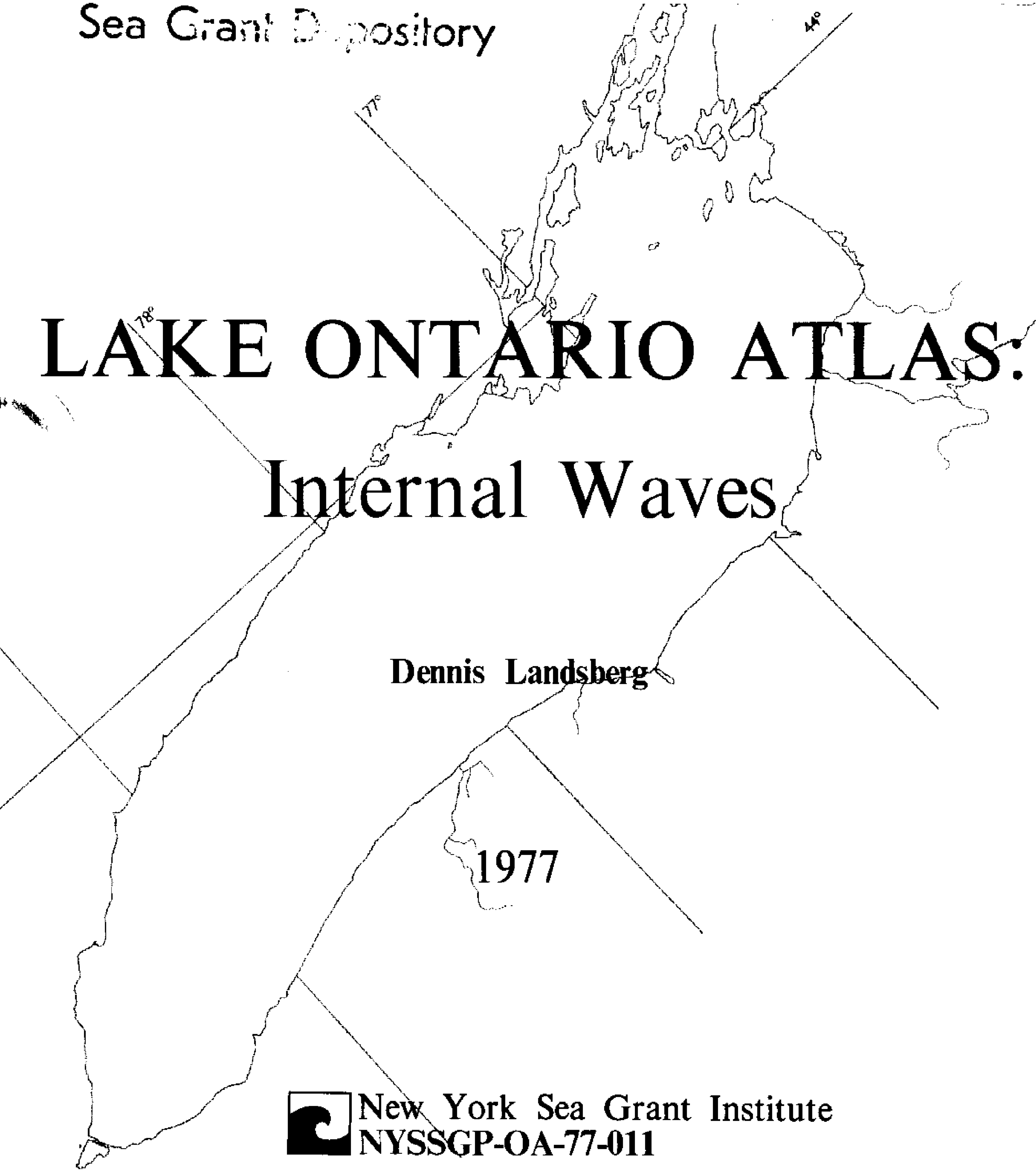


# LAKE ONTARIO ATLAS:

## Internal Waves

**Dennis Landsberg**

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## ABSTRACT

The long internal waves in the offshore zone of Lake Ontario consist of Poincaré waves, one for each mode of motion, which stand across the lake and either stand or vary exponentially along it. The wavelengths of these waves are proportional to the basin width. The periods approach but never exceed the inertial period, and the lower mode oscillations travel faster and decay more slowly than the higher mode oscillations. The first and third modes are the most commonly observed in Lake Ontario, but the periods of all modes are so close together that they are difficult to separate and form an interference pattern.

In the nearshore zone the internal waves consist of two oppositely rotating internal Kelvin waves and quasi-geostrophic waves (formed from combinations of Poincaré waves which satisfy the nearshore boundary conditions) of near inertial period and wavelength which depends on the internal Rossby radius of deformation. The sloping bottom in the nearshore zone causes the period of the internal Kelvin waves to be about 30 days, and the currents associated with these waves can be considered steady. However, when the internal Rossby radius of deformation equals and then exceeds the effective shelf width, resonance occurs between the quasi-geostrophic waves and the internal Kelvin waves, and the latter rotates with a period of 12 to 16 days. This is believed to cause the observed cyclonic resultant transport in Lake Ontario in summer in the nearshore zone.

Short internal waves are present in both zones of Lake Ontario and are concentrated at frequencies near to but less than the Brunt-Väsälä frequency. The Brunt-Väsälä period in Lake Ontario in summer is three to eight minutes. These waves are believed to be formed by shear instability at the nodes of Poincaré waves or thermal surges.

# INTERNAL WAVES IN LAKE ONTARIO

## Introduction

The study of the dynamics of large lakes has lagged behind the other fields of limnology. The reason for this is the difficulty of measuring and predicting the transfer of energy and momentum from the atmosphere to a lake, and the distribution of these quantities within the lake. The purpose of this monograph is to examine one aspect of lake dynamics, internal waves, for Lake Ontario. The present state of knowledge will be reviewed from a non-mathematical point of view, giving the important concepts and findings. The bibliography will direct the reader to more rigorous treatments of specific topics.

### 1. What are internal waves?

Internal waves are periodic vertical displacements of the thermocline of a lake. They may involve the entire lake, the boundaries of the lake, or small sections of the interior of the lake. Periods of motion may last from minutes to many days. The occurrence of internal waves requires the presence of a thermocline separating the warm surface water or epilimnion from the cold deeper water or hypolimnion of the lake. The epilimnion is slightly less dense than the hypolimnion because of the temperature difference. The reader is advised to refer to the monograph on temperature for information on thermocline formation and structure. The topic of surface waves is treated in a separate monograph and will not be dealt with here. Before discussing internal waves, the reasons for studying them and their causes will be reviewed.

### 2. The importance of studying internal waves

The study of internal waves has added considerably to the body of scientific knowledge concerning lake dynamics. Motions in the epilimnion of Lake Ontario cause movement of the thermocline. Since changes in the thermocline depth are often two or three orders of magnitude larger than displacements of the lake surface, they are more easily measured. In addition, the motion of the thermocline can be used to determine the structure of currents in the lake. This is important since temperature measurements are considerably easier and cheaper to make than current measurements.

Internal waves play an important role in the distribution of energy within the lake. In the spring and summer, solar energy warms the surface of Lake Ontario. The application of severe wind stresses mixes this energy downward into the lake. Internal waves are an important part of this mixing process and serve to lower the depth of the thermocline.

There is considerable evidence that internal waves in the nearshore zone of Lake Ontario are linked to the water movements. Interactions between Kelvin waves and baroclinic geostrophic waves appear to be responsible for the cyclonic transport pattern which is observed in the lake in summer.

Internal waves cause upwelling and downwelling of the thermocline in Lake Ontario. This affects the temperature and quality of water which is present at beaches, power plant sites and municipal water intakes. Severe upwelling can cause the hypolimnion water to be brought to the surface of the lake. Hence, the distribution of temperature, chemicals and nutrients in the lake is strongly affected by internal waves. This influence causes spatial variation in the biological productivity in Lake Ontario.

Before proceeding with the discussion of internal waves, their causes must be understood.

3. The causes of internal waves

A. The effect of a steady wind

Internal waves are the direct result of surface motions which are brought about primarily by the wind. Consider the simple case of a steady wind blowing across Lake Ontario from west to east. The force of the wind drags warm surface water eastward, piling it at the eastern end of the lake. This causes the water level at the eastern end of the lake to be higher for two reasons. The first is the addition of more water. The second is the fact that the warm water which has been concentrated at the eastern end of the lake is less dense than the cooler water below it and therefore occupies a greater volume. The increase in water level is partially compensated for by westward flow in the hypolimnion, as shown in Figure 1. This motion tends to lower the thermocline at the downwind end of the lake and raise it at the upwind end of the lake. Since the density difference between the epilimnion and hypolimnion waters is much smaller than that between air and water at the surface, a change in surface level of two or three centimeters (about 1 in) may result in a change in thermocline depth of several meters (yds).

If the steady wind were suddenly stopped, the force of gravity would tend to return the lake to a condition of level surface and level thermocline. What actually happens is that the water sloshes back and forth, much like water in a bathtub when set in motion, until frictional forces stop the movement. The thermocline level changes in the opposite sense. Small increases in the surface water level result in large decreases of the thermocline depth as previously explained, and visa versa.

### B. Wind variation

At first glance, the study of internal waves appears to be a simple matter. However, there are several phenomena which make internal waves a complex feature of lake dynamics. The wind, which is the primary driving force for all motions in the lakes, is anything but steady. In addition to major changes in speed and direction brought about by the passage of weather systems, there are fluctuations at all scales of motion. This can be seen by watching a flag fluttering in the breeze, the complex pattern of ripples on the surface of a pond or the smoke rising from a campfire. The wind stress acting on the surface of Lake Ontario is highly variable in space and time for both speed and direction. The resulting perturbations of lake level, and the internal waves which are formed, are equally complex.

### C. Seasonal variation of stability

The transfer of energy and momentum from the air to a lake, and the distribution of that energy and momentum in the lake, are dependent upon the stability of the lake both of itself, and with respect to the air above it. When momentum is transferred by the wind to an isothermal lake in winter, (a lake whose temperature is relatively uniform throughout) that momentum is transmitted deep into the lake. Initially, thermal energy is mixed by the wind in the same manner. In spring, the surface of the lake heats up quite rapidly. This warm water, being less dense than the water below it, exhibits bouyancy and hence resists the forces of mixing. Energy and momentum which are added to the lake tend to be trapped in this layer, and as the temperature difference between the upper and lower layers increases, a thermocline forms. A thermocline is simply a relatively narrow layer of water separating the warm epilimnion water from the cold hypolimnion water as shown in Figure 2. Consequently, the rate of vertical temperature change across the thermocline (or thermal gradient) is quite large. Some definitions of the thermocline require a minimum thermal gradient as part of the definition. This quantity can vary considerably with the location and size of the lake. For Lake Ontario, a vertical change of 10°C per 5 m (18°F/16 ft) would represent a relatively strong thermocline. The greater the thermal gradient, the more effectively transferred energy and momentum are confined to the epilimnion, and the more stable the lake becomes. The concentration of momentum in the epilimnion results in more warm water being transported downwind for a given wind stress, and thereby results in larger amplitude internal waves.

Similarly, if the lake surface temperature is cooler than the air above it, stability impedes the transfer of energy and momentum to the lake. When the lake surface is warmer than the air, more energy and momentum is transferred to it, and the wave heights increase for a given wind speed. The amount of momentum transferred to a lake by a given wind stress (wind stress is proportional to the velocity squared) is usually expressed in terms of a wind stress coefficient which depends on the stability of the air-water interface and the surface roughness of the lake. This constant value is only a crude estimate, owing to the variability of the factors involved.

The internal waves in Lake Ontario have a seasonal variance which depends on both the stability of the lake, and the stability of the air-water interface. These two factors limit the transfer of energy and momentum to the lake and also affect the distribution of these quantities in the lake.

D. Spatial variation of stability

The stability of Lake Ontario varies spatially in any given season. One reason for this is differential heating. In spring, land surrounding the lake warms more rapidly than the lake causing the water near shore to heat more rapidly as well. It was shown earlier that heat initially added to the lake was mixed downward by the wind. In shallow areas near shore, where there is less water, the lake heats up faster than it does in the center. In the spring, surface temperatures near shore may reach 15°C (59°F), while those at the center of the lake are still 4°C (39°F) or lower as shown in Figure 3 (from Webb 1974). In summer, epilimnion water will extend all the way to the bottom of the lake near shore, as can be seen in Figure 2.

The mean winds over Lake Ontario are eastward. Since they transport epilimnion waters eastward, the surface temperature is warmer and the thermocline deeper at the eastern end of the lake as shown in Figure 4. The difference in thermocline depth can be as great as 14 meters (45 ft) in the monthly mean.

These two effects cause considerable variation in the stability of the lake, and of the air-water interface. Even if the winds were steady, this would cause differences in energy and momentum transfer to the lake between different points on the lake's surface.

E. The effect of basin shape

The basin of Lake Ontario plays an important role in the formation of internal waves. Motions are constrained by the boundaries of the lake which permits the concentration of warm water at the downwind end of the lake when the wind blows across it. The lake boundaries also affect rotary motions, but these will be discussed in the next section.

Lake Ontario has its own unique frequencies of motion, just like each string on a guitar vibrates at its own particular frequency. There are actually two sets of frequencies of motion for Lake Ontario; one for the east-west axis and one for the north-south axis. If the major wind variations are such that they are in phase with one of these natural frequencies, resonance occurs and the amplitudes of the internal waves will be considerably increased.

## F. Coriolis force

The study of internal waves in Lake Ontario is further complicated by the fact that the earth rotates. To an observer standing on the earth's surface, there is no effect on stationary objects, but objects or parcels of fluid in motion will deviate to the right when looking in the downstream direction in the northern hemisphere. The value of the coriolis force at the latitude of Lake Ontario is approximately $10^{-4}$ $sec^{-1}$. This is inconsequential for brief motions such as the flight of a baseball, but the coriolis force equals unity for motions which last three hours. It therefore has an effect on fluid motions in large lakes such as Lake Ontario when these motions last for several hours or more. When an eastward wind stress is applied to Lake Ontario, the warm water is actually transported southward as well as eastward. Figure 4 shows that the maximum difference in thermocline depth occurs between the southeast and northwest portions of the lake. The small area of warm water in southwest Lake Ontario is caused by outflow from the Niagara River.

Flow of this type is known as geostrophic flow. Consider a current flowing in an isothermal lake with a shore to its right looking downstream. The effect of the coriolis force is to transport surface water toward shore such that the water level is higher at the right of the current than at the left looking downstream. This continues until the pressure exerted by the piled up water in the offshore direction balances the coriolis force in the onshore direction as shown in Figure 5. The current is then in geostrophic equilibrium, and for an isothermal lake the flow is referred to as barotropic geostrophic flow.

For a stratified lake, there is an additional height differential between the right and left sides of the current looking downstream. Since epilimnion water is transported to the right, the depth of the thermocline increases to the right of the current (downwelling) and decreases to the left of the current (upwelling) as shown in Figure 6. The warmer water to the right of the current is less dense, and therefore occupies a larger volume than the colder water to the left of the current. This larger volume causes the water level to be higher at the right of the current than at the left, creating a pressure gradient (or force) away from shore. The balance between this thermally induced pressure gradient and the coriolis force is called baroclinic geostrophic equilibrium, and the flow is referred to as baroclinic geostrophic flow.

When a wind stress is applied to the lake causing transport of surface waters, there is a change in the water level pattern of the lake, and of the thermocline depth as well. This is known as geostrophic adjustment. If the adjustment time is small in comparison to the duration of the flow being considered, then the flow is termed quasi-geostrophic (the flow is near enough to geostrophic equilibrium to be considered in equilibrium for the purposes of the discussion). If the changes in the flow are so rapid that geostrophic adjustment is always incomplete, then the flow is termed ageostrophic. Since this monograph deals with internal waves, the terms just discussed will be used only in reference to baroclinic geostrophic flow.

Next consider a parcel of fluid in the center of the lake which is set in motion by a wind stress. The coriolis force is constantly deviating its motion to the right such that it moves in a clockwise spiral pattern. If the wind stress were removed, the parcel of fluid would move in a circle known as the inertial circle. At the latitude of Lake Ontario, the inertial circle has a radius of about one kilometer (0.6 mi), and the time required for a parcel of water to complete one rotation is called the inertial period which is about 17.5 hours.

If the parcel of fluid is moved nearer to the coast, the circle becomes an ellipse whose major axis is parallel to shore, and whose minor axis becomes smaller the closer to shore the parcel is moved. If the parcel is brought near enough to shore, only motions parallel to the shoreline or bottom contours of the lake occur. This is because water cannot flow across the boundaries of the lake.

### G. The nearshore and offshore zones of Lake Ontario

The boundaries of Lake Ontario have been shown to have an important effect on motions within the lake. Water movements near the shore are primarily translational and parallel to the shoreline or bottom contours, while motions occuring away from the constraints of the shoreline are rotational because of the effect of the coriolis force. But what distance from shore is far enough to be away from its influence, and is Lake Ontario large enough for rotational effects to be dominant at its center? Mortimer (1974) stated that a lake must have a width five times greater than the radius of the inertial circle for rotational effects to be significant and 20 times greater for rotational effects to be dominant. Since the radius of the inertial circle for Lake Ontario is about one kilometer (0.6 mi) and its width is about 60 kilometers (36 mi) this is clearly the case.

Since these two regimes of motion are present in Lake Ontario, it is convenient to study them separately. They shall be referred to as the nearshore zone (that portion of the lake in which motions are strongly influenced by lake boundaries) and the offshore zone (that portion of the lake which responds as if part of an infinite body of water). There have been many definitions of the extent of the nearshore zone. For example, Blanton (1974) noted that in current studies on the north shore of Lake Ontario, motions shifted quite suddenly from translational to rotational at six kilometers (3.6 mi) from shore.

A mathematical definition of the nearshore zone was presented by Allen (1975). In its simplest form, this relation states that a shelf width for a body of water is that distance ($x_2 - x_1$) measured perpendicular to shore over which the change in the lake depth ($h_2 - h_1$) equals the average depth $(h_2 + h_1)/2$ as shown in Figure 7. The bottom slope in Lake Ontario is shallower for the north shore than for the south shore, so that the shelf width is about six kilometers (3.6 mi) for the north shore and about four kilometers (2.4 mi) for the south shore.

It is preferable to call this shelf width an effective shelf width. In Lake Ontario, the thermocline often intersects the surface or bottom of the lake because of upwelling or downwelling. Since the generation of internal waves requires the presence of both an epilimnion and a hypolimnion, the actual shelf width must be measured outward from the point where the thermocline intersects the surface or bottom of the lake. The additional distance is usually less than two kilometers (1.2 mi) except for cases of extreme upwelling. The actual shelf width will be used to define the limits of the nearshore zone. It is a useful definition for the study of internal waves since it takes into account variations in the thermal structure of the lake as well as the actual boundaries.

4. Approach

The above seven phenomena make the study of internal waves the complex subject that it is. Now that the causes of internal waves have been examined, the different forms of internal waves will be discussed. This study is divided into four sections. The first section deals with internal seiches. These are oscillations involving the entire basin. In the second section, long internal waves in the offshore zone will be considered. This includes Poincaré waves, which are actually higher mode seiches, and wave interactions which result in standing waves across the basin. In the third section, long internal waves in the nearshore zone will be treated. This includes Kelvin waves and quasi-geostrophic waves as well as how their interactions affect transport in the nearshore zone. Finally the fourth section deals with short internal waves.

In each section, both theoretical and observational aspects will be considered. Analytical and numerical models will be presented where appropriate. Only the cases of a single thermocline will be examined since significant multiple thermoclines are rarely important in Lake Ontario.

## Internal Seiches

1. Description

The internal seiche, as described earlier, is the simplest response to a wind stress over a stratified lake. A laboratory model of an internal seiche including a thermocline layer is presented by Mortimer (1954) and shown in Figure 8. The action of the wind stress is to drag the epilimnion in the downwind direction with a resultant upwind return flow in the lower layers as exemplified by Figure 8a. When the wind subsides, the layers slide back and forth over each other as shown in Figures 8d and 8e until frictional damping stops the sloshing motion after many oscillations. Oscillations occur about a nodal line in the center of the channel and perpendicular to the long axis of the lake where the thermocline depth does not change. Maximum amplitudes occur at the ends of the basin.

Under normal conditions, the different density layers in the lake slide over each other smoothly. However, for fluids in which the Reynold's number (a non-dimensional parameter relating scale length, velocity, density and viscosity) is high and the vertical density gradient is small, as occurs in lakes, instability can develop as in Figure 8b. The smooth sliding of the layers breaks down quite suddenly into large eddies (or whirls) which then transfer their energy into smaller eddies. The action of this mechanism is to dissipate energy (by friction and by overcoming the force of bouyancy) and to thoroughly mix the layers of water involved resulting in a lowering of the thermocline depth. This phenomena occurs when the Richardson number, a non-dimensional parameter relating gravity, lake stability, and the square of the vertical velocity gradient (or shear) is of order unity. The theoretical aspects of instability were discussed by Batchelor (1953) and applied to thermoclines in lakes by Mortimer (1963a). When the wind subsides suddenly a thermal surge can form as shown in Figure 8c. This is the internal equivalent to tidal bores in rivers and estuaries and is marked by a rapid vertical rise in the thermocline depth at a given cross section of the lake which moves across the lake.

2. Higher order modes of the internal seiche

The internal seiche just described is the simplest form of internal seiche, the uninodal seiche. This is the most commonly observed seiche in small lakes and narrow channels where the lake width is only a few times the inertial radius. Csanady (1973) states that for narrow lakes and channels the fundamental free mode (where fundamental refers to the uninodal seiche and free refers to the natural oscillation of the thermocline once the wind dies down) is not too different from the forced solution (the thermocline pattern which is forced to occur by the action of the wind). However, in wide channels or bodies of water the forced solution, which is short in comparison to basin dimensions, can excite many modes of motion. Only the odd modes of motion (1, 3, 5...) actually appear because the wind causes the initial thermocline pattern to be asymmetric. Instead of one nodal line as in the case of the uninodal seiche, there are three lines for the trinodal case, five lines for the quintinodal case, etc. Maximum amplitudes occur at the antinodal lines, the lines midway between the nodal lines. The maximum amplitude decreases as the number of nodes increases. The internal seiches oscillate more slowly and are out of phase with the surface seiches which are treated in the monograph section on surface waves. The higher mode seiches travel more slowly and are damped out by friction more rapidly that the lower mode seiches. It should be noted that a basin which is "wide" when dealing with internal oscillations may not be wide when surface motions are considered.

3. The transverse internal seiche

In Lake Ontario, the theoretical study of the internal seiche is complicated by the effects of the earth's rotation, as discussed by Taylor (1920). However, Rao (1966) has shown that if the length to width ratio of a basin is large, (this ratio is 5:1 for Lake Ontario, which is suitably large) the end effects on transverse oscillations are minor, and the transverse seiche can be studied as if it occurred in an infinitely long rotating channel. A transverse seiche is one which is excited across the width of a lake, rather than across its length. Rao's results were generated for surface oscillations but apply equally well to the case of internal oscillations.

The transverse seiche for Lake Ontario was studied by Csanady (1973). The theoretical amplitudes of motion for the free modes of motion and for the forced solution at one shore are shown in Table 1. The first two modes have been observed in the field. The higher modes probably exist but are too small to be easily measured. The periods of motion of these seiches are of the order of the inertial period, but never exceed it. The transverse seiches can also be excited by a longitudinal wind stress, but the amplitudes are larger and 90° out of phase with those excited by north-south winds. Away from shore in large basins such as Lake Ontario, the forced solution is unimportant. The free oscillations cancel each other because of the alternating signs. This forms a pattern of standing waves known as Poincaré waves which will be treated in the third section. Because the periods of the transverse seiches are near to the inertial period, these modes of motion are most easily excited by north-south wind impulses lasting eight or 24 hours (1/2 or 1 1/2 times the inertial period).

TABLE 1 Amplitudes of free and forced oscillations for the transverse internal seiche in Lake Ontario

| node n | thermocline oscillation (cm) |
|---|---|
| 1 | 26.5 |
| 3 | -17.4 |
| 5 | 10.3 |
| 7 | - 6.4 |
| 9 | 5.0 |
| Forced | 84.2 |

## 4. Rotating internal seiches

For narrow lakes and channels, the coriolis force has little effect, and the surface and internal seiches oscillate back and forth along the axis of the lake. The internal seiche oscillation is slower and out of phase with the surface seiche oscillation, and of larger amplitude as described earlier. Consider a constant depth elliptical basin aligned with its major axis in the east-west direction with a strong thermocline and variable width. As the basin width is increased, the coriolis force causes a change in the nature (but not the period) of the internal seiches. The effect on the uninodal seiche will be examined. If wind blows over the hypothetical lake from west to east, the warm epilimnion surface water is transported eastward by the action of the wind stress and southward by the coriolis force. When the wind subsides, the force of gravity acts to move the warm water piled up at the south shore of the lake northward to return the thermocline to level (equilibrium) state. But the coriolis force deviates this motion to the right in the northern hemisphere resulting in the warm water being piled up at the eastern shore. From here gravity acts to restore thermocline equilibrium by causing westward flow of the epilimnion water, but the coriolis force deviates this motion as well and the warm water becomes concentrated at the northern shore. From the northern shore, gravity forces the warm water southward, and the coriolis force deviates it to the western shore. In this manner, the surface rotating uninodal seiche rotates counterclockwise around the edge of the basin instead of oscillating back and forth along it. The thermocline depth increases in response to the addition of epilimnion water in a given region of the lake. This response, the internal seiche, travels around the lake in the same manner as the surface seiche, but more slowly.

One cycle of a rotating uninodal internal seiche is shown in Figure 9, which is taken from Mortimer (1974). The rotating uninodal internal seiche oscillates in a counterclockwise direction around a nodal point in the center of the basin, just as the non-rotating internal seiche oscillates about a nodal line. The maximum amplitude of the rotating internal seiche moves counterclockwise around the edge of the lake, while for the non-rotating seiche it alternates between the eastern and western ends of the basin.

The maximum amplitude of the uninodal rotating internal seiche, and the maximum currents associated with it occur at the shore of the lake, and decrease exponentially outward from the shore such that the lakeward pressure gradient force is always in balance with the shoreward directed coriolis force. For lakes of sufficient width, it will have little effect in the central regions of the lake. (Similarly, for narrow lakes there is little change in amplitude across the width of the basin and rotational effects can be ignored.) Figure 9 represents a lake medium width where the amplitude reaches zero at the lake's center. The rate of decrease of amplitude in the offshore direction is more rapid for the internal rotating seiche than for the surface rotating seiche. Mortimer (1974) described this decrease in amplitude for Lake Michigan, but the argument is equally applicable to Lake Ontario. The amplitude of the rotating internal seiche was found to decrease by 50 percent at four kilometers (2.4 mi) from shore and by 95 percent at 20 kilometers (12 mi) from shore. The rotating uninodal surface seiche is of significant amplitude across the entire basin. Therefore the rotating uninodal internal seiche in wide basins such as Lake Ontario (width 60 km or 36 mi) is only important in the nearshore zone and is referred to as the internal Kelvin wave. Current motions associated with Kelvin waves are in the along shore direction. The internal Kelvin wave will be discussed as part of the total long internal wave response of the lake in the third section, and also in the fourth section concerning long internal waves in the nearshore zone. Suitable modifications will be made to the theory of internal Kelvin waves to account for sloping bottom contours (since the depth of the real lake is not constant).

Similarly, higher mode rotating internal seiches oscillate around multiple nodal points (3, 5, 7...) corresponding to the multiple nodal lines in the non-rotating case. This is exemplified by Figure 10 from a constant depth circular lake model after Csanady (1968) showing the forced lake response and higher mode internal wave responses which are free oscillations. Maximum amplitudes occur at the anti-nodal points between nodal points, and these motions are important away from shore and the dominance of the internal Kelvin wave. These higher modes of oscillation interact to form standing waves known as Poincaré waves as described earlier, and these will be treated in the third section concerning long internal waves in the offshore zone. Since the motions associated with higher mode internal rotating seiches are rotary (the Kelvin wave rotates but since it only has one nodal point, motions are everywhere parallel to shore) these seiches cannot exist near shore, since motions perpendicular to the lake boundaries are impossible. However, combinations of higher mode seiches can produce motions which are possible in the nearshore zone and these will be discussed in the third section.

To summarize, in Lake Ontario the internal seiches rotate. The uninodal internal seiche, or internal Kelvin wave is the dominant internal mode of oscillation in the nearshore zone but its amplitude decreases exponentially away from shore. Interactions between higher mode internal seiches form quasi-geostrophic waves which also appear in the nearshore zone. Interactions of higher mode internal seiches form Poincaré waves which are the dominant internal waves in the offshore zone of Lake Ontario.

## Model Internal Wave Solution for Large Rotating Basins and Long Internal Waves in the Offshore Zone of Lake Ontario

1. Model long internal wave solution for the complete Lake Ontario basin

A model for the internal wave solution in a large rectangular rotating basin has been discussed by Mortimer (1963a 1963b 1971 1974), and will be summarized here since it is a good approximation of the long internal waves in Lake Ontario. The basic wave form in the offshore solution is the Sverdrup wave which was first described by Sverdrup (1926) and is shown in Figure 11. This is a progressive wave (it translates linearly with time like surface waves approaching a beach) with level crests and troughs, but the currents associated with it rotate in clockwise ellipses, instead of moving in straight lines as in the case of non-rotating waves. The major axes of the ellipses are in the direction of wave travel, and fluid particles traverse the ellipses once for each Sverdrup wave cycle. The wave speed is greater than in the case of a non-rotating wave.

An upper limit exists for the period of these waves. As the period of the waves increases, the amplitude decreases and the elliptic current paths approach circles. This is similar to inertial type motion with most of the energy in the form of kinetic energy instead of potential energy. Periods greater than the inertial period are precluded since the wave speed would approach infinity. This sets an upper limit on the period of long internal waves in the offshore zone of large lakes. Since the inertial period for Lake Ontario is about 17.5 hours, this implies that these are fast moving waves (since they must cover the 300 km [180 mi] length of Lake Ontario in less than 17.5 hr). The inertial period limit does not apply to waves in the nearshore zone where waves are not dominated by rotation.

It is obvious from Figure 11 that the inclusion of boundaries would disturb the flow pattern of the progressive Sverdrup waves. However, it is possible to combine Sverdrup waves of wavelength comparable to the Lake Ontario basin dimensions to form a standing (stationary as opposed to progressive) wave pattern which satisfies the boundary condition at the shores of the lake; no motion perpendicular to the lake boundaries is permitted at the boundaries. These waves have periods which are close to but less than (90-99% of) the inertial period. The simplest combination would be a pair of oppositely directed Sverdrup waves traveling along the basin from east to west and west to east. This would produce standing Sverdrup waves with horizontal crests which Mortimer refers to as Class IIc waves.

