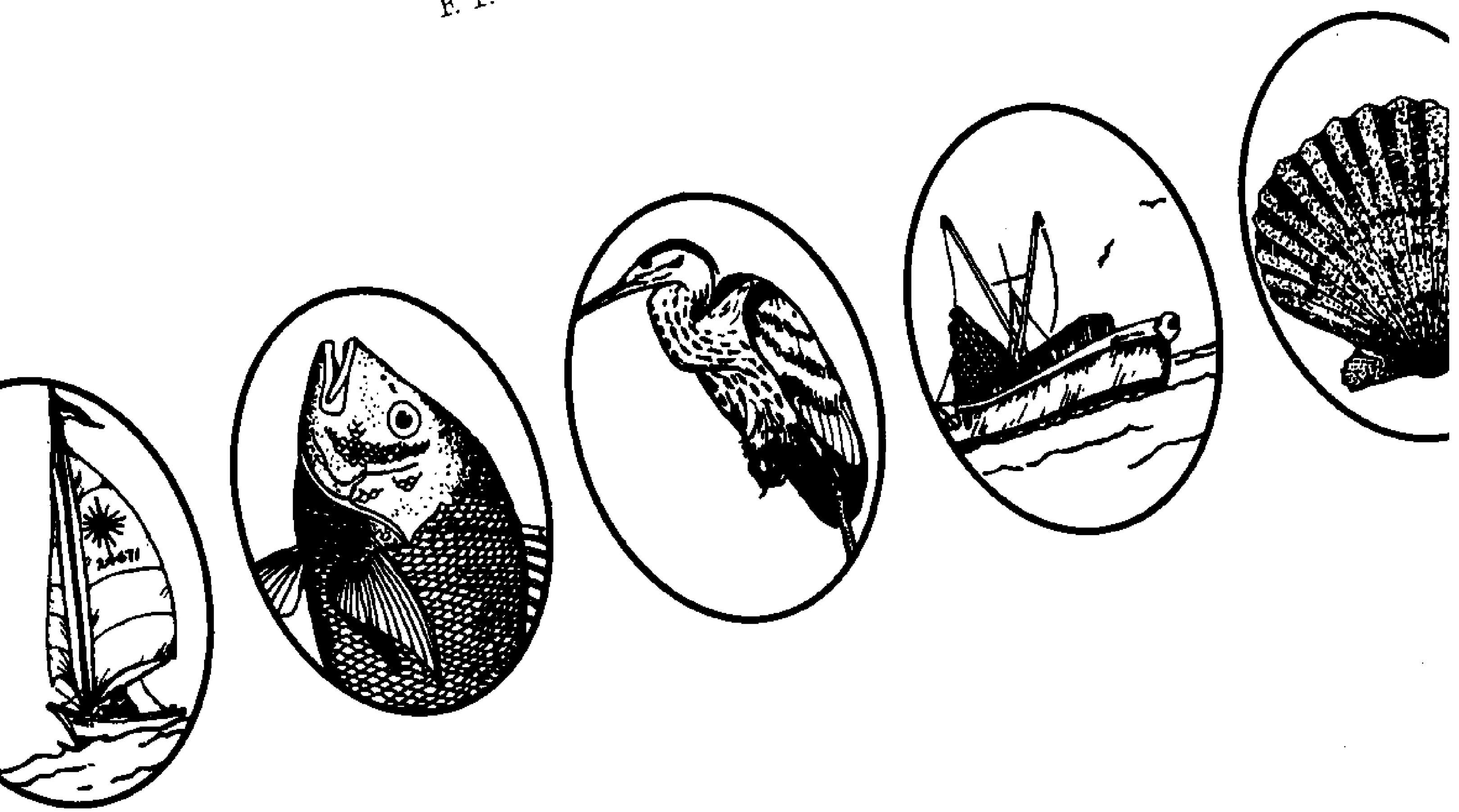

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SUBMARINE WASTEWATER OUTFALL
NEAR FIELD FLOW DYNAMICS AND MIXING

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This work was sponsored by the Office of Sea Grant, NOAA, U.S. Department of Commerce, under Grant No. 04-8-M01-66, and the North Carolina Department of Administration.

Sea Grant College Working Paper 78-4 October, 1978

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ABSTRACT

A model is developed for the flow dynamics and mixing in the near-field of a multiple port outfall diffuser. The model employs the equations for gross conservation of mass, momentum and buoyancy of the fluid in the discharge jet or plume. These equations are obtained by assuming flow similarity and then integrating over the cross-sectional area of the plume. Two distinct interactions of the discharge jets or plumes are included in the model. These are the interaction of the individual round jets from the diffuser ports and the merging of the plume from either side of the diffuser, over the top of the diffuser. The resulting equations are closed by the "entrainment assumption" and solved numerically. Results provide the velocity, width and dilution of the jet or plume.

Calculations were made for a number of cases where experimental results were available. The model gives reasonable agreement with the experiments over a wide range of discharge conditions and over the complete range of flow patterns. In most cases it slightly underestimates the mixing or dilution. Therefore, the model should be useful in determining the minimum dilution that can be expected from any marine outfall.

In view of the extensive analysis on this particular problem, there has been surprisingly little experimental work. Thus, there was not a great deal of data with which to compare the present model. Additional data on the process by which the plumes merge is especially needed. Moreover, while dilution is the parameter of greatest interest to the engineer, it is shown that it is a fairly insensitive measure of flow dynamics in the mixing zone. Therefore, the measurement of some other quantities such as the velocity or plume width would be of more use. This is particularly true when attempting to understand the flow-dynamics in the near-field and when comparing results from different models or situations.

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NOTATION

The following symbols are used in this paper.

b= Jet or plume width.

F_o= Densimetric froude number based on port diameter.

g= Acceleration of gravity.

H_m= Height of plume use before merging.

K= Plume translation coefficient.

L= Port spacing (c.f. Figure 1).

n= Coordinate normal to two-dimensional plume axis.

r= Coordinate normal to round jet or plume axis.

s= Coordinate along jet or plume axis.

u= Jet or plume centerline velocity.

V_e= Entrainment velocity.

V_n= Plume translation velocity.

x= Horizontal coordinate.

y= Vertical coordinate.

α= Entrainment coefficient.

θ= Angle of plume axis with horizontal.

λ= Spreading coefficient (λ^2= Schmidt number).

ρ_a= Density of background ocean water.

$\rho(s)$=Density of fluid in jet or plume.

ρ_o= Density of ocean water at discharge point.

INTRODUCTION

The primary purpose of the diffuser, which is located on the end of the outfall pipe, is to mix the effluent being discharged with the ambient or surrounding water as much and as rapidly as possible. The benefits of this mixing are lower concentration of the effluent, even near the outfall, and a diminished health hazard. In addition there is a possible increase in die-off of enteric microorganisms as a result of their low concentrations. In order to do this a diffuser uses the momentum and buoyancy of the discharged effluent as the driving forces and attempts to direct these forces in a way to produce maximum mixing. These mechanisms are effective only as long as they are the significant forces producing the flow. Therefore, diffuser design can be effective in increasing the mixing in that area near the diffuser where the discharge momentum and buoyancy are still the dominant forces. This region is usually referred to as the "near field" mixing zone. Clearly, once these forces are dissipated, the path and further dilution of the effluent are controlled by the local (or ambient) conditions in the ocean. The purpose of the work reported here is to provide an analytical technique for describing the flow dynamics in the "near-field" mixing zone, and to predict effluent concentrations in this region.

The complete design or feasibility analysis of a marine outfall is obviously quite complicated. In addition to the primary objective of maximum mixing, the diffuser is also subjected to both economic and structural constraints: what is the cost of providing this mixing and will the diffuser withstand all necessary forces? Thus one simply cannot take the approach that a certain dilution is required and then determine the appropriate configuration. Rather a more productive approach, and the one employed here, is to consider a proposed or probable design and to analyze the mixing and dispersion provided by the specific design. The work begins with the assumption that the design is, or can be made, structually sound and that it is economically practical. Then, after analysis of the "near field" flow dynamics, if it is determined that the design does not provide adequate mixing or that it results in an adverse environmental or public impact, the design must be altered and subsequently re-analyzed.

Before proceeding further with analysis of the flow dynamics and mixing, some consideration must be given to the discharge of toxic materials from the outfall. No amount of mixing with ocean water, no matter how extensive, can completely offset the continued discharge of toxic materials. Therefore it is necessary to assure that the discharge will not increase, or add to the toxic substances in the ocean. All kinds of water, including rain and drinking water, contain some identifiable (trace) quantities of heavy metals and possibly other toxic materials. This also applies to the ocean, which contains most known toxic substances in very small, but often measurable quantites. The usual approach (which is supported here) is to assure that the outfall will not increase the concentration of any toxic material already in the ocean, and that it will not indroduce any new toxic substances.

Generally, with domestic sewage typical of that from eastern North Carolina treatment plants, this can be achieved without difficulty. Barber, et.al. (1977) investigated a possible increase in heavy metal concentration due to the treated effluents of Beaufort, Morehead City, New Bern and Newport. The researchers found no significant amounts of toxic materials. Moreover, toxic materials from new industrial sources are required by the Environmental Protection Agency (EPA) to be removed by pre-treatment before discharge into a municipal waste treatment facility. Using federal guidelines as a standard (EPA, 1977), it can be shown that all known toxic substances can be reduced to essentially ambient levels by correct outfall design. This conclusion is supported by the recent investigation of Pearce and Smallwood (1978).

Assuming no toxic materials of significance are being released, the major concern then becomes the health effects of enteric microorganisms. These organisms decay with time and, in addition, they are not harmful in sufficiently low concentrations. Because of this, adequate dilution with the ambient ocean water will usually prevent them from causing any environmental impact or becoming a public health hazard. The health hazard can also be removed by sufficient disinfection, but there are indications that most common disinfectants may be toxic themselves. Therefore it is very important that the diffuser provide the maximum mixing possible, and that this mixing and the resulting dilution be accurately predicted. This will enable, among other things, use of a minimum amount of disinfection.

