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Fisheries Stock Assessment User's Manual

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INTRODUCTION

Fisheries Stock Assessment User's Manual Part I: An Introduction to Basic Methods & Models

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Preface

This manual was developed based on material presented during several stock assessment training courses sponsored by the Atlantic States Marine Fisheries Commission, and supplemented by class lecture notes of Dr. Joseph T. DeAlteris, Professor of Fisheries and Aquaculture, University of Rhode Island. It is intended to be used as a reference for stock assessment biologists and students, and as a course handbook for future ASMFC stock assessment training courses. It may be used alone, or in conjunction with Part II of this manual (scheduled to be printed in Fall 2000) which covers several more technical, advanced stock assessment models.

Acknowledgements

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INTRODUCTION

INTRODUCTION TO FISH STOCK ASSESSMENT, FISHERIES MANAGEMENT, FISHERIES AND FISHERY-DEPENDENT DATA, AND RESEARCH SURVEYS AND FISHERY-INDEPENDENT DATA

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Background

A fish population is a group of interbreeding fish that is characterized by its own birth rate, growth rate, age structure, and death rate. A fish stock is often referred to as that portion or subset of a fish population that is subject to exploitation or harvest. Fish stocks may respond differently to exploitation because of differences in reproductive, growth, and natural mortality rates. Therefore, fish stocks are considered discrete units for management purposes.

The purpose of fish stock assessment is to evaluate the status of a fish stock and to predict how the stock will respond to various exploitation or harvest scenarios. The current status of a stock is characterized by estimating stock parameters such as mortality (natural and fishing), abundance, biomass, age structure, and growth rate. The future status of a stock is predicted by modeling the process of stock change over time in response to management, using the previously estimated stock parameters.

Fisheries management is the process by which we attempt to control fish stock abundance by regulating harvest. Fisheries management decisions are made in an attempt to meet pre-determined objectives concerning future stock status based on biological, sociological, economic, and political inputs.

The history of fish stock assessment and fishery resource management began with the erroneous assumption that the ocean's resources were unlimited. Thomas Huxley concluded in 1884 that fish were so abundant and fecund, and man's ability to harvest them was so limited, that fish populations were immune to man's activities. Shortly thereafter, at the turn of the century, the International Council for the Exploration of the Sea (ICES) initiated the collection of commercial catch data to respond to concerns of overfishing and depleted fish stocks. World Wars I and II allowed worldwide fish stocks to rebuild, but overfishing in the last fifty years has driven stocks to record low levels.

Fisheries Management

The Magnuson Fishery Conservation and Management Act, enacted in 1976, empowered the federal government to regulate fishing from 3 to 200 miles off the coasts of the United States. The Act created eight regional fishery management councils that are charged with the responsibility of developing fishery management plans (FMPs) for stocks within their region. Council members include representatives from each state who then represent the regulatory, recreational, commercial, and conservation constituencies. Each council has an executive director and staff to assist in the preparation of FMPs.

NMFS is mandated by Congress to collect and analyze data on the status of the fishery resources off the coasts of the United States and on the fisheries. NMFS then provides this information to the management councils for their use.

Additionally, the councils have committees and panels that provide further technical assistance to the council staff and members on scientific and sociological issues related to the FMPs. Rules for the development of FMPs are referred to as the 602 guidelines, and provide directions for the definition of overfishing, the establishment of measures to prevent overfishing, and the development of a program for rebuilding a stock if overfishing already exists. Public input and comment is sought throughout the FMP development process. FMPs are modified through plan amendments that also allow for public input and comment. However, if conditions in the fishery are changing rapidly, framework action notices are used to allow management to keep pace with an evolving fishery.

The original Magnuson Act and the recently re-authorized Magnuson-Stevens Act provide national standards for the management of fishery resources. The Act has many standards it attempts to achieve, including promote conservation and utilization of the fishery resources based on the best scientific information available, seek to promote optimum sustainable yield while preventing overfishing, and protect the habitats for fishery resources. The full text can be found at *www.nmfs.gov/sfa/magact.*

After the fishery management plan amendment or notice action has proceeded through the regulatory process, it is published in the federal register. Management measures become federal regulations which are enforced by NMFS law enforcement agencies, the U.S. Coast Guard, and others. Violations are subject to civil and criminal sanctions. Civil sanctions include written warnings, fines issued by Notices of Violations and Assessment (NOVA), forfeiture of seized property including catch, vessels, and equipment, and finally, permit sanctions.

In addition to the regional management councils, there are three regional interstate fishery management commissions established by federal law: the Atlantic States Marine Fisheries Commission (ASMFC), the Gulf State Marine Fisheries Commission, and the Pacific States Marine Fisheries Commission. These commissions include three representatives from each state in the region, again representing various constituencies. Recently, these commissions were charged by Congress to promote and encourage management of interjurisdictional marine resources.

The Atlantic Coastal Fisheries Cooperative Management Act passed by Congress in 1993 charged the ASMFC with the responsibility of developing FMPs for transboundary, migratory coastal species. For example, in 1998, the ASMFC developed a FMP for American lobster, a resource harvested from Maine to Virginia.

The main management strategies used to control harvest rates include restricting effort and restricting harvest. U.S. fisheries have traditionally been open-entry or open-access fisheries. Since the passage of the Magnuson Act, there has been steady growth in the harvesting capacity. Thus, as we enter the twenty-first century, there is excess capacity or over-capitalization in our fisheries, resulting in overfishing of limited resources. To limit or restrict overfishing, management has responded in some fisheries by issuing seasonal or annual total allowable catch (TAC) regulations (*i.e.* restricting harvest). These quotas result in "derby fisheries" where individual fishermen attempt to catch as much as they can, as quickly as they can, until the quota is reached and the fishery closed. These derbies result in temporary market gluts and lower prices paid for catch to fishermen. Other methods to control fishing mortality include limiting effort by closing fishing areas during specific times to protect spawning aggregation of fish or nursery areas, allowing vessels only limited number of days at sea, restricting the vessel size, horsepower, or the amount of gear fished.

The most controversial effort-control measure, however, is limited entry. This is a fundamental change in the traditional open-access fishery management policy in the U.S. Limited entry begins with a moratorium on new licenses. A related issue is the transferability of licenses, *i.e.*, can an individual sell his license, or can potential new entrants to the fishery apply to a lottery to enter the fishery, as existing participants leave the fishery.

Another aspect of limited entry is the provision for property rights through individual transferable quotas (ITQ). After controlling access to the fishery with a moratorium on new licenses, fishermen are individually awarded a portion or allocation of the TAC each year, and that share is transferable to other fishermen via direct sale. Thus larger, more efficient harvesters are able to purchase the shares of the smaller, less efficient harvesters. This results in consolidation of harvesting capacity and increased economies of scale. Typically, limits are placed on the total number of shares an individual or corporation may acquire so as to avoid monopoly situations.

Fish Stock Assessment

The most recent Report on the Status of Fisheries of the United States published every year by the National Marine Fisheries Service (NMFS) indicates that 98 fish stocks nationwide are considered overfished. Fisheries managers have the responsibility to properly manage these fish stocks for the long-term benefit of both the fish stocks and the human population. Management decisions are made based on information derived through the various methods of fish stock assessment. Used properly, these methods will allow overfished stocks to rebuild and will ensure harvest pressure does not exceed sustainable levels.

A stock assessment report typically includes the following sections:

1. Description of the fisheries that interact with the stock and the presentation of fishery-dependent data (landings, effort, etc.).

2. Results of research surveys that provide fishery-independent data on abundance and samples for biological analysis.

3. Life history characteristics of the resource including natural mortality, growth, and maturity.

4. Population and fishery parameters that may include stock-recruitment relationships, estimation of exploitation rates, yield per recruit and spawning stock/egg per recruit models, surplus production models, and stock abundance indices.

5. Biological reference points based on the previous models and analyses.

6. Review of management objectives and alternative actions to achieve a sustainable fishery.

Fisheries and Fishery-Dependent Data

A wide array of gear types are used to harvest fishery resources commercially and recreationally. The principal gears are: hook and line, pots and traps, trawls and dredges, seines, and gillnets.

Hooks and Line Gear

Hook and line fishing methods have evolved from the simple act of attaching bait to a line, lowering that line into the sea, then carefully retrieving bait with a prey still attached feeding. This method of fishing is referred to as bobbing and is practiced today in Chesapeake Bay by recreational fishermen using a chicken neck attached to a line for the purpose of harvesting blue crabs.

The modern bent hook is believed to have evolved from a natural thorn hook, and from stone and carved shell hooks. The simplest form of hook and line fishing is the handline. It consists of a line, sinker, leader, and at least one hook. There are both recreational and commercial handline fisheries in the U.S. In the New England area, handlines are used to harvest bluefin tuna from small vessels. In fact, although this is a technologically sophisticated fishery with fish finding and navigation electronics, it is still conducted by individual or pairs of fishermen in small boats (< 10 m), so it may be considered an artisinal fishery. Recreationally, handlines are used in ice fishing.

The most basic pole and line fishery is a bamboo pole with a short line and hook attached. Recreationally, these are used to catch small fish in a wide variety of fisheries. The addition of a reel to store the line was a significant improvement to pole and line gear, and is again used in recreational and commercial fisheries. The reel, pole, and line gear is probably the most widely used recreational fishing gear; it is used in freshwater and marine fisheries in a wide variety of forms from fly fishing to offshore trolling for large pelagic billfish.

With the guiding philosophy that if one hook is good, many hooks are better, commercial fishermen developed bottom longline gear (Figure 1). The principle element of this gear is the mainline or groundline that can extend up to 50 km in length. Branching off the mainline at regular intervals are leaders or snoods, and hooks. Anchors hold each end of the mainline in place and surface buoys attached via float lines to the anchors mark the location of the gear. The mainline was initially constructed of natural fiber lines, which was replaced by a hard-lay, twisted, tarred nylon, and now monofilament and wire cables are typically used. Leaders were initially tied to the mainline, and now they typically snap-on to the mainline allowing separate storage of the hooks and leaders and the mainline. All bottom-set, longline gear, is considered fixed and passive because once deployed the gear does not move and the fish voluntarily takes the hook.

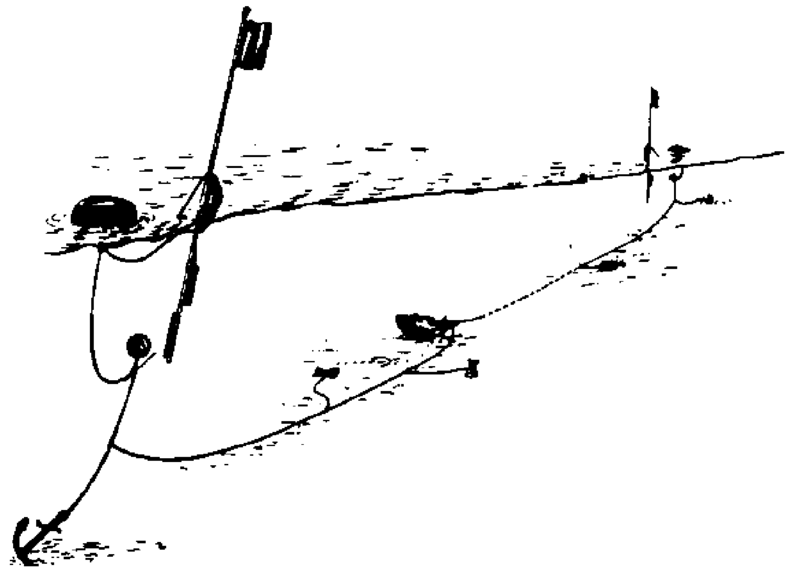

Figure 1. Bottom longline gear (USDOI Circular 48).

On the east coast of the U.S., there is an active pelagic longline fishery for large highly migratory pelagic species, in particular, swordfish, tuna, and shark. A typical vessel, 20 m in length, fishes a 100 km mainline and about 500 hooks on a 12 hour soak. The gear is fixed with respect to the water, but can drift over the seabed as much as 100 km in an overnight-soak. The gear is passive, in that fish are attracted to the hook with bait, light sticks, and sometimes noise makers, and voluntarily take the gear.

The art of attracting fish or squid to a lure with hooks moving up and down is called jigging. Jigging is conducted by hand, with a reel, pole and line, or using jigging machines that are programmed to move the lure in a particular way. Finfish usually take the hook with their mouth, but are occasionally snagged. In contrast, squid are almost always snagged by the hooks. Thus, jigs are classified as either active or passive depending on the methods of capture.

Pots and Traps

The essential element of any pot or trap fishing gear is a non-return device that allows the animal to voluntarily enter the gear, but makes escape difficult, if not impossible. The terminology used to identify pots and traps is also confusing, as both terms have been applied to the small portable, 3-dimensional gear. In this manual, a pot is defined as a small, portable, 3-dimensional device, whereas a trap is identified as a large, permanent, 2-dimensional gear.

The principle of operation of pot gear is that animals enter the device seeking food, shelter, or both. The non-return device, while allowing the animal to enter the gear, restricts escape. The holding area retains the catch until the gear is retrieved. Bait is placed in a bag or cage within the pot. Culling rings or escape vents are added to the exterior wall of the pot to allow for the release of undersize sub-legal animals. Finfish, shellfish, and crustaceans are all harvested with pots in the estuarine, coastal, and offshore waters of the U.S.

The blue crab fisheries conducted in the inshore waters of the mid and south Atlantic regions use a wire mesh pot (Figure 2). The design of the pot incorporates two sections, an "upstairs" and "downstairs." Crabs attracted by bait enter the "downstairs" via one of two to four entrance funnels. Once in the pot, the escape reaction is to swim upward, so a partition with two funnels separates the two sections. The "upstairs" section serves to hold the catch for harvest. Escape vents or cull rings may be installed in the pot to reduce juvenile bycatch. Crab pots are usually fished as singles and are hauled by hand from small boats or with a pot hauler in larger vessels. Crab pots are generally fished after an overnight soak, except early and late in the season.

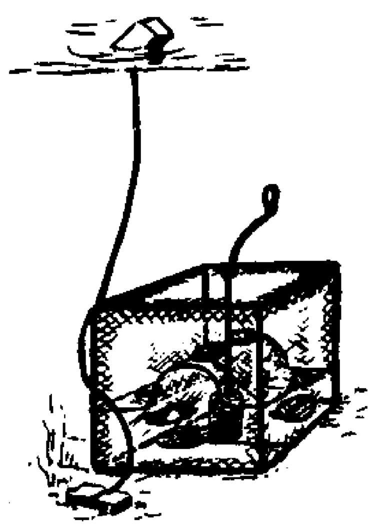

Figure 2. Crab pot (Sundstrom 1957).

Traps are generally a large scale, 2-dimensional device that uses the seabed and sea surface as boundaries for the vertical dimension. The gear is fixed, that is it is installed at a location for a season, and is passive, as the animals voluntarily enter the gear. Traps consist of a leader or fence that interrupts the coast parallel to the migratory pattern of the target prey, a heart or parlor that leads fish via a funnel into the bay section, and a bay or trap section that serves to hold the catch for harvest by the fishermen. The non-return device is the funnel linking the heart and bay sections. The bay, if constructed of webbing, is harvested by concentrating the catch in one corner, a process referred to as "bagging" or "hardening" the net. The catch is removed by "brailing" with a dip net. The advantages of traps are that the catch is alive when harvested, resulting in high quality; that the gear is very fuel efficient; and that there is the potential for very large catches. The disadvantages are that the initial cost of the gear is high, that there is competition for space by other users of the estuarine and coastal ecosystem, and finally that the fish must pass by the gear to be captured, so any alterations in migratory routes will radically affect catch.

On the mid-Atlantic coastal plain, large traps constructed of webbing hang from stakes that are pounded into the unconsolidated seabed and are locally referred to as "pound nets" (Figure 3).

These traps are usually set at points or capes that fish tend to migrate around. The leader sections are 100 to 600 m in length, starting in shallow water (< 2 m), and ending in water depths of 10 to 15 m. The heart sections lead to single or double funnels that lead into the bay section. The gear captures both pelagic and demersal species.

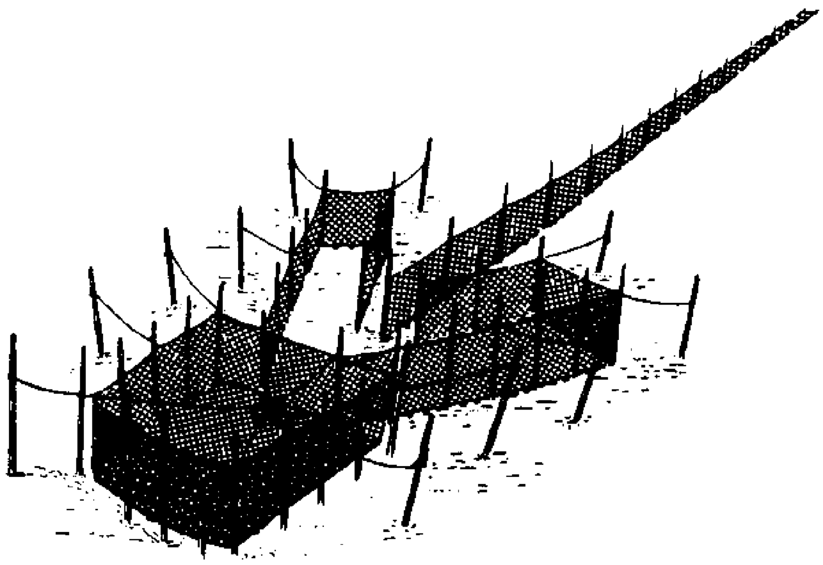

Figure 3. Pound net (Sundstrom 1957).

Dragged Gear

Fishing gear that is dragged or towed over the seabed or through the water is referred to as mobile gear. The dragged gears include a bag constructed of webbing or rings and chain links that collect the catch. These are exclusively active fishing gears, in that the animals do not voluntarily enter the gear, but are either swept up from the seabed or filtered from the water by the gear. Towed gear evolved from the need of man to harvest more efficiently, and that required collecting from more water or the bottom. Towed gear was initially deployed from hand-powered boats, then sailing vessels, and finally from large ships with engines greater than 1000 horsepower. Mechanization of these fisheries with engines and winches enabled larger gear to be towed faster and handled with less labor. The earliest dragged gear was probably some form of small rake used to collect shellfish towed by a hand-paddled canoe. As we enter the twenty-first century, the largest gear is a pelagic fish trawl with a mouth opening in excess of 100 x 100 m, towed by a vessel larger than 100 m in length with an engine of 2000 horsepower or more.

Dredges are rake-like devices, equipped with bags to collect the catch. They are typically used to harvest molluscan shellfish from the seabed, but occasionally are used to target crustacean, finfish, and echinoderm species. Dredges are designed to harvest both epifauna and infauna; however, the specific design details of the gear are very different.

In estuarine water, dredges are used to harvest oysters. The oyster is a sessile organism, generally growing in reef-like habitats. The oyster dredge consists of a steel frame 0.5 to 2.0 m in width, with an eye and "nose" or "tongue," and a blade with teeth (Figure 4). Attached to the frame is the tow chain or wire, and a bag to collect the catch. The bag is constructed of rings and chain-links on the bottom to reduce the abrasive effects of the seabed, and twine or webbing on top. The dredge is towed slowly (< 1 m/sec) in circles, from vessels 7 to 30 m in length. Compared to shaft tongs or patent tongs, the oyster dredge is very efficient. In many regions, oyster dredging is allowed only on private or leased oyster beds, and prohibited on public beds. However, in the Maryland portion of Chesapeake Bay, dredging is permitted on public beds, but only under sail, so as to maintain inefficiency, thus allowing for a traditional fishery.

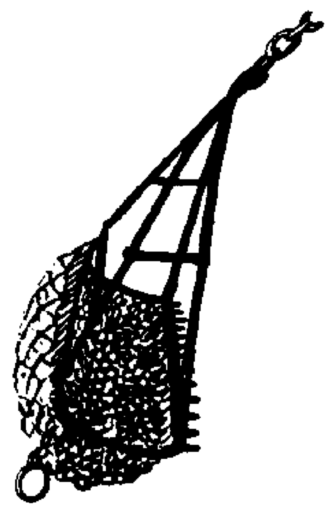

Figure 4. Oyster dredge (Sundstrom 1957).

Blue crabs are harvested during the winter months with large dredges similar to oyster dredges. The blue crab, susceptible to pots during the active summer months, are dormant in the winter months, and burrow into soft estuarine bottoms. Stern-rig dredge boats (≈ 15 m in length) tow two dredges in tandem from a single chain warp. The dredges are equipped with long teeth (10 cm) that rake the crabs out of the bottom. This same gear is used to harvest wheik in the summer and mussels in the fall from Chesapeake Bay.

Again, as fishermen sought to increase efficiency and tow vessels became larger, dredges evolved into beam trawls so as to capture finfish. The steel frame became larger and lighter, and the bag became larger and funnel shaped, so as to concentrate the catch in a cylindrical-shaped, webbing section, referred to as the codend. The first beam trawls were towed by sailing vessels, but today large beam trawls with mouth openings of 15 to 20 m, are towed from both sides of modern, high horsepower trawlers.

Otter trawls developed as fishermen sought to further increase the horizontal opening of the trawl mouth, but without the cumbersome rigid beam (Figure 5). In the late 1880s, Musgrave invented the otter board, a water-plane device that when used in pairs, each towed from a separate wire, served to open the net mouth horizontally and hold the net on the bottom. Initially, all otter boards were connected to the wing ends of the trawl, as they are today in the shrimp trawl fishery. In the 1930s, the Dan Leno gear developed by Frenchmen, Vigarnon and Dahl, allowed the otter boards (doors) to be separated from the trawl wing ends using cables or "ground gear."

This technology increased the effective area swept by trawls from the distance between the net wings to the distance between the doors. The ground gear can be as long as 200 m, thus increasing the area swept by the trawl by as much as three fold.

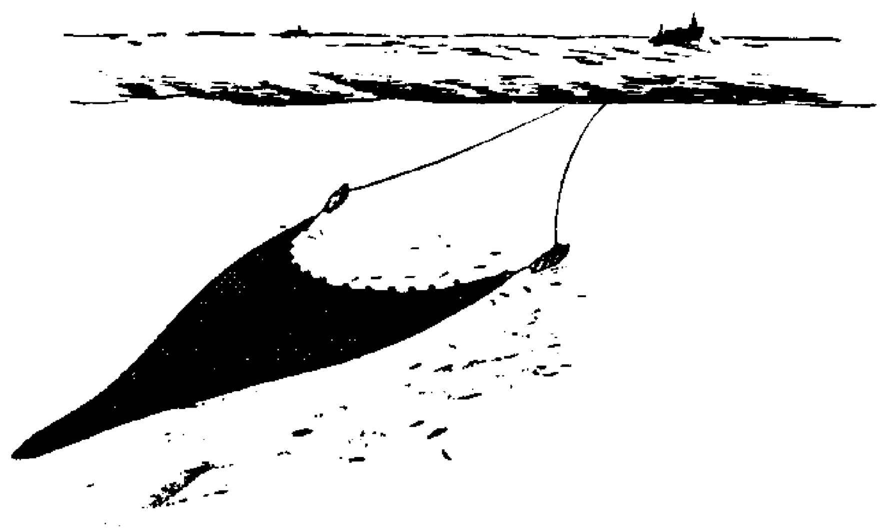

Figure 5. Otter trawl (Sundstrom 1957).

Bottom trawl fisheries are prosecuted for demersal species on all coasts of the U.S. In the northeast, vessels from 15 to 50 m fish in waters ranging from 10 to 400 m in depth. Small mesh nets are used to capture northern shrimp, whiting, butterfish, and squid. Large mesh trawls are used to harvest cod, haddock, flounder, and other large species. These trawls are typically rigged with long ground wires that create sand clouds on the seabed, herding the fish into the trawl mouth. In the southeast and Gulf coast areas, small mesh trawls are used to harvest shrimp.

Pelagic fishes are harvested using off-bottom or midwater trawl nets. The nets must be aimed or directed at specific concentrations of fish. Therefore, the fishermen must be able to identify the location of fish both laterally and vertically, and to direct the pelagic trawl to that position.

Seines

Fishing gear that is used to encircle marine resources either on the seabed or in the water column are classified as surround gear. Because the area enclosed by the gear is limited, the gear is directed or aimed at identified concentrations of fish. Surround gear are often referred to as seine nets.

The simplest form of seine is a single wall of webbing without a bag, connected at each end to poles that are handled by fishermen. The net is pulled through the shallow water collecting finfish, crustaceans, mollusks, etc., and finally dragged up onto a beach where the catch is sorted. The webbing is of variable mesh size, but is usually very small, (about 0.5 cm), as the gear is typically used to harvest bait fish for recreational hook and line fisheries. Typically a recreational or subsistence beach seine is about 20 m in length and 1.0 to 1.5 m in height with a

1.0 cm mesh size. Commercial beach seines range in length from 200 to 400 m and are equipped with a bag in the center or side.

The long-haul seine is set and hauled in shallow water estuaries from a boat (about 15 m). The net is a single wall of small mesh webbing (< 5 cm) and is usually greater than 400 m in length and about 3 m in depth. The end of the net is attached to a pole driven into the bottom and the net is set in a circle so as to surround fish feeding on the tidal flat. After closing the circle, the net is hauled into the boat, reducing the size of the circle, and concentrating the fish. Finally, the live fish are brailed or dip-netted out of the net.

Seine nets are also used on pelagic fishes. However, the net must be designed to close at the bottom. The nets are floating, that is the buoyancy on the float line exceeds the weight of the webbing and leadline. The gear fishes from the air-sea interface to the depth of the webbing. The gear is set in a circle around an identified school of pelagic fishes then closed off on the bottom, so as to prevent the escape of the fish.

The purse seine is closed using a continuous purse line (Figure 6). Functionally, purse seines are used to surround a concentration of fish, then the purse seine is hauled in so as to close the bottom of the net.

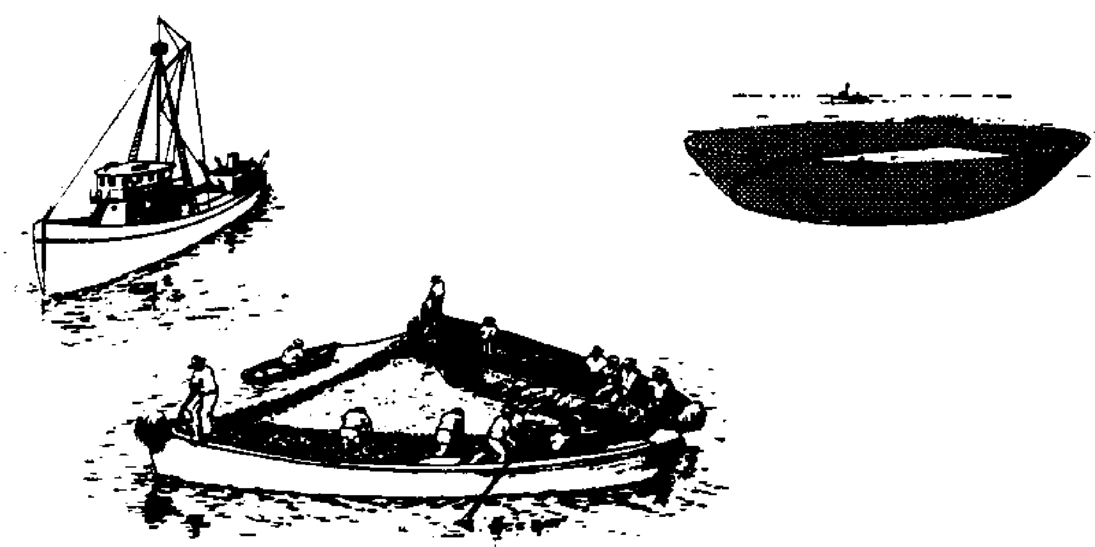

Figure 6. Purse seine (Sundstrom 1957).

The puretic power block developed in the early 1950s, was a significant mechanization of the purse seine fishery. The V-shaped sheave, attached to a beam end and powered by a hydraulic motor, has replaced 10 to 20 men that used to haul in the long wings of the small seines (300 m) used to harvest menhaden in Chesapeake Bay. The largest purse seines now used on tuna, fish in the open ocean and are more than 2000 m in length and 200 m in depth. Without the power block, these fisheries would not have developed.

Gillnets

Gillnets include a group of fishing gear types where animals are captured by a wall of webbing in the water column or on the bottom. The animals are captured by wedging, gilling, or tangling.

Gillnets operate principally by wedging and gilling fish, and secondarily by entangling. The nets are a single wall of webbing with float and lead lines. The nets are designed and rigged to operate as either sink or floating nets, and are anchored or drifting. The webbing is usually monofilament nylon due to its transparency; but multifilament, synthetic or natural fibers are also used.

Anchored sink gillnets are used to harvest demersal fish along all coasts of the U.S. The nets are rigged so that the weight of the leadline exceeds the buoyancy of the floatline, thus the net tends the seabed and fishes into the near bottom water column (Figure 7). Anchors are used at either ends of the net to hold the gear in a fixed location. The nets vary in length from 100 to 200 m and in depth from 2 to 10 m. Multiple nets are attached together to form a string of nets, up to 2000 m in length. In shallow water, sink gillnets may fish from bottom to surface, if the webbing is of sufficient depth.

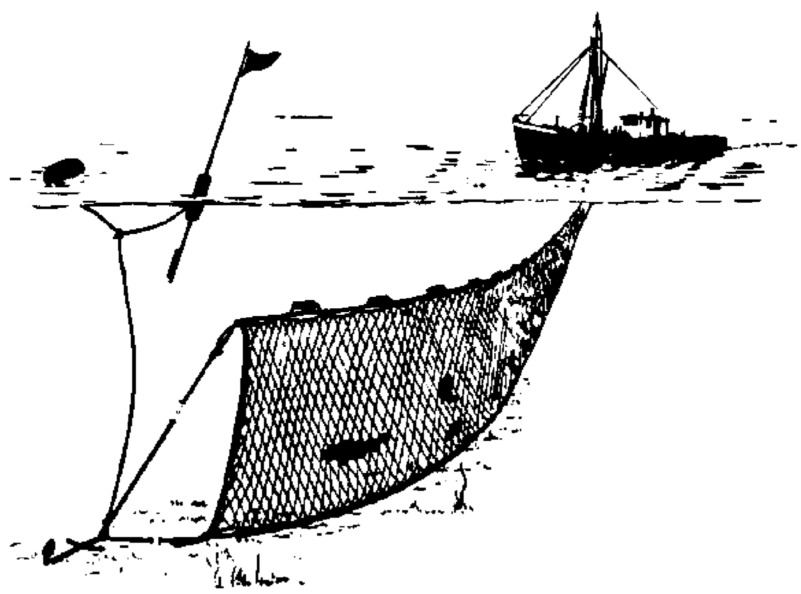

Figure 7. Sink gillnet (Sundstrom 1957).

Gillnets are also designed so as to float from the sea surface and extend downward into the water column and are used to catch pelagic fish. In this case, the buoyancy of the floatline exceeds the weight of the leadline. Floating gillnets are anchored at one end or set-out to drift usually with the fishing vessel attached at one end. Anchored floating gillnets are used in shad fisheries on the east coast. Offshore, large mesh drift nets are set for swordfish and other large pelagic fishes.

Fishery-Dependent Data and Analyses

The National Marine Fisheries Service (NMFS) and state agencies collect catch and effort data on the recreational and commercial fisheries, so as to monitor the status of the fishery resource stocks and to estimate fishing mortality. From these data and analyses, and in conjunction with fishery-independent data sources (scientific surveys) and analyses, fishery scientists are able to predict the outcomes of various management alternatives.

In the commercial fisheries, landings data is collected from fishermen's logbooks and trip tickets, dockside interviews by port agents, monthly summaries from dealers, or other means. However, landings data may not be entirely representative of the actual catch, due to at-sea discards. Data on discards is collected by at-sea observers who sample the entire catch, then note discards and landings. Sea-sampling is usually only conducted on a subset of the fishing fleet due to the high cost of staffing these programs, but the observed discard rates are extrapolated to the entire fleet, so as to develop complete estimates of age/size-specific catch. Discards are prorated into the landings based on their age/size and gear-specific survival probability. Effort in commercial fisheries is often based on license data according to gear type, vessel tonnage, days at sea or fishing, or the amount of gear set and soak time. Within a specific fishery, there is a standardized unit of effort, for example, one day fishing by bottom trawl for a 50 to 99 ton vessel. Other classes of trawl vessels, both smaller and larger, are then compared to the standard vessel in terms of catchability and rated accordingly. An example of fishery-dependent data can be seen in Figure 8. The Potomac River blue crab harvest time series of both catch and effort was obtained from historical commercial harvest and license data from the Potomac River Fisheries Commission.

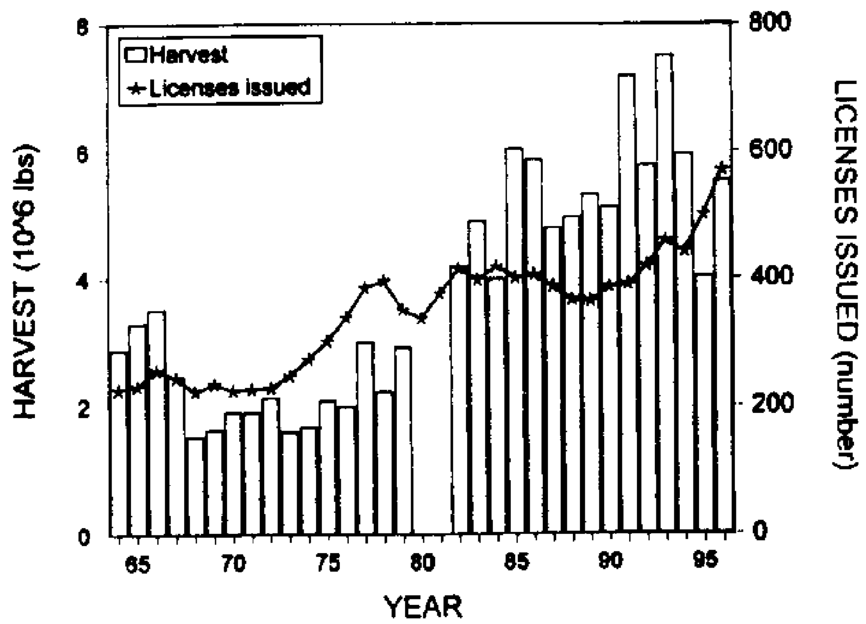

Figure 8. Potomac River blue crab harvest and licenses issued.

Recreational fisheries landing statistics are collected by port-based samplers who conduct intercept interviews with fishermen returning from a day of fishing at sea. These data are supplemented with the Marine Recreational Fishery Statistics Survey (MRFSS). The MRFSS is a series of surveys initiated by NMFS starting in 1979 to obtain standardized and comparable estimates of participation, effort, and catch by recreational anglers in the marine waters of the United States. The MRFSS collects recreational fisheries data using both dockside intercept and telephone surveys. The intercept survey collects data on the number, weights, and lengths of fish caught by species, state and county of residence, and avidity level (e.g. trips per year, mode of fishing, and primary area of fishing). The telephone survey collects data on the presence of marine recreational anglers in the household, number of anglers per household, number of fishing trips in a 2-month period, the mode of each trip, and the locations (county) of each trip. The estimated number of marine recreational fishing trips in Rhode Island from the MRFSS Survey can be seen in a time series of recreational effort (Figure 9).

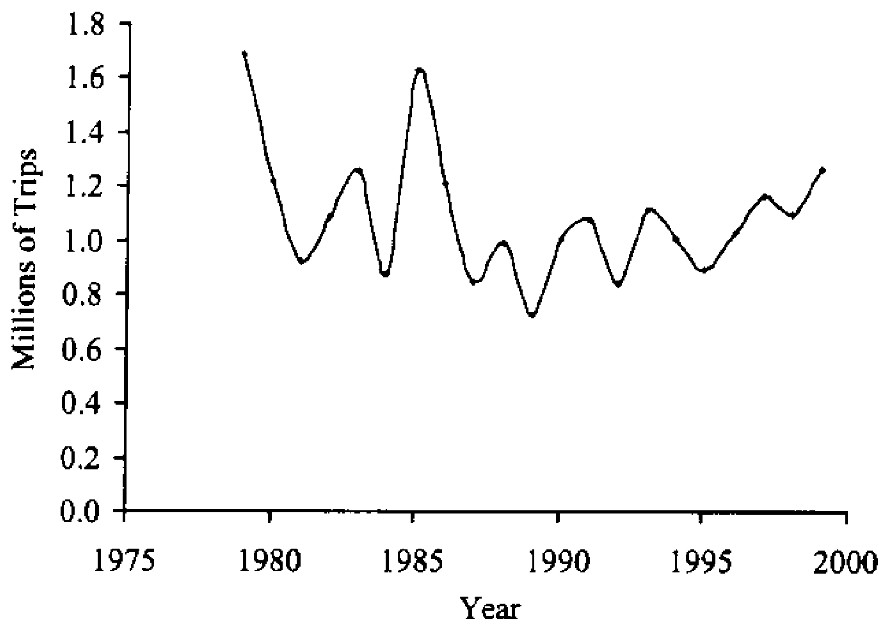

Figure 9. Estimated number of marine recreational fishing trips in Rhode Island from MRFSS Survey.

Research Surveys

NMFS and state agencies also collect and analyze data on fishery resources independent of the recreational and commercial harvesting sectors. NMFS utilizes a fleet of research vessels operated by the National Oceanographic and Atmospheric Administration (NOAA) to collect this data. Surveys conducted by NMFS range from marine mammal population counts to plankton surveys. The trawl surveys for fish provide an independent index of relative abundance of species taken by the sampling gear that can be compared to fishery catch per unit effort, also an index of relative abundance. When the two indices together track trends of increasing or decreasing abundance, there is greater confidence in the conclusions drawn from these analyses.

The fishery-independent surveys also provide biological samples for the study of age and growth, mortality, fecundity, and other life history characteristics, in addition to allowing for the collection of oceanographic data that is used to develop ecological models relating fish abundance and distribution to environmental conditions.

Fishery-independent surveys follow a rigorous methodology that is designed to result in statistically valid samples, taken in a consistent and reproducible manner. The protocol for bottom trawl surveys usually follows a random stratified design. The continental shelf water is divided into similar strata by latitude and depth zone, so as to reduce sample variability within strata and therefore increase the precision of abundance estimates. Within strata, station locations are selected randomly, so as to remove possible biases and to meet statistical design requirements. Survey data is used to develop a fishery-independent index of relative abundance (CPUE), so temporal consistency in sampling is extremely important. Considerable effort is expended to ensure that each tow of the trawl is exactly the same as every other tow within each survey, and between past and future surveys. Small changes in sampling method or gear may result in substantial changes in catchability of that gear, so any changes are avoided or investigated thoroughly via paired comparison methods prior to implementation.

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MATH & BIOSTATISTICS

MATHEMATICS AND BIOSTATISTICS REVIEW

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Note: This chapter is adapted from materials presented in Chapter 2 of Gulland (1969) and Chapter 2 of Sparre *et al.* (1989). Additional references are provided at the end of the chapter.

Functions

If to each value of x there corresponds one or more values of a variable y, then y is a function of x, and we write $y = f(x)$ where f symbolizes the function (Figure 1). The relation $y = f(x)$ is defined as a continuous set of points forming a line or curve. Note the ordinate or y-axis evaluates the dependent variable y, and the abscissa or x-axis evaluates the independent variable x.

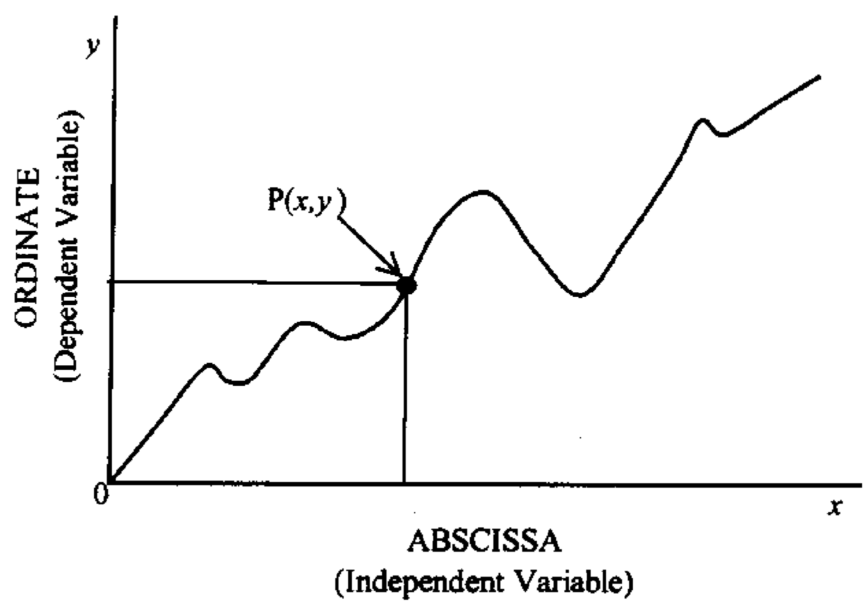

Figure 1. Function: $y = f(x)$.

A linear function can be described by:

$$y = ax + b,$$

where a and b are constants (Figure 2).

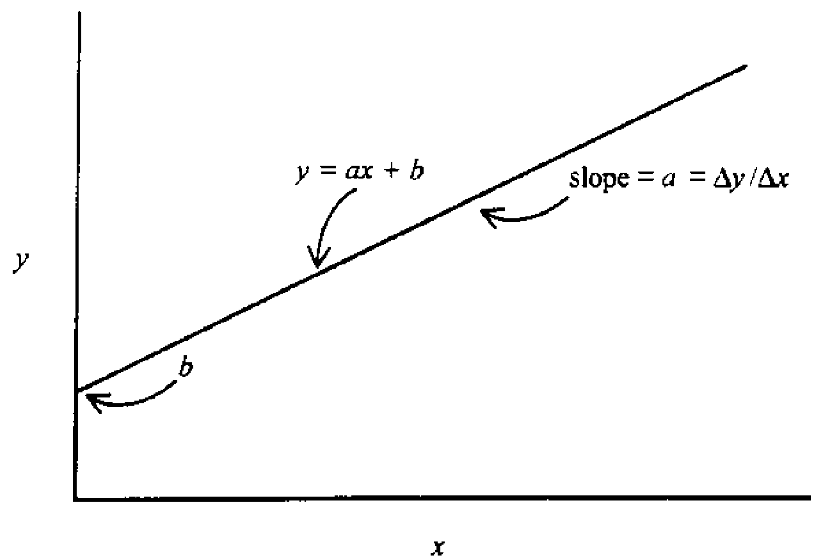

Figure 2. Linear function: $y = ax + b$.

The slope of the line is a and the y-axis intercept is b. The slope is defined as the change in y divided by the change in x. Mathematically, this is expressed as $\Delta y / \Delta x$ or dy/dx. Special cases of the linear function occur when $b = 0$ ($y = ax$; *i.e.*, the function intersects the origin), (Figure 3a) and when $a = 1$ and $b = 0$ ($y = x$) (Figure 3b).

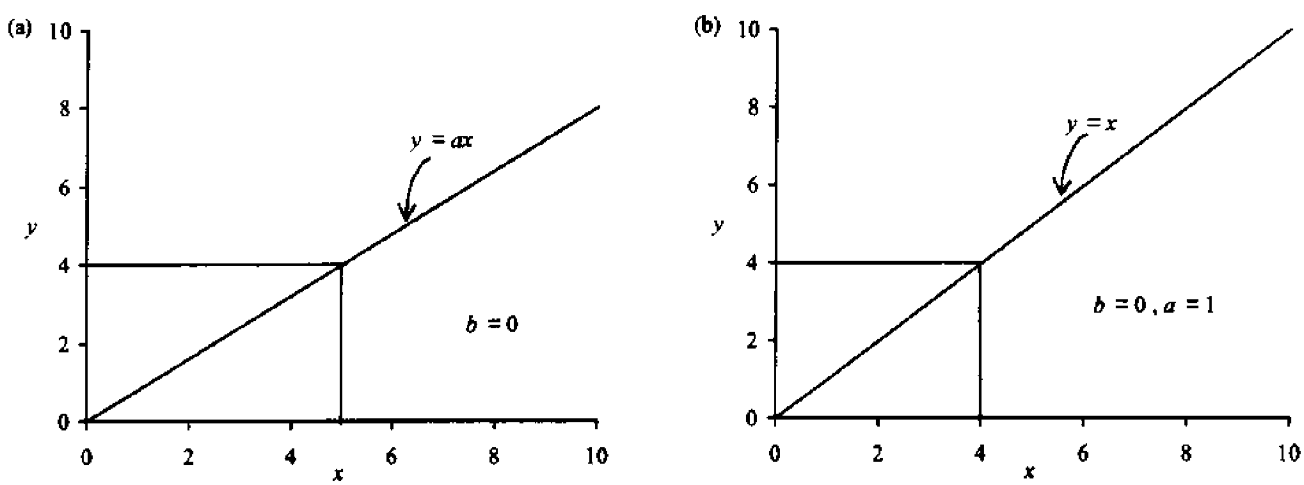

Figure 3. Special cases of linear functions:
(a) $y = ax$
(b) $y = x$.

Note the effect of value of the slope a on the linear function (Figure 4). The function trends upward if $a > 0$, is a horizontal line parallel to the abscissa if $a = 0$, and trends downward if $a < 0$.

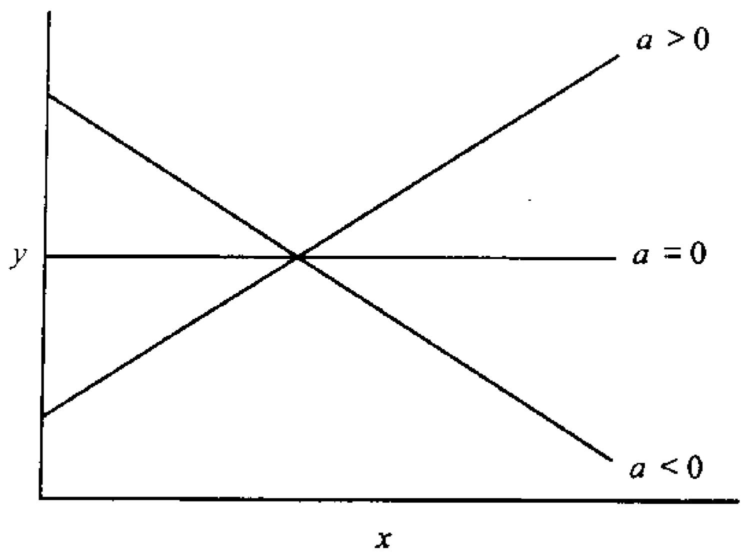

Figure 4. Effect of the slope a on the linear function.

A more complex function is the second order polynomial, or quadratic function described by

$$y = ax^2 + bx + c$$
or
$$y = A(x - x_0)(x - x_1)$$

where x_0 and x_1 are x-axis intercepts and A determines whether the curve is concave upward or downward (Figure 5).

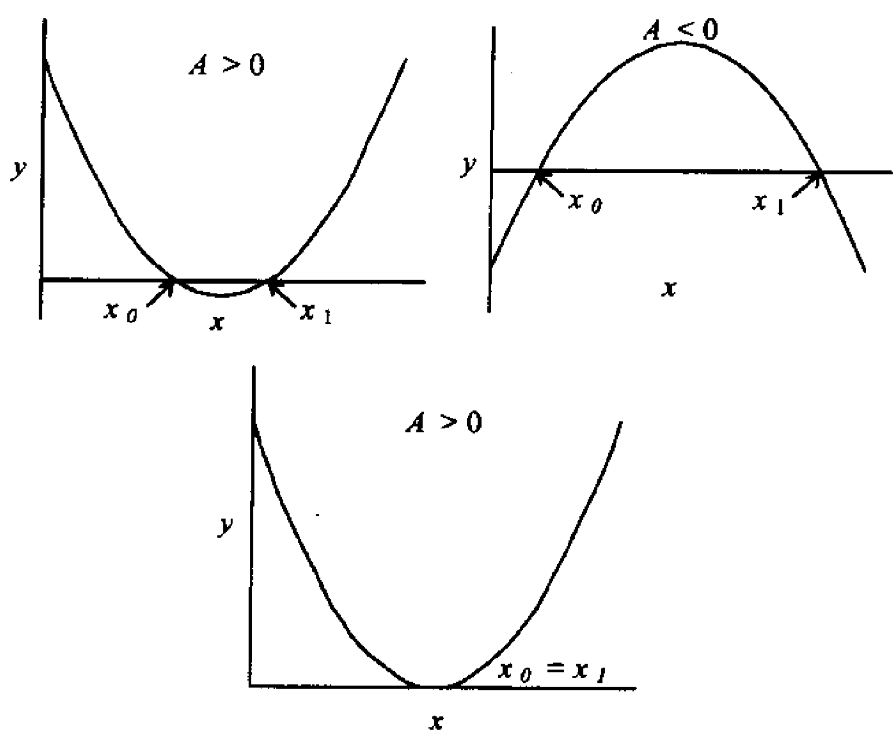

Figure 5. The quadratic function: $y = A(x\text{-}x_0)(x\text{-}x_1)$.

The parabola is a special case of the second order polynomial, where $A < 0$, $x_0 = 0$, and $x_1 = b$ (Figure 6).