The boundary conditions at the shores of the lake can be approximately met by one pair of oppositely directed Sverdrup waves traveling along the lake and another pair traveling across it. The wavelengths of these waves are closely related to the respective length and width of the lake, and the wave periods are slightly less than the inertial period. The waves interact producing a pattern of standing Poincaré waves (similar to the progressive waves described by Poincaré [1910]) which are referred to by Mortimer as Class IIa waves with undulating crests. A unit cell (or uninodal) Poincaré wave is shown in Figure 12. The unit cell has five nodal points; one at each corner and one in the center. Current flow reaches a maximum at the center, and all current vectors are in the same direction at any given time. The current pattern executes one clockwise rotation for each wave cycle as shown in Figure 12. Because of the cellular nature of the Poincaré wave, wave patterns of any combination of transverse and longitudinal nodalities can be built by arraying unit cells. The cell widths and lengths can also be varied to conform to any basin shape. Figure 13 is an example of a trinodal Poincaré wave constructed from nine unit cells.

Two sets of Sverdrup waves will not exactly satisfy the boundary conditions at the ends of the basin because the currents at the edges of the Poincaré wave cells are not zero. For waves of near inertial frequency and low nodality, these edge currents are small enough to be ignored. But for nodalities of five or higher, the edge currents are significant. An exact solution satisfying the boundary conditions identically requires at least one exponential type Poincaré wave which Mortimer refers to as Class IIb, and which is shown in Figure 14. The amplitudes of these waves vary exponentially along the basin (but are constant across it) thereby satisfying the zero velocity boundary condition at the ends of the basin when combined with the other waveforms.

As described earlier, the Kelvin wave is the dominant wave in the nearshore zone, and the only waveform which satisfies the lake boundary conditions of itself. Figure 15 shows a Kelvin wave in the nearshore zone of an infinitely wide basin with all currents directed along the shores. Since the Kelvin waves are the major feature in the nearshore and the Poincaré waves are most important in the offshore zone, a transition exists as shown in Figure 16. It should be noted that interaction between these wave forms is necessary to satisfy the conditions of Kelvin wave reflection at the ends of the basin.

The exact solution for internal waves in the rectangular rotating constant depth basin consists of two Kelvin waves rotating in opposite directions and a set of Poincaré waves (one for each nodality, e.g., 1, 3, 5..., at least one of which is exponential) which stand across the basin and either stand or vary exponentially along the basin. Figures 17a and 17b show trinodal and quintinodal Poincaré waves in combination with Kelvin waves. This is not presented to confuse the reader, but rather to demonstrate the complexity of the wave pattern when even two types of waves are present. In Lake Ontario, the Kelvin waves, uninodal, trinodal and perhaps quintinodal Poincaré waves all exist simultaneously, with the additional presence of short internal waves. The effects of these waves must be sorted out in order to properly analyze any data which are gathered in Lake Ontario.

The internal wave pattern is further complicated by the fact that wind stresses constantly excite new waves before the old ones have died out. In addition, waves in the nearshore zone are modified by the sloping lake bottom, and geostrophic currents which form there as a result of the piling of warm water caused by the wind stresses.

2. Observations of long internal waves in the offshore zone of Lake Ontario

Large scale measurements of the internal waves in Lake Ontario have not been frequent. Much of the information is based on water intake temperature data which only indicate when an internal wave reaches the shore. Figure 18 shows near inertial temperature oscillations at the Toronto and Rochester water intakes. The problem with fixed depth temperature recordings is that when upwelling brings the thermocline near to the sensor depth at one station, downwelling often drops the thermocline well below sensor depth at the opposite shore. The latter sensor is high up in the epilimnion and shows little temperature variation. There are short periods of time in Figure 18 (e.g., 8-11 August 1952 and 2-4 August 1953) when both sensors are near thermocline depth and the oscillations at both stations are nearly in phase.

These traces display two significant features. The first is that the amplitudes of the oscillations vary with time. This variation is probably caused by interactions between two or more different modes of oscillation. Such an interaction would result in periods of oscillation which are a weighted average (depending on the amplitude of the oscillation in each mode of motion) of the periods of oscillation of the waves present and a strongly varying amplitude as the wave crests and troughs added or cancelled. More intense storms result in stronger excitation of the higher mode shorter period oscillations which lowers the period of the observed motion. Unfortunately, the periods of the various modes of oscillation are so close that unless the modes of motion are known, they cannot be separated in the data.

The second important feature to be noted in Figure 18 is that the periods of oscillation tend to increase with time. The 1951 trace varies from 15 to 17 hours, and the 1953 trace varies from 14 to 16 1/2 hours. Since these periods are estimates, it is likely that the transition occurs smoothly rather than suddenly. Each excited mode of wave motion decays frictionally with time, but the higher modes of motion decay faster than the lower mode longer period motions. This explains the shift toward longer period oscillations and supports the hypothesis that the observed internal waves are the product of complex wave interactions.

Spectral analyses are not very useful for this type of data. Most of the data records are too short, and even when longer records are available the energy peaks are so close together that a single wide peak results at just below the inertial frequency. However, if oscillations continue for a long time undisturbed, all near inertial motions except for the uninodal Poincaré wave will dissipate. An example of this is the 22-25 August 1952 trace in Figure 18, where a relatively constant amplitude 17 hour oscillation is present. There is also some evidence of eight to nine hour oscillations, which are probably also caused by interactions between different mode Poincaré waves.

It is apparent that the best way to determine more information about internal waves in Lake Ontario is by continual temperature transects across the lake. This was done as part of the 1972 International Field Year for the Great Lakes program by the NOAA ship Researcher towing an undulating underwater temperature and pressure sensor described by Mortimer (1972), by the Cape Fear Technical Institute Ship the Advance II using mechanical bathythermographs and by the Canada Centre for Inland Waters ship R/V Limnos towing an undulating body called the "Batfish" equipped with an electronic bathythermograph and also towing a chain of temperature sensors. These data are still being analyzed, and it will be one to two years before the study is complete. However, some preliminary findings are available.

Figures 19 through 21, taken from Mortimer and Cutchin (1974) show three cross lake transects taken by the Researcher on 10 August 1972. A passing storm caused an eastward wind stress which set up the strong downwelling along the southern (left hand side of the figure) shore of Lake Ontario in Figure 19. Note that the isotherms are nearly vertical near the south shore. A thermal surge formed which moved northward across the lake at 2.3 kilometers per hour (1.4 mi/hr) which is close to the predicted propagation speed for long internal waves. In Figure 20, the thermal surge is nearly half way across the lake and somewhat reduced in amplitude. In Figure 21, the internal surge has moved still farther across the lake and a new smaller surge is forming on the south shore about 15 hours after the first surge. In the first section it was noted that in small lakes, when the thermocline displacement is large compared to lake depth, nonlinear effects and instability can cause internal surges to form on the internal seiches. Although Poincaré waves stand across the basin while internal seiches in small lakes are along the basin, Mortimer and Cutchin suggest that the situations are analagous. For thermocline displacements which are large compared to lake depth, internal surges form on the Poincaré waves. While the waves are strongly affected by the earth's rotation, it appears that the surges are not.

Since it took the Researcher about three and one half hours to complete a transect, and this is about 20 percent of the inertial period, numerical computations must be used to compensate for ship movements to give an instantaneous picture of the thermocline structure. The various lake motions can then be filtered from the data so that the internal wave variation in time over a cross section of Lake Ontario can be examined. Preliminary results show that the technique is quite promising. Examples of this type of analysis were not available at the time of publication, but will be published by Mortimer when the research is completed.

## Long Internal Waves in the Nearshore Zone of Lake Ontario

### 1. Description of internal waves in the nearshore zone

There are three types of large scale internal motions which occur in the nearshore zone of Lake Ontario. The first of these is the Kelvin wave which was described in the third section and is shown in Figure 15. For a constant depth basin with a thermocline depth of about 20 meters (65 ft) and epilimnion and hypolimnion temperatures of 20°C (68°F) and 6°C (43°F) respectively, the internal Kelvin wave in Lake Ontario travels at a speed of 1.6 kilometers per hour (about 1 mi/hr). These are typical conditions for mid-summer in Lake Ontario. At this speed the Kelvin wave would complete one cycle of the lake in 12 to 16 days.

The effects of a sloping bottom in the nearshore zone are quite complex, and are described by Csanady (1971). In the region of the nearshore zone, defined by the shelf width, (where the Kelvin wave "feels" bottom) a series of waves is produced such that a wave variation exists as one moves shoreward from the edge of the nearshore zone. The closer to shore, the more complex the wave pattern. The wave period also increases from 12 to 16 days to about 30 days.

When this situation exists, the current pattern associated with the internal Kelvin waves can be considered steady. Since passing weather systems cause wind stress impulses which set up new Kelvin waves about every five to eight days, each wave only exists for a fraction of a wave cycle before being disrupted by the next storm passage. Most storm systems travel from west to east causing eastward wind stress impulses over the lake. For this reason, the initial Kelvin wave set up with downwelling on the south shore, upwelling on the north shore and eastward coastal currents along both shores is a regular feature of the nearshore pattern. Many models have been developed to describe this pattern, but these are discussed in detail in the monograph on currents and will not be treated here.

The sloping bottom along the shores strongly affects the upwelling-downwelling pattern of the initial Kelvin wave set up. The depth of the epilimnion water in the nearshore zone is limited by the bottom depth, so that when upwelling occurs there is less water to be transported out of this region before the hypolimnion water surfaces. On the downwelling shore, there is less room for the epilimnion water that is transported into this region resulting in further concentration of the warm water and a sharper thermal gradient. This concentration accelerates the coastal currents associated with the Kelvin waves, and these currents form the second type of large scale internal motions found in the nearshore zone of Lake Ontario.

The third feature of the oscillatory motions in the nearshore zone of Lake Ontario is quasi-geostrophic waves. As stated earlier, these are formed by combinations of Poincaré waves which satisfy the boundary conditions near the shores of the lake. The period of these waves is slightly less than the inertial period, as in the case of Poincaré waves in the offshore zone. The wavelength is dependent on the depth of the thermocline (De), the average density or temperature (T) in the epilimnion (Te) and hypolimnion (Th) the force of gravity and the coriolis force. These factors are combined to form the internal Rossby radius of deformation (Ri), which is the appropriate scale length for these waves. Table 2 shows the variation of internal Rossby radius of deformation with temperature and thermocline depth for conditions found in Lake Ontario in spring and summer.

TABLE 2 Variation of the internal Rossby radius of deformation with strength of stratification and depth of thermocline

| Te | | Th | | De | | Ri | |
|---|---|---|---|---|---|---|---|
| °C | °F | °C | °F | m | ft | km | mi |
| 6(2) | 43(36) | 4 | 39 | 10 | 32.5 | 0.54 | 0.32 |
| | | | | 20 | 65.0 | 0.77 | 0.46 |
| | | | | 30 | 97.5 | 0.94 | 0.56 |
| | | | | 40 | 130.0 | 1.08 | 0.65 |
| 10 | 50 | 4 | 39 | 10 | 32.5 | 1.63 | 0.98 |
| | | | | 20 | 65.0 | 2.30 | 1.40 |
| | | | | 30 | 97.5 | 2.82 | 1.69 |
| | | | | 40 | 130.0 | 3.25 | 1.95 |
| 20 | 68 | 4 | 39 | 10 | 32.5 | 4.20 | 2.50 |
| | | | | 20 | 65.0 | 5.87 | 3.52 |
| | | | | 30 | 97.5 | 7.19 | 4.32 |
| | | | | 40 | 130.0 | 8.31 | 4.99 |
| 20 | 68 | 6 | 43 | 10 | 32.5 | 4.12 | 2.47 |
| | | | | 20 | 65.0 | 5.82 | 3.49 |
| | | | | 30 | 97.5 | 7.13 | 4.28 |
| | | | | 40 | 130.0 | 8.24 | 4.94 |

When the internal Rossby radius of deformation, the length associated with the quasi-geostrophic waves, is approximately equal to (within 25%) the effective shelf width, the distance over which the internal Kelvin wave feels bottom, interactions between the two wave forms can occur. This results in energy transfer from the quasi-geostrophic wave to the internal Kelvin wave. The coastal currents are accelerated causing increased downwelling on the righthand shore looking downstream and further increase in the Rossby radius of deformation. When this value exceeds that of the effective shelf width, the internal Kelvin wave accelerates to a velocity which is faster than that at which the temperature pattern can adjust (e.g., the internal Kelvin wave becomes ageostrophic). The internal Kelvin wave then rotates around the basin with a period of 12 to 16 days. The quasi-geostrophic pattern, which is then referred to as "bottom-trapped", lags behind the rotating Kelvin wave. When the internal Rossby radius of deformation is less than the effective shelf width, strong interactions do not occur. The theoretical aspects of these interactions are discussed by Allen (1975) and Wang (1975). There is now evidence that these wave interactions alter the water movement patterns in the nearshore zone of Lake Ontario in summer. This will be discussed in the fourth section on wave interactions. However, first the relationship between thermocline structure, internal waves and coastal currents will be examined.

## 2. Geostrophic adjustment

The concepts of geostrophic adjustment and baroclinic geostrophic flow were presented in the first section. These ideas were developed theoretically by Rossby (1937 1938), Stommel (1948), Charney (1955) and Mihaljan (1963) for the case of oceanic currents and applied to Lake Ontario by Csanady (1972a 1972b). As stated earlier, when a current is flowing in Lake Ontario, warm epilimnion water is transported to the right of the current looking downstream. This results in downwelling to the right of the current and upwelling to its left. The leftward pressure gradient thus established balances the rightward coriolis force, and the current flows in a straight line. If one moves far enough to the right or left of the current, the thermocline returns to its undisturbed level. This often results in weak counter-currents along the boundaries of strong currents.

Since the coriolis force is essentially constant for Lake Ontario, a relation exists between the temperature pattern and the current pattern. It is possible to work backwards and compute a baroclinic geostrophic current cross section from the temperature cross section. If the flow pattern has been relatively constant for several days and the thermocline is strong, then agreement between computed and measured currents will be good. However, under normal conditions the current patterns are constantly changing and geostrophic adjustment is never complete. Also, the measured currents contain both baroclinic and barotropic components, and only the former is calculated. Despite these drawbacks, baroclinic geostrophic current calculations are useful for determining information about current structure and the amount of flow in baroclinic and barotropic modes, especially in the mean. These calculations are discussed in detail in the monograph on currents, and are only mentioned here because results from baroclinic geostrophic computations will be used in the next subsection.

The coastal currents often meander. Consider a current in geostrophic equilibrium which is suddenly accelerated by the wind. It will turn to the right until enough warm water has been transported to the right to restore geostrophic balance by increasing the pressure gradient. In actual flows, the equilibrium point is passed. The pressure gradient force finally overcomes the coriolis force and inertia and turns the current to the left. In this manner, coastal currents meander at about the inertial period. The thermocline pattern oscillates in response to the current motion, with downwelling occurring on the side to which the current is moving and upwelling on the opposite side. If the currents are fast enough, eddies (or whirls) are shed from the currents as they meander. The mechanism is similar to the shear mechanism which causes eddies to form on the thermocline when large amplitude internal seiches are set up in small lakes. In this case it is the rapid change in horizontal velocity with distance that causes the eddies to form. Stability is not important since density differences across the horizontal boundaries of the current are small.

The data presented in this section are taken from the 1972 International Field Year for the Great Lakes Program which was conducted on Lake Ontario. Figures 22 and 23 show the location of US and Canadian measurement stations. A complete set of figures are available in Landsberg and Scott (1976).

Set up by wind, which alters the thermocline pattern in the nearshore zone, also effects the strength and locations of the coastal currents. Figure 25b shows five temperature cross sections, three on the south shore and two on the north shore of Lake Ontario for 2 June 1972 after an eastward wind stress. Note that downwelling has caused wedge shaped thermoclines near shore at Olcott and Oswego which extend quite deeply into the lake. Upwelling on the north shore has caused shallow lens shaped thermoclines which are displaced from shore at Oshawa and Presqu'ile. Figure 25a shows cross sections of along shore velocity. Values are in centimeters per second (10 cm/sec is about 0.3 ft/sec). Solid lines refer to eastward currents (out of the page) while dashed lines refer to westward currents (into the page). The table at the lower right is a summary of instantaneous volume transport which is computed by multiplying the velocities times the cross sectional areas over which they are present. Values are in $10^4$ $m^3/sec$ ($10^4$ $m^3/sec$ is about $3.4 \times 10^5$ $ft^3/sec$). The table in Figure 25a is a summary of transports computed from the baroclinic geostrophic currents, while the table in Figure 25c is the difference between these values, an approximation of the barotropic geostrophic transport. The strongest currents in Figure 25b are contained within the epilimnion water at all stations as can be seen by comparison with Figure 25a. They are deeper, stronger and nearer to shore on the south shore than on the north shore in agreement with the temperature pattern. The computed baroclinic geostrophic currents in Figure 25c are strongly related to the temperature cross sections from which they were computed. These current values are weaker than those of the measured currents because of the weak spring thermocline indicating that most energy is in the barotropic mode. This is not the case in summer as will be seen in the fourth section under number 3.

Internal waves often appear in temperature cross sections as exemplified by the Olcott cross section in Figure 28b. In Figure 28c it can be seen that the internal waves have caused alternating bands of eastward and westward flow which do not appear in the measured currents in Figure 28a.

3. Wave interactions in the nearshore zone

Since weather systems pass over Lake Ontario every several days and most of them exert eastward wind stresses on the lake, the Kelvin wave set up pattern is a common one. This includes downwelling on the south shore, upwelling on the north shore and eastward coastal currents along both shores. However, the mean transport pattern in summer in Lake Ontario is cyclonic as originally determined by Harrington (1895); eastward on the south shore and westward on the north shore. The mean temperature pattern is a downwelled pattern along both shores, although the downwelling is stronger along the south shore.

Many investigators have suggested that the cyclonic mean transport is caused by a steady thermally driven background circulation, and that passing storms are just short term perturbations on the flow pattern. Since the nearshore zones warm faster in spring and remain warmer throughout the summer, a cyclonic mean circulation would be consistent with geostrophic equilibrium. This concept was suggested by Millar (1952), Rodgers and Anderson (1963) and most recently by Pickett and Richards (1975). This argument is reasonable in the offshore zone, but is the thermally driven circulation strong enough to overcome wind induced transport in the nearshore zone?

If the cyclonic mean transport is caused by a steady thermal circulation, that circulation should be strongest in the spring when the maximum differential heating between the nearshore and offshore zones occurs. Furthermore, the strongest temperature gradient should exist along the north shore where the flow is being forced upwind. However, the strongest temperature gradient occurs in the southeast portion of the lake as shown in Figure 30, and the mean transport in spring is not cyclonic. Figure 24 shows the resultant transport for three one month intervals (or "alert" periods) beginning on 15 May, 15 July and 15 August 1972 for five cross sections of Lake Ontario as part of the IFYGL program. These means were determined by averaging the transport values calculated from once or twice daily cross sectional measurements of velocity and temperature over the alert periods. These data are summarized in reports by Csanady and Pade (1972) and Scott et al. (1973). The measured transport (u) is determined from the alongshore component of velocity, while the baroclinic geostrophic (or thermal) transport ($u_g$) is computed from the temperature cross sections. Values are in $10^4$ $m^3/sec$ ($10^4$ $m^3/sec$ is about $3.4 \times 10^5$ $ft^3/sec$) and positive and negative values refer to eastward and westward transport respectively. Since the small boats used to make the measurements could not operate in severe weather conditions, a fair weather bias exists which actually favors thermally driven steady state circulation since the strongest wind driven flows were not measured.

Figure 24 shows that while the resultant baroclinic geostrophic transport is cyclonic in all three alert periods (except for Olcott in the second alert) the resultant measured transport was cyclonic in the second and third alerts but not in the first. This occurred despite the fact that the mean eastward wind stress was only half as strong in the first alert as in the second and third alerts. Westward or upwind transport is a regular feature of the north shore as described by Csanady and Scott (1974), Arajs and Faroqui (1974), Blanton (1974 1975) and Landsberg and Scott (1976).

If the cyclonic resultant transport is not caused by a steady thermally driven circulation, then the major driving force must be the wind stress. Since a steady wind driven flow pattern does not produce cyclonic mean transport in the nearshore zone of Lake Ontario, this line of reasoning leads to the possibility that periodic motions resulting from internal wave interactions is the cause of the observed transport pattern. In the remainder of this section the response of the lake to wind stress impulses in the first and second alerts will be examined to determine whether internal wave interactions are responsible for the cyclonic transport pattern observed in summer in Lake Ontario. This discussion is taken from Landsberg (1975).

### A. The response of Lake Ontario to an eastward wind stress impulse in spring

On 1 June 1972 an eastward wind stress impulse was applied to Lake Ontario in conjunction with a passing weather system. The winds gradually diminished until 6 June when another weather system passed over the lake. Cross sections of alongshore velocity (u) temperature (T) and baroclinic geostrophic velocity ($u_g$) for 2 June are shown in Figures 25a, 25b and 25c. The cross section locations can be found in Figure 24. There is strong downward tilting of the isotherms on the south shore and strong upward tilting of the isotherms on the north shore. The coastal currents are eastward at all cross sections and are affected by the temperature pattern. Transport is greatest where the most warm water is present. It is somewhat weak compared to transport in summer, especially at Rochester and Oswego, since these cross sections are located near the deepest part of the lake which hasn't warmed up too much as yet. The baroclinic geostrophic currents are similar to the measured currents. They are stronger on the south shore, especially at Olcott in response to the strong thermal gradient.

The cross sections of u and T for 3 June appear in Figures 26a and 26b. The wind stress has lessened somewhat but is still eastward resulting in increased downward tilt to the isotherms at Rochester and Olcott, but the isotherm uptilt has decreased at Presqu'ile. Transport is still strongly eastward along the north shore, but eastward transport has decreased by 80 percent at Presqu'ile in conjunction with a decrease in isotherm uptilt and the formation of a westward countercurrent.

By 5 June the eastward wind stress has dropped to nearly zero and the current and temperature patterns are considerably different as shown in Figures 27a and 27b. Eastward transport is about the same at Oswego but has decreased at Rochester where upward tilt of the isotherms at stations 27 to 30 has reduced the area of eastward coastal current. A westward countercurrent is now present at Rochester. A similar situation is occurring at Olcott where westward flow is present lakeward of station 48. On the north shore the relaxation of eastward wind stress has caused the return of warm water to the nearshore zone resulting in relatively flat thermoclines with the beginning of downward tilt near shore. Strong westward currents are now the dominant flow feature at Oshawa and Presqu'ile. Computed baroclinic geostrophic transport is still eastward at all cross sections, but westward baroclinic geostrophic currents are now present along the north shore as shown in Figure 27c. A new eastward wind stress impulse on 6 June returned the current and temperature cross sections to a pattern similar to that of 2 June.

The resultant transports for the period 2-6 June were calculated for comparison with the resultant transport for the first alert and appear in Table 3. Positive values refer to eastward transport. Both measured and baroclinic geostrophic transports are eastward for this "event" in response to the eastward wind stress, but baroclinic geostrophic transports are larger on the south shore where downwelling occurred. The resultant transports for an event caused by a westward wind stress impulse from 26-29 May 1972 are also shown. The resultant measured transports are westward, while the baroclinic geostrophic transports are westward except for Olcott. The discrepancy at Olcott is caused by westward flow originating at Oshawa which traveled around the western end of the lake reaching Olcott by 29 May as eastward flow. The effect on resultant measured transport is small, but baroclinic geostrophic transport on the south shore is so weak under westward flow that one day of reversal is enough to change the sign of the resultant. Most importantly, when the transport is westward, the baroclinic geostrophic transports are larger on the north shore where downwelling occurs.

TABLE 3 Resultant measured and baroclinic geostrophic alongshore transport for 26-29 May, 2-6 June and 15 May through 15 June 1972 ($10^4$ $m^3$/sec which is about $3.4 \times 10^5$ $ft^3$/sec)

| | | Oswego | Rochester | Olcott | Oshawa | Presqu'ile |
|---|---|---|---|---|---|---|
| 5/26-29 | $u$ | --- | -0.51 | -0.41 | -0.48 | -0.50 |
| | $u_g$ | -0.02 | -0.06 | 0.02 | -0.26 | -0.24 |
| 6/2-6 | $u$ | 1.05 | 1.32 | 1.30 | 0.50 | 0.95 |
| | $u_g$ | 0.29 | 0.67 | 1.63 | 0.03 | 0.09 |
| 5/15-6/15 | $u$ | 1.87 | 1.07 | 0.55 | 0.33 | 0.21 |
| | $u_g$ | 0.54 | 0.62 | 0.89 | -0.08 | -0.03 |

It appears that in the spring, resultant measured transport is eastward because the average wind stress is eastward. Resultant baroclinic geostrophic transport is cyclonic because the baroclinic geostrophic currents are much stronger on the shore where downwelling is occurring; i.e., eastward wind stress produces large eastward baroclinic geostrophic currents on the south shore, while westward wind stress produces large westward baroclinic geostrophic transport. The transport patterns are wind driven and not steady state thermally driven.

In the spring, the thermal bar effectively separates the nearshore zone from the rest of the lake creating channel-like flow and limiting the amount of energy and momentum which can be concentrated near shore by downwelling. Because of this, the internal Rossby radius of deformation remains smaller than the effective shelf width (which is 4-6 km or 2.4-3.6 mi) and resonance between internal Kelvin waves and quasi-geostrophic waves does not occur. While the waves were present, they did not alter the eastward wind-driven transport significantly. Their effect does show up in the resultant baroclinic geostrophic transport.

When a wind stress impulse relaxed, the flow along the lefthand shore looking downwind returned to geostrophic equilibrium faster than the flow along the righthand shore. The coriolis force and differential heating acted to concentrate the warm water along the righthand shore, while along the lefthand shore differential heating acted to warm the hypolimnion water near shore and flatten the upwelled thermocline.

The two shores responded independently to wind stress for the most part, although flow originating along one shore could travel around the end of the lake and reach the other shore. Countercurrents in association with geostrophic adjustment always appear such that the flow with the shore to its right looking downstream will be nearest to shore. For adjustment after an eastward wind stress the westward currents appear near shore on the north shore and away from shore on the south shore. This supports the idea that geostrophic adjustment occurs along the two shores independently. Countercurrents form because when the wind stress relaxes the adjustment process exceeds the equilibrium point (downwelling becomes upwelling) causing transverse thermocline oscillations. The thermocline oscillates across the "channels" formed between the shore and the thermal bar with periods on the order of several days.

### B. The response of Lake Ontario to an eastward wind stress impulse in summer

An eastward wind stress impulse began on 23 July 1972, and the cross sections of u, T and $u_g$ for this date appear in Figures 28a, 28b and 28c. Eastward coastal currents are present at all cross sections, but those on the south shore are stronger, nearer to shore and extend to a greater depth. The temperature cross sections in Figure 28b show that 22°C (72°F) water is present at the surface along the south shore, while 20°C (68°F) water is present at the surface on the north shore. Downwelling is present at all south shore cross sections and increases from west to east. The 10°C (50°F) isotherm is at a depth of 18 meters (58.5 ft) at

Olcott and Rochester and 25 meters (81 ft) at Oswego. The internal Rossby radius of deformation along the south shore is large enough for resonance between internal Kelvin waves and quasi-geostrophic waves to occur. The depth of the 10°C (50°F) isotherm on the north shore is about 10 meters (32.5 ft). The baroclinic geostrophic currents are primarily eastward as shown in Figure 28c.

The set up during the summer is somewhat different from that which occurs in the spring. At that time, the warm water is constrained by the thermal bar on the north shore and forms a lens shaped thermocline, while in the summer it spreads southward into the lake forming a thin layer about 10 meters (32.5 ft) in thickness. This results in very shallow coastal currents along the north shore. The constraint along the south shore (the lake boundary) is the same as in the spring, but now warm water from the center of the lake can be transported shoreward, instead of just the warm water between the thermal bar and shore. Consequently, the coastal currents on the south shore extend deeper into the lake and are quite fast.

Resonance between the internal Kelvin waves and quasi-geostrophic waves appears to occur, causing energy and momentum transfer to the Kelvin wave. Once formed, the internal Kelvin wave and associated current pattern rotate cyclonically within a period of 12 to 16 days. The initial set up of the rotating current pattern or "ROCUP" is shown in Figure 29a. Both measured and baroclinic geostrophic transports are eastward at all cross sections. The nodes of the ROCUP are believed to be at the eastern and western ends of the lake. The fastest moving water or the ROCUP crest is located at the south shore, while slower moving water or the ROCUP trough is at the north shore.

The cross sections of u, T and $u_g$ for 27 July appear in Figures 30a, 30b and 30c. A node of the ROCUP has passed Olcott causing the flow direction to reverse to westward. The reversal occurred near shore, unlike the countercurrents along the south shore in spring. The isotherms are beginning to show upward tilt in response to the reversal of flow direction. Downwelling is continuing at Rochester and Oswego where the depth of the 10°C (50°F) isotherm has reached 30 meters (97.5 ft) and 42 meters (136.5 ft) respectively. The strength of the eastward currents has increased at both cross sections, but westward flow is now present as well. Dramatic upwelling has occurred along the north shore dropping surface temperatures from 20°C (68°F) to 6 to 14°C (42-57°F). Strong advection of warm water from north to south across the width of the lake has occurred causing a strong southward shift of the epilimnion.

The other node of the ROCUP appears to be on the north shore resulting in weak westward transport, but the warm water has been displaced so far to the south that it will take several days for geostrophic adjustment to occur, as can be seen by examining the baroclinic geostrophic currents in Figure 30c. Since the internal Rossby radius of deformation is now greater than the effective shelf width of four kilometers (2.4 mi) on the south shore as shown by Table 2, the quasi-geostrophic waves have become bottom trapped causing the rotation of the temperature field to lag behind that of the current field.

The position of the ROCUP on 27 July is shown in Figure 29b after completion of about one-fourth of rotation. The fastest flow or ROCUP crest is at Oswego. A node of the ROCUP is in the vicinity of Oshawa and Presqu'ile. The transport values are weakly westward there although the ROCUP crest has not reached the north shore as yet. Baroclinic geostrophic transports are still eastward along the north shore because the temperature pattern has not adjusted to the westward currents. The other node of the ROCUP has passed Olcott where both measured and baroclinic geostrophic transports are now westward. The ROCUP axis and direction of rotation are shown.

By 31 July the wind stress is weak but still eastward. The cross sections of u, T and $u_g$ for 31 July are shown in Figures 31a, 31b and 31c. The isotherms are tilting upward sharply on the south shore. The 10°C (50°F) isotherm is at 10 to 15 meters (32.5-49 ft) at Olcott and Rochester, while at Oswego it has risen from 42 meters (136.5 ft) to 25 meters (81 ft) in four days. Shallow westward coastal currents are present at Olcott as well as Rochester, where the flow has reversed direction in two days. Eastward transport at Oswego has been reduced to nearly zero with the approach of a node of the ROCUP. Strong downward tilt of the isotherms in present at Presqu'ile where the 10°C (50°F) isotherm depth has changed from 10 meters (32.5 ft) two days previously to a range of 20 to 30 meters (65-97.5 ft) on 31 July. Surface water with a temperature of 20°C (68°F) has returned to Presqu'ile. The temperature field at Oshawa will not adjust for another three days.