Proceeding the detailed description of the model used for the near-field calculations, a brief discussion of the overall operation and design of a typical outfall is given. As previously mentioned, the model requires that a specific design be available for the analysis. This means a specific diffuser geometry and the discharge parameters for each of the diffuser ports. Site data required are the depth of the water and the slope of the bottom at the diffuser location, as well as the density of the seawater. Previous work by Sorrell (1978) has attempted to establish typical diffuser configurations for the Southeastern coast and the range of discharge parameters that are likely to be utilized. That work also includes a method for numerical analysis of the internal hydraulics of the diffuser, so that the discharge parameters can be accurately calculated when a specific design is given. The method employs a technique first developed by Brooks (1970), and a excellent summary and overview of the problem has been given by Koh and Brooks (1975).

As previously mentioned, the near-field mixing zone of the diffuser is that region where the flow dynamics are either completely or partically controlled by the discharge conditions and the diffuser. Conversly the far-field is that region where the mixing and/or flow dynamics are controlled primarily by ocean currents and eddy motion (turbulence). In some situations, high volume thermal discharges also may modify the far-field. Thus, the flow in both the near and far-fields may be at least altered by the discharge. This is partically true for high volume thermal discharges in very shallow water. This is not the situation with wastewater discharges in the ocean, however, and thus the far-field is not considered here. Moreover, design or feasibility calculations are normally for "worse case" conditions or the set of circumstances that produces the least mixing. Usually this is zero current, although the recent work by Nospal and Tatinclaux (1976) has shown that may not always be true for thermal discharges. Based on these observations the worst case would be in the near-field and with zero background current. The present work provides a calculation procedure for this situation.

In order to better understand various aspects of the model a qualitative description the flow dynamics produced by a typical diffuser with representative discharge parameters is given. The expected diffuser configuration is shown graphically in Figures 1 and 2. As is indicated in Figure 1, the discharge from a multiple port (multi-port) diffuser initally consists of a series of round horizontal jets. These jets are driven outward by the discharge momentum and upward because the wastewater is less dense than the ocean. As the round jets entrain fluid, their diameter increases and ultimately the individual jets begin to interfere with each other. It is generally accepted (Koh and Fan, 1970; Jirka and Harleman, 1973; Shirnazi and Davis, 1972) that the individual round jets interact to form a two-dimensional or slot jet. The situation is depicted in Figures 1 and 2, where the resulting flow field, after interaction of the round jets, is a two-dimensional buoyant plume rising from either side of the diffuser pipe. These plumes continue to entrain fluid as they rise. Because there is a limited volume of background fluid between the two plumes, they move toward each other as required by the conservation of mass of the fluid between the two plumes. If the receiving water is sufficiently deep, the two-dimensional plumes eventually will merge together as they rise thus forming a single vertically rising plume. It is a plume in the true sense, because the horizontal discharge momentum has been canceled and the only driving mechanism is the buoyancy of the less dense effluent. The merging or joining of these two-dimensional plumes has been observed in the field and has been demonstrated by the laboratory experiments of Liseth (1970).

Therefore, although the effluent discharged from a multi-port diffuser begins initially as a series of round horizontal jets, these jets usually interact very quickly to form a two-dimensional plume. The plumes on either side of the diffuser then merge over the diffuser and result in a single vertically rising plume. If the diffuser is located in deep water, the details or fine structure of the flow patterns do not greatly alter the final flow geometry or dilution of the effluent as it reaches the surface (or a terminal level). This has been demonstrated by the numerical experiments of Wallis (1977). And, in fact, Koh and Brooks (1975) have utilized this observation to obtain preliminary estimates of dilution by simply considering only a vertical plume. This is accurate because the discharge momentum is quickly dissipated by turbulent entrainment and, thus, the primary long-term driving mechanism is buoyancy.

Southeastern coastal waters are relatively shallow, however, and thus the round jet interaction and the subsequent merging of the two-dimensional plumes may not have been completed before the rising effluent reaches the ocean surface. Here the dilution, velocity and configuration of the discharge plume does depend on the degree of interference that has occured between the jets. Therefore, in southeastern coastal waters an accurate calculation of the mixing requires an analysis of the round jet interference, and of the merging

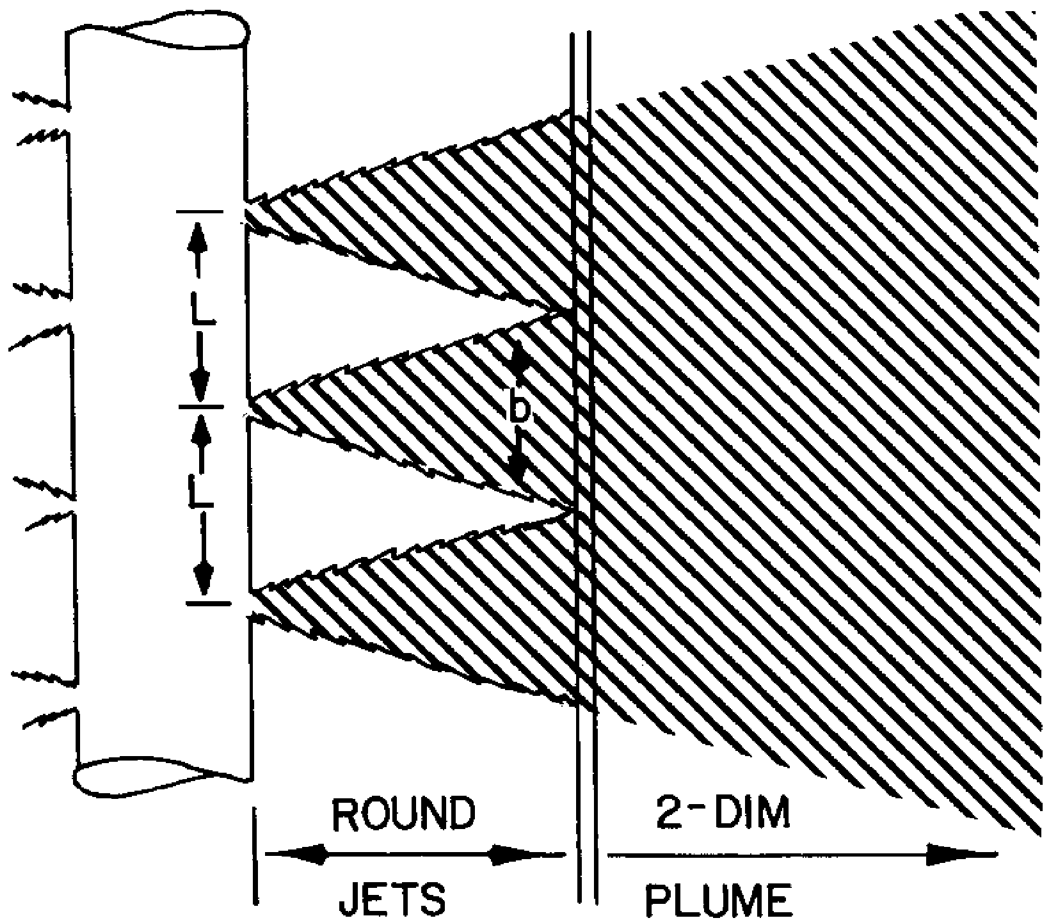

Figure 1. Schematic of round jet interference (Plain view).

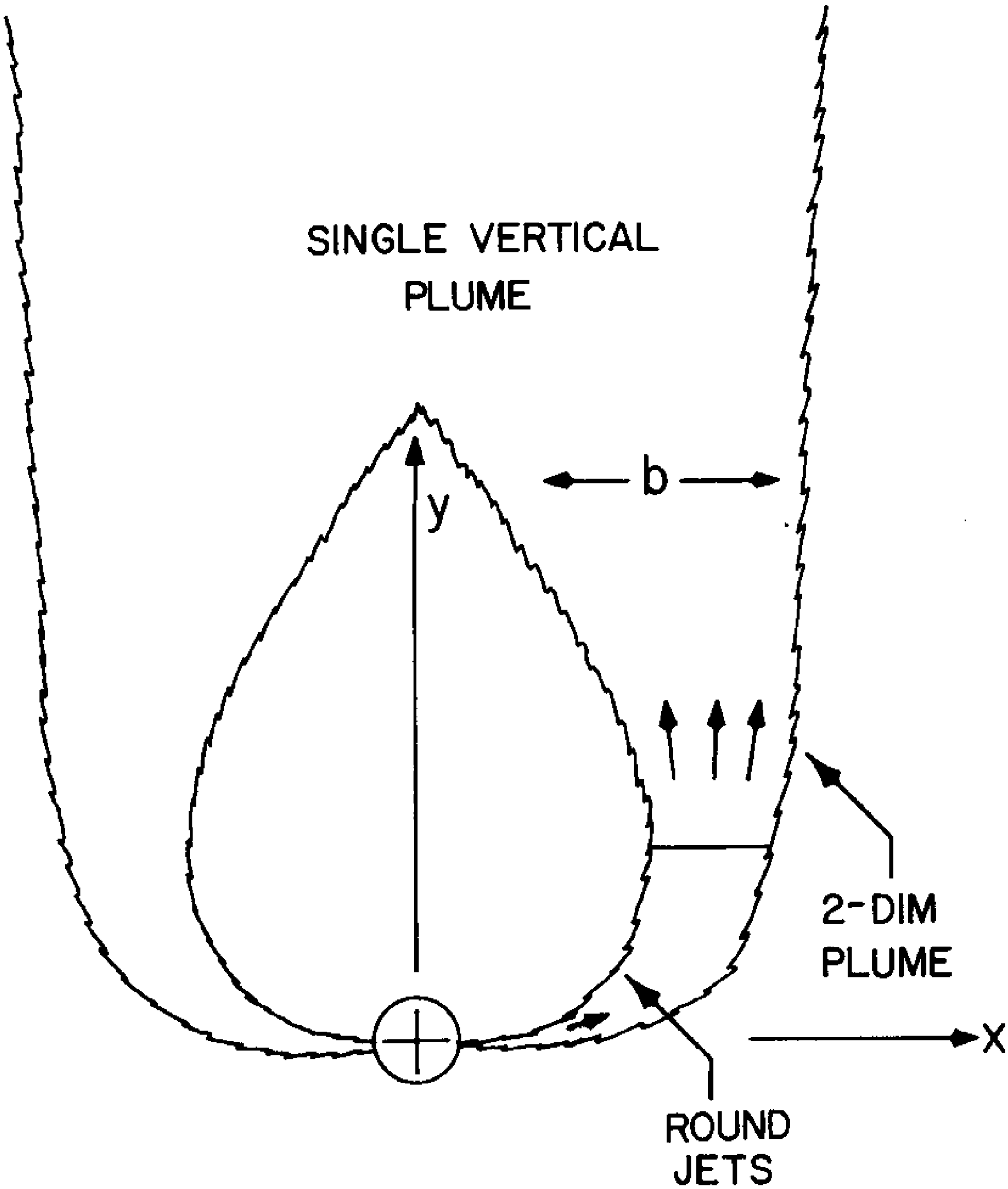

Figure 2. Schematic of two-dimensional plume merging to single vertical plume (end view).

and the two-dimensional plume. This paper provides a rather simple model for these two processes, but one that agrees reasonably well with all experimental data available. The procedure thus provides a complete calculation of the near-field flow patterns or dynamics and the mixing or dilution that would occur from any multi-port diffuser operating in southeastern coastal waters.