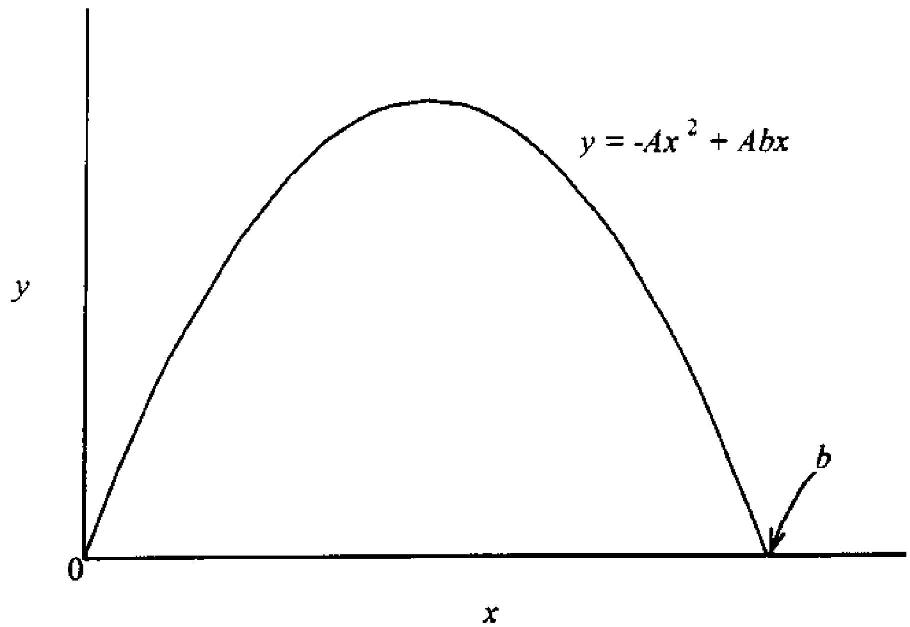

Figure 6. Parabola: $y = -Ax^2 + Abx$.

An exponential function is described by

$$y = a^x$$

where a is a constant raised to an exponential power, x (Figure 7). As with the linear function, x is the independent variable, and y is the dependent variable.

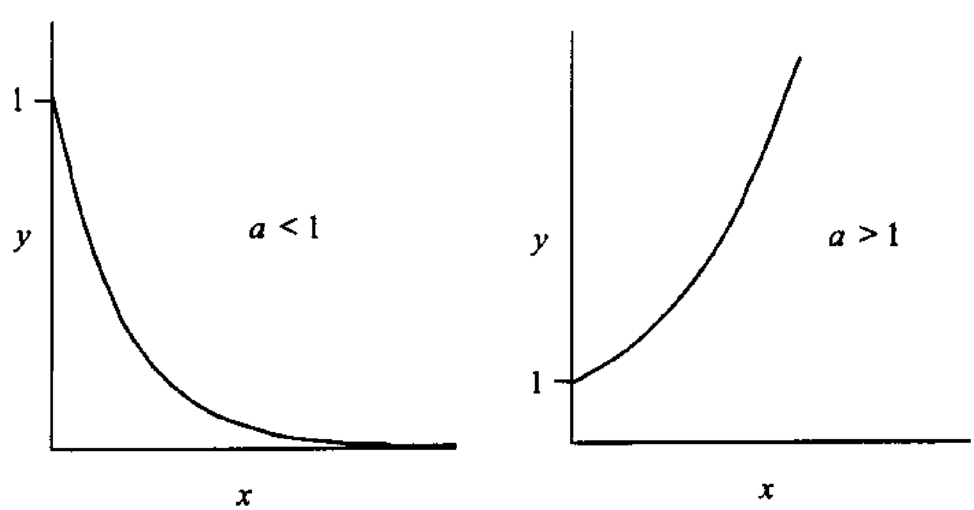

Figure 7. Exponential functions: $y = a^x$.

The exponential function increases to infinity if $a > 1$. The function decreases asymptotically and approaches the x-axis if $a < 1$. An asymptotic function is described by a curve that approaches a singular value on the y-axis as the values on the x-axis become larger and larger.

Another example of an asymptotic function is

$$y = 1 - e^{-x}$$

where the function approaches $y = 1$ as x increases in value (Figure 8).

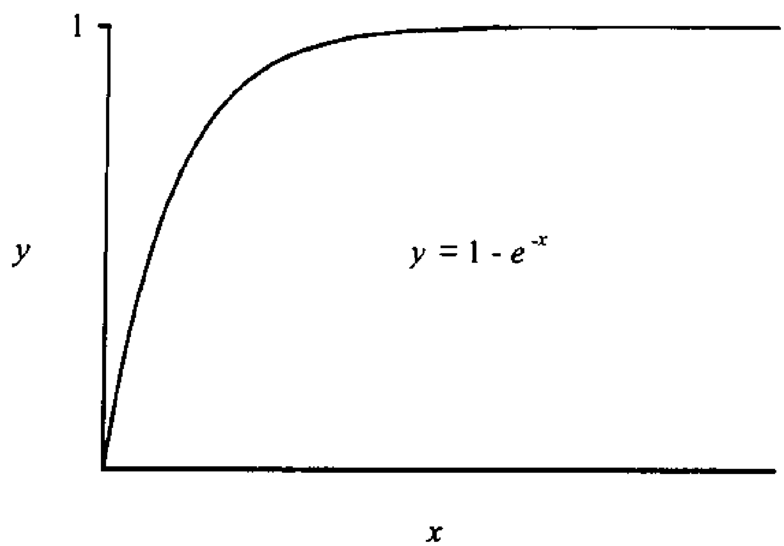

Figure 8. Asymptotic function: $y = 1 - e^{-x}$.

A power function is described by

$$y = x^N$$

where N is a constant (Figure 9).

If $N > 1$, then the curve ascends rapidly. If $N = 1$, then $y = x$ is a straight line. If $N < 1$, then the curve ascends slowly.

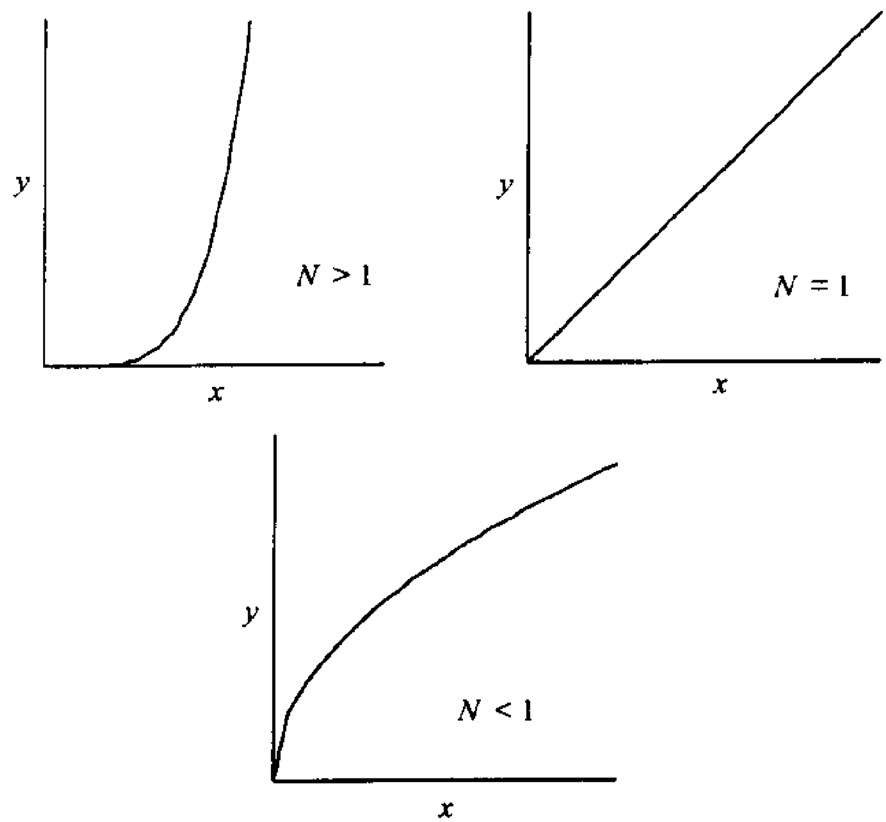

Figure 9. Power functions: $y = x^N$.

Powers and Logarithms

A power is represented by two numbers and is expressed as a^N, where a is the base and N is the exponent. The following laws apply to powers:

$$
\begin{aligned}
&a^0 = 1\\
&a^{-M} = 1/a^M\\
&(a^N)\cdot(a^M) = a^{(N+M)}\\
&(a^N)/(a^M) = a^{(N-M)}\\
&(a^M)\cdot(b^M) = (a\cdot b)^M\\
&(a^M)/(b^M) = (a/b)^M\\
&(a^N)^M = a^{NM}\\
&a^{M/N} = \sqrt[N]{a^M}.
\end{aligned}
$$

The inverse function of a power is a root, and the inverse function of an exponential is a logarithm. For example, consider the exponential function

$$y = a^x.$$

Taking the logarithm to the base a of both sides of the equation yields

$$\log_a y = x(\log_a a) = x(1)$$
or
$$x = \log_a y.$$

The bases most commonly used for logs are 2, 10, and e where $e \approx 2.72$. Log base 2 $(\log_2)$ is the basis of binary algebra; base 10 $(\log_{10})$ is the basis of the numerical system; and base e $(\log_e)$ is the basis of the Napierian system.

Log_e is referred to as the natural log and can be abbreviated as "ln." The natural log has convenient properties in calculus, and the value of e is described by

$$e = \lim_{h \to \infty}\left(1 + \frac{1}{h}\right)^h$$
$$= 1 + \left(\frac{1}{1!}\right) + \left(\frac{1}{2!}\right) + \left(\frac{1}{3!}\right) + \ldots = 2.7182$$

where "!" indicates "factorial" (*i.e.*, a series product of descending integers); for example,

$$4! = (4)(3)(2)(1) = 24.$$

The following laws apply to logarithms:

$$\log_{10} 10 = 1$$
$$\log_e e = \ln e = 1$$
$$\log_{10} 1 = \ln 1 = 0$$
$$\log_a(x) + \log_a(y) = \log_a(x \cdot y)$$
$$\log_a(x) - \log_a(y) = \log_a\left(\frac{x}{y}\right)$$
$$\log_a\left(x^M\right) = M \log_a(x).$$

Transforming or Linearizing Functions

Non-linear functions can sometimes be simplified for evaluation or fitting models by transforming or linearizing the function. Consider the negative exponential, which is frequently used in population dynamics to describe the survival of animals in a stock as a function of time, and a specified mortality coefficient. The function, shown in Figure 10a, has the form:

$$y = Ae^{-zx}.$$

The estimation of parameters A and z from (x, y) data points requires non-linear regression techniques that until recently were not readily available. However, a linear function can be obtained by transforming the original function using natural logarithms. This simplified function (Figure 10b) can be analyzed using linear regression techniques:

$$\ln y = \ln A - zx \ln e$$
$$= \ln A - zx$$

which is in the linear form of

$$y = b - ax.$$

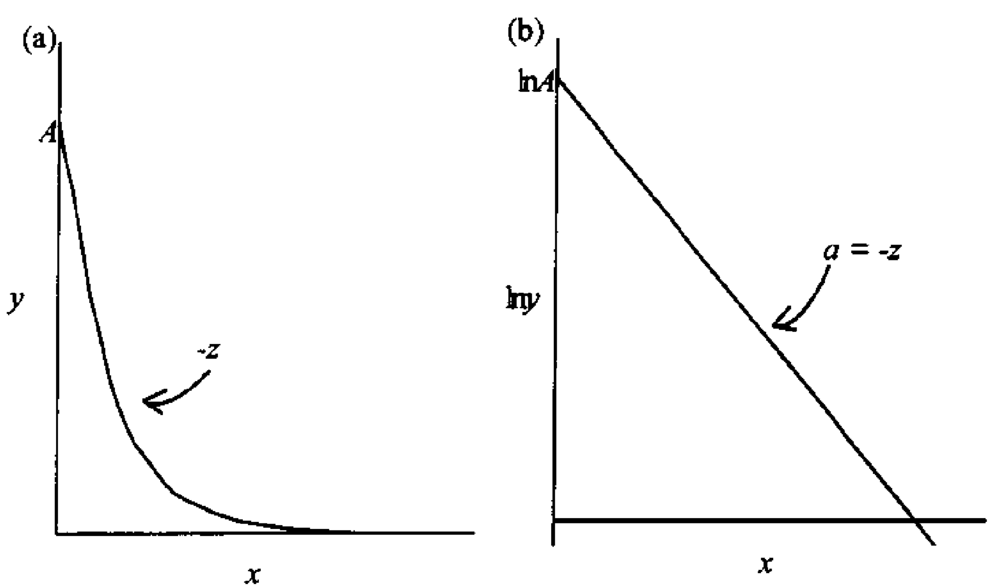

Figure 10. Non-linear and linear forms of $y = Ae^{-zx}$.

The parameter A is obtained by taking the inverse natural log of the y-intercept (b) in the linearized form. The parameter z is obtained directly from the slope a in the linearized form.

A parabolic or dome shaped function is also frequently used in population dynamics to describe the relationship between yield and effort or yield and stock biomass. The general form of the function is:

$$y = bx - ax^2$$

(Figure 11a). Estimation of parameters a and b from (x, y) data points requires non-linear regression techniques. However, transforming the function by dividing both sides of the equation by x, results in the linear form

$$y/x = b - ax$$

(Figure 11b). This form is amenable to simplified regression techniques. The difficulty with this methodology, however, is that the dependent variable (y) has been confounded with the presence of the independent variable (x).

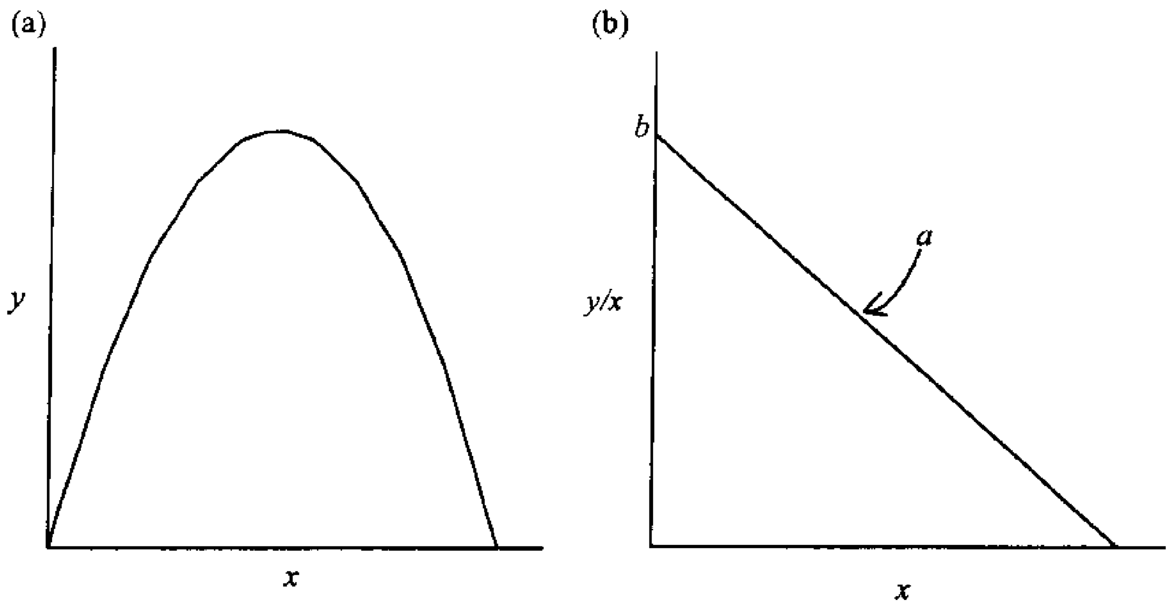

Figure 11. Non-linear and linear forms of $y = bx - ax^2$.

Differential Calculus

Differential calculus is concerned with rates of change. We often refer to a derivative which is a function that measures the rate of change of a quantity. For example, a derivative function may indicate how fast y changes with respect to x at any point x (*i.e.*, the instantaneous slope) (Figure 12). The derivative is indicated by:

$$dy/dx \quad \text{or} \quad y' = f'(x).$$

The slope or derivative of a linear function (*i.e.*, a straight line) is a constant. However, for a parabolic function, the slope is initially positive, decreases to zero at the apex of the parabola, and becomes increasingly negative as it approaches the x-axis intercept.

The following are functions and their derivatives:

Function	**Derivative**
a (constant)	0
ax	a
ax^N	aNx^{N-1}
$\ln x$	$1/x$

$\ln(g(x))$	$g'(x)/g(x)$
e^x	e^x
$e^{g(x)}$	$e^{g(x)}g'(x)$
e^{kx}	ke^{kx}
$g(x)+h(x)$	$g'(x)+h'(x)$
$g(x)-h(x)$	$g'(x)-h'(x)$
$kg(x)$	$kg'(x)$
$(1/k)g(x)$	$(1/k)g'(x)$
$u(x)\cdot v(x)$	$v(x)u'(x)+u(x)v'(x)$
$u(x)/v(x)$	$[v(x)u'(x)-u(x)v'(x)]/[v(x)]^2$.

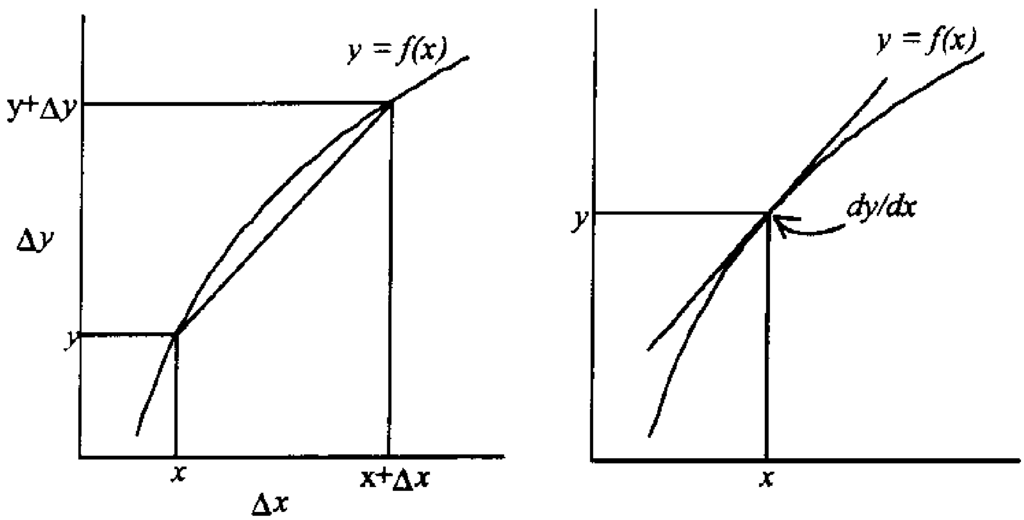

Figure 12. Derivative of the function $y = f(x)$.

Examples of differential calculus are the following:

$$\frac{d}{dx}(x) = 1$$
$$\frac{d}{dx}(3x) = 3$$
$$\frac{d}{dx}(3x+2) = 3+0 = 3$$
$$\frac{d}{dx}(3x^2) = 3\cdot 2x^{2-1} = 6x$$
$$\frac{d}{dx}(3x\cdot x) = (3x\cdot 1)+(3\cdot x) = 6x.$$

Example 1: Given the parabolic function,

$$y = bx - ax^2$$

find the x and y values for the maximum of the function (Figure 13a).

Solution: Given the shape of a parabola, the function is at a maximum when the slope = 0. Take the derivative of the function and set it equal to zero (Figure 13b),

$$\frac{dy}{dx} = y' = b - 2ax = 0$$

$$b = 2ax$$
$$x = \frac{b}{2a}\ .$$

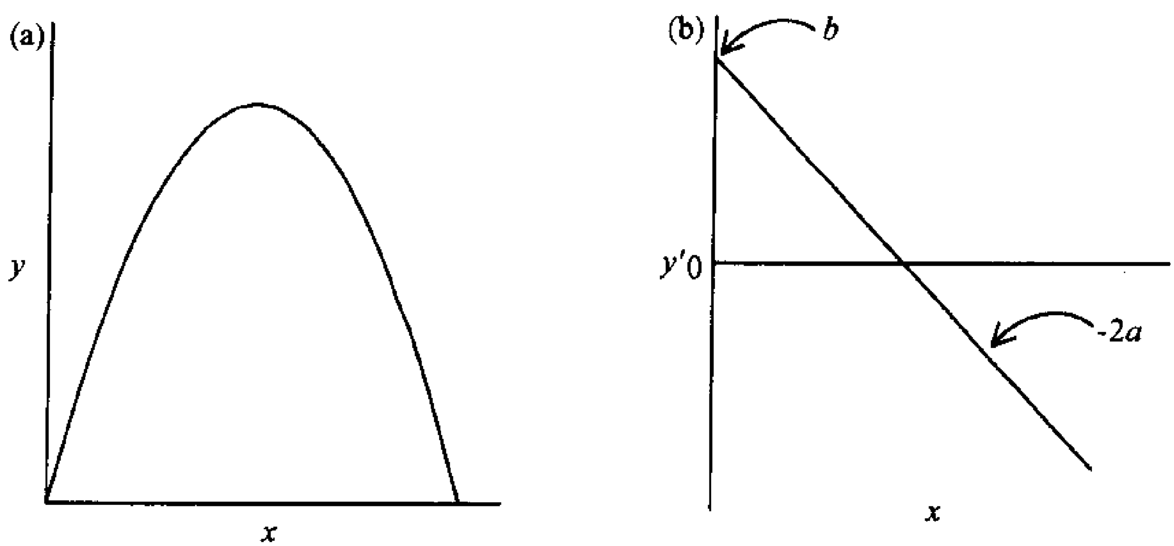

Figure 13. Functions: (a) $y = bx - ax^2$
(b) $y' = b - 2ax$.

Substituting this value of x into the original function to determine the y-max value,

$$y = b\left(\frac{b}{2a}\right) - a\left(\frac{b}{2a}\right)^2$$
$$y = \frac{b^2}{2a} - \frac{ab^2}{4a^2}$$
$$y = \frac{2b^2}{4a} - \frac{b^2}{4a} = \frac{b^2}{4a}\ .$$

Thus, the coordinates of the y-max for the parabolic function are:

$$x = \frac{b}{2a}$$
$$y = \frac{b^2}{4a}\ .$$

Integral Calculus

Integral calculus is concerned with summing quantities that are changing and can be thought of as the inverse of differentiation (*i.e.*, the anti-derivative). An integral can be definite or indefinite. The definite integral is conceptually equivalent to the area under a curve and between specified limits a and b on the curve or function. The area bound by the curve $y = f(x)$, the x-axis, the lower bound $x = a$, and the upper bound $x = b$ (Figure 14). The definite integral of $f(x)$ between a and b is indicated by:

$$\int_{x=a}^{b} f(x)dx$$

where $f(x)dx$ is the integral and $[a, b]$ is the range of integration.

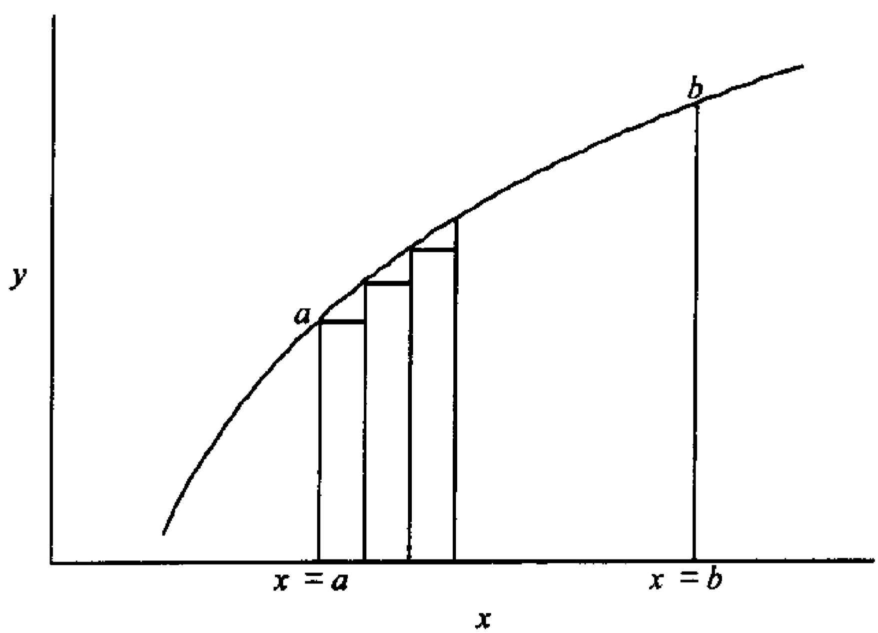

Figure 14. The integral of a function: $y = f(x)$.

It follows that the solution to the definite integral of $f(x)$ between a and b is:

$$\int f(x)dx = [F(x)]\Big|_a^b = F(b) - F(a)$$

where $F(x)$ is a function such that $F'(x) = f(x)$ for $x = a$ and b.

An indefinite integral is a function $F(x)$ such that $F'(x) = f(x)$ (*i.e.*, $F(x)$ is the anti-derivative of $f(x)$). $F(x) + c$, where c is a constant, is also an indefinite integral of $f(x)$ because $[F(x)+c]' =$ $F(x) = f(x)$.

The following are some rules of integration:

$$\int a dx = ax$$
$$\int af(x)dx = a\int f(x)dx$$
$$\int x^M dx = x^{(M+1)} \Big/ (M+1) \qquad M \neq -1$$
$$\int a^x dx = a^x \Big/ \ln(a) \qquad a > 0, a \neq 1$$
$$\int e^x = e^x$$
$$\int e^{ax} = e^{ax} \Big/ a$$
$$\int 1\Big/ x \, dx = \ln(x)$$
$$\int \ln(x)dx = x\ln(x) - x$$
$$\int [f(x) + g(x)]dx = \int f(x)g'(x) - \int g(x)f'(x)$$
$$\int f(x)g'(x) = f(x)g(x) - \int f'(x)g(x)dx.$$

Example 2: Determine the area under the function

$$y = 3x^2 + 5$$

for $x = 2$ to 4.

Solution:

The equation $y = 3x^2 + 5$ is a power function that intersects the y-axis at a value of 5.

The integral of the function is:

$$dy \Big/ dx = 3x^2 + 5$$
$$dy = (3x^2 + 5)dx$$
$$\int_2^4 dy = \int_2^4 [x^3 + 5x]dx = [x^3 + 5x]\Big|_2^4$$

$$= (64 + 20) - (8 + 10)$$
$$= 66.$$

That is, the area under the curve $y = 3x^2 + 5$ between x values of 2 and 4 is 66.

Differential Equations

A differential equation is a function that includes a derivative. Conceptually, it is an equation that includes one variable that changes with respect to another variable:

$$dy/dx = f(x) \text{ or } dy/dx = ay.$$

Typically, differential equations are used to describe rate processes, such as the decay of radioactive materials or the decline in population numbers as a function of stock size. The decline in population numbers as a function of stock size is described by:

$$dN/dt = -ZN$$

where N is the population size,
 Z is the mortality rate, and
 dN/dt is the rate of change of the population size over time.

The equation states that the rate of change of population size is equal to the product of the instantaneous mortality coefficient and the population size.

Solving differential equations is generally complex, but there are some simple solution techniques. The rate equation is solved by the separation of variables technique. The generalized rate equation is:

$$dy/dx = ay.$$

Rearranging and separating variables:

$$1/y\, dy = adx.$$

Integrating both sides of the equation:

$$\int 1/y\, dy = \int adx$$
$$\ln y \Big|_{y_0}^{y_x} = ax \Big|_0^x.$$

Evaluating the integrals and rearranging:

$$\ln(y_x) - \ln(y_0) = ax - a * 0$$
$$\ln(y_x) - \ln(y_0) = ax$$

$$\ln\left(\frac{y_x}{y_0}\right) = ax$$
$$\frac{y_x}{y_0} = e^{ax}$$
$$y_x = y_0 e^{ax}.$$

Applying this solution technique to the population rate equation:

$$\frac{dN}{dt} = -ZN$$
$$\frac{1}{N}dN = -Zdt$$
$$\int \frac{1}{N}dN = \int -Zdt$$
$$\ln N \Big|_{N_0}^{N_t} = -Zt \Big|_0^t$$
$$\ln(N_t - N_0) = -Zt - Z*0$$
$$N_t = N_0 e^{-Zt}.$$

Thus the solution to the rate differential equation is the exponential decay equation. Therefore, we use differential equations to find the value of a quantity (e.g., population size N) when we know how fast it is changing (e.g., $\frac{dN}{dt}$). Note that the derivative of this equation $\left(\frac{d}{dt}\right)$ provides the original differential equation

$$\frac{d}{dt}(N_t) = \frac{d}{dt}\left(N_0 e^{-Zt}\right)$$
$$\frac{dN_t}{dt} = N_0 e^{-Zt}(-Z).$$

Substituting $$N_0 = N_t e^{Zt}$$

yields: $$\frac{dN_t}{dt} = N_t(-Z)$$
$$\frac{dN}{dt} = -ZN.$$

Descriptive Statistics

Statistics can be used to describe the properties of a set of data. Descriptive statistics are used to characterize the central tendencies of the data and the variability around those central measures. If the data are a random sample from a large population, the descriptive statistics of the data set can be used to make inference to the properties of the population sampled.

The mean, median, and mode are used to describe the central tendencies of the data. The mean $(\bar{x})$ is calculated as follows:

$$\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i$$

where x_i are individual values in the data, and
n is the number of data points.

The median is the value half way between extremes in a ranked data set (*i.e.*, 50% of the values are less than and 50% are greater than the median value). The mode is the data point with the greatest number of observations.

Measures of the variability in the data include the following:

Estimated Variance: $s^2 = \left(\frac{1}{(n-1)}\right)\sum_{i=1}^{n}(x_i - \bar{x})^2$

Standard Deviation: $s = \sqrt{\frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n-1}}$

Standard Error: $S.E. = \frac{s}{\sqrt{n}}$.

The Coefficient of Variation ($C.V.$) is a measure of variability relative to the mean

$$C.V. = \frac{s}{\bar{x}} \ .$$

Example 3: Consider a sample of 20 measures of fork length (cm) for fish taken from a RI salt pond.

15.5	18.2
16.3	19.3
18.3	17.9
17.3	16.5
15.8	20.4
14.9	17.8
16.7	19.7
17.3	18.4
16.2	18.6
17.8	17.4

Calculating the descriptive statistics provides:

Sample size = n = 20

Mean = $\bar{x}$ = 17.5 cm

Variance = s^2 = 2.0 cm^2

Standard Deviation = s = 1.4 cm

$$C.V. = s/\bar{x} = 1.4/17.5 = 0.08$$

$S.E. = 0.32$

Median = 17.6.

If the data are grouped into integer categories (Table 1, e.g., any value from 17.0 to 17.9 is assigned to integer category 17), then seven groups emerge.

Table 1. Length data grouped into integer categories.

Group	Number of Observations
14	1
15	2
16	4
17	6
18	4
19	2
20	1

These data can be plotted as a length-frequency histogram (Figure 15), and the mode of the distribution is 17 cm.

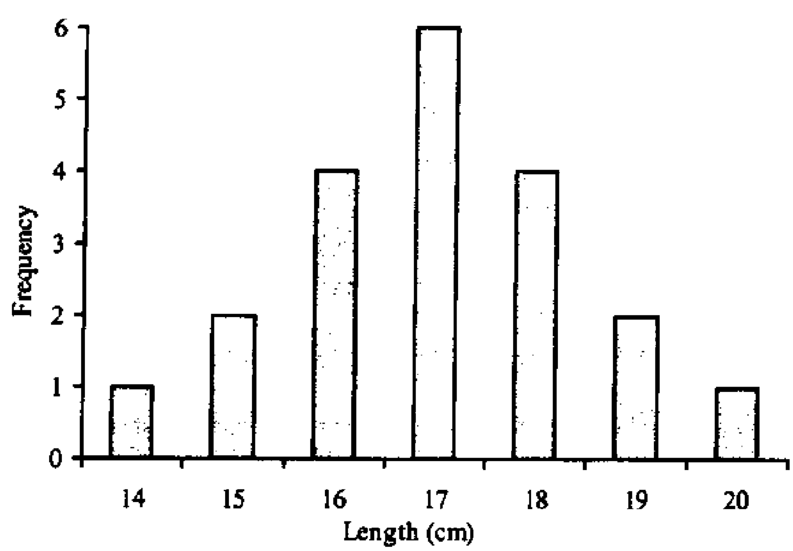

Figure 15. Length-frequency distribution.

Hypothesis Testing

Statistics are also used to compare data sets to evaluate hypotheses. The null or "no difference" hypothesis is considered with tests of significance. Two groups of data can be compared using the descriptive statistics for each group, or more appropriately using a *t*-test that pools the variance in the data sets. Groups of three or more can be evaluated using an Analysis of Variance (ANOVA).

The most rudimentary comparison between two groups of data compares the Confidence Intervals (*C.I.*) around the mean of each data set. If the *C.I.*s overlap, then we fail to reject the null hypothesis; that is, there is no detectable difference between the two groups at the stated significance level. If the *C.I.*s do not overlap, then there is a detectable difference at the stated significance level, and the null hypothesis is rejected.

The *C.I.* for a univariate data set is calculated using the mean, the standard error and a *t* statistic. The *t* statistic is used assuming the data are normally distributed and provides a factor to account for the probability of drawing the incorrect conclusion in the test. The *t* statistic is determined based on sample size and the specified significance level, but stabilizes at about 2 for large sample sizes and a confidence level of 95% (α=0.05). Tables of values for *t* statistics can be found in most introductory statistics books. *C.I.*s are calculated as follows:

$$C.I. = \bar{x} \pm \left(t_{(n-1)(\alpha/2)}\right)(S.E.)$$

where $\bar{x}$ is the mean,

S.E. is the standard error, and

$t_{(n-1)(\alpha/2)}$ is the *t* statistic for sample size *n* and significance level $1-\alpha$.

For the previous fork length data set, the *C.I.* is as follows:

$$17.5 \pm (2)(0.32) = 17.5 \pm 0.64 = 16.9 \le \bar{x} \le 18.1 \text{ cm.}$$

Example 4: Compare the fish length data from Example 3 to a similar data set collected at the same time from a different location. The null hypothesis is that there is no difference in the mean fish size between the two locations. If the mean fork length $(\bar{x})$ from the second pond is 16.4 cm and the *S.E.* is 0.30, then the *C.I.* is

$$16.4 \pm 0.60 = 15.8 \le \bar{x} \le 17.0.$$

The *C.I.*s overlap and we therefore conclude that there is no detectable difference in mean fork length between the two data sets at the $\alpha = 0.05$ level. This can be shown graphically as a bar plot with *C.I.*s around the mean (Figure 16).

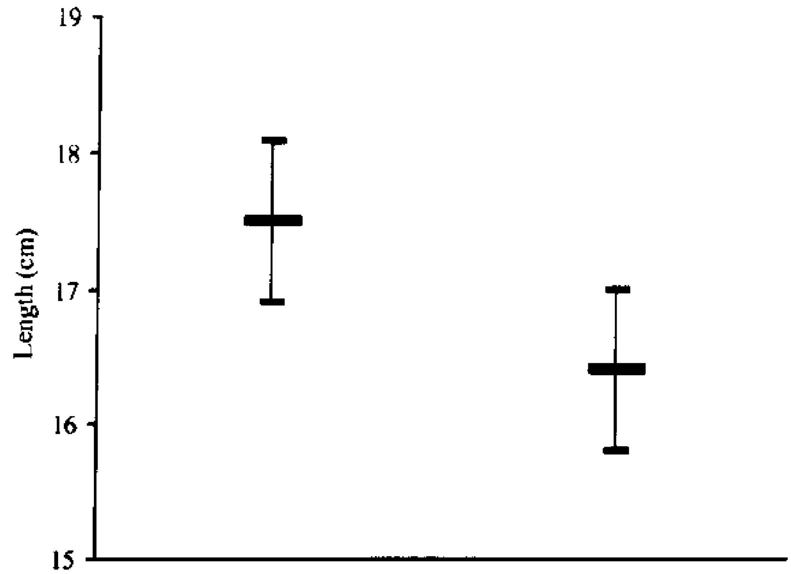

Figure 16. Mean and confidence intervals of fish fork lengths.

Fitting Models to Data

Considerable effort in fish stock assessment is devoted to fitting models to data to make predictions. The concept is referred to as regression analysis. The procedure fits a specified model (function) to a data set by estimating values for model parameters that minimize the sum of squared errors between observed and predicted values. The particular parameters that minimize the sum of the squared errors are considered the "best fit" for that model.

Linear regression analysis is the simplest form of model fitting. More complex functions are sometimes transformed to linear functions for analysis.

Example 5: Consider the estimation of a weight-length relationship for a given species of fish. The general model is as follows:

$$W = aL^b$$

where W = the weight in grams,
L = length in centimeters,
a = a unit conversion coefficient, and
b = the volumetric expansion coefficient.

The function is linearized by taking the natural logarithm of both sides of the equation

$$\ln W = \ln(a) + b * \ln(L)$$

which is analogous to the linear model: $y = a' + b'x$.

Given the following length and weight data, estimate the parameters a and b for this fish species:

W (gr)	L (cm)	ln(W)	ln(L)
9710	100	9.18	4.61
6020	85	8.70	4.44
3610	72	8.19	4.28
2620	65	7.87	4.17
1150	50	7.05	3.91
680	42	6.52	3.74
360	35	5.89	3.56

Using linear regression on the log transformed data, the values of the linearized parameters are:

$$a' = -5.1949$$
$$b' = 3.1273.$$

The non-linear value of a is the anti-ln (a'), and the value of b remains the same. Thus, the final value of the parameters in the model are:

$$W = 0.005545L^{3.1273}.$$

The linear regression and non-linear regression are plotted in Figure 17.

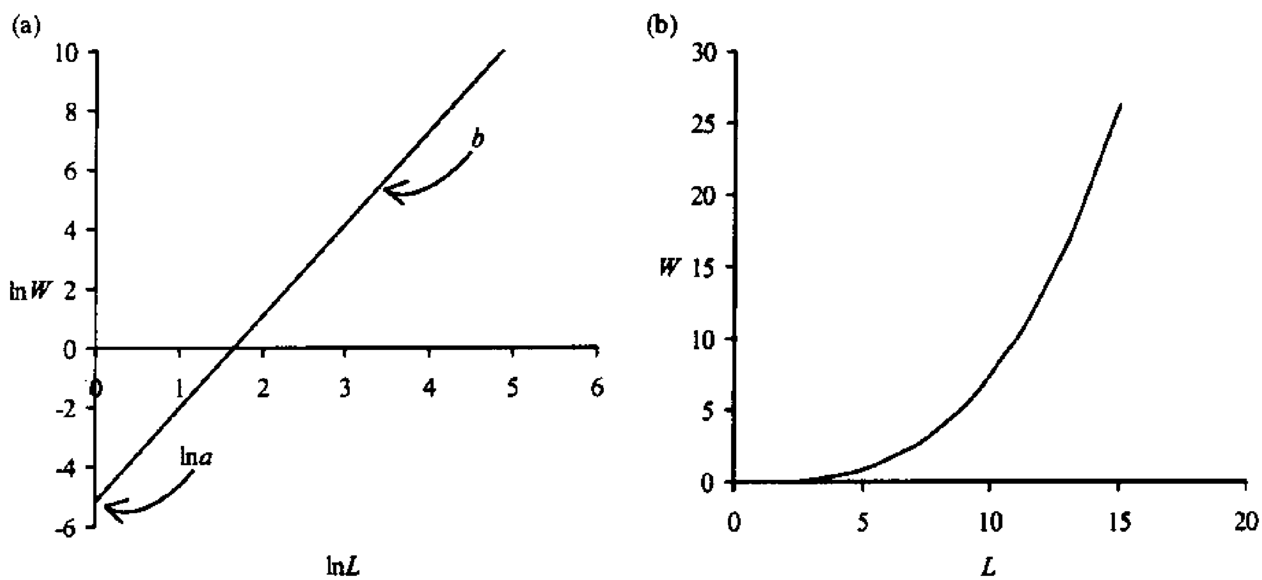

Figure 17. Linear (a) and non-linear (b) models for the weight-length relationship: $W = 0.005545L^{3.1273}$.

Exercises

1. Given the following data points (x, y values), plot the points, fit the linear model $y = ax + b$ and obtain the best estimates of the parameters m and b. On the same graph, plot the predicted model.

x	5	10	15	20	25	30
y	17	32	45	65	57	72

2. Calculate the following.

10^3 10^{-1} 4^0 $8^{2/3}$ $25^{-1/2}$

3. Calculate the following.

$\mathrm{Log}_{10}(42.5)$ $\ln(2.52)$

4. Determine the value of x.

$0.70 = e^{-x}$ $10^4 = e^x$

5. Calculate dy/dx for the following functions.

$y = 3$
$y = e^x$
$y = 4 - 6x$
$y = 5x^2 - 2x$

6. Integrate the following.

$\int_1^2 x^2 dx$

$\int_0^2 e^x dx$

$\int_2^4 (3 + 2x) dx$

7. Compare the following length-frequency distribution using univariate descriptive statistics for each data set. Plot both L-F distributions as histograms on the same graph. Compare the means and the distributions around the means using confidence intervals.

Length (cm)	10	15	20	25	30	35	40	45	50
Group A	3	7	18	29	21	12	6	1	0
Group B	0	5	12	23	32	15	7	2	1

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GROWTH

GROWTH

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Introduction

The prediction of the length or weight of an aquatic animal as a function of age is a critical aspect of fish stock assessment. The growth of a fish, crustacean, or mollusc is rapid at a young age, slows at middle age, and stops at old age. The growth of an individual animal can be quite variable depending on food supply, environmental conditions, and genetic background. Therefore, the analysis of the age and growth of an aquatic animal requires large sample sizes.

Von Bertalanffy (1938) proposed a simple asymptotic function or model to describe the growth of fish by length, (*i.e.*, a curve for which the slope continuously decreases with increasing age, approaching an upper asymptote parallel to the x-axis) (Figure 1). Curves of weight at age also approach an upper asymptote, but form an asymmetrical sigmoid shape with an inflection occurring at a weight equal to about one third of the asymptotic weight (Figure 2).

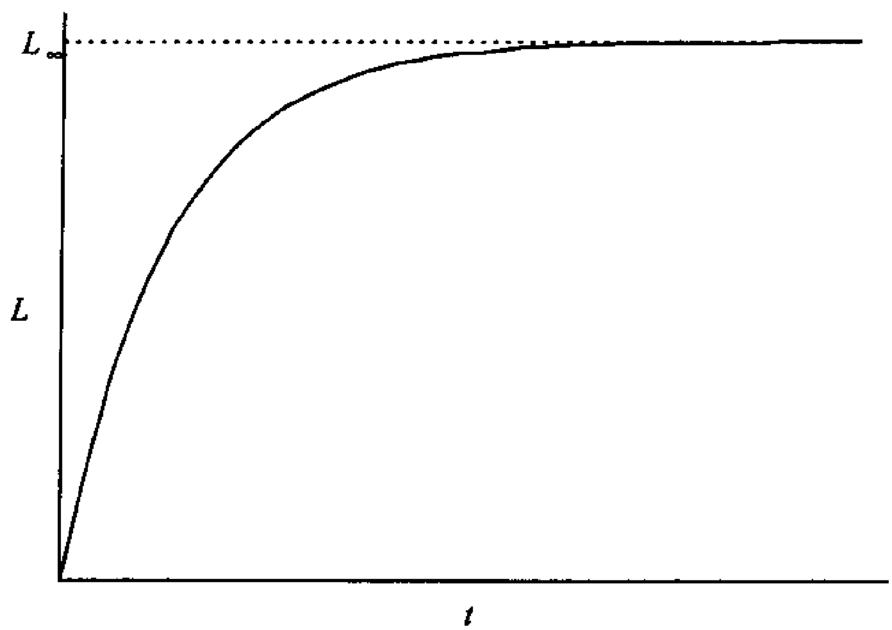

Figure 1. Growth curve for length where L_∞ is the maximum length that can be achieved.

Input data for growth models may include length, weight, or age measurements. Length measurements may include total length, fork length, depth, girth, width, and height. Weight measurements may include total body weight, wet weight, dry weight, organ weight, shell weight, and meat weight. Age can be determined by counting growth rings that form in fish hard parts including scales, otoliths, and fin spines. Growth rings result from seasonal variation in growth. Ages can also be inferred from multi-modal length-frequency distributions (e.g., for tropical fish species that exhibit little seasonal variation in growth and for some crustaceans) using graphical methods and computer based analysis (Figure 3).

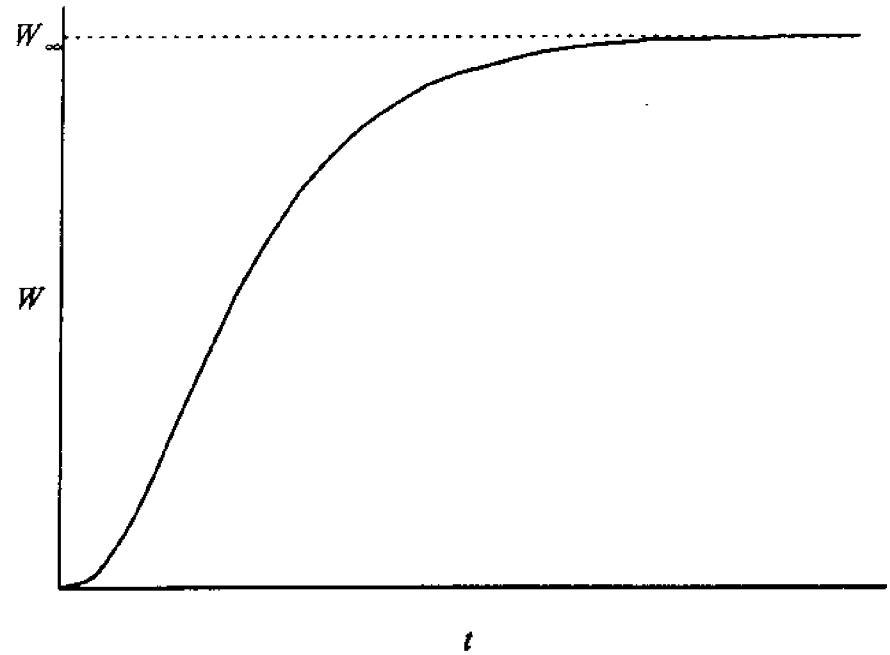

Figure 2. Growth curve for weight where W_∞ is the maximum weight that can be achieved.

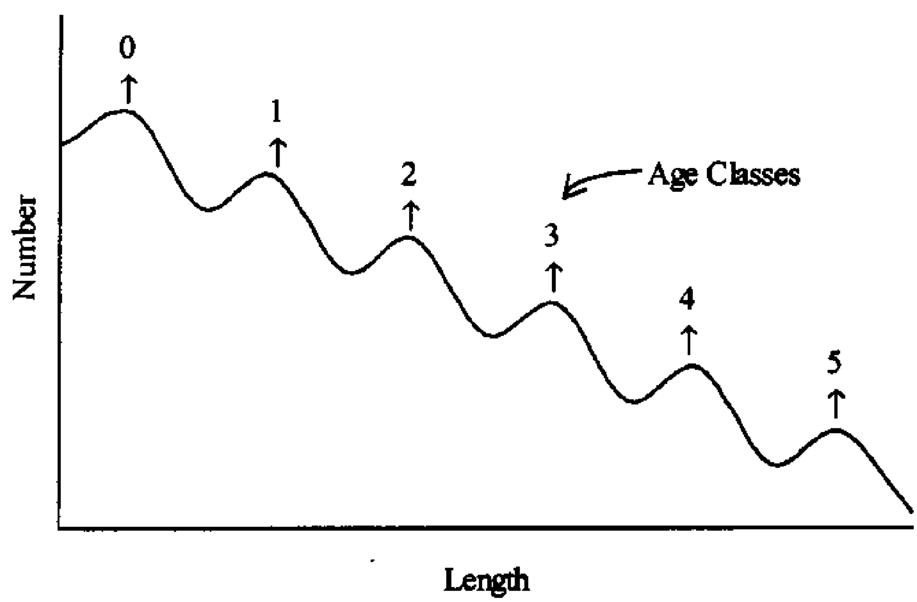

Figure 3. Multi-modal length-frequency distribution.

von Bertalanffy Growth Equation

Estimating Length

The von Bertalanffy growth function states that the rate of growth dL/dt is linearly related to length by the growth coefficient K (Figure 4):

$$dL/dt = K(L_\infty - L_t) = KL_\infty - KL_t$$

where L_∞ is asymptotic length (*i.e.*, the value of L for which growth is zero), and
t is age.

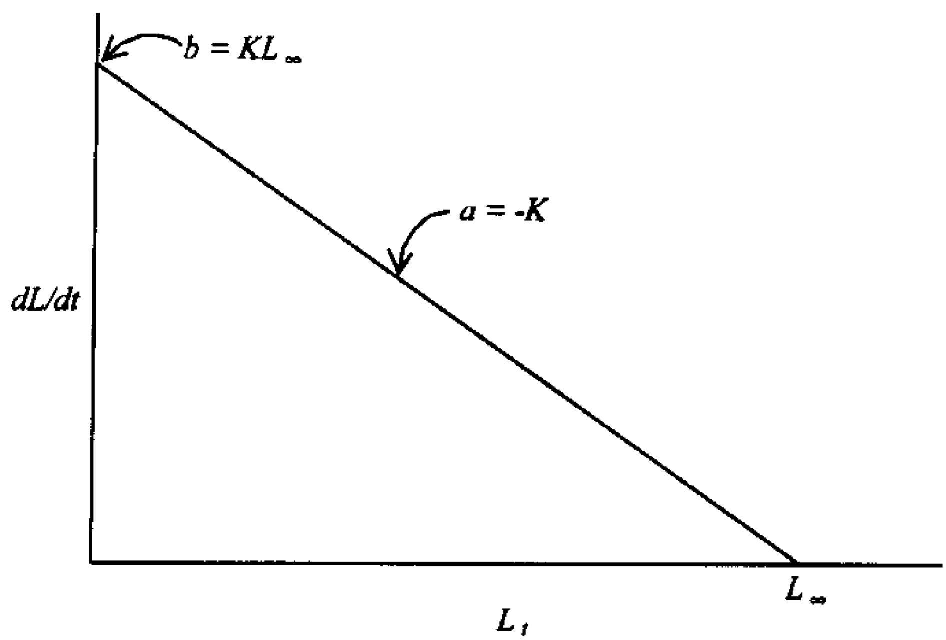

Figure 4. The linear relationship between growth rate (*dL/dt*) and length (L_t).