The currents at Oshawa and Presqu'ile are strongly westward and extend to a greater depth than those on the south shore. Westward flow is occurring along both shores despite the presence of eastward wind stress. The baroclinic geostrophic transport is westward at Olcott and Rochester but still eastward at Oswego. On the north shore, baroclinic geostrophic flow is strongly westward at Presqu'ile and has become westward at Oshawa.

The position of the ROCUP on 31 July is shown in Figure 29c. It appears to have completed nearly one-half the cycle in eight days. Transport is westward along both shores, but now it is stronger on the north shore. The baroclinic geostrophic transport pattern lags the measured transport pattern, as shown by the fact that the baroclinic geostrophic transport is now stronger than the measured transport at Oswego (after passage of the ROCUP crest) and quite weak at Oshawa.

The resultant transports for the period 23-31 July were calculated for comparison with the resultant transports for the second alert and appear in Table 4. For the event, measured resultant transport is eastward at Oswego and Rochester, and westward at Oshawa and Presqu'ile. A ROCUP node passes Olcott very early in the event causing reversal of flow direction and westward resultant transport. The baroclinic mode of motion lags behind the barotropic (or ageostrophic) mode causing eastward resultant baroclinic geostrophic transport at Olcott and Oshawa.

TABLE 4 Resultant measured and baroclinic geostrophic alongshore transport for 23-31 July and 15 July through 15 August 1972 ( $10^4$ $m^3/sec$ = about $3.4 \times 10^5$ $ft^3/sec$)

| | | Oswego | Rochester | Olcott | Oshawa | Presqu'ile |
|---|---|---|---|---|---|---|
| 7/23-31 | $u$ | 3.26 | 2.06 | -0.84 | -0.57 | -2.17 |
| | $u_g$ | 5.47 | 2.03 | 0.47 | 0.18 | -0.79 |
| 7/15-8/15 | $u$ | 2.60 | 2.10 | 0.10 | -0.34 | -1.49 |
| | $u_g$ | 3.28 | 2.04 | -0.06 | -0.37 | -1.71 |

For the entire alert, resultant transport values are small at Olcott and Oshawa. A node of the ROCUP passes Olcott first causing early reversal to westward flow and a weak resultant eastward transport. Oshawa is the last coastal chain to be passed by a node, so reversal to westward flow occurs late in an event and the resultant westward transport is small. Resultant baroclinic geostrophic transport for the alert increases from west to east on both shores of the lake because the resultant eastward wind stress causes a mean thermocline depth which increases from the northwest to the southwest portion of the lake as shown in Figure 4.

The cyclonic resultant transport appears to be caused by north-south shear in the coastal currents combined with the formation of rotating current patterns. There are four mechanisms which lead to this north-south shear (increase in eastward transport from north to south when an eastward wind stress is applied). The first of these is river flow. Primary river inflows are from the Niagara, Genesee and Oswego rivers, all of which are on the south shore, while the primary outflow is the St. Lawrence river at the eastern end of the lake. Li et al. (1974) have used a rotating model of Lake Ontario to show that this river flow follows the south coast of the lake eastward and then turns northward to the St. Lawrence river. Whether or not this exact course is taken, an eastward pressure gradient results which produces eastward transport in the southern portion of Lake Ontario of 0.5 to 0.8 x $10^4$ $m^3/sec$ (1.7-2.7 x $10^5$ $ft^3/sec$) as determined from data by DeCooke and Witherspoon (1974). Another possible mechanism is the fact that the warmer water along the southern coast of Lake Ontario results in increased air-water instability leading to greater energy transfer to this region of the lake for a given wind stress. These are probably not the most important factors since both occur in the spring as well.

The most important cause of the north-south shear in the coastal currents is the southward transport of warm water during eastward wind stresses (also known as Ekman transport). First, it serves to concentrate the warm water along the southern coast. Second, the warm epilimnion water which is transported southward has a higher eastward momentum than the cooler hypolimnion water beneath it. This creates a north-south flux of eastward momentum.

Steady state north-south shear in the coastal currents is not sufficient to produce cyclonic resultant transport. If this occurred, the flow pattern in Figure 32a would result. However, this shear lowers the depth of the thermocline along the righthand shore looking downwind increasing the internal Rossby radius of deformation and permitting resonance between the internal Kelvin waves and the quasi-geostrophic waves to occur. The initial set up is shown in Figure 32b where the north-south shear results in larger eastward transport on the south shore than on the north shore. When the internal Kelvin wave and accompanying ROCUP have rotated through one-half the cycle, transport is westward along both shores but larger along the north shore as shown in Figure 32c. If flow over one or more wave cycles is averaged, the cyclonic resultant transport pattern in Figure 32d is obtained. If north-south shear were not present in the coastal currents, the resultant transport would be zero.

Resonance can also occur along the north shore during westward wind stress impulses, but this is less likely to occur for four reasons. First, westward wind stresses are less frequent over Lake Ontario than eastward wind stresses. Second, the mean thermocline depth is shallower along the north shore, so that the downwelling must be stronger to obtain a given thermocline depth and internal Rossby radius of deformation for resonance. Third, the bottom slope on the north shore is shallower than on the south shore so that the effective shelf width is six kilometers (3.6 mi) instead of the four kilometers (2.4 mi) on the south shore. Therefore, a greater thermocline depth is required for resonance. Fourth, the average epilimnion temperature is colder on the north shore which also increases the thermocline depth required for resonance.

The presence of ROCUPS can cause strong upwind flow and thermocline slope opposite to what would be expected from the prevailing wind conditions. This must therefore be taken into account during any planning for which temperatures and flow patterns are important.

## Short Internal Waves

Not all internal waves are of basin size dimensions. Waves can exist over a wide range of frequencies and wavelengths. The theoretical upper frequency limit for internal waves in a stratified fluid was determined to be the Brunt-Väsälä frequency by Groen (1948). This frequency is proportional to the square root of the gravitational acceleration, density and the vertical rate of change of density in a lake. It is evident from research conducted to date that most of the energy associated with the short internal waves is found near the Brunt-Väsälä frequency, which can be considered a free mode of motion for internal oscillations. The Brunt-Väsälä period for internal waves in Lake Ontario in summer is about three to eight minutes.

Short internal waves were observed in large lakes by Bryson and Ragotzkie (1960). The waves moved to the right of the wind so that the origin was not at the downwind end of the lake where the kinetic energy in surface waves is at a maximum. Each isotherm was found to move independently and slightly differently, as would be expected since the vertical rate of change of density, and hence the Brunt-Väsälä frequency varies with depth. Both progressive waves with periods of a few minutes and standing waves were observed. Mortimer et al. (1968) measured short internal waves in Lake Michigan. The waves were concentrated near the thermocline and most of the observed waves had frequencies of 0.3 to 0.5 times the Brunt-Väsälä frequency (N). Waves of frequency greater than N were not observed.

Although the author could not find the reference, studies by Hollan (1966) showed the presence of internal waves with frequencies near the Brunt-Väsälä frequency in the Baltic. Neshyba et al. (1972) found an energy peak at 0.6 N which dropped rapidly to zero as the frequency approached N in the Arctic. Laboratory experiments by Schooley and Hughes (1972) produced waves with frequencies of 0.5 to 0.7 N. Wavelengths were found to increase with distance from the source of the disturbance, which is to be expected since waves with longer wavelengths travel faster and move greater distances before being damped out by friction than waves with shorter wavelengths. Results of these experiments indicate that short internal waves may be generated in regions of shear instability as occur when thermal surges form or at the nodes of Poincaré waves. This idea is supported by Hollan (1972) who observed internal waves in groups whose period decreased with time. It was suggested that a series of recorders could be used to determine the source of short internal waves. Further support of this theory is given in preliminary analysis of IFYGL data in Lake Ontario by Mortimer and Cutchin (1974). After passing through a thermal surge, the wavelength of short internal waves were found to increase with distance from the surge.

## Summary

The long internal waves found in Lake Ontario are of two forms. In the offshore zone where motions are rotational and Poincaré waves with periods approaching the inertial period are the dominant wave form. The mean circulation pattern is cyclonic. In the nearshore zone, motions are constrained by the walls of the basin and rotating internal Kelvin waves with periods of 12 to 16 days are present. Wave interactions cause rotating current patterns whose resultant circulation is cyclonic, although instantaneous nearshore flow patterns are rarely cyclonic. The short internal waves have periods of three to eight minutes. The frequencies approach the Brunt Väsälä frequency, which is the theoretical upper limit for short internal waves. Internal waves are an important energy dissipation mechanism in Lake Ontario and aid in energy transfers between the epilimnion and the hypolimnion of the lake.

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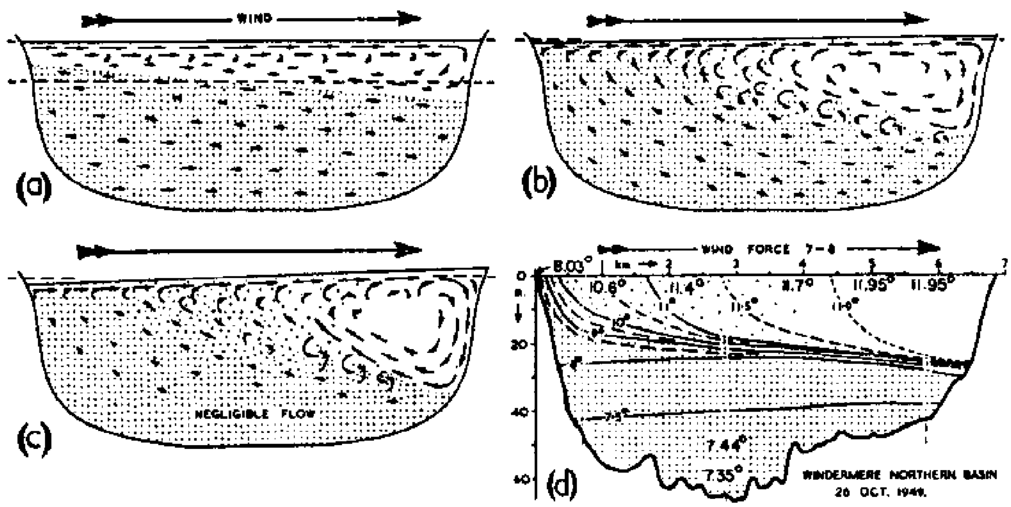
Figure 1    Two dimensional representation of the effects of wind on a stratified lake.






Two-dimensional representation of the effects of wind on a stratified Lake (MORTIMER 1961): (a) to (c) stages in the development of the epilimnetic wedge and of shear instability at the downwind end of the thermocline; (d) isotherm distribution in Windermere, northern basin, after about 12 hours of relatively steady wind. The layer initially below the thermocline is shown stippled, and the relative speed and direction of flow is roughly indicated by arrows. (From Mortimer 1974)

Figure 2 Cross section of Lake Ontario during stratification (showing epilimnion, hypolimnion, and thermocline).

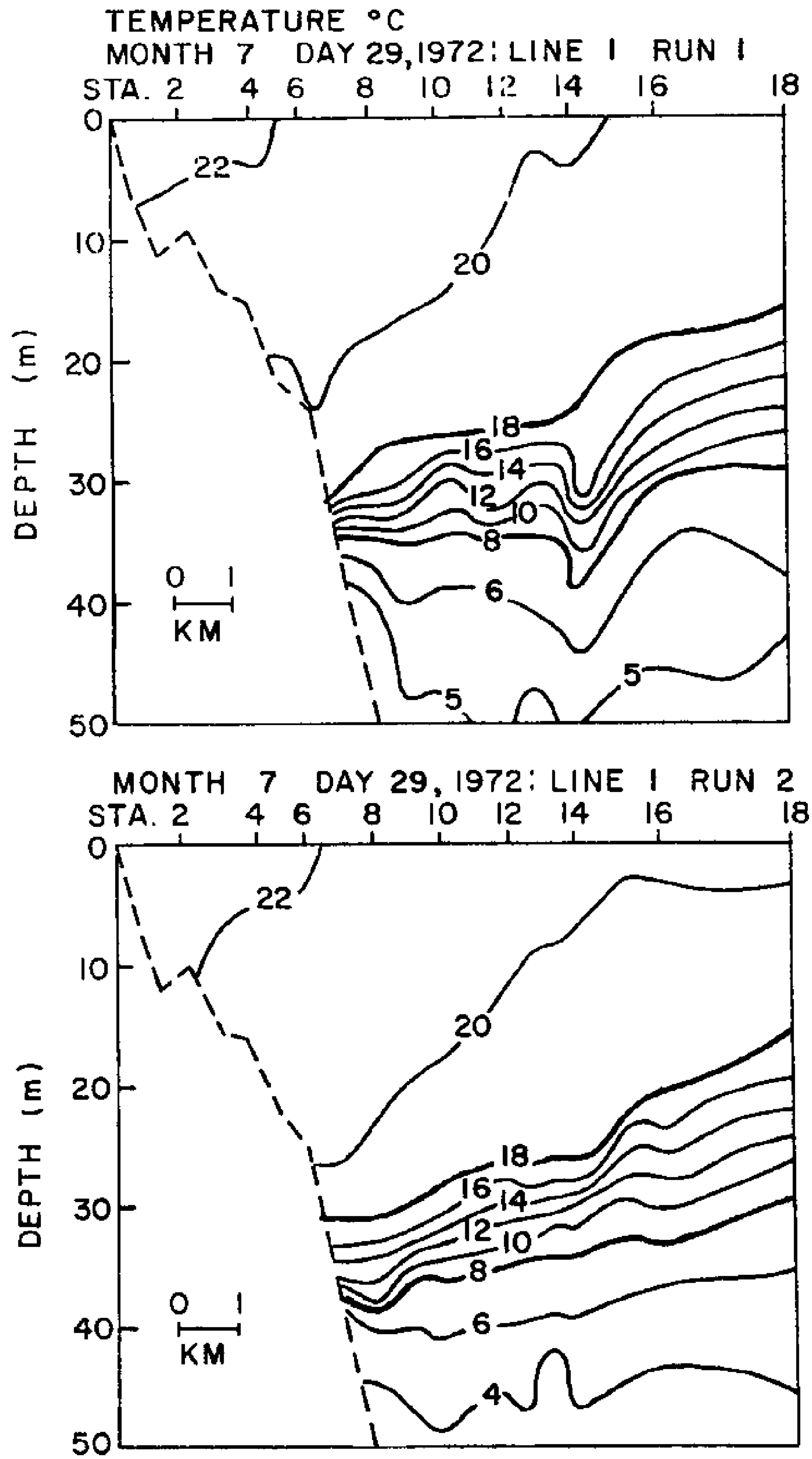
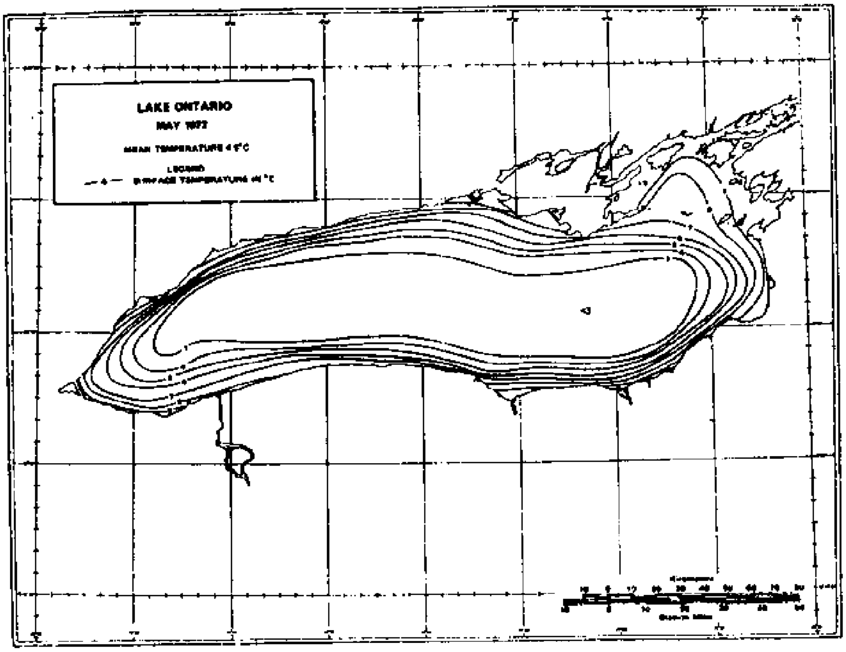
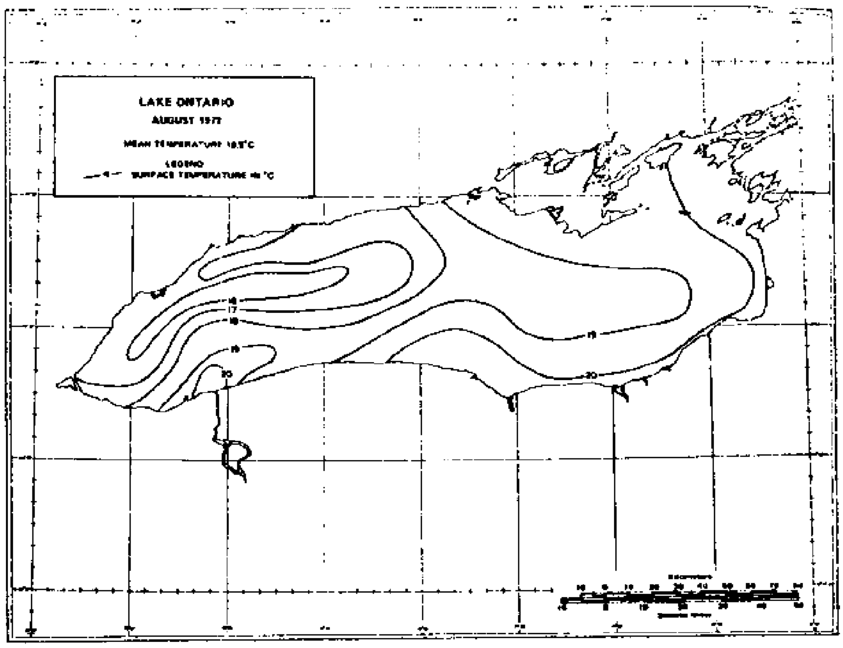


Figure 3 Mean surface temperatures for Lake Ontario in May 1972.



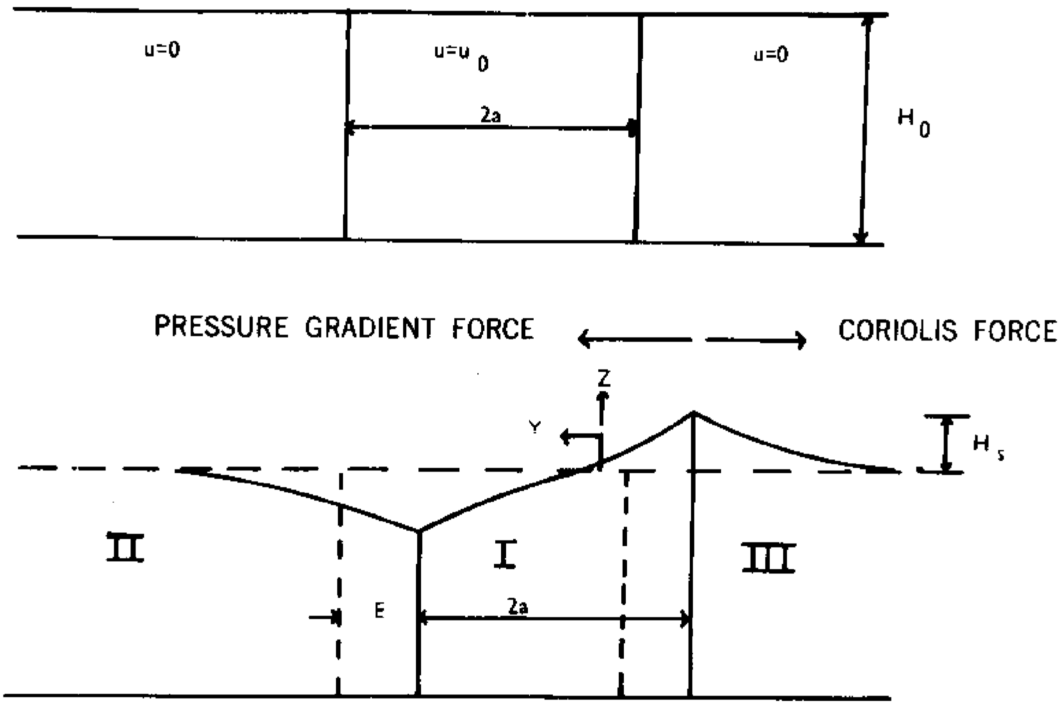

(From Webb 1974)

Figure 4 Mean surface temperatures for Lake Ontario in August 1972.



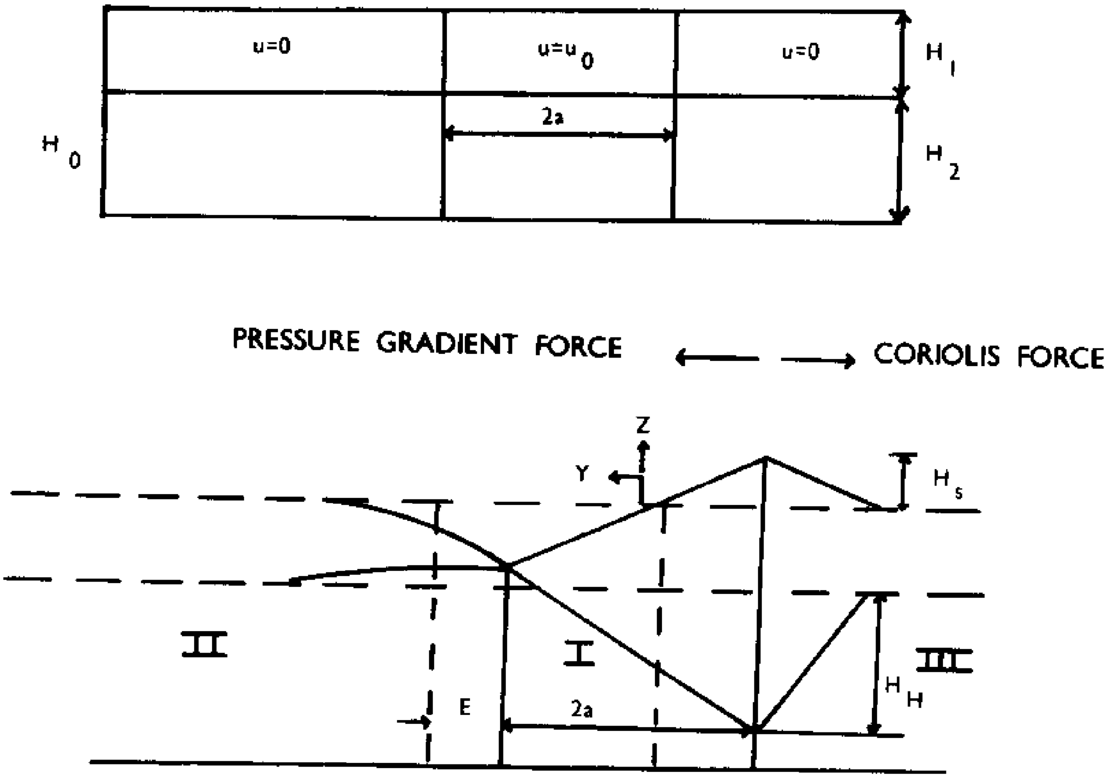

(From Webb 1974)

Figure 5 Barotropic geostrophic adjustment for a current flowing in an isothermal lake.




a= HALF WIDTH OF INITIAL CURRENT OF VELOCITY u=u 0
H 0= LAKE DEPTH
H s= VERTICAL DISPLACEMENT OF LAKE SURFACE ( EXAGGERATED )
E= HORIZONTAL DISPLACEMENT OF CURRENT

Figure 6 Baroclinic geostrophic adjustment for a current flowing in a stratified lake.




a=HALFWIDTH OF INITIAL CURRENT VELOCITY u=u o

H o =LAKE DEPTH

E=HORIZONTAL DISPLACEMENT OF CURRENT

H s =VERTICAL DISPLACEMENT OF LAKE SURFACE (EXAGGERATED)

H h =VERTICAL DISPLACEMENT OF THERMOCLINE

Figure 7 Effective and actual shelf widths for a stratified lake with a sloping bottom.

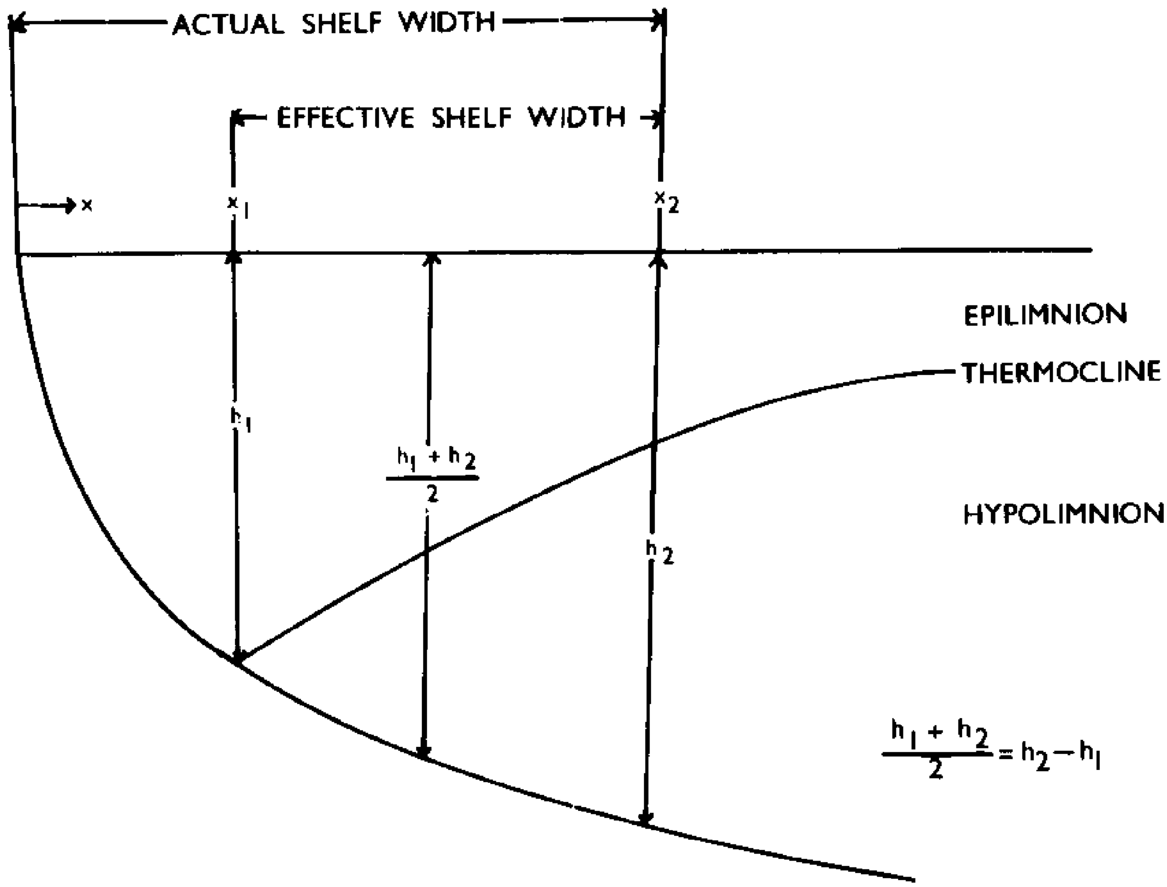
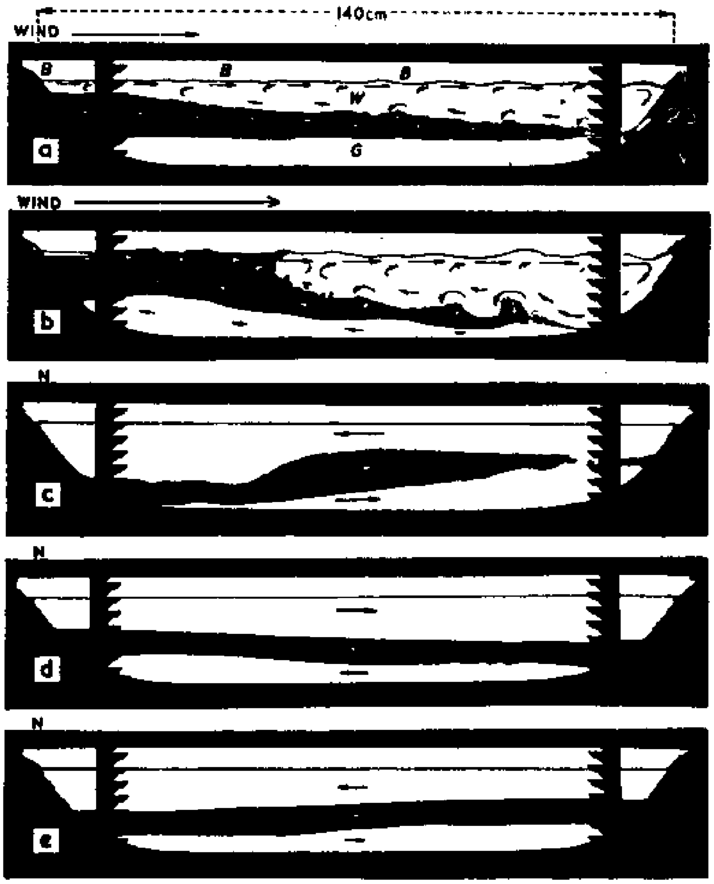
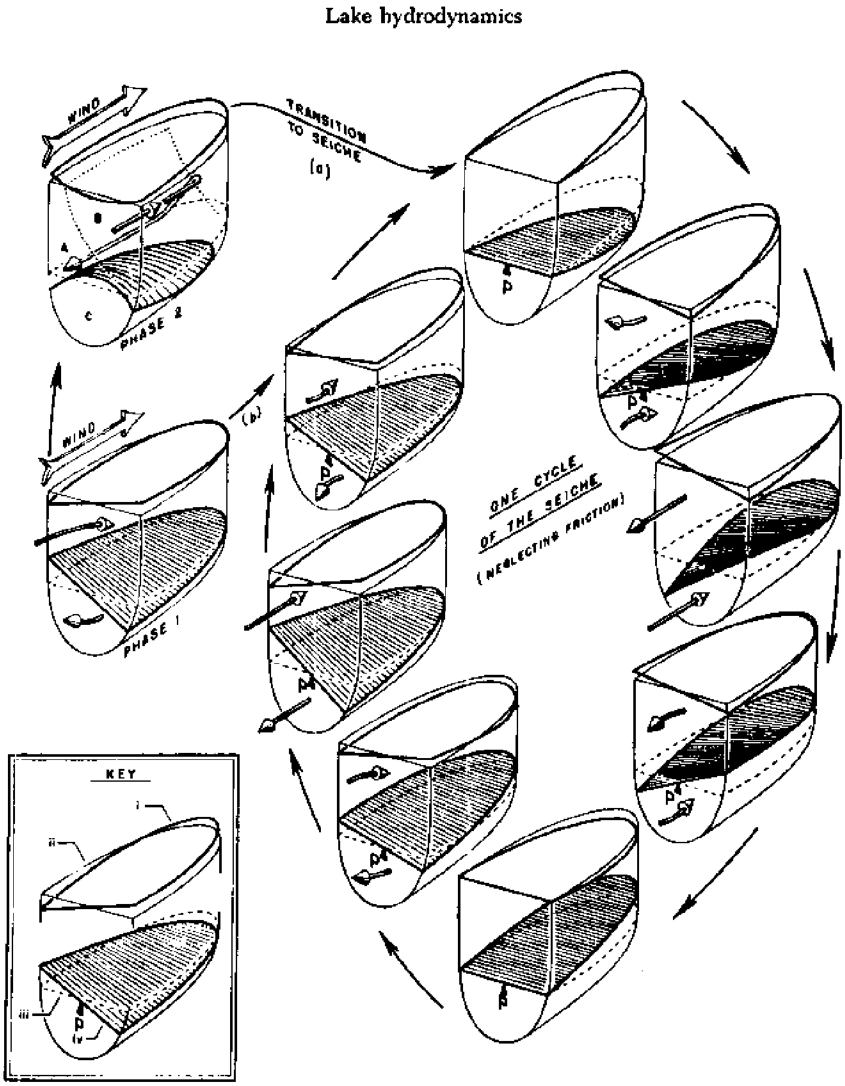


Figure 8 Wind effects and subsequent internal seiche in a laboratory model of a stratified lake.