MODEL

The model utilized has its origin in the classical work by Morton, Taylor and Turner (1956), and by Priestley and Ball (1955). They assume quasi-similarity in the sense that the mean flow is self-similar and that the turbulent entrainment or the turbulence itself can be directly related to the mean flow. In this approach the equations of motion are integrated over the cross-sectional area of the jet or plume to give gross-flux equations, for example the equations used by Morton, Taylor and Turner (1956) are conservation of mass, momentum and buoyancy. They close the system of equations by the "entrainment assumption," which asserts that the velocity at which the background fluid is drawn into the jet or plume (the entrainment velocity) by turbulent diffusion can be related to the mean flow parameters. One of the first applications of this approach to outfalls was by Fan (1967) and by Fan and Brooks (1969). There have been numerous other approaches and improvements using basically this method. Morton (1971) gives an excellent discussion of the various different conservation equations that have been used and the particular advantages of each. When using the entrainment assumption the entrainment velocity is usually related to the mean flow velocity at the jet or plume centerline by use of an entrainment coefficient. The entrainment velocity being the entrainment coefficient times the mean velocity at the centerline. The question of the functional relation between the entrainment coefficient and the mean flow quantities has been considered in detail by List and Imberger (1973). At approximately the same time Jirka and Harleman (1973) suggested a specific relation for the two-dimensional buoyant jet, based in part on the work of Fox (1970). In the work considered here the discharge momentum is very quickly dissipated, and for most of the calculation the discharge is essentially a plume. Therefore, the present calculation uses a constant value of entrainment coefficient, typical of that recommended by Brooks (1973) for plumes. The actual values are given in Table 1.

TABLE 1

Values of Entrainment Coefficient α and Spreading Ratio λ Used for the Calculations.

Round Jets		
	Entrainment Coefficient	$\alpha_1 = 0.082$
	Spreading Ratio	$\lambda_1 = 1.16$
2-Dimensional Plume		
	Entrainment Coefficient	$\alpha_2 = 0.14$
	Spreading Ratio	$\lambda_2 = 1.0$

When the assumptions and restrictions that follow are considered, the use of a constant entrainment coefficient seems appropriate.

The discharge leaves the diffuser as a series of round jets, and the model starts by applying the procedure to the individual round jets. Initial conditions are the jet diameter and velocity, as computed from the hydraulic analysis (c.f. Sorrell, 1978). These are corrected for the transition from approximately uniform flow out of the port to the assumed Gaussian velocity profile by using the results of Abbertson et. al. (1950). This results in a velocity profile of the form:

$$u=u(s)\ e^{-r^2/\lambda b^2} \tag{1}$$

where u= centerline velocity, s= distance along the jet or plume axis, r= the coordinate normal to the plume axis and b = the (local) plume diameter. In order to allow for different rates of spreading (different effective diameters) between mass and momentum, the density profiles are represented by:

$$\frac{\rho_a-\rho\ (s,r)}{\rho_o} = \frac{\rho_a-\rho(s)}{\rho_o} \quad e{-r^2/(\lambda b)^2} \tag{2}$$

where ρo = the ambient or background density at the discharge location, ρ_a= the ambient density, $\rho(s)$ the density of the fluid in the jet or plume, and λ_1= the ratio between the mass transfer diameter and the momentum transfer diameter of the jet or plume. As such λ_1^2 is the turbulent Schmidt number which is assumed constant and is usually found to be somewhat larger than one.

These profiles are then integrated over the area of the jet or plume to yield gross flux conservation equations. In the present approach equations for conservation of mass, momentum and buoyancy are obtained. Brooks (1973) gives a summary of this calculation and a report by Ditmars (1969) provides a detailed description of the procedure as applied to a single round jet. After integration and some subsequent manipulation, one obtains a set of total differential equations for the centerline velocity, u, the jet or plume diameter, b, the buoyancy $\rho-\rho$ (s), the angle of the jet or plume axis with the horizontal, θ and the x and y coordinates of the axis. These equations are

$$\frac{du}{ds} = \frac{2g\lambda_1{}^2(\rho a\ -\ \rho)\sin\theta}{u\quad \rho o} - \frac{2\alpha_1 u}{b} \tag{3}$$

$$\frac{du}{ds} = 2\alpha_1\ \frac{g\lambda_1{}^2\ \ (\rho a\ -\ \rho)\ \sin\theta}{u^2 \qquad \rho o} \tag{4}$$

$$\frac{d(\rho a-\rho)}{ds} = \frac{1\ +\lambda_1{}^2}{\lambda_1{}^2} \quad \frac{d\rho a}{dy} \quad \sin\theta \frac{2\lambda_1(\rho a-\rho)}{b} \tag{5}$$

$$\frac{d\theta}{ds} = \frac{2g\lambda_1{}^2(\rho a-\rho)\ \cos\theta}{u^2 \qquad \rho o} \tag{6}$$

$$\frac{dx}{ds} = \cos\theta \tag{7}$$

$$\frac{dy}{ds} = \sin\theta \tag{8}$$

Using the previously established initial conditions these equations are then integrated numerically. Two numerical procedures were used. One was an extropolation algorithm developed by Bulirsch and Stoer (1966) and the other was a modified Adams predictor-corrector algorithm developed by Gear (1971). Both algorithms were available as packaged computer sub-routines and were called from computer memory. In the sample program provided in Appendix I, the Bulirsh and Stoer alogrithm is referred to as DREBS and the Gear alogrithm is referred to as DVOGER. The calculation proceeds assuming a round jet until the diameter of the jet, b, equals some fraction of the port spacing L (c.f. Figure 1). At this time the computation switches the model from a round jet to a two-dimensional or slot plume. This approach to round jet interference has been suggested previously by Cedewall (1971) and utilized by Jirka and Harleman (1971). An alternate criteria for transition has been suggested by Koh and Fan (1970) based on equal entrainment rates. While this may be more satisfactory from a philosophical view, the result is essentially the same as the previously proposed criteria. Implict in this procedure is the assumption that the transition from a self-similar round plume to a self-similar two-dimensional plume is quite rapid.

The model thus produces transition from round to slot plumes when the round plume width equals some fraction of the port spacing. Present calculations were made using equivalence of the two, b=L. The transition provides a slot plume whose initial width is that of the round plume before transition, which is also the port spacing L. (c.f. Figure 1). The initial centerline velocity and concentration in the slot plume are assumed to be equal to those of the round plume immediately before transition. The initial conditions for the slot plume are thus established.

If merging of the slot plumes is not a consideration, the integral technique can then be used to continue the calculation. The equations and procedure have been summarized by Brooks (1973). A detailed description with constant entrainment coefficient is given by Sotil (1971) and with a variable entrainment coefficient by Jirka and Harleman (1973). However, if the merging of the two slot plumes from either side of the diffuser is to be considered, some modification of this procedure is required.

In order to model the merging of the slot plumes, the following mechanism is postulated. Before the round plumes begin to interact, the volume of background fluid between these plumes is basically unrestricted and thus there is very little, if any, tendency for the round plumes on either side of the diffuser to move toward each other. After formation of the two-dimensional or slot plume, flow of the background fluid into the region between the plumes can occur only at the ends of the diffuser and is thus severely restricted. Accordingly, merging of the two-dimensional plumes is assumed to begin after the transition from the round to the two-dimensional plumes. When this occurs the two-dimensional plume is shifted, normally to the centerline, toward the diffuser. This is because the background fluid between the two plumes is restricted in volume and is being entrained into the plume. Moreover it is argued that the shift of the plumes (toward each other, see Figure 2) is directly related to the entrainment velocity. The present model assumes that the plumes move toward each other at some fraction of the entrainment velocity. Implicit in this is the assumption that the entrainment velocity is unchanged by the merging process.

The gross-flux equation for the slot plume is thus modified by inclusion of a velocity normal to the plume axis. This velocity is

$$V_n = KV_e \tag{9}$$

where V_n is the velocity of the plume normal to its axis (see Figure 2), V_e is the entrainment velocity and K is some constant less than or equal one. Using the concept of an entrainment coefficient, α_2, the equation becomes:

$$V_n = K\,\alpha_2\,u \tag{10}$$

where u is the centerline velocity. It is reasonable to expect the merging to become more pronounced (larger shift normal to the plume axis) as the plume rises. Therefore, the original intention was to vary K through the calculation, and to determine the most accurate way to do this from experiments. However, as discussed in the next section, due to a lack of relevant experimental results, simply selecting K = 1 was all that reasonably could be done.