Note that this function is in the form of the linear model:

$$y = ax + b$$

where $y \equiv dL/dt$
$x \equiv L_t$,
$a \equiv -K$, and
$b \equiv KL_\infty$.

The growth rate equation is a differential equation that is solved by the separation of variables technique. Rearranging and integrating yields:

$$\int\left[\frac{1}{(L_\infty - L_t)}\right]dL = \int K dt.$$

Based on integral tables,

$$-\ln[L_\infty - L_t]\Big|_{L_0}^{L_t} = Kt\Big|_{t_0}^{t}.$$

Assuming $t_0 = 0$ and $L_0 = 0$ and substituting,

$$-\ln\left[\frac{(L_\infty - L_t)}{L_\infty}\right] = Kt.$$

Taking the inverse natural log, or exponential, and rearranging yields:

$$\frac{(L_\infty - L_t)}{L_\infty} = e^{-Kt}$$
$$L_\infty - L_t = L_\infty e^{-Kt}$$
$$-L_t = \left(L_\infty e^{-Kt} - L_\infty\right)$$
$$L_t = L_\infty\left(1 - e^{-Kt}\right).$$

Recall that a simplifying assumption was that at $t_0 = 0$, $L_t = 0$. In reality, most fish at age 0 have a finite length. Therefore, the equation is corrected by specifying t_0, which is the age when length is equal to zero (Figure 5):

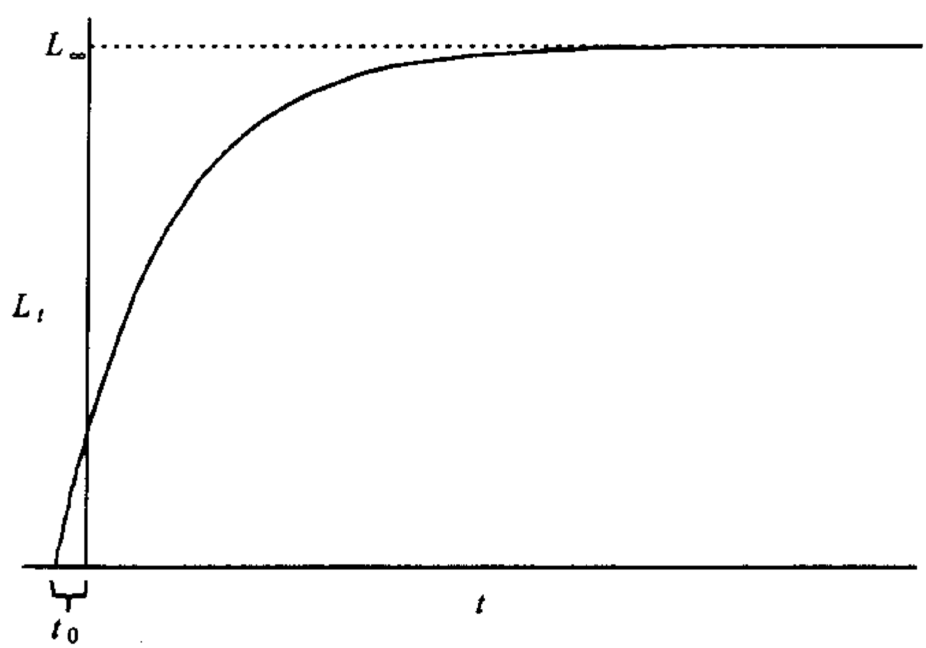

Figure 5. The corrected von Bertalanffy growth function for length $L_t = L_\infty\left(1 - e^{-K(t-t_0)}\right)$.

Using the mean of length at age data for young fish, t_0 is estimated by rearranging the von Bertalanffy function, solving for t_0, and substituting values for t and L_t at the youngest age:

$$t_0 = t + \frac{1}{K} \ln\left(\frac{L_\infty - L_t}{L_\infty}\right).$$

The effect of the growth coefficient K on the growth curve for a given L_∞ is shown in Figure 6. A value of $K = 0.2$ results in a gently ascending curve, whereas a value of $K = 1.0$ results in a rapidly rising curve.

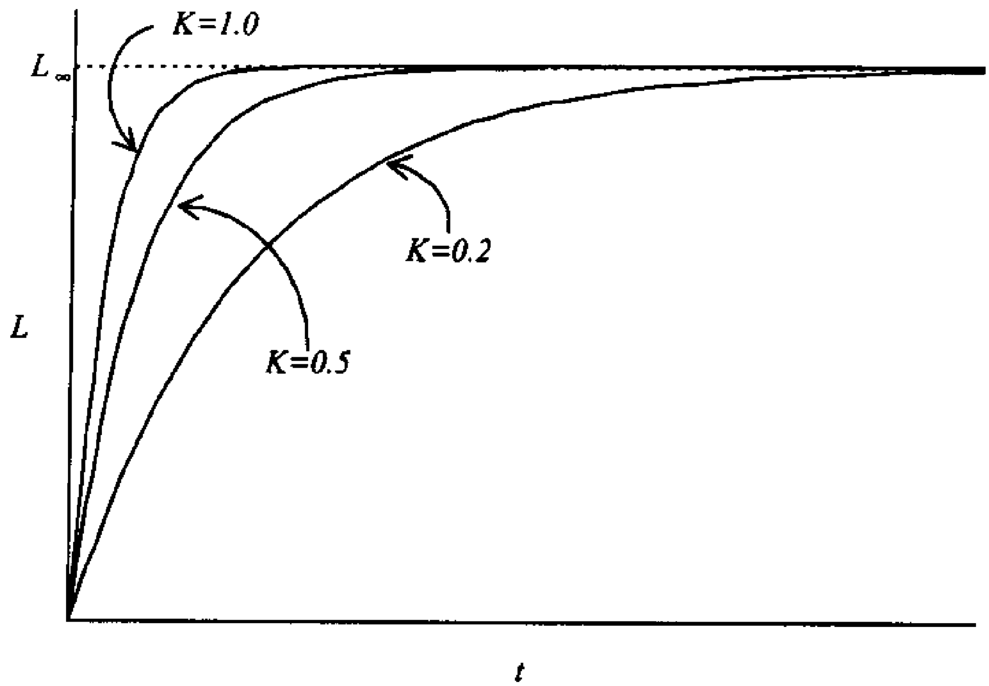

Figure 6. The effect of the value of the growth coefficient (*K*) on the growth curve.

Estimating Weight

The relation between the weight and length of an aquatic animal is expressed as:

$$W_t = aL_t^b$$

where a is a unit conversion coefficient
b is a volumetric expansion coefficient.

Von Bertalanffy assumed a value of $b = 3$ (*i.e.*, isometric or proportionally equal growth in length, breadth, and depth) and proposed the following:

$$W_t = W_\infty\left[1 - e^{-K(t-t_0)}\right]^3.$$

Estimating Age

The age of a fish at any length can be estimated by rearranging the von Bertalanffy equation:

$$t = \frac{1}{K}\ln\left(\frac{L_\infty}{L_\infty - L_t}\right) + t_0 .$$

The span of age $(t_2 - t_1)$ between two lengths L_1, L_2 is estimated as follows:

$$t_2 - t_1 = \frac{1}{K}\ln\left[\frac{(L_\infty - L_1)}{(L_\infty - L_2)}\right].$$

Estimating Growth Equation Parameters

Gulland-Holt Method

There are several classical methods for estimating von Bertalanffy growth model parameters using linear regression techniques. The Gulland-Holt method, based on the original rate equation, assumes uniform growth over an interval between two ages and plots that growth increment against mean length between the two ages (Gulland and Holt 1959).

That is:

$$dL/dt = K(L_\infty - L_t)$$

or

$$\Delta L/\Delta t = KL_\infty - KL_t$$

which has the form of the linear model:

$$y = b + ax$$

where K = slope

L_∞ = y-intercept/K.

Example 1: Given the following set of age and length data, where length represents the mean of a large number of fish measured at each age, calculate the animal growth increment or rate and the mean length-at-age using the Gulland-Holt method.

Age (t)	Length (cm) (L_t)	Growth Increment (cm) ($L_{t+1} - L_t$)	Mean Length (cm) ($L_{t+1} - L_t$)/2
1	25	11	30.5
2	36	6	39.0
3	42	5	44.5
4	47	4	49.0
5	51	2	52.0
6	53	1	53.5
7	54		

The data are fit by a linear model using a regression of the annual growth increment versus mean length (Figure 7). The value of K can be estimated from the negative of the slope (a); in this case $a = -0.4$, so $K = 0.4$. The y-intercept (b), gives an estimate of KL_∞ which in this example equals 22.6 cm. Therefore,

$$L_\infty = b/K = 22.6/0.4 = 56.5 \text{ cm.}$$

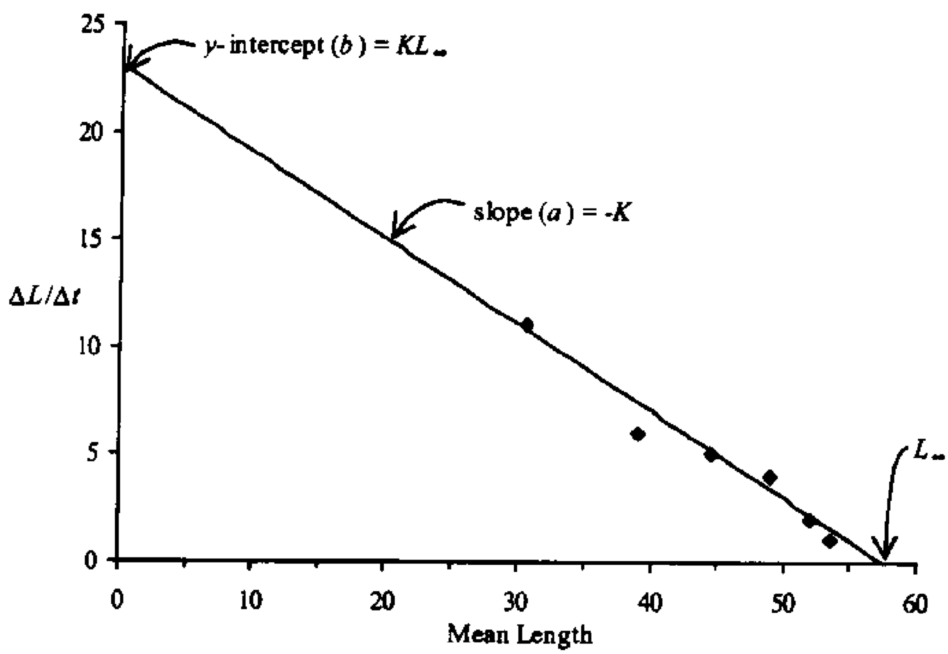

Figure 7. Gulland-Holt plot of growth rate against mean length.

The x-axis intercept (*i.e.*, when $(\Delta L/\Delta t) = 0$), is a verification of the value of L_∞. The value of t_0 is estimated by taking the mean of t_0 obtained for the ages and lengths of the youngest fish when substituted into the rearranged von Bertalanffy function with the parameters $K = 0.4$ and $L_\infty = 56.5$ cm:

for $t = 1$ and $L_t = 25$,

$$t_0 = 1 + 1/0.4 \ln\left(\frac{56.5 - 25}{56.5}\right) = -0.46,$$

and for $t = 2$ and $L_t = 36$,

$$t_0 = 2 + \frac{1}{0.4}\ln\left(\frac{56.5-36}{56.5}\right) = -0.53.$$

The mean t_0 is -0.495.

In summary, the von Bertalanffy growth model parameters for this length and age data are:

$K = 0.4$
$L_\infty = 56.5$ cm
$t_0 = -0.495$.

Ford-Walford Plot

An alternative method to determine the parameters K and L_∞ is the Ford-Walford plot (Walford 1946). This method requires equal time increments (T) between obtaining measures of fish length. The length of a fish at the later time (L_{t+T}) is plotted on the y-axis against the length of the fish at an earlier time (L_t) on the x-axis.

The derivation of the Ford-Walford plot is based on the von Bertalanffy function for ages t and $t+T$:

$$L_t = L_\infty\left(1-e^{-Kt}\right) \text{ and } L_{t+T} = L_\infty\left(1-e^{-K(t+T)}\right).$$

Subtract L_t from L_{t+T} and solve for L_{t+T}:

$$\begin{aligned} L_{t+T} - L_t &= L_\infty\left[1-e^{-K(t+T)}\right] - L_\infty\left[1-e^{-Kt}\right] \\ L_{t+T} - L_t &= L_\infty e^{-Kt}\left[1-e^{-KT}\right] \\ L_{t+T} &= L_\infty e^{-Kt}\left[1-e^{-KT}\right] + L_t. \end{aligned}$$

Substituting $L_\infty e^{-Kt} = -(L_t - L_\infty)$ into the above equation and simplifying yields:

$$\begin{aligned} L_{t+T} &= -(L_t - L_\infty)(1-e^{-KT}) + L_t \\ L_{t+T} &= L_\infty\left(1-e^{-KT}\right) + L_t e^{-KT}. \end{aligned}$$

Note that this equation has the form of the linear model $y = b + ax$
where $y = L_{t+T}$,
$x = L_t$,
$a = e^{-KT}$, and
$b = L_\infty\left(1-e^{-KT}\right)$.

The plot of L_t versus L_{t+T} forms a straight line regression that intersects the 45 ° line (Figure 8). The 45 ° line describes the function $L_{t+T} = L_t$, which indicates no growth between the earlier and later measurements. The intersection between the no growth function and the data regression line is L_∞.

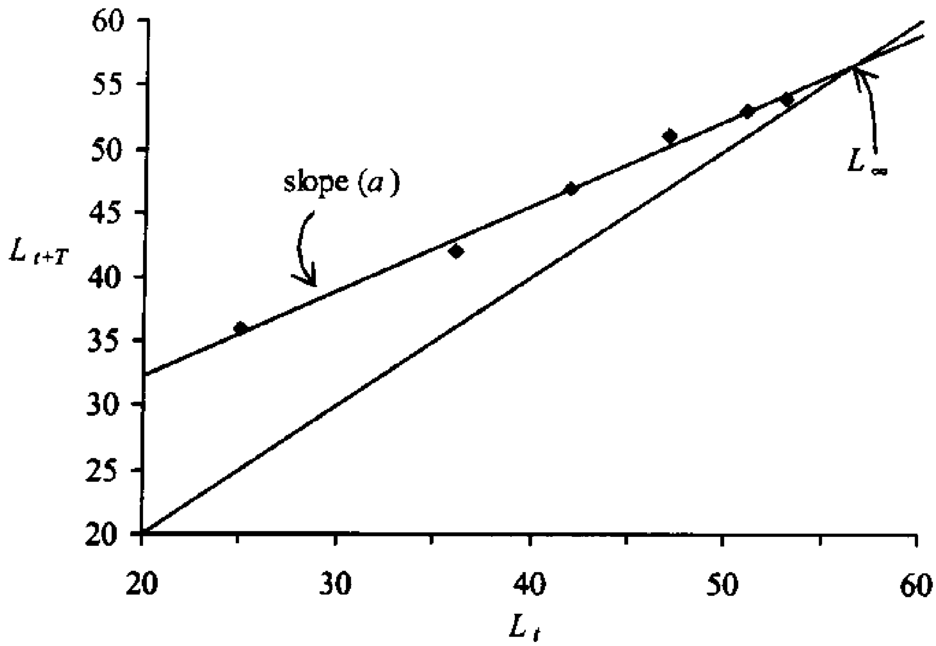

Figure 8. Ford-Walford plot of growth at time $t+T$ versus length at time t.

The slope of the regression line is $a = e^{-KT}$. The value of K can be obtained by rearranging this equation to

$$K = -\left(\ln(a)/T\right).$$

If $T = 1$, this equation simplifies to $K = -\ln(a)$.

Example 2: Given the age-length data in Example 1, use the Ford-Walford plot method to estimate K and L_∞.

The data are fit by a linear model using a regression of the length of a fish at the later time versus length of the fish at an earlier time (Figure 8). To solve for L_∞, a 45° line (line of no growth) is added to the graph and then the no growth equation ($y = x$) and the regression line ($y = 18.94 + 0.667x$) are set equal to each other:

$$x = 18.94 + 0.667x$$
$$0.333x = 18.94$$
$$x = 18.94/0.333$$
$$x = 56.88.$$

Therefore, the value of L_∞ is 56.88.

The value of K is obtained using the equation:

$$K = -\ln(a).$$

The slope a is 0.667, therefore $K = -\ln(0.667) = 0.4$. The t_0 value is calculated in the same manner as before.

In summary, the von Bertalanffy growth model parameters for this length and age data are:

$$K = 0.4$$
$$L_\infty = 56.8 \text{ m}$$
$$t_0 = -0.476.$$

Non-linear Regression Methods

A more direct method of fitting the von Bertalanffy growth function to length or weight and age data is to use non-linear regression methods. Non-linear regression methods use a computer algorithm to iteratively fit new parameter values to the model until the sum of the squared differences between the observed and predicted values is minimized. The **Solver** routine in *Microsoft Excel* is a useful tool to conduct such an analysis on data entered in a spreadsheet.

Example 3: The following spreadsheet was constructed in *Excel* given the previous length-age data set, and estimated values for K and L_∞ of 0.4 and 57, respectively. Note that the values of L_∞ and K are set aside as adjustable cells (cells C12 and C13), so that **Solver** can change the values as it attempts to minimize the sum of the squared differences (cell D9).

Column	A	B	C	D
Rows 1	AGE	L_{obs}	L_{pre}	$(L_{obs}-L_{pred})^2$
2	1	25	-	-
3	2	36	-	-
4	3	42	-	-
5	4	47	-	-
6	5	51	-	-
7	6	53	-	-
8	7	54	-	-
9				SUM(D2:D8)
10				
11				
12		K =	-	
13		L_∞ =	-	
14				
15				

Gompertz Growth Equation

Another growth curve that is sometimes used to describe the increase in fish weight with age is the Gompertz function (Ricker 1979):

$$W_t = W_0 e^{[G(1-e^{gt})]}$$

where W_t is weight at time t,

t is the age of fish,

W_0 is a hypothetical weight at $t = 0$,

G is an instantaneous growth parameter when $t = 0$, and

g is the instantaneous rate of decrease of the instantaneous rate of growth (G).

The Gompertz function has a sigmoid shape with an upper asymptote of $W_0 e^g$ at $t = \infty$ and a lower asymptote of $W = 0$ at $t = -\infty$. The inflexion point is $\frac{1}{e}(W_\infty)$.

The Gompertz function is used primarily to describe data on weight at age, especially at young ages where growth is rapid. Note that there are three parameters to be estimated (W_0, G, and g), requiring non-linear estimation techniques for large data sets.

Exercises

1. Given the following age-length data set for mackerel, determine K, L_∞, and t_0 using Ford-Walford and non-linear methods.

Age	Length
1	15.1
2	22.7
3	27.5
4	32.3
5	34.8
6	37.1

2. Given the following lengths at age for herring, determine K, L_∞, and t using Gulland-Holt and non-linear methods.

Age	Length
3	25.7
4	28.4
5	30.2
6	31.7
7	32.8
8	33.7
9	34.4
10	34.9
11	35.6
12	36.0
13	35.9
14	37.0
15	37.7

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ESTIMATION OF MORTALITY

ESTIMATION OF MORTALITY RATES

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Introduction

The general diagram of the dynamics of exploited fish stocks can be represented by an input-output diagram (Figure 1). Recruits into the stock and growth add to the total abundance and weight of the stock and are therefore considered inputs. Total losses from the stock are measured in two terms and are considered outputs. Natural mortality (M) is a measure of mortality resulting from natural causes (e.g., diseases, pollution, predation, aging), and fishing mortality (F) is a measure of mortality attributable to human harvest and discards.

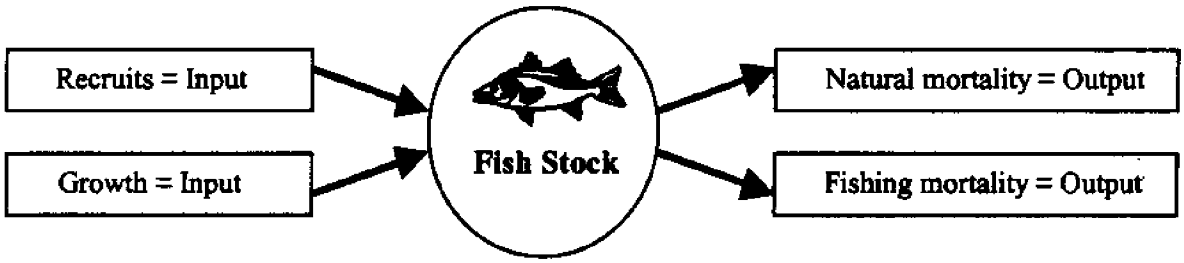

Figure 1. Schematic diagram of inputs and losses to a stock.

Mortality represents losses to a stock and is expressed as the rate of change of the size of a stock or a portion of the stock (e.g., cohort). It is generally most convenient to deal with instantaneous rates of change; *i.e.*, the rate at which the numbers in the population are decreasing. The term "instantaneous" infers that the number of fish that die in an "instant" is at all times proportional to the number present. The rate of loss can be expressed as:

$$\frac{dN}{dt} = -ZN$$

where $Z = F + M$ is defined as the total instantaneous mortality coefficient. Rearranging this equation:

$$-\left(\frac{dN}{ZN}\right) = dt$$

or

$$-\left(\frac{1}{ZN}\right)dN = dt \cdot$$

Integrating the left side of this equation between N_0 and N_t yields:

$$\int_{N_0}^{N_t} \frac{1}{-ZN} dN = -\left(\frac{1}{Z}\right) \ln\left(\frac{N_t}{N_0}\right)$$

and integrating the right side between t_0 and t yields:

$$\int_{t_0}^{t} dt = t - t_0 ,$$

which is equal to t when $t_0 = 0$. Therefore,

$$\ln\left(\frac{N_t}{N_0}\right) = -Zt \cdot$$

Rearranging and solving for N_t yields:

$$N_t = N_0 e^{-Zt} .$$

This solution is known as the exploited cohort equation or decay equation because it describes the decline in numbers over time. The parameter N_0 is the number of animals in the population at time 0 and N_t is the number present at time t. The parameter Z is the total instantaneous mortality rate which can be separated into natural (M) and fishing (F) mortality (*i.e.*, $Z = F + M$). Figure 2 shows the relationship between N_t and N_0 over time for various levels of mortality.

Example 1: If the instantaneous mortality rate is 2 (*i.e.* $Z = 2$) and the initial population size (N_0) is 1 million fish, how many will be alive at the end of the year.

If the year is apportioned into 365 days (*i.e.*, the "instant" time is one day), then 2/365 or 0.548% of the population will die each day. On the first day of the year, 5,480 fish will die (1,000,000 x 0.00548), leaving 994,520 alive. Similarly, 5,450 fish will die on the second day. If p is the proportion of fish that dies every day, then $q = 1-p$ is the proportion of fish that survives. Table 1 describes the decrease in numbers over time.

At the end of the year, [1,000,000 x $(1-0.00548)^{365}$] = 134,566 fish remain alive = N_t.

Table 1. Proportional decrease in population over time.

Time	Number of fish dead	Number surviving
1^{st} day	PN_0	qN_0
2^{nd} day	$p*q*N_0$	q^2N_0
3^{rd} day	$p*q^2*N_0$	q^3N_0
....	....	....
n^{th} day	$p*q^{n-1}*N_0$	q^nN_0

If we had instead selected a smaller 'instant' of time, say an hour, 0.0228% of the population would have died by the end of the first time interval (an hour), leaving [1,000,000 x $(1-0.000228)^{8760}$] or 135,673 fish alive at the end of the year.

As the 'instant' of time becomes shorter and shorter, the exact answer to the number of animals surviving after 1 year is determined using the survival equation mentioned above, or, in this example:

$$N_t = N_0 e^{-Zt} = 1{,}000{,}000 * e^{-2} = 135{,}335 \text{ fish.}$$

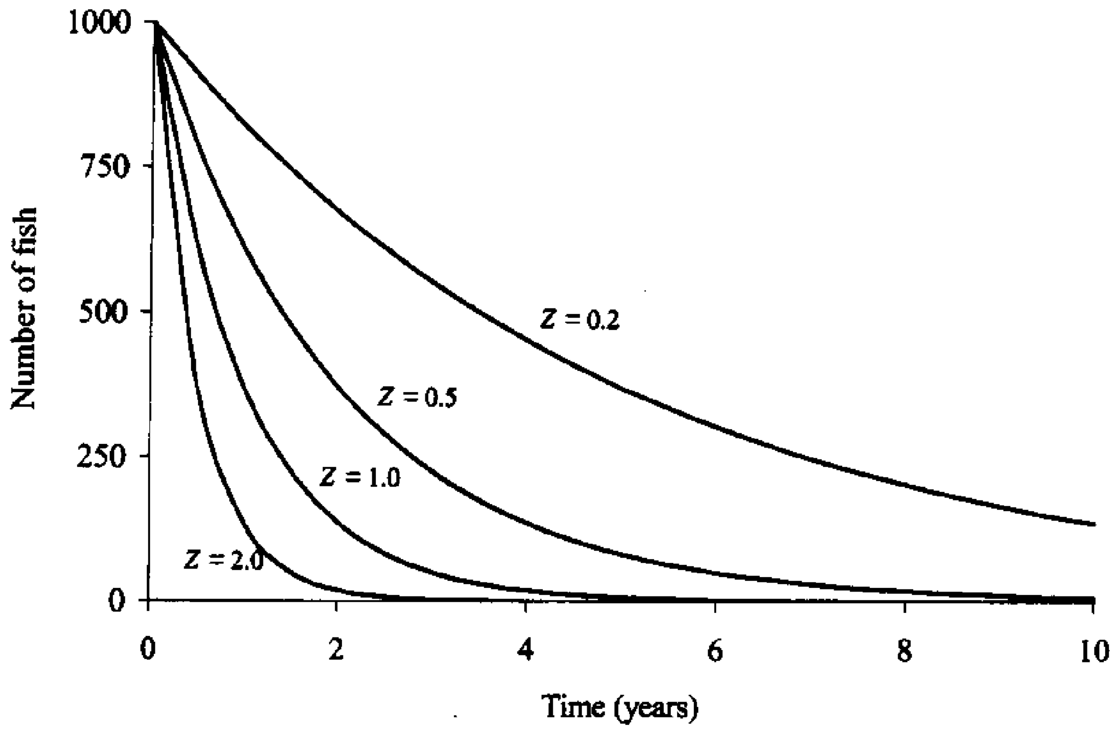

Figure 2. Exponential decay curves for $Z = 0.2, 0.5, 1.0,$ and 2.0 with recruitment $N_0 = 1000$ fish.

The slope of the curve is also called the survival rate (S), and is equivalent to the number at the end of time t divided by the number at the start of time t:

$$S = \frac{N_t}{N_0} = e^{-Zt} \ .$$

Solving for Z gives:

$$Z = -\frac{\ln(S)}{t} \ .$$

The total instantaneous rate of mortality equals the sum of the instantaneous rates of natural and fishing mortality ($Z = F + M$). Similarly, the total survival rate (S_T), equals the product of the survival rates from each source of mortality (*i.e.*, natural and fishing).

$$N_t = N_0 e^{-Zt} \Rightarrow N_t = N_0 e^{-(F+M)t}$$

and

$$S_T = S_F \times S_M \ .$$

The exact formulation used to calculate mortality rates depends on the relationship between natural and fishing mortality. A Type 1 fishery exists when natural mortality occurs at a time of the year other than the period of harvest (Figure 3). A Type 2 fishery exists when fishing and natural mortality occur simultaneously (Figure 4). Type 2 fisheries are more common.

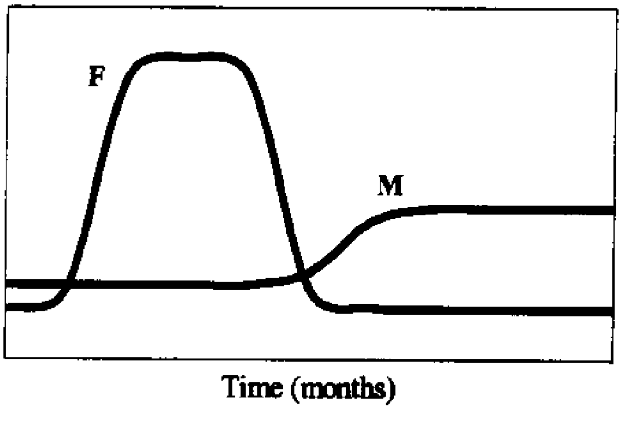

Figure 3. Type 1 fishery.

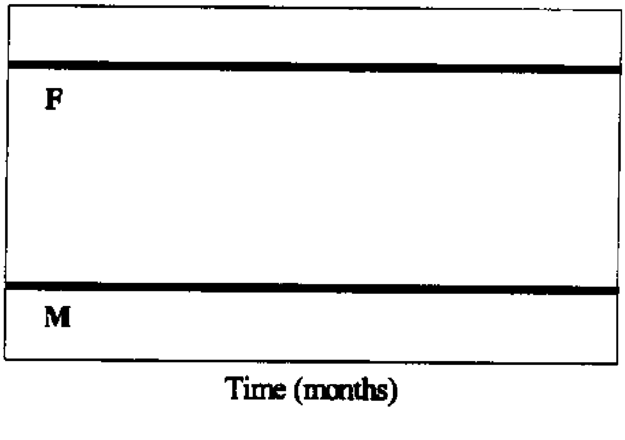

Figure 4. Type 2 fishery.

For Type 2 fisheries, the natural and fishing mortality rates can be added together to determine total mortality rate, which can then be used in computations. For Type 1 fisheries, however, population size must be computed in two steps using the two mortality rates separately. Under the scenario shown in Figure 3, fishing mortality early in the year would reduce the population size upon which natural mortality would occur at the end of the year. To find the end of the year population size, apply fishing mortality to the initial population size to find a mid-year population size. The mid-year population size is then the starting population size upon which natural mortality is applied.

Annual mortality can also be measured in annual rates of exploitation, which can be defined as the annual percent removal rate (U). Exploitation rate can also be defined as the finite proportion of the population harvested (U = Catch/population size).

In a Type 1 fishery, the exploitation rate U is calculated as

$$U = 1-e^{-Z}.$$

In a Type 2 fishery, the exploitation rate is calculated as

$$U = (F*A)/Z$$

where $A = 1-e^{-Z}$ is the annual mortality.

Example 2: Consider a population of N_0 = 1000 fish at the start of the year. At the end of the year, N_t = 358 fish. During the year, 321 fish were caught. Calculate S, Z, A, U, F, and M during the year.

Solution:

$$S = \frac{N_t}{N_0} = \frac{358}{1000} = 0.358$$
$$Z = -\ln(S) = -\ln(0.358) = 1.027$$
$$A = 1 - S = 0.642$$
$$U = \frac{C}{N_0} = \frac{321}{1000} = 0.321$$
$$F = \frac{Z*U}{A} = \frac{(1.027*0.321)}{0.642} = 0.514$$
$$M = Z - F = 1.027 - 0.514 = 0.513.$$

Estimating Total Mortality from Catch Curve Analysis

Edser (1908) was the first to point out that when catches of North Sea Plaice were grouped into size-classes of equal breadth, the plot of the logarithms of the numbers of fish in each class had a steeply ascending limb, a dome shaped upper portion, and a long descending right limb, which was nearly straight through its entire length. This was soon recognized as a convenient method of representing catches graphically, and later became known as Catch Curve Analysis (CCA). This analysis is simply a graphical representation of the numbers of survivors plotted against age.

In CCA, a linear regression of catch as a function of age is fit using the function

$$\ln(N_t) = aX + b$$

where X is time in years.

The absolute value of the slope, a, is equal to the total mortality Z. The variable b is the y-intercept. It is important that CCA be performed only on the portion of the stock that is fully recruited to the fishing gear. Also, the plus group is often not considered in catch curve analyses.

CCA is most appropriate for data from a single year class collected over time. If CCA is used for a single year's catch, it should only be done when there is no interannual trend in recruitment.

Example 3: Perform a catch curve analysis using the catch-at-age data for striped bass (*Morone saxatilis*) on the Atlantic coast from Maine to North Carolina (both landings and discards) reported by the Atlantic States Marine Fisheries Commission.

Age	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15+
Numbers	0.5	98	658	664	551	476	456	216	143	71	44	48	13	4.6	2.6
Ln(N)	-0.69	4.59	6.48	6.49	6.31	6.16	6.12	5.37	4.90	4.26	3.77	3.86	2.58	1.52	0.95

Figures 5 and 6 show the results of a catch curve analysis using this data. The CCA suggests that the average total mortality in 1996 on fully recruited cohorts is 0.51 (absolute value of -0.5104). The Atlantic States Marine Fisheries Commission assumed a natural mortality M between 0.15 and 0.2. Therefore, fishing mortality (F) was estimated to range between 0.31 and 0.36. The fisheries management target is set at $F = 0.31$.

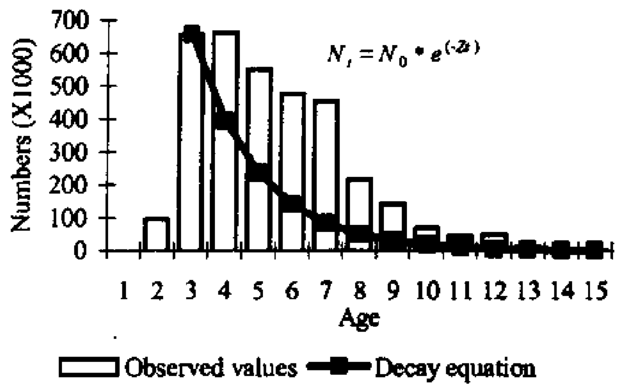

Figure 5. Observed and predicted population size.

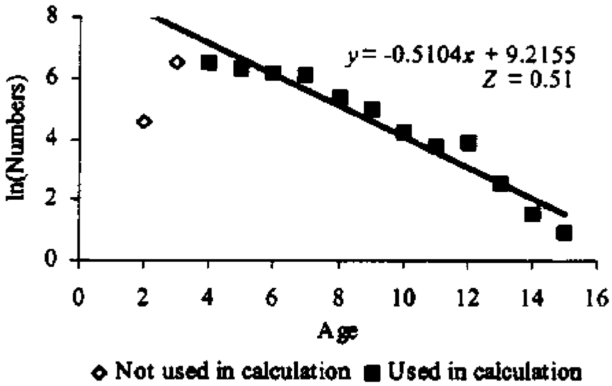

Figure 6. Catch curve analysis.

Catch curves are very simple to calculate, but they hide numerous assumptions that one has to consider when interpretting the results. Baranov (1914) described the assumptions involved in the interpretation of the catch curve analysis:

1. The survival rate is uniform with age, over the range of age-groups in question.
2. Since the survival rate is the complement of total mortality rate, and total mortality is composed of fishing and natural mortality, this will usually mean that each of these, individually, is uniform.

3. There has been no change in mortality rate with time.
4. The sample is taken randomly from the age-groups involved.
5. The age-groups in question were equal in numbers at the time each was being recruited to the fishery (constant recruitment).

If these conditions are satisfied, the right limb is a curve of survivorship which is both age-specific and time-specific. Two principal exceptions should always be kept in mind: 1) the decrease in vulnerability to fishing with age and 2) the consequent tendency toward an increase in survival rate will not be reflected in the catch curve and, in some instances, will introduce a bias in the estimates.

The most common application of the catch curve is estimating mortality on a cohort from research survey data. If we collect a random sample using a trawl from a fish stock at a fixed time t, the mean catch at age per tow from one year to another can be used to estimate total mortality. The sample is characterized by its catchability which can be defined as the "catch capacity of the gear per one unit of effort". The relationship between CPUE or survey index (I) and the stock (N) can be written as:

$$I_{a,t} = qN_{a,t}$$

where $I_{a,t}$ is the survey index or mean catch per tow,
q is the catchability coefficient, and
$N_{a,t}$ is the cohort size at age a and time t.

The total mortality can be calculated from data for two or more consecutive years as follows:

$$Z = -\ln\left(\frac{N_{a+1,t+1}}{N_{a,t}}\right).$$

Estimating Total and Fishing Mortality from Tagging Experiments

One method used to estimate parameters of a fish population is by tagging or marking a representative sample of the population, releasing them, and resampling at a later date to see what fraction of the population is tagged.

Tagging fish was first done to study movement and migration of individuals, but Petersen (1896) realized that tagging could also be used to measure population size and mortality rates. The principal kinds of estimates that can be obtained from marking studies are:

1. Rate of exploitation
2. Size of the population
3. Survival rate of the population from one time interval to the next; most usefully, between times one year apart
4. Rate of recruitment to the population (Ricker 1975).

Not all mark and recapture experiments provide all this information. Estimating population size in a marine environment using mark-recapture techniques can be difficult, sometimes resulting in biased or imprecise estimates because of small capture and recapture probabilities. However, these problems are not encountered as often when estimating mortality rate.

One Time Releases or Single Census (Petersen Type)

Prior to a fishing season, C_1 fish are captured, marked and released; subsequently a sample of C_2 fish of which R were previously marked, is taken during the fishing season. Estimates of population size and exploitation rate of the population are given by:

$$\hat{N} = \frac{C_2}{U} = \frac{C_1 * C_2}{R} \quad \text{and} \quad U = \frac{R}{C_1}.$$

Variance for these estimates can be estimated with the following equations:

$$V\hat{a}r(\hat{N}) = \frac{C_1^2 C_2 (C_2 - R)}{R^3} \quad \text{and} \quad V\hat{a}r(U) = \frac{R(C_2 - R)}{C_1^2 C_2}.$$

The exploitation rate is converted to an instantaneous rate using either

$$U = 1 - e^{-Z}$$

for a Type 1 fishery, or

$$U = \frac{F}{Z}(1 - e^{-Z})$$

for a Type 2 fishery. If natural mortality (M) is known (or assumed), estimates of fishing mortality can be found by substituting $Z = F + M$ into either of these equations.

Example 4: The Northeast Utilities Service Company (NUSCO) in Waterford, CT has collected and tagged lobsters in Long Island Sound since 1978. Commercial fishers and others recaptured lobsters and returned the tags to the NUSCO. Recapture data for individual years of tagging were used to determine annual exploitation rates. The results of this study are shown in the following table. Given this information, calculate fishing mortality rate for each year, assuming a natural mortality rate of $M = 0.15$.

Because fishing mortality and natural mortality occur at the same time, the instantaneous rate of mortality will be calculated using the Type 2 fishery equation:

$$U = \frac{F}{Z}(1 - e^{-Z}).$$

Setting $M = 0.15$, we can iteratively solve for F in *Microsoft Excel* using the **Goal Seek** or **Solver** procedures. For example in 1996, $U = 0.16$; thus $F = 0.19$ with an $M = 0.15$. In 1997, $U = 0.27$ therefore $F = 0.34$ and $M = 0.15$.

Year	Tagged	Recaptured	Rate of Exploitation
1986	5797	1194	0.21
1987	5680	1356	0.24
1988	6836	1727	0.25
1989	6436	1235	0.19
1990	5741	1066	0.19
1991	6136	1109	0.18
1992	9126	1842	0.20
1993	8177	1708	0.21
1994	7533	1974	0.26
1995	5307	963	0.18
1996	6221	997	0.16
1997	6102	1665	0.27

Recaptures in a Series of Years from a Single Year Release

In more complex tagging experiments, a known number of tagged fish are released at one time and recaptured over a period of several years. When more than two years of recaptures are available from a single release, S and Z can be estimated using a method similar to Catch Curve Analysis.

If N_0 fish are tagged initially, the number of tagged fish remaining in the population at the beginning of the r^{th} time interval is N_r, where

$$N_r = N_0 e^{[-(F+M)T]}$$

where F is fishing mortality rate,
M is natural mortality rate,
N_0 is the total number of fish tagged, and
T is time.

The number of tagged fish captured in the r^{th} time interval (n_r) is given by:

$$n_r = \left[F/(F+M)\right][N_r]\left[1 - e^{-(F+M)T}\right].$$

Substituting for N_r yields

$$n_r = \left[F/_{(F+M)} \right] [N_0] [e^{-(F+M)T}] [1 - e^{-(F+M)T}] \ .$$

Taking the natural log of both sides and substituting Z for $(F+M)$ gives:

$$\ln(n_r) = -ZT + \left[\ln\left(\frac{FN_0}{Z} \right) + \ln\left(1 - e^{-ZT}\right) \right] \ .$$

This equation can be rewritten in linear form as

$$\ln(n_r) = aT + b$$

where a is the negative of total mortality ($-Z$) (since a will be negative)
T is time in years.

The natural log of recaptures plotted against time provides a negatively sloped linear function, similar to CCA.

Example 5: A sample of 1000 winter flounder were tagged in Narragansett Bay in the winter of 1990. Returns from those tagged fish from 1991 to 1994 are shown below. Determine estimates of total mortlaity (Z) and fishing mortality (F) over the time period, assuming a natural mortality rate of $M = 0.30$.

Year	n_r	$\ln(n_r)$
1991	270	5.6
1992	36	3.6
1993	6	1.8
1994	1	0

Conducting a linear regression of $\ln(n_r)$ as a function of time in years (Figure 7) provides an estimate of total mortality of $Z = 1.8$. If $M = 0.3$, then $F = 1.5$, an extremely high rate of fishing mortality.

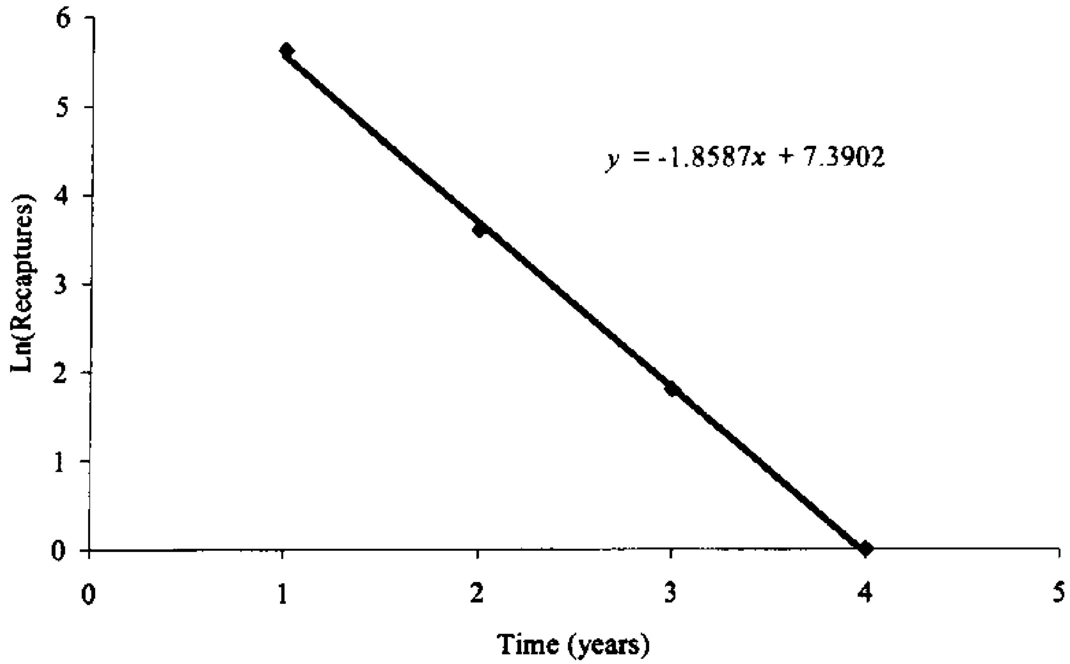

Figure 7. Ln(recaptures) versus time for tagged winter flounder.

These types of estimates hold assumptions of equal survivorship, complete reporting, and no tag loss or tag mortality. In the real world, these assumption are rarely satisfied and should be investigated by conducting parallel studies such as a tag reward program and special laboratory work on tag retention and tag induced mortality. Non-random mixing of tagged and untagged fish and emigration can also introduce bias and should be investigated.

Estimating Natural Mortality from Fishing Mortality and Effort Data

Paloheimo (1980) introduced a relationship between independent estimates of total mortality and fishing effort as follows:

$$F = qf$$

and

$$Z = F + M = qf + M$$

where f is fishing effort, and
q is a catchability coefficient.

This is in the linear form of $y = ax + b$. Plotting Z against f and fitting the best line through the data, the resulting slope is an estimate of q and the intercept is an estimate of natural mortality M

(Figure 8). These estimates, particularly M, should be treated with a little caution because the true fishing effort is not always accurately estimated by the available figures of nominal effort.

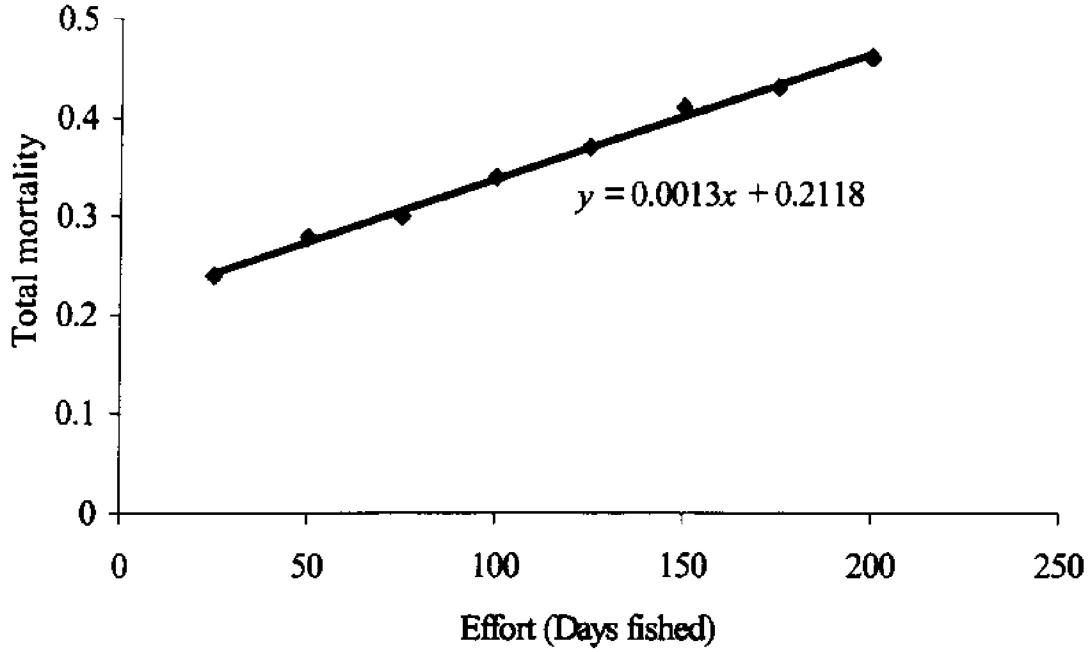

Figure 8. Plot of $Z = qf + M$. In this example, $q = 0.0013$ and $M = 0.2118$.

Example 6: Solve for M using the time series of effort and total mortality data in the kingfish fisheries off the coast of Thailand from 1966 to 1974.

Solution: Plotting the values of Z against f given in the table below results in Figure 9. The y-intercept estimate from the least squares regression equation gives an estimate of $M = 2.05$

Year	Effort (f) (x1000 days)	Z
1966	2.08	2.41
1967	2.08	2.69
1968	3.50	2.72
1969	3.60	2.62
1970	3.80	3.73
1971	-	-
1972	7.19	3.68
1973	9.94	4.61
1974	6.06	3.30

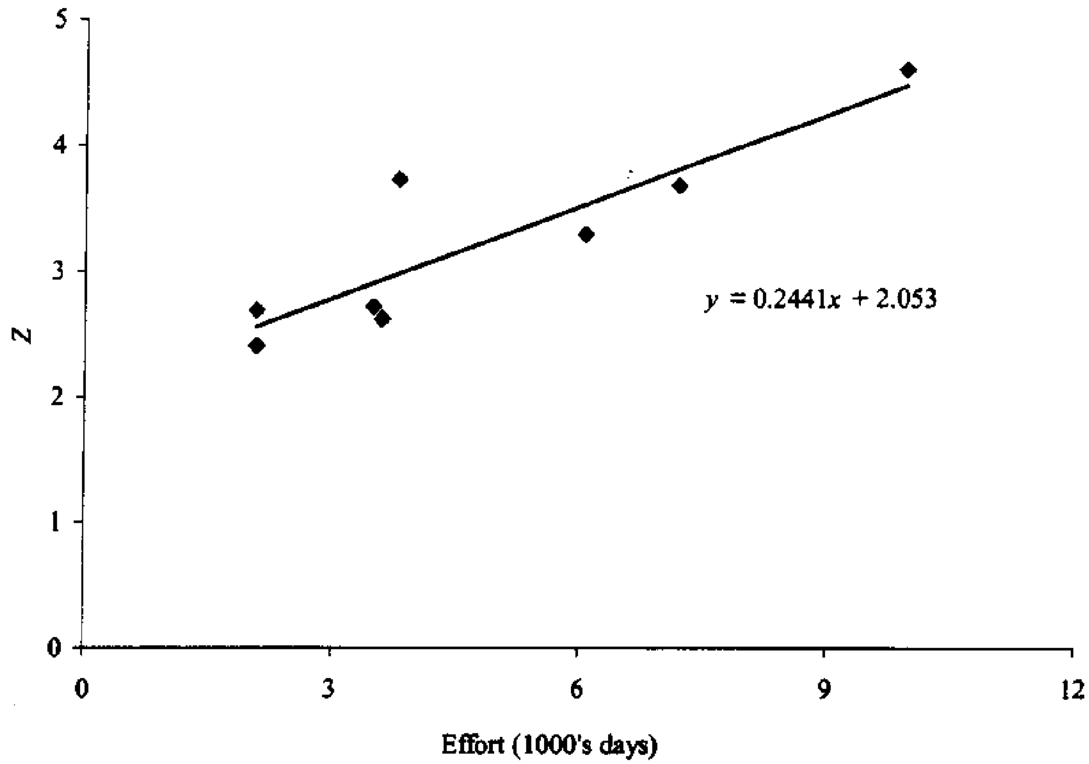

Figure 9. Linear regression of Z against f to solve for M.