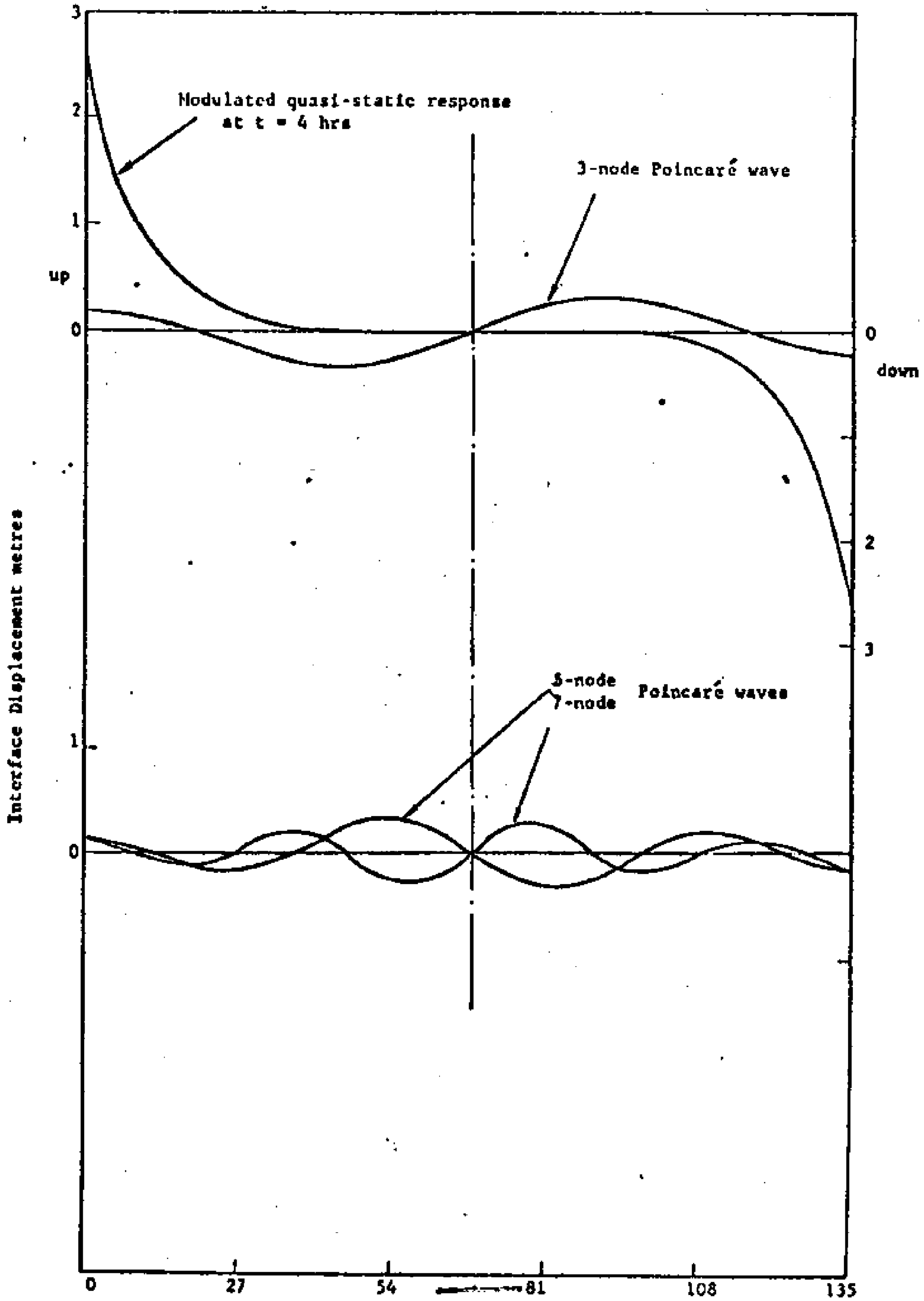

Wind effects and a subsequent internal seiche in a laboratory model of a stratified lake with three homogeneous land (relatively) immiscible layers of density 1.00 (water), 1.04 (phenol) and 1.06 (a solution of glycerine and water), from MORTIMER (1954). The top two sections show progress in redistribution of the layers under wind stress, with the onset of "instability" at the base of the "epilimnion". The bottom three sections show phases in the ensuing internal seiche, which initially shows marked nonlinear effects.

Figure 9 An internal seiche in a rotating two-layered model of "medium" width.




An (amphidromic) internal seiche in a rotating two-layered model of "medium" width. In the inset "key" diagram: the oscillating lake surface (i) is shown as a heavy line (this is the surface signature of the internal seiche mode, i.e., is not a surface seiche mode); the equilibrium lake surface position (ii) is shown by a thin line; the equilibrium interface position (iii) is shown by a broken line; and the oscillating interface (iv) is shown as a shaded surface. The two diagrams at upper left illustrate a hypothetical distribution of the layers during the application of the wind stress, which set the seiche in motion. B and A respectively indicate the wind-driven surface and return currents in the upper layer, both deflected to the right by the Coriolis force. C indicates the lower layer, the greater part of which has become displaced out of the half-basin shown. Eight phases of one oscillation cycle of the (first mode) internal seiche are shown. Directions of flow in the upper and lower layer are shown by heavy arrows, the nodal amphidromic point P, the only point of zero elevation change, takes the place of the nodal line in the non-rotating model. Around this point, the internal seiche and its surface (out of phase) counterpart rotate clockwise. (From Mortimer 1974)

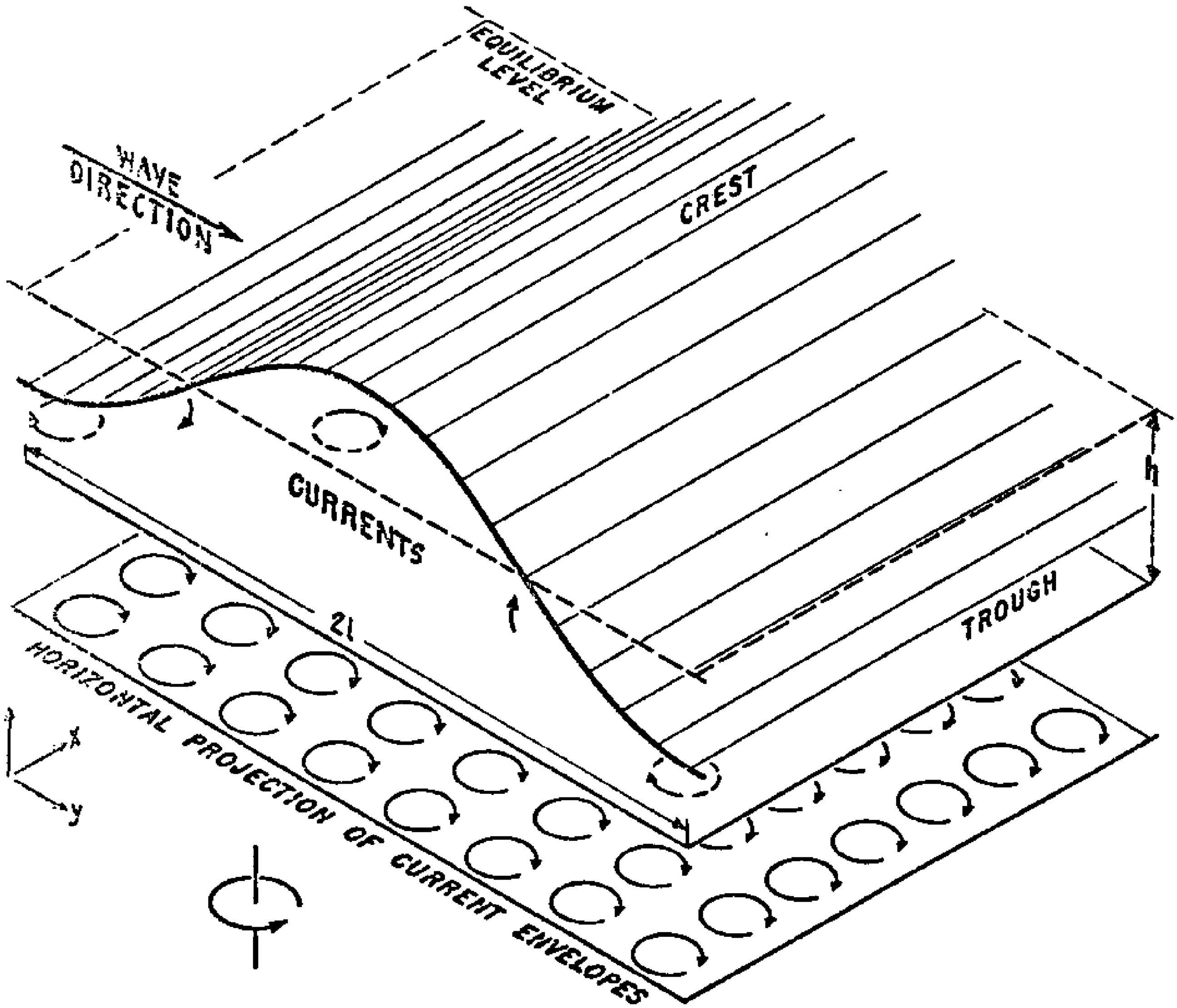
Figure 10 Thermocline movements in Csanady Model Great Lake.




Therinocline movements in Csanady Model Great Lake. Contribution to amplitudes from wave-like and quasi-static modes along diameter perpendicular to wind.

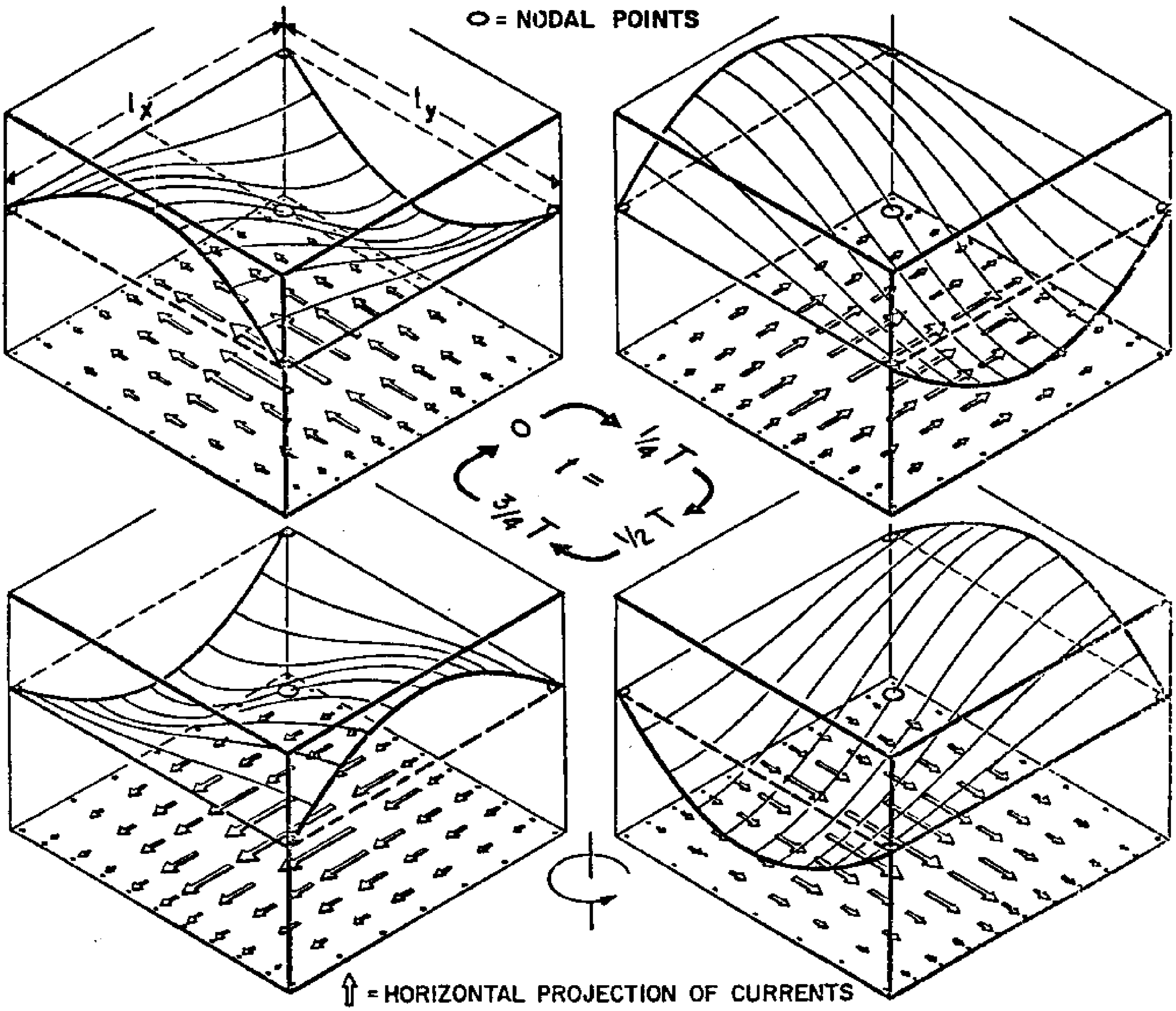
(From G. Csanady 1968)

Figure 11 Representation of one wavelength of a Sverdrup wave in a constant depth (counterclockwise) rotating ocean of infinite extent.



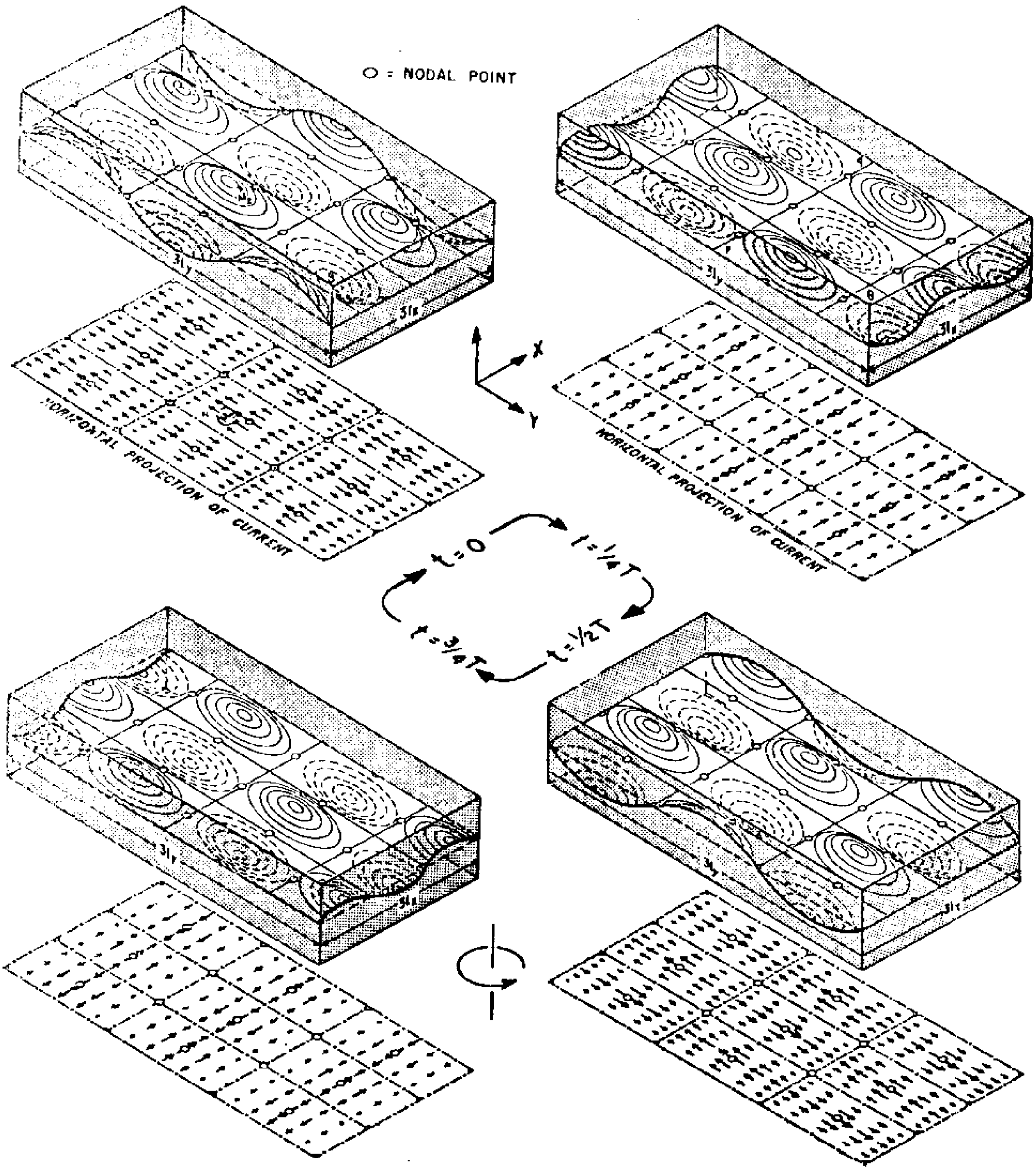

Diagrammatic representation of one wavelength of a Sverdrup wave in a constant-depth, (counterclockwise) rotating ocean of infinite extent. The current directions everywhere undergo one clockwise rotation per wave cycle, and the horizontal projections of the current envelopes (i.e., the track traced out by a water particle) are ellipses. (From Mortimer 1963b)

Figure 12 Four phases in oscillation of a "unit cell" standing Poincaré wave.



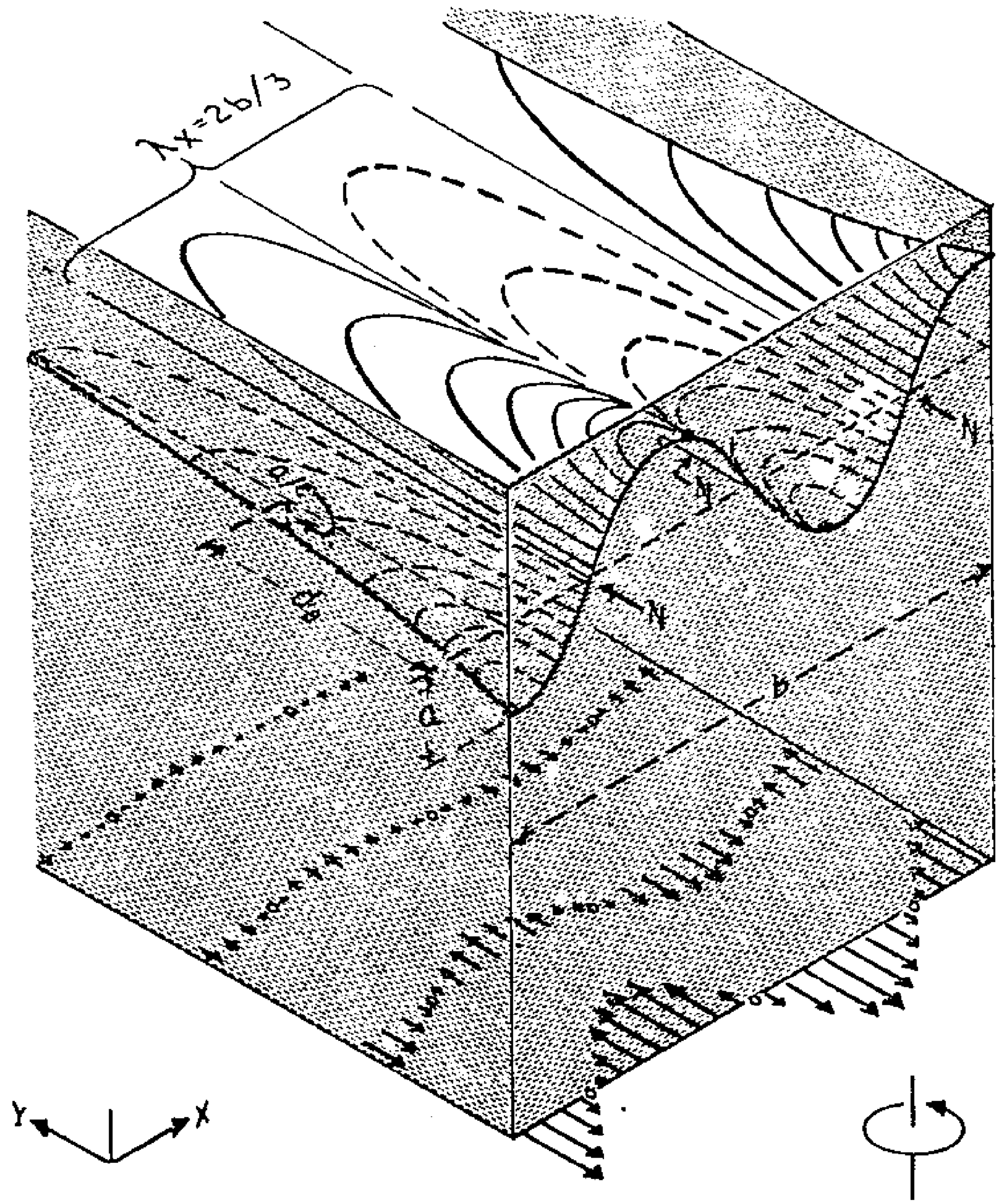

Constant-depth model; counterclockwise rotation;
wave period, T. (From Mortimer 1963b)

Figure 13  Four phases in one cycle of oscillation of a trinodal standing Poincaré wave of long wavelength.




Constant-depth model; counterclockwise rotation; wave period, T. (From Mortimer 1963b)

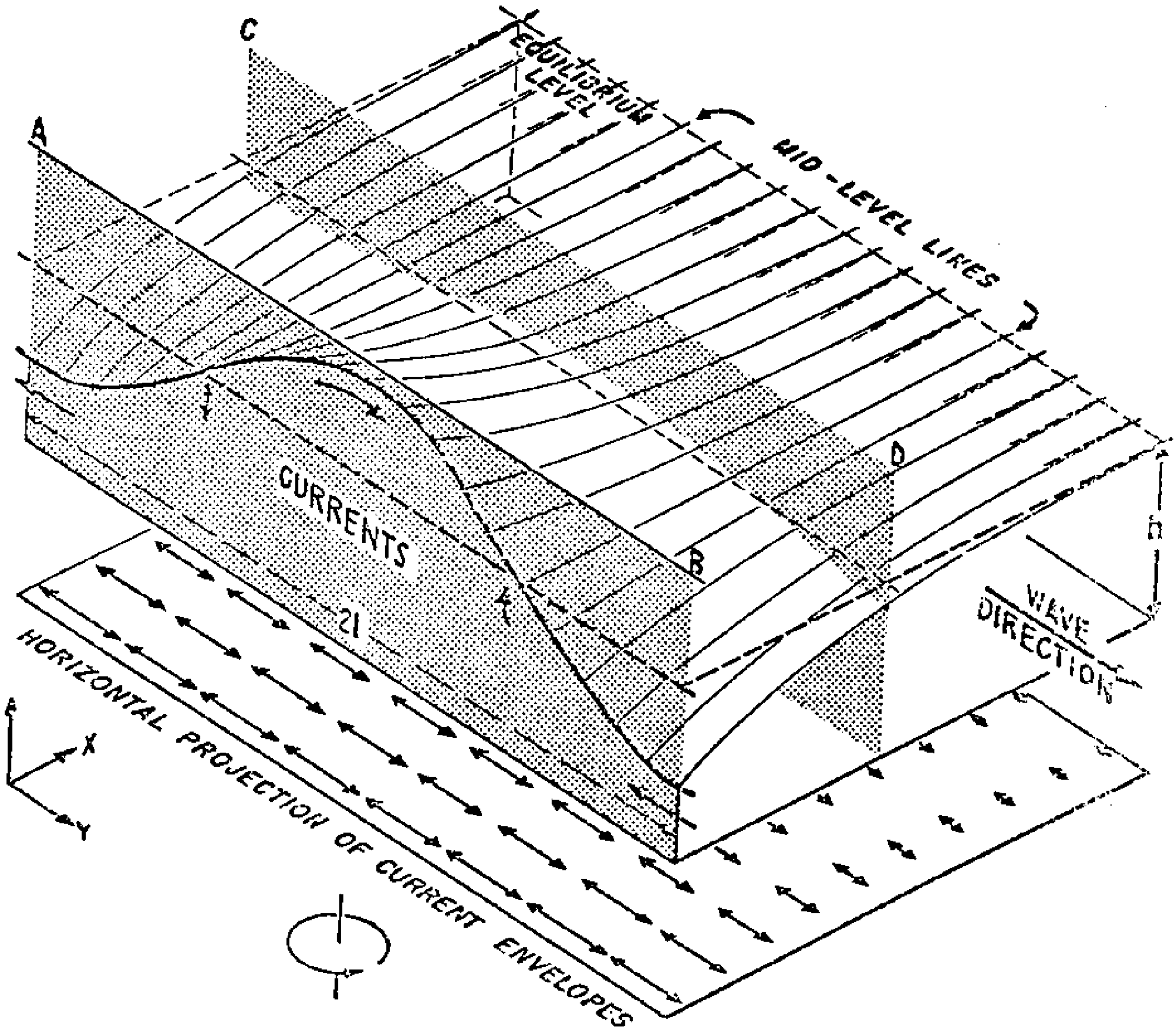
Figure 14   Exponential Poincaré wave solution of transverse nodality 3.




Exponential Poincaré wave solution of transverse nodality $n_x$=3, at a transverse boundary in a basin (or gulf) of width b. The waveform is standing in the x-direction; the amplitude, a at the transverse boundary (y=0), decreases exponentially in the y-direction; $d_e$ is the distance at which amplitude has fallen to a/e. N indicates a nodal line.

The existence of current components, here illustrated normal to the transverse boundary, demonstrated that the single exponential Poincaré wave is not itself a possible solution in the basin (or gulf) model. For this model, the complete solution is a combination of a set of P waves ($n_x$=1 to∞) and a pair of K waves, all of the same frequency, coincident with one of the basin eigenfrequencies, as explained in the text. (From Mortimer 1971)

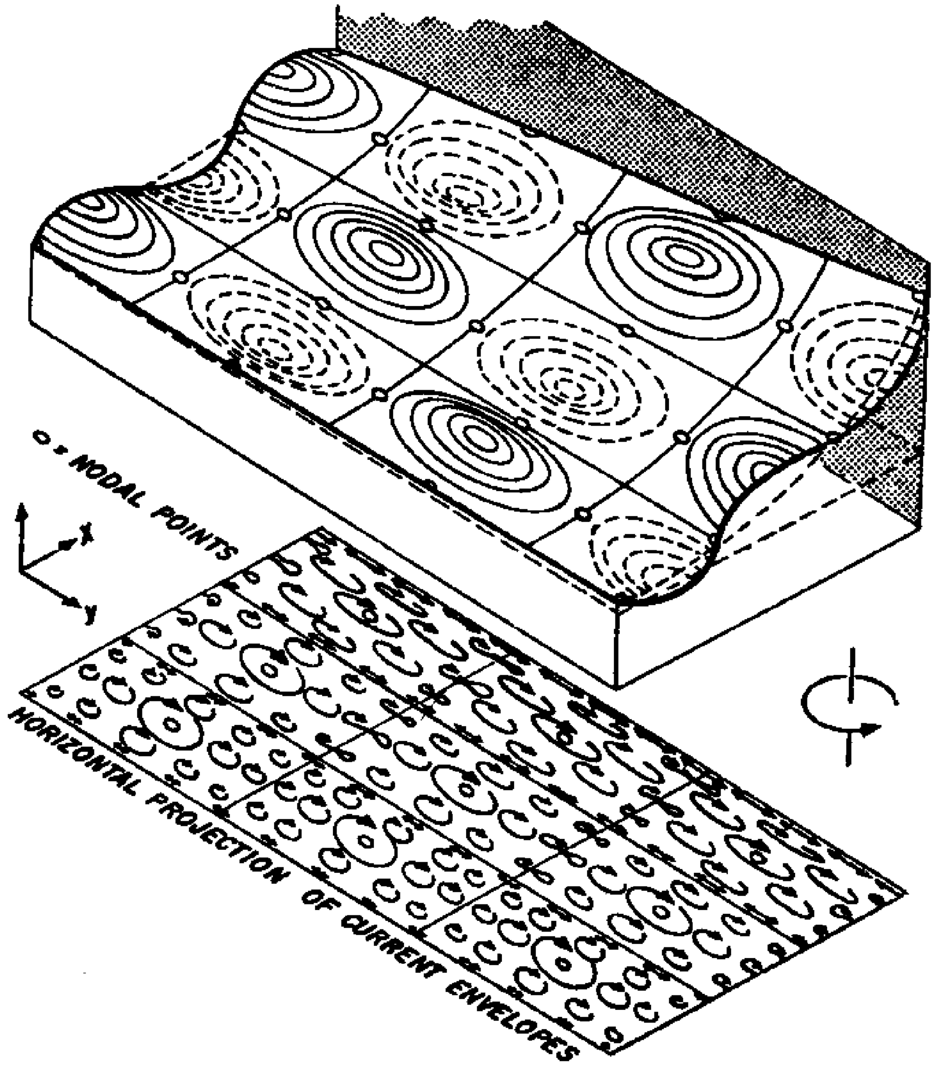
Figure 15 Representation of one wavelength of a Kelvin wave.




Diagrammatic representation of one wavelength of a Kelvin wave travelling along the shore, AB, of a (counterclockwise) rotating, semi-infinite model of constant depth. A barrier, CD, can be inserted parallel to AB without disturbing the wave, to yield a model of a Kelvin wave in a constant-depth channel.