Again Gaussian profiles are assumed for the velocity and density distributions in the two-dimensional plume. That is the two-dimensional velocity profile is given by

$$u = u(s)\; e^{-n^2/b^2} \tag{11}$$

where u= the center velocity as before, n= the coordinate normal to the plume axis, and b is the (two-dimensional) plume width. The profile of density deficiency with respect to the ambient density is given by

$$\frac{\rho_a - \rho(s,n)}{\rho_o} = \frac{\rho_a - \rho(s)}{\rho_o} \quad e^{-n^2/\lambda^2 b)^2}$$

where the density symbols, ρ, take the same meaning as in equation (2). Again λ_2^2 is the turbulent Schmidt, but in this case for a two-dimensional plume rather than the round jet or plume. Values that were used for α_1, and α_2 and of λ_1 and λ_2 are provided in Table I; these values have been suggested by Brooks (1973). Using these expressions the two-dimensional plume is also integrated over its area to yield gross flux conservation equations. These equations, including the terms to account for the merging of the plumes are as follows:

$$\frac{du}{ds} = \frac{\sqrt{2}\; g\lambda_1{}^2}{u}\frac{(\rho a - \rho)\; \sin\theta}{\rho o} - \frac{2\alpha_1 u}{b} \tag{12}$$

$$\frac{db}{ds} = \frac{4a_2}{\sqrt{\pi}} \; \frac{\sqrt{2}g\lambda_2}{u^2}\frac{(\rho a - \rho)b\; \sin\theta}{\rho o} \tag{13}$$

$$\frac{d(\rho a - \rho)}{ds} = \frac{1 + \lambda_2^{\,2}}{\lambda_2^{\,2}} \; ^{1/2} \; \frac{d\rho a}{dy} \sin\theta - \frac{2a_2(\rho a - \rho)}{\sqrt{\pi}\beta} \qquad (14)$$

$$\frac{d\theta}{ds} = \frac{\sqrt{2}g\lambda}{\rho} (\rho\alpha - \rho) \frac{\cos\theta}{u^2} + \frac{K\alpha_2}{(x^2 + y^2} \; ^{1/2} \qquad (15)$$

$$\frac{dx}{ds} = \cos\theta - k\alpha_2 \sin\theta \qquad (16)$$

$$\frac{dy}{ds} = \sin\theta + k\alpha_2 \cos\theta \qquad (17)$$

All symbols in these equations have been defined previously. The terms on the right of equations 15, 16 and 17 (enclosed by the square brackets) are the additional terms added to model the merging process.

The trajectory of the two-dimensional plume is computed and the inner and outer edge of the plume calculated at each step. This edge is defined as the width where velocity has decreased to $1/e^2$ of centerline velocity. The inside edge of the plume is that side closest to the diffuser. If the inside edges of both plumes overlap, the plumes are then assumed to have merged. Because the flow pattern is symmetric this occurs at x=0. Therefore the merging criteria is when the inner edge of the plume intersects the y-axis. After this occurs the model makes the transition to a single vertically rising plume. The initial conditions for the vertical plume are an initial width of twice the single plume (two individual plumes have merged), and initial velocity and concentration equal that of the two-dimensional plumes before merging. The calculation is then completed, i.e. carried to the surface or to the computed terminal level, as a single vertically rising plume. Numerically this is a special case of the two-dimensional plume and is quite easily carried out. Figure three shows a graph of the previously defined edges of a plume which was drawn by computer. The calculation is for 9.15 cm (0.30 ft.) diameter ports, spaced 731 cm (24 ft.) apart on each side of the diffuser and with a discharge velocity of 192 cm/s (6.3 ft.s). The discharge is fresh water into weakly stratified seawater. The seawater density at the diffuser is 1.025 gm/cm^3. The transition from round to a two-dimensional plume occurs at a height of 2130 cm (70 ft.) above the bottom. The above parameters were chosen to model the Orange County, California, marine outfall. The actual computer program for this case is given in Appendix I of this report. Results from this program are given in Appendix II.

In summary the model considers two distinct interactions (1) The interaction of the individual round jets from the diffuser ports to form a two-dimensional plume. The interaction criteria is based on a comparison of the round jet width to spacing or distance between jets. (2) Merging of the two-dimensional plumes rising on either side of the diffuser. Merging criteria is based on overlapping of the inner edge of the plumes.

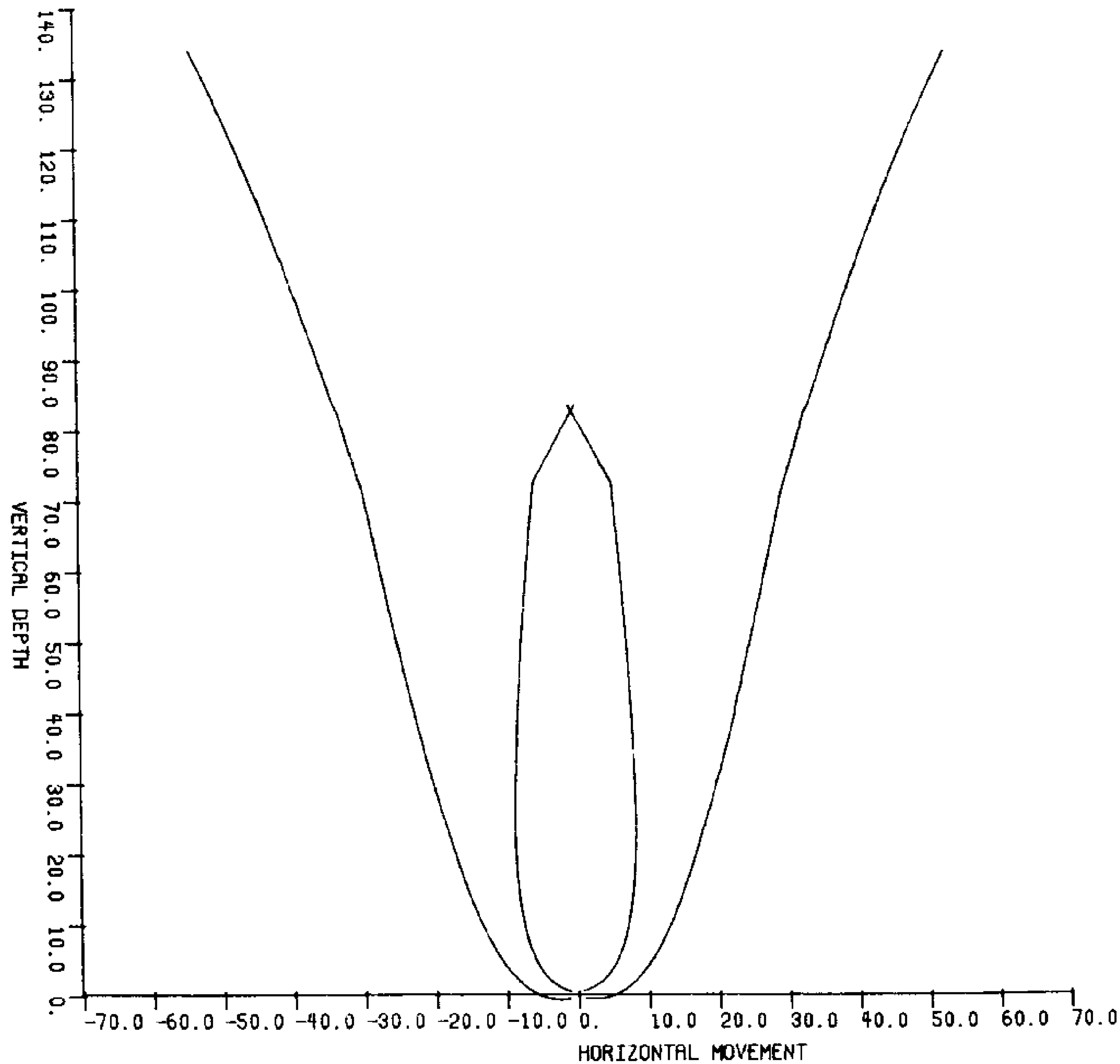

Figure 3. Computer plot of calculated plume geometry

RESULTS

Because of the large number of parameters that are used in a complete calculation, it was not considered feasible to give general results either in tabular or graphical form. Rather specific results are given for prescribed discharge conditions, stratification and diffuser parameters (diameter, spacing). One of the first objectives was to compare computed and experimental results. This was done not only to investigate the validity of the model, but also to try to determine the proper range of values for K as defined by Equation 9. It is rather surprising that, in view of the large quantity of high quality analysis devoted to this problem, only a meager amount of experimental work has been undertaken.

Of the experiments reported in the literature, those by Liseth (1970) seem the only ones suited for the present purposes. These experiments were on the discharge from a model of a multi-port diffuser and were conducted in the laboratory. Liseth (1970) reports results from a large number of experiments in which he measured dilution at the jet or plume centerline. In addition some concentration profiles normal to the axis of the diffuser were also measured and pictures were taken of the flow pattern. These were the only data found from which the location that the plumes merged could be determined. In order to evaluate several choices for K, the numerical model was run in all the cases where there was experimental data for the height at which the plume merged. The results indicated that a value of K=1 gave reasonable agreement. Because this was a larger value than expected an additional approach was used.

In a number of the experiments the port spacing (L in Figure 1.) was sufficiently close that the round jets to interact very quickly. In this situation transition into the two-dimensional plume occurs very near the diffuser and the subsequent merging into a single vertical plume should be primarily because of entrainment into the two-dimensional plumes. Thus the merging must occur relatively independently of the round jet interaction. This was modeled numerically by assuming an equivalent two-dimensional plume initally, that is, beginning at the diffuser. This model should merge at least as quickly as the experiments, and thus the value of K determined in this manner should be a minimum value. This value so determined was approximately K=1. On the other hand the maximum value that one would expect from consideration of mass conservation is K=1, therefore this value was chosen for the subsequent calculations.

It is rather surprising that, in order to achieve merging of the two-dimensional plumes as rapidly as observed experimentally, they must move toward each other with a velocity at least as large as the entrainment velocity. This observation is relatively independent of the processes that occur in the round jet or plumes, and thus it would appear to validate the assumption that the merging begins only after transition to the two-dimensional plume. It would therefore rule out some motion toward each other before transition to two-dimensional plumes as an explanation of the relatively large value required for K. A possible explanation is the generation of a pressure field that would promote merging in addition to that which occurs from simple mass conservation. Clearly the model is too crude and the experiments too sparse to explore this further.

TABLE II

Comparison of Computed and Experimental Values

Diffuser Characteristics		
Port Diameter = 0.373 Cm.		
Port Spacing = 10 Cm.		
Discharge Velocity = 84 Cm/s.		Froude Number = 24
Computed Values		
Round Jets Merge S = 25 Cm.	Height = 12 Cm.	
Vertical Plume S = 106 Cm.	Height = 92 Cm.	
Comparison At S = 40	Height = 25	
	Measured	Calculated
Width	22-24 Cm.	19.5 Cm.
Dilution	41-43	34

The calculation procedure, therefore, gives a reasonable value for the amount of mixing that will occur over the complete range of possible flow patterns. Moreover, when compared with experiments, the computed values of dilution are consistently slightly lower than the measured values. Because of this, the model should be quite useful in the evaluation of the environmental or public health impact from any marine outfall. The dilution as computed by the present model should always be conservative or slightly less than that which actually will occur.