Other Methods for Estimating Natural Mortality

Natural mortality is difficult to measure directly. There is, however, a loose relationship between natural mortality and fish life history. In general, fish with early maturity, a high growth rate, and low longevity have high natural mortality. This includes pelagic fish such as anchovies, mackerel, and herring. On the contrary, fish that mature late, have a slow growth rate, and live longer have low natural mortality. This includes demersal fish such as tautog, cod, sturgeon, and haddock.

Based on this and other generalizations about fish life history, several methods have been introduced that provide rough estimates of a species' natural mortality. The values for M obtained from these methods may not be accurate. Natural mortality is influenced by many factors other than life history. However, these methods can be used to get a handle on *relative* rates of natural mortality.

For example, Hoenig (1983) proposed the following formula that relates M to a species' longevity

$$M = \frac{2.98}{T_{max}} \approx \frac{3}{T_{max}}$$

where T_{max} is the maximum age or longevity.

Pauly (1980) proposed a formula for tropical species that relates natural mortality with variables such as the growth parameter (*K*) and temperature (*T*). Gunderson (1980) relates natural morality to female gonadosomatic index.

Table 2 shows values of *M* for several species based on Hoenig's $3/T_{max}$ equation. It is important to remember that values derived from these methods represent only relative rates of natural mortality and may not be entirely accurate.

Table 2. Values of natural mortality derived using Hoenig (1983).

Species	Longevity (years)	*M* (Hoenig 1983)
Croaker	5	0.60
Menhaden	8	0.375
Bluefish	8	0.375
Cod	20	0.15
Striped bass	30	0.10
Tautog	30	0.10
Red Drum	50	0.06
Atlantic Sturgeon	60	0.05

Exercises

1. Given an initial population of $N_0 = 25{,}000$ fish, a survival rate of $S = 0.47$, and a commercial harvest of 10,500 fish, determine N_t, U, Z, F and M during the first year for both a Type 1 and Type 2 fishery. For the Type 1 fishery, assume fishing occurs only in the second half of the year.

2. Weakfish caught by the NEFSC autumn bottom trawl survey were aged by applying annual age-length keys from pooled commercial and research samples to survey caught fish. Catch-at-age (expressed as CPUE) for the 1985 and 1990 year classes is shown below. Estimate total and fishing mortality for the two different years, assuming $M = 0.25$.

	Number at age					
Year	**0**	**1**	**2**	**3**	**4**	**5**
1985	10.39	4.12	0.93	0.06	0.03	
1990	3.45	0.73	0.13	0.06	0.019	0.013

Weakfish CPUE-at-age from NEFSC autumn bottom trawl survey.

3. The following data are taken from the Cooperative Striped Bass Tagging Program, conducted by the U.S. Fish and Wildlife Service and the Atlantic States Marine Fisheries Commission. The purpose of the program is to monitor mortality and migration of striped bass for the major producer areas (Hudson River, Chesapeake Bay, and Delaware Bay). This program comprises 4 critical operations: tagging fish, recovering tags, managing records of releases and recoveries, and analyzing recovery data. Total releases of tagged striped bass have exceeded 170,000 fish in ten years , through the participation of 10 states. Analysis of these data is performed on an annual basis by the Atlantic States Marine Fisheries Commission tagging group. Data from the Hudson River portion of this program are shown in the following table. Using this data, derive estimates of total and fishing mortality for the years 1990, 1993, and 1996. Natural mortality for striped bass is $M = 0.15$.

Release and recapture matrix of striped bass in the Hudson River from 1988 and 1996.

Year of release	Number released	Recaptures 1988	1989	1990	1991	1992	1993	1994	1995	1996
1988	227	25	31	18	11	10	5	4	1	4
1989	387		41	29	17	9	6	8	4	0
1990	446			62	31	27	14	9	4	1
1991	364				38	31	12	10	9	4
1992	699					90	58	35	21	13
1993	537						73	36	24	18
1994	381							43	33	26
1995	462								50	34
1996	683									88

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GEAR SELECTIVITY

SELECTIVITY OF MARINE FISH HARVESTING GEARS: GENERAL THEORY, SIZE SELECTION EXPERIMENTS AND DETERMINATION OF SIZE SELECTION CURVES

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Background

Since the 1970's, considerable progress has been made in defining the selection characteristics of various fish harvesting gears. Fishery managers and fishing gear technologists have investigated the subtle characteristics of species-specific size selection as a function of mesh size and shape in trawls, mesh size and hanging ratio in gill nets, hook size and style in longlines, and mesh size and funnel opening size in traps, so as to provide improved management of fishery stocks harvested with these gear types.

Literature Review

The study of size selection characteristics of fish harvesting gear began in the early 1900's, with an application toward fishery management (Baranov 1918 *in* Baranov 1976). In the late 1950's, the International Commission for the Northwest Atlantic Fisheries (ICNAF) co-sponsored a special scientific meeting on the selectivity of fishing gear (Anonymous 1963), and research summarized in the proceedings of that meeting were the basis for three decades of progress. The size selectivity of all fish harvesting gear can be classified broadly into two types of probability distributions (Clark 1960, Holt 1963, Pope et al. 1975):

1. A sigmoid curve, increasing from some positive value less than one to one as a function of fish size. This curve is represented by a logistic cumulative distribution function (LCDF). The selection characteristics of this curve are that all fish smaller than a particular size (L_1) are not captured ($P = 0$); that all fish larger than a particular size (L_2) are captured ($P = 1$); and that fish of a certain size (L_{50}) between L_1 and L_2 have a 50 percent probability of capture ($P = 0.5$) if encountering the gear.

2. A dome-shaped curve, increasing from some positive value less than one to one, then decreasing again as a function of fish size. This curve is represented by a truncated, rescaled normal probability density function (NPDF). The characteristics of this curve are that all fish smaller than a particular size (L_1) and larger than another particular size (L_2) are not captured, and that fish of a certain size (L_{opt}) between L_1 and L_2 have a 100 percent probability ($P = 1.0$) of capture if encountering the gear.

Fish size selection by a trawl codend may be modeled by a LCDF. Early work by Clark (1963) estimated sigmoid selection curves for groundfish species in the Northwest Atlantic. In the 1970s and 1980s additional research provided species- and mesh size-specific selection curves (Smolowitz 1983). More recent work has attempted to further define codend selectivity as a function of mesh shapes (square versus diamond) and to relate mesh shape to codend escape survival (DeAlteris and Reifsteck 1993). For fish selection by trawl codends, the following generalizations may be made: (1) larger meshes retain fewer small fish, shifting the selection curve to the right; (2) square mesh codends steepen the selection curve and shift it slightly to the right, as compared to a codend of similar mesh size of diamond shape.

Fish size selection by a gillnet may sometimes be modeled by a NPDF (Hamley 1975). Early work by Regier and Robson (1966) established an experimental methodology to describe the parameters of a normal distribution used to characterize the selectivity of the gillnet. Later work by Borgstrom (1989) and Hamley and Regier (1973) further defined the application of the NPDF to gillnet selection. More recently, Lazar and DeAlteris (1993), presenting the results of an analysis of gillnet selection in the Gulf of Maine groundfish fishery, used a truncated two-term gram Charlier series model to define in greater detail the shape of the selection curve.

Fish size selection by a longline with hooks may also be modeled by a sigmoid curve (McCracken 1963 and Saetersdal 1963). Ralston (1982), investigating the Hawaiian deep-sea handline fishery, concluded that a sigmoid curve most accurately described the selective properties of the gear in that fishery. Similar results were reported by Bertrand (1988) in his analysis of hook selectivity in the handline fishery of the Saya de Malha Banks (Indian Ocean). In contrast, Ralston (1990), investigating the size selection of snappers by hook and line gear, concluded that neither distribution model in its simplest form depicted hook selectivity. Otway and Craig (1993), studying the effects of hook size in catches of undersize snapper, also determined that neither the normal nor the logistic model was appropriate.

Fish size selection by traps has also been investigated. Stevenson and Stuart-Sharkey (1980) tested the effect of three different mesh sizes and found that increasing the mesh size led to a significant reduction in the number of smaller fish caught. Ward (1988), reporting on the results of mesh size experiments in the Bermuda trap fisheries, developed sigmoid-shape selection curves for the dominant species. However, as noted by Ward, since the traps had very large funnel openings relative to the maximum fish size in the population, nothing prevented entry by even the largest fish. Bohnsack et al. (1989) investigated the effect of fish trap mesh size on reef fish off southeastern Florida and found that larger meshes retained fewer small fish. It is clear that the mesh covering a trap will affect the retention of the smaller fish. If there is no restriction to entry by the largest fish in the population, then the selection curve may be sigmoid. However, the traps with the highest catch efficiency will have funnel openings small enough so as to impede the exit of captured fish that would otherwise be retained by the mesh size. Therefore these traps may have a dome-shaped selection curve.

General Theory

Logistic Cumulative Distribution Function

The size selection characteristics of trawl codend meshes and some hooks can be represented by a logistic cumulative distribution function (LCDF) (Figure 1) of the form:

$$PL_L = \left(1 + e^{(-\alpha 2 * (L - L_{50}))}\right)^{-1}$$

where PL_L is the probability of retention at length (L)
$\alpha 2$ is the steepness of the curve, and
L_{50} is the length at 50% selection.

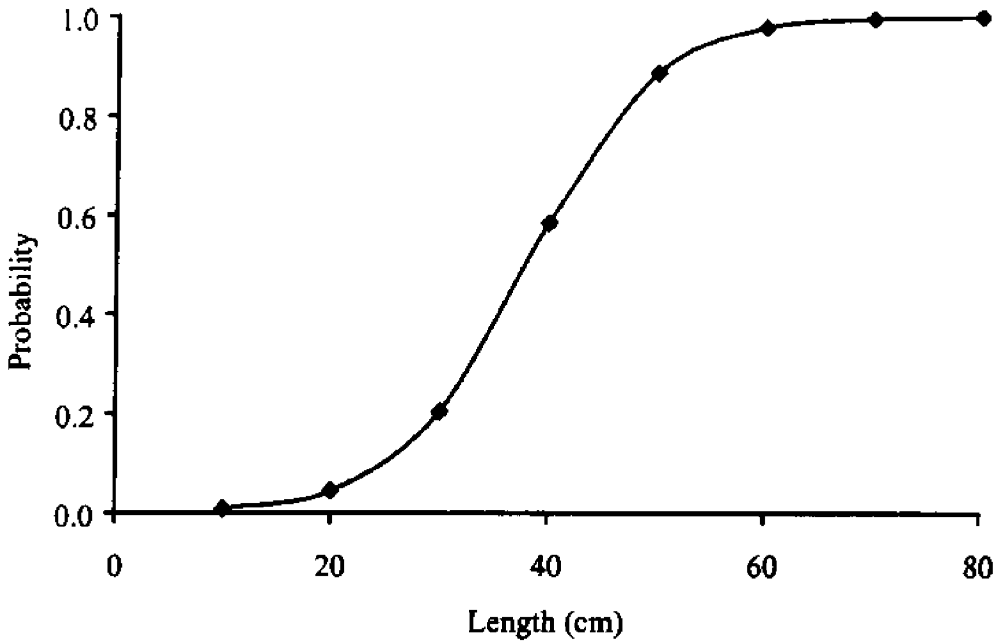

Figure 1. Probability of selection following a logistic cumulative distribution function.

This equation is a specialized form of the general LCDF equation:

$$PL_L = \left(1 + e^{(-(\alpha + \beta L))}\right)^{-1}$$

where α is $(-\alpha 2 * L_{50})$ or $(-\beta * L_{50})$, and
β is identical to $\alpha 2$, the steepness of the curve.

The terms α and β can be determined using:

1. Non-linear regression of data relating PL_L and L, or

2. Linear regression (Figure 2) of the linearized LCDF using the equation:

$$\ln(P/(1 - P)) = \alpha + (\beta * L).$$

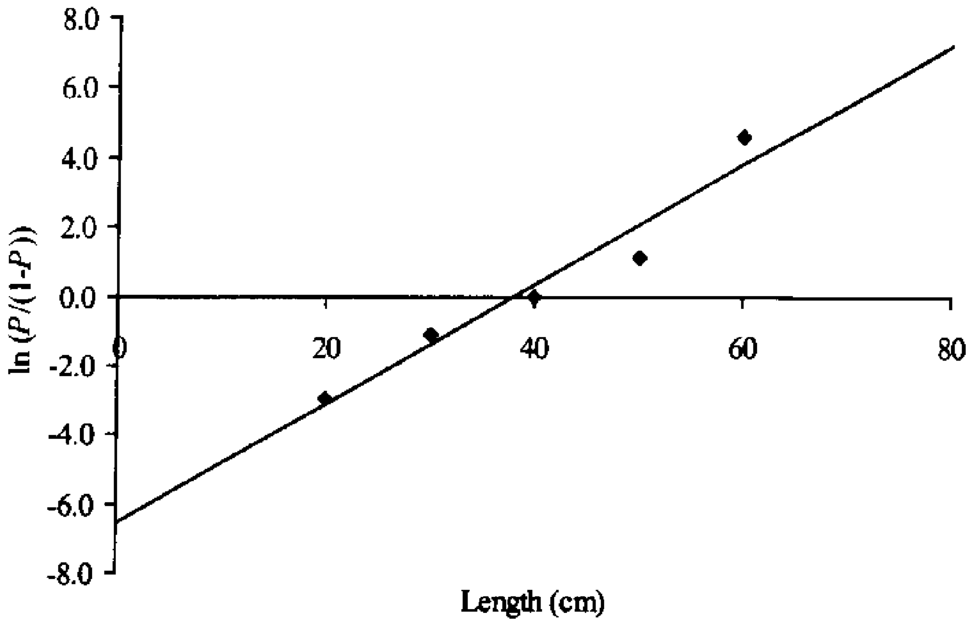

Figure 2. Regression of ln(*P*/(1-*P*)) vs *L*.

The LCDF curve is unique for a particular fish species, mesh size, and mesh shape. The selection factor (*SF*) is defined as:

$$SF = L_{50}/ml$$

where *ml* is the stretched mesh length.

The selection range (*SR*) is a measure of the steepness of the LCDF curve, and is described by:

$$SR = L_{75} - L_{25}$$

where L_{75} is the length at $P = 0.75$, and
L_{25} is the length at $P = 0.25$.

Using the selection factor, the L_{50} of other mesh sizes can be determined, resulting in a family of selection curves for a given species and mesh shape (Figure 3).

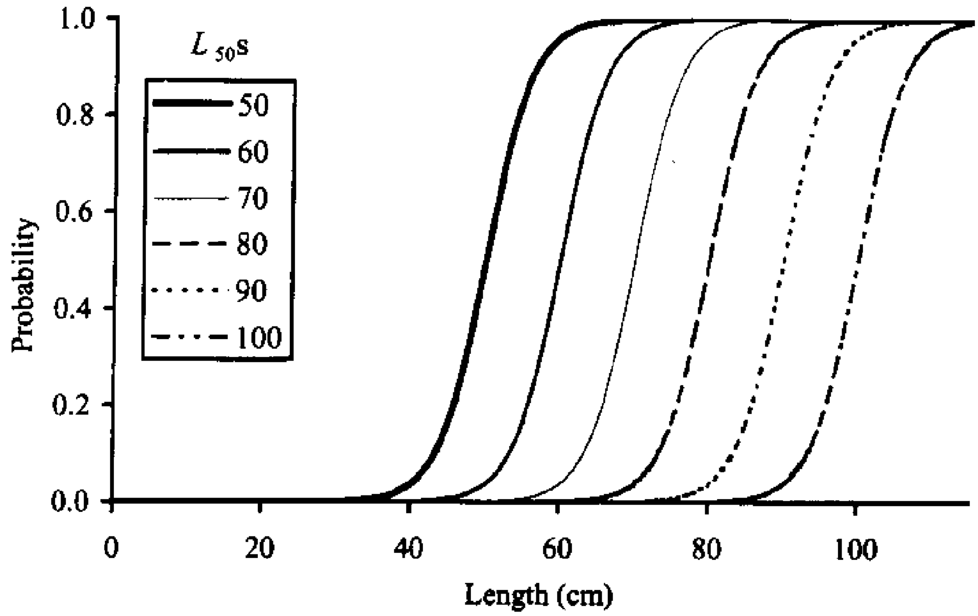

Figure 3. Logistic cumulative distribution function (LCDF) selectivity curves.

Normal Probability Distribution Function

The size selection characteristics of gillnets and some traps are represented by a truncated, scaled normal probability distribution function (NPDF) (Figure 4):

$$PN_L = e^{\left[-\left(L-L_{opt}\right)^2/\left(2*SD^2\right)\right]}$$

where PN_L is the probability of capture at length (L),
SD is the standard deviation, and
L_{opt} is the length of maximum selection probability.

The parameters L_{opt} and SD which define the NPDF can be determined by comparing the catches of two similar gears (A and B) that overlap in length-frequency distributions (Holt 1963). The method regresses the natural log of the ratio of the catches of the two gears at given lengths against lengths (Figure 5) using the linear model:

$$y = a + bL$$

where y is the $\ln(C_B/C_A)$,
a is the y-intercept, and
b is the slope.

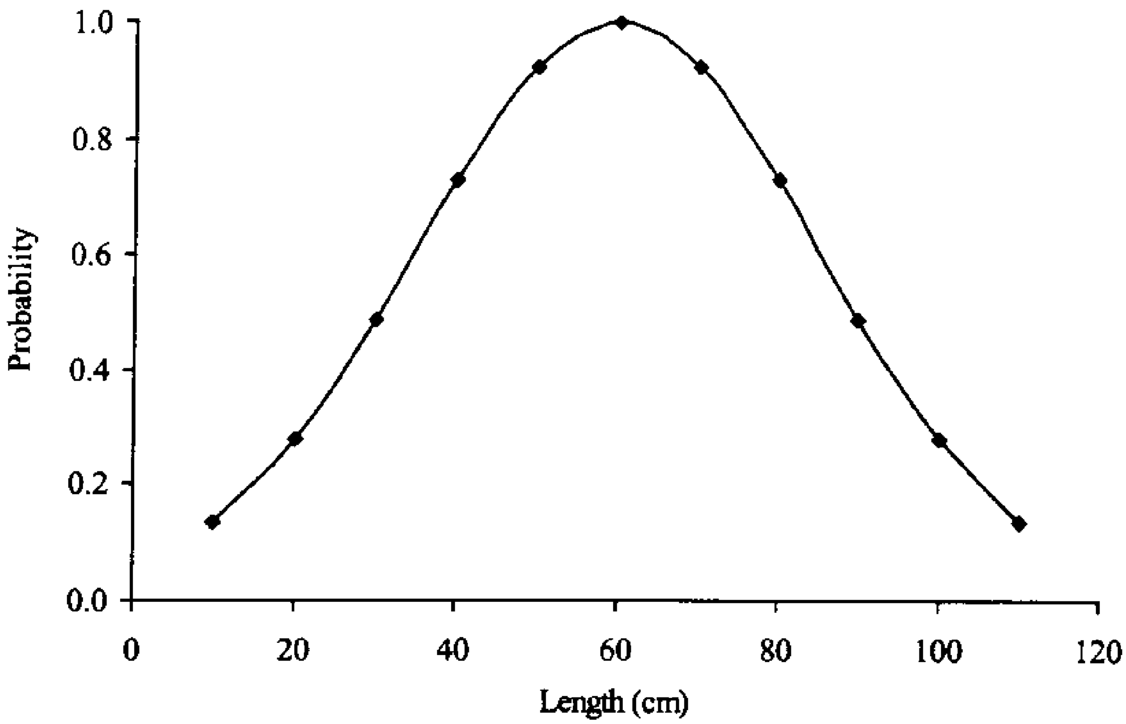

Figure 4. Probability of selection following a normal probability distribution function.

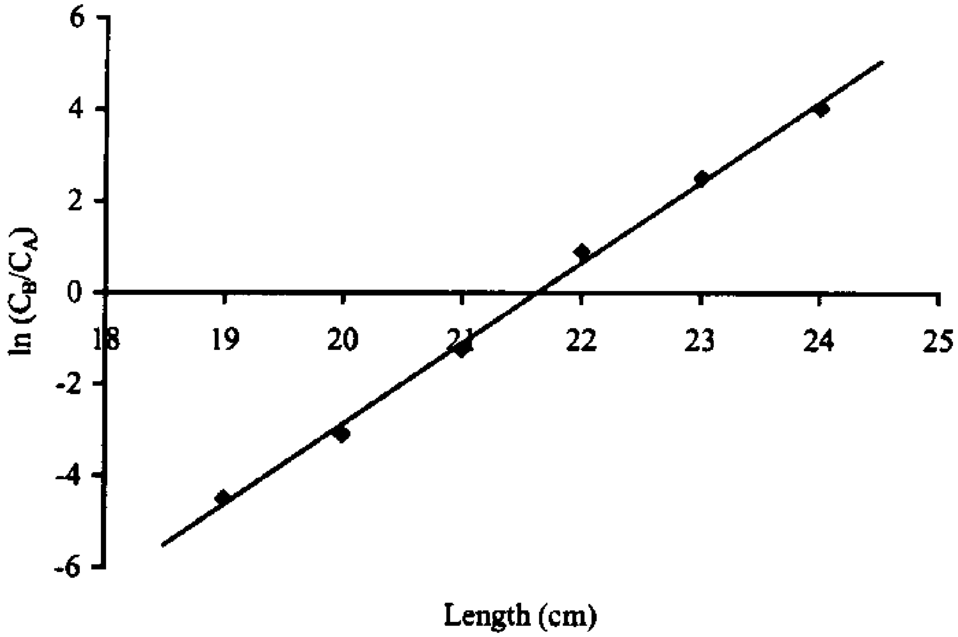

Figure 5. Linear regression of ln(C_B/C_A) for two similar gears that follow NPDF.

The values of L_{opt} and SD for the two gears can be determined with the following equations using the parameters a and b, and the mesh sizes of the two gears ml_A, and ml_B:

$$L_{opt}A = -2*[(a*ml_A)/(b*(ml_A + ml_B))]$$

$$L_{opt}B = -2*[(a*ml_B)/b*(ml_A + ml_B))]$$

$$SD = [-2*a*(ml_B - ml_A))/(b^2*(ml_A + ml_B))]^{1/2}.$$

The selection factor SF is:

$$SF = L_{opt}/ml$$

where ml is the mesh size.

Using the selection factor, the L_{opt} of other mesh sizes can be determined, resulting in a family of selection curves for a given species (Figure 6).

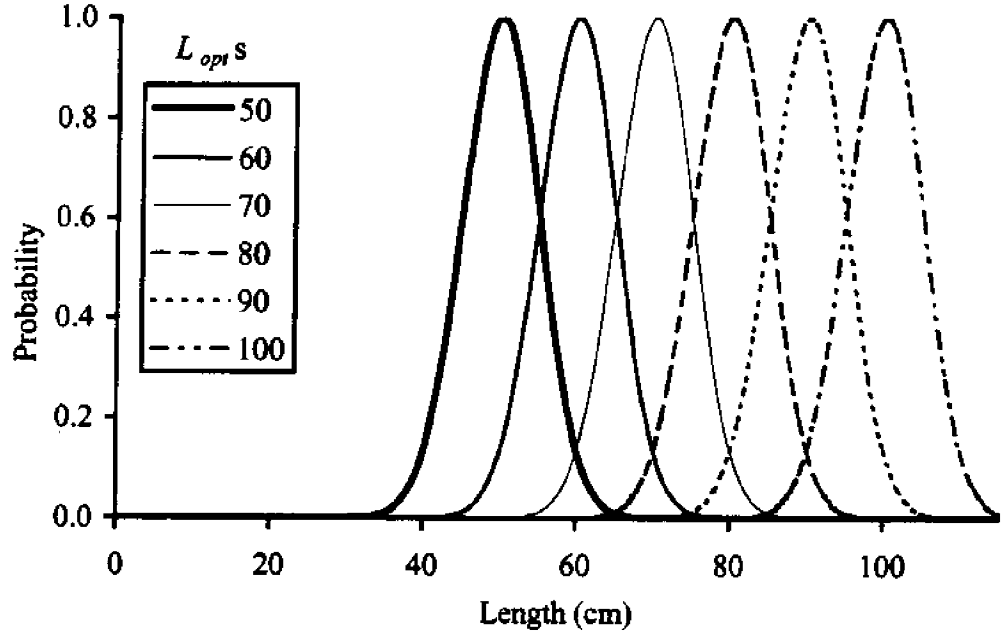

Figure 6. Normal probability distribution function (NPDF) selectivity curves.

Field Experiments and Estimation of Size Selection Curves

The selection characteristics of fish harvesting gears are usually determined using comparative fishing trials.

For LCDF selection, the probability of capture or retention approaches 100% ($P = 1$) for the largest fish in the population, and the smallest fish in the population have a probability of capture or retention approaching 0% ($P = 0$). Therefore, the comparative fishing experiment compares the catch of a relatively larger hook or mesh to the catch of a small hook or mesh that captures or retains all the fish that would encounter the larger gear, but be only partially captured or retained.

In the case of a trawl codend mesh experiment, the comparative trials are conducted using:

1. A covered codend, where the catch retained in the codend is compared to the catch of the cover and the codend.

2. A trouser trawl with two codends, small mesh and experimental, where the catch retained in the experimental codend is compared to the catch retained in the small mesh codend.

3. Alternate paired tows aboard a single or paired vessels where the catches of the small mesh codends are compared to the catches of the large mesh codends.

Example 1: Covered codend experiment for an idealized roundfish.

The catches by length (cm) for a mesh cover and an experimental codend (12 cm, diamond mesh) are shown below. Solve for L_{50}, selection factor, selection range, and the parameters α and β that define the LCDF.

L (cm)	Cover	Codend	Sum	Ratio (*P*)
10	10	0	10	0.00
20	19	1	20	0.05
30	75	25	100	0.25
40	200	200	400	0.50
50	100	300	400	0.75
60	2	198	200	0.99
70	0	50	50	1.00
80	0	5	5	1.00

The selection curve is plotted, and graphically the L_{50} is estimated to be 40 cm.

The selection factor is 40 cm/12 cm = 3.3

The selection range is L_{75}-L_{25} = 20 cm

The parameters (α and β) defining the LCDF are determined indirectly using linear regression on the transformed equation or directly using non-linear regression. These results are illustrated in Table 1 and Figure 7.

Table 1. Results of the covered codend experiment.

L (cm)	Cover	Codend	Sum	Ratio (*P*)	ln (*P*/1-*P*)	Predicted *P*
10	10	0	10	0.00		0.008
20	19	1	20	0.05	-2.94	0.042
30	75	25	100	0.25	-1.10	0.198
40	200	200	400	0.50	0.00	0.582
50	100	300	400	0.75	1.10	0.887
60	2	198	200	0.99	4.60	0.978
70	0	50	50	1.00		0.996
80	0	5	5	1.00		0.999

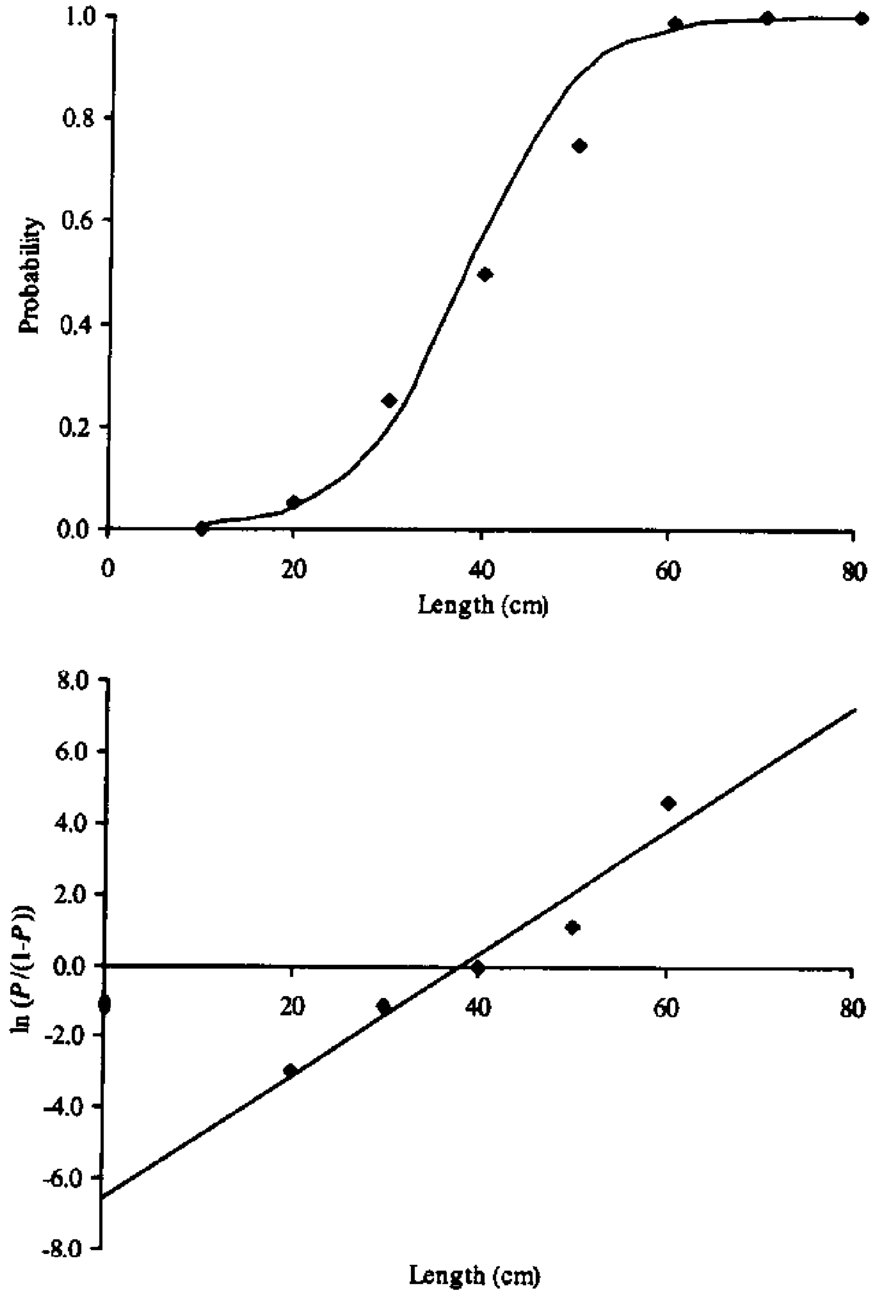

Figure 7. Results of the covered codend experiment.

For NPDF selection, the probability of retention or capture approaches 100% ($P = 1$) for a particular size of fish, then decreases to 0% ($P = 0$) for smaller and larger fish. Therefore, a comparative fishing experiment compares the catch of a particular size mesh in a gillnet or combination of wall mesh and entrance funnel in a trap to similar gears smaller or larger. The length-frequency distributions for the catches of the two gears must overlap for comparison to be effective. In the case of the gillnet, a dome-shaped selection curve reflects capture by wedging or gilling. The catch comparison assumes that the two nets with different mesh sizes have similar fishing power and standard deviations for the selection curves.

Example 2: Gillnet catch comparison experiment for two nets.

Nets A and B have mesh sizes of 8.1 and 9.1 cm respectively. Catches from these two nets are shown below. Determine the selection parameters a and b, the L_{opt}s for both nets, the standard deviation, and selection factor. Plot the results using the NPDF model.

Length	# Captured *A*	# Captured *B*	ln(*B*/*A*)
18	20	0	-
19	90	1	-4.5
20	199	9	-3.1
21	182	53	-1.2
22	119	290	-0.9
23	29	357	2.5
24	4	225	4.0
25	0	101	-

Plotting the length-frequency distributions for the catches of the two nets, the required overlap is observed (Figure 8a).

Regressing the ln(B/A) against L (Figure 8b) and fitting the model $y = bx + a$ results in coefficients:

a = -38.1
b = 1.76.

Following the method of Holt (1963):

$L_{opt} A$ = 20.4 cm
$L_{opt} B$ = 23. 0 cm
SD = 1.44
SF = 2.5.

Applying these values to the parameters of the NPDF model results in the selection curves shown in Figure 8c.

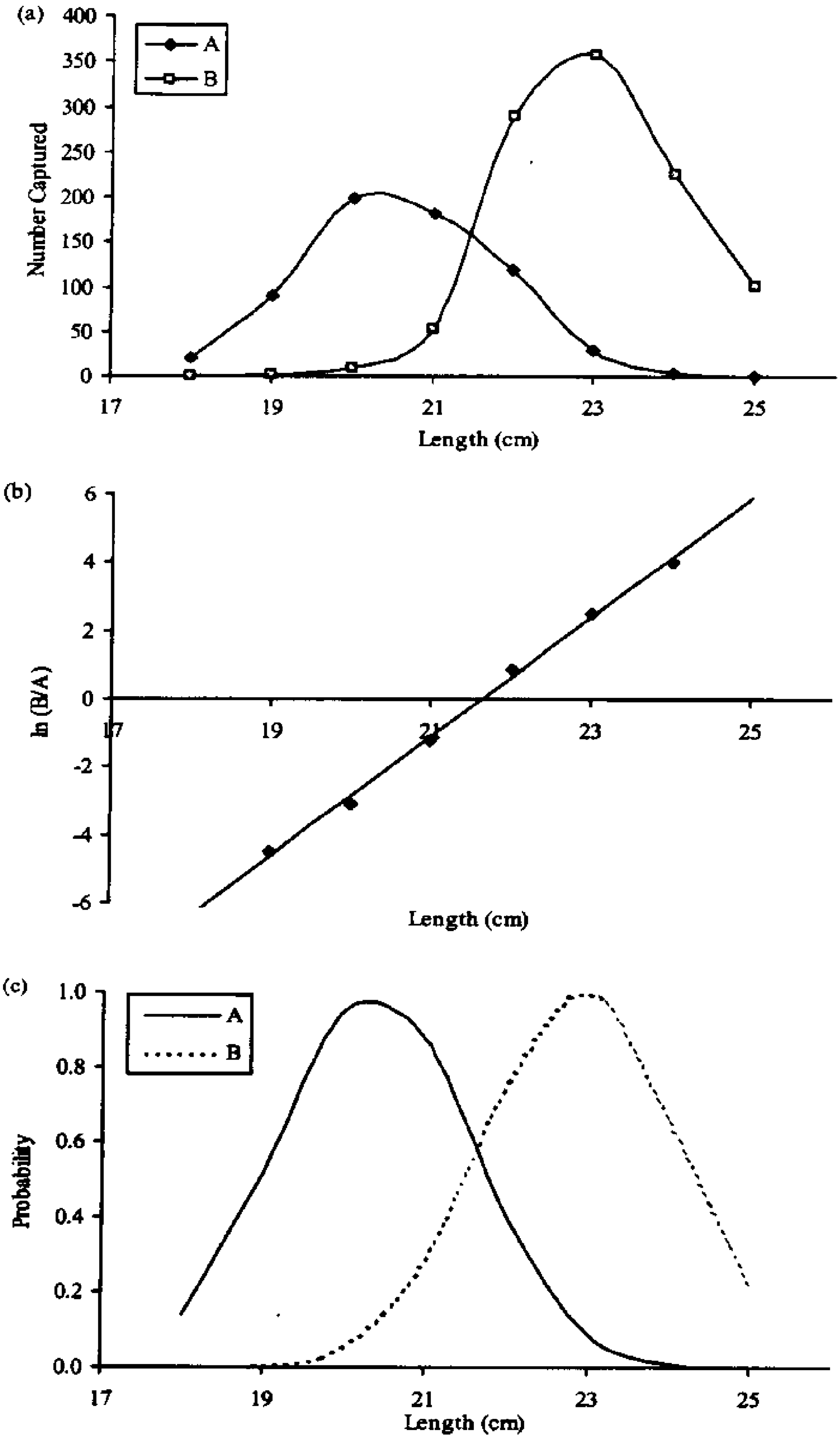

Figure 8. Analysis and results of NPDF selectivity analysis.

Exercises

TRAWL CODEND SELECTION PROBLEM

COVERED CODEND EXPERIMENT

Yellowtail flounder on Georges Bank

Codend = 14 cm, diamond mesh; Cover = 5 cm, square mesh

Fish Length (cm)	Codend	Cover
10-12	0	0
13-15	0	50
16-18	10	102
19-21	20	90
22-24	33	60
25-27	48	43
28-30	107	21
31-33	95	5
34-36	87	0
37-39	60	0
40-42	60	0
43-45	20	0
46-48	12	0
49-51	2	0
52-54	0	

1. Determine the selection curve by linear regression on natural log transformed data.

2. Based on the selection curve, estimate the L_{50}, SF, SR for yellowtail flounder, using a 14 cm diamond mesh codend.

TRAWL CODEND SELECTION PROBLEM

ALTERNATE TOW EXPERIMENT

Cod on Georges Bank

Exp Trawl = 14 cm, diamond mesh; Lined Trawl = 5 cm

Fish Length (cm)	Lined Trawl	Exp Trawl
11-15	0	0
16-20	5	0
21-25	7	0
26-30	2	0
31-35	10	0
36-40	30	2
41-45	36	6
46-50	42	8
51-55	50	20
56-60	83	35
61-65	64	42
66-70	53	47
71-75	42	38
76-80	19	20
81-85	11	10
86-90	7	7
91-95	6	5
96-100	4	3
101-105	1	1
106-110	0	0

1. Determine the selection curve by non-linear regression of PL_L versus L.

2. Based on the selection curve, estimate the L_{50}, SF, SR for cod using a 14 cm diamond mesh codend.

GILLNET SELECTION PROBLEM FOR COD

Fish Length (cm)	Webbing A 13.6 cm	Webbing B 14.8 cm	Webbing C 16.0 cm
46	0	0	0
48	5	0	0
50	26	0	0
52	52	1	0
54	102	16	4
56	295	131	17
58	309	362	95
60	118	326	199
62	79	191	202
64	27	111	133
66	14	44	52
68	8	14	25
70	7	8	15
72	0	1	5
74	0	0	1
76	0	0	0

1. Determine parameters *a* and *b* for each paired comparison.
2. Determine *SD* for each paired comparison.
3. Determine L_{opt} for 13.6, 14.8, 16.0 cm webbing.
4. Average the parameters *a*, *b*, and *SD* resulting from the two paired comparisons.
5. Plot *L-F* and selectivity curve for each webbing.

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YIELD PER RECRUIT

DEVELOPMENT AND APPLICATION OF YIELD PER RECRUIT AND SPAWNING STOCK PER RECRUIT MODELS

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Background

Models of yield per recruit (YPR) and spawning stock biomass per recruit (SPR or SSBPR) used in the analysis of fish population dynamics have sometimes assumed knife-edge selection at a single length or age. The purpose of this chapter is to review the analytical solution to the YPR model assuming knife-edge selection, and to integrate gear-specific size selection into YPR and SPR discrete time models. The development of a generalized model applied to a hypothetical or "idealized" roundfish is the prelude to the application of the model to specific marine fish species using actual selectivity data for harvesting gear, either presently used in the fisheries or proposed for future use.

Yield per recruit (YPR) models are useful to fishery resource managers for predicting the effects of alterations in harvesting activity on the yield available from a given year-class or cohort (Gulland 1983). Two elements that define the model and that are usually regulated by resource managers are fishing mortality (F) and the pattern of harvesting activity on different sizes of fish. Often the latter element has been simplified by assuming knife-edge selection (100% vulnerability at age of first capture), so that Beverton and Holt's (1957) analytical solution to the yield equation could be applied (Gulland 1969, 1983; Pauly 1984; Ricker 1975; Saila et al. 1988; Sparre et al. 1989). While this assumption may be appropriate for size selection that follows a logistic distribution function, as is sometimes observed in a trawl codend, the Beverton-Holt yield equation does not incorporate recent advances in understanding the size selection processes of the principal gear types used on groundfish (trawls, traps, gillnets, and longlines).

Analytical Solution

To predict the yield from a given number of recruits in a single cohort of fish, parameters characterizing the life history of the fish species and affecting the harvest of the stock must be specified. While the life history parameters affect the potential biomass available from the cohort, harvest related factors are controlled by fisheries management to ultimately affect the yield taken from the biomass. The biological or life history parameters affecting the potential maximum biomass and the timing of the maximization are:

> K is the instantaneous growth coefficient,
> M is the instantaneous natural mortality coefficient, and
> W_∞ is the maximum weight an individual fish may attain.

The fishery related factors affecting the maximum potential yield are:

> t_c is the age at which fish enter the fishery (controlled by mesh size in a trawl fishery), and

F is the instantaneous fishing mortality coefficient.

If R recruits from a cohort at time $t = 0$, then the numbers of fish caught (dC_t) and the yield in weight (dY_t) from that catch can be defined in short time intervals (t, $t + dt$) by:

$$dC_t = F_t * N_t * dt$$

and

$$dY_t = F_t * N_t * W_t dt$$

where N_t is the number of fish alive at age t,
F_t is the fishing mortality coefficient, which may vary with age, and
W_t is the average weight of an individual fish at age t.

The total catch in numbers (C) from a cohort or yield in weight (Y) results from the integration of the previous differential equations from the age at which the fish remaining in the cohort enter the fishery (t_c) to some limiting age t_L:

$$C = \int_{tc}^{tL} dC_t = \int_{tc}^{tL} F_t N_t dt$$

$$Y = \int_{tc}^{tL} dY_t = \int_{tc}^{tL} F_t N_t W_t dt.$$

Making the following assumptions simplifies the problem:

$$F_t = 0 \text{ and } t < t_c$$
$$F_t = F = \text{constant for } t \geq t_c$$
$$Z_t = M \text{ for } t < t_c$$
$$Z_t = F + M \text{ for } t \geq t_c$$
$$N_t = R\ e^{-M(t-t_c)} \text{ for } t < t_c$$

where Z_t is the total mortality coefficient, and
R is the total number of recruits in the cohort.

Therefore,

$$N_t = R'\ e^{[-(F+M)(t-t_c)]} \text{ for } t \geq t_c$$

where R' is the number of fish recruiting to the fishery at time $t = t_c$, and therefore,

$$R' = R\ e^{[-Mt_c]}.$$

Thus the total number caught is:

$$C = \int_{t_c}^{t_L} R' * F e^{[-(F+M)(t-t_c)]} dt$$

$$= R'(F/(F+M))(e^{[-(F+M)(t_L-t_c)]})$$

or

$$C = R(F/(F+M))(e^{[-M(t_c)]})(1 - e^{[(F+M)(t_L-t_c)]})$$

and ignoring the last term if $t_L >> t_c$

$$C = R(F/(F+M))e^{[-M(t_c)]}$$

or

$$C = (F/(F+M))R'.$$

Recall that Yield = (Catch)*(Weight), and that the von Bertalanffy growth equation describes individual fish growth as a function of time:

$$W_t = W_\infty[1 - e^{(-Kt)}]^3.$$

This equation is expanded to:

$$W_t = W_\infty \sum_{n=0}^{3} U_n e^{(-nKt)}$$

where $U_0 = 1$, $U_1 = -3$, $U_2 = 3$, $U_3 = -1$.

Incorporating the simplified catch equation and individual fish growth equation into the simplified yield equation results in:

$$Y = F * R\ e^{[-M(t_c)]} W_\infty \sum_{n=0}^{3} \left[U_n e^{(-nKt)} / (F + M + n * K)\right].$$

Yield per recruit is obtained by normalizing the total yield by the number of recruits (R') in the cohort.

Beverton and Holt (1957) noted several important results from the yield per recruit analysis. First is the ratio of the growth parameter (*K*) to the natural mortality coefficient (*M*), which estimates the potential of a fish to complete its potential growth before dying of natural mortality.

If M/K is small ($M/K \leq 0.5$), then growth is high relative to mortality, and the cohort will reach maximum biomass at a larger size relative to the maximum size, or the stock (in the absence of fishing) will contain relatively larger fish. From a fishery perspective, management should maximize the size or age of entry to the fishery (t_c), with only light fishing mortality on smaller fish.

If M/K is large ($M/K \geq 2.0$), then natural mortality exceeds growth, indicating many fish will die before completing their potential growth. Again, from a fishery perspective, management should allow heavy fishing with a small size (age) at first capture, so as to harvest the maximum biomass before they die of natural causes.

The yield equation is separated into two parts that characterize the fish stock as a constant and the fishing as a variable. Two additional terms are defined:

Exploitation Ratio $E = \dfrac{F}{(F + M)}$, and

Relative Size at First Capture $c = \dfrac{l_c}{l_\infty}$.

The yield equation can then be written as:

$$Y = Y' \left[R * W_\infty e^{(-Mt_c)}\right]$$

where

$$Y' = E(1-c)^{(M/K)} \sum_{n=0}^{3} \left[\left(U_n (1-c)^n\right) / 1 + (n * K/M)(1-E) \right].$$

Beverton and Holt (1957) provide tabulated yield values (Y') for a series of values of *M/K* from 0.25 to 5.00 for various values of E and *c*. Tables 1 and 2 illustrate the effect of *M/K*, *c*, and *E* on Y'. Note that for small *M/K* ratios (0.5), maximum yield is achieved at higher values of *c*; whereas if *M/K* is larger (2.0), then maximum yield is achieved at lower values of *c*.

The Beverton and Holt analytical solution to the yield per recruit (YPR) problem, assuming knife-edge selection, is applied using Tables 1 and 2 or by direct calculation on a computer. With a simple algorithm on a spreadsheet program, a YPR curve is estimated for a particular age or length of entry into the fishery. (Instructions for creating similar tables can be found on the "Answers to Exercises" disk in the file "Chapter 6 – Tables 1&2.xls.")