(From Mortimer 1963b)

Figure 16 Representation of the region between an offshore standing Poincaré wave pattern and a nearshore Kelvin wave pattern.

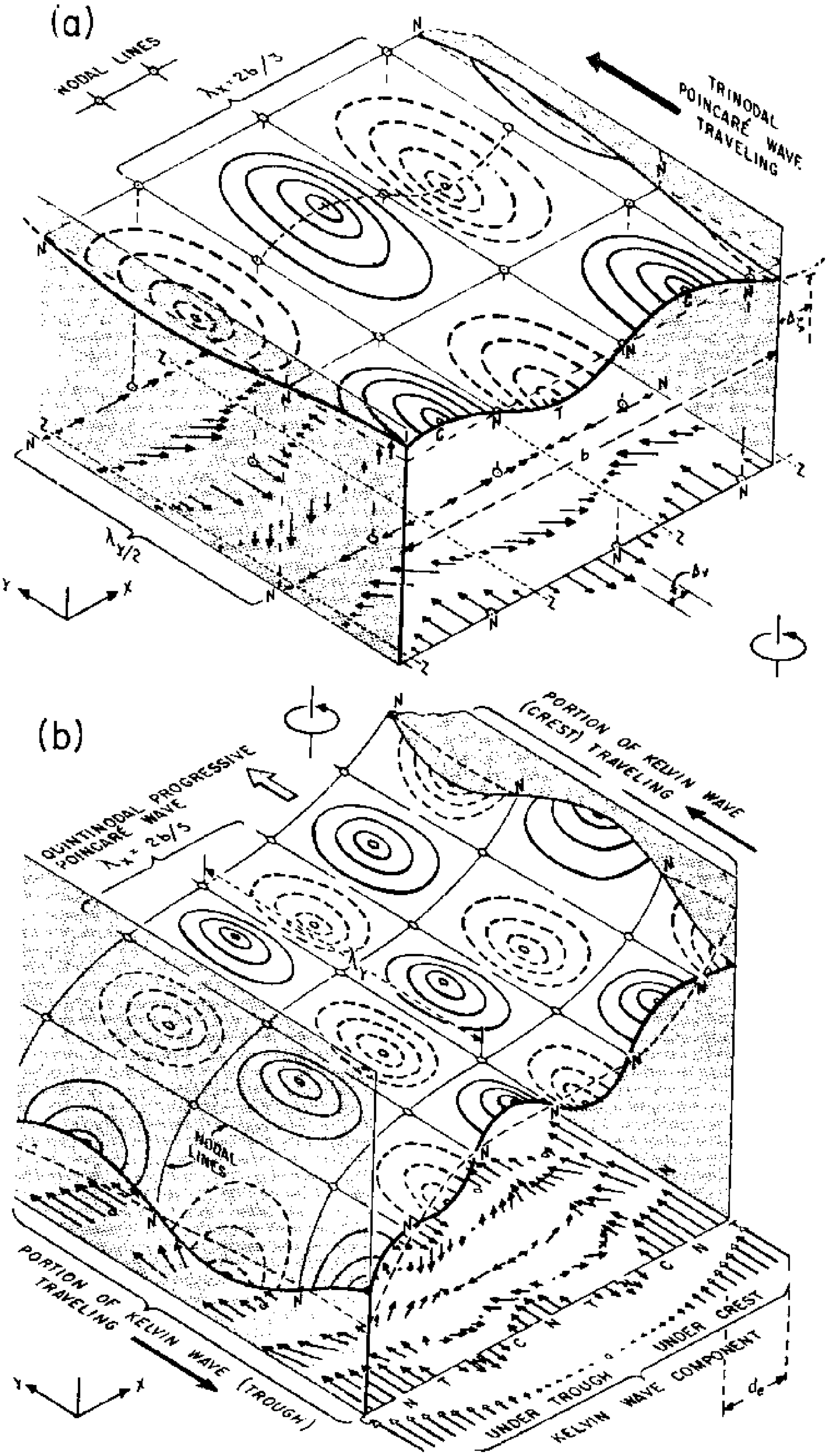



Diagrammatic representation of the combination--in a region of a constant-depth, (counterclockwise) rotating, semi-infinite model--between an "offshore" standing Poincaré wave pattern and an "inshore" Kelvin wave pattern. (From Mortimer 1963b)

Figure 17 Distribution along a channel of wave elevation and current vectors in Poincaré waves:

a) Progressive wave of nodality 3.

b) Progressive wave of nodality 5 combined with a pair of oppositely directed Kelvin waves.




Distribution, along a channel of width b, of wave elevation and current vectors in Poincare waves of $\lambda x/\lambda y=1/2$: (a) a progressive P wave of transverse nodality $n_x=3$; and (b) a progressive P wave of $n_x=5$, combined with a pair of Kelvin waves traveling in opposing directions along the channel sides (at distance $d_e$ from the channel side, the K wave amplitude is 1/e of that at the side). The following letters refer to the P wave: N, nodal lines, longitudinal or transverse; C and T, crests and troughs; Z lines of zero current. (From Mortimer 1971)

Figure 18 Lake Ontario periodic fluctuations in temperature (hourly readings or means) at water intakes of the municipal filtration plants at Toronto, Ontario and Rochester, NY. (From Mortimer 1971)

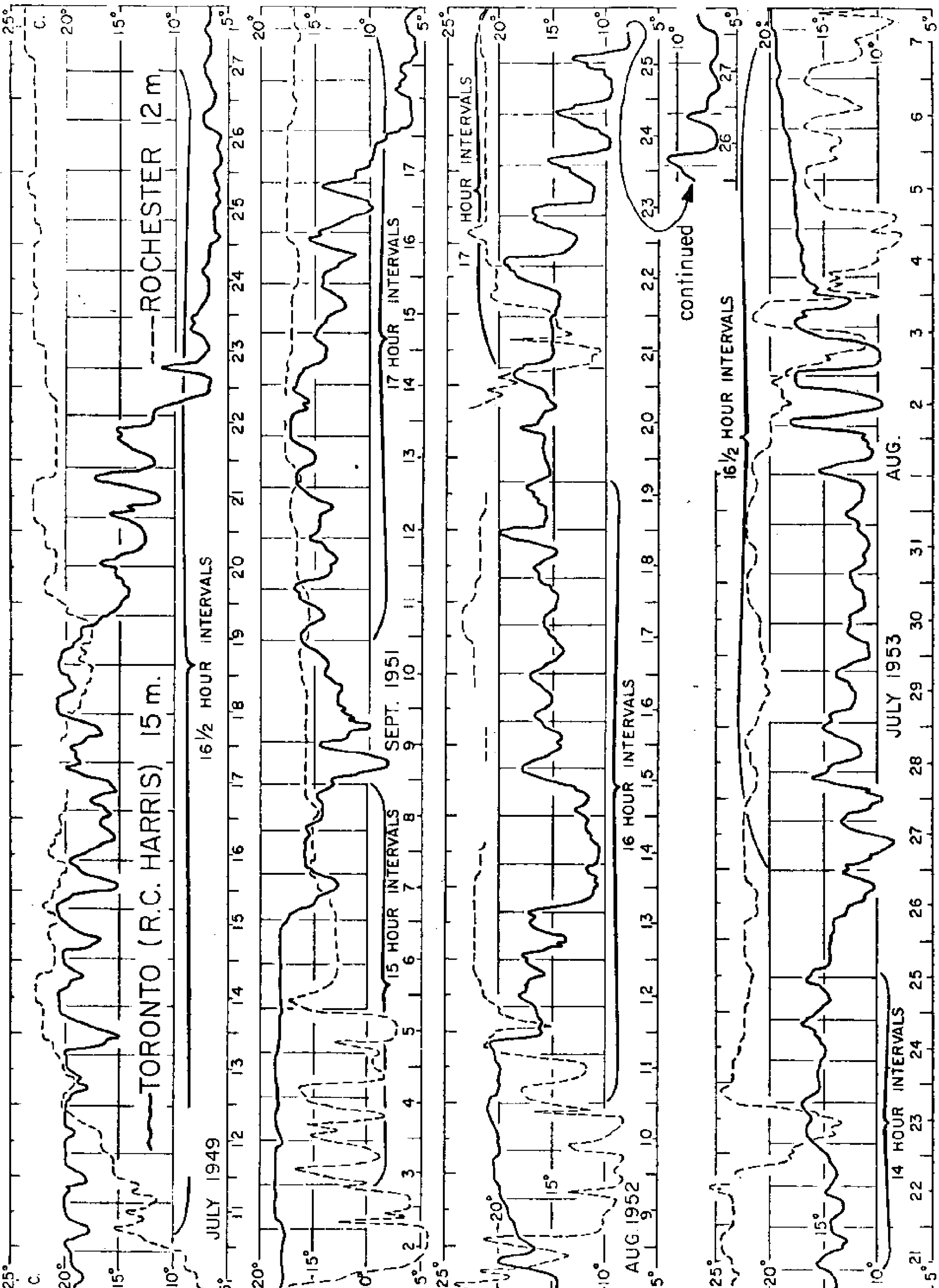
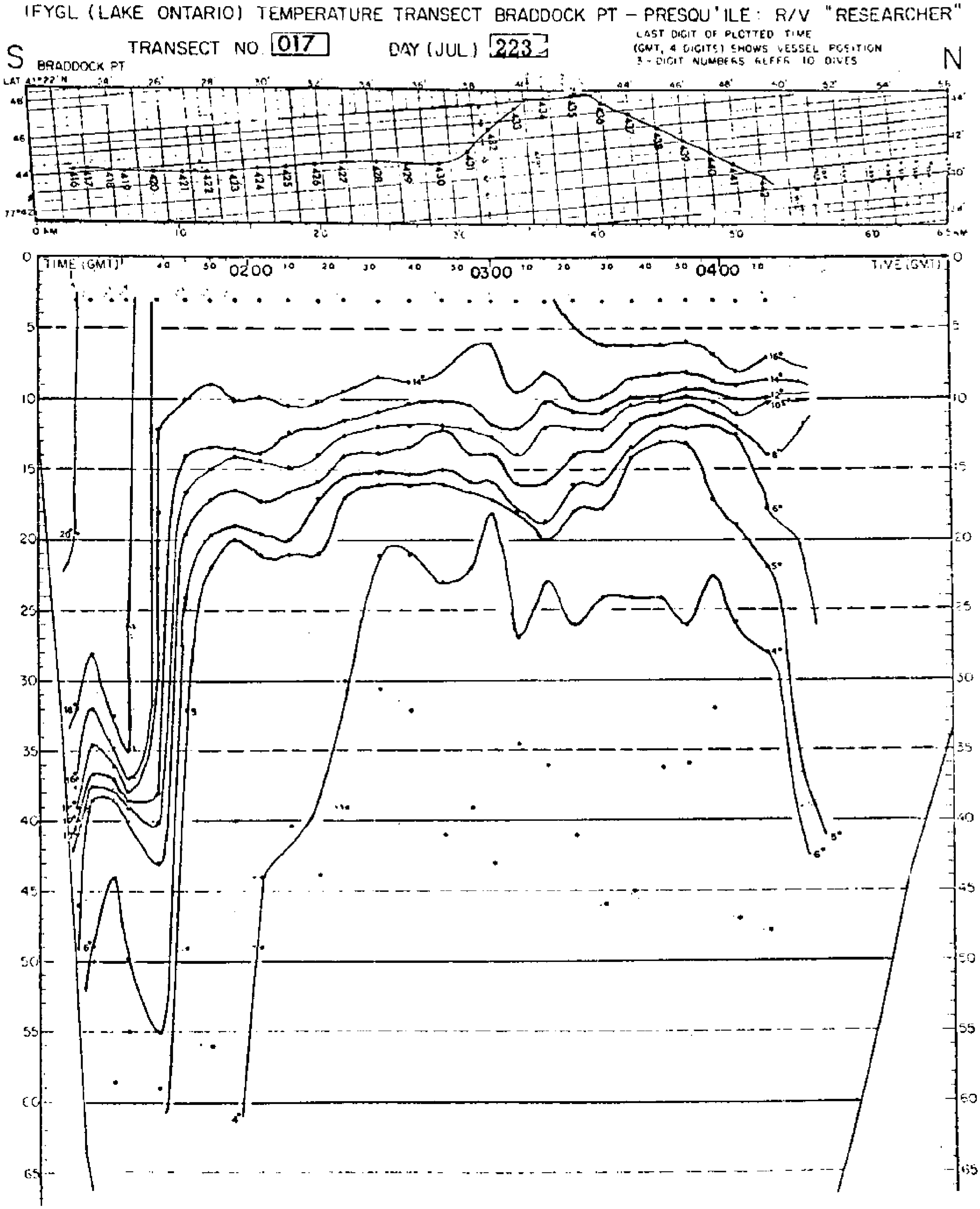
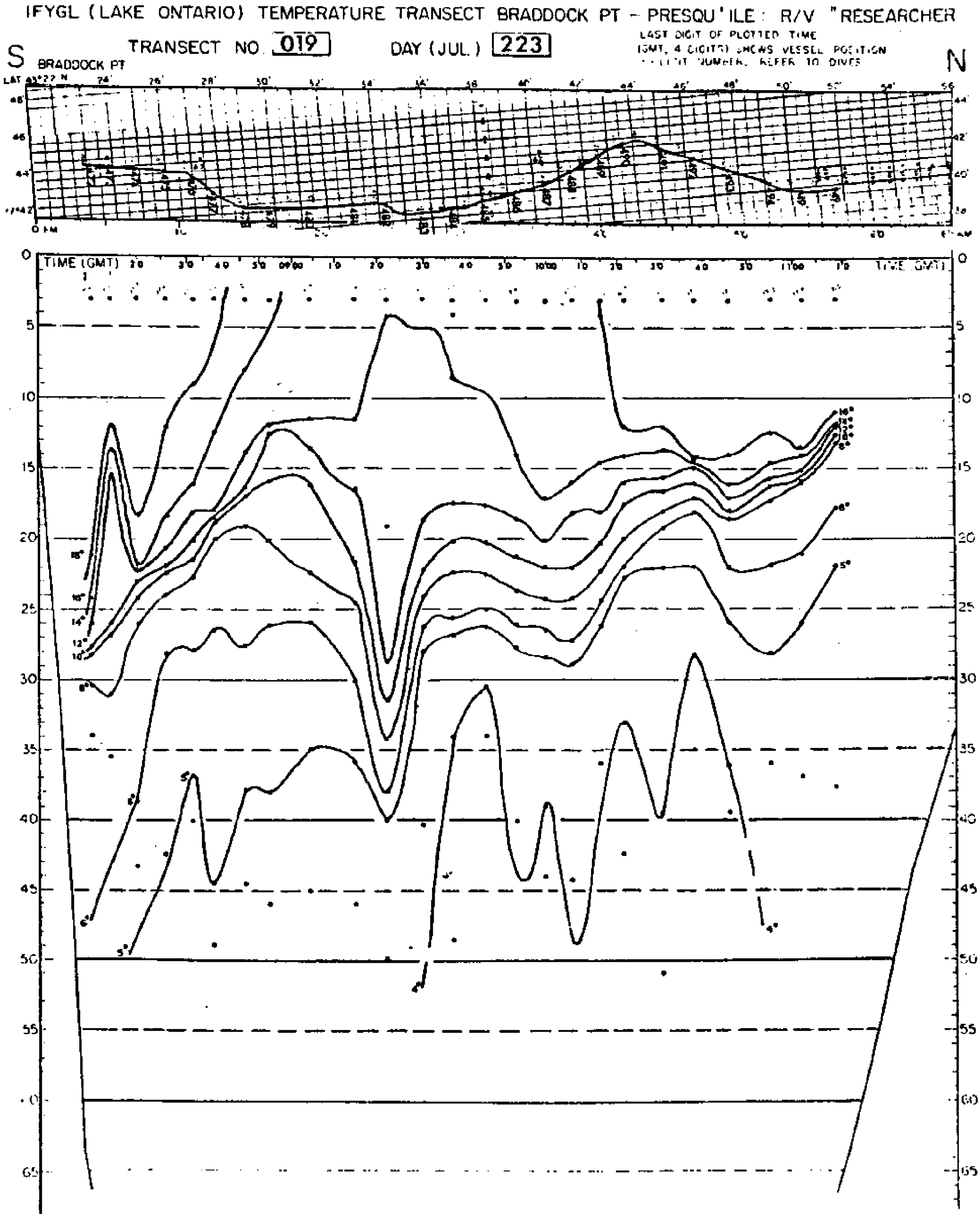


Figure 19 Temperature distribution across transect of Lake Ontario from Braddock Point to Presqu'ile for 10 August 1972, 0100-0400 GMT.



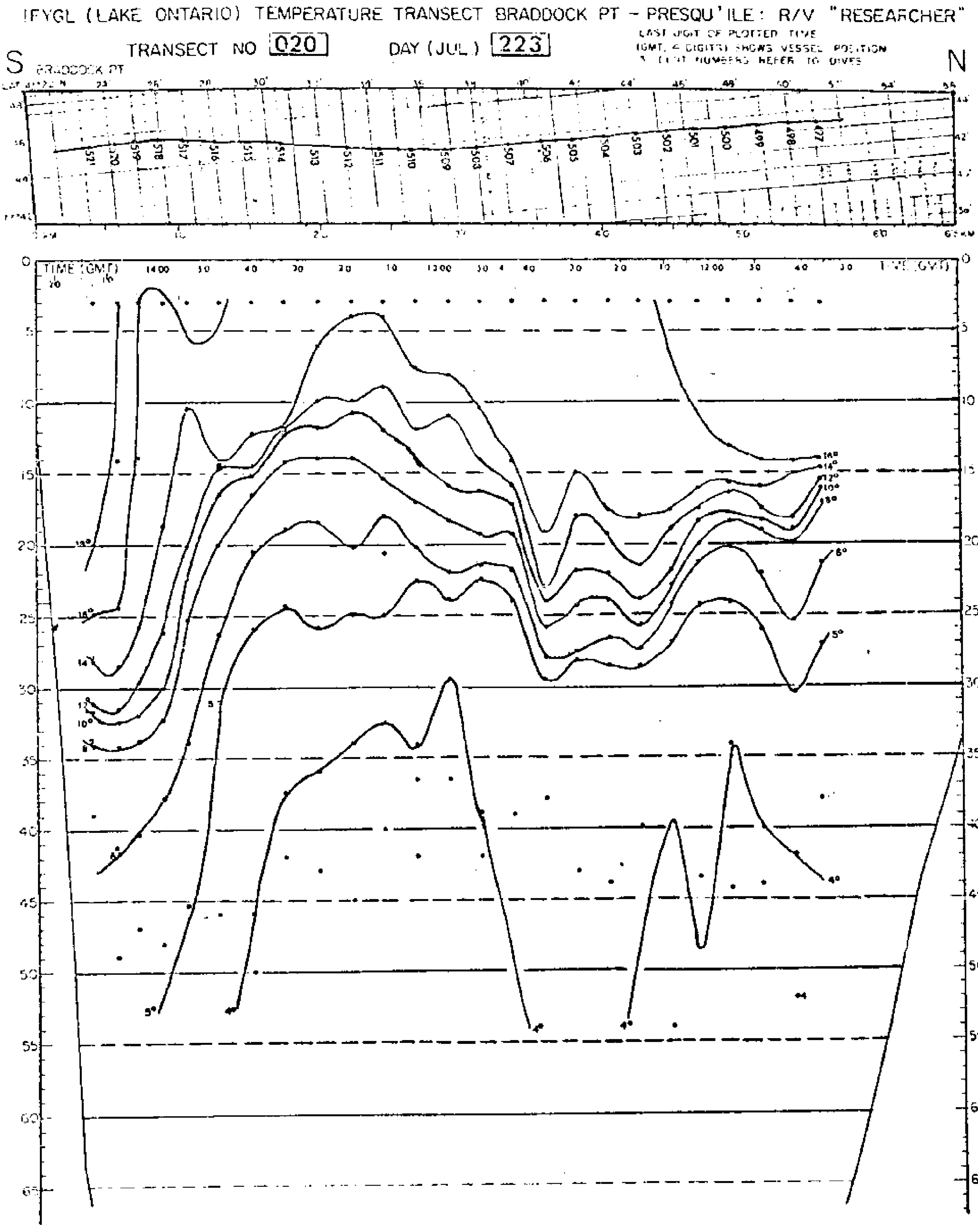

(From Mortimer and Cutchin 1974)

Figure 20  Temperature distribution across transect of Lake Ontario from Braddock Point to Presqu'ile for 10 August 1972, 0800-1100 GMT.

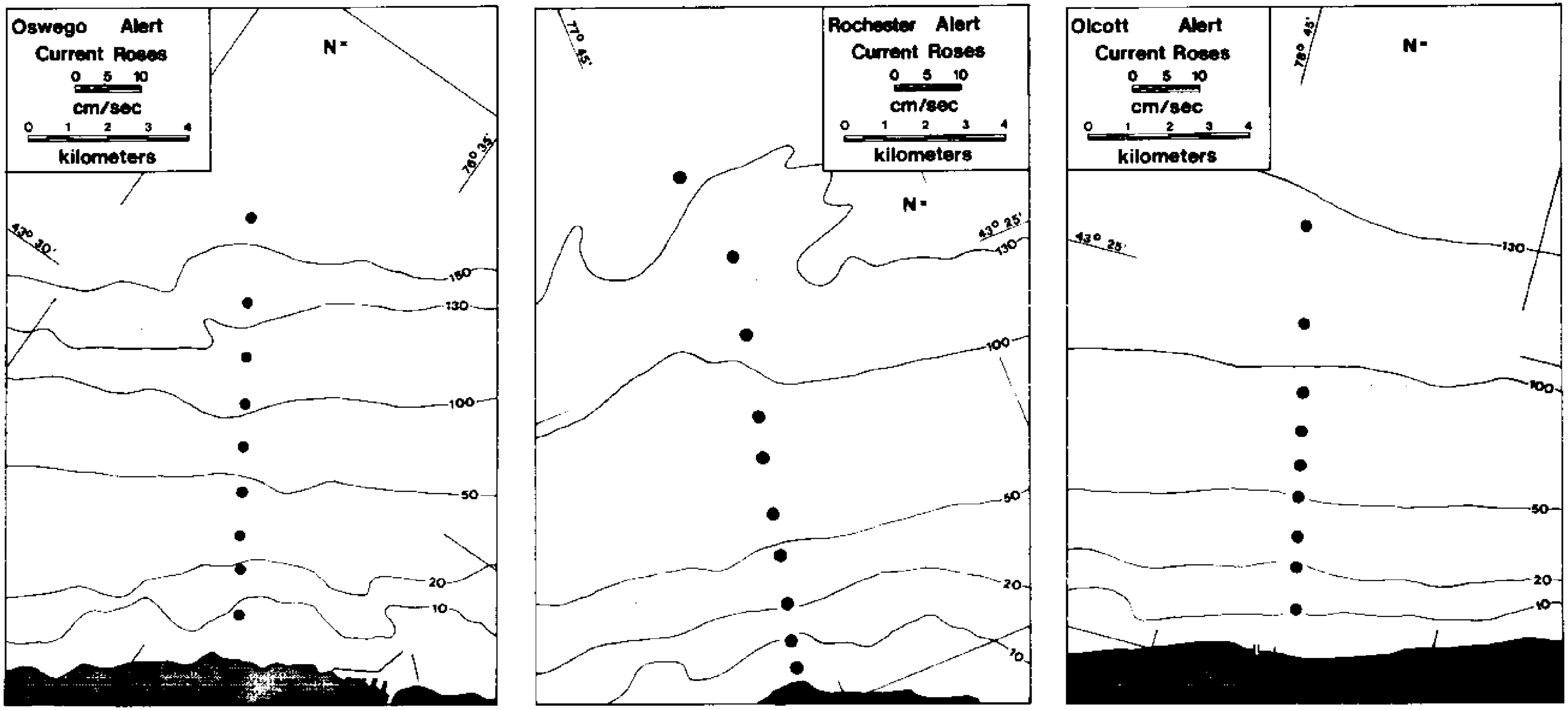



(From Mortimer and Cutchin 1974)

Figure 21 Temperature distribution across transect of Lake Ontario from Braddock Point to Presqu'ile for 10 August 1972, 1130-1430 GMT.




(From Mortimer and Cutchin 1974)




Oswego coastal chain

Rochester coastal chain

Olcott coastal chain

Figure 22 United States Coastal Chains, International Field Year for the Great Lakes, 1972.

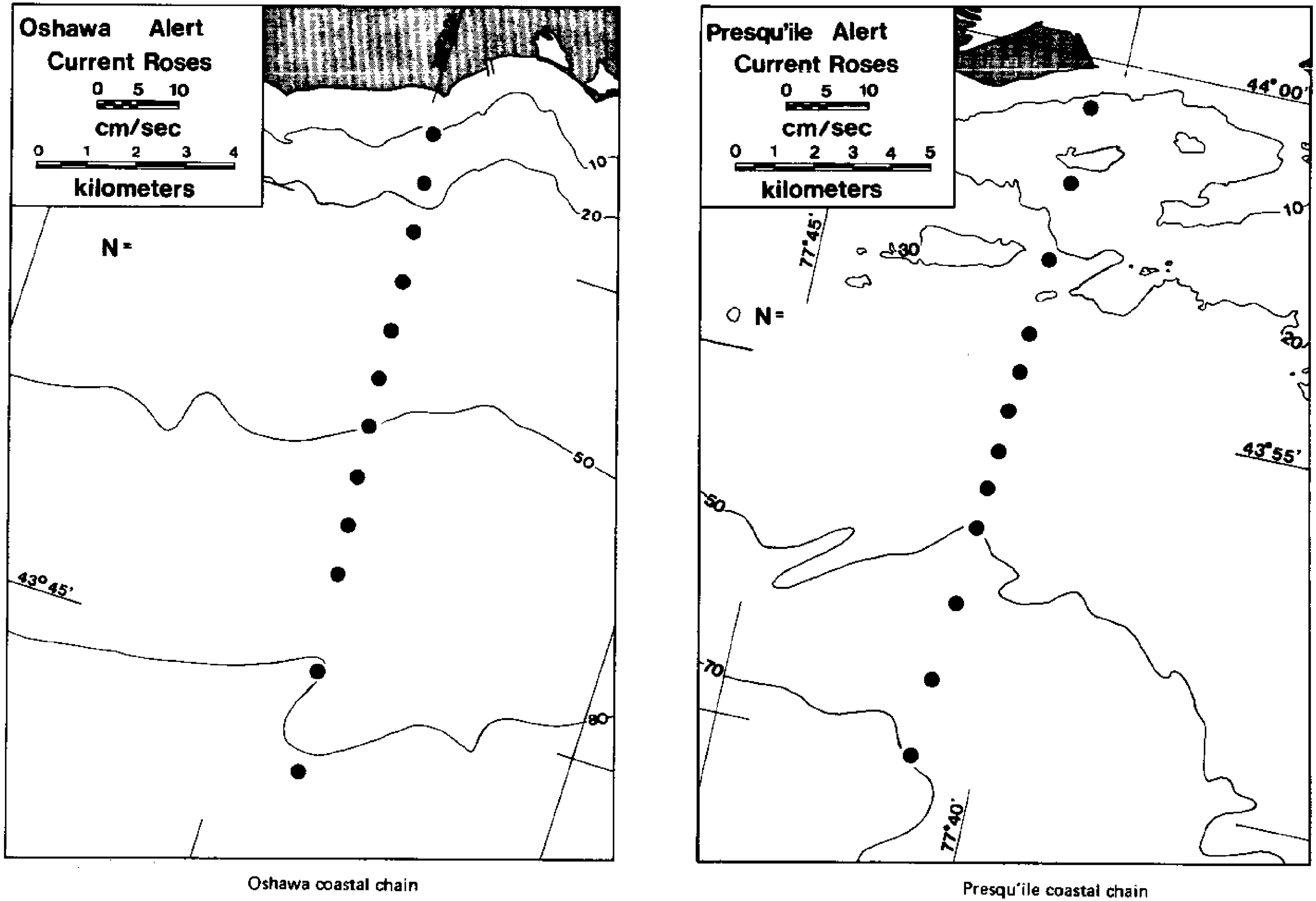
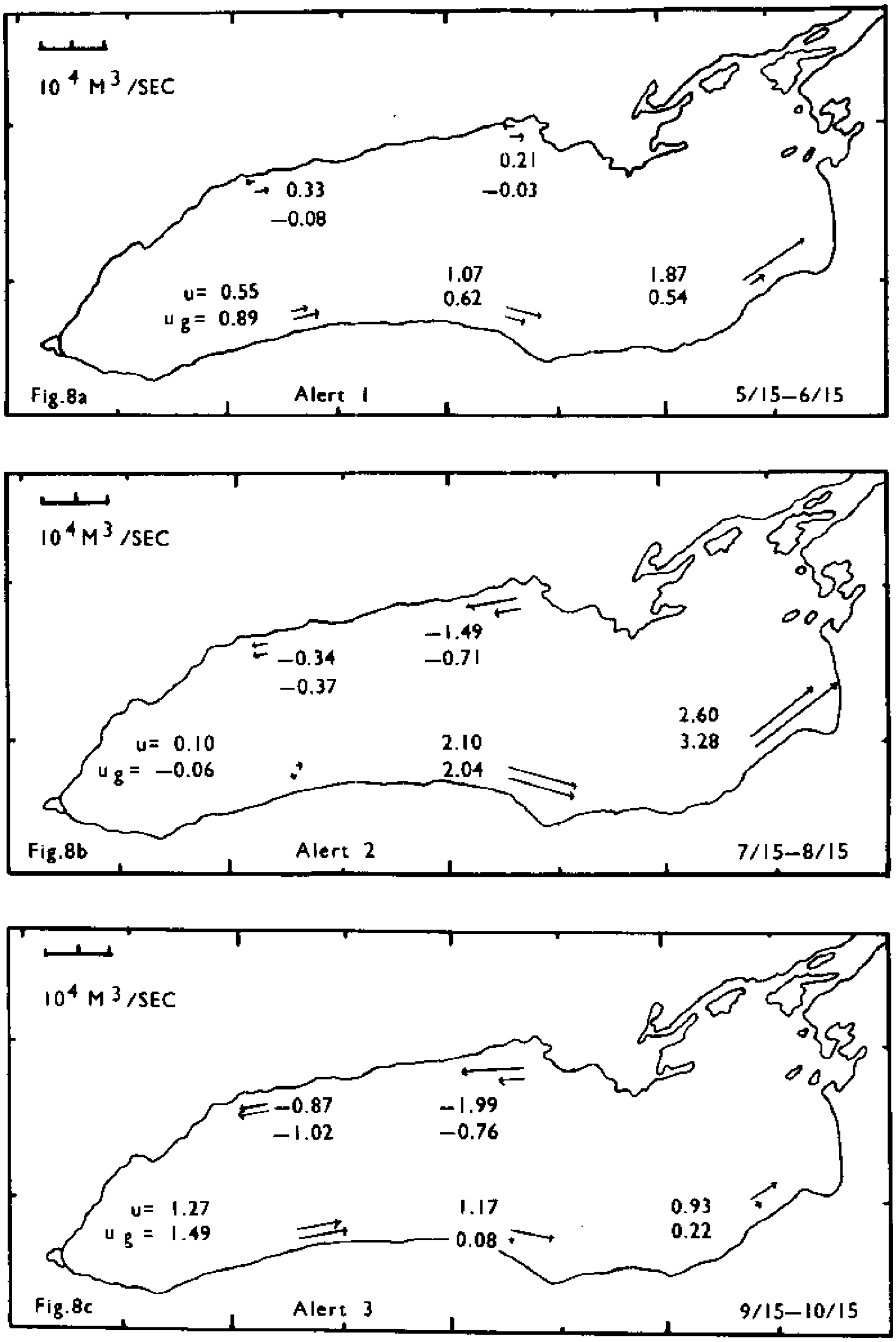