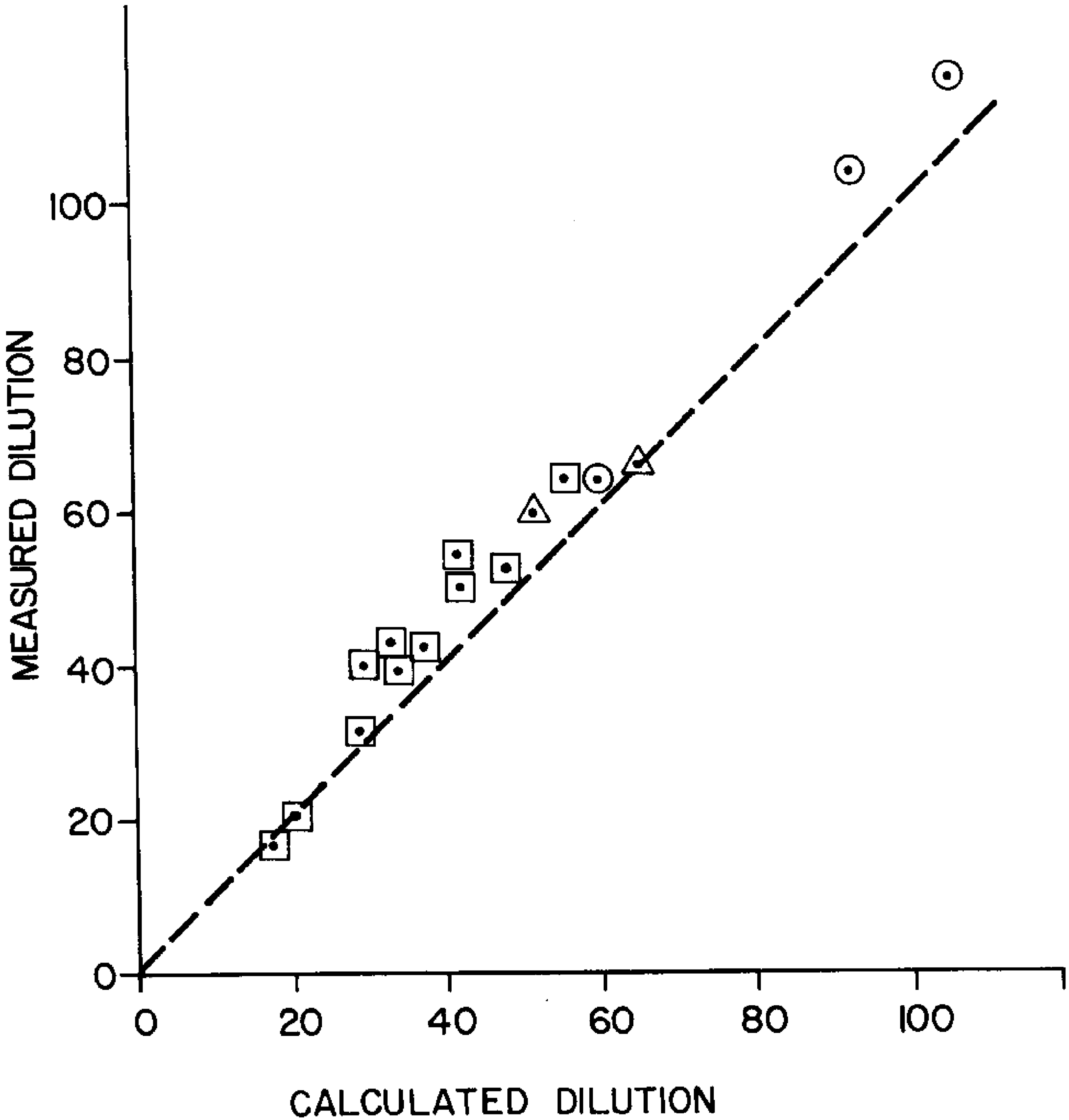

Figure 4. Comparison of calculated and measured values of dilution
Single round jet
Round jet plus two-dimensional plume
Round jet plus two-dimensional plume plus vertical plume

SUMMARY AND CONCLUSIONS

The discharged effluent in the near-field or mixing zone from a multiport diffuser was modeled. The model uses the gross-flux equations of mass momentum and buoyancy and is closed by the entrainment assumption. The interaction of the round jets from the individual diffuser ports was modeled by a transition from round jets to a single two-dimensional plume. This was assumed to occur when the width of the jet reaches some fraction of the port spacing. The resulting two-dimensional plumes from either side of the diffuser then merge over the top of the diffuser. This is believed to occur because the volume of background fluid available to be entrained into the plumes is restricted by the plumes themselves. It is modeled by assuming the plumes translate toward each other at some fraction of the entrainment velocity.

Results calculated by this model were compared with previously reported experimental data. It was found that translating the plumes at the entrainment velocity yields good agreement. Although this is a higher velocity than expected from physical reasoning, it does appear to provide the best agreement. This may be because too few experimental data points were available to permit an accurate comparison or because an additional mechanism, other than entrainment of the background fluid, causes the plumes to merge together.

The dilution as computed by the model was compared with experimentally measured values over as wide a range of discharge conditions as experimental data permitted. In view of the extensive analysis on this particular problem, there has been surprisingly little experimental work. Thus, there was not a great deal of data with which to compare the model. Additional data on the process by which the plumes merge is especially needed. Moreover, it is concluded that while dilution is the parameter of greatest interest to the engineer, it is a fairly insensitive measure of the flow dynamics in the mixing zone. Therefore the measurement of the velocity profile or possibly some other parameter would by more useful in understanding the details of the near field and in comparing different results.

The present model does give reasonable agreement with the experiments over a fairly wide range of discharge conditions and over the complete range of flow patterns. In most cases it slightly underestimates the dilution. Thus, the model should be a useful tool in helping design engineers determine the minimum mixing that will occur in the discharge from any marine outfall used in southeastern coastal waters.

APPENDIX I

Computer Program

```
C     DIA=DIAMETER OF PORT OR EQUIVALENT DIAMETER
C     THETA=ANGLE OF WASTE DISCHARGE FROM PORT IN DEGREES
C     STEP=STEP SIZE
C     DEND=DISCHARGE DENSITY
C     VISC=KINEMATIC VISCOSITY OF DISCHARGING FLUID
C     ALPHAR=ENTRAINMENT COEFF. FOR ROUND JET
C     ALPHAS=ENTRAINMENT COEFF. FOR SLOT JET
C     DAMDAR=SCHMIDT NO. FOR ROUND JET
C     DAMDAS=SCHMIDT NO. FOR SLOT JET
C     DEN(I)=DENSITY OF AMBIENT FLUID AT POSITION I
C     YDEN(I)=VERTICAL POSITION OF AMBIENT FLUID WITH DENSITY DEN(I)
C     PTSPAC=PORT SPACING
C     W=JET LOCATION
C     FRB=FROUDE NO. BASED ON THE EQUIVALENT SLOT WIDTH
C     INTER=A NUMBERING SYSTEM USED TO LIMIT THE NUMBER OF TIMES A TITLE
C         IS PRINTED
C     K=A NUMBERING SYSTEM TO DEVELOP A TWO NUMBER ARRAY TO POSITION THE
C         OUTER BOUNDARIES OF THE JET
C     I=A NUMBERING SYSTEM TO DETERMINE THE POSITION OF THE JET WITH
C         RESPECT TO THE DENSITY LEVELS
C     COUNT=A NUMBERING SYSTEM USED TO PROVIDE INTEGER VALUES WITH WHICH
C         TO COMPARE THE INDEPENDENT VARIABLE S
C     DELTA=THE WHOLE NUMBER INCREMENT BY WHICH COUNT IS ADVANCED AFTER
C         EACH CYCLE
C     INTRA=A NUMBERING SYSTEM TO LIMIT THE PRINTING OF THE TITLE FOR
C         ESTABLISHMENT OF A PLUME
      EXTERNAL DFUN
      EXTERNAL DCON
      EXTERNAL DFN
      DOUBLE PRECISION YMAX(6),WK(102),Y(8,6),H,HMIN,HMAX,EPS,T,ERROR(6)
      DOUBLE PRECISION YE(9)
      DOUBLE PRECISION Z(3),T,H,HMIN,EPS,V(3),WG(87)
      DOUBLE PRECISION R(3)
      DIMENSION YA(8,6)
      DOUBLE PRECISION Y1(200),Y2(200),Y3(200),Y4(200)
      DOUBLE PRECISION X1(200),X2(200),X3(200),X4(200)
      DIMENSION DENR(20)
      COMMON/BLOCK A/GRAV,VISC,VEL,DIA,ALPHAR,ALPHAS,DAMDAR,DAMDAS
      COMMON/BLOCK B/DEN(4),YDEN(4)
      COMMON/BLOCK C/I
      COMMON/BLOCK D/WIDT
      NT=1
      DENR(NT)=1.0375
      READ(1,500) VEL,DIA,THETA,DEPTH,STEP
  500 FORMAT(5F10.2)
      READ(1,501)DEND,VISC,ALPHAR,ALPHAS,DAMDAR,DAMDAS
  501 FORMAT(2F10.5,2F10.3,2F10.3)
      READ(1,502)PTSPAC
  502 FORMAT(F10.3)
      WRITE(3,99)
   99 FORMAT(1H1)
      WRITE(3,100)
  100 FORMAT(/////,35X,'  C O M P O S I T E   B O U Y A N T   J E T   I N
     -  A'//,//,42X,'V A R I B L E   D E N S I T Y   F L U I D ',//)
      WRITE(3,101)
  101 FORMAT(55X,'INITIAL CONDITIONS',/)
```