$E = F/(F+M)$	Rate of Exploitation																	
L_c/L_∞	0.05	0.10	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50	0.55	0.60	0.65	0.70	0.75	0.80	0.85	0.90
0.98	0.006927	0.013843	0.020748	0.027640	0.034517	0.041379	0.048223	0.055046	0.061845	0.068618	0.075359	0.082063	0.088725	0.095336	0.101887	0.108363	0.114750	0.121024
0.96	0.009596	0.019164	0.028699	0.038199	0.047659	0.057075	0.066440	0.075748	0.084991	0.094158	0.103239	0.112219	0.121081	0.129802	0.138356	0.146708	0.154812	0.162610
0.94	0.011514	0.022977	0.034382	0.045722	0.056991	0.068179	0.079276	0.090270	0.101145	0.111886	0.122471	0.132874	0.143064	0.153003	0.162642	0.171921	0.180760	0.189057
0.92	0.013027	0.025976	0.038837	0.051600	0.064255	0.076788	0.089183	0.101421	0.113482	0.125338	0.136959	0.148306	0.159333	0.169983	0.180184	0.189845	0.198852	0.207056
0.90	0.014272	0.028436	0.042479	0.056389	0.070149	0.083741	0.097143	0.110331	0.123275	0.135938	0.148280	0.160247	0.171778	0.182794	0.193199	0.202873	0.211663	0.219375
0.88	0.015322	0.030503	0.045529	0.060381	0.075040	0.089481	0.103678	0.117598	0.131203	0.144447	0.157277	0.169625	0.181412	0.192539	0.202886	0.212300	0.220591	0.227522
0.86	0.016220	0.032266	0.048119	0.063757	0.079155	0.094284	0.109110	0.123593	0.137686	0.151334	0.164469	0.177012	0.188864	0.199907	0.209992	0.218938	0.226518	0.232453
0.84	0.016997	0.033785	0.050340	0.066637	0.082646	0.098331	0.113652	0.128563	0.143006	0.156915	0.170212	0.182800	0.194565	0.205365	0.215028	0.223343	0.230049	0.234827
0.82	0.017674	0.035102	0.052257	0.069108	0.085621	0.101754	0.117462	0.132687	0.147366	0.161420	0.174759	0.187271	0.198824	0.209255	0.218369	0.225924	0.231628	0.235129
0.80	0.018267	0.036249	0.053916	0.071234	0.088162	0.104653	0.120654	0.136101	0.150920	0.165022	0.178303	0.190638	0.201877	0.211837	0.220297	0.226991	0.231596	0.233733
0.78	0.018787	0.037251	0.055357	0.073066	0.090333	0.107105	0.123321	0.138910	0.153787	0.167855	0.180997	0.193071	0.203911	0.213314	0.221038	0.226790	0.230222	0.230934
0.76	0.019245	0.038127	0.056608	0.074644	0.092185	0.109171	0.125535	0.141198	0.156066	0.170030	0.182961	0.194704	0.205075	0.213853	0.220774	0.225522	0.227727	0.226974
0.75	0.019453	0.038523	0.057171	0.075349	0.093004	0.110076	0.126491	0.142168	0.157009	0.170898	0.183701	0.195256	0.205370	0.213814	0.220313	0.224540	0.226117	0.224623
0.74	0.019648	0.038894	0.057694	0.076001	0.093759	0.110902	0.127356	0.143034	0.157833	0.171635	0.184296	0.195649	0.205493	0.213589	0.219654	0.223351	0.224293	0.222055
0.72	0.020004	0.039565	0.058636	0.077165	0.095089	0.112340	0.128834	0.144476	0.159157	0.172744	0.185086	0.195999	0.205268	0.212638	0.217806	0.220418	0.220073	0.216344
0.70	0.020317	0.040151	0.059450	0.078157	0.096206	0.113520	0.130010	0.145574	0.160092	0.173423	0.185402	0.195834	0.204490	0.211097	0.215337	0.216841	0.215199	0.209988
0.68	0.020593	0.040662	0.060151	0.078999	0.097134	0.114473	0.130922	0.146369	0.160687	0.173724	0.185306	0.195224	0.203234	0.209051	0.212341	0.212724	0.209783	0.203111
0.66	0.020836	0.041106	0.060752	0.079708	0.097895	0.115226	0.131600	0.146898	0.160983	0.173696	0.184850	0.194226	0.201567	0.206573	0.208899	0.208156	0.203926	0.195823
0.64	0.021049	0.041491	0.061264	0.080297	0.098508	0.115802	0.132072	0.147192	0.161018	0.173381	0.184083	0.192894	0.199546	0.203727	0.205082	0.203215	0.197714	0.188219
0.62	0.021235	0.041823	0.061697	0.080781	0.098989	0.116220	0.132362	0.147280	0.160824	0.172814	0.183044	0.191273	0.197223	0.200571	0.200953	0.197971	0.191223	0.180383
0.60	0.021398	0.042107	0.062059	0.081171	0.099353	0.116500	0.132491	0.147187	0.160428	0.172028	0.181770	0.189405	0.194643	0.197154	0.196567	0.192486	0.184521	0.172389
0.58	0.021539	0.042350	0.062357	0.081477	0.099614	0.116656	0.132479	0.146935	0.159857	0.171052	0.180294	0.187325	0.191847	0.193521	0.191975	0.186813	0.177667	0.164303
0.56	0.021661	0.042554	0.062599	0.081709	0.099783	0.116704	0.132342	0.146543	0.159134	0.169912	0.178645	0.185067	0.188870	0.189714	0.187220	0.181003	0.170716	0.156183
0.54	0.021767	0.042724	0.062791	0.081875	0.099870	0.116656	0.132095	0.146030	0.158278	0.168631	0.176849	0.182659	0.185747	0.185767	0.182342	0.175097	0.163715	0.148081
0.52	0.021856	0.042864	0.062939	0.081982	0.099886	0.116524	0.131754	0.145411	0.157308	0.167230	0.174930	0.180127	0.182505	0.181712	0.177378	0.169137	0.156706	0.140043
0.50	0.021933	0.042978	0.063046	0.082038	0.099839	0.116318	0.131329	0.144701	0.156241	0.165728	0.172909	0.177497	0.179171	0.177580	0.172357	0.163155	0.149728	0.132109
0.48	0.021997	0.043067	0.063119	0.082048	0.099736	0.116048	0.130832	0.143914	0.155092	0.164142	0.170804	0.174788	0.175769	0.173396	0.167310	0.157184	0.142813	0.124315
0.46	0.022050	0.043135	0.063161	0.082018	0.099585	0.115723	0.130274	0.143060	0.153875	0.162487	0.168634	0.172020	0.172319	0.169183	0.162262	0.151251	0.135992	0.116692
0.44	0.022094	0.043185	0.063175	0.081952	0.099391	0.115349	0.129664	0.142151	0.152601	0.160778	0.166413	0.169210	0.168842	0.164963	0.157236	0.145381	0.129292	0.109268
0.42	0.022129	0.043218	0.063165	0.081857	0.099162	0.114935	0.129009	0.141196	0.151282	0.159026	0.164157	0.166375	0.165354	0.160755	0.152252	0.139595	0.122734	0.102067
0.40	0.022156	0.043236	0.063135	0.081734	0.098902	0.114487	0.128318	0.140205	0.149928	0.157243	0.161877	0.163528	0.161871	0.156574	0.147328	0.133915	0.116341	0.095109
0.38	0.022177	0.043242	0.063086	0.081590	0.098616	0.114010	0.127598	0.139185	0.148548	0.155440	0.159585	0.160681	0.158407	0.152437	0.142481	0.128355	0.110128	0.088412
0.36	0.022192	0.043236	0.063022	0.081426	0.098307	0.113509	0.126855	0.138143	0.147150	0.153626	0.157292	0.157847	0.154973	0.148357	0.137725	0.122932	0.104112	0.081989
0.34	0.022202	0.043222	0.062945	0.081246	0.097982	0.112991	0.126093	0.137086	0.145742	0.151808	0.155006	0.155035	0.151582	0.144345	0.133073	0.117658	0.098305	0.075854
0.32	0.022208	0.043199	0.062857	0.081053	0.097641	0.112458	0.125320	0.136020	0.144331	0.149995	0.152736	0.152254	0.148243	0.140411	0.128534	0.112544	0.092718	0.070013
0.30	0.022210	0.043169	0.062760	0.080850	0.097290	0.111915	0.124538	0.134951	0.142921	0.148193	0.150489	0.149512	0.144963	0.136566	0.124119	0.107600	0.087358	0.064475
0.28	0.022209	0.043134	0.062655	0.080638	0.096931	0.111365	0.123753	0.133882	0.141519	0.146408	0.148272	0.146817	0.141752	0.132815	0.119834	0.102831	0.082233	0.059244
0.26	0.022205	0.043094	0.062545	0.080420	0.096566	0.110812	0.122967	0.132818	0.140130	0.144646	0.146089	0.144173	0.138614	0.129167	0.115686	0.098245	0.077347	0.054321
0.24	0.022199	0.043050	0.062430	0.080197	0.096198	0.110258	0.122184	0.131762	0.138756	0.142909	0.143947	0.141586	0.135555	0.125625	0.111681	0.093846	0.072704	0.049708
0.22	0.022191	0.043004	0.062311	0.079972	0.095829	0.109705	0.121407	0.130718	0.137403	0.141203	0.141848	0.139061	0.132579	0.122194	0.107822	0.089636	0.068303	0.045402
0.20	0.022182	0.042955	0.062190	0.079745	0.095460	0.109156	0.120638	0.129689	0.136072	0.139531	0.139797	0.136600	0.129690	0.118878	0.104111	0.085617	0.064145	0.041399
0.18	0.022171	0.042904	0.062069	0.079519	0.095093	0.108613	0.119880	0.128677	0.134766	0.137894	0.137796	0.134208	0.126891	0.115677	0.100551	0.081790	0.060229	0.037695
0.16	0.022160	0.042853	0.061946	0.079293	0.094730	0.108077	0.119134	0.127683	0.133488	0.136297	0.135847	0.131884	0.124182	0.112595	0.097140	0.078152	0.056550	0.034281
0.14	0.022148	0.042801	0.061824	0.079069	0.094371	0.107549	0.118401	0.126710	0.132240	0.134739	0.133953	0.129632	0.121566	0.109631	0.093880	0.074703	0.053104	0.031150
0.12	0.022136	0.042749	0.061703	0.078848	0.094018	0.107030	0.117683	0.125759	0.131021	0.133223	0.132113	0.127451	0.119042	0.106784	0.090767	0.071439	0.049885	0.028290
0.10	0.022124	0.042697	0.061583	0.078631	0.093672	0.106522	0.116981	0.124830	0.129834	0.131748	0.130328	0.125342	0.116610	0.104053	0.087801	0.068354	0.046885	0.025691
0.00	0.022063	0.042450	0.061016	0.077605	0.092045	0.104150	0.113716	0.120530	0.124367	0.125000	0.122216	0.115837	0.105764	0.092045	0.075000	0.055411	0.034843	0.016071

Table 1. Yield per recruit for $M/K = 0.5$.

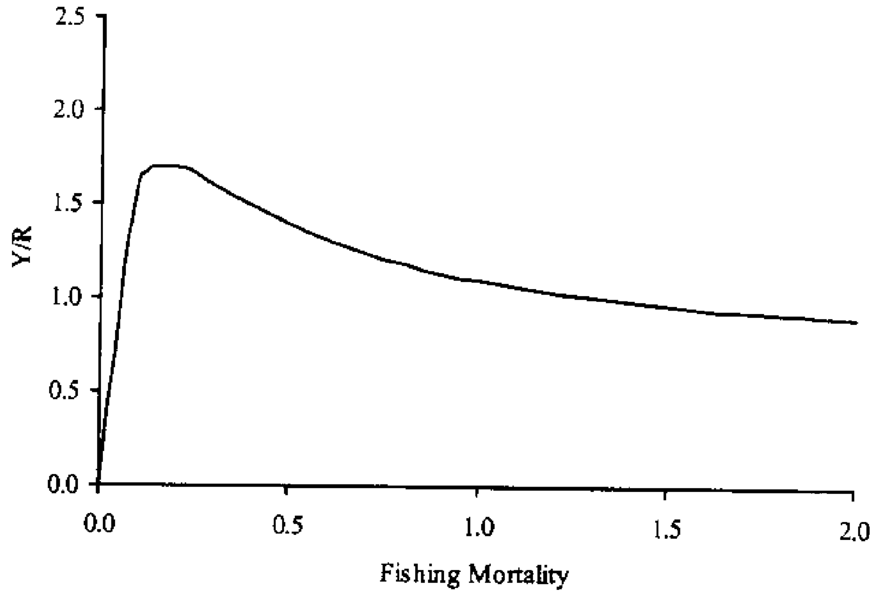
$E = F/(F+M)$	Rate of Exploitation																	
L_c/L_∞	0.05	0.10	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50	0.55	0.60	0.65	0.70	0.75	0.80	0.85	0.90
0.98	0.000019	0.000038	0.000058	0.000077	0.000096	0.000115	0.000134	0.000153	0.000172	0.000191	0.000209	0.000228	0.000247	0.000266	0.000284	0.000303	0.000321	0.000340
0.96	0.000074	0.000147	0.000220	0.000293	0.000366	0.000439	0.000511	0.000583	0.000654	0.000726	0.000797	0.000867	0.000937	0.001007	0.001077	0.001145	0.001214	0.001282
0.94	0.000159	0.000317	0.000475	0.000632	0.000788	0.000943	0.001097	0.001250	0.001402	0.001554	0.001704	0.001852	0.002000	0.002146	0.002291	0.002434	0.002576	0.002716
0.92	0.000271	0.000540	0.000808	0.001074	0.001338	0.001600	0.001860	0.002117	0.002373	0.002626	0.002876	0.003123	0.003368	0.003610	0.003848	0.004083	0.004314	0.004541
0.90	0.000406	0.000808	0.001208	0.001604	0.001996	0.002385	0.002769	0.003150	0.003526	0.003897	0.004264	0.004625	0.004981	0.005331	0.005675	0.006012	0.006343	0.006666
0.88	0.000560	0.001114	0.001663	0.002206	0.002743	0.003274	0.003798	0.004315	0.004825	0.005327	0.005820	0.006305	0.006781	0.007247	0.007704	0.008149	0.008584	0.009006
0.86	0.000729	0.001451	0.002163	0.002867	0.003562	0.004246	0.004920	0.005584	0.006236	0.006876	0.007503	0.008118	0.008718	0.009304	0.009874	0.010428	0.010965	0.011485
0.84	0.000912	0.001812	0.002699	0.003573	0.004434	0.005281	0.006113	0.006929	0.007728	0.008510	0.009274	0.010019	0.010744	0.011448	0.012130	0.012789	0.013424	0.014034
0.82	0.001104	0.002191	0.003261	0.004313	0.005347	0.006360	0.007353	0.008324	0.009273	0.010197	0.011097	0.011971	0.012817	0.013635	0.014423	0.015179	0.015903	0.016592
0.80	0.001303	0.002584	0.003842	0.005076	0.006284	0.007467	0.008622	0.009748	0.010844	0.011909	0.012940	0.013937	0.014898	0.015821	0.016705	0.017549	0.018349	0.019105
0.78	0.001507	0.002985	0.004433	0.005851	0.007235	0.008586	0.009901	0.011179	0.012419	0.013618	0.014774	0.015887	0.016953	0.017971	0.018940	0.019856	0.020719	0.021526
0.76	0.001713	0.003390	0.005029	0.006629	0.008188	0.009704	0.011175	0.012600	0.013976	0.015301	0.016574	0.017791	0.018951	0.020051	0.021090	0.022064	0.022972	0.023813
0.75	0.001816	0.003592	0.005327	0.007017	0.008661	0.010258	0.011806	0.013301	0.014742	0.016127	0.017453	0.018719	0.019920	0.021056	0.022124	0.023121	0.024045	0.024895
0.74	0.001920	0.003795	0.005623	0.007403	0.009132	0.010808	0.012429	0.013993	0.015497	0.016939	0.018316	0.019626	0.020866	0.022033	0.023125	0.024141	0.025077	0.025931
0.72	0.002125	0.004195	0.006209	0.008164	0.010058	0.011888	0.013651	0.015344	0.016966	0.018512	0.019981	0.021369	0.022674	0.023892	0.025021	0.026059	0.027003	0.027852
0.70	0.002327	0.004589	0.006784	0.008908	0.010958	0.012933	0.014828	0.016640	0.018367	0.020006	0.021552	0.023003	0.024356	0.025607	0.026755	0.027796	0.028728	0.029552
0.68	0.002525	0.004973	0.007342	0.009627	0.011826	0.013935	0.015951	0.017871	0.019690	0.021405	0.023014	0.024511	0.025895	0.027161	0.028308	0.029333	0.030234	0.031011
0.66	0.002717	0.005345	0.007879	0.010317	0.012654	0.014887	0.017012	0.019025	0.020923	0.022700	0.024355	0.025882	0.027278	0.028541	0.029668	0.030657	0.031506	0.032218
0.64	0.002903	0.005702	0.008393	0.010974	0.013438	0.015783	0.018004	0.020097	0.022057	0.023881	0.025565	0.027104	0.028495	0.029735	0.030823	0.031756	0.032536	0.033162
0.62	0.003080	0.006042	0.008882	0.011594	0.014174	0.016618	0.018921	0.021079	0.023087	0.024941	0.026636	0.028170	0.029537	0.030737	0.031767	0.032626	0.033316	0.033840
0.60	0.003250	0.006365	0.009341	0.012174	0.014858	0.017388	0.019759	0.021967	0.024007	0.025874	0.027565	0.029074	0.030400	0.031541	0.032495	0.033262	0.033846	0.034251
0.58	0.003410	0.006669	0.009771	0.012712	0.015487	0.018089	0.020514	0.022757	0.024813	0.026677	0.028346	0.029815	0.031082	0.032145	0.033006	0.033665	0.034126	0.034398
0.56	0.003560	0.006952	0.010169	0.013207	0.016059	0.018720	0.021185	0.023447	0.025503	0.027348	0.028978	0.030389	0.031581	0.032551	0.033302	0.033837	0.034162	0.034288
0.54	0.003701	0.007215	0.010535	0.013657	0.016574	0.019279	0.021769	0.024036	0.026077	0.027887	0.029462	0.030800	0.031899	0.032761	0.033388	0.033785	0.033961	0.033930
0.52	0.003831	0.007456	0.010869	0.014062	0.017030	0.019767	0.022266	0.024524	0.026534	0.028294	0.029799	0.031048	0.032041	0.032780	0.033269	0.033516	0.033534	0.033337
0.50	0.003951	0.007677	0.011169	0.014421	0.017427	0.020182	0.022678	0.024911	0.026877	0.028571	0.029992	0.031138	0.032011	0.032614	0.032955	0.033042	0.032892	0.032524
0.48	0.004061	0.007875	0.011436	0.014735	0.017767	0.020526	0.023005	0.025201	0.027108	0.028724	0.030046	0.031077	0.031817	0.032274	0.032455	0.032374	0.032050	0.031508
0.46	0.004161	0.008053	0.011670	0.015005	0.018050	0.020801	0.023250	0.025395	0.027230	0.028755	0.029967	0.030870	0.031468	0.031768	0.031782	0.031527	0.031026	0.030309
0.44	0.004250	0.008210	0.011873	0.015231	0.018278	0.021008	0.023416	0.025497	0.027249	0.028671	0.029762	0.030527	0.030972	0.031108	0.030949	0.030517	0.029838	0.028947
0.42	0.004330	0.008347	0.012045	0.015415	0.018453	0.021151	0.023505	0.025512	0.027170	0.028478	0.029438	0.030056	0.030342	0.030307	0.029972	0.029360	0.028504	0.027445
0.40	0.004400	0.008465	0.012187	0.015559	0.018576	0.021232	0.023523	0.025445	0.026998	0.028183	0.029004	0.029469	0.029588	0.029380	0.028865	0.028075	0.027046	0.025825
0.38	0.004461	0.008563	0.012301	0.015665	0.018652	0.021256	0.023472	0.025300	0.026740	0.027795	0.028469	0.028774	0.028725	0.028340	0.027647	0.026681	0.025485	0.024111
0.36	0.004513	0.008645	0.012388	0.015735	0.018682	0.021225	0.023359	0.025085	0.026404	0.027321	0.027844	0.027986	0.027764	0.027203	0.026334	0.025197	0.023842	0.022328
0.34	0.004557	0.008709	0.012449	0.015772	0.018671	0.021144	0.023188	0.024805	0.025997	0.026771	0.027139	0.027114	0.026720	0.025985	0.024944	0.023644	0.022140	0.020499
0.32	0.004593	0.008758	0.012488	0.015777	0.018621	0.021017	0.022965	0.024466	0.025526	0.026154	0.026364	0.026173	0.025608	0.024701	0.023495	0.022041	0.020402	0.018650
0.30	0.004623	0.008793	0.012505	0.015754	0.018536	0.020849	0.022695	0.024076	0.025001	0.025480	0.025530	0.025174	0.024441	0.023369	0.022005	0.020409	0.018648	0.016803
0.28	0.004645	0.008815	0.012503	0.015706	0.018420	0.020645	0.022384	0.023642	0.024429	0.024758	0.024649	0.024130	0.023233	0.022003	0.020492	0.018766	0.016900	0.014982
0.26	0.004662	0.008825	0.012484	0.015635	0.018277	0.020410	0.022039	0.023170	0.023817	0.023997	0.023732	0.023053	0.021999	0.020620	0.018974	0.017132	0.015179	0.013208
0.24	0.004674	0.008825	0.012449	0.015544	0.018110	0.020147	0.021664	0.022668	0.023176	0.023207	0.022789	0.021956	0.020753	0.019234	0.017466	0.015525	0.013504	0.011502
0.22	0.004681	0.008816	0.012402	0.015437	0.017923	0.019863	0.021265	0.022142	0.022512	0.022397	0.021831	0.020851	0.019508	0.017861	0.015985	0.013963	0.011893	0.009882
0.20	0.004684	0.008799	0.012343	0.015316	0.017720	0.019561	0.020849	0.021599	0.021833	0.021577	0.020867	0.019749	0.018275	0.016515	0.014545	0.012460	0.010362	0.008365
0.18	0.004683	0.008775	0.012275	0.015183	0.017504	0.019245	0.020420	0.021046	0.021147	0.020754	0.019909	0.018660	0.017068	0.015207	0.013161	0.011031	0.008927	0.006966
0.16	0.004680	0.008747	0.012200	0.015042	0.017279	0.018921	0.019984	0.020488	0.020460	0.019937	0.018964	0.017595	0.015897	0.013949	0.011845	0.009689	0.007599	0.005696
0.14	0.004674	0.008714	0.012119	0.014895	0.017048	0.018592	0.019545	0.019930	0.019780	0.019133	0.018040	0.016562	0.014771	0.012752	0.010606	0.008444	0.006388	0.004564
0.12	0.004667	0.008678	0.012035	0.014743	0.016814	0.018262	0.019108	0.019379	0.019111	0.018348	0.017146	0.015569	0.013698	0.011624	0.009454	0.007305	0.005303	0.003575
0.10	0.004659	0.008640	0.011948	0.014591	0.016580	0.017934	0.018676	0.018838	0.018459	0.017589	0.016286	0.014623	0.012686	0.010572	0.008395	0.006276	0.004347	0.002733
0.00	0.004610	0.008445	0.011520	0.013853	0.015470	0.016404	0.016696	0.016397	0.015569	0.014286	0.012634	0.010714	0.008640	0.006539	0.004545	0.002797	0.001421	0.000508

Table 2. Yield per recruit for $M/K = 2.0$.

Example 1: Consider the development of a harvesting strategy for a roundfish species where $K = 0.2$, $M = 0.1$, $W_\infty = 10$ kg, and $t_0 = 0$. Should the mesh size in the trawl fishery be regulated to allow entry into the fishery at age 3 or age 5? To what level should fishing mortality be set for these ages so as to maximize yield (F_{MAX}).

The result of those calculations are illustrated as Figure 1. At a $t_c = 3$, the maximum yield per recruit is 1.67 kg at $F = 0.19$. In contrast if the $t_c = 5$, the maximum yield per recruit is 2.02 kg at an $F = 0.31$, an increase of 21%. Interestingly, if t_c is set to 10 years, maximum YPR is achieved at 2.36 kg with an $F = 2.0$. Thus, for fish species where M/K is small (0.5), substantially greater yields, about 40%, are realized by delaying entry into the fishery.

M	=	0.1
W_∞	=	10
K	=	0.2
t_0	=	0
t_c	=	3
t_r	=	0
N_{tr}	=	100000
t_y	=	20
M/K	=	0.50



M	=	0.1
W_∞	=	10
K	=	0.2
t_0	=	0
t_c	=	5
t_r	=	0
N_{tr}	=	100000
t_y	=	20
M/K	=	0.50

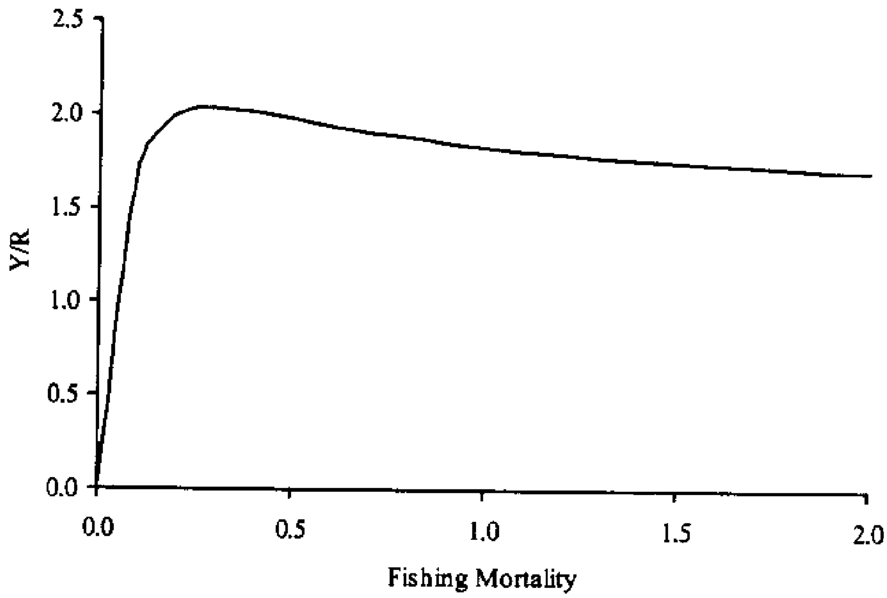


Figure 1. Yield per recruit as a function of fishing mortality;
(A) $t_c = 3.0$
(B) $t_c = 5.0$.

Discrete Time Model

A discrete time model (DeAlteris and Riedel 1996) was developed to incorporate more complex size selection patterns than the knife edge selection assumed in the Beverton-Holt model. The methodology is based on a computer spreadsheet. The time step is set at 0.1 years, over the range of 0 to 30 years.

The length of the fish (L) at age (t) is calculated using a simplified ($t_0 = 0$) von Bertalanffy growth equation:

$$L_t = L_\infty\left(1 - e^{(-Kt)}\right)$$

where L_∞ is the maximum length, and
K is the instantaneous growth rate.

The weight of the fish (W) at age t is determined using a length-weight relationship.

$$W_t = (aL_t)^b$$

where a is the L-W conversion factor, and
b is the L-W growth factor.

The percent maturity (P_t) of individuals in the cohort at age is expressed using a LCDF:

$$P_t = \left(1 + e^{(-\alpha 1*(t-\beta 1))}\right)^{-1}$$

where α is the steepness of the curve, and
β is the age at 50% maturity.

The number of individuals (N_t) remaining in the unfished cohort at age t is determined using an instantaneous natural decay function incremented at the time step of t years:

$$N_t = N_{(t-1)}e^{(-tM)}$$

where M is the instantaneous natural mortality, and
N_0, the initial cohort size, is 1000 individuals.

The biomass (B_t) of the individuals remaining in the unfished cohort at age t is calculated:

$$B_t = N_t * W_t$$

and the unfished spawning stock biomass (UFSSB) of the individuals remaining in the cohort at age t is determined:

$$UFSSB_t = P_t * B_t \, .$$

Based on gear selection literature, trawls and hooks are assumed to possess qualitatively similar size selection characteristics, which can be represented by LCDF of individual fish length (PL_t):

$$PL_L = \left(1 + e^{(-\alpha 2 * (L - L_{50}))}\right)^{-1}$$

where α = steepness of the curve, and
L_{50} = length at 50% selection.

Gillnets and traps are assumed to possess qualitatively similar size-selection characteristics which can be represented by a truncated, scaled NPDF of individual fish length (PN_t):

$$PN_L = e^{\left[-\left(L - L_{opt}\right)^2 / \left(2 * SD^2\right)\right]}$$

where SD = standard deviation, and
L_{opt} = length of maximum selection.

Applying length-specific susceptibility to fishing (PN_L or PL_L) at a specified level of fishing mortality (F) and including natural mortality (M), the number of individuals remaining in the fished cohort (NF_t) at each time step (t) is calculated as:

$$NF_t = NF_{(t-1)} e^{\{-[((PN_L or PL_L)(F)) + M] * t\}} .$$

Thus, the yield of the fished cohort (Y_t) from each time-step is:

$$Y_t = \left[(PN_L \quad or \quad PL_L)(F) \Big/ ((PN_L \quad or \quad PL_L)(F)) + M \right] * (NF_{(t-1)} - NF_t) * (W_t)$$

and the spawning stock biomass of the fished cohort (SSB_t) at each time step is simply:

$$SSB_t = (NF_t) * (W_t) * (P_t).$$

Given these equations and specific values of L_∞, K, a, b, α, β, and M, the total biomass and spawning stock biomass of the unfished cohort are determined. With the specification of fishing conditions (F, α, L_{50}, SD, and L_{opt}) total yield and spawning stock biomass of the fished cohort are determined. By evaluating a wide range of L_{50}, L_{opt} and F values, the resulting matrix of data, expressed as a percentage of the maximum value, is contoured to produce isopleth diagrams of yield per recruit (YPR) and spawning stock biomass per recruit (SPR).

The effect of the selectivity function's shape on YPR and SPR is evaluated by specifying a range of steepness and standard deviations for the LCDF and NPDF while holding other factors constant.

Example 2: Evaluate the effects of increasing size at entry from 50 to 100 cm in 10 cm increments on the yield and SSB of an idealized roundfish harvested by both trawls and gill nets. For trawls, the steepness of the LCDF curve is 0.33. For gill nets, the standard deviation of the NPDF is 5. The specifications for the idealized roundfish used in this analysis are: L_∞ = 100 cm, K = 0.2, a = 0.00001, b = 3, α = 1, β = 3, and M = 0.2.

Based on these values, the characteristics of the individuals and the cohort of idealized roundfish are shown in Figures 2 and 3. An individual idealized roundfish reaches an asymptotic maximum length and weight of 100 cm and 10 kg. Maturation is assumed to occur rapidly, with 50% of the cohort mature at an age of 3 years and a length of about 45 cm. Based on an initial cohort of 1000, the number of individuals in the unfished cohort is reduced to about 5% of the initial number by the age of 16 years, although the model is extended to an age of 30 years when only a single fish remains. Biomass of the cohort peaks at an age of 6.3 years and an individual fish length of 75 cm.

The LCDF and NPDF for size selection are shown in Figures 4A and 4B. The L_{50}s for the LCDF ranged from 50 to 100 cm, and a representative steepness of 0.33 is specified. The L_{opt}s ranged from 50 to 100 cm, and a representative standard deviation of 5 is specified.

The spreadsheet program is now run for a range of fishing mortality values from 0 to 0.5 at 0.1 intervals and 0.5 to 4.0 at 0.5 intervals, calculating YPR and SPR values for both types of selection functions at each of the six L_{50} and L_{opt} values. The resulting isopleth diagrams for YPR and SPR are shown in Figures 5 and 6 for the LCDF and NPDF, respectively.

Evaluating the isopleth diagrams for the LCDF, it is clear that maximum YPR will be realized at an L_{50} of 80 cm and at fishing mortalities of 3.0 and greater. Operating the fishery in this range will provide a relative SPR of 35% at F = 3.0, decreasing to 25% at F = 4.0.

Evaluating the isopleth diagrams of the NPDF, it is clear again that maximum YPR will be realized at a L_{50} of 80 cm and at fishing mortalities of 2.0 and greater. Operating the fishery in this range will provide a spawning stock biomass of 30% at F = 2.0, decreasing to 26% at F = 4.0.

The effect of the shape of the selection curve on the YPR and SPR is evaluated at an L_{50} or L_{opt} of 80 cm and an F value of 3.0. Steepness values ranging from 0.13 to 2.00 are specified for the LCDF (Figure 7). Increasing the steepness of the selection curve effects both the YPR and SPR. Lower values for the steepness parameter results in a 100% increase in YPR and 50% reduction of the SPR. Standard deviation values ranging from 2 to 10 are specified for the NPDF (Figure 8). Increasing the standard deviation of the selection curve results in 50% reduction in SPR and a 300% increase in YPR.

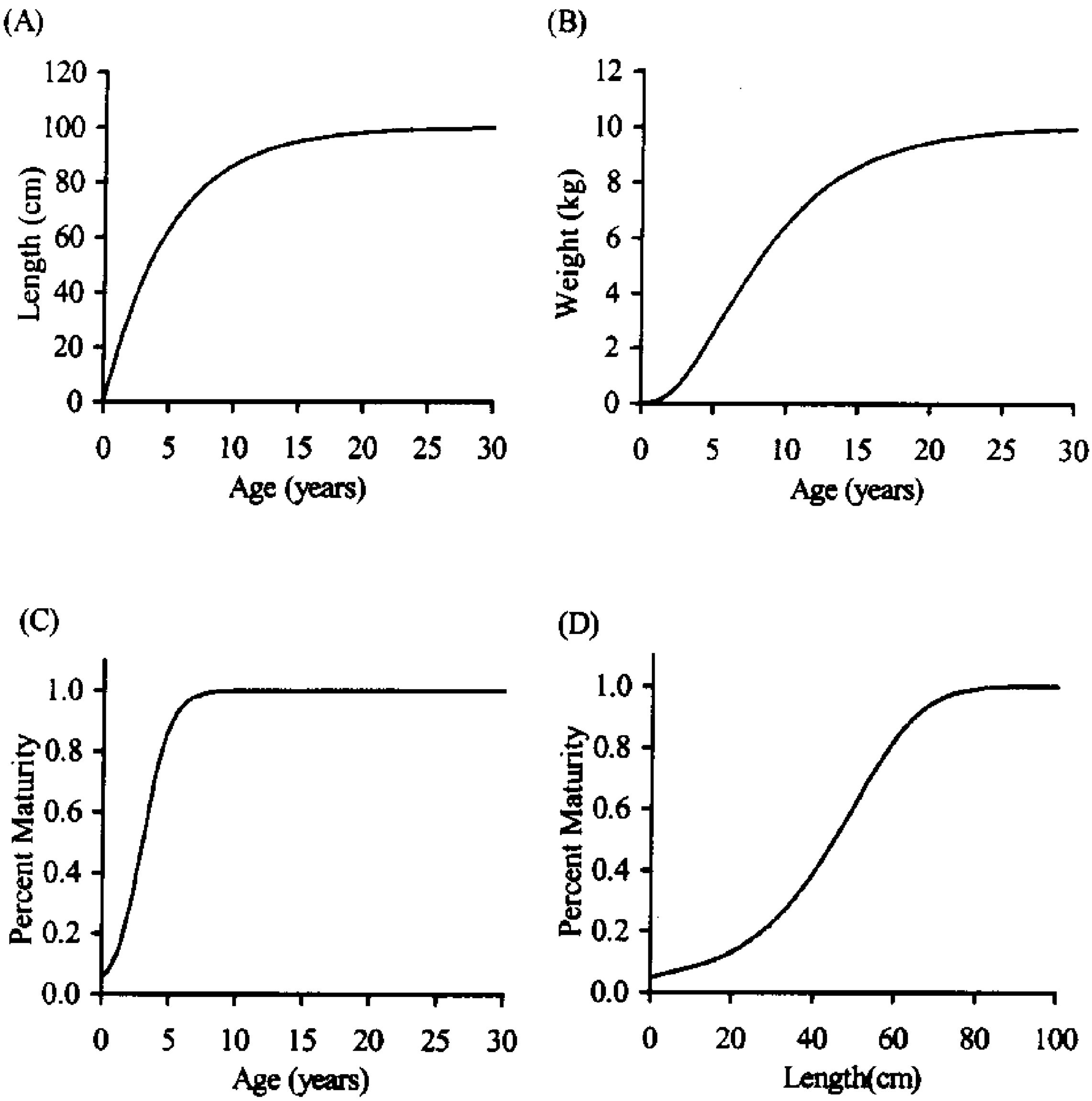


Figure 2. Functional characteristics of an unfished cohort of an idealized roundfish;
(A) Length as a function of age,
(B) Weight as a function of age,
(C) Percent Maturity as a function of age, and
(D) Percent Maturity as a function of length.

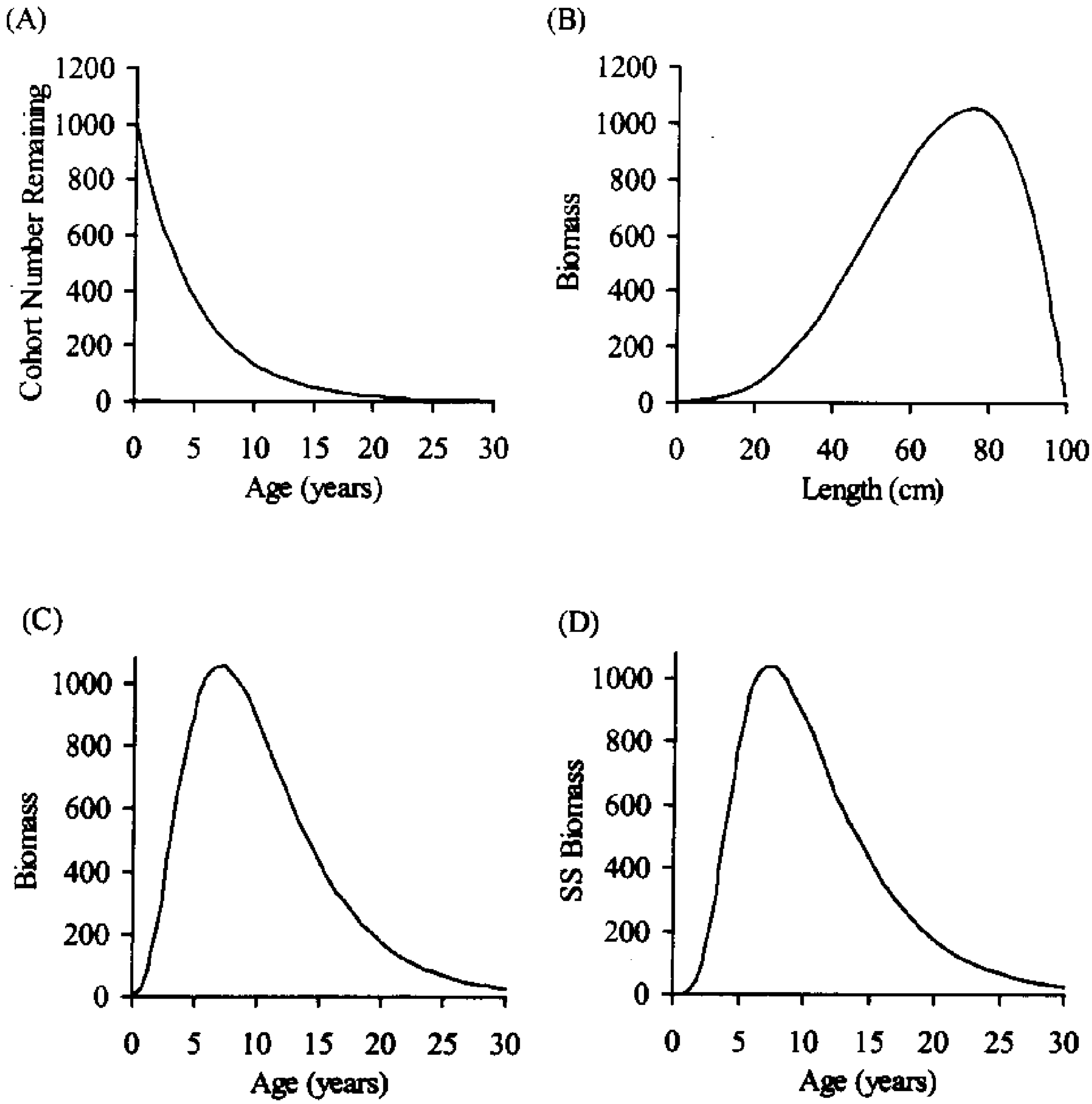


Figure 3. Functional characteristics of an unfished cohort of an idealized roundfish;
(A) Numbers as a function of age,
(B) Biomass as a function of length,
(C) Biomass as a function of age, and
(D) Spawning Biomass as a function of age.

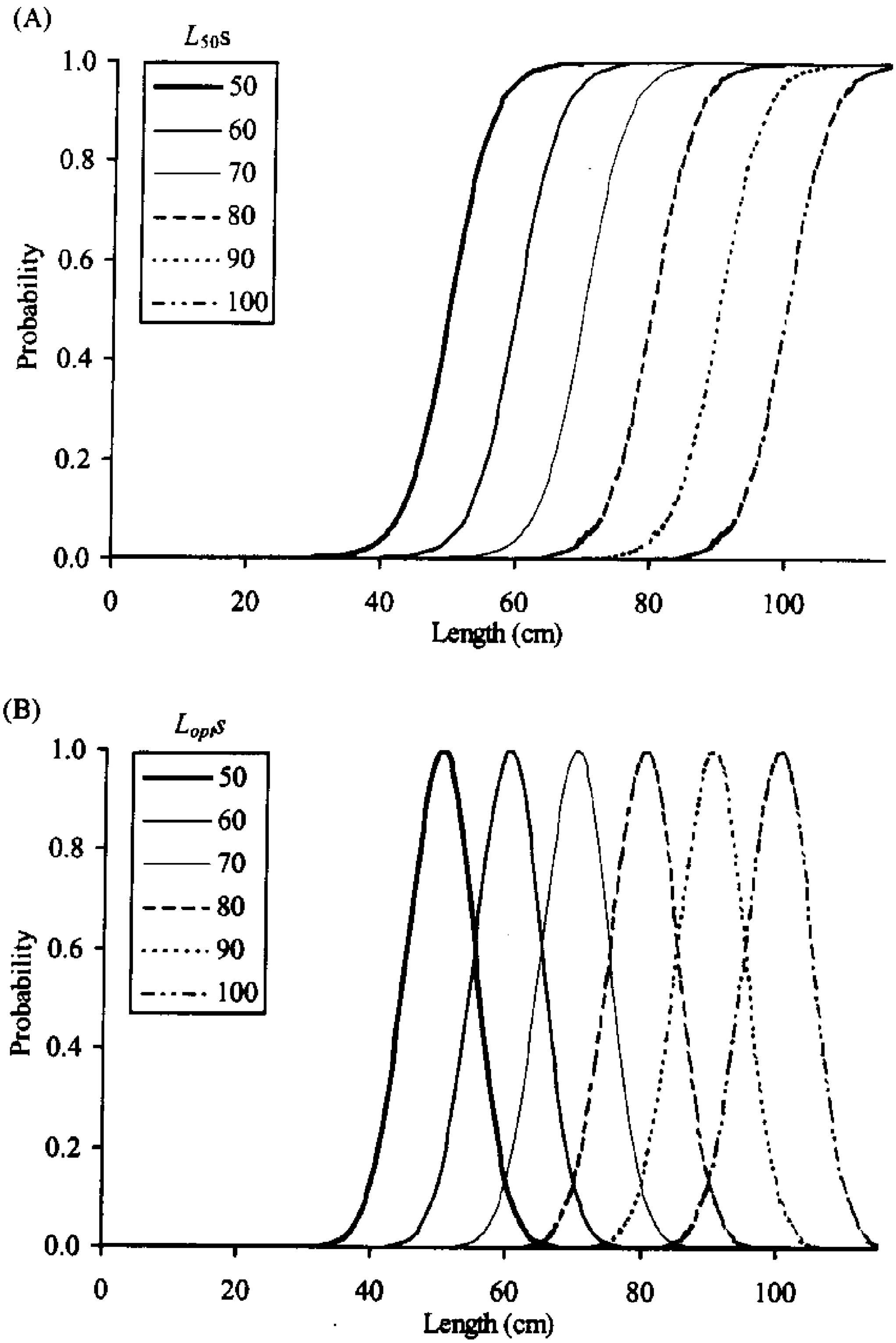


Figure 4. Selection characteristics of harvesting gears used on the cohort of idealized roundfish;
(A) Logistic cumulative distribution function (LCDF) selectivity curves for L_{50}s from 50-100 cm and
(B) Normal probability density function (NPDF) selectivity curves for L_{opt}s from 50-100 cm.

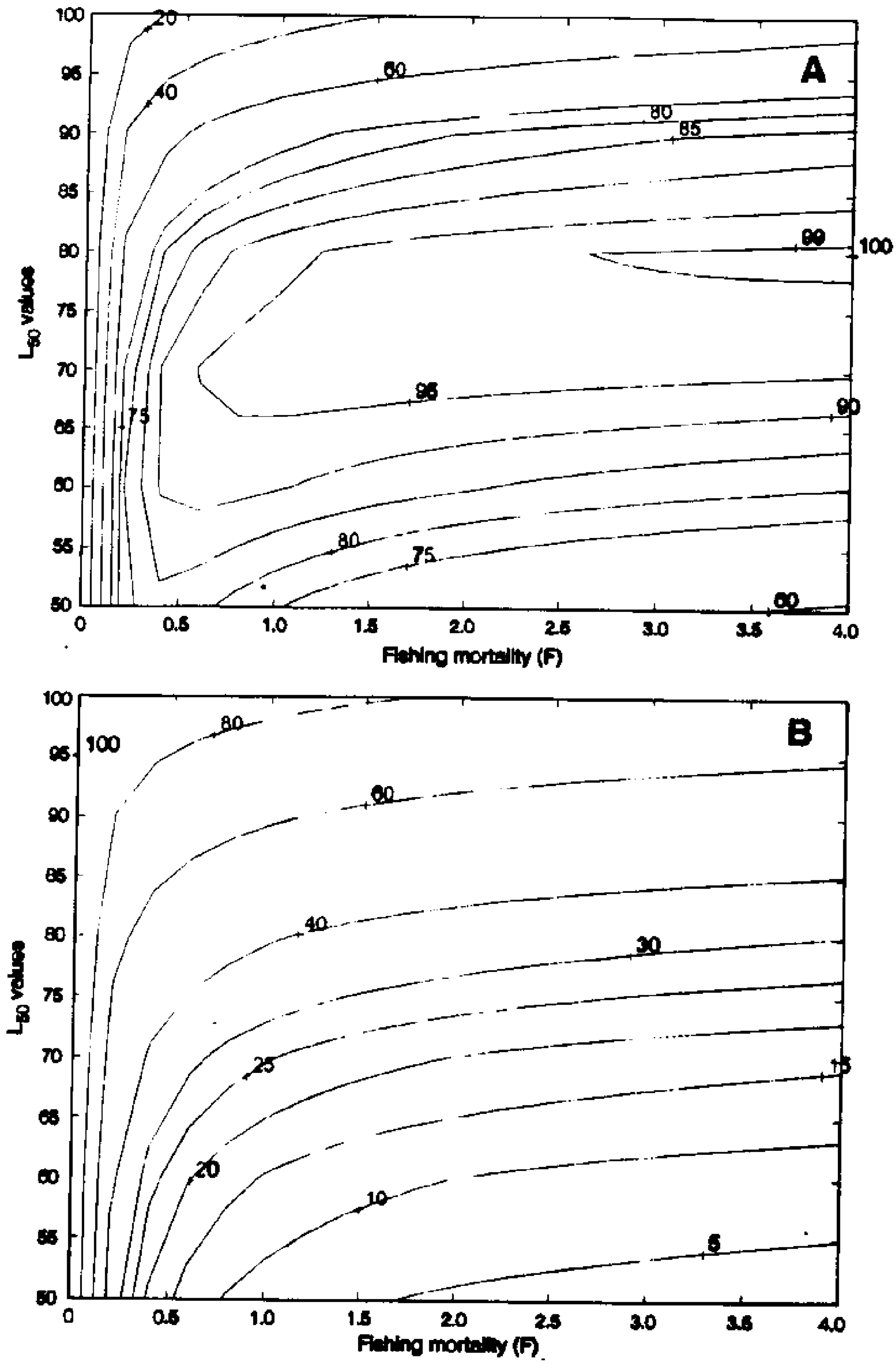
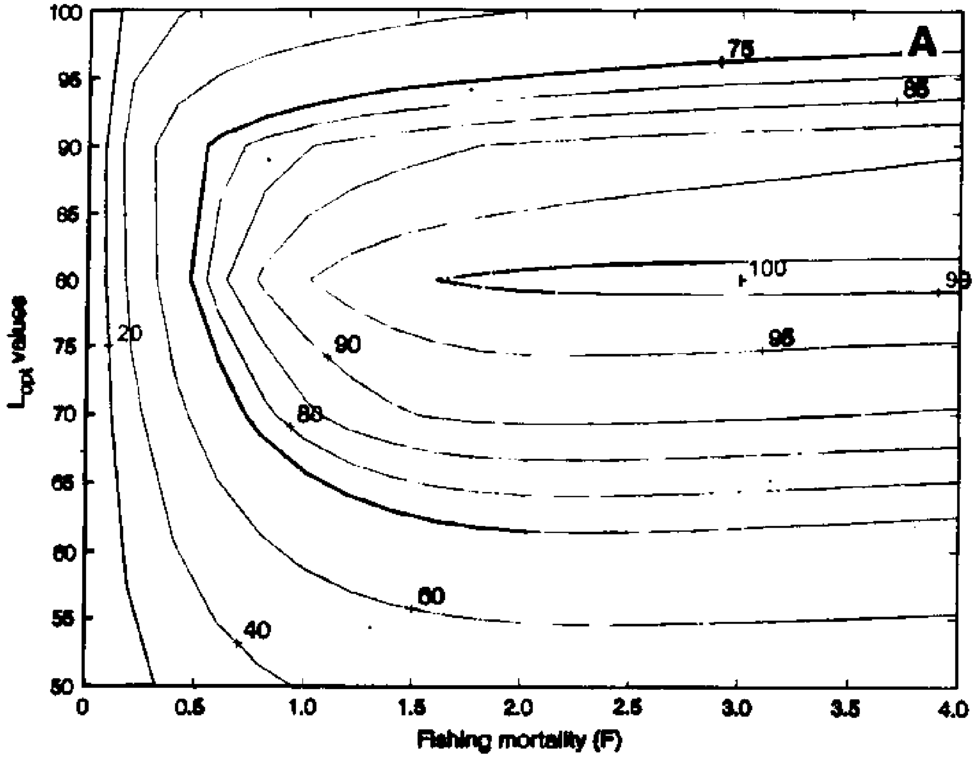


Figure 5. Isopleth diagrams expressed as a percentage of maximum for size selection based on a LCDF:
(A) Yield per Recruit (YPR) and
(B) Spawning stock biomass per recruit (SSBPR).



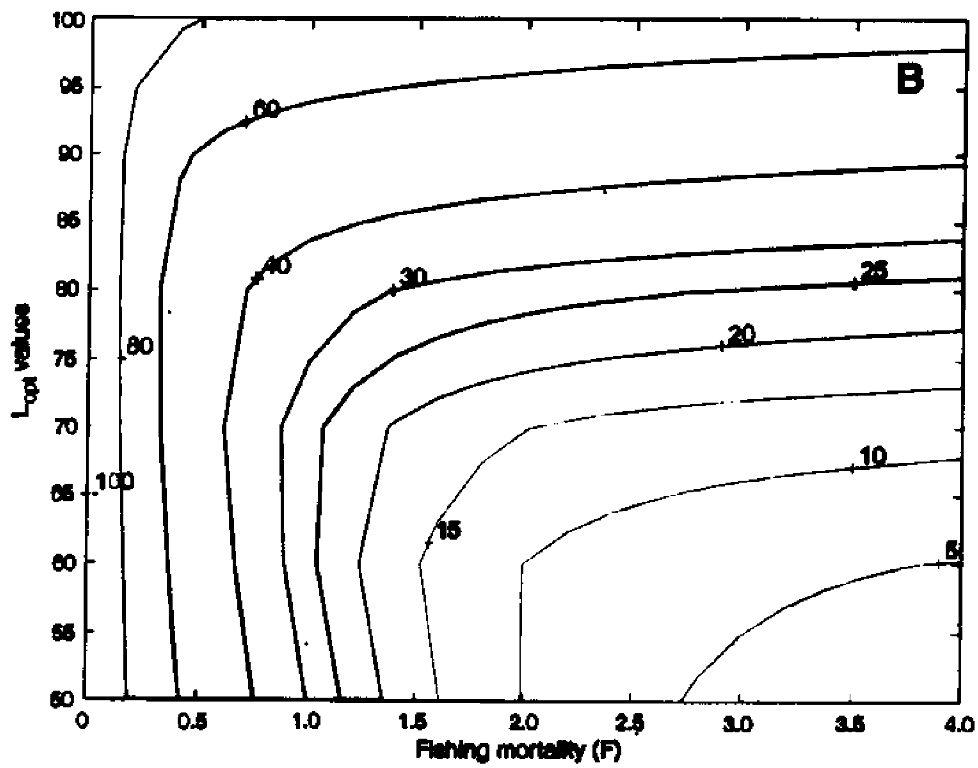


Figure 6. Isopleth diagrams expressed as a percentage of maximum for size selection based on a NPDF:
(A) Yield per recruit (YPR) and
(B) Spawning stock biomass per recruit (SSBPR).