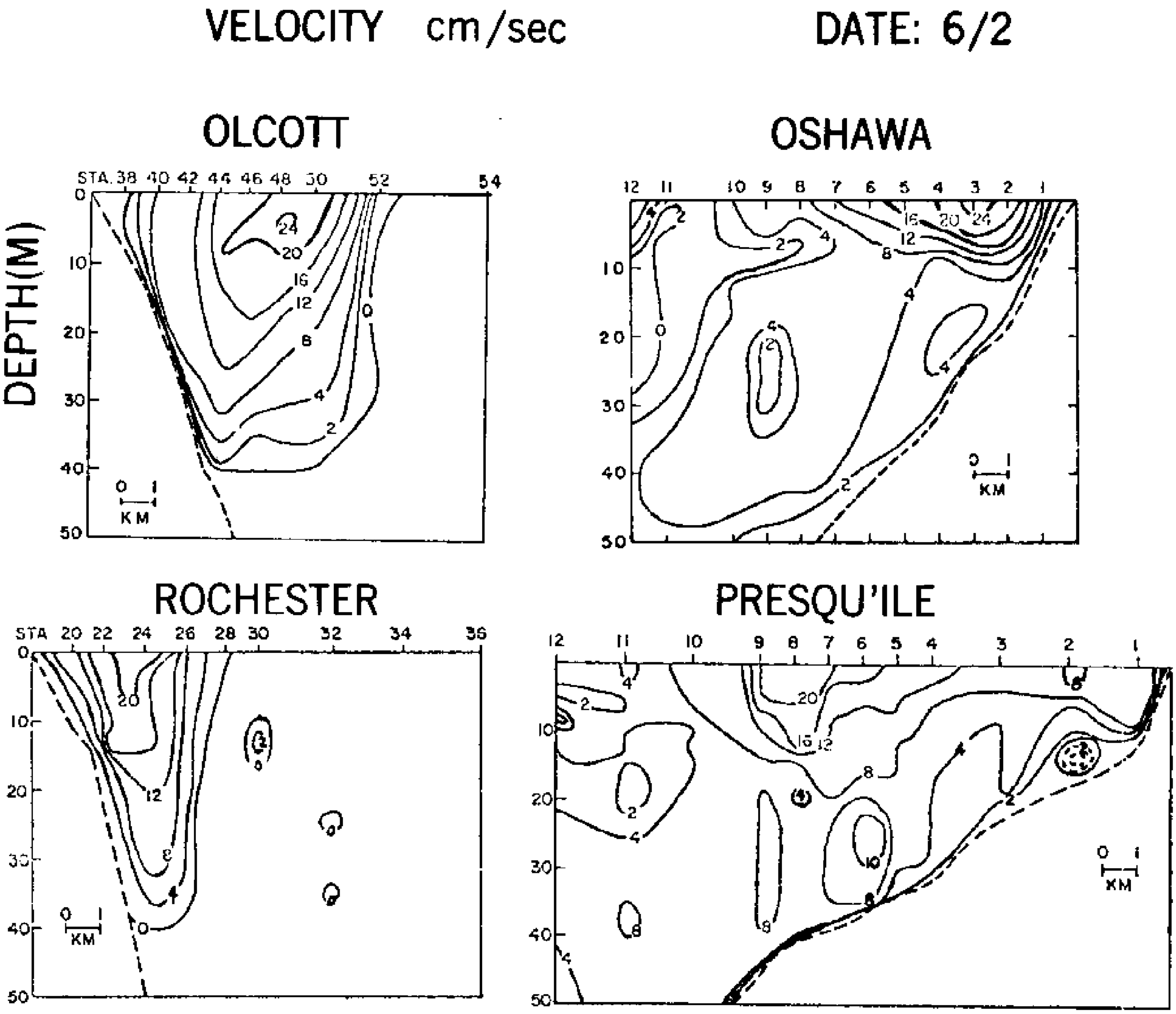

Figure 23 Canadian Coastal Chains, International Field Year for the Great Lakes, 1972.

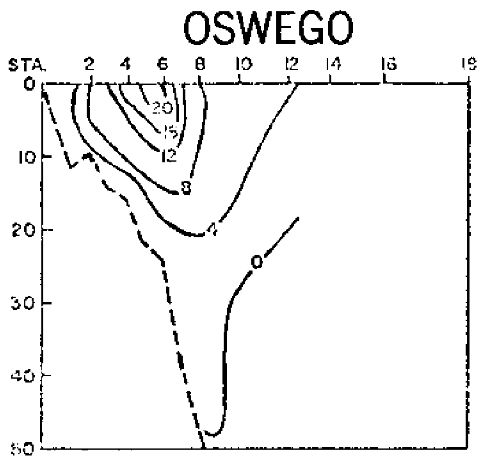
Figure 24 Resultant measured and baroclinic geostrophic transport for all three alerts.




(From Landsberg 1975)

Figure 25a Cross sections of longshore component of velocity for 2 June 1972.



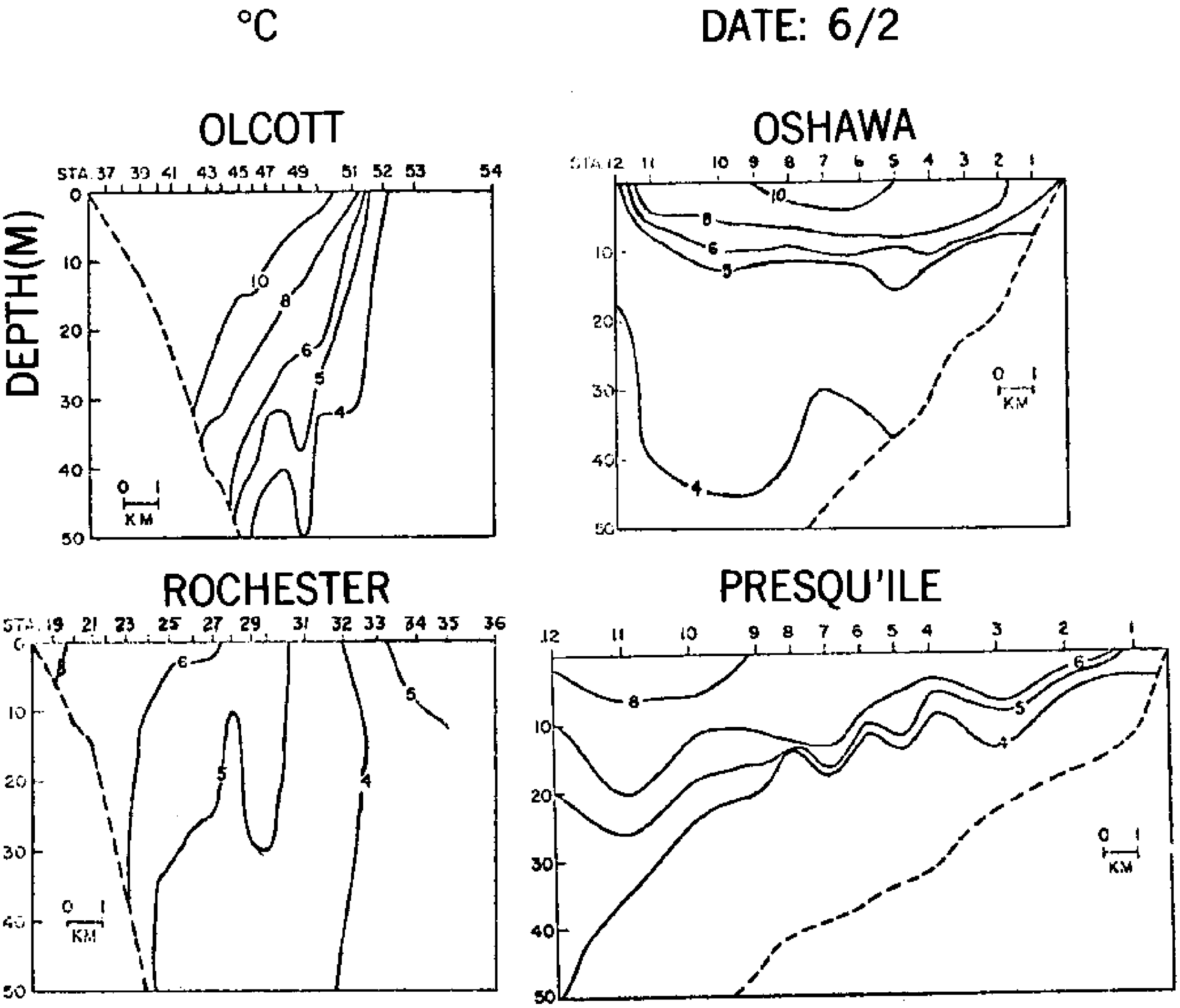

DAILY LONGSHORE VELOCITY TRANSPORT (u)

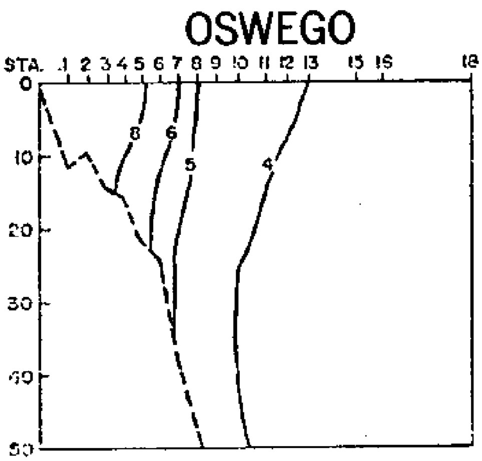
($10^4$ $M^3$/SEC)

| LINE | POS | NEG | TOT |
|---|---|---|---|
| OSWEGO - 1 | 0.84 | 0.0 | 0.84[6] |
| 2 | 0.73 | 0.0 | 0.73[7] |
| ROCHESTER | 1.04 | -0.01 | 1.03[7] |
| OLCOTT - 1 | 2.34 | -0.02 | 2.31[8] |
| 2 | 1.58 | -0.02 | 1.56 |
| OSHAWA | 2.52 | -0.01 | 2.51 |
| PRESQU'ILE | 4.15 | -0.03 | 4.11 |

(From Landsberg 1975)

Figure 25b Cross sections of temperature for 2 June 1972.



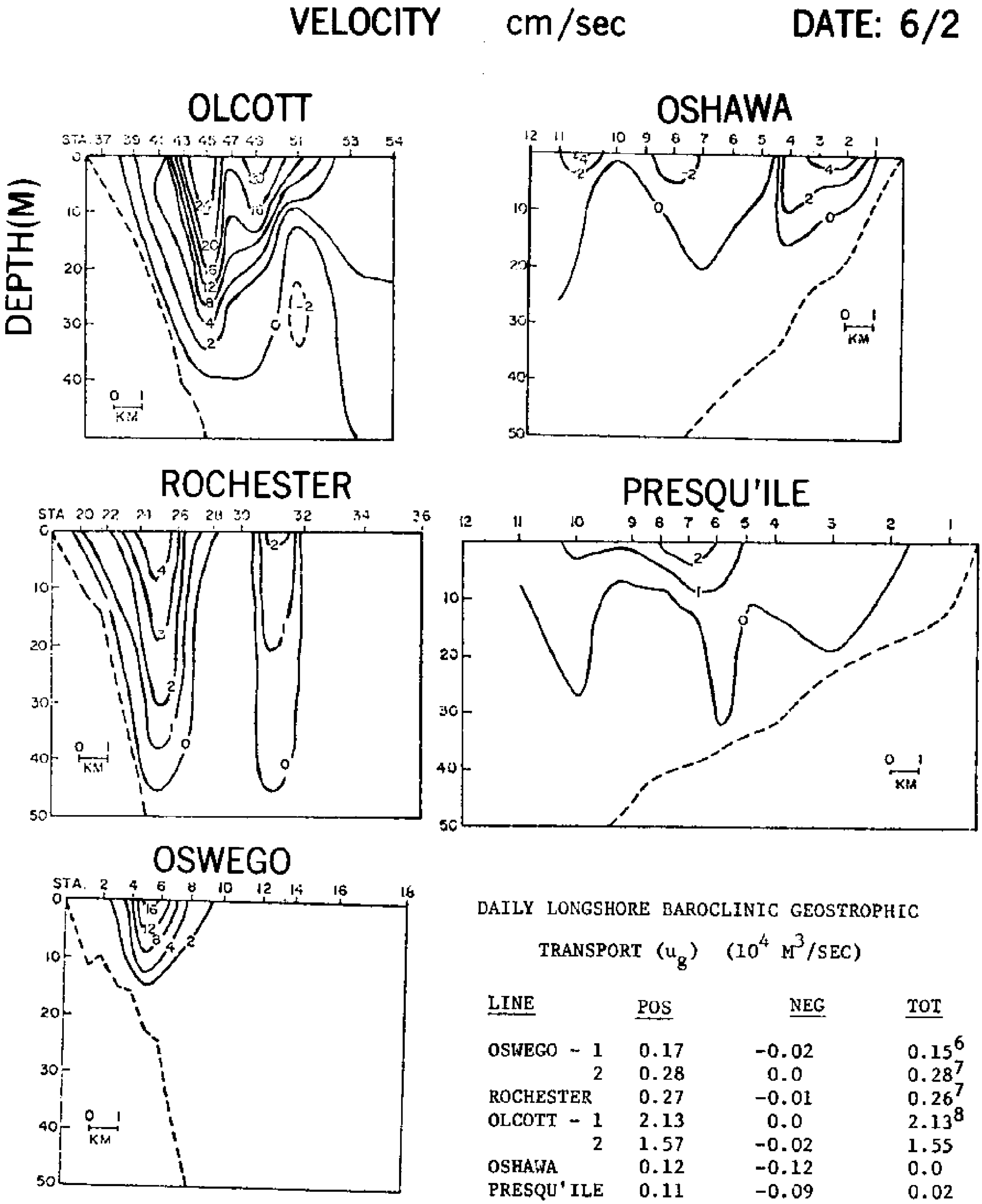

DAILY LONGSHORE TRANSPORT DIFFERENCE $(u-u_g)$

$(10^4\ M^3/SEC)$

| LINE | POS | NEG | TOT |
|---|---|---|---|
| OSWEGO - 1 | 0.70 | 0.0 | $0.70^{6}$ |
| 2 | 0.45 | 0.0 | $0.45^{7}$ |
| ROCHESTER | 0.77 | 0.0 | $0.78^{7}$ |
| OLCOTT - 1 | 0.21 | -0.02 | $0.18^{8}$ |
| 2 | 0.01 | 0.0 | 0.01 |
| OSHAWA | 2.40 | 0.11 | 2.52 |
| PRESQU'ILE | 4.04 | 0.06 | 4.10 |

(From Landsberg 1975)

Figure 25c Cross sections of longshore baroclinic geostrophic velocity for 2 June 1972.

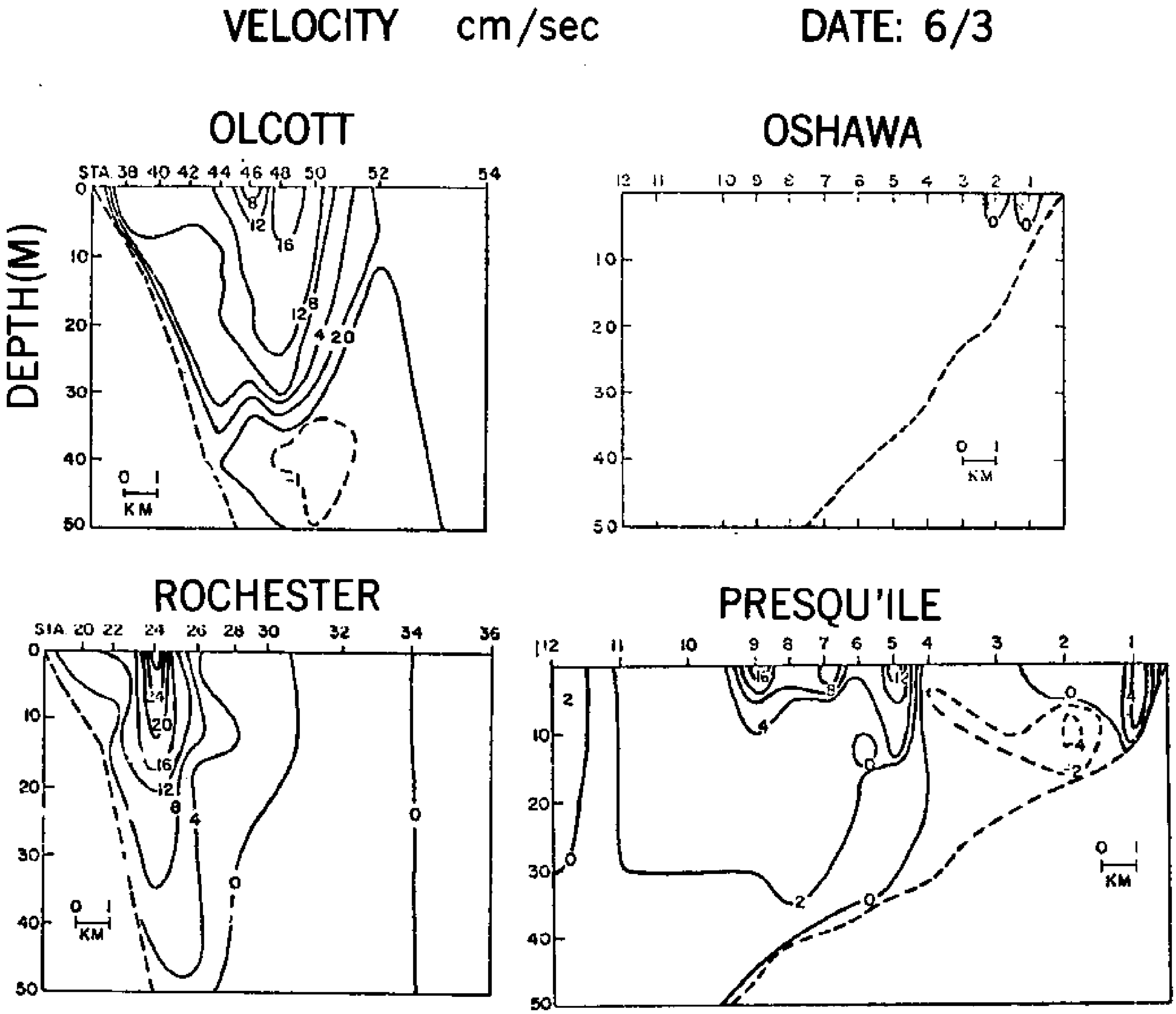


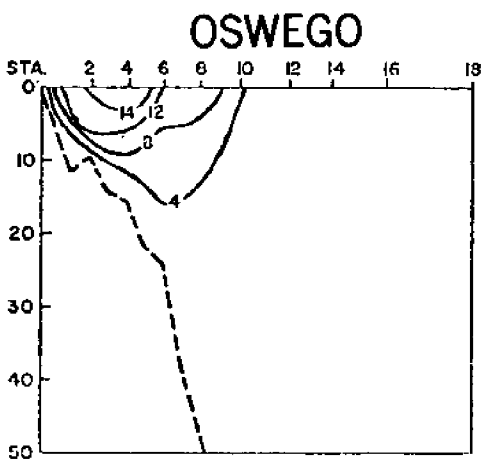

DAILY LONGSHORE BAROCLINIC GEOSTROPHIC TRANSPORT ($u_g$) ($10^4$ $M^3$/SEC)

| LINE | POS | NEG | TOT |
|---|---|---|---|
| OSWEGO - 1 | 0.17 | -0.02 | 0.15[6] |
| 2 | 0.28 | 0.0 | 0.28[7] |
| ROCHESTER | 0.27 | -0.01 | 0.26[7] |
| OLCOTT - 1 | 2.13 | 0.0 | 2.13[8] |
| 2 | 1.57 | -0.02 | 1.55 |
| OSHAWA | 0.12 | -0.12 | 0.0 |
| PRESQU'ILE | 0.11 | -0.09 | 0.02 |

(From Landsberg 1975)

Figure 26a Cross sections of longshore component of velocity for 3 June 1972.



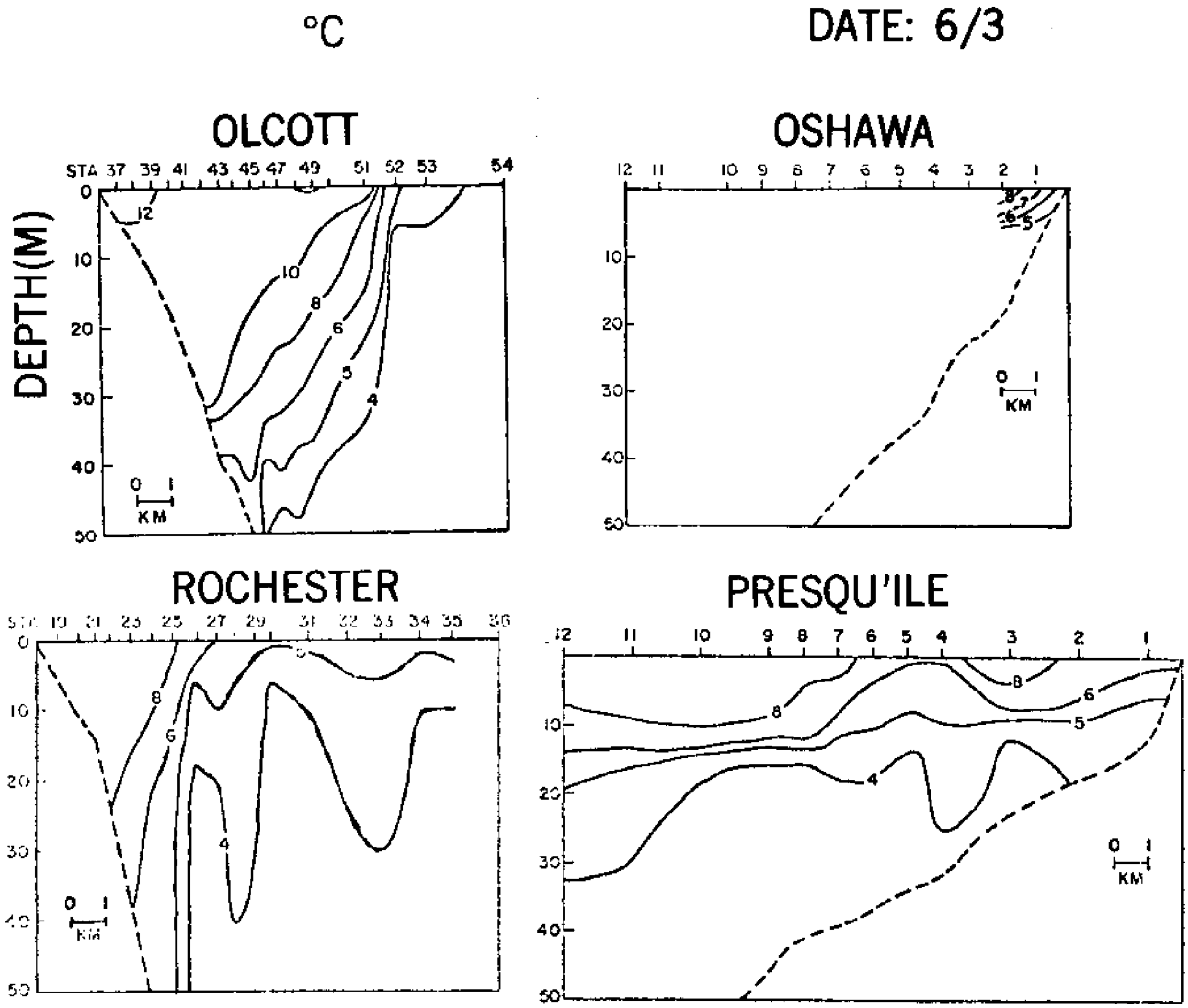

DAILY LONGSHORE VELOCITY TRANSPORT (u)

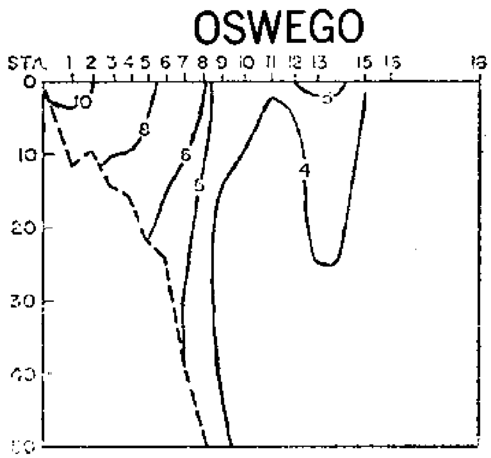
($10^4$ $M^3$/SEC)

| LINE | POS | NEG | TOT |
|---|---|---|---|
| OSWEGO - 1 | 0.76 | 0.0 | 0.76[7] |
| 2 | 0.72 | 0.0 | 0.72[7] |
| ROCH. - 1 | 1.35 | 0.0 | 1.35[8] |
| 2 | 1.35 | 0.0 | 1.35[8] |
| OLCOTT - 1 | 1.75 | -0.05 | 1.70 |
| 2 | 1.80 | 0.0 | 1.80[8] |
| OSHAWA | 0.0 | 0.0 | 0.0 [2] |
| PRESQU'ILE | 1.10 | -0.28 | 0.82 |

(From Landsberg 1975)

Figure 26b Cross sections of temperature for 3 June 1972.



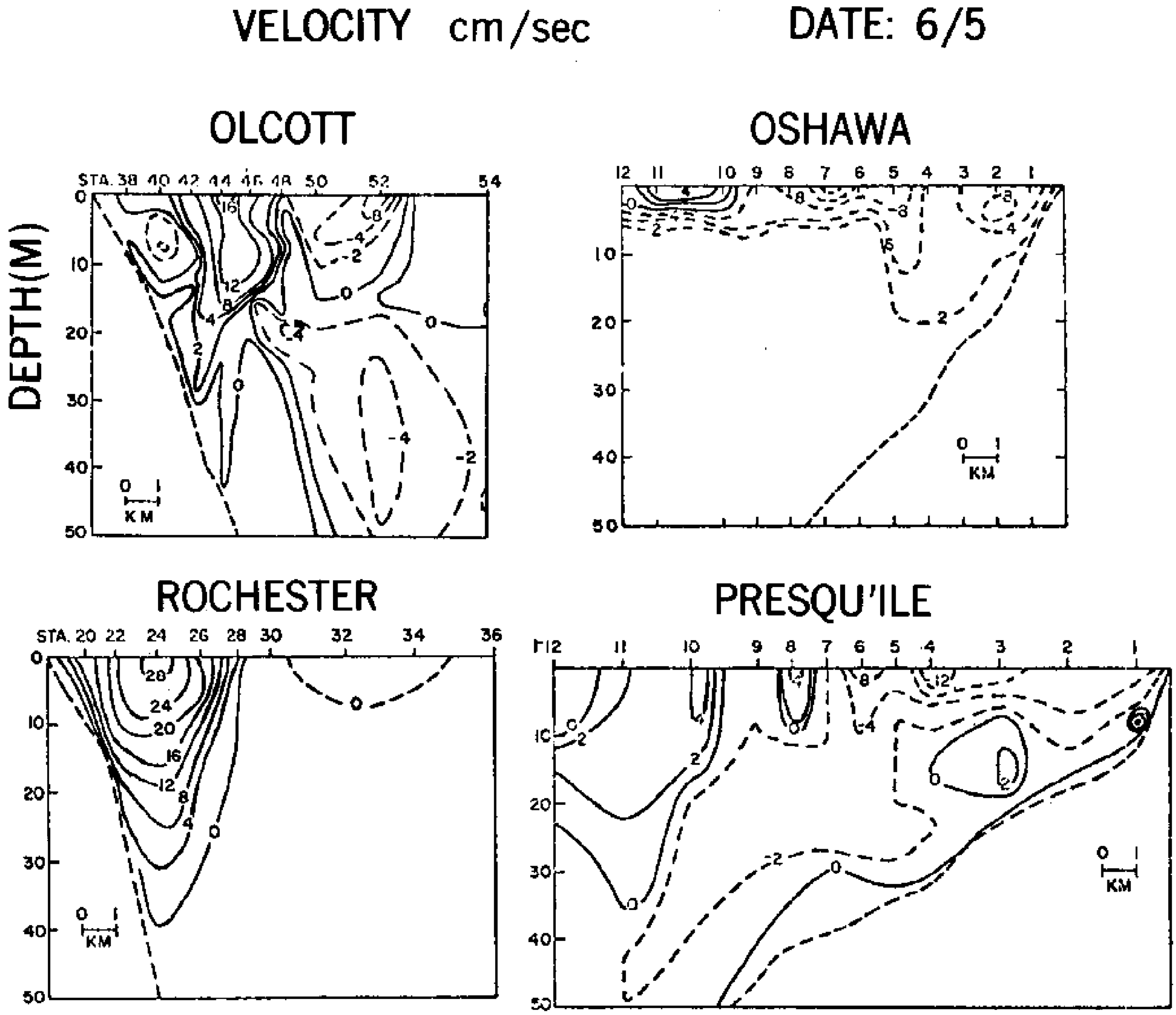

DAILY LONGSHORE TRANSPORT DIFFERENCE $(u-u_g)$

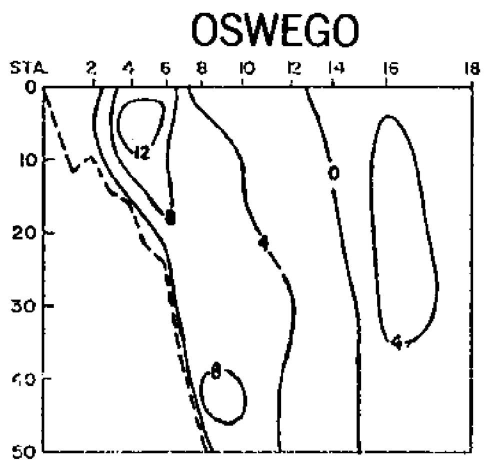
$(10^4\ M^3/SEC)$

| LINE | POS | NEG | TOT |
|---|---|---|---|
| OSWEGO - 1 | 0.47 | 0.01 | 0.47[7] |
| 2 | 0.51 | 0.0 | 0.51[7] |
| ROCH. - 1 | 0.84 | 0.01 | 0.84[8] |
| 2 | 0.66 | 0.01 | 0.67[8] |
| OLCOTT - 1 | -0.17 | -0.04 | -0.22 |
| 2 | 0.47 | 0.0 | 0.47[8] |
| OSHAWA | -0.01 | 0.0 | -0.01[2] |
| PRESQU'ILE | 0.94 | -0.23 | 0.71 |

(From Landsberg 1975)

Figure 27a Cross sections of longshore component of velocity for 5 June 1972.