A-2

```
      WRITE(3,102)VEL,ALPHAR
  102 FORMAT(/,30X,'VELOCITY',8X,F10.5,14X,'ENTRAINMENT COEFFICIENTS',2X
     -,F10.5,3X,'ROUND JET')
      WRITE(3,503)ALPHAS
  503 FORMAT(95X,F10.5,3X,'SLOT JET')
      WRITE(3,103)DIA,VISC
  103 FORMAT(30X,'JET DIAMETER',4X,F10.5,14X,'KINEMATIC VISCOSITY',8X,E1
     -0.2,/)
      WRITE(3,105)DEPTH,DEND
  105 FORMAT(30X,'DEPTH',11X,F10.5,14X,'DISCHARGE DENSITY',9X,F10.5,/)
      W=0.0
      WRITE(3,106)W,DAMDAR
  106 FORMAT(30X,'JET LOCATION (Y)',1X,F10.6,13X,'SCHMIDT NUMBER',12X,F1
     -0.5,3X,'ROUND JET')
      WRITE(3,504)DAMDAS
  504 FORMAT(96X,F10.5,3X,'SLOT JET')
      WRITE(3,107)STEP
  107 FORMAT(30X,'STEP SIZE',7X,F10.5,14X,'GRAVITY',22X,' 32.2',//)
      WRITE(3,505)PTSPAC,THETA
  505 FORMAT(30X,'PORT SPACING',4X,F10.5,14X,'THETA',21X,F10.5,//)
      DEN(1)=1.0251
      YDEN(1)=0.0
      DEN(2)=1.0251
      YDEN(2)=20.0
      DEN(3)=1.02498
      YDEN(3)=25.0
      DEN(4)=1.0249
      YDEN(4)=150.0
      INTER=1
      COEF1=SQRT((1.+DAMDAR**2)/(2*DAMDAR**2))
      BATA=((2.*DAMDAS**2.)/(1.+DAMDAS**2.))**.5
      K=1
      DENDIF=DEN(1)-DEND
      GRAV=32.2
      I=1
      Y(1,1)=VEL
      Y(1,2)=DIA*(2./3.14159)**.5
      Y(1,3)=DENDIF*COEF1
      Y(1,4)=THETA*3.1415927/180.
      Y(1,5)=0.
      Y(1,6)=0.
      VELR=VEL
      DIAR=Y(1,2)
      BO=Y(1,2)
      WIDT=2.8282828*Y(1,2)
      T=0.
      N=6
      JSTART=0
      NTH=0
      MAXDER=7
      H=.1
      HMAX=0.2
      HMIN=0.000001
      EPS=.002
      DO 110 J=1,6
      YMAX(J)=1.
```

A-3

```
0083      110 CONTINUE
0084          COUNT=1.
0085          DELTA=1.
0086          FRB=VEL/SQRT(GRAV*DIA*DENDIF/DEN(1))
0087          WRITE(3,120)FRB
0088      120 FORMAT(//,30X,'FROUDE NO.(B)',13X,F10.5,//)
0089          WRITE(3,108)
0090      108 FORMAT(//,48X,'DENSITY',/)
0091          WRITE(3,109)
0092      109 FORMAT(5X,'S',11X,'VELOCITY',8X,'WIDTH',10X,'DIFFERENCE',5X,'THETA
             C',10X,'X CORD',9X,'Y CORD',8X,'DILUTION',///)
0093          INTRA=1
0094        5 IF(COUNT.LT.3) GO TO 91
0095          K1=K-1
0096          IF(X2(K1).LE.0.0) GO TO 80
0097       91 IF(WIDT.GE.PTSPAC) GO TO 50
0098       90 CALL DVOGER(DEQN,Y,T,N,MTH,MAXDER,JSTART,H,HMIN,HMAX,EPS,YMAX,ERRO
             -R,WK,IER)
0099          DAMDA=DAMDAR
0100          GO TO 51
0101       50 CALL DVOGER(DFUN,Y,T,N,MTH,MAXDER,JSTART,H,HMIN,HMAX,EPS,YMAX,ERRO
             +R,WK,IER)
0102          DAMDA=DAMDAS
0103          GO TO 51
0104       80 IF(INTRA.GT.1)GO TO 86
0105          INTRA=2
0106          Z(1)=Y(1,1)
0107          Z(2)=Y(1,2)*2.
0108          Z(3)=Y(1,3)
0109          Z10=Z(1)
0110          Z20=Z(2)
0111          T1=Y2(K1)*100.
0112          MNO=T1
0113          T2=MNO
0114          T=T2/100.
0115          MI=T
0116          N=3
0117          JSTART=0
0118          H=0.01
0119          HMIN=0.00001
0120          C=MI+0.999
0121          EPS=0.0001
0122          JM=6
0123          IND=3
0124          DO 200 M=1,3
0125          V(M)=1.
0126      200 CONTINUE
0127       86 CALL DRESS(DFN,Z,T,N,JM,IND,JSTART,H,HMIN,EPS,R,V,WG,IER)
0128          Z(4)=3.14159/2.
0129          Z(5)=0.0000
0130          Z(6)=T
0131          DAMDA=DAMDAS
0132          IF(T.GE.C) GO TO 6
0133          IF(IER.EQ.129) GO TO 1000
0134          H=0.01
0135          GO TO 7
```

A-4

```
      6 H=0.2
      7 L=T
        L=L+1
        C1=L
        CH2=C1-T
        IF(CH2.LE.0.005) GO TO 11
        IF(T.GE.DEPTH) GO TO 15
        GO TO 86
     11 WIDT=2.828428*Z(2)
        DILUT=Z(1)*Z(2)/Z10/Z20*DILUS
        THET=Z(4)*180./3.1415927
        DISTX=1.41414*Z(2)
        DISTY=0.00
        X1(K)=Z(5)+DISTX
        Y1(K)=Z(6)-DISTY
        X4(K)=Z(5)-DISTX
        Y4(K)=Z(6)+DISTY
        WRITE(3,970) T,Z(1),WIDT,Z(3),THET,Z(5),Z(6),DILUT
    970 FORMAT(F10.4,7F15.6)
        K=K+1
        J=I+1
        IF(YDEN(J)-Z(6))43,43,44
     43 I=I+1
     44 CONTINUE
        IF(Z(5).GE.100.) GO TO 15
        IF(Z(6).GT.DEPTH) GO TO 15
        IF(Z(6).LE.0.0) GO TO 15
        IF(T.GT.200.) GO TO 15
        GO TO 86
     51 IF(IER.EQ.34) GO TO 16
        IF(IER.EQ.33) GO TO 16
        IF(IER.EQ.35) GO TO 16
        GO TO 17
     16 WRITE(3,600) (ERROR(I),I=1,6),(YMAX(I),I=1,6)
    600 FORMAT(12F10.5)
        GO TO 1000
     17 CH2=COUNT-T
        S=T
        IF(T.GT.COUNT) GO TO 20
        IF(CH2.LE.0.005) GO TO 9
        GO TO 5
     20 E=T-COUNT
        DO 30 J=1,6
        YE(J)=0.
        JFINAL=JSTART+1
        DO 31 M=1,JFINAL
        YE(J)=YE(J)+Y(M,J)*(-E/H)**(M-1)
     31 CONTINUE
        YA(1,J)=YE(J)
     30 CONTINUE
        S=COUNT
        GO TO 10
      9 DO 12 J=1,6
        YA(1,J)=Y(1,J)
     12 CONTINUE
     10 WIDT=2.828428*YA(1,2)
```

```
      IF(WIDT.GE.PTSPAC) GO TO 60
      COEF2=(1.+DAMDA**2)/DAMDA**2
      DILUT=YA(1,1)*YA(1,2)*YA(1,2)/(COEF2*VEL*B0*B0)*2.
      VELR=YA(1,1)
      DIAR=YA(1,2)
      DILUR=DILUT
      GO TO 61
   60 DILUT=YA(1,1)*YA(1,2)/VELR/DIAR*DILUR
      DILUS=DILUT
   61 THET=YA(1,4)*180./3.1415927
      COUNT=COUNT+DELTA
      IF(WIDT.GE.PTSPAC) GO TO 70
      GO TO 75
   70 IF(INTER.GT.1) GO TO 75
      INTER=2
      WRITE(3,71)
   71 FORMAT(////,30X,'MERGING OF ROUND JETS HAS OCCURRED',/)
      FRBL=YA(1,1)/SQRT(GRAV*WIDT*DENDIF/DEN(1))
      WRITE(3,72)FRBL,FRD
   72 FORMAT(//,20X,'LOCAL SLOT FROUDE NO.(B)',5X,F10.5,20X,'INITIAL FRO
     -UDE NO.(3)',5X,F10.5,/)
   75 CONTINUE
      IF(K.GT.200.) GO TO 13
      DISTX=1.41414*YA(1,2)*COS(1.57079-YA(1,4))
      DISTY=1.41414*YA(1,2)*SIN(1.57059-YA(1,4))
      X1(K)=YA(1,5)+DISTX
      Y1(K)=YA(1,6)-DISTY
      X2(K)=YA(1,5)-DISTX
      Y2(K)=YA(1,6)+DISTY
      X3(K)=-X2(K)
      Y3(K)=Y2(K)
      X4(K)=-X1(K)
      Y4(K)=Y1(K)
      WRITE(3,900)S,YA(1,1),WIDT,YA(1,3),THET,YA(1,5),YA(1,6),DILUT
  900 FORMAT(F10.4,7F15.6)
      IF(X2(K).LE.0.0)GO TO 81
      GO TO 85
   81 IF(INTER.GT.2) GO TO 85
      INTER=3
      WRITE(3,82)
   82 FORMAT(//,30X,'VERTICAL PLUME ESTABLIHED',/)
      FRBL=YA(1,1)/SQRT(GRAV*WIDT*DENDIF/DEN(1))
      WRITE(3,84)FRBL,FRD
   84 FORMAT(//,20X,'LOCAL SLOT FROUDE NO.(B)',5X,F10.5,20X,'INITIAL FRO
     -UDE NO.(3)',5X,F10.5,/)
   85 CONTINUE
      K=K+1
   13 CONTINUE
      J=I+1
      IF(YDEN(J)-Y(1,6))41,41,40
   41 I=I+1
   40 CONTINUE
      IF(Y(1,6).GE.DEPTH) GO TO 15
      IF(Y(1,6).LE.0.0) GO TO 1000
      IF(T.GT.200.) GO TO 1000
      GO TO 5
```