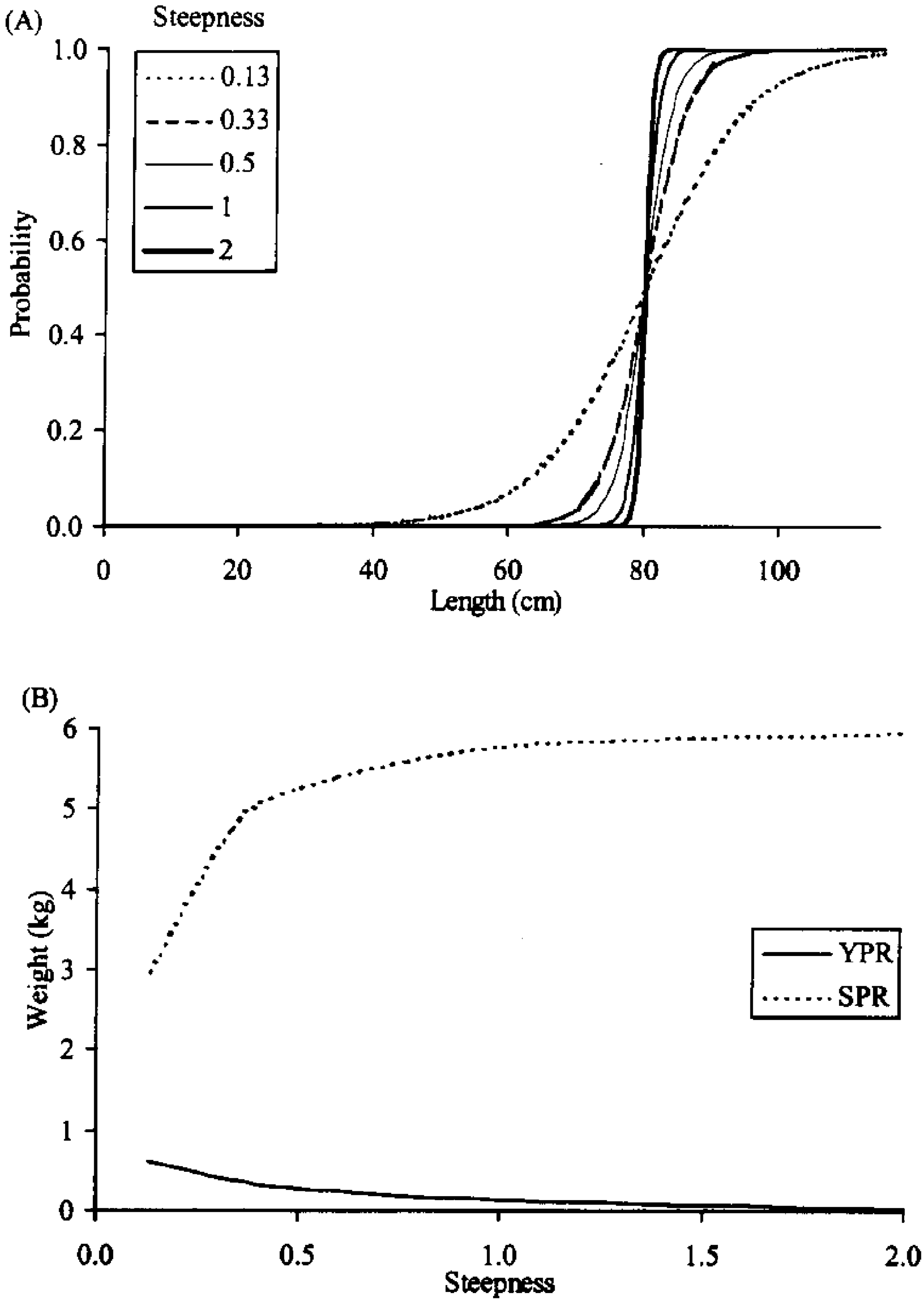


Figure 7. Effect of the steepness of the LCDF on the cohort of an idealized roundfish:
(A) Size selectivity curve and
(B) YPR and SPR at L_{50} = 80 cm and F = 3.0.

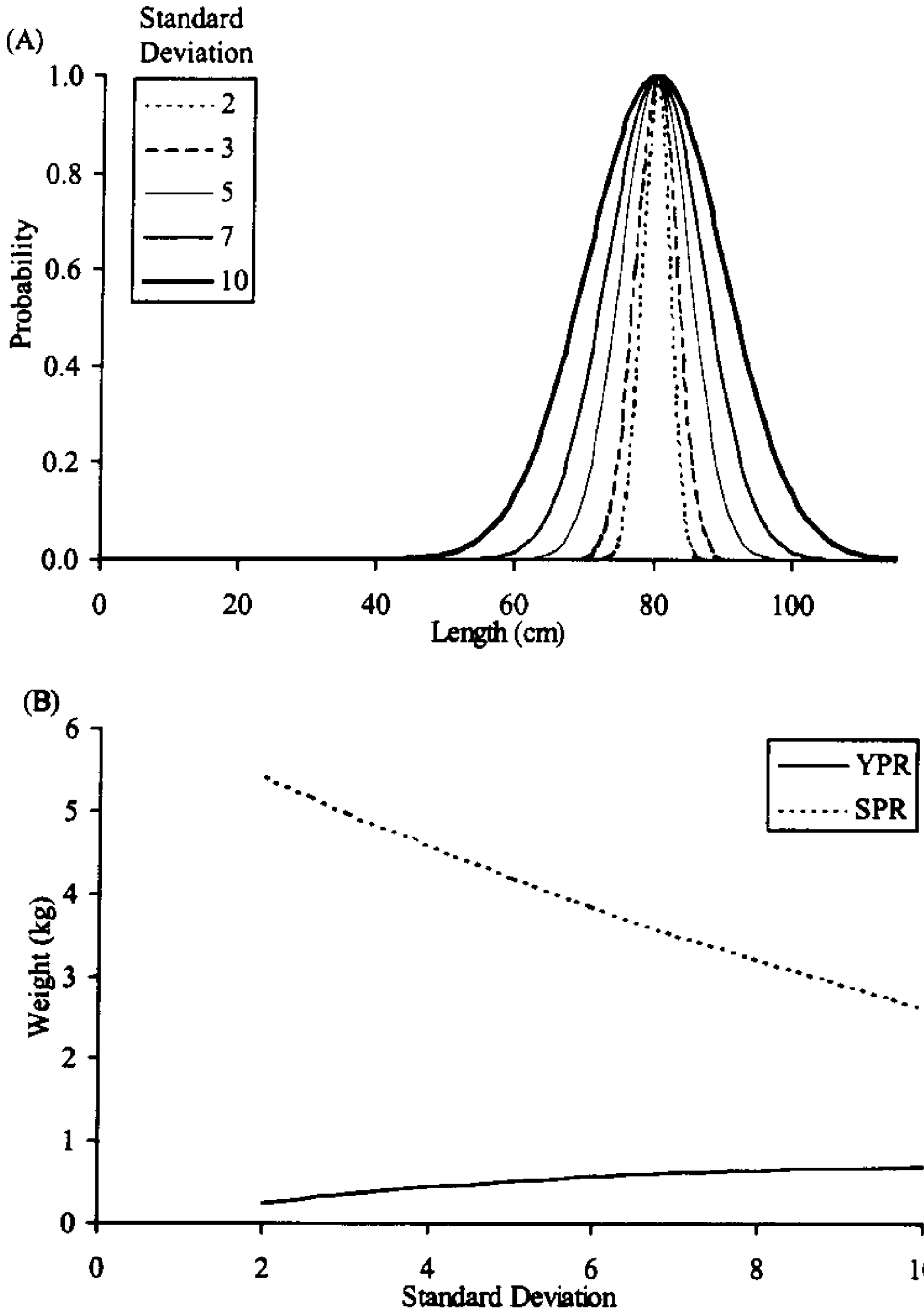


Figure 8. Effect of the standard deviation of the NPDF for the cohort of an idealized roundfish:
(A) Size selectivity curve and
(B) YPR and SPR at L_{opt} = 80 cm and F = 3.0.

Exercises

Northwest Atlantic groundfish species have markedly different growth and mortality rates as indicated in the following table.

Species	K	M	M/K	W_∞	L_∞
cod	0.12	0.2	1.7	33.7	148
haddock	0.38	0.2	0.5	4.4	74
silver hake	0.18	0.4	2.2	2.0	65
winter flounder	0.37	0.2	0.5	3.5	63
yellowtail flounder	0.63	0.2	0.3	0.9	46
plaice	0.17	0.2	1.2	2.4	65
summer flounder	0.21	0.2	1.0	7.6	84

Given this variability in M/K ratios and L_∞, there is the need to have different harvesting strategies in terms of age at entry into the fishery and target fishery mortality levels to maximize yield. The implementation of these strategies requires differing mesh size regulations for a trawl fishery so as to control age at entry, or retention by the gear.

1. Using the Beverton-Holt analytical solution to the yield per recruit problem, compare the harvesting strategies (age at entry to the fishery, and fishing mortality) to maximize yield for cod, silver hake and yellowtail flounder. Note that silver hake and cod have a M/K ratio of about 2.0, while yellowtail flounder has a M/K ratio of less than 0.5. Assuming that the selection factor for diamond mesh trawl codends are 3.7, 3.5, and 2.6 for cod, silver hake, and yellowtail flounder, respectively, and that management seeks to match trawl selection (L_{50}) to YPR_{MAX}, targets, determine the appropriate mesh size for each species. Recall that the simplified von Bertalanffy age-length relationship is $L_t = L_\infty\left(1 - e^{[-K*t]}\right)$.

2. Using the discrete YPR and SSB model for summer flounder, compare the yield and spawning stock biomass curves for gillnets and trawls if gear regulations are set so as to achieve L_{50}s and L_{opt}s, of 35 cm as in the present regulations, and 55 cm as may be a future target.

Notes

A. Summer flounder maturity parameters are $\alpha = 5$ and $\beta = 2.37$ and length-width relationship parameters are $a = 0.000$ and $b = 3.07$.

B. NPDF SD is 5 and the LCDF steepness 0.33.

C. Develop the discrete time YPR and SSB per recruit curves at F intervals of 0.1 in the range of 0.0 to 0.5, and intervals of 0.5 in the range of 0.5 to 3.0.

D. When evaluating the SSB curve, note the 20% of virgin SSB line.

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PRODUCTION MODELS

PRODUCTION MODELS

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Introduction

Thus far, the growth and mortality of a single cohort of animals has been considered. Based on species-specific life history characteristics, a model for a harvesting strategy that includes age at entry into the fishery and a fishing mortality rate has been derived for a cohort of animals. The yield per recruit (YPR) analysis is used to set targets or reference points for the harvest of a fishery resource to maximize yield and prevent growth overfishing. The corollary to the YPR analysis is the spawning stock biomass or egg per recruit analysis that is used to ensure that a minimum percentage of the virgin spawning stock biomass or egg production remains in the stock, so as to prevent recruitment overfishing. While these analyses are useful to set harvesting targets assuming a healthy stock and consistent recruitment to that stock, clearly the intensity of fishing must also be regulated with respect to the status of the stock, that is the abundance of animals in the stock relative to maximum number of animals of a particular species that the ecosystem can support. This concept is fundamental in ecology and refers to the carrying capacity of the environment.

There are two broad categories of models used to assess that status of fish stocks:

1. Global models, known as production or biomass dynamic models which do not distinguish between recruitment, growth, and mortality as contributing factors to overall changes in population abundance but consider only their resultant effect as a single function of the population size. These models do not rely on age structure, and are particularly useful when age data is not available or when the catch cannot be aged. These models are simple in their concept and use, and require a minimum of data.

2. Structural models, known as age-structured models include cohort analysis or virtual population analysis, which divide the catch into age groups and provide estimates of time specific biomass and fishing mortality at age. These models require more data and can be complicated when allowing calibration using independent information on abundance-at-age of the stock.

This chapter develops the concepts and application of global models, which include the surplus or stock production models that assume quasi-equilibrium conditions between yield and effort. These models are the precursors of the true biomass dynamic models that consider time-history trends in biomass indices and catch.

Population Growth and Regulation

The simplest model of population growth over time assumes birth (b) and death (d) rates are consistent over all ranges of population density (*i.e.* these rates are density independent).

Assuming that the population is closed, spatially homogeneous, and that there is no age structure, the time history of that population is described by:

$$dN/dt = (b-d)N = rN$$

where r is the intrinsic rate of growth or decay of the population.

If the birth rate exceeds death rate, then dN/dt is positive ($r > 0$); if the birth rate equals death rate then dN/dt is zero ($r = 0$); and if death rate exceeds the birth rate, then dN/dt is negative ($r < 0$). Note that this relationship is a differential equation, but it also fits the linear model $y = ax$ (Figure 1).

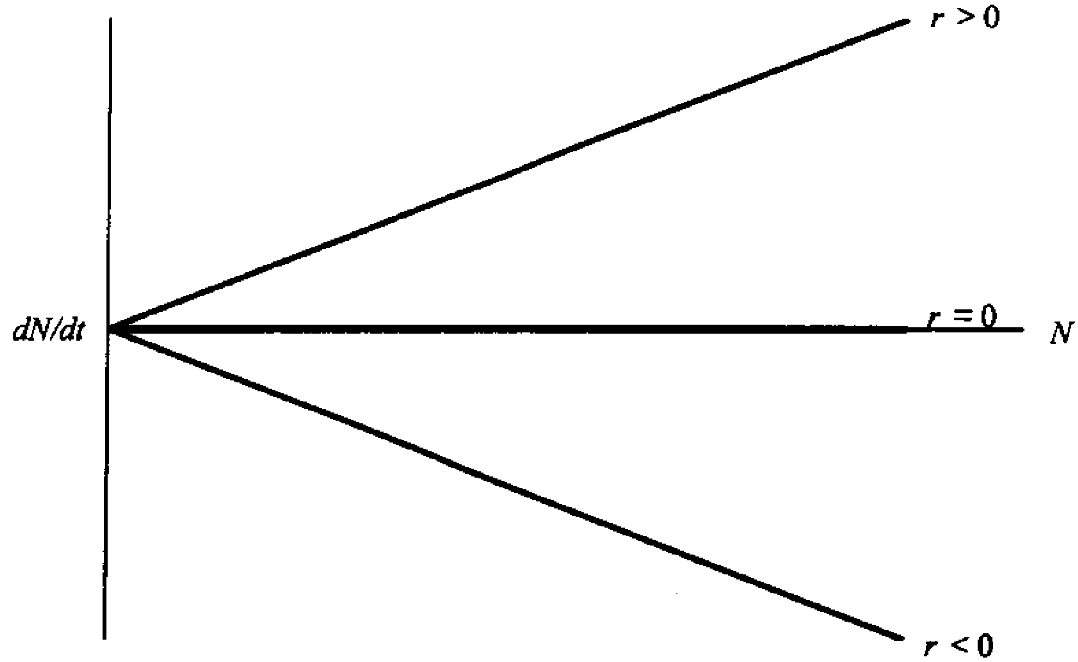

Figure 1. The rate of change of population number (dN/dt) as a function of population size (N) for $r > 0$, $= 0$, and < 0.

Note also that if the differential equation is rearranged as follows:

$$\left(1/N\right) dN/dt = (b-d) = r.$$

The "per capita" rate of change of a population in this density independent model is a constant (r). This differential equation is solved by the separation of variables method,

$$\int 1/N \, dN = \int (b-d)dt$$
$$N_t = N_0 e^{(b-d)t}.$$

The trajectory of a population with density independent birth and death rates is shown in Figure 2. If $r > 0$, then population numbers grow exponentially; if $r < 0$, population numbers decay as a negative exponential to 0, and if $r = 0$ the population remains in a neutral equilibrium, where any perturbation will disturb the balance, and the population will grow exponentially or decline toward extinction.

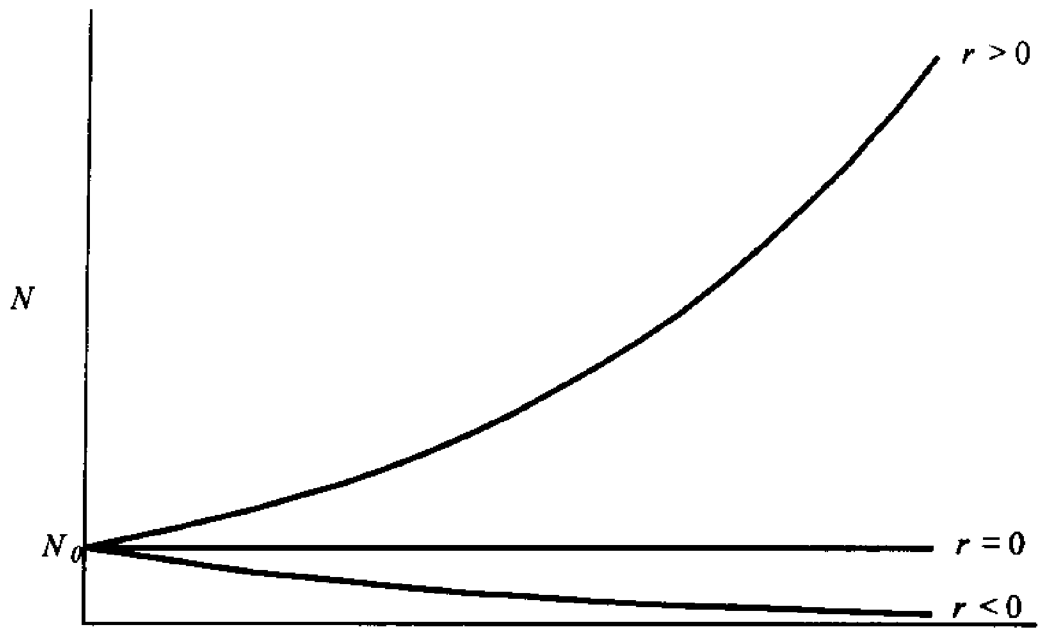

Figure 2. Population trajectories for density independent growth where $r > 0$, $= 0$, and < 0.

In summary, while a model that includes only density independent terms is conceptually simple, it is incapable of producing a stable population. Therefore, density dependence must be introduced into the model to regulate population growth.

A reasonable approach for the addition of density dependence in the birth and death rates is to express these as linear functions:

$$b = b_0 - b_1 N$$
$$d = d_0 + d_1 N$$

where b_0 and d_0 are the rates at $N = 0$, and
b_1 and d_1 are the population dependent coefficients.

These functional relationships are shown in Figure 3, and are incorporated into the basic population growth equation as follows:

$$dN/_{dt} = [(b_0 - b_1 N) - (d_0 - d_1 N)]N$$

or rearranging:

$$\frac{dN}{dt} = [(b_0 - d_0) - (b_1 + d_1)N]N.$$

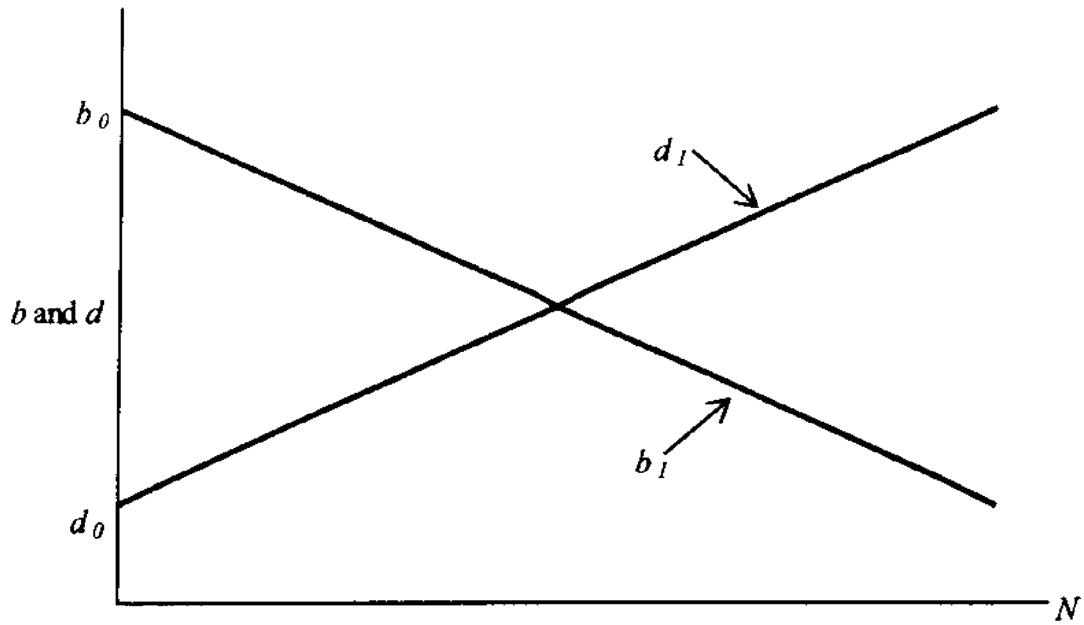

Figure 3. Birth (*b*) and death (*d*) rates incorporating density dependence.

Replacing the initial birth and death rate difference with α and the density dependent birth and death rate coefficients with β results in the following:

$$\frac{dN}{dt} = (\alpha - \beta N)N = \alpha N - \beta N^2.$$

Note that this equation has the form of a parabolic function (Figure 4). Additionally,

at $N = 0$, $\quad \frac{dN}{dt} = 0$

at N_{MAX}, $\quad \frac{dN}{dt} = 0$

$$N_{MAX} = \frac{\alpha}{\beta} = \frac{(b_0 - d_0)}{(b_1 + d_1)}$$

and the population size (N) that achieves the maximum rate of population change is determined by taking the derivative of the function and setting it equal to zero:

$$\frac{d}{dN}[(\alpha - \beta N)N] = 0$$
$$\alpha - 2\beta N = 0$$
$$N = \tfrac{1}{2}\left(\frac{\alpha}{\beta}\right) \text{ or } \tfrac{1}{2} \text{ carrying capacity.}$$

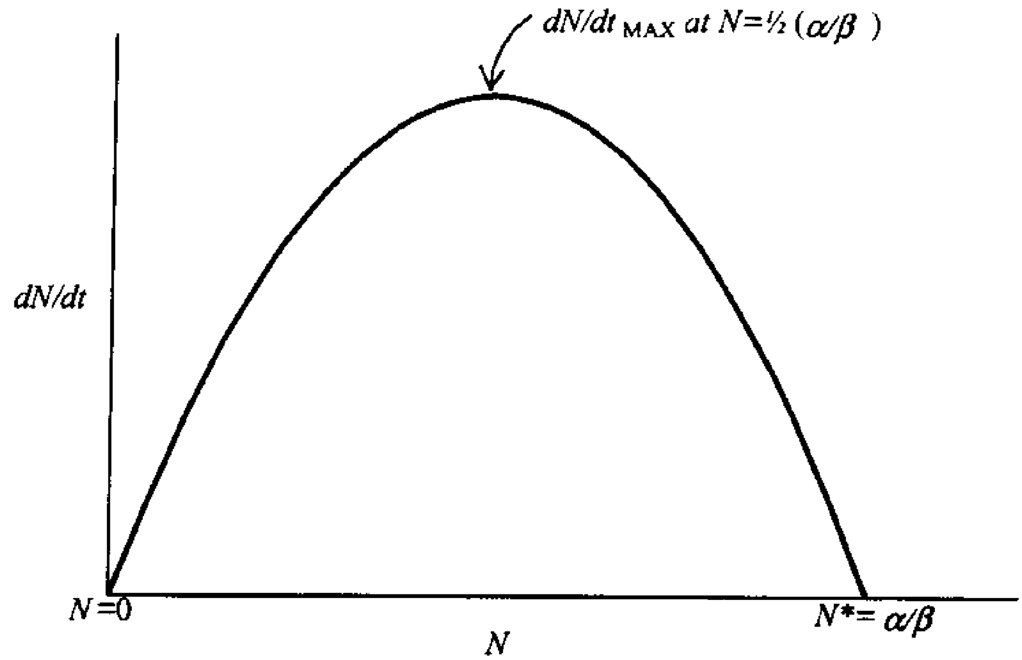

Figure 4. Parabolic model relating density dependent rate of population change to population size.

The differential equation relating dN/dt to N is solved using the separation of variables method:

$$\int\left[1/(\alpha-\beta N)N\right]dN = \int dt$$

$$N_t = [\alpha/\beta] \Big/ \left[((\alpha-\beta N)/\beta N_0)e^{-\alpha t}+1\right]$$

substituting $$N_{MAX} = \left(\alpha/\beta\right)$$

and $$\gamma = (\alpha-\beta N_0) \Big/ (\beta N_0)$$

yields: $$N_t = N_{MAX} \Big/ \left[1+\gamma e^{-\alpha t}\right].$$

Note that this equation has the form of a generalized logistic function (Figure 5).

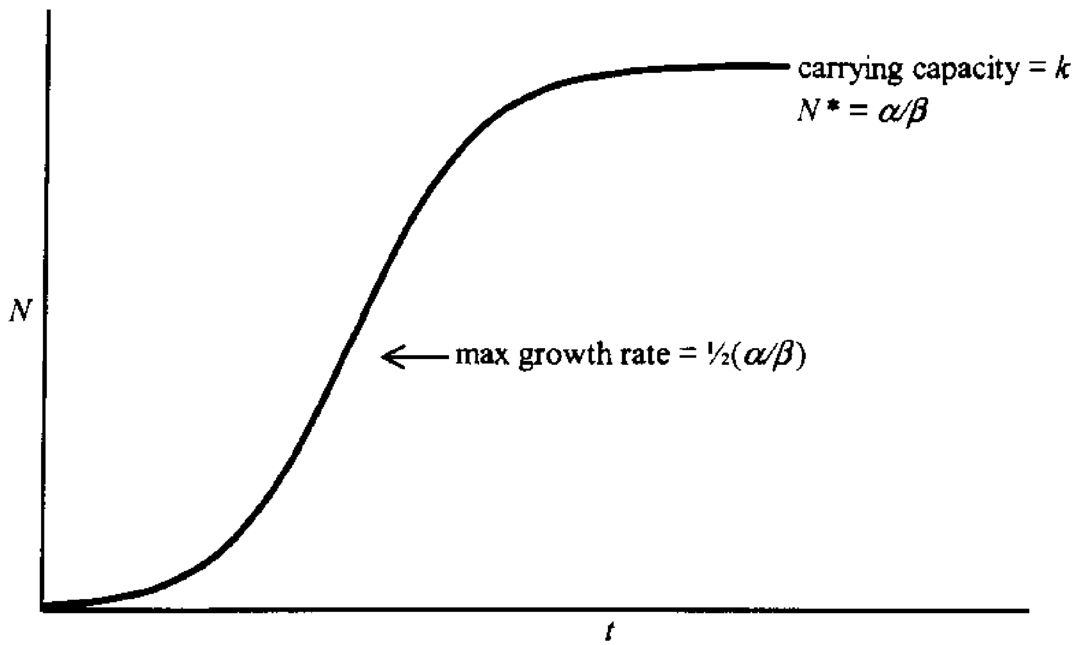

Figure 5. Logistic model describing population number as a function of time.

Anthropogenic mortality or harvesting is added to the model of population regulation as follows:

$$dN/_{dt} = (\alpha - \beta N)N - aN$$

where a is a coefficient or rate of anthropogenic removal that can be equated to a fishing mortality rate.

Surplus Production Model

Russell (1935) advanced the mass-balance concept, arguing that fish stocks fluctuate in abundance according to imbalance between additions and losses to the stock and this balance can be summarized as follows:

New Biomass = Old Biomass + Recruitment + Growth - Catch - Natural mortality.

If the sum of the recruitment and growth is larger than the sum of catch and natural mortality, a stock increases in abundance; if the losses exceed the additions, the stock declines. Grouping terms relating to natural processes (recruitment, growth, and natural mortality) and referring to them collectively as *Surplus Production* yields:

New Biomass = Old Biomass + Surplus Production - Catch.

For a stock to remain at a given level of biomass (New Biomass = Old Biomass), the fishery removal (catch) should not be larger than the surplus production of the stock. To rebuild a stock, catch must be lower than the surplus production.

These types of models are attractive in stock assessment in that not only do they have biological soundness but also require minimal amounts of data. The basic set of data required for surplus production models is a time series of catch and fishing effort.

Schaefer Model

The surplus production model is derived from the density dependent population growth model by replacing population number (N) with biomass (B),

$$dB/dt = (\alpha - \beta B)B - FB\,.$$

At equilibrium $dB/dt = 0$, therefore:

$$(\alpha - \beta B^*)B^* = F^* B^*$$
$$(\alpha - \beta B^*) = F^*$$
$$B^* = (\alpha - F^*)/\beta$$

where B^* represents equilibrium levels of biomass at specific equilibrium levels of fishing mortality (F^*).

Recalling that catch or anthropogenic removal is the product of biomass and fishing mortality then equilibrium yield values are defined as:

$$Y^* = F^* B^*\,.$$

Substituting for equilibrium biomass and expanding results in the basic surplus production model:

$$Y^* = F^*\left(\frac{\alpha - F^*}{\beta}\right) = \left[\left(\alpha/\beta\right) - \left(1/\beta\right)F^*\right]F^*\,.$$

Substituting:

a for $\left(\alpha/\beta\right)$
b for $\left(1/\beta\right)$, and
f for F assuming catchability is constant (based on $F = fq$)

results in the generalized Schaefer Model (1954):

$$Y = (a - bf)f = af - bf^2 .$$

This model has the parabolic or dome shaped form that relates yield (Y) to fishing effort (f) (Figure 6). The level of effort required to achieve Maximum Equilibrium Yield (MEY) is determined by taking the derivative of the function and setting it equal to zero:

$$\frac{d}{df}\left[af - bf^2\right] = 0$$
$$a - 2bf = 0$$
$$f_{MEY} = \frac{a}{2b} .$$

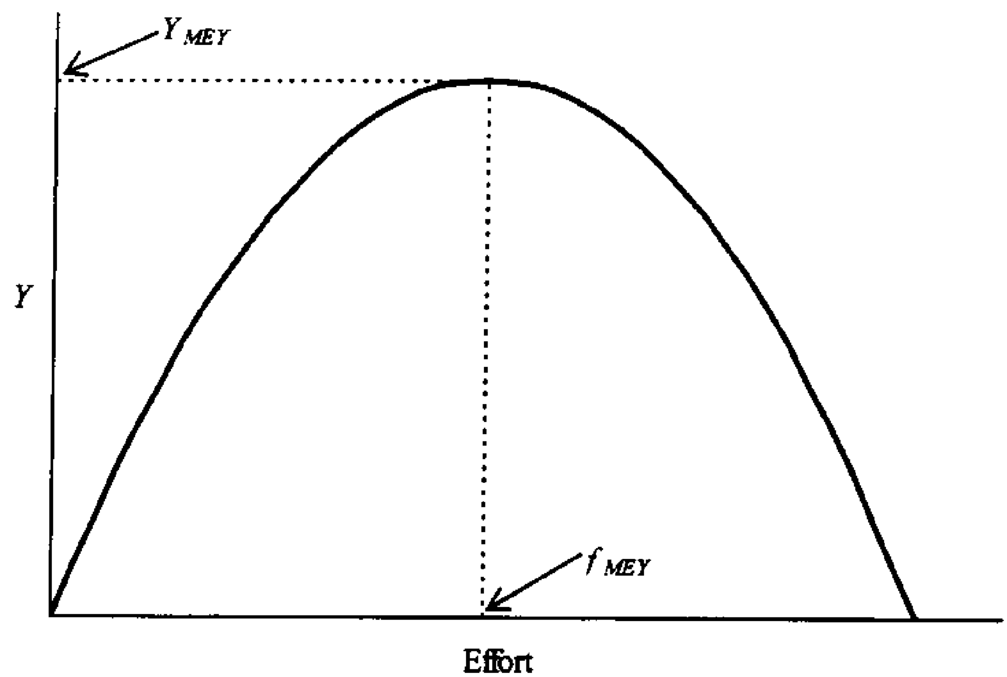

Figure 6. Schaefer model relating yield to effort.

MEY is then determined by substituting f_{MEY} back into the original equation:

$$Y_{MEY} = a\left(\frac{a}{2b}\right) - b\left(\frac{a}{2b}\right)^2$$
$$= \frac{a^2}{2b} - \frac{ba^2}{4b^2}$$
$$Y_{MEY} = \frac{2a^2b}{4b^2} - \frac{a^2b}{4b^2} = \frac{a^2b}{4b^2} = \frac{a^2}{4b} .$$

Notice that the MEY occurs at the point of maximum growth (one-half carrying capacity).

The parameters a and b of the Schaefer model are initially estimated by linearizing the function:

$$Y/f = a - bf,$$

and using linear regression on CPUE $\left(Y/f\right)$ versus effort (f). Non-linear, best-fit estimation of the parameters is accomplished using **Solver** in *Microsoft Excel*, with parameter starting values from the linearized estimation.

Fox Model

An alternative model to fitting the relationship between catch and effort was introduced by Fox in 1970 which assumes that a stock would respond to intense fishing by maximizing productivity thus the yield would never reach zero. This model also assumes that CPUE would decline as effort increases and provides an estimate of the MEY usually close to the Schaefer model. The Fox model has the form

$$Y = fe^{(c-df)}.$$

This model is linearized to:

$$\ln\left(Y/f\right) = c - df.$$

MEY for the Fox model is estimated by again taking the derivative of the function, setting it equal to zero to solve for f at MEY, and finally, substituting that back into the original equation:

$$f_{MEY} = 1/d$$
$$Y_{MEY} = \left(1/d\right)e^{(c-1)}.$$

Example 1: Consider the following time series of catch and effort for a trawl fishery. Determine the MEY for this fishery by fitting the Schaefer and Fox models. Recommend a level of effort to achieve MEY.

Year	Catch	Effort
1988	50	623
1989	49	628
1990	47.5	520
1991	45	513
1992	51	661
1993	56	919
1994	66	1158
1995	58	1970
1996	52	1317

1. The time history of the catch and effort data is shown in Figure 7.

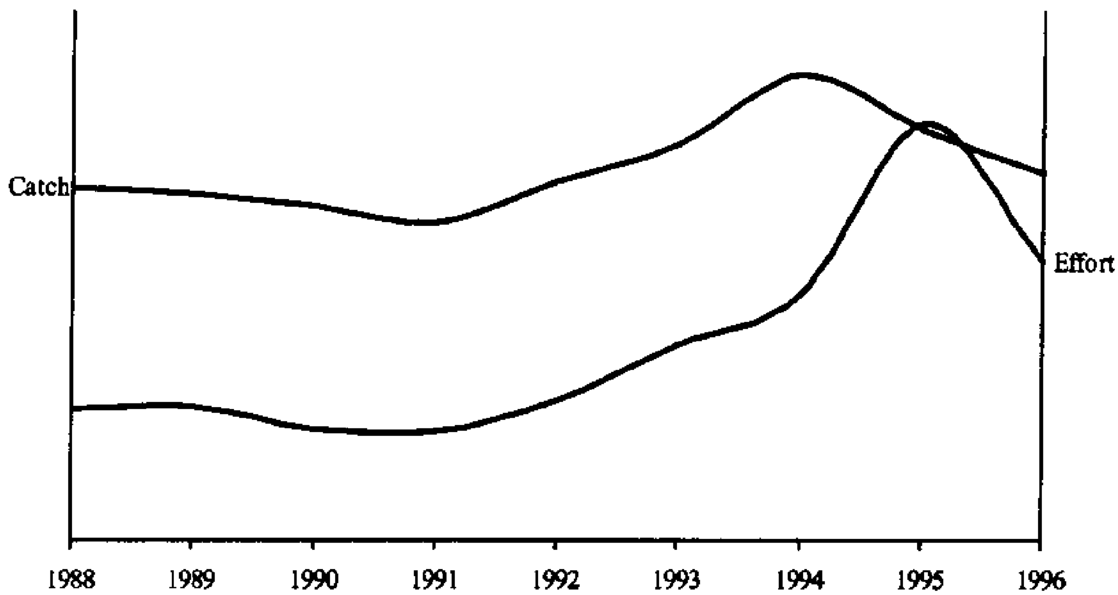

Figure 7. Time history of catch and effort.

2. To fit the linearized Schaefer model, calculate CPUE as ratios of catch/effort (Table 1), then use linear regression of CPUE versus effort (Figure 8).

Table 1. Trawl catch and effort data with CPUE.

Year	Catch	Effort	CPUE
1988	50	623	0.080257
1989	49	628	0.078025
1990	47.5	520	0.091346
1991	45	513	0.087719
1992	51	661	0.077156
1993	56	919	0.060936
1994	66	1158	0.056995
1995	58	1970	0.029442
1996	52	1317	0.039484

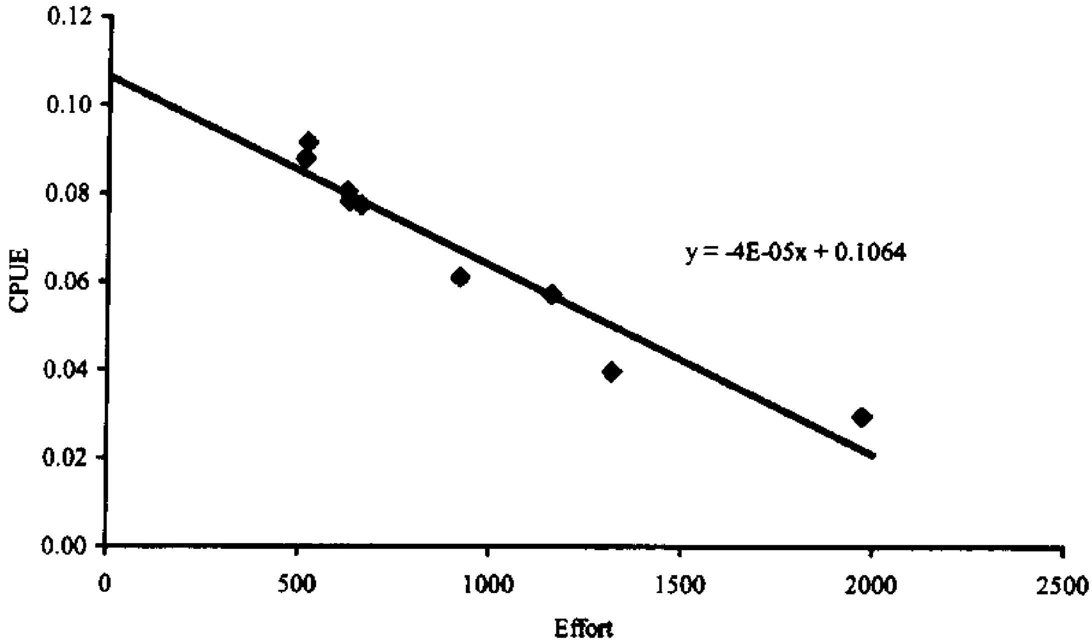

Figure 8. Linearization of the Schaefer model and the best fit regression.

3. Using the *a* and *b* coefficients from the linear regression as starting values, use **Solver** to fit the non-linear model (Figure 9). (Note the Sum of the Squared Residuals (SSR).)

4. Using the best fit coefficients, estimate MEY and the recommended level of effort to achieve MEY.

5. To fit the linearized Fox model, determine the ln(CPUE) (Table 2), and then linearly regress ln(CPUE) versus effort (Figure 10).

Table 2. Trawl catch and effort data with the ln(CPUE).

Year	Catch	Effort	ln(CPUE)
1988	50	623	-2.52252
1989	49	628	-2.55072
1990	47.5	520	-2.39310
1991	45	513	-2.43361
1992	51	661	-2.56193
1993	56	919	-2.79793
1994	66	1158	-2.86479
1995	58	1970	-3.52535
1996	52	1317	-3.23187

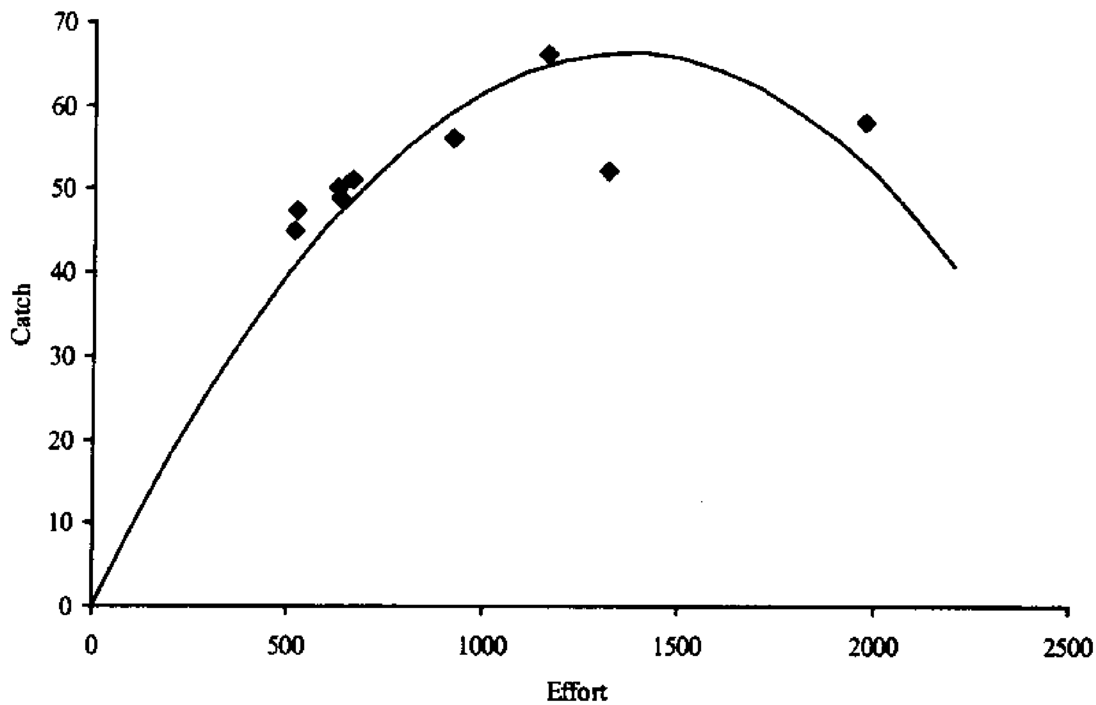

Figure 9. Non-linear Schaefer model best fit to the catch and effort data.

6. Using the *c* and *d* coefficients from the linear regression as starting values, use Solver to fit the non-linear model (Figure 11). (Note the Sum of Squared Residuals (SSR).)

7. Using the best fit coefficients, estimate the MEY and recommend level of effort to achieve MEY.

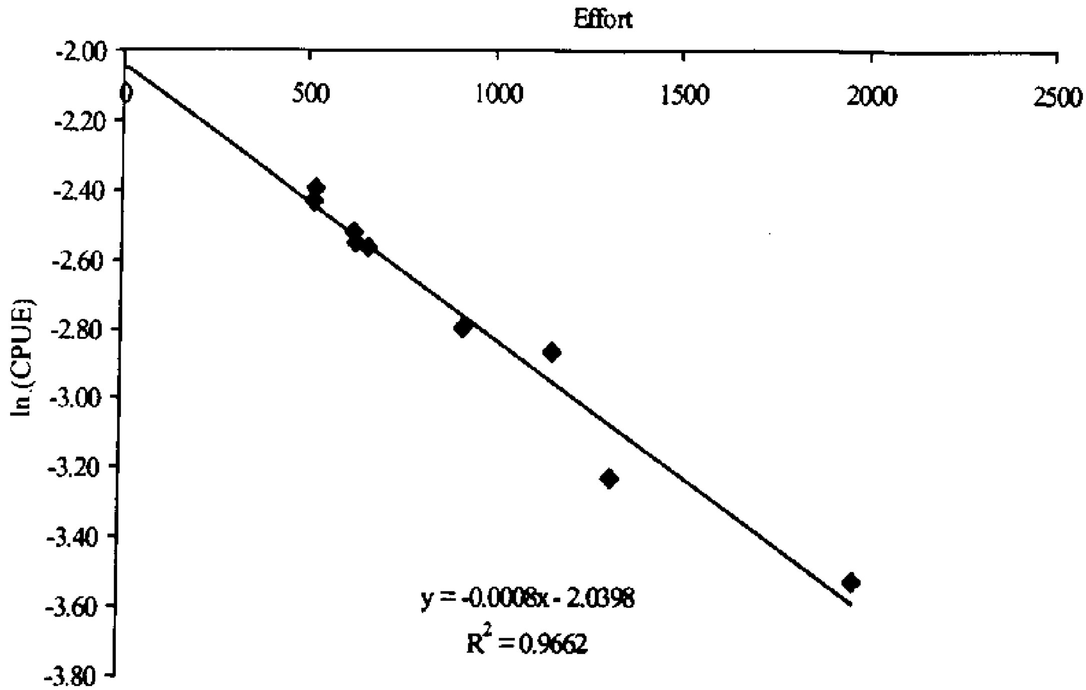

Figure 10. Linearization of the Fox model and the best fit regression.

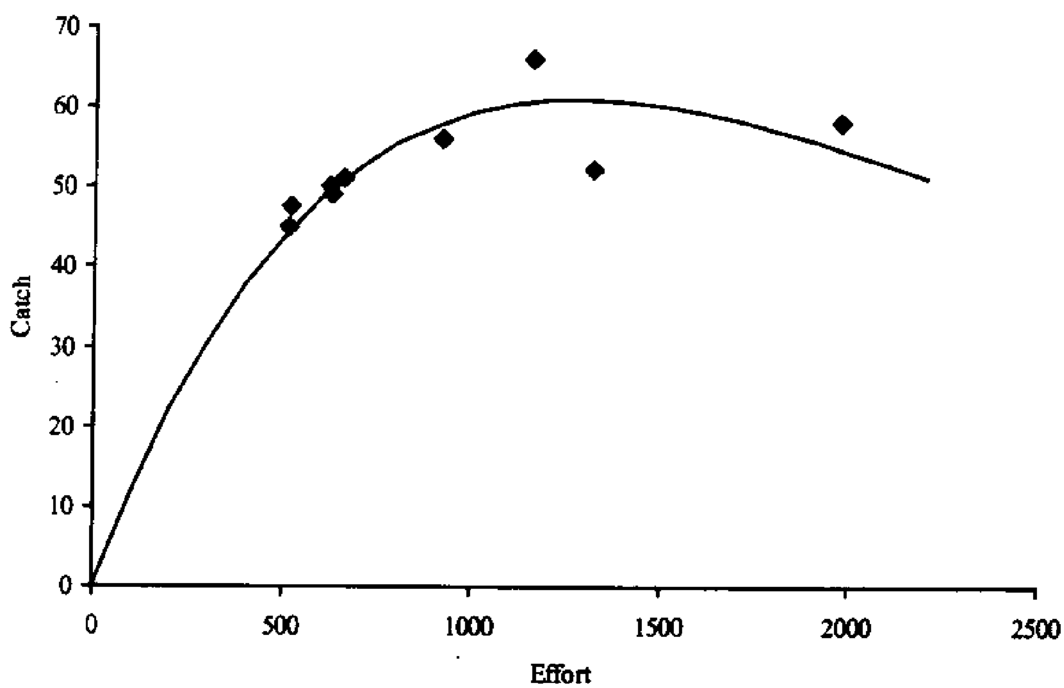

Figure 11. Non-linear Fox model best fit to the catch and effort data.

Exercises

Given the following catch and effort data for the trawl fishery on this pelagic fish species for the period 1976 to 1995,

Year	Catch (10^6 kg)	Effort (10,000 days)
1976	104	0.13
1977	282	0.28
1978	348	0.39
1979	507	0.51
1980	548	0.72
1981	602	0.98
1982	584	1.12
1983	542	0.96
1984	521	1.03
1985	487	1.09
1986	472	1.15
1987	416	1.22
1988	298	1.32
1989	150	1.50
1990	72	1.52
1991	81	1.53
1992	60	1.56
1993	82	1.58
1994	75	1.60
1995	71	1.62

A. Plot the trajectories of catch and effort. Describe the time history of the fishery.

B. Estimate the parameters of Schaefer and Fox Surplus Production models for the data using linear regression.

C. Use **Solver** to improve the parameter estimates for the Schaefer and Fox models.

D. Estimate Y_{MEY} and f_{MEY} for both models, compare graphic and empirical estimates.

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STOCK & RECRUITMENT

STOCK AND RECRUITMENT

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Introduction

Recruitment processes include those factors that affect the growth and survival of fish between the egg and the age that fish enter either the spawning stock biomass portion of the population or become vulnerable to harvesting. Consider a basic two-stage life history model for fish:

$$\text{RECRUITS} \underset{\text{RECRUITMENT}}{\overset{F+M}{\rightleftarrows}} \text{ADULTS.}$$

Recruits, having entered the fishery, are subject to the fishing (*F*) and natural mortality (*M*), and once mature contribute to egg production before being harvested or dying of natural causes. Eggs must hatch, releasing larvae that metamorphose into juvenile fish which must survive to the recruit stage. The development and survival of the egg, larval, and juvenile life stages are affected by predation, genetic fitness, nutrition, and environmental factors.

The purpose of investigating stock-recruitment (*S-R*) relationships for fishery resources is to be able to predict the number of recruits to the fishery at a future date based on estimates of the present spawning stock abundance. In reality, stock-recruitment relationships are used by resource managers as a rationale for regulating fishing mortality so as to avoid low stock sizes that may lead to recruitment failure and stock collapses.

Biological Processes

Density independent mortality in the stock-recruitment relationship implies that the probability of eggs surviving to the recruit stage is independent of the spawning stock size or number of eggs produced. Biologically, this is a simple and reasonable assumption, but within limits. No population can reproduce with the same average probability of success as stock size increases indefinitely. Eventually, every population becomes limited by resources available. Compensation is the reduction in recruits-per-spawner as spawning stock size increases. The result is that the *S-R* curve rises less steeply at higher stock sizes, asymptotes, and can eventually fall off at the highest stock levels. Density dependent factors include maturation and fecundity, growth, predation, and cannibalism. Depensation is an increase in recruits-per-spawner as spawning stock increases.

Measurement of Spawning Stock and Recruitment

Spawning stock is measured by the following:

1. Number of females alive at each age times fecundity at age,

2. Number of individuals alive at each age times fecundity at age,
3. Total biomass of individuals at or above the age of first reproduction, and
4. Index and abundance of the population in the year eggs are deposited.

Recruitment is measured by the following:

1. Recruits to the fishery determined by Virtual Population Analysis (VPA) from catch-at-age data and
2. Juvenile / pre-recruit surveys.