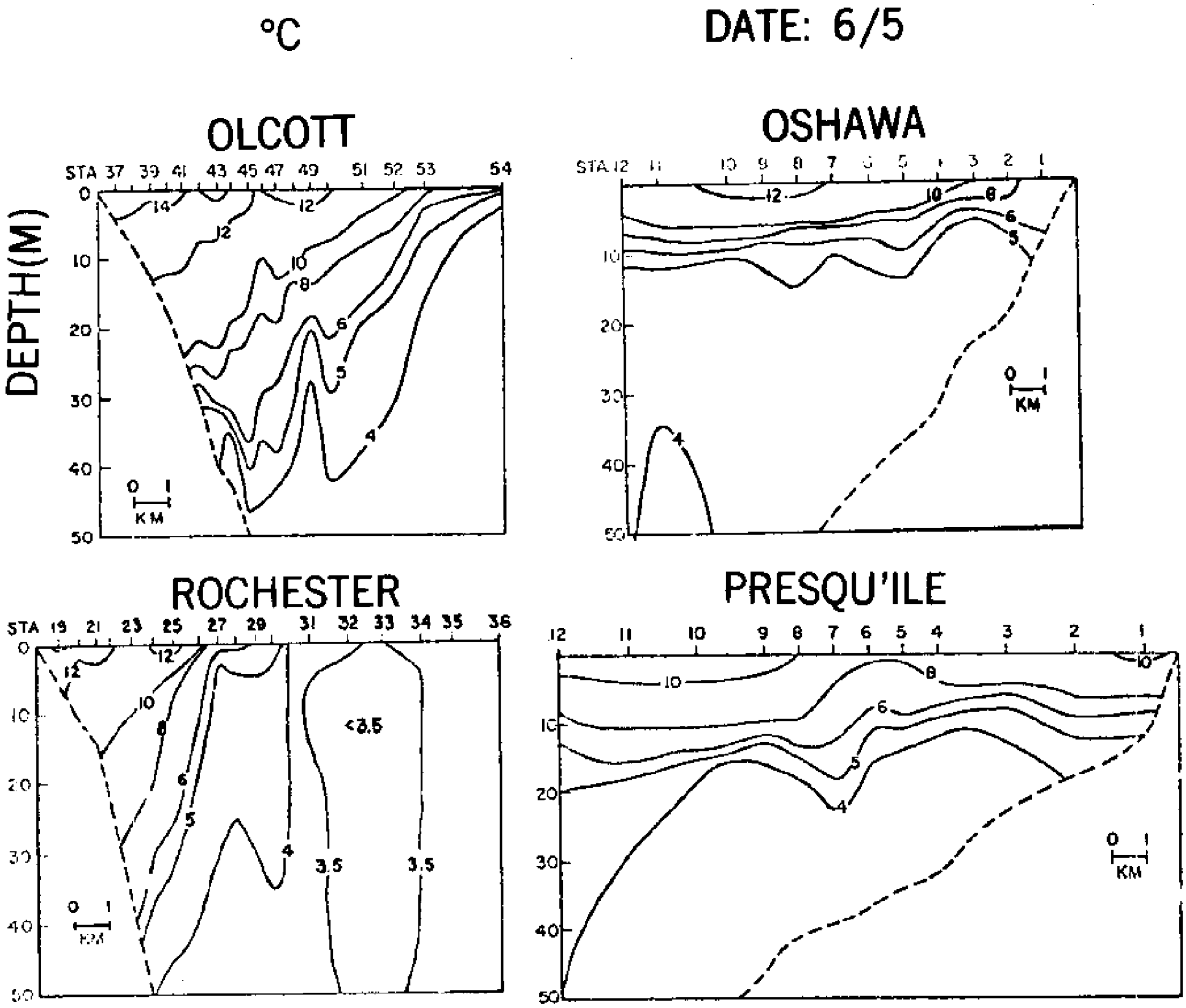

DAILY LONGSHORE VELOCITY TRANSPORT (u)

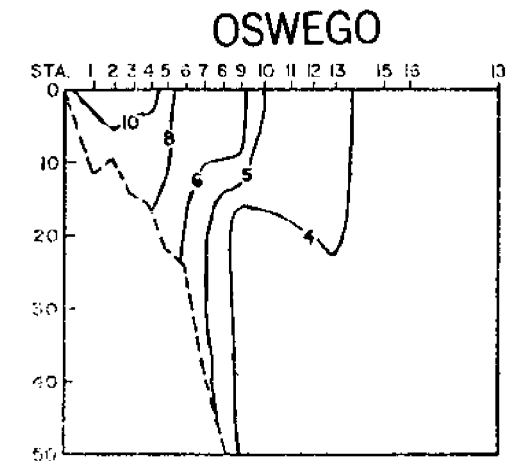
($10^4$ $M^3$/SEC)

| LINE | POS | NEC | TOT |
|---|---|---|---|
| OSWEGO - 1 | 1.37 | 0.0 | 1.37[8] |
| 2 | 1.30 | 0.0 | 1.30[8] |
| ROCH. - 1 | 1.44 | -0.02 | 1.42[8] |
| 2 | 1.19 | -0.02 | 1.16 |
| OLCOTT - 1 | 0.54 | -0.53 | 0.02 |
| 2 | 0.37 | -0.25 | 0.12 |
| OSHAWA | 0.04 | -0.53 | -0.49 |
| PRESQU'ILE | 0.40 | -0.89 | -0.49 |

(From Landsberg 1975)

Figure 27b Cross sections of temperature for 5 June 1972.



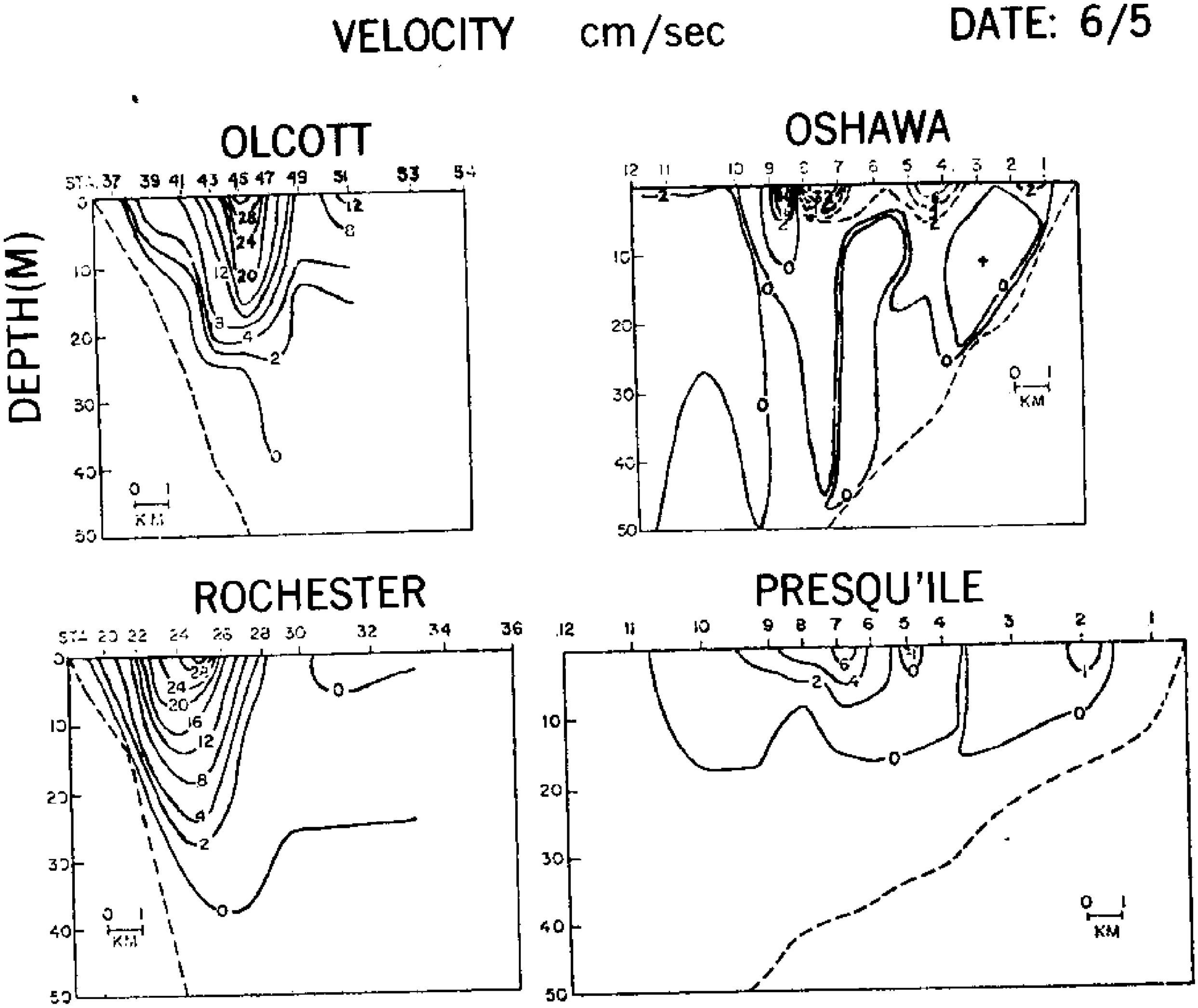

DAILY LONGSHORE TRANSPORT DIFFERENCE ($u-u_g$)

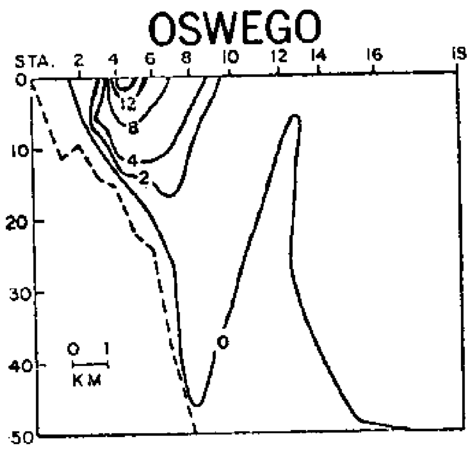
($10^4$ $M^3$/SEC)

| LINE | | POS | NEG | TOT |
|---|---|---|---|---|
| OSWEGO - | 1 | 1.02 | 0.01 | $1.03^{8}$ |
| | 2 | 1.01 | 0.03 | $1.04^{8}$ |
| ROCH. - | 1 | 0.49 | 0.01 | $0.50^{8}$ |
| | 2 | -0.07 | -0.01 | -0.09 |
| OLCOTT - | 1 | -0.77 | -0.49 | -1.25 |
| | 2 | -0.84 | -0.24 | -1.07 |
| OSHAWA | | -0.14 | -0.39 | -0.54 |
| PRESQU'ILE | | 0.22 | -0.81 | -0.59 |

(From Landsberg 1975)

Figure 27c Cross sections of longshore baroclinic geostrophic velocity for 5 June 1972.



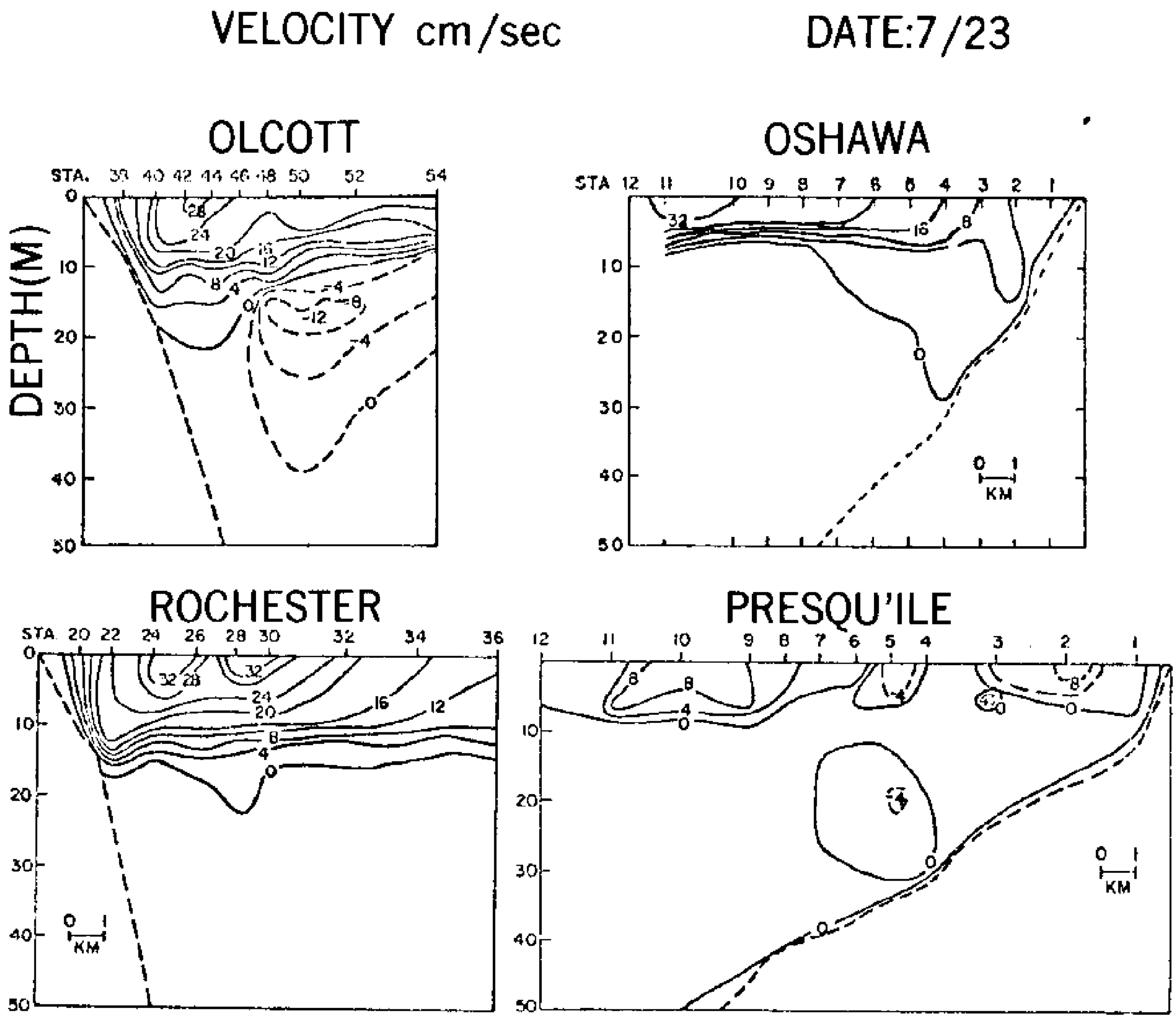



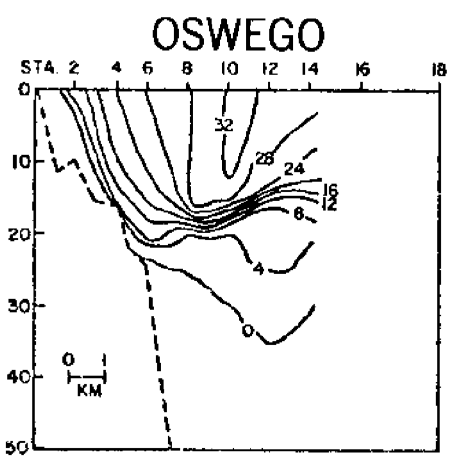

DAILY LONGSHORE BAROCLINIC GEOSTROPHIC TRANSPORT ($u_g$) ($10^4$ $M^3$/SEC)

| LINE | POS | NEG | TOT |
|---|---|---|---|
| OSWEGO - 1 | 0.35 | -0.01 | 0.34$^{8}$ |
| 2 | 0.29 | -0.03 | 0.26$^{8}$ |
| ROCH. - 1 | 0.95 | -0.03 | 0.92$^{8}$ |
| 2 | 1.26 | -0.01 | 1.25 |
| OLCOTT - 1 | 1.31 | -0.04 | 1.27 |
| 2 | 1.21 | -0.01 | 1.20 |
| OSHAWA | 0.18 | -0.14 | 0.04 |
| PRESQU'ILE | 0.18 | -0.08 | 0.10 |

(From Landsberg 1975)

Figure 28a  Cross sections of longshore component of velocity for 23 July 1972.



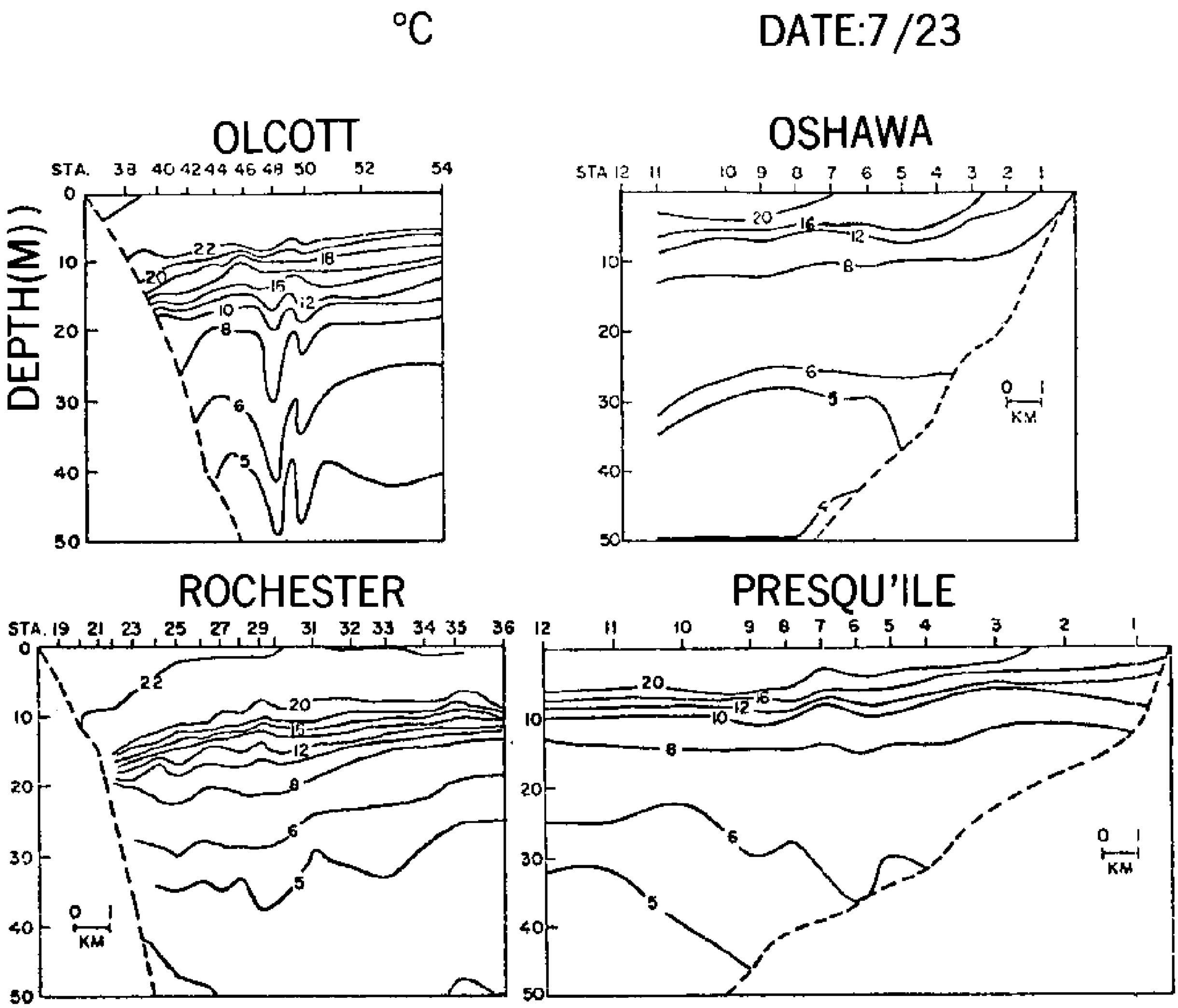

DAILY LONGSHORE VELOCITY TRANSPORT (u)

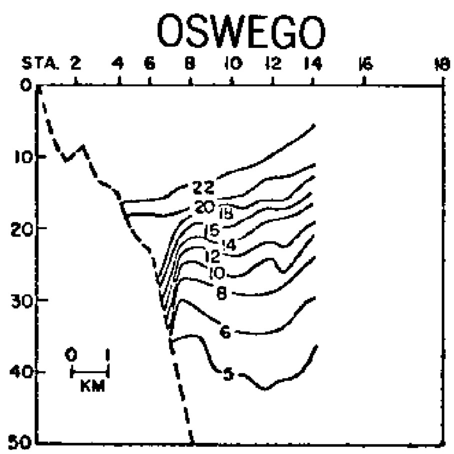
($10^4$ $M^3$/SEC)

| LINE | POS | NEG | TOT |
|---|---|---|---|
| OSWEGO | 2.85 | -0.01 | 2.84[7] |
| ROCHESTER | 2.72 | -0.02 | 2.70 |
| OLCOTT | 1.73 | -0.58 | 1.16 |
| OSHAWA | 1.44 | -0.02 | 1.42[9] |
| PRESQU'ILE | 0.50 | -0.26 | 0.24 |

(From Landsberg 1975)

Figure 28b Cross sections of temperature for 23 July 1972.



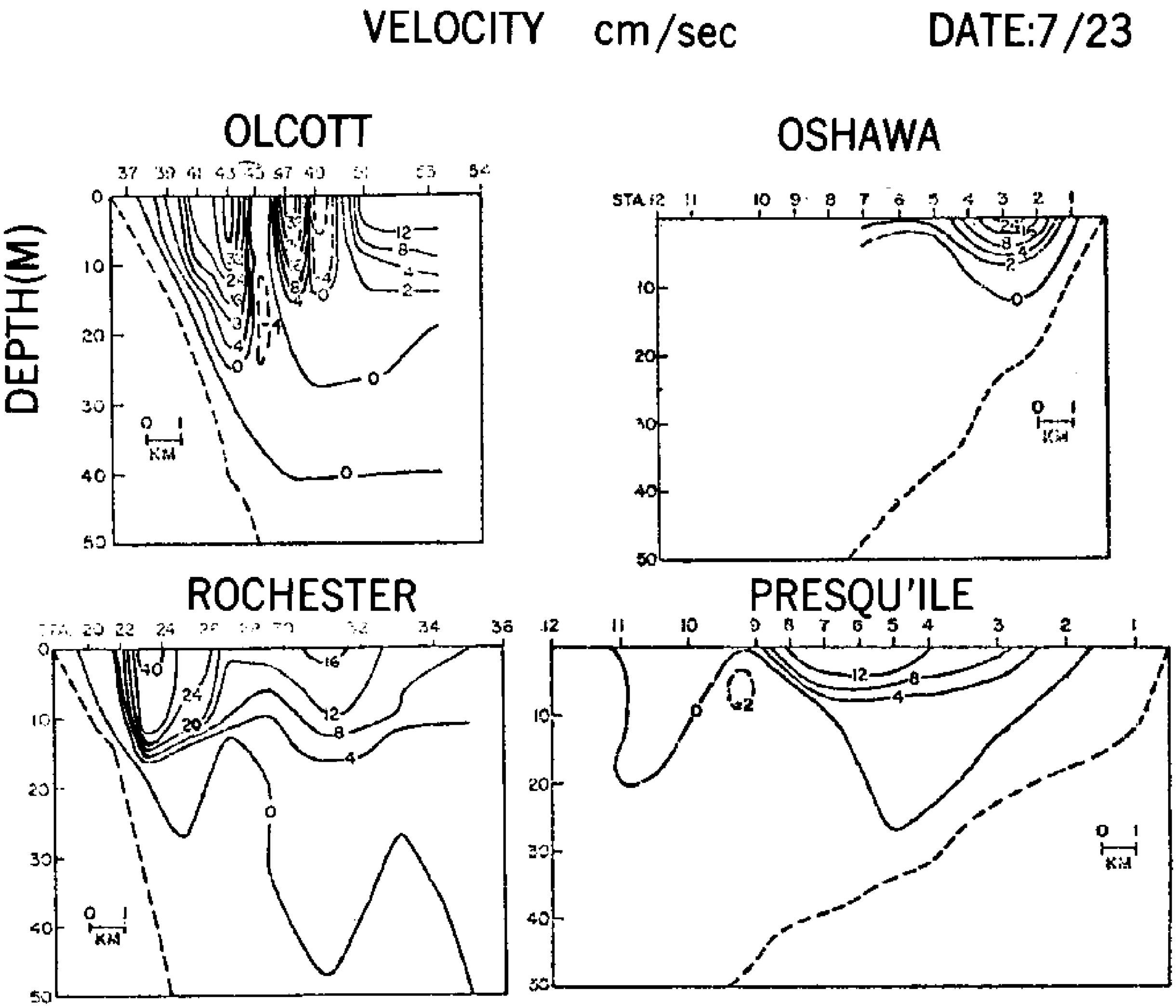

DAILY LONGSHORE TRANSPORT DIFFERENCE $(u-u_g)$

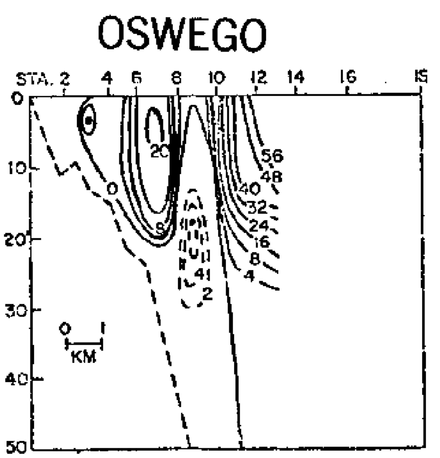
$(10^4\ M^3/SEC)$

| LINE | POS | NEG | TOT |
|---|---|---|---|
| OSWEGO | 0.22 | 0.09 | 0.32[7] |
| ROCHESTER | 0.52 | 0.0 | 0.52 |
| OLCOTT | 0.17 | -0.55 | -0.38 |
| OSHAWA | 0.69 | 0.07 | 0.76[9] |
| PRESQU'ILE | -0.07 | 0.0 | -0.07 |

(From Landsberg 1975)

Figure 28c  Cross sections of longshore baroclinic geostrophic velocity for 23 July 1972.




DAILY LONGSHORE BAROCLINIC GEOSTROPHIC TRANSPORT ($u_g$) ($10^4$ $M^3$/SEC)

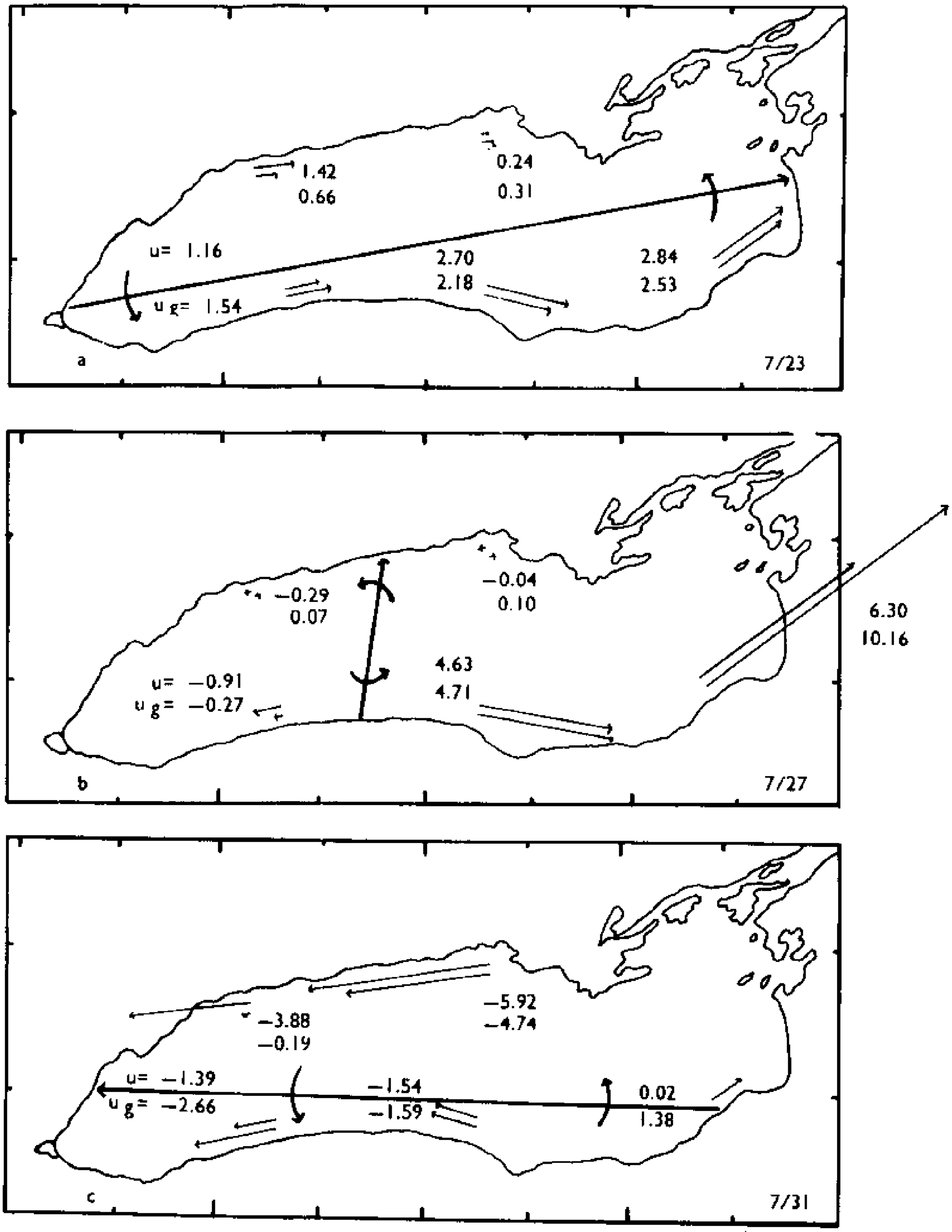
| LINE | POS | NEG | TOT |
|---|---|---|---|
| OSWEGO | 2.63 | -0.10 | 2.53[7] |
| ROCHESTER | 2.20 | -0.02 | 2.18 |
| OLCOTT | 1.56 | -0.03 | 1.54 |
| OSHAWA | 0.75 | -0.09 | 0.66[9] |
| PRESQU'ILE | 0.57 | -0.26 | 0.31 |

(From Landsberg 1975)

Figure 29 a) Measured and baroclinic geostrophic longshore transport for 23 July 1972.

b) Measured and baroclinic geostrophic longshore transport for 27 July 1972.