A-6

```
      15 CONTINUE
    1000 CONTINUE
         STOP
         END
```

```
      SUBROUTINE DFN(Y,T,N,DY)
      DOUBLE PRECISION Y(3),T,H,HMIN,EPS,S(3),WK,DY(3)
      COMMON/BLOCK A/GRAV,VISC,VEL,DIA,ALPHAR,ALPHAS,DAMDAR,DAMDAS
      COMMON/BLOCK B/DEN(4),YDEN(4)
      COMMON/BLOCK C/I
      ALPHA=ALPHAS
      DAMDA=DAMDAS
      COEF1=1.414*GRAV*DAMDA/DEN(1)
      K=I+1
      COEF3=ALPHA/1.7725
      GRAD=(DEN(K)-DEN(I))/(YDEN(K)-YDEN(I))
      DY(1)=COEF1*Y(3)/Y(1)-2.*COEF3*Y(1)/Y(2)
      DY(2)=4*COEF3-COEF1*Y(2)*Y(3)/Y(1)**2
      DY(3)=SQRT((1.+DAMDA**2)/DAMDA**2)*GRAD-2*COEF3*Y(3)/Y(2)
      RETURN
      END
```

```
      SUBROUTINE DFUN(YP,TP,M,DY,PW,IND)
      DOUBLE PRECISION YP(8,6),TP,DY(6),PW(M,M)
      COMMON/BLOCK A/GRAV,VISC,VEL,DIA,ALPHAR,ALPHAS,DAMDAR,DAMDAS
      COMMON/BLOCK B/DEN(4),YDEN(4)
      COMMON/BLOCK C/I
      COMMON/BLOCK D/WIDT
      ALPHA=ALPHAS
      DAMDA=DAMDAS
      IND=0
      COEF1=1.414*GRAV*DAMDA/DEN(1)
      K=I+1
      GRAD=(DEN(K)-DEN(I))/(YDEN(K)-YDEN(I))
      COEF3=ALPHA/1.7725
      DY(1)=COEF1*YP(1,3)*DSIN(YP(1,4))/YP(1,1)-2*COEF3*YP(1,1)/YP(1,2)
      DY(2)=4.*COEF3-COEF1*YP(1,2)*YP(1,3)*DSIN(YP(1,4))/YP(1,1)**2
      DY(3)=SQRT((1.+DAMDA**2)/DAMDA**2)*DSIN(YP(1,4))*GRAD-2.*COEF3*YP(
     -1,3)/YP(1,2)
      POSVEC=DSQRT(YP(1,5)**2+YP(1,6)**2)
      IF(POSVEC.GT.1.0) GO TO 10
      DY(4)=COEF1*YP(1,3)*DCOS(YP(1,4))/(YP(1,1)**2)
      GO TO 20
   10 DY(4)=COEF1*YP(1,3)*DCOS(YP(1,4))/(YP(1,1)**2)+(ALPHA/POSVEC)
   20 DY(5)=DCOS(YP(1,4))-ALPHA*DSIN(YP(1,4))
      DY(6)=DSIN(YP(1,4))+ALPHA*DCOS(YP(1,4))
      RETURN
      END
```

A-9.

```
      SUBROUTINE DEQN(YP,TP,M,DY,PW,IND)
      DOUBLE PRECISION YP(8,6),TP,DY(6),PW(M,M)
      COMMON/BLOCK A/GRAV,VISC,VEL,DIA,ALPHAR,ALPHAS,DAMDAR,DAMDAS
      COMMON/BLOCK B/DEN(4),YDEN(4)
      COMMON/BLOCK C/I
      COMMON/BLOCK D/WIDT
      ALPHA=ALPHAR
      DAMDA=DAMDAR
      IND=0
      COEF1=GRAV*(DAMDA**2)/DEN(1)
      COEF2=(1.+(DAMDA**2))/(DAMDA**2)
      K=I+1
      GRAD=(DEN(K)-DEN(I))/(YDEN(K)-YDEN(I))
      DY(1)=2.*COEF1*YP(1,3)*DSIN(YP(1,4))/YP(1,1)-2.*ALPHA*YP(1,1)/YP(1
     -,2)
      DY(2)=2.*ALPHA-YP(1,2)*COEF1*YP(1,3)*DSIN(YP(1,4))/YP(1,1)**2
      DY(3)=(COEF2*DSIN(YP(1,4))*GRAD)-(2.*ALPHA*YP(1,3)/YP(1,2))
      DY(4)=2.*COEF1*YP(1,3)*DCOS(YP(1,4))/YP(1,1)**2
      DY(5)=DCOS(YP(1,4))
      DY(6)=DSIN(YP(1,4))
      RETURN
      END
```

APPENDIX II

Sample Computer Output for Graphical results given in Figure 3.

3-1

C O M P O S I T E B O U Y A N T J E T I N A

V A R I B L E D E N S I T Y F L U I D

INITIAL CONDITIONS

VELOCITY	6.32000	ENTRAINMENT COEFFICIENTS	0.08200	ROUND JET
			0.14000	SLOT JET
JET DIAMETER	0.30000	KINEMATIC VISCOSITY	0.10E-04	
DEPTH	150.00000	DISCHARGE DENSITY	0.99910	
JET LOCATION (Y)	0.0	SCHMIDT NUMBER	1.16000	ROUND JET
			1.00000	SLOT JET
STEP SIZE	0.20000	GRAVITY	32.2	
PORT SPACING	24.00000	THETA	0.0	

FROUDE NO.(8) 12.76811

S	VELOCITY	WIDTH	DENSITY DIFFERENCE	THETA	X CORD	Y CORD	DILUTION
1.0000	3.757546	1.139692	0.014397	3.945012	0.999299	0.031599	1.933043
2.0000	2.701997	1.594885	0.010227	9.782489	1.992032	0.149507	2.722107
3.0000	2.155293	2.030462	0.007911	17.129601	2.984287	0.379334	3.519314
4.0000	1.949531	2.432304	0.006424	25.258935	3.396047	0.740080	4.333706
5.0000	1.675756	2.792615	0.005379	33.305962	4.767212	1.229391	5.176012
6.0000	1.576511	3.114300	0.004597	40.606659	5.564932	1.831299	6.056295
7.0000	1.518606	3.407170	0.003988	46.868805	6.286003	2.523444	6.982241
8.0000	1.482403	3.681767	0.003498	52.083638	6.934416	3.284280	7.958577
9.0000	1.457343	3.946164	0.003098	56.398209	7.517644	4.096300	8.988230
10.0000	1.437803	4.205581	0.002764	59.961365	8.043989	4.946383	10.071949
11.0000	1.420913	4.463101	0.002484	62.928009	8.521265	5.825010	11.209976
12.0000	1.405277	4.720439	0.002245	65.420975	8.956310	6.725331	12.401911
13.0000	1.390215	4.978497	0.002040	67.536697	9.354926	7.642387	13.647099
14.0000	1.375445	5.237708	0.001863	69.349762	9.721967	8.572549	14.944723
15.0000	1.360863	5.498244	0.001709	70.917587	10.061440	9.513116	16.293869
16.0000	1.346452	5.760138	0.001574	72.284821	10.376850	10.462060	17.693680
17.0000	1.332232	6.023347	0.001454	73.486221	10.670916	11.417826	19.143326
18.0000	1.318232	6.287789	0.001349	74.549286	10.946074	12.379210	20.641891
19.0000	1.304425	6.553372	0.001255	75.495634	11.204353	13.345257	22.188644
20.0000	1.291018	6.820000	0.001171	76.343536	11.447509	14.315347	23.782776
21.0000	1.277851	7.087570	0.001095	77.106552	11.677020	15.288545	25.423584
22.0000	1.264999	7.355997	0.001027	77.796738	11.894136	16.264664	27.110567

24.0000	1.240373	7.995104	0.000939	78.995407	12.215339	18.223677	30.419290
25.0000	1.224405	8.153640	0.000330	79.518763	12.432177	19.206260	32.340216
26.0000	1.216854	8.455749	0.000312	79.999451	12.659905	20.193445	34.304749
27.0000	1.204333	8.710921	0.000739	80.436569	12.829726	21.175919	36.212139
28.0000	1.190331	8.994998	0.000061	80.820709	12.992440	22.152542	38.160522
29.0000	1.174318	9.288629	0.000588	81.158768	13.149042	23.150238	40.147125
30.0000	1.157136	9.592361	0.000519	81.456406	13.300119	24.138763	42.169403
31.0000	1.137960	9.905766	0.000454	81.717987	13.445376	25.128006	44.224731
32.0000	1.118237	10.225851	0.000423	81.952209	13.596216	26.117905	46.312103
33.0000	1.099890	10.543664	0.000404	82.176980	13.726256	27.108322	48.427734
34.0000	1.082850	10.859127	0.000385	82.392151	13.860497	28.099274	50.573120
35.0000	1.066561	11.172465	0.000368	82.598389	13.991091	29.090714	52.748291
36.0000	1.052092	11.485879	0.000352	82.796097	14.118191	30.082596	54.953156
37.0000	1.038127	11.793560	0.000338	82.935321	14.241940	31.074905	57.187622
38.0000	1.024970	12.101678	0.000324	83.166779	14.362490	32.067627	59.451691
39.0000	1.012535	12.408394	0.000311	83.340546	14.479943	33.060699	61.745178
40.0000	1.000749	12.713856	0.000298	83.507156	14.594456	34.054123	64.068024
41.0000	0.989548	13.018195	0.000287	83.666855	14.706139	35.047867	66.420166
42.0000	0.978874	13.321536	0.000276	83.819962	14.815110	36.041901	68.801392
43.0000	0.968679	13.623996	0.000265	83.965927	14.921478	37.036240	71.211487
44.0000	0.958916	13.925681	0.000255	84.107727	15.025351	38.030823	73.650482
45.0000	0.949550	14.226688	0.000246	84.242950	15.126925	39.025665	76.118011
46.0000	0.940545	14.527111	0.000237	84.372803	15.226000	40.020737	78.614044
47.0000	0.931871	14.827038	0.000229	84.497528	15.322963	41.016022	81.138397
48.0000	0.923499	15.126544	0.000220	84.617355	15.417803	42.011505	83.690795
49.0000	0.915406	15.425704	0.000213	84.732529	15.510602	43.007202	86.271149
50.0000	0.907569	15.724592	0.000205	84.843201	15.601439	44.003067	88.879242
51.0000	0.899968	16.023270	0.000198	84.949738	15.690398	44.999100	91.514801
52.0000	0.892586	16.321793	0.000192	85.052170	15.777520	45.995300	94.177704
53.0000	0.885406	16.620239	0.000185	85.150772	15.862905	46.991639	96.867767
54.0000	0.878413	16.918555	0.000179	85.245743	15.946607	47.988129	99.584717
55.0000	0.871593	17.217102	0.000173	85.337204	16.028687	48.984756	102.328339
56.0000	0.864936	17.515610	0.000168	85.425339	16.109192	49.981506	105.098511
57.0000	0.858428	17.814240	0.000162	85.510361	16.188217	50.978378	107.894867
58.0000	0.852060	18.113052	0.000157	85.592270	16.265778	51.975372	110.717285
59.0000	0.845823	18.412064	0.000152	85.671219	16.341934	52.972473	113.565613
60.0000	0.839707	18.711349	0.000147	85.747452	16.416748	53.969666	116.439423
61.0000	0.833704	19.010941	0.000142	85.821045	16.490250	54.966965	119.338654
62.0000	0.827808	19.310867	0.000138	85.892075	16.562500	55.964340	122.263031
63.0000	0.822011	19.611176	0.000133	85.960693	16.633530	56.961823	125.212280
64.0000	0.816306	19.911911	0.000129	86.026901	16.703400	57.959381	128.186218
65.0000	0.810689	20.213120	0.000125	86.090866	16.772125	58.957016	131.184570
66.0000	0.805152	20.514832	0.000121	86.152725	16.839752	59.954727	134.207153
67.0000	0.799691	20.817078	0.000118	86.212570	16.906326	60.952499	137.253601
68.0000	0.794301	21.119904	0.000114	86.270386	16.971863	61.950348	140.323853
69.0000	0.788979	21.423355	0.000110	86.326263	17.036423	62.948273	143.417450
70.0000	0.783717	21.727463	0.000107	86.380341	17.100021	63.946243	146.534363
71.0000	0.778514	22.032272	0.000103	86.432648	17.162734	64.944275	149.674133
72.0000	0.773365	22.337799	0.000100	86.483307	17.224472	65.942368	152.836670
73.0000	0.768267	22.644119	0.000097	86.532272	17.285395	66.940506	156.021637
74.0000	0.763215	22.951233	0.000094	86.579758	17.345459	67.938705	159.228760
75.0000	0.758208	23.259201	0.000091	86.625610	17.404709	68.936951	162.457794
76.0000	0.753242	23.568069	0.000088	86.670059	17.463181	69.935242	165.708466
77.0000	0.748313	23.877869	0.000085	86.713028	17.520889	70.933563	168.980560