Basic Principles of the *S-R* Relationship

Ricker (1975) proposed some basic tenets for the *S-R* relationship:

1. The curve must start at the origin, that is at $S(0)$, $R(0)$.
2. At no point after $S(0)$ will there be a $R(0)$ (*i.e.* at high densities recruitment will not go to zero).
3. The rate of recruitment (R/S) should decrease continuously with increasing stock size; that is the highest rate of recruitment should be at the lowest stock level.
4. Recruitment must exceed parental stock size over some part of the range of S (when R and S are in the same units), otherwise stock collapse would result from any perturbation to the system.

Beverton-Holt Model

The Beverton-Holt *S-R* relationship (1957) is based on the assumption that juvenile competition results in a mortality rate that is linearly dependent upon the number of fish alive in the cohort at any time.

$$dN/dt = -(q + pN)N$$

where N is the number alive in the cohort at time t,
q is a density-independent mortality rate, and
pN is a mortality rate component that is proportional to the density of the cohort at time t.

The Beverton-Holt *S-R* relationship is asymptotic. If R and S are in the same units and $R = S$ at replacement (S_r), then:

$$R = S \Big/ \left[1 - A\left(1 - S/S_r\right)\right]$$

where A is the shape of the curve and has values of $0 \rightarrow 1$ (Figure 1).

If R and S are in different units, then:

$$R = 1/\left(\alpha + \beta/S\right) = S/\alpha S + \beta$$

where $\beta = 1 - A$ and
$\alpha = A/S_r$.

Note that as $S \rightarrow \infty$, $R = 1/\alpha$.

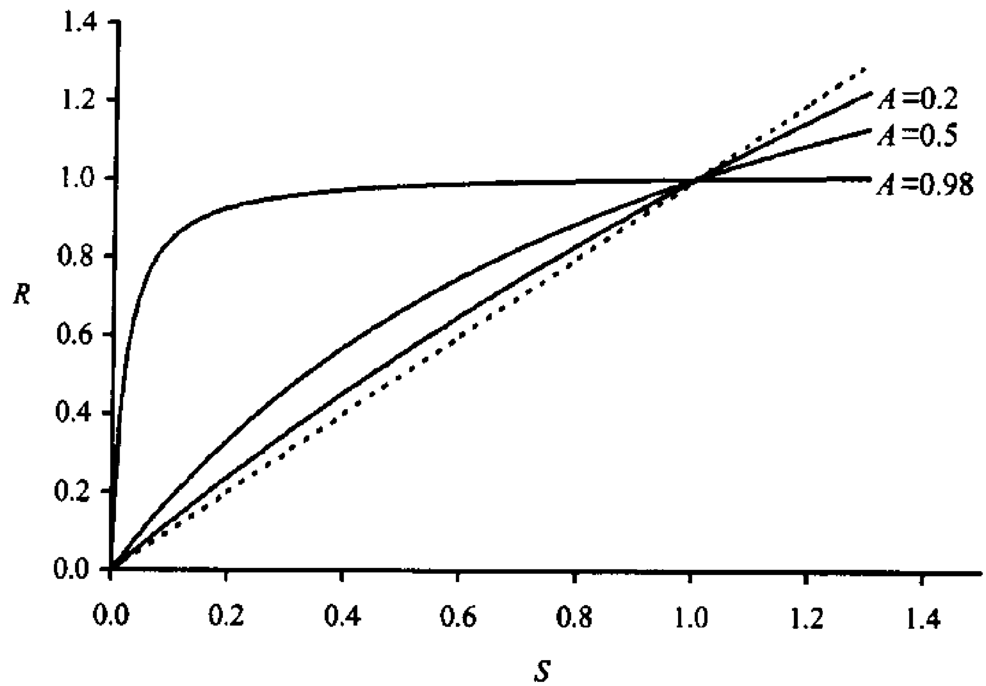

Figure 1. The Beverton-Holt *S-R* relationship with changing values of *A*.

Ricker Model

The Ricker *S-R* relationship (1954, 1975) is based on the assumption that the mortality rate of eggs and juveniles is proportional to the initial cohort size. In other words, mortality is spawning stock dependent rather than cohort size dependent as in the Beverton-Holt model.

$$dN/dt = -(q + pS)N$$

where N is the cohort size at any time prior to recruitment,
S is the initial spawning stock size,
q is the density independent mortality rate, and
$(q + pS)$ is the mortality rate for the cohort.

The Ricker model

$$R = \alpha S e^{-\beta S}$$

where α is the recruits per spawners (R/S, the slope) at low stock sizes, and
β is the shape of the curve

results in recruitment declining at high stock levels (Figure 2).

When spawning stock and recruits are measured in the same units, the replacement level ($R = S$) and $S_r = \ln\alpha/\beta$.

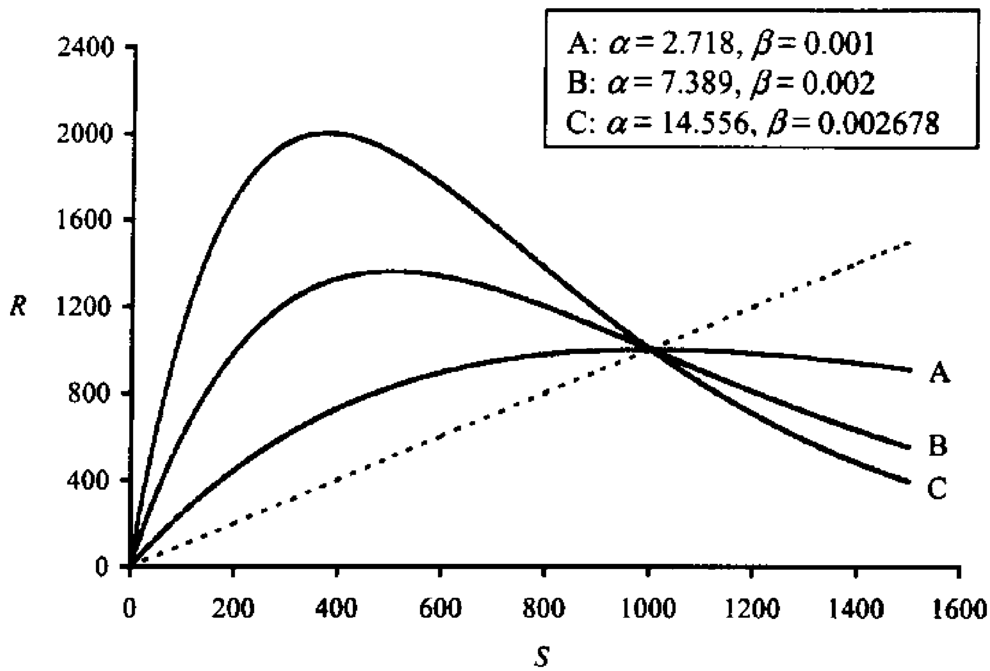

Figure 2. The Ricker *S-R* relationship at various *a* and *b* values.

Shephard Model

The Shepherd *S-R* relationship (1982) is a more versatile form of the *S-R* (Figure 3). It can accommodate both the Beverton-Holt and Ricker *S-R* relationships:

$$R = \alpha S \Big/ \left[1 + \left(S/K\right)^{\beta}\right]$$

where α is the slope at the origin,
β describes the shape of the curve and provides for the degree of compensation, and
K is the threshold stock biomass above which density dependent effects overcome density independent effects.

The degree of compensation (β) measures the power of the density dependent effects to compensate for changes of stock size. If $\beta < 1$, recruitment continues to increase when biomass increases, indefinitely. If $\beta = 1$, then at large stock sizes density dependent effects compensate exactly for increases in biomass, leading to asymptotically constant recruitment. If $\beta > 1$, the density dependent processes are so strong that they over-compensate for changes in biomass, leading to decreased recruitment at higher stock sizes. The threshold biomass (K) is the biomass at which recruitment is reduced to half the level it would have had under density-independent process alone.

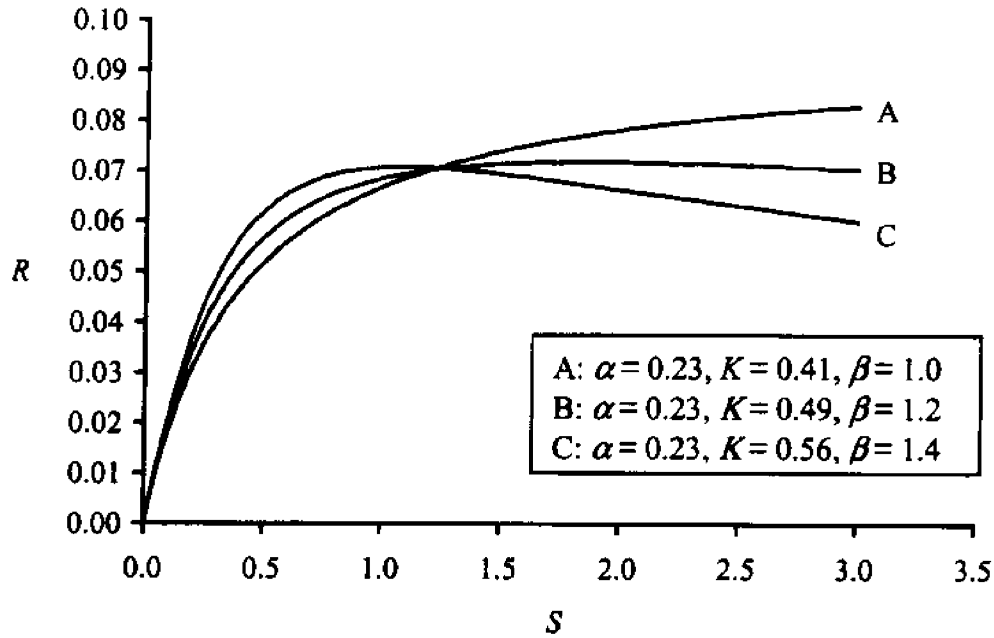

Figure 3. The Shepard *S-R* relationship at various α, K, and β values.

Estimation of *S-R* Parameters

The general form of Beverton-Holt model can be rearranged as follows:

$$R = \frac{1}{\left[\alpha + \frac{\beta}{S}\right]} = \frac{S}{\alpha S + \beta}$$

$$\frac{S}{R} = \beta + \alpha S.$$

In the rearranged form, the Beverton-Holt *S-R* relationship conforms to the basic linear model:

$$y = ax + b$$

where $y = S/R$
$a = \text{slope} = \alpha$
$x = S$
$b = \text{intercept} = \beta.$

The α and β parameters are estimated by regressing *S/R* against *S*. These values are then used to plot a predicted Beverton-Holt *S-R* curve, or can be used as starting values for non-linear regression methods.

Example 1: Given the following stock recruitment data, solve for α and β using both the linear and non-linear methods of the Beverton-Holt model.

Year	Stock	Recruitment	S/R
1	8.8	7.1	1.239
2	7.4	6.4	1.156
3	4.5	6.4	0.703
4	13.2	7.0	1.886
5	14.6	7.7	1.896
6	7.0	7.0	1.000
7	3.1	5.4	0.574
8	7.7	6.1	1.262
9	10.7	6.8	1.574
10	8.6	6.0	1.433
11	15.4	6.2	2.484
12	2.0	3.5	0.571

The data are fit to a linear model using a regression of *S/R* versus *S* (Figure 4). The values of α and β are obtained directly from the slope and *y*-intercept values where $\alpha = a$ and $\beta = b$. Therefore, $\alpha = 0.1319$ and $\beta = 0.1824$. Non-linear regression is performed using **Solver** in *Microsoft Excel* and the linear regression parameter values as starting values. The new parameter values are $\alpha = 0.1262$ and $\beta = 0.2167$. The Beverton-Holt model is graphed onto the original data using the non-linear regression parameter values (Figure 5).

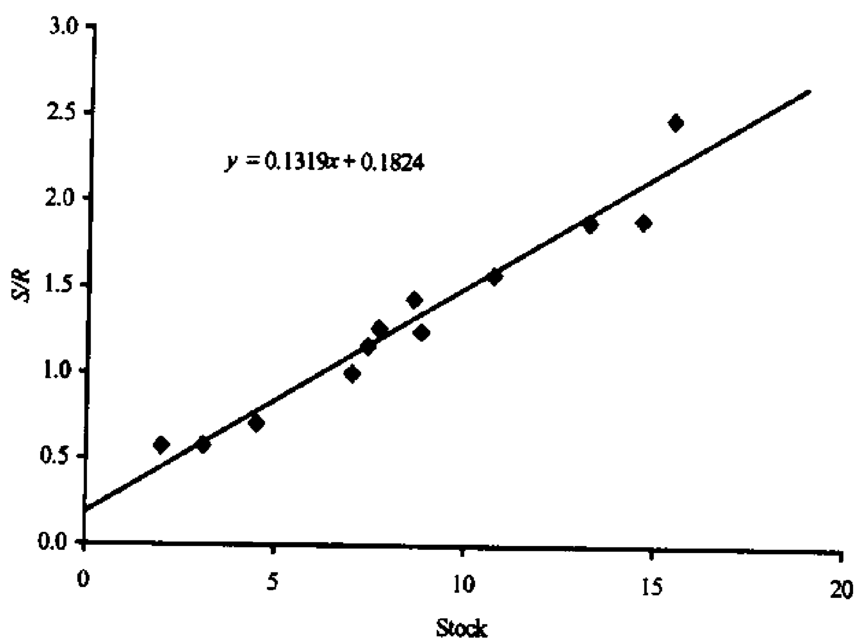

Figure 4. Application of *S-R* data in the linearized Beverton-Holt *S-R* relationship.

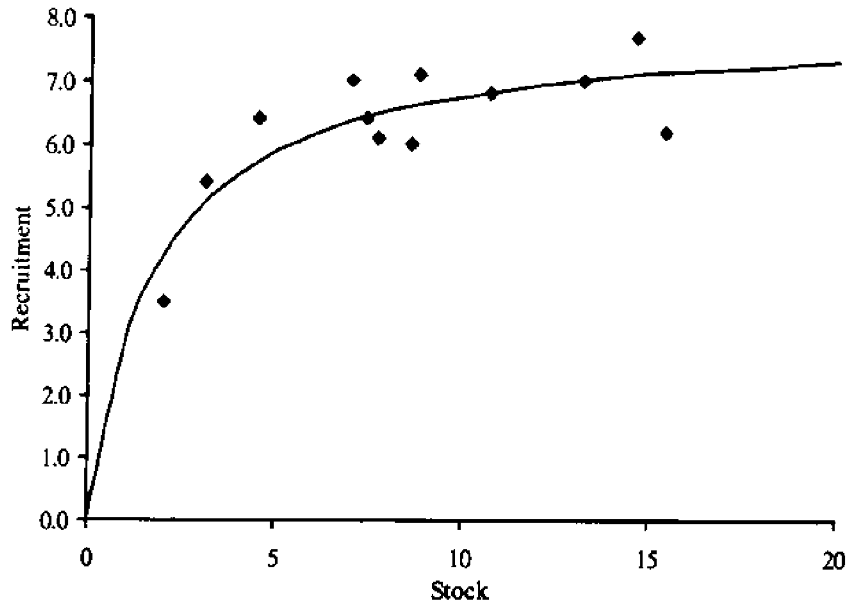

Figure 5. Application of *S-R* data in the Beveton-Holt *S-R* relationship.

The general form of the Ricker model is rearranged as follows:

$$R = \alpha S e^{-S\beta}$$
$$R/S = \alpha e^{-S\beta}$$
$$\ln(R/S) = \ln\alpha - \beta S.$$

In the rearranged form, the Ricker *S-R* relationship conforms to the basic linear model:

$$y = ax + b$$

where $y = \ln(R/S)$
$a = \text{slope} = \beta$
$x = S$
$b = \text{intercept} = \ln\alpha$

The α and β parameters are estimated by regressing $\ln(R/S)$ against S. After taking the anti-ln of $\ln\alpha$, these values are then used to plot a predicted Ricker *S-R* curve, or can be used as starting values for non-linear regression methods.

Example 2: Utilizing the same data as in Example 1, use the Ricker model to solve for α and β performing both linear and non-linear regression.

The data are fit to a linear model using a regression of $\ln(R/S)$ versus S (Figure 6). The value of α is equal to the inverse ln of the y-intercept, therefore $\alpha = e^{0.7427} = 2.1016$. The value of β is equal to the negative slope, therefore, $\beta = 0.1071$. Non-linear regression is performed using **Solver** in *Excel* and the linear regression parameter values as starting values. The new parameter values are $\alpha = 1.9704$ and $\beta = 0.1015$. The Ricker model is graphed onto the original data using the non-linear regression parameter values (Figure 7).

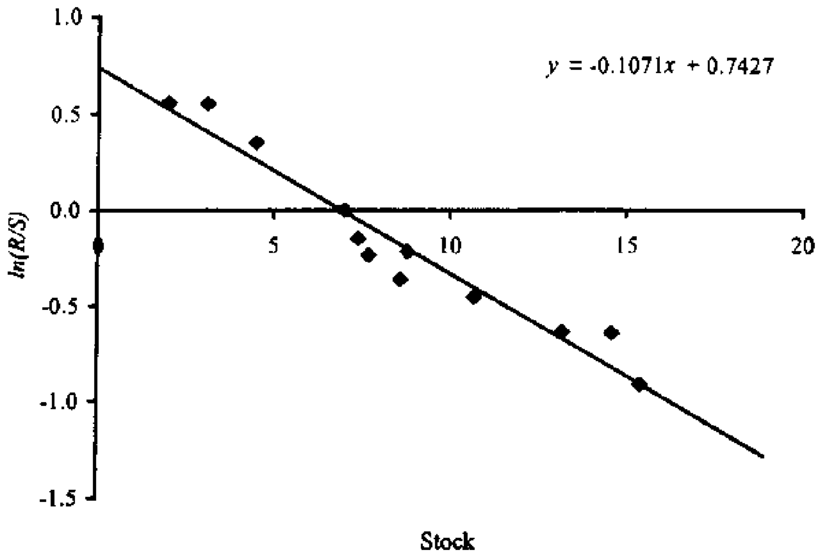

Figure 6. Application of *S-R* data in the linearized Ricker *S-R* relationship.

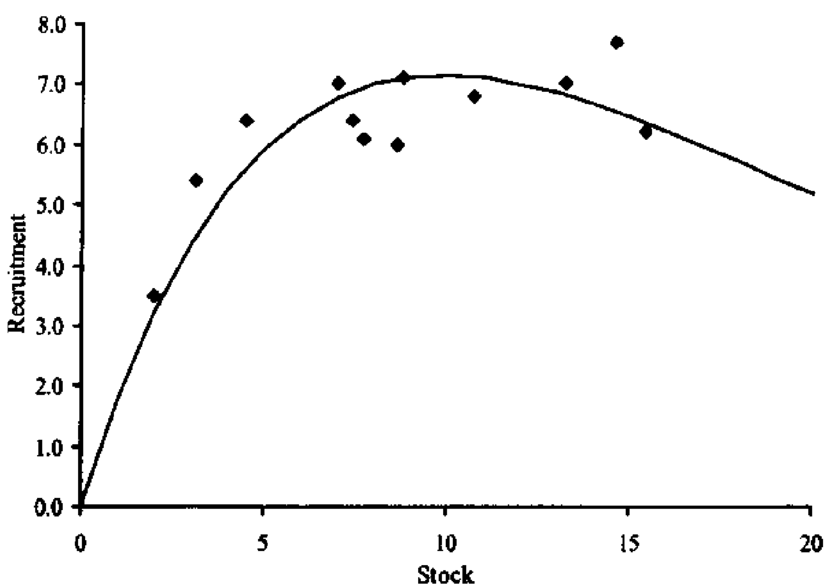

Figure 7. Application of *S-R* data in the Ricker *S-R* relationship.

The general form of the Shepherd model is:

$$R = \alpha S \Big/ \left[1 + \left(S/K\right)^{\beta}\right] .$$

Shepherd suggest values for β of slightly less than 1 for pelagic fish, about 1 for flatfish, and greater than 1 for those species which cannibalism is believed to be significant. The value of a is estimated by drawing a straight-line through the origin and determining the slope of that line. The parameter K is estimated by choosing "typical" current values of stock and recruitment through which the curve should pass, then estimating values of K from the following:

$$K = S^* \Big/ [(\alpha S^*/R^*) - 1]^{1/\beta} .$$

Spawning Stock-Per-Recruit and Steady State

The reciprocal of recruits per unit spawning stock is spawning stock per recruit (SSBPR). Recall that this is the corollary output of the yield per recruit model when considering the effects of exploitation. The SSBPR is a measure of survival in the population at various levels of exploitation (F). The intersection of these straight-line functions representing various levels of fishing mortality with the S-R curve represent equilibrium points (Figure 8).

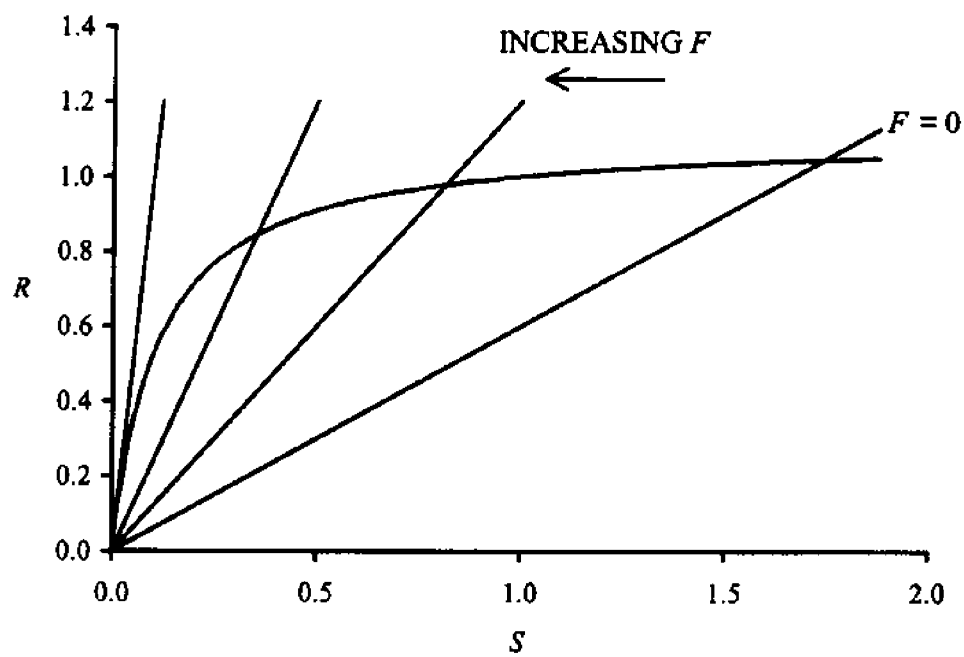

Figure 8. Intersection of SSBPR functions at various fishing mortality levels (F), with a Beverton-Holt S-R relationship.

At the highest level of fishing mortality, there is no intersection with the *S-R* curve, leading to stock collapses.

Exploited Population Trajectories

The two-stage life history trajectory for an exploited population can be described on a *S-R* / SSBPR plot (Figure 9) where the relationship between the recruit stage and spawning stock stage is described by the straight line with a slope dependent of the level of *F*, and the *S-R* curve. At a fixed exploitation rate the stock will return to that intersection point.

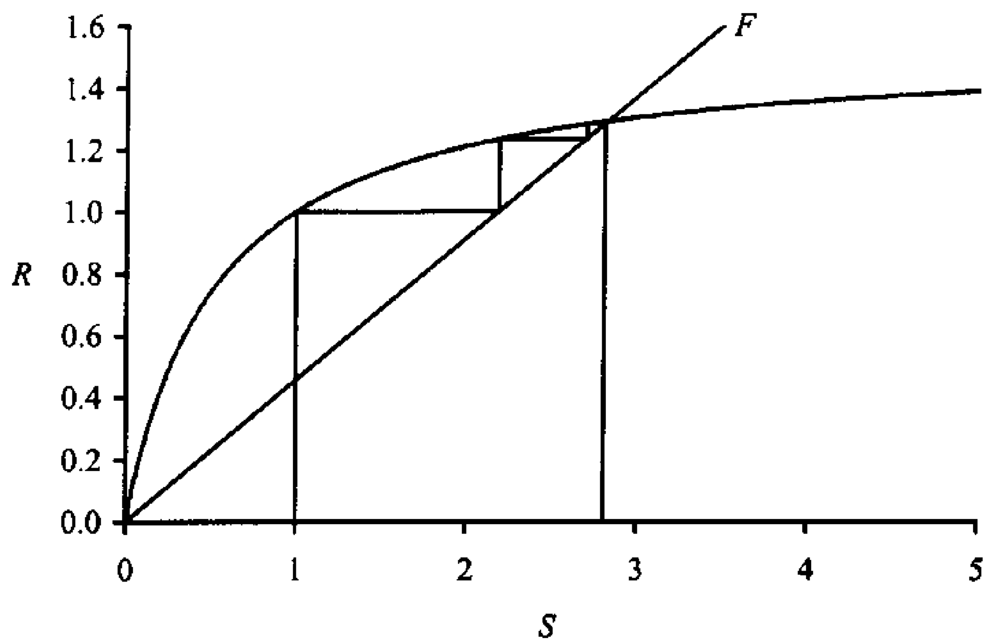

Figure 9. Two-stage life history trajectory based on the intersection of *S-R* and SSBPR relationships.

Environmental Effects on the *S-R* Relationship with Exploitation

Environmental factors can modify the *S-R* relationship markedly reducing the level of recruitment available for any stock level. However, if the effects of exploitation are superimposed on the *S-R* relationship, the disastrous effects of negative environmental factors and high fishing mortality are evident with the lack of stable equilibrium points (Figure 10). Note that at high levels of fishing mortality and low stock levels, there is no intersection between the unfavorable environment *S-R* curve, and the high fishing mortality curve, leading to stock collapse.

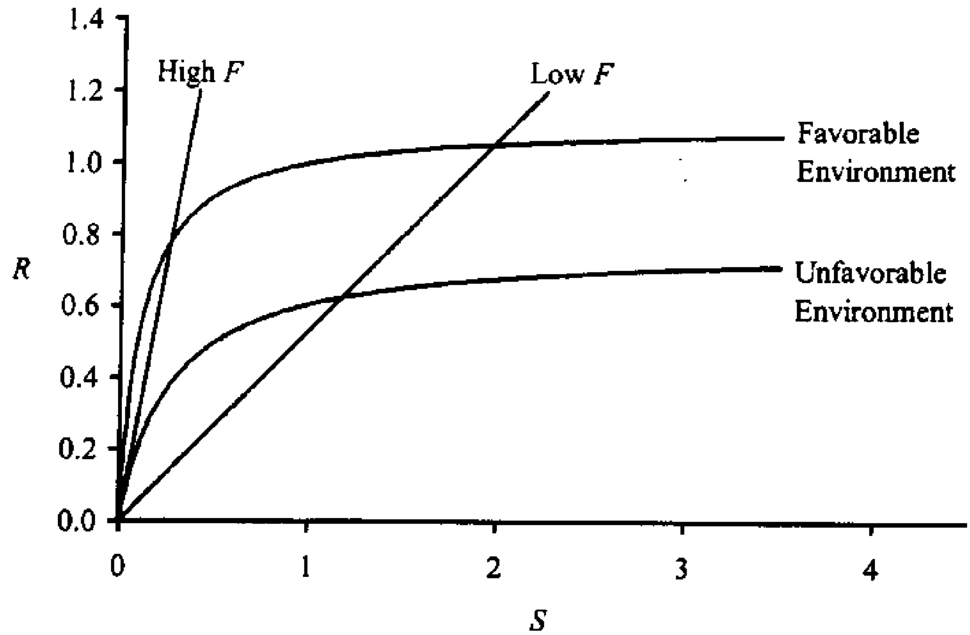

Figure 10. Effect of environmental suitability on the *S*-*R* and SSBPR relationships.

Exercises

1. Given the following stock-recruitment data, for each species:
   (a) Plot the time history of spawning stock size and recruitment.
   (b) Estimate the parameters of a Beverton-Holt stock-recruitment model using both the linear and non-linear regression methods.
   (c) Estimate the parameters of a Ricker stock-recruitment model using both the linear and non-linear regression methods.
   (d) Describe and interpret the models for each species over their time history.

	Shad		Salmon	
Year	S	R	S	R
1921			150	449
1922			40	228
1923			70	199
1924			106	81
1925			162	161
1926			253	146
1927			87	162
1928			109	263
1929			90	159
1930			109	117
1931			87	258
1932			74	254
1933			97	219
1934			145	126
1935			88	125
1936			137	135
1937			126	133
1938			123	159
1939			71	183
1940	212	193	88	86
1941	284	188	93	57
1942	237	156	63	69
1943	229	135	92	150
1944	200	113	77	114
1945	123	83	66	126
1946	101	103	44	82
1947	75	129	48	77
1948	86	114	75	81
1949	75	97		
1950	65	80		
1951	89	85		
1952	120	128		
1953	74	163		
1954	77	166		
1955	59	169		
1956	138	157		

	Yellowtail Flounder		Blue Crab	
year	S	R	S	R
1956			1.6	2.8
1957			0.5	4.8
1958			0.2	0.3
1959			0	4
1960			0.3	0.5
1961			0.1	0
1962			0.2	1
1963			0.5	0.4
1964			0.2	8.6
1965			0.4	3.7
1966			0.8	3.1
1967			1.6	0.5
1968			3	15.2
1969			1.5	1.5
1970			4.7	17.8
1971			3	7.1
1972			2.1	7.5
1973			0.1	4
1974			0.1	0.7
1975			0	0.7
1976	25.5	50.3	0.1	5.7
1977	18.0	57.1	0.2	6.3
1978	11.9	20.1	0.2	6.4
1979	13.5	14.1	0.4	0.7
1980	9.2	50.5	0.4	12.3
1981	6.7	26.8	0.6	9.3
1982	11.6	23.8	0.6	5
1983	12.9	56.2	0.4	10.5
1984	12.4	20.4	0.3	4.5
1985	16.4	7.4	0.2	8.7
1986	11.4	9.4	0.6	8.5
1987	3.3	17.3	0.4	12.7
1988	2.7	6.3	3.8	17.1
1989	3.8	6.2	4.5	24.3
1990	2.6	17.2	7.1	11.3
1991	2.2	6.6	4	16.8

1957	151	123		
1958	149	119		
1959	154	133		
1960	149	129		
1962	84	105		
1963	101	112		
1964	130	138		
1965	121	144		
1966	133	109		
1967	110	111		
1968	115	128		
1969	160	168		
1970	160	227		

1992	5.1	6.2	1.5	16.2
1993	4.3	16.8	1.2	19.4
1994	3.5	4.9	0.5	12.5
1995	3.7	5.2		

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YPR TABLE INSTRUCTIONS

APPENDIX 1

INSTRUCTIONS FOR YIELD PER RECRUIT TABLES

To obtain Tables 1 and 2, use the spreadsheet program in the MK worksheet.

Simply change *M/K* (cell C4) to the desired value.

For example, to calculate Table 1, set cell C4 to 0.5 and to calculate Table 2, set cell C4 to 2.

M/K ——> 2

Rate of Exploitation

E=(F/(F+M)) ——>

L_c/L_∞	0.05	0.1	0.15	0.2	0.25	0.3	0.35	0.4	0.45	0.5	0.55	0.6	0.65	0.7	0.75	0.8	0.85	0.9	0.95	1
0.98	0.000019	0.000038	0.000058	0.000077	0.000096	0.000115	0.000134	0.000153	0.000172	0.000191	0.000209	0.000228	0.000247	0.000266	0.000284	0.000303	0.000321	0.000340	0.000358	0.000376
0.96	0.000074	0.000147	0.000220	0.000293	0.000366	0.000439	0.000511	0.000583	0.000654	0.000726	0.000797	0.000867	0.000937	0.001007	0.001077	0.001145	0.001214	0.001282	0.001349	0.001416
0.94	0.000159	0.000317	0.000475	0.000632	0.000788	0.000943	0.001097	0.001250	0.001402	0.001554	0.001704	0.001852	0.002000	0.002146	0.002291	0.002434	0.002576	0.002716	0.002854	0.002990
0.92	0.000271	0.000540	0.000808	0.001074	0.001338	0.001600	0.001860	0.002117	0.002373	0.002626	0.002876	0.003123	0.003368	0.003610	0.003848	0.004083	0.004314	0.004541	0.004765	0.004984
0.90	0.000406	0.000808	0.001208	0.001604	0.001996	0.002385	0.002769	0.003150	0.003526	0.003897	0.004264	0.004625	0.004981	0.005331	0.005675	0.006012	0.006343	0.006666	0.006982	0.007290
0.88	0.000560	0.001114	0.001663	0.002206	0.002743	0.003274	0.003798	0.004315	0.004825	0.005327	0.005820	0.006305	0.006781	0.007247	0.007704	0.008149	0.008584	0.009006	0.009416	0.009813
0.86	0.000729	0.001451	0.002163	0.002867	0.003562	0.004248	0.004920	0.005584	0.006236	0.006876	0.007503	0.008118	0.008718	0.009304	0.009874	0.010428	0.010965	0.011485	0.011986	0.012467
0.84	0.000912	0.001812	0.002699	0.003573	0.004434	0.005281	0.006113	0.006929	0.007728	0.008510	0.009274	0.010019	0.010744	0.011448	0.012130	0.012789	0.013424	0.014034	0.014617	0.015173
0.82	0.001104	0.002191	0.003261	0.004313	0.005347	0.006360	0.007353	0.008324	0.009273	0.010197	0.011097	0.011971	0.012817	0.013635	0.014423	0.015179	0.015903	0.016592	0.017247	0.017864
0.80	0.001303	0.002584	0.003842	0.005076	0.006284	0.007467	0.008622	0.009748	0.010844	0.011909	0.012940	0.013937	0.014898	0.015821	0.016705	0.017549	0.018349	0.019105	0.019816	0.020480
0.78	0.001507	0.002985	0.004433	0.005851	0.007235	0.008586	0.009901	0.011179	0.012419	0.013618	0.014774	0.015887	0.016953	0.017971	0.018940	0.019856	0.020719	0.021526	0.022276	0.022968
0.76	0.001713	0.003390	0.005029	0.006629	0.008188	0.009704	0.011175	0.012600	0.013976	0.015301	0.016574	0.017791	0.018951	0.020051	0.021090	0.022064	0.022972	0.023813	0.024584	0.025285
0.74	0.001920	0.003795	0.005623	0.007403	0.009132	0.010808	0.012429	0.013993	0.015497	0.016939	0.018316	0.019626	0.020866	0.022033	0.023125	0.024141	0.025077	0.025931	0.026704	0.027393
0.72	0.002125	0.004195	0.006209	0.008164	0.010058	0.011888	0.013651	0.015344	0.016966	0.018512	0.019981	0.021369	0.022674	0.023892	0.025021	0.026059	0.027003	0.027852	0.028605	0.029263
0.70	0.002327	0.004589	0.006784	0.008908	0.010958	0.012933	0.014828	0.016640	0.018367	0.020006	0.021552	0.023003	0.024356	0.025607	0.026755	0.027796	0.028728	0.029552	0.030265	0.030870
0.68	0.002525	0.004973	0.007342	0.009627	0.011826	0.013935	0.015951	0.017871	0.019690	0.021405	0.023014	0.024511	0.025895	0.027161	0.028308	0.029333	0.030234	0.031011	0.031665	0.032198
0.66	0.002717	0.005345	0.007879	0.010317	0.012654	0.014887	0.017012	0.019025	0.020923	0.022700	0.024355	0.025882	0.027278	0.028541	0.029668	0.030657	0.031506	0.032218	0.032792	0.033235
0.64	0.002903	0.005702	0.008393	0.010974	0.013438	0.015783	0.018004	0.020097	0.022057	0.023881	0.025565	0.027104	0.028495	0.029735	0.030823	0.031756	0.032536	0.033162	0.033640	0.033974
0.62	0.003080	0.006042	0.008882	0.011594	0.014174	0.016618	0.018921	0.021079	0.023087	0.024941	0.026636	0.028170	0.029537	0.030737	0.031767	0.032626	0.033316	0.033840	0.034203	0.034415
0.60	0.003250	0.006365	0.009341	0.012174	0.014858	0.017388	0.019759	0.021967	0.024007	0.025874	0.027565	0.029074	0.030400	0.031541	0.032495	0.033262	0.033846	0.034251	0.034485	0.034560
0.58	0.003410	0.006669	0.009771	0.012712	0.015487	0.018089	0.020514	0.022757	0.024813	0.026677	0.028346	0.029815	0.031082	0.032145	0.033006	0.033665	0.034126	0.034398	0.034490	0.034418
0.56	0.003560	0.006952	0.010169	0.013207	0.016058	0.018720	0.021185	0.023447	0.025503	0.027348	0.028978	0.030389	0.031581	0.032551	0.033302	0.033837	0.034162	0.034288	0.034227	0.033999
0.54	0.003701	0.007215	0.010535	0.013657	0.016574	0.019279	0.021769	0.024036	0.026077	0.027887	0.029462	0.030800	0.031899	0.032761	0.033388	0.033785	0.033961	0.033930	0.033708	0.033319
0.52	0.003831	0.007456	0.010869	0.014062	0.017030	0.019767	0.022266	0.024524	0.026534	0.028294	0.029799	0.031048	0.032041	0.032780	0.033269	0.033516	0.033534	0.033337	0.032948	0.032396
0.50	0.003951	0.007677	0.011169	0.014421	0.017427	0.020182	0.022678	0.024911	0.026877	0.028571	0.029992	0.031138	0.032011	0.032614	0.032955	0.033042	0.032892	0.032524	0.031956	0.031250
0.48	0.004061	0.007875	0.011436	0.014735	0.017767	0.020526	0.023005	0.025201	0.027108	0.028724	0.030046	0.031077	0.031817	0.032274	0.032455	0.032374	0.032050	0.031508	0.030780	0.029904
0.46	0.004161	0.008053	0.011670	0.015005	0.018050	0.020801	0.023250	0.025395	0.027230	0.028755	0.029967	0.030870	0.031468	0.031768	0.031782	0.031527	0.031026	0.030309	0.029413	0.028363
0.44	0.004250	0.008210	0.011873	0.015231	0.018278	0.021008	0.023416	0.025497	0.027249	0.028671	0.029762	0.030527	0.030972	0.031108	0.030949	0.030517	0.029838	0.028947	0.027889	0.026714
0.42	0.004330	0.008347	0.012045	0.015415	0.018453	0.021151	0.023505	0.025512	0.027170	0.028478	0.029438	0.030056	0.030342	0.030307	0.029972	0.029360	0.028504	0.027445	0.026232	0.024923
0.40	0.004400	0.008465	0.012187	0.015559	0.018576	0.021232	0.023523	0.025445	0.026998	0.028183	0.029004	0.029469	0.029588	0.029380	0.028865	0.028075	0.027046	0.025825	0.024468	0.023040
0.38	0.004461	0.008563	0.012301	0.015665	0.018652	0.021256	0.023472	0.025300	0.026740	0.027795	0.028469	0.028774	0.028725	0.028340	0.027647	0.026681	0.025485	0.024111	0.022623	0.021093
0.36	0.004513	0.008645	0.012388	0.015735	0.018682	0.021225	0.023359	0.025085	0.026404	0.027321	0.027844	0.027986	0.027764	0.027203	0.026334	0.025197	0.023842	0.022328	0.020725	0.019110
0.34	0.004557	0.008709	0.012449	0.015772	0.018671	0.021144	0.023188	0.024805	0.025997	0.026771	0.027139	0.027114	0.026720	0.025985	0.024944	0.023644	0.022140	0.020499	0.018798	0.017121
0.32	0.004593	0.008758	0.012488	0.015777	0.018621	0.021017	0.022965	0.024466	0.025526	0.026154	0.026364	0.026173	0.025608	0.024701	0.023495	0.022041	0.020402	0.018650	0.016870	0.015152
0.30	0.004623	0.008793	0.012505	0.015754	0.018536	0.020849	0.022695	0.024076	0.025001	0.025480	0.025530	0.025174	0.024441	0.023369	0.022005	0.020409	0.018648	0.016803	0.014966	0.013230
0.28	0.004645	0.008815	0.012503	0.015706	0.018420	0.020645	0.022384	0.023642	0.024429	0.024758	0.024649	0.024130	0.023233	0.022003	0.020492	0.018766	0.016900	0.014982	0.013109	0.011380
0.26	0.004662	0.008825	0.012484	0.015635	0.018277	0.020410	0.022039	0.023170	0.023817	0.023997	0.023732	0.023053	0.021999	0.020620	0.018974	0.017132	0.015179	0.013208	0.011323	0.009625
0.24	0.004674	0.008825	0.012449	0.015544	0.018110	0.020147	0.021664	0.022668	0.023176	0.023207	0.022789	0.021956	0.020753	0.019234	0.017466	0.015525	0.013504	0.011502	0.009628	0.007985
0.22	0.004681	0.008816	0.012402	0.015437	0.017923	0.019863	0.021285	0.022142	0.022512	0.022397	0.021831	0.020851	0.019508	0.017861	0.015985	0.013963	0.011893	0.009882	0.008043	0.006478
0.20	0.004684	0.008799	0.012343	0.015316	0.017720	0.019561	0.020849	0.021599	0.021833	0.021577	0.020867	0.019749	0.018275	0.016515	0.014545	0.012460	0.010362	0.008365	0.006584	0.005120
0.18	0.004683	0.008775	0.012275	0.015183	0.017504	0.019245	0.020420	0.021046	0.021147	0.020754	0.019909	0.018660	0.017068	0.015207	0.013161	0.011031	0.008927	0.006966	0.005265	0.003921
0.16	0.004680	0.008747	0.012200	0.015042	0.017279	0.018921	0.019984	0.020488	0.020460	0.019937	0.018964	0.017595	0.015897	0.013949	0.011845	0.009689	0.007599	0.005696	0.004096	0.002890
0.14	0.004674	0.008714	0.012119	0.014895	0.017048	0.018592	0.019545	0.019930	0.019780	0.019133	0.018040	0.016562	0.014771	0.012752	0.010606	0.008444	0.006388	0.004564	0.003084	0.002029
0.12	0.004667	0.008678	0.012035	0.014743	0.016814	0.018262	0.019108	0.019379	0.019111	0.018348	0.017146	0.015569	0.013698	0.011624	0.009454	0.007305	0.005303	0.003575	0.002232	0.001338
0.10	0.004659	0.008640	0.011948	0.014591	0.016580	0.017934	0.018676	0.018838	0.018459	0.017589	0.016286	0.014623	0.012686	0.010572	0.008395	0.006276	0.004347	0.002733	0.001539	0.000810
0.00	0.004610	0.008445	0.011520	0.013853	0.015470	0.016404	0.016696	0.016397	0.015569	0.014286	0.012634	0.010714	0.008640	0.006539	0.004545	0.002797	0.001421	0.000508	0.000077	0.000000

ANSWERS TO EXAMPLES

APPENDIX 2

ANSWERS TO EXAMPLES

Consider a sample of 20 measures of fork length for fish taken from a RI salt pond.

Fork lengths (cm)
15.5
16.3
18.3
17.3
15.8
14.9
16.7
17.3
16.2
17.8
18.2
19.3
17.9
16.5
20.4
17.8
19.7
18.4
18.6
17.4

The mean, standard deviation, and confidence level can be determined directly in *Excel*. Go to **Tools**, **Data Analysis**, **Descriptive Statistics**, select the input and output ranges and check the boxes for **Summary Statistics** and **Confidence Level for Mean.**

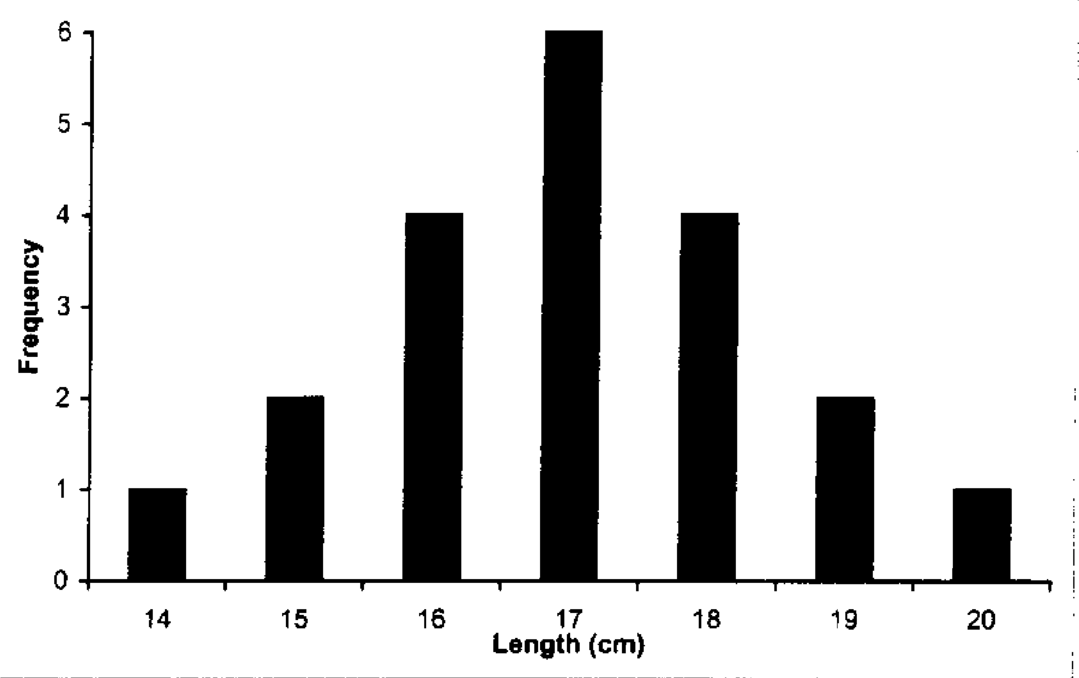
Column1	
Mean	17.515
Standard Error	0.318241467
Median	17.6
Mode	17.3
Standard Deviation	1.423219109
Sample Variance	2.025552632
Kurtosis	-0.309554994
Skewness	0.126320369
Range	5.5
Minimum	14.9
Maximum	20.4
Sum	350.3
Count	20
Confidence Level(95.0%)	0.666087253

The coefficient of variance ($C.V.$) can be calculated by using the equation $C.V. = {}^{s}/_{\bar{x}}$ therefore $C.V.$ = 1.4/17.5 = 0.08.

If the data are grouped into integer categories (e.g. any value from 17.0 to 17.9 is assigned to integer category 17), then seven groups emerge.

Group	# of Observations
14	1
15	2
16	4
17	6
18	4
19	2
20	1

These data can be plotted as a length-frequency histogram and the mode of the distribution is evident at 17 cm.



Consider the estimation of a weight-length relationship for a given species of fish.
The general model is as follows:

$$W = aL^b$$

where
- W = the weight in grams
- L = length in centimeters
- a = a unit conversion coefficient
- b = the volumetric expansion coefficient

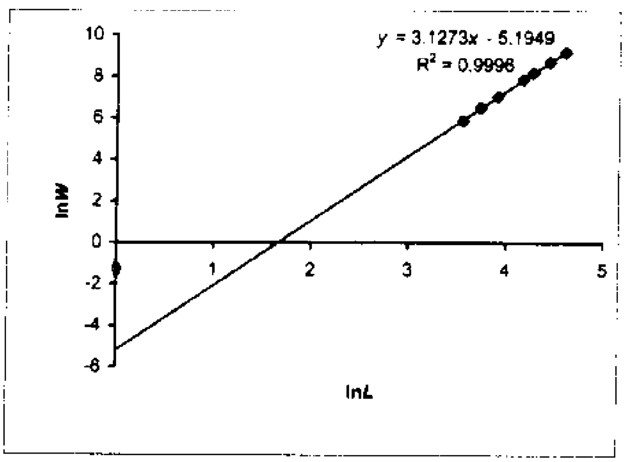
The function is linearized by taking the natural logarithm of both sides of the equation

$$\ln W = \ln(a) + b * \ln(L)$$

which is analogous to the linear model: $y = a' + b'x$

Given the following data, estimate the parameters a and b for this fish species.

W (gr)	L (cm)	ln(W)	ln(L)
9710	100	9.18	4.61
6020	85	8.70	4.44
3610	72	8.19	4.28
2620	65	7.87	4.17
1150	50	7.06	3.91
680	42	6.52	3.74
360	35	5.89	3.56



There are 2 methods to calculate the slope and y-intercept:

(1) Go to **Tools, Data Analysis, Regression**, select the input and output ranges, then click OK. The two numbers that are important are the **X variable 1** which is the slope and intercept which is the y-**intercept.**

(2) Select the **data series** on the graph, right click, and choose **Add Trendline.** Choose "linear" for the **Type**. In the **Options** tab, click "display equation on chart".