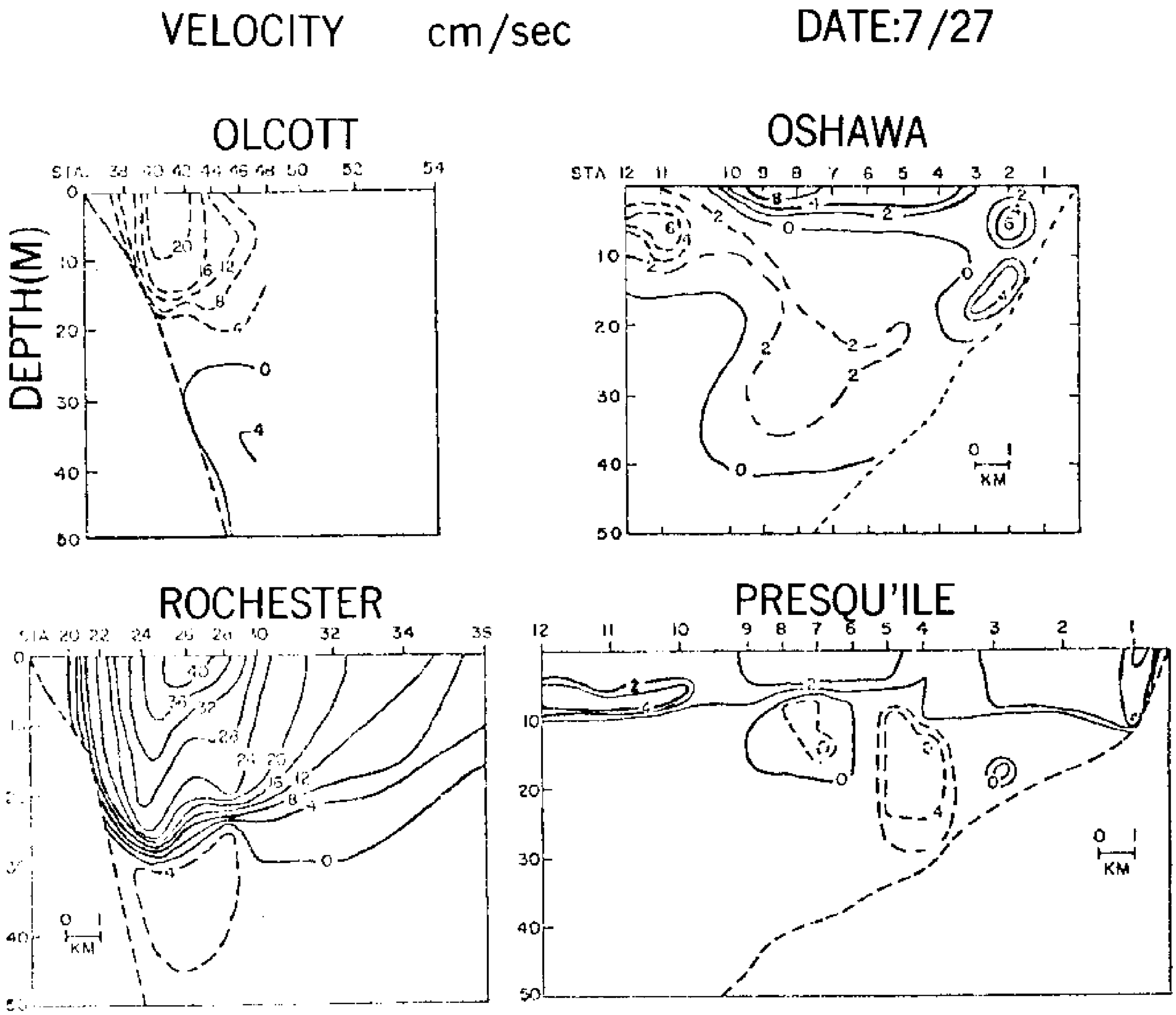
c) Measured and baroclinic geostrophic longshore transport for 31 July 1972.

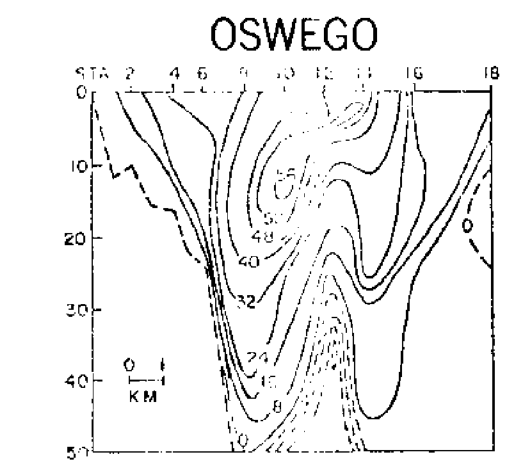



Units $10^4 M^3/sec = 3.4x10^5 ft^3/sec$

(From Landsberg 1975)

Figure 30a  Cross sections of longshore component of velocity for 27 July 1972.



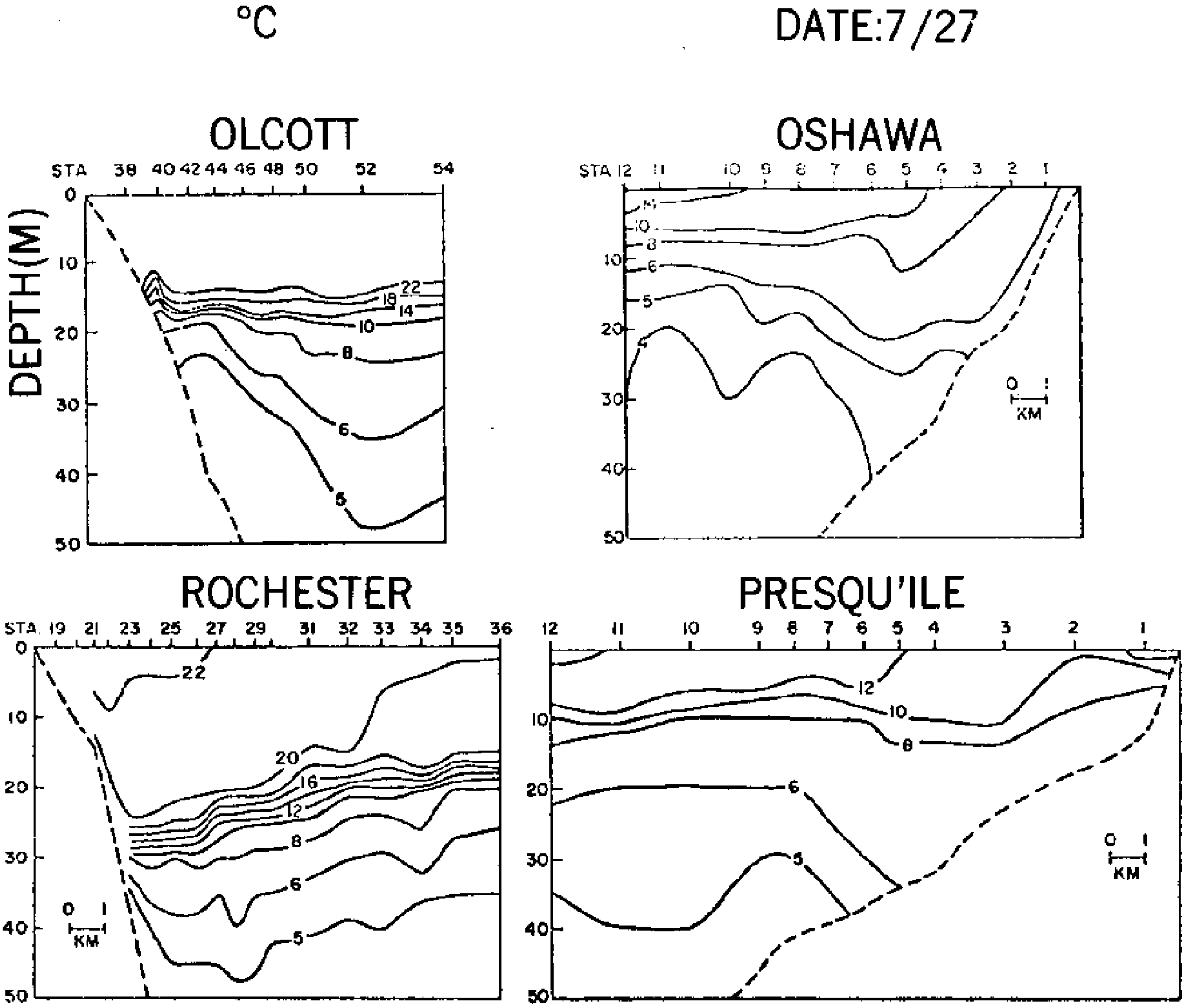

DAILY LONGSHORE VELOCITY TRANSPORT (u)

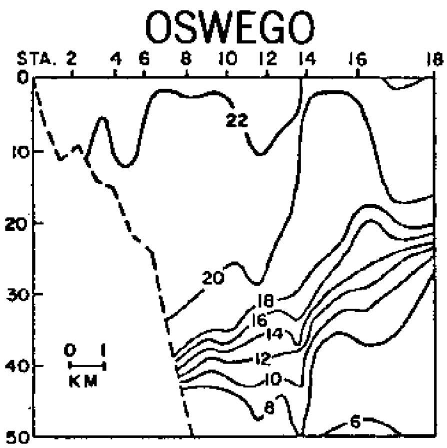
($10^4$ $M^3$/SEC)

| LINE | POS | NEG | TOT |
|---|---|---|---|
| OSWEGO - 1 | 6.89 | -0.59 | 6.30 |
| 2 | 1.57 | -0.94 | 0.63 |
| ROCH. - 1 | 4.87 | -0.24 | 4.63 |
| 2 | 3.08 | -0.06 | 3.02 |
| OLCOTT | 0.08 | -0.98 | -0.91 |
| OSHAWA | 0.18 | -0.48 | -0.29 |
| PRESQU'ILE | 0.29 | -0.34 | -0.04 |

(From Landsberg 1975)

Figure 30b Cross sections of temperature for 27 July 1972.



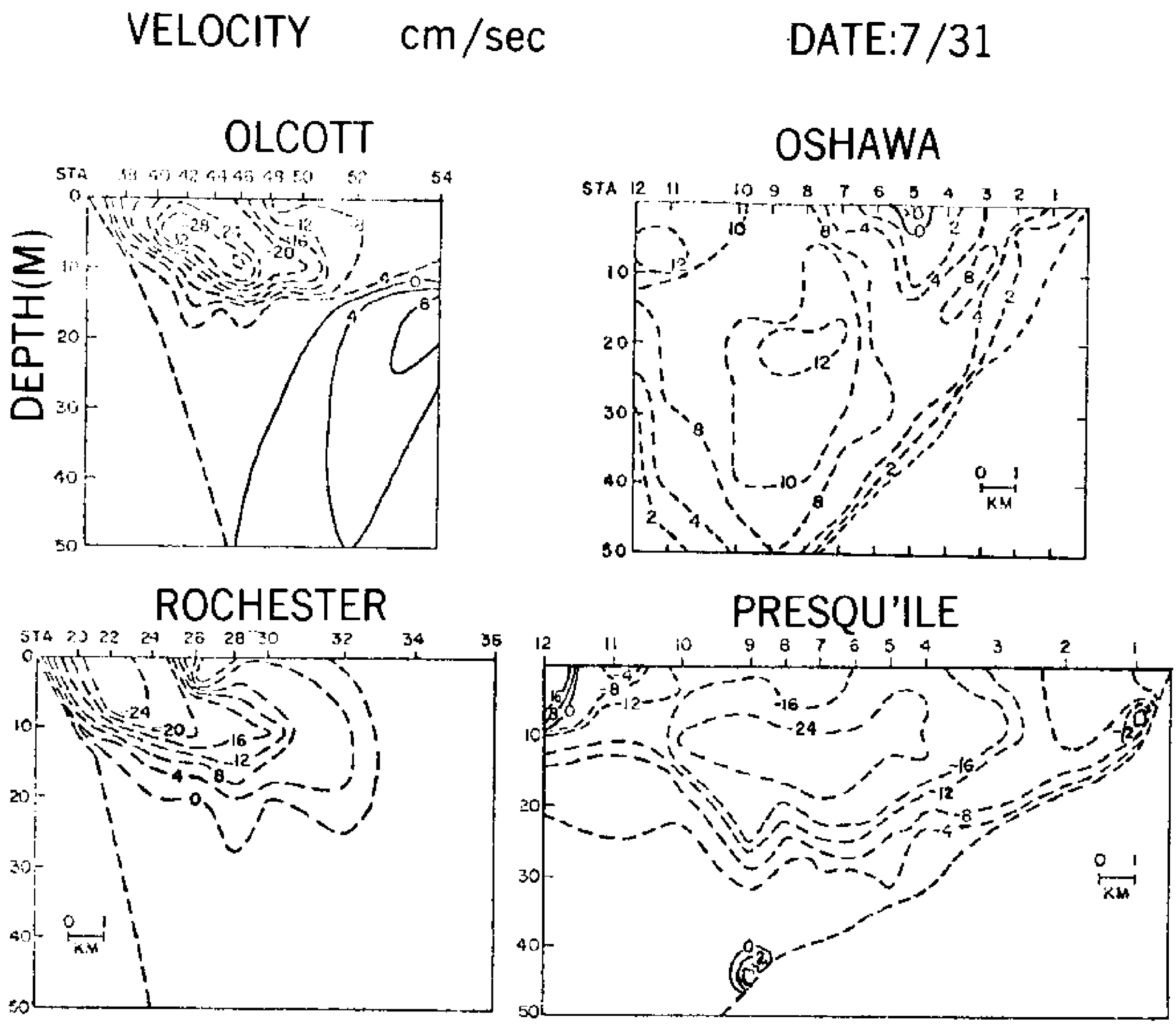

DAILY LONGSHORE TRANSPORT DIFFERENCE $(u-u_g)$

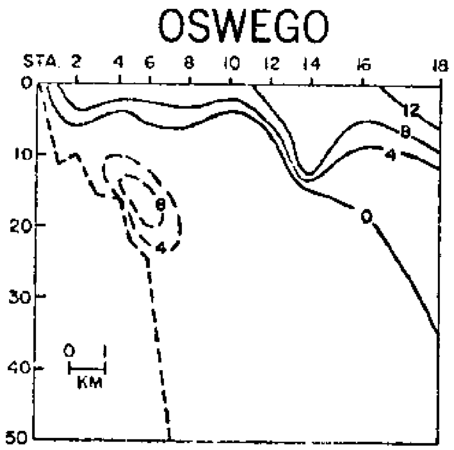
($10^4$ $M^3$/SEC)

| LINE | POS | NEG | TOT |
|---|---|---|---|
| OSWEGO - 1 | -3.28 | -0.58 | -3.86 |
| 2 | -0.37 | -0.94 | -1.31[5] |
| ROCH. - 1 | -0.15 | 0.07 | -0.08 |
| 2 | -1.56 | -0.02 | -1.58 |
| OLCOTT | -1.66 | 1.03 | -0.63 |
| OSHAWA | -0.02 | -0.35 | -0.37 |
| PRESQU'ILE | -0.12 | -0.03 | -0.14 |

(From Landsberg 1975)

Figure 31a Cross sections of longshore component of velocity for 31 July 1972.



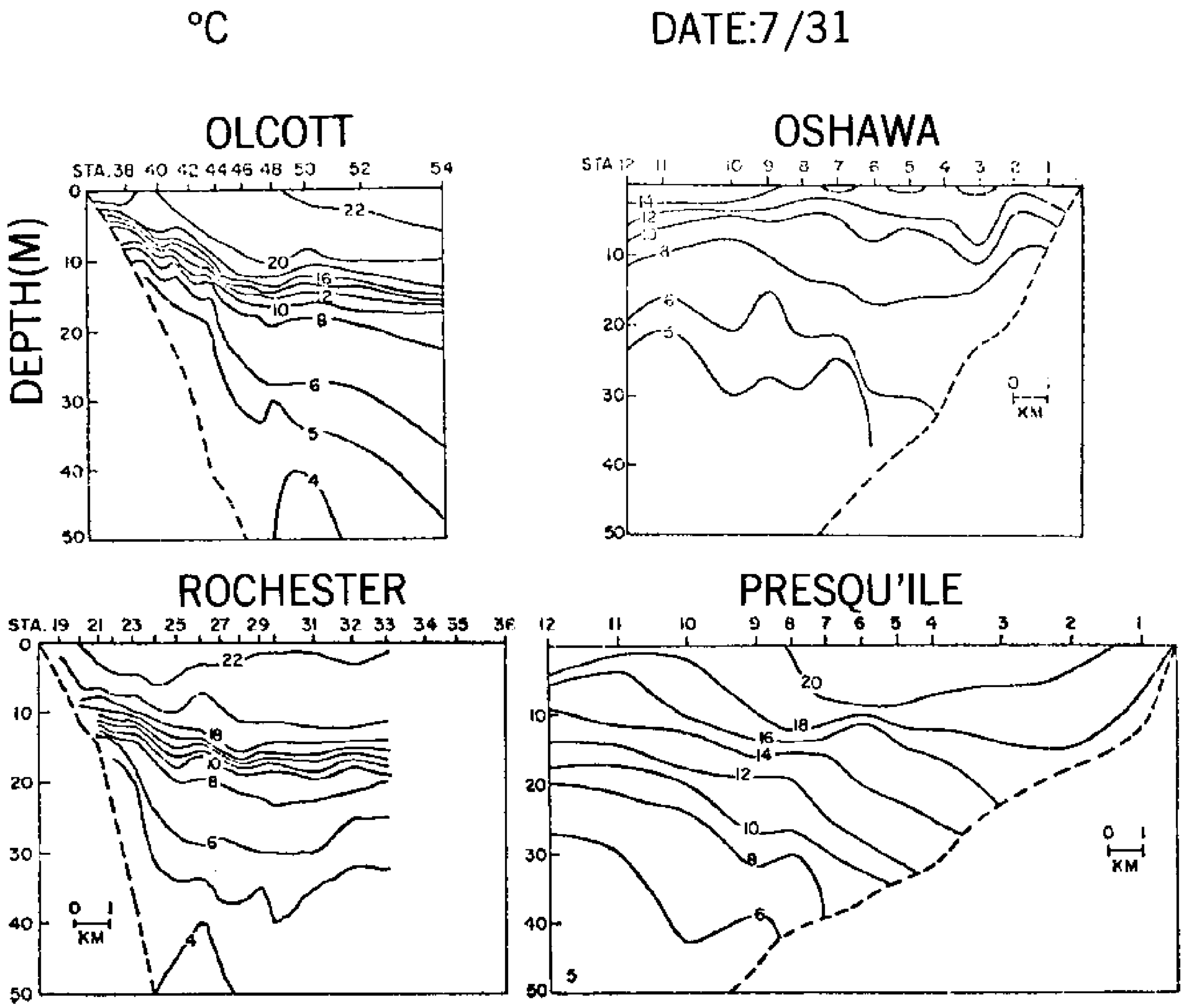

DAILY LONGSHORE VELOCITY TRANSPORT (u)

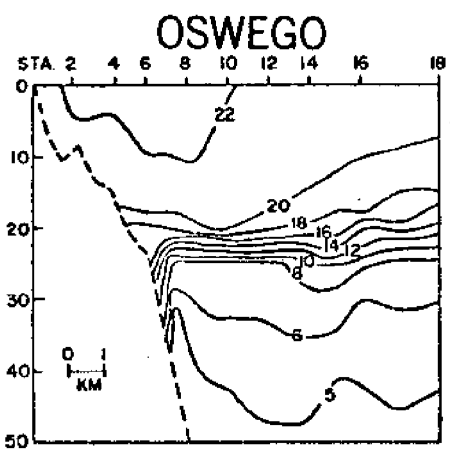
($10^4$ $M^3$/SEC)

| LINE | POS | NEG | TOT |
|---|---|---|---|
| OSWEGO | 0.39 | -0.36 | 0.02 |
| ROCHESTER | 0.08 | -1.62 | -1.54 |
| OLCOTT - 1 | 0.62 | -2.01 | -1.39 |
| 2 | 0.35 | -2.28 | -1.93 |
| OSHAWA | 0.0 | -3.88 | -3.88 |
| PRESQU'ILE | 0.17 | -6.09 | -5.92 |

(From Landsberg 1975)

Figure 31b Cross sections of temperature for 31 July 1972.



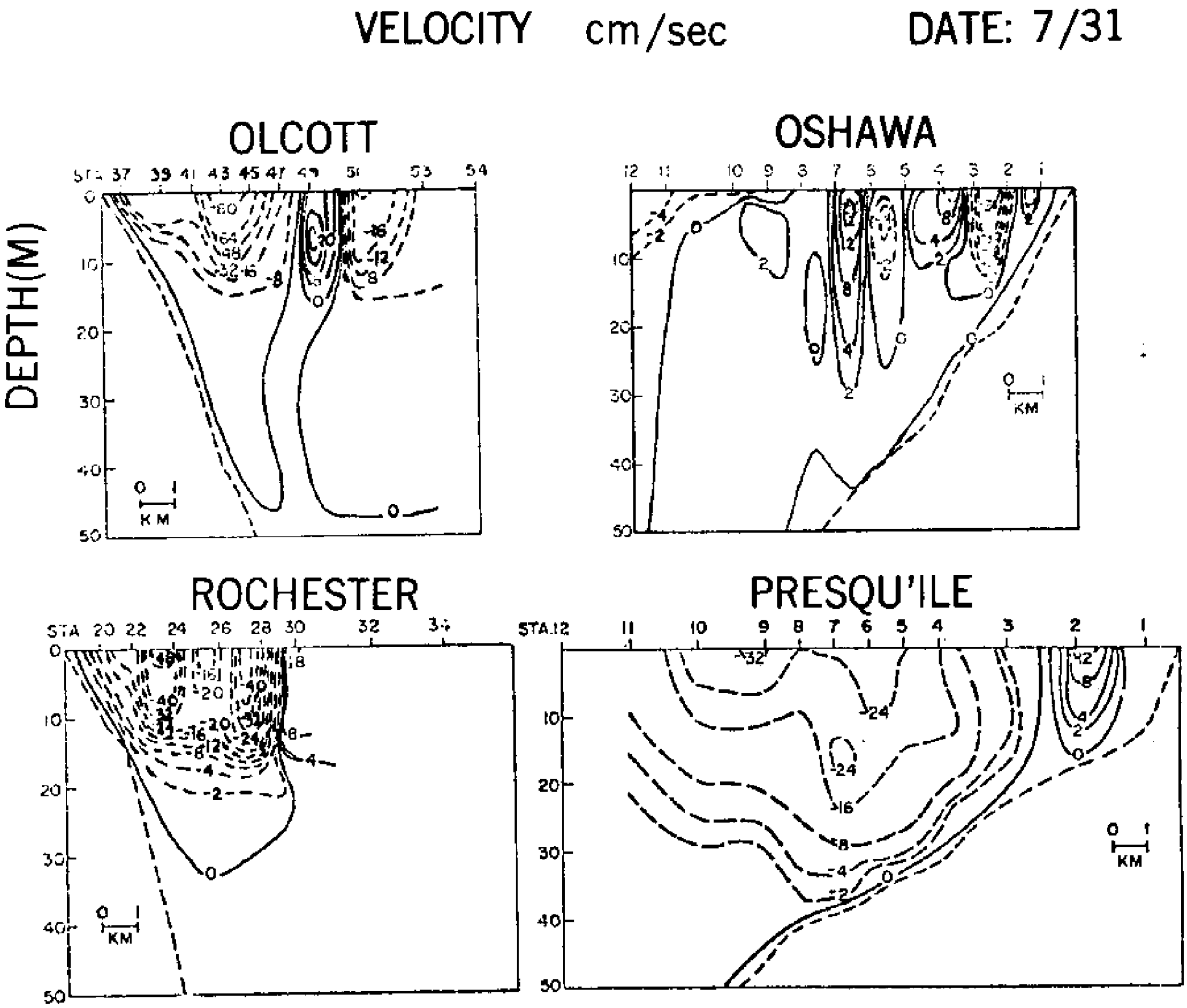

DAILY LONGSHORE TRANSPORT DIFFERENCE $(u-u_g)$

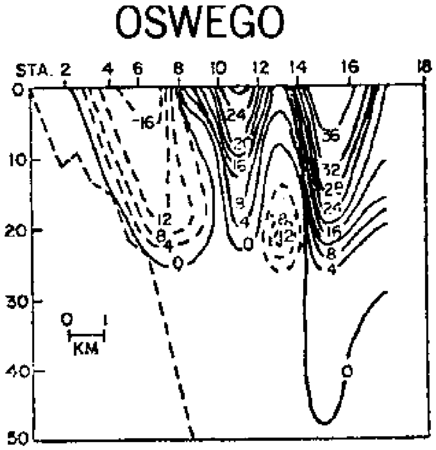
$(10^4\ M^3/SEC)$

| LINE | POS | NEG | TOT |
|---|---|---|---|
| OSWEGO | -1.67 | 0.32 | -1.36 |
| ROCHESTER | -0.25 | 0.30 | 0.05 |
| OLCOTT - 1 | 0.55 | 0.72 | 1.27 |
| 2 | 0.28 | -0.33 | -0.05 |
| OSHAWA | -0.32 | -3.37 | -3.69 |
| PRESQU'ILE | -0.27 | -0.91 | -1.18 |

(From Landsberg 1975)

Figure 31c  Cross sections of longshore baroclinic geostrophic velocity for 31 July 1972.



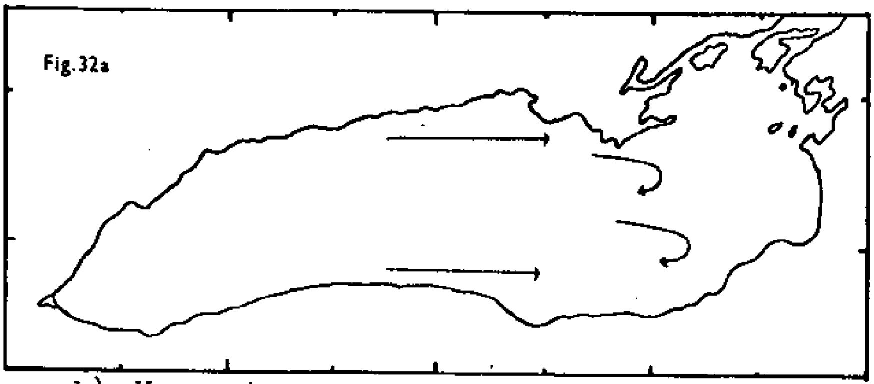

DAILY LONGSHORE BAROCLINIC GEOSTROPHIC TRANSPORT ($u_g$) ($10^4$ $M^3$/SEC)

| LINE | POS | NEG | TOT |
|---|---|---|---|
| OSWEGO | 2.06 | - .68 | 1.38 |
| ROCHESTER | 0.33 | -1.92 | -1.59 7 |
| OLCOTT - 1 | 0.07 | 2.73 | -2.66 |
| 2 | 0.07 | -1.95 | -1.88 |
| OSHAWA | 0.32 | -0.51 | -0.19 |
| PRESQU'ILE | 0.44 | -5.18 | -4.74 |

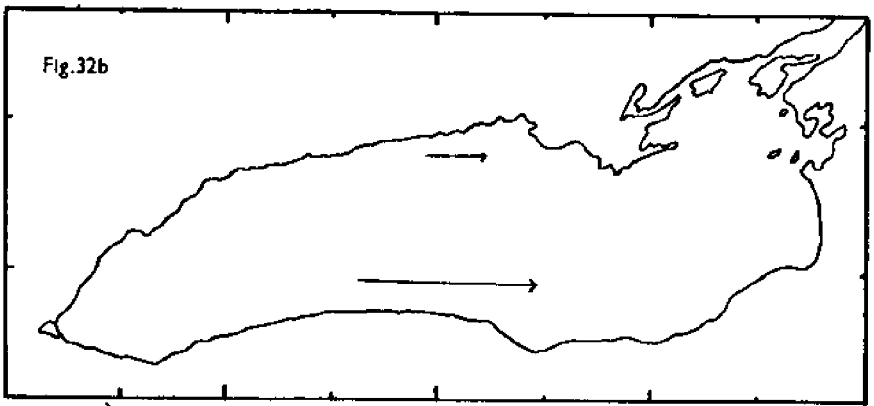
(From Landsberg 1975)

Figure 32 Schematics of Lake Ontario transport pattern for eastward wind stress:

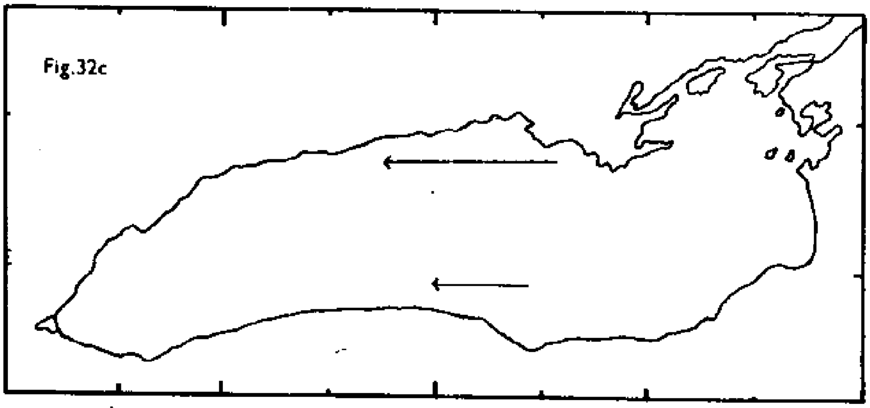
a) No waves.



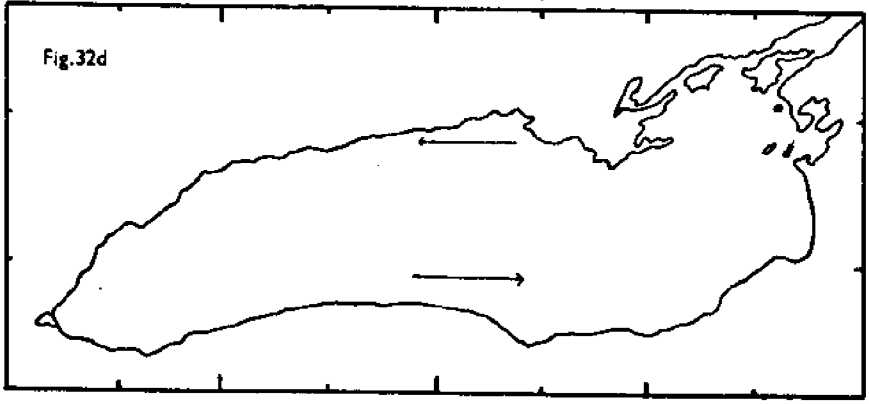

b) Wave set up.




c) Wave rotated through 1/2 cycle.




d) Average of several wave cycles.




(From Landsberg 1975)