MERGING OF ROUND JETS HAS OCCURRED

LOCAL SLOT FROUDE NO.(B) 0.16726 INITIAL FROUDE NO.(B) 12.76811

78.0000	0.743420	24.188629	0.000083	86.754684	17.577866	71.931946	170.060272
79.0000	0.735783	24.812775	0.000080	86.860611	17.529735	72.936172	172.656281

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80.0000	0.727386	25.541895	0.000078	86.987045	17.443549	73.942245	[illegible]
81.0000	0.719397	26.266846	0.000076	87.111145	17.355301	74.948120	178.805695
82.0000	0.712403	26.988739	0.000073	87.233078	17.264916	75.953812	181.831329
83.0000	0.705359	27.707489	0.000071	87.352753	17.172195	76.959305	184.826675
84.0000	0.698830	28.423294	0.000069	87.470261	17.077499	77.964615	187.792847
85.0000	0.692196	29.136353	0.000067	87.585663	16.980759	78.969711	190.731140
86.0000	0.686133	29.846978	0.000065	87.699051	16.882004	79.974625	193.642838
87.0000	0.680121	30.555084	0.000064	87.810410	16.781281	80.979340	196.529063
88.0000	0.674439	31.261154	0.000062	87.919830	16.678635	81.983871	199.390823
89.0000	0.663971	31.965302	0.000060	88.027222	16.574081	82.988190	202.229034
90.0000	0.663700	32.667740	0.000058	88.132843	16.467651	83.992325	205.044525
91.0000	0.658611	33.368652	0.000057	88.236664	16.359390	84.996246	207.838104

VERTICAL PLUME ESTABLIHED

LOCAL SLOT FROUDE NO.(B) 0.12616 INITIAL FROUDE NO.(B) 12.76311

86.0000	0.657533	67.210373	0.000056	89.999878	0.0	86.000000	208.531464
87.0000	0.656903	67.721802	0.000054	89.999878	0.0	87.000000	209.917099
88.0000	0.656229	68.238403	0.000053	89.999878	0.0	88.000000	211.301346
89.0000	0.655511	68.760208	0.000052	89.999873	0.0	89.000000	212.684143
90.0000	0.654730	69.287201	0.000051	89.999878	0.0	90.000000	214.065369
91.0000	0.653945	69.819397	0.000049	89.999878	0.0	91.000000	215.444946
92.0000	0.653102	70.356796	0.000048	89.999878	0.0	92.000000	216.822784
93.0000	0.652216	70.899445	0.000047	89.999878	0.0	92.999999	218.198792
94.0000	0.651290	71.447342	0.000046	89.999878	0.0	93.999999	219.572906
95.0000	0.650325	72.000534	0.000045	89.999878	0.0	94.999999	220.945023
96.0000	0.649321	72.559036	0.000043	89.999878	0.0	95.999999	222.315048
97.0000	0.648278	73.122910	0.000042	89.999878	0.0	96.999999	223.682933
98.0000	0.647198	73.692169	0.000041	89.999878	0.0	97.999999	225.048569
99.0000	0.646080	74.266876	0.000040	89.999878	0.0	98.999999	226.411876
100.0000	0.644925	74.847076	0.000039	89.999878	0.0	99.999999	227.772812
101.0000	0.643733	75.432831	0.000038	89.999878	0.0	100.999999	229.131271
102.0000	0.642505	76.024185	0.000037	89.999878	0.0	101.999999	230.487157
103.0000	0.641242	76.621216	0.000035	89.999878	0.0	102.999999	231.840453
104.0000	0.639743	77.223984	0.000034	89.999878	0.0	103.999999	233.191025
105.0000	0.638509	77.832565	0.000033	89.999878	0.0	104.999999	234.538834
106.0000	0.637241	78.447052	0.000032	89.999878	0.0	105.999999	235.883774
107.0000	0.635537	79.067490	0.000031	89.999878	0.0	106.999999	237.225615
108.0000	0.634399	79.694016	0.000030	89.999878	0.0	107.999999	238.564835
109.0000	0.632927	80.326675	0.000029	89.999878	0.0	108.999999	239.900602
110.0000	0.631421	80.965607	0.000028	89.999878	0.0	109.999999	241.233607
111.0000	0.629882	81.610886	0.000027	89.999878	0.0	110.999998	242.563240
112.0000	0.628303	82.262619	0.000026	89.999878	0.0	111.999998	243.889587
113.0000	0.626701	82.920944	0.000025	89.999878	0.0	112.999998	245.212570
114.0000	0.625061	83.585953	0.000024	89.999878	0.0	113.999998	246.532120
115.0000	0.623397	84.257797	0.000023	89.999878	0.0	114.999998	247.848190
116.0000	0.621380	84.936584	0.000022	89.999878	0.0	115.999998	249.160690
117.0000	0.619939	85.622467	0.000021	89.999878	0.0	116.999998	250.469559
118.0000	0.618166	86.315567	0.000020	89.999878	0.0	117.999998	251.774719
119.0000	0.616359	87.016068	0.000019	89.999878	0.0	118.999998	253.076096
120.0000	0.614519	87.724106	0.000018	89.999878	0.0	119.999998	254.373642
121.0000	0.612645	88.439850	0.000017	89.999878	0.0	120.999998	255.667267
122.0000	0.610739	89.163452	0.000016	89.999878	0.0	121.999998	256.956787
123.0000	0.608798	89.895126	0.000015	89.999878	0.0	122.999998	258.242439
124.0000	0.606825	90.635040	0.000014	89.999878	0.0	123.999998	259.523025
125.0000	0.604817	91.383392	0.000013	89.999878	0.0	124.999998	260.801925
126.0000	0.602775	92.140381	0.000012	89.999878	0.0	125.999998	262.074019
127.0000	0.600701	92.906235	0.000011	89.999878	0.0	126.999997	263.342773
128.0000	0.598592	93.681168	0.000010	89.999878	0.0	127.999997	264.606934
129.0000	0.596449	94.465393	0.000009	89.999878	0.0	128.999997	265.866659
130.0000	0.594272	95.259186	0.000008	89.999879	0.0	129.999997	267.122179

132.0000	0.5[illegible]312	[illegible].870450	0.000000	89.999878	0.0	131.999997	269.618403
133.0000	0.587530	97.700470	0.000005	89.999878	0.0	132.999997	270.859619
134.0000	0.585212	98.535126	0.000004	89.999878	0.0	133.999997	272.095947
135.0000	0.582858	99.380722	0.000003	89.999878	0.0	134.999997	273.327143
136.0000	0.580468	100.237579	0.000002	89.999878	0.0	135.999997	274.553467
137.0000	0.578042	101.106033	0.000001	89.999878	0.0	136.999997	275.774653
138.0000	0.575579	101.986420	0.000001	89.999878	0.0	137.999997	276.990967
139.0000	0.573079	102.879105	-0.000000	89.999878	0.0	138.999997	278.201660
140.0000	0.570542	103.784485	-0.000001	89.999878	0.0	139.999997	279.407227
141.0000	0.567966	104.702927	-0.000002	89.999878	0.0	140.999997	280.607422
142.0000	0.565352	105.634888	-0.000003	89.999878	0.0	141.999997	281.802246
143.0000	0.562699	106.580780	-0.000004	89.999878	0.0	142.999996	282.991211
144.0000	0.560007	107.541061	-0.000005	89.999878	0.0	143.999996	284.174805
145.0000	0.557275	108.516235	-0.000006	89.999878	0.0	144.999996	285.352539
146.0000	0.554502	109.506790	-0.000007	89.999878	0.0	145.999996	286.524658
147.0000	0.551688	110.513275	-0.000008	89.999878	0.0	146.999996	287.690674
148.0000	0.548832	111.536255	-0.000009	89.999878	0.0	147.999996	288.850830
149.0000	0.545935	112.576294	-0.000009	89.999878	0.0	148.999996	290.004883
150.0000	0.542994	113.634033	-0.000010	89.999878	0.0	149.999996	291.152832

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