From **Tools, Data Analysis, Regression:**

SUMMARY OUTPUT

Regression Statistics	
Multiple R	0.9998102
R Square	0.99962044
Adjusted R Square	0.99954452
Standard Error	0.02546791
Observations	7

ANOVA

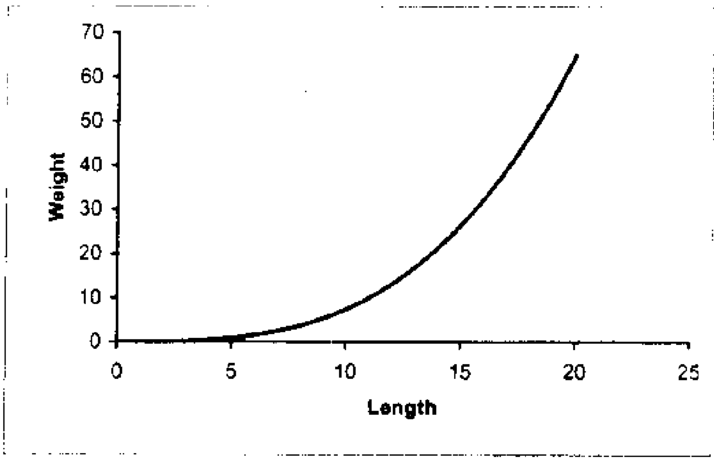
	df	*SS*	*MS*	*F*	*Significance F*
Regression	1	8.534273326	8.534273	13168.04	9.53121E-10
Residual	5	0.003240526	0.000648		
Total	6	8.537513852			

	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*	*Lower 95.0%*	*Upper 95.0%*
Intercept	-5.19492453	0.112166392	-46.31486	8.86E-08	-5.483254381	-4.906594686	-5.483254381	-4.906594686
X Variable 1	3.12732012	0.027252845	114.7521	9.53E-10	3.057264566	3.197375673	3.057264566	3.197375673

The y-intercept = $\ln a$, therefore $a = e^{-5.19} = 0.005545$
The slope = b, therefore $b = 3.1273$
The final value of the parameters in the model are: $W = 0.005545L^{3.1273}$

To create the non-linear curve, create a data series of lengths and solve for weight using the length-weight equation above and the parameter values obtained.

a	0.005545
b	3.1273

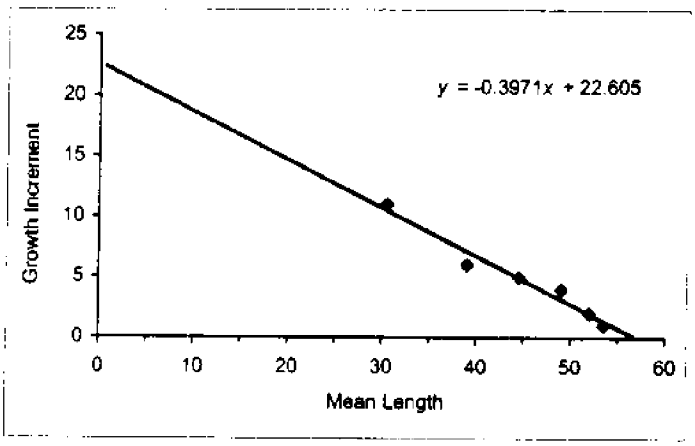


Length	Weight
0	0
1	0.005545
2	0.048452106
3	0.172187988
4	0.423373588
5	0.850727448
6	1.504575397
7	2.436557423
8	3.699430449
9	5.346925712
10	7.433840436
11	10.01495252
12	13.14695151
13	16.88638148
14	21.29059293
15	26.41750175
16	32.32555367
17	39.07369321
18	46.72133629
19	55.3283459
20	64.95501029

Given the following set of age and length data where length represents the mean of a large number of fish measured at each age, calculate the animal growth increment or rate, and the mean length-at age.

Age (t)	Length (cm) (L_t)
1	25
2	36
3	42
4	47
5	51
6	53
7	54

Age (t)	Length (cm) (L_t)	Growth Increment (cm) ($L_{t+1} - L_t$)	Mean Length (cm) ($L_{t+1} - L_t$)/2
1	25	11	30.5
2	36	6	39.0
3	42	5	44.5
4	47	4	49.0
5	51	2	52.0
6	53	1	53.5
7	54		



From **Tools, Data Analysis, Regression:**

SUMMARY OUTPUT

Regression Statistics	
Multiple R	0.980967873
R Square	0.962278348
Adjusted R Square	0.952847935
Standard Error	0.769768984
Observations	6

ANOVA

	df	*SS*	*MS*	*F*	*Significance F*
Regression	1	60.46315618	60.46315618	102.0398936	0.000540452
Residual	4	2.370177156	0.592544289		
Total	5	62.83333333			

	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*	*Lower 95.0%*	*Upper 95.0%*
Intercept	22.60493425	1.787153436	12.6485694	0.000224958	17.64299056	27.56687793	17.64299056	27.56687793
X Variable 1	-0.397130747	0.039314116	-10.10147977	0.000540452	-0.506284457	-0.287977036	-0.506284457	-0.287977036

Solving for t_0

Use the rearranged von Bertalanffy equation to solve for t_0

For t = 1 and L_t = 25 $$t_0 = 1 + \frac{1}{0.4}\ln\left(\frac{56.5-25}{56.5}\right) = -0.46,$$

For t = 2 and L_t = 36 $$t_0 = 2 + \frac{1}{0.4}\ln\left(\frac{56.5-36}{56.5}\right) = -0.53,$$

mean = -0.495

a	-0.40
K (= -a)	0.40
b	22.61
L_∞ (= b/K)	56.51
t_0	-0.495

Non-linear Method

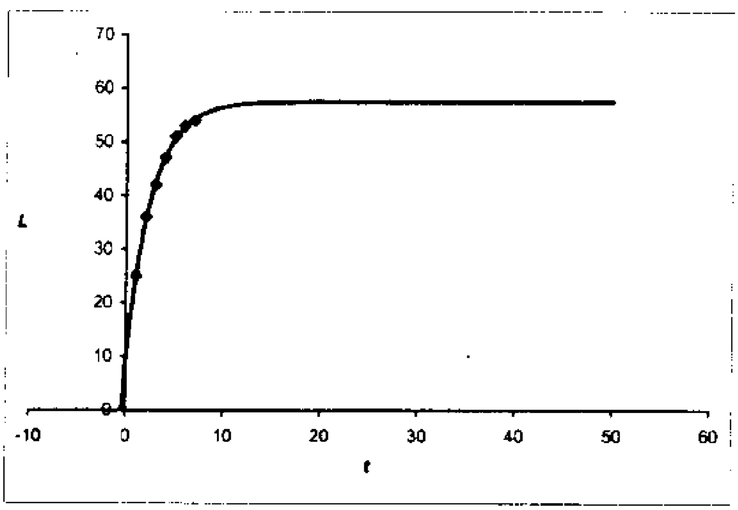
Non-linear regression can be calculated using the values obtained from the linear regression as initial parameter values, and then using **Solver** to adjust those values to provide the best fitting values.

The equation to calculate $L_{predicted}$ is the von Bertalanffy growth equation:

$$L_t = L_\infty\left(1 - e^{-K(t-t_0)}\right)$$

		Before Solver		After Solver	
Age	L_{obs}	L_{pred}	$(L_{obs}-L_{pred})^2$	L_{pred}	$(L_{obs}-L_{pred})^2$
1	25	25.43	0.19	25.13	0.02
2	36	35.68	0.10	35.47	0.28
3	42	42.55	0.30	42.51	0.26
4	47	47.15	0.02	47.30	0.09
5	51	50.24	0.58	50.55	0.20
6	53	52.30	0.48	52.77	0.06
7	54	53.69	0.10	54.27	0.07
			1.78		0.97

	Before Solver	After Solver
a	-0.4	-0.4
b	22.61	22.61
K	0.4000	0.3853
L_∞	56.51	57.47519802
t_0	-0.495	-0.491991597



To plot the von Bertalanffy curve, create a series of ages (t) and solve for L_t using the von Bertalanffy equation and the parameter values obtained from **Solver**.

a	-0.4
b	22.61
K	0.3853
L_∞	57.47519802
t_0	-0.491991597

t	L_t
-0.491991597	0
0	9.926005937
1	25.1316262
2	35.47468481
3	42.51016592
4	47.29579077
5	50.55103433
6	52.76529285
7	54.27145996
8	55.2959742
9	55.99286196
10	56.46689397
11	56.78933664
12	57.00866629
13	57.15785714
14	57.25933667
15	57.32836771
16	57.37532216
17	57.40726117
18	57.4289665
19	57.44376435
20	57.45381643
21	57.46065399
22	57.46530498
23	57.46846865
24	57.47062061
25	57.47208441
26	57.4730801
27	57.47375738
28	57.47421808
29	57.47453145
30	57.47474461
31	57.4748896
32	57.47498823
33	57.47505532
34	57.47510095
35	57.47513199
36	57.4751531
37	57.47516747
38	57.47517724
39	57.47518388
40	57.4751884
41	57.47519148
42	57.47519357
43	57.47519499
44	57.47519596
45	57.47519662
46	57.47519706
47	57.47519737
48	57.47519757
49	57.47519772
50	57.47519781

Microsoft Excel 8.0a Answer Report
Worksheet: [Example 2.xls]Example 1
Report Created: 3/27/00 2:28:05 PM

Target Cell (Min)

Cell	Name	Original Value	Final Value
F85	$(L_{obs}-L_{pred})^2$	1.78	0.97

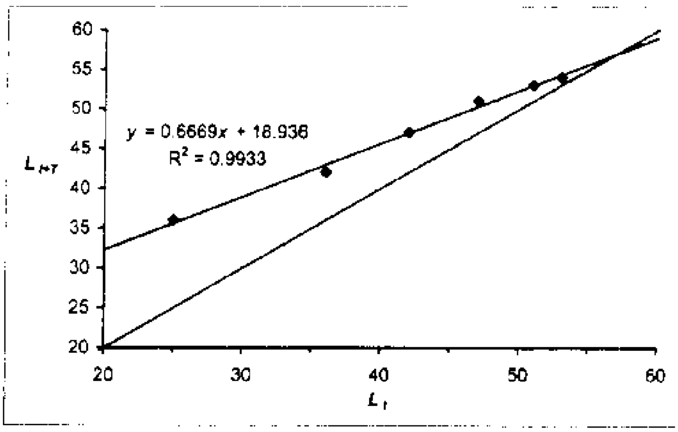
Adjustable Cells

Cell	Name	Original Value	Final Value
D90	K After **Solver**	0.4000	0.3853
D91	L_∞ After **Solver**	56.51	57.47519802
D92	t_0 After **Solver**	-0.495	-0.491991597

Constraints
NONE

Example 2: Given the following set of age and length data where length represents the mean of a large number of fish measured at each age, use the Ford-Walford plot method to estimate K and L_∞.

Age	L_t	L_{t+1}
1	25	36
2	36	42
3	42	47
4	47	51
5	51	53
6	53	54
7	54	



There are 2 methods to calculate the slope and y-intercept:

(1) Go to **Tools, Data Analysis, Regression**, select the input and output ranges, then click OK. The two numbers that are important are the X variable 1 which is the slope and intercept which is the y-intercept.

(2) Select the data series on the graph, right click, and choose **Add Trendline.** Choose "linear" for the **Type.** In the **Options** tab, click "display equation on chart."

To plot the 45° line, simple create a data series with 2 pairs of points that have equal x and y values.

45° Line	
0	0
60	60

From **Tools, Data Analysis, Regression:**

SUMMARY OUTPUT

Regression Statistics	
Multiple R	0.99665587
R Square	0.99332292
Adjusted R Square	0.99165365
Standard Error	0.64189675
Observations	6

ANOVA

	df	*SS*	*MS*	*F*	*Significance F*
Regression	1	245.1852076	245.1852076	595.06432	1.67561E-05
Residual	4	1.648125756	0.412031439		
Total	5	246.8333333			

	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*	*Lower 95.0%*	*Upper 95.0%*
Intercept	18.9359129	1.186584037	15.95834121	9.014E-05	15.64142067	22.2304052	15.64142067	22.2304052
X Variable 1	0.6668682	0.027337453	24.39394021	1.676E-05	0.590967103	0.74276929	0.590967103	0.742769294

To solve for L_∞ set the regression line equation and the 45° line equation equal to each other:

Regression Line
$y = 18.94 + 0.667x$

45° Line
$y = x$

$x = 18.94 + 0.667x$
$0.333x = 18.94$
$x = 18.94/0.333$
$x = 56.88$

To solve for K, use the equation:

$K = -\ln(a)$

$K = -\ln(0.667)$
$K = 0.405$

a	0.667
b	18.94
K	0.405
L_∞	56.88

Solving for t_0

Use the rearranged von Bertalanffy equation to solve for t_0

For t = 1 and L_t = 25 $\quad t_0 = 2 + \frac{1}{0.4}\ln\left(\frac{56.88-36}{56.88}\right) = -0.51,$

For t = 2 and L_t = 36 $\quad t_0 = 1 + \frac{1}{0.4}\ln\left(\frac{56.88-25}{56.88}\right) = -0.45,$

mean = -0.476

Non-linear Method

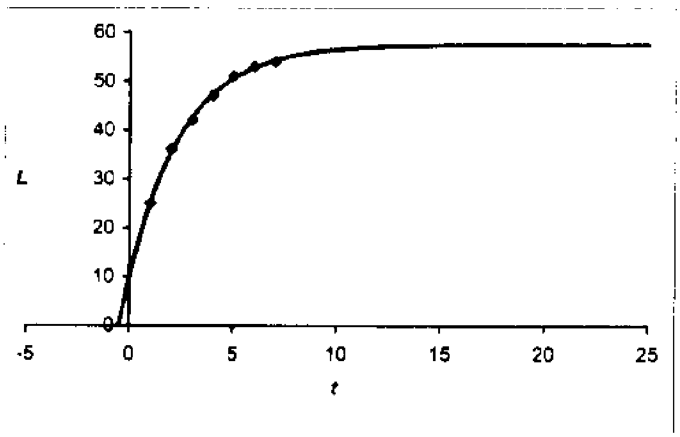
Non-linear regression can be calculated using the values obtained from the linear regression as initial parameter values, and then using **Solver** to adjust those values to provide the best fitting values.

The equation to calculate $L_{predicted}$ is the von Bertalanffy growth equation:

$$L_t = L_\infty\left(1 - e^{-K(t-t_0)}\right)$$

		Before Solver		After Solver	
Age	L_{obs}	L_{pred}	$(L_{obs}-L_{pred})^2$	L_{pred}	$(L_{obs}-L_{pred})^2$
1	25	25.59	0.35	25.13	0.02
2	36	36.01	0.00	35.47	0.28
3	42	42.96	0.93	42.51	0.26
4	47	47.60	0.36	47.30	0.09
5	51	50.69	0.10	50.56	0.20
6	53	52.75	0.06	52.77	0.06
7	54	54.13	0.02	54.27	0.07
			1.81		0.97

	Before Solver	After Solver
a	0.667	0.667
b	18.94	18.94
K	0.4050	0.3854
L_∞	56.88	57.47507346
t_0	-0.476	-0.491986901



To plot the von Bertalanffy curve, create a series of ages (t) and solve for L_t using the von Bertalanffy equation and the parameter values obtained from **Solver**.

a	0.667
b	18.94
K	0.3854
L_∞	57.4750735
t_0	-0.4919869

x	y
-0.491986901	0.00
0	9.93
1	25.13
2	35.47
3	42.51
4	47.30
5	50.56
6	52.77
7	54.27
8	55.30
9	55.99
10	56.47
11	56.79
12	57.01
13	57.16
14	57.26
15	57.33
16	57.38
17	57.41
18	57.43
19	57.44
20	57.45
21	57.46
22	57.47
23	57.47
24	57.47
25	57.47

Microsoft Excel 8.0a Answer Report
Worksheet: [Example 2.xls]F-W
Report Created: 3/27/00 2:21:09 PM

Target Cell (Min)

Cell	Name	Original Value	Final Value
F101	$(L_{obs}-L_{pred})^2$	1.81	0.97

Adjustable Cells

Cell	Name	Original Value	Final Value
D106	K After **Solver**	0.4050	0.3854
D107	L_∞ After **Solver**	56.88	57.47507346
D108	t_0 After **Solver**	-0.476	-0.491986901

Constraints
NONE

Consider a population of N_0 = 1,000 fish at the start of the year. At the end of the year, N_t = 358 fish. During the year, 321 fish were caught. Calculate S, Z, A, U, F, and M during the year.

Given data

$N_0 = 1,000$
$N_t = 358$
$C = 321$

S

$S = N_t / N_0$
= 358 / 1000
0.358

Z

$Z = -\ln(S)$
= -ln(0.358)
1.027222293

A

$A = 1 - S$
= 1 - 0.358
0.642

U

$U = C / N_0$
= 321 / 1000
0.321

F

$U = (F * A) / Z$

therefore

$F = (U * Z) / A$
= (0.321 * 1.027) / 0.642
0.513611146

M

$Z = F + M$

therefore

$M = Z - F$
= 1.027 - 0.51
0.513611146

Perform a catch curve analysis using the catch-at-age data for striped bass (*Morone saxatilis*) on the Atlantic coast from Maine to North Carolina (both landings and discards) reported by the Atlantic States Marine Fisheries Commission.

Age	Numbers	Ln(*N*)
1	0.5	-0.69
2	98	4.58
3	658	6.49
4	664	6.50
5	551	6.31
6	476	6.17
7	456	6.12
8	216	5.38
9	143	4.96
10	71	4.26
11	44	3.78
12	48	3.87
13	13	2.56
14	4.6	1.53
15	2.6	0.96

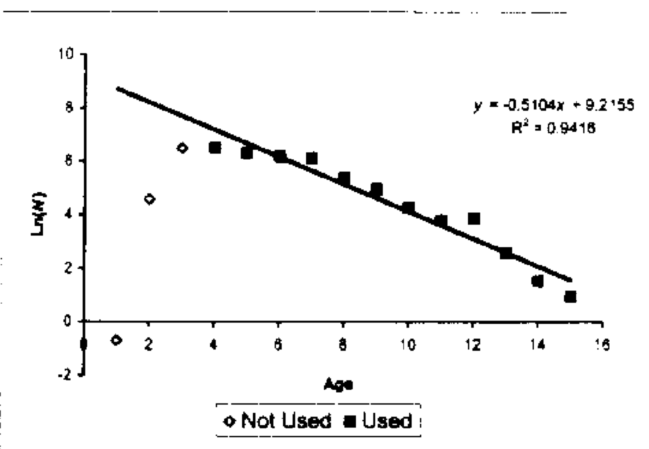
Plot the Ln(Number of fish) against time using only the fully recruited ages (ages 4 and up).

There are 2 methods to calculate the slope and *y*-intercept:

(1) Go to **Tools, Data Analysis, Regression**, select the input and output ranges, then click OK. The two numbers that are important are the X variable 1 which is the slope and intercept which is the *y*-intercept.

(2) Select the data series on the graph, right click, and choose **Add Trendline.** Choose "linear" for the **Type.** In the **Options** tab, click "display equation on chart."

Age	Numbers	Ln(*N*) (not used)	Ln(*N*) (used)
1	0.5	-0.69	
2	98	4.58	
3	658	6.49	
4	664		6.50
5	551		6.31
6	476		6.17
7	456		6.12
8	216		5.38
9	143		4.96
10	71		4.26
11	44		3.78
12	48		3.87
13	13		2.56
14	4.6		1.53
15	2.6		0.96



From **Tools, Data Analysis, Regression**:

SUMMARY OUTPUT

Regression Statistics	
Multiple R	0.970339958
R Square	0.941559633
Adjusted R Square	0.935715597
Standard Error	0.480852953
Observations	12

ANOVA

	df	*SS*	*MS*	*F*	*Significance F*
Regression	1	37.25284748	37.25284748	161.1146009	1.71996E-07
Residual	10	2.312195622	0.231219562		
Total	11	39.5650431			

	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*	*Lower 95.0%*	*Upper 95.0%*
Intercept	9.215531611	0.406442282	22.67365387	6.27385E-10	8.309921616	10.12114161	8.309921616	10.12114161
X Variable 1	-0.510401237	0.040210944	-12.69309285	1.71996E-07	-0.599996819	-0.420805655	-0.599996819	-0.420805655

The CCA suggests that average total mortality in 1996 on fully recruited cohorts is 0.51 (absolute value of -0.5104). The Atlantic States Marine Fisheries Commission assumed a natural mortality *M* of between 0.15 and 0.2. Therefore, fishing mortality, *F*, was estimated to range from 0.31 to 0.36. The fisheries management target is set at *F* = 0.31.

The Northeast Utilities Service Company (NUSCO) in Waterford, CT has collected and tagged lobsters in Long Island Sound since 1978. Commercial fishers and others recaptured lobsters and returned the tags to the NUSCO. Recapture data for individual years of tagging were used to determine annual exploitation rates. The results of this study are shown in the table below. Given this information, calculate fishing mortality rate for each year, assuming a natural mortality rate of *M* = 0.15.

Year	Number Tagged	Number Recaptured	Rate of Exploitation
1986	5797	1194	0.21
1987	5680	1356	0.24
1988	6836	1727	0.25
1989	6436	1235	0.19
1990	5741	1066	0.19
1991	6136	1109	0.18
1992	9126	1842	0.20
1993	8177	1708	0.21
1994	7533	1974	0.26
1995	5307	963	0.18
1996	6221	997	0.16
1997	6102	1665	0.27

Because fishing mortality and natural mortality occur at the same time, the instantaneous rate of mortality will be calculated using the Type 2 fishery equation.

$$U = (F / Z) * (1 - e^{-Z})$$

To solve, set *M* = 0.15. Create columns called "Solver Exploitation" and "Fishing Mortality (*F*)." In the "Solver Exploitation" column, enter the exploitation equation using cell values where appropriate, including the cell for *F*. Under the **Tools** menu, use the **Solver** function. Set the "Solver Exploitation" column equal to the "Rate of Exploitation" column, by changing the value in the *F* column. **Solver** iteratively solves for *F*. Repeat for each year.

M = 0.15

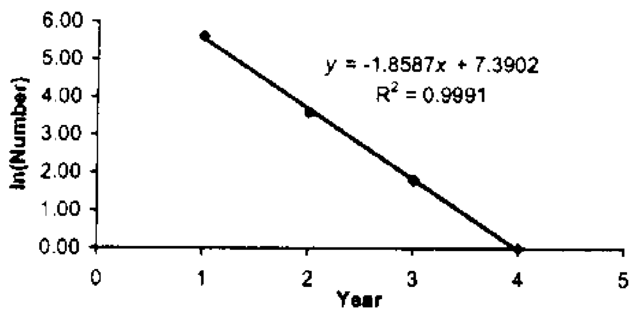
Year	Number Tagged	Number Recaptured	Rate of Exploitation	Solver Exploitation	*F*
1986	5797	1194	0.21	0.20918553	0.25431598
1987	5680	1356	0.24	0.23998242	0.29769718
1988	6836	1727	0.25	0.2499764	0.31217705
1989	6436	1235	0.19	0.1895022	0.22751774
1990	5741	1066	0.19	0.1895022	0.22751774
1991	6136	1109	0.18	0.17962501	0.2143288
1992	9126	1842	0.20	0.19935631	0.24084656
1993	8177	1708	0.21	0.20918553	0.25431598
1994	7533	1974	0.26	0.25996883	0.32686111
1995	5307	963	0.18	0.17962501	0.2143288
1996	6221	997	0.16	0.1598088	0.18836771
1997	6102	1665	0.27	0.26995943	0.34175474

A sample of 1000 winter flounder were tagged in Narragansett Bay in the winter of 1990. Returns from those tagged fish from 1991 to 1994 are shown in the table below. Determine estimates of total mortality (*Z*) and fishing mortality (*F*) over the time period, assuming a natural mortality rate of *M* = 0.30.

Year	Number	ln(Number)
1991	270	5.60
1992	36	3.58
1993	6	1.79
1994	1	0.00

The solution to this example is similar to the Catch Curve Analysis.
Plot the natural log of recaptures against time.

Time	ln(Number)
1	5.60
2	3.58
3	1.79
4	0.00



There are 2 methods to calculate the slope and *y*-intercept:

(1) Go to **Tools, Data Analysis, Regression**, select the input and output ranges, then click OK. The two numbers that are important are the X variable 1 which is the slope and intercept which is the *y*-intercept.

(2) Select the data series on the graph, right click, and choose **Add Trendline.** Choose "linear" for the **Type.** In the **Options** tab, click "display equation on chart."

From **Tools, Data Analysis, Regression:**

SUMMARY OUTPUT

Regression Statistics	
Multiple R	0.999567896
R Square	0.999135978
Adjusted R Square	0.998703967
Standard Error	0.086423126
Observations	4

ANOVA

	df	*SS*	*MS*	*F*	*Significance F*
Regression	1	17.27387556	17.27387556	2312.756161	0.000432104
Residual	2	0.014937913	0.007468957		
Total	3	17.28881347			

	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*	*Lower 95.0%*	*Upper 95.0%*
Intercept	7.390181428	0.10684628	69.81994475	0.000205072	6.934761325	7.845601531	6.93476133	7.845601531
X Variable 1	-1.85870263	0.038649597	-48.0911236	0.000432104	-2.02499844	-1.69240663	-2.0249984	-1.69240663

Y = -1.8587*X* + 7.3902

therefore

Z = 1.8587

Assuming *M* = 0.30, fishing mortality is found using the equation

Z* = *F* + *M

F = *Z* - *M*
F = 1.8587 - 0.30
1.5587

Solve for *M* using the time series of effort and total mortality data in the kingfish fishery off the coast of Thailand from 1966 to 1974.

Year	Effort (1000 days)	Z
1966	2.08	2.41
1967	2.08	2.69
1968	3.50	2.72
1969	3.60	2.62
1970	3.80	3.73
1971	–	–
1972	7.19	3.66
1973	9.94	4.61
1974	6.06	3.30

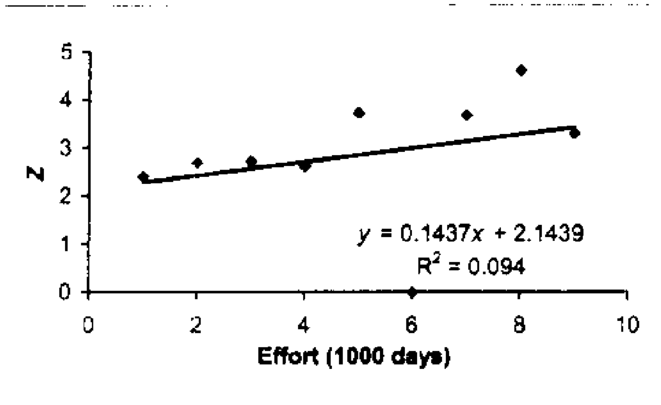
To solve, plot the values of *Z* against effort using the equation

$$Z = qf + M$$

There are 2 methods to calculate the slope and *y*-intercept:

(1) Go to **Tools, Data Analysis, Regression**, select the input and output ranges, then click OK. The two numbers that are important are the X variable 1 which is the slope and intercept which is the *y*-intercept.

(2) Select the data series on the graph, right click, and choose **Add Trendline**. Choose "linear" for the **Type**. In the **Options** tab, click "display equation on chart."



Data for regression

Effort (1000 days)	Z
2.08	2.41
2.08	2.69
3.50	2.72
3.60	2.62
3.80	3.73
7.19	3.66
9.94	4.61
6.06	3.30

From **Tools, Data Analysis, Regression:**

SUMMARY OUTPUT

Regression Statistics	
Multiple R	0.890274856
R Square	0.792589317
Adjusted R Square	0.758020869
Standard Error	0.36985708
Observations	8

ANOVA

	df	*SS*	*MS*	*F*	*Significance F*
Regression	1	3.136434444	3.136434444	22.92811452	0.003036798
Residual	6	0.820765556	0.136794259		
Total	7	3.9572			

	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*	*Lower 95.0%*	*Upper 95.0%*
Intercept	2.052953774	0.27659031	7.422363317	0.000307641	1.376161171	2.729746378	1.376161171	2.729746378
X Variable 1	0.2440681	0.05097561	4.788331079	0.003036798	0.119355183	0.368821016	0.119355183	0.368821016

$y = 0.2441x + 2.053$

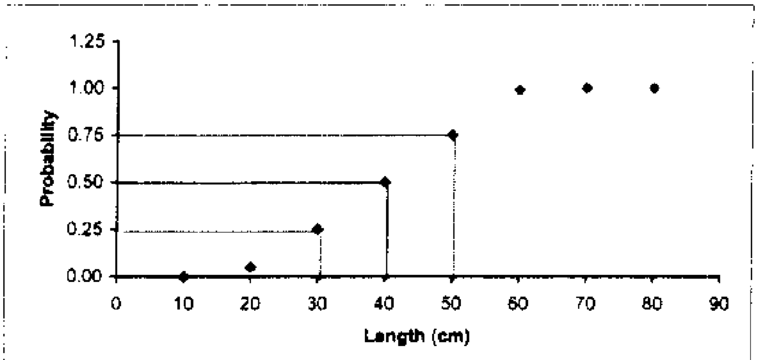
therefore

$M = 2.053$

Catches by length (cm) for a mesh cover and an experimental cod-end (12 cm, diamond mesh) are shown in the table below. Solve for L_{50}, selection factor, selection range, and the parameters α and β that define the LCDF.

Length (cm)	Cover	Cod-end	Sum	Ratio (*P*)
10	10	0	10	0.00
20	19	1	20	0.05
30	75	25	100	0.25
40	200	200	400	0.50
50	100	300	400	0.75
60	2	198	200	0.99
70	0	50	50	1.00
80	0	5	5	1.00

Plot the probability of capture (*P*) against length and estimate L_{50} from the plot.



Graphically, the L_{50} is estimated to be 40 cm.
L_{25} and L_{75} are estimated to be 30 and 50 cm, respectively

SF

SF = L_{50} / ml
= 40/12
SF = 3.333333333

SR

SR = L_{75} - L_{25}
= 50 - 30
SR = 20

The parameters α and β defining the LCDF can be determined indirectly using the linear regression equation

$$\ln(P / (1-P)) = \alpha + (\beta * L)$$

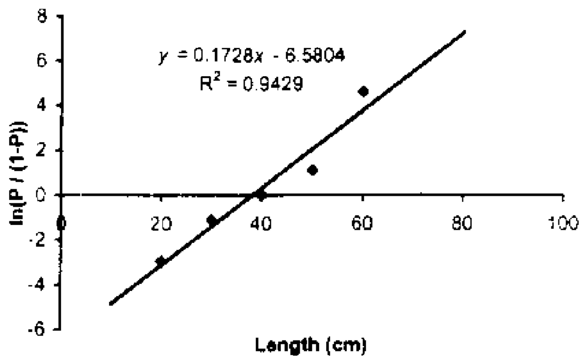
Plot the values of ln(*P* / (1-*P*)) versus length.

Length (cm)	Cover	Cod-end	Sum	Ratio (*P*)	Ln(*P* / (1-*P*))
10	10	0	10	0.00	
20	19	1	20	0.05	-2.94
30	75	25	100	0.25	-1.10
40	200	200	400	0.50	0.00
50	100	300	400	0.75	1.10
60	2	198	200	0.99	4.60
70	0	50	50	1.00	
80	0	5	5	1.00	

There are 2 methods to calculate the slope and y-intercept:

(1) Go to **Tools, Data Analysis, Regression**, select the input and output ranges, then click OK. The two numbers that are important are the X variable 1 which is the slope and intercept which is the y-intercept.

(2) Select the data series on the graph, right click, and choose **Add Trendline**. Choose "linear" for the **Type**. In the **Options** tab, click "display equation on chart."



From **Tools, Data Analysis, Regression**:

SUMMARY OUTPUT

Regression Statistics	
Multiple R	0.971043981
R Square	0.942926412
Adjusted R Square	0.923901883
Standard Error	0.776014538
Observations	5

ANOVA

	df	*SS*	*MS*	*F*	*Significance F*
Regression	1	29.84720011	29.84720011	49.56371861	0.005889071
Residual	3	1.806595688	0.602198563		
Total	4	31.65379579			

	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*	*Lower 95.0%*	*Upper 95.0%*
Intercept	-6.58040072	1.041132756	-6.32042426	0.00800601	-9.893752917	-3.267048524	-9.893752917	-3.267048524
X Variable 1	0.172763422	0.024539734	7.040150468	0.005889071	0.094666962	0.250859883	0.094666962	0.250859883

$y = 0.1728x - 6.5804$

therefore

$\alpha =$ -6.5804
$\beta =$ 0.1728

The selectivity parameters can also be determined directly using the non-linear regression equation.

$$PL_L = 1 / [1 + e^{(-\alpha 2 * (L - L50))}]$$

Use the value for L_{50} determined above and guess at a value for $\alpha 2$
Use this equation to solve for expected values of PL_L.

Find the Sum of Squares for the observed and expected PL_L values. Use **Solver** to minimize the Sum of Squares by changing the guessed values of L_{50} and $\alpha 2$. The new values for $\alpha 2$ and L_{50} are the solutions to the non-linear regression.

Length (cm)	Cover	Cod-end	Sum	Observed Ratio (P)	Before Solver Expected P	Before Solver Squared differences	After Solver Expected P	After Solver Squared differences
10	10	0	10	0.0000	0.0006	3.05564E-07	0.0200	0.000401766
20	19	1	20	0.0500	0.0067	0.001875509	0.0700	0.000401292
30	75	25	100	0.2500	0.0759	0.030325373	0.2171	0.001084296
40	200	200	400	0.5000	0.5000	0	0.5051	2.64255E-05
50	100	300	400	0.7500	0.9241	0.030325373	0.7898	0.001586964
60	2	198	200	0.9900	0.9933	1.09372E-05	0.9326	0.003294843
70	0	50	50	1.0000	0.9994	3.05564E-07	0.9807	0.000370641
80	0	5	5	1.0000	1.0000	2.06097E-09	0.9947	2.81268E-05
					Sum Sq. =	0.062537807	**Sum Sq. =**	0.007194353

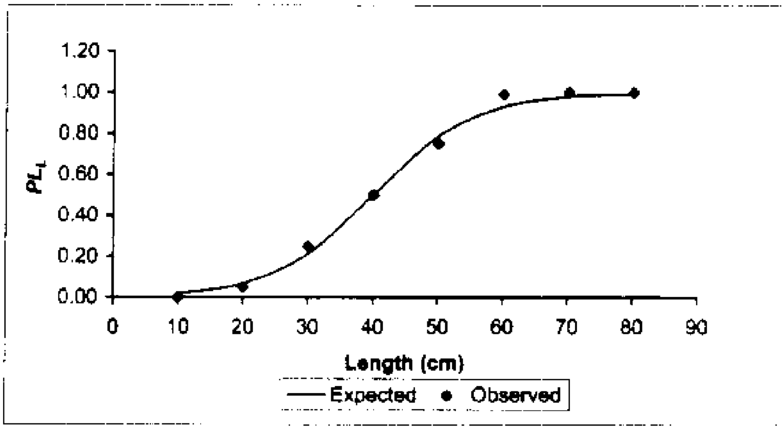
	Before Solver	After Solver
L_{50}	40	39.84223301
$\alpha 2$	0.25	0.130337644

L_{50} =	40
$\alpha 2 = \beta$ =	0.1304

$\alpha = -\alpha 2 \cdot L_{50}$
$= -40 \cdot 0.13$
$\alpha = -5.2157$

Use the new values to plot the selectivity curve.

Length (cm)	Observed P	Expected P
10	0	0.02
20	0.05	0.07
30	0.25	0.21
40	0.5	0.50
50	0.75	0.79
60	0.99	0.93
70	1	0.98
80	1	0.99



Microsoft Excel 8.0 Answer Report
Worksheet: [Chapter 5 - Selectivity Examples.xls]Example 1
Report Created: 5/10/00 9:48:25 AM

Target Cell (Min)

Cell	Name	Original Value	Final Value
I136	Sum Sq.	0.062537807	0.007194353

Adjustable Cells

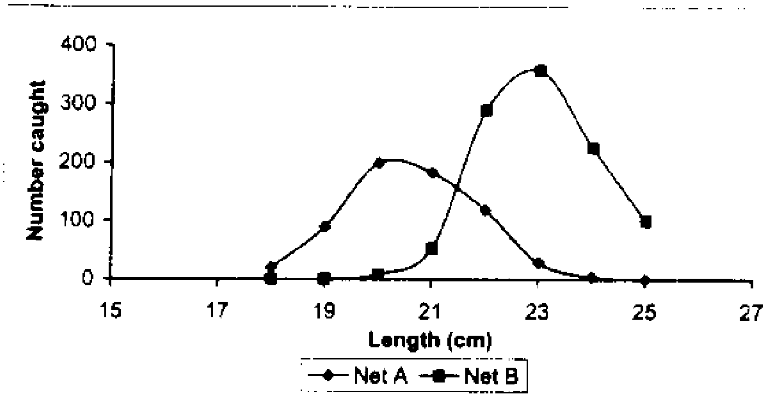
Cell	Name	Original Value	Final Value
H139	L_{50} After **Solver**	40	39.84223301
H140	α2 After **Solver**	0.25	0.130337844

Constraints
NONE

Nets A and B have mesh sizes of 8.1 and 9.1 cm respectively. Catches from these two nets are shown in the table below. Determine the selectivity parameters *a* and *b*, the L_{opt}s for both nets, the standard deviation and selection factor. Plot the results using the NPDF model.

Length (cm)	# Caught Net A	# Caught Net B	ln(B/A)
18	20	0	
19	90	1	-4.50
20	199	9	-3.10
21	182	53	-1.23
22	119	290	0.89
23	29	357	2.51
24	4	225	4.03
25	0	101	

Plotting the length frequency distribution for both nets, the required overlap is observed.

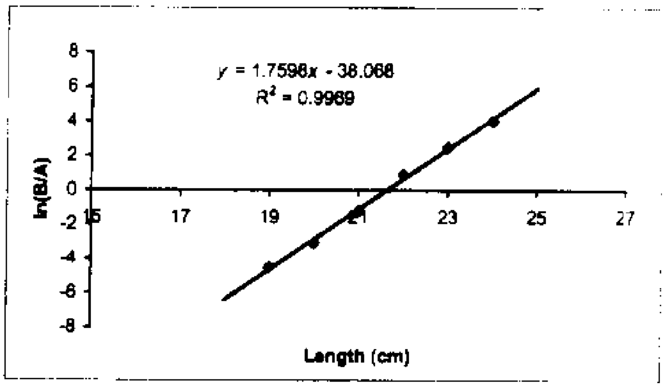


To determine the selectivity parameters, plot ln(B/A) against length.

There are 2 methods to calculate the slope and *y*-intercept:

(1) Go to **Tools, Data Analysis, Regression**, select the input and output ranges, then click OK. The two numbers that are important are the X variable 1 which is the slope and intercept which is the *y*-intercept.

(2) Select the data series on the graph, right click, and choose **Add Trendline**. Choose "linear" for the **Type**. In the **Options** tab, click "display equation on chart."



From **Tools, Data Analysis, Regression**:

SUMMARY OUTPUT

Regression Statistics	
Multiple R	0.998425922
R Square	0.996854321
Adjusted R Square	0.996067901
Standard Error	0.206769956
Observations	6

ANOVA

	df	*SS*	*MS*	*F*	*Significance F*
Regression	1	54.19411688	54.19411688	1267.585532	3.71463E-06
Residual	4	0.171015259	0.042753815		
Total	5	54.36513214			

	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*	*Lower 95.0%*	*Upper 95.0%*
Intercept	-38.06825433	1.066038044	-35.71003356	3.67048E-06	-41.02805657	-35.10845209	-41.02805657	-35.10845209
X Variable 1	1.75977461	0.049427473	35.60316744	3.71463E-06	1.62254166	1.897007561	1.62254166	1.897007561

$y = 1.7598x - 38.068$

therefore

$a =$ -38.068
$b =$ 1.7598

L_{opt}s, standard deviation, and selection factors are found following the method of Holt (1963)

L_{opt} A

$L_{opt}A = -2 * [(a*ml_A) / (b*(ml_A + ml_B))]$
$= -2*((-38.068*8.1) / (1.7598*(8.1+9.1)))$
$L_{opt}A =$ 20.37432901

L_{opt} B

$L_{opt}B = -2 * [(a*ml_B) / (b * (ml_A + ml_B))]$
$= -2*((-38.068*9.1) / (1.7598*(8.1+9.1)))$
$L_{opt}B =$ 22.88967827

SD

$SD = [-2*a*(ml_B - ml_A) / ((b^2)*(ml_A) + ml_B))]$
$= (-2*(-38.068)*(9.1-8.1)) / ((1.7598^2)*(8.1+9.1))$
SD = 1.429338141

SF

$SF = L_{opt} / ml$

$SF_A = 20.37 / 8.1$
$SF_A =$ 2.51534926

$SF_B = 22.89/91.$
$SF_B =$ 2.51534926

Consider the development of a harvesting strategy for a roundfish species where

K	0.2
M	0.1
W_∞	10 kg
t_0	0

and the question is whether to regulate mesh size in the trawl fishery to allow entry into the fishery at age 3 or 5, and at what level to set fishing mortality at these ages so as to maximize yield (F_{MAX}).

Using the spreadsheet program in the Example 1 worksheets, insert the appropriate numbers for each variable (cells D1 to D9), as well as, for U (cells F2 to F5) and n (cells G2 to G5).

The program automatically solves for yield per recruit (cells K23 to K223) and graphs F versus YPR.

Included are two worksheets, Example 1 (tc=3) is the solution when t_c = 3 and Example 1 (tc=5) is when t_c = 5.

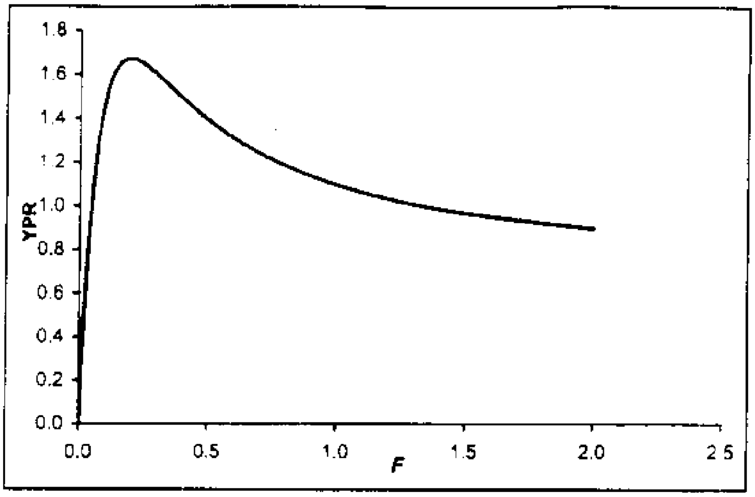
The problem answer can be found in the Conclusion worksheet.

Beverton-Holt
Yield Per Recruit

$t_c = 3$

M	0.1
W_∞	10
K	0.2
t_0	0
t_c	3
t_r	0
N_R	100000
t_λ	20
M/K	0.50

U	n
1	0
-3	1
3	2
-1	3



F	FW_EXP(..)	C	Y1	Y2-0	Y2-1	Y2-2	Y2-3	Yield	YPR
0	0	0	0	8.173164759	-5.45465671	1.806797571	-0.23613967	0	0
0.01	0.074081822	5,697	7,408	7.689784893	-5.28376222	1.771426525	-0.232814	29,223	0.292225763
0.02	0.148163644	10,742	14,816	7.249760743	-5.12278197	1.737407262	-0.22958068	53,856	0.538546007
0.03	0.222245466	15,220	22,225	6.84845655	-4.9709309	1.704664593	-0.22643591	74,580	0.745801187
0.04	0.296327288	19,207	29,633	6.481781589	-4.82749865	1.673128693	-0.22337611	91,981	0.919810431
0.05	0.37040911	22,766	37,041	6.146122227	-4.6918416	1.642734646	-0.22039788	106,553	1.065525288
0.06	0.444490932	25,951	44,449	5.838282785	-4.56337582	1.613422036	-0.217498	118,716	1.187160162
0.07	0.518572754	28,809	51,857	5.555434043	-4.44157072	1.585134569	-0.21467344	128,830	1.288302975
0.08	0.592654577	31,381	59,265	5.29506836	-4.32594362	1.557819739	-0.21192128	137,201	1.372009095
0.09	0.666736399	33,703	66,674	5.054960533	-4.21605473	1.531428518	-0.20923879	144,088	1.440881049
0.1	0.740818221	35,805	74,082	4.83313365	-4.11150287	1.505915082	-0.20662335	149,714	1.497136213
0.11	0.814900043	37,712	81,490	4.627829268	-4.01192154	1.481236567	-0.20407249	154,266	1.54266429
0.12	0.888981865	39,448	88,898	4.437481349	-3.91697546	1.45735279	-0.20158383	157,908	1.579076126
0.13	0.963063687	41,033	96,306	4.260693474	-3.82635755	1.434226138	-0.19915514	160,775	1.607745191
0.14	1.037145509	42,484	103,715	4.096218893	-3.73978613	1.411821282	-0.19678427	162,984	1.629842827
0.15	1.111227331	43,815	111,123	3.942943064	-3.65700252	1.390105047	-0.19446918	164,637	1.646368208
0.16	1.185309153	45,040	118,531	3.799868337	-3.57776884	1.369046249	-0.19220792	165,817	1.658173812
0.17	1.259390975	46,171	125,939	3.666100525	-3.50186604	1.348615549	-0.18999865	166,599	1.665987091
0.18	1.333472797	47,216	133,347	3.540837109	-3.4290922	1.328785317	-0.18783959	167,043	1.670428896
0.19	1.407554619	48,186	140,755	3.423356885	-3.35926089	1.309529512	-0.18572904	167,203	1.672029168
0.2	1.481636441	49,087	148,164	3.313010645	-3.29219962	1.290823571	-0.18366539	167,124	1.671240279
0.21	1.555718263	49,926	155,572	3.209214159	-3.22774967	1.272644309	-0.1816471	166,845	1.668448408
0.22	1.629800085	50,710	162,980	3.111439115	-3.16576244	1.254969817	-0.17967268	166,398	1.663983219
0.23	1.703881908	51,444	170,388	3.019208881	-3.1061014	1.237779385	-0.17774072	165,813	1.658126113
0.24	1.77796373	52,132	177,796	2.932092014	-3.04863925	1.221053415	-0.17584986	165,112	1.651117257
0.25	1.852045552	52,778	185,206	2.849697599	-2.99325768	1.204773351	-0.17399881	164,316	1.643161583
0.26	1.926127374	53,386	192,613	2.771670956	-2.93984662	1.18892161	-0.17218633	163,443	1.634433898
0.27	2.000209196	53,959	200,021	2.697689838	-2.8883035	1.173481518	-0.17041121	162,508	1.625083249
0.28	2.074291018	54,501	207,429	2.627461064	-2.83853263	1.158437258	-0.16867233	161,524	1.615236648
0.29	2.14837284	55,014	214,837	2.560717531	-2.79044469	1.143773807	-0.16696857	160,500	1.605002251
0.3	2.222454662	55,499	222,245	2.497215562	-2.74395618	1.129476894	-0.16529888	159,447	1.594472075
0.31	2.296536484	55,960	229,654	2.436732554	-2.69898898	1.115532961	-0.16366226	158,372	1.583724321
0.32	2.370618306	56,399	237,062	2.379064876	-2.65546992	1.101929071	-0.16205773	157,283	1.572825357
0.33	2.444700128	56,815	244,470	2.324026007	-2.61333041	1.088652967	-0.16048435	156,183	1.561831415
0.34	2.51878195	57,213	251,878	2.27144487	-2.5725061	1.075692938	-0.15894124	155,079	1.550790048
0.35	2.592863772	57,592	259,286	2.221164346	-2.53293654	1.063037831	-0.15742751	153,974	1.539741368
0.36	2.666945594	57,954	266,696	2.173039953	-2.49456491	1.050677013	-0.15594235	152,872	1.528719115
0.37	2.741027417	58,300	274,103	2.126938651	-2.45733775	1.038600339	-0.15448494	151,775	1.517751566
0.38	2.815109239	58,631	281,511	2.082737787	-2.42120471	1.026798123	-0.15305452	150,686	1.50686232
0.39	2.889191061	58,949	288,919	2.040324139	-2.38611834	1.015261116	-0.15165036	149,607	1.496070967
0.4	2.963272883	59,253	296,327	1.999593063	-2.3520339	1.003980479	-0.15027172	148,539	1.485393661
0.41	3.037354705	59,546	303,735	1.960447727	-2.31890912	0.992947762	-0.14891792	147,484	1.474843819
0.42	3.111436527	59,827	311,144	1.922798418	-2.2867041	0.98215488	-0.14758829	146,443	1.46443154
0.43	3.185518349	60,097	318,552	1.886561921	-2.25538109	0.9715941	-0.1462822	145,417	1.45416597
0.44	3.259600171	60,357	325,960	1.851660962	-2.22490438	0.961256013	-0.14499902	144,406	1.444053616
0.45	3.333681993	60,607	333,368	1.818023699	-2.19524017	0.951139524	-0.14373816	143,410	1.434099613
0.46	3.407763815	60,848	340,776	1.785583268	-2.16635642	0.941231635	-0.14249904	142,431	1.424307757

0.47	3.481845637	61,081	348,185	1.754277369	-2.13822274	0.931528426	-0.1412811	141,468	1.414680702
0.48	3.555927459	61,306	355,593	1.724047893	-2.11081031	0.922023044	-0.1400838	140,522	1.405220131
0.49	3.630009281	61,523	363,001	1.694840579	-2.08409176	0.912709688	-0.13890663	139,593	1.395926904
0.5	3.704091103	61,733	370,409	1.666604716	-2.05804108	0.903582598	-0.13774907	138,680	1.386801185
0.51	3.778172925	61,935	377,817	1.639292854	-2.03263356	0.894636242	-0.13661065	137,784	1.377842548
0.52	3.852254748	62,131	385,225	1.612860554	-2.00784568	0.885865303	-0.13549089	136,905	1.369050074
0.53	3.92633657	62,321	392,634	1.587266158	-1.98365504	0.877264673	-0.13438934	136,042	1.36042243
0.54	4.000418392	62,505	400,042	1.562470576	-1.96004033	0.868829439	-0.13330555	135,196	1.351957937
0.55	4.074500214	62,684	407,450	1.538437097	-1.93698122	0.860554876	-0.13223911	134,365	1.343654634
0.56	4.148582036	62,856	414,858	1.515131207	-1.91445834	0.852436436	-0.13118959	133,551	1.335510326
0.57	4.222663858	63,024	422,266	1.492520436	-1.8924532	0.844469742	-0.1301566	132,752	1.327522626
0.58	4.29674568	63,187	429,675	1.470574206	-1.87094816	0.83665058	-0.12913976	131,969	1.319688899
0.59	4.370827502	63,345	437,083	1.449263698	-1.84992637	0.828974888	-0.12813867	131,201	1.31200679
0.6	4.444909324	63,498	444,491	1.428561728	-1.82937171	0.821438754	-0.12715299	130,447	1.304473253
0.61	4.518991146	63,647	451,899	1.408442635	-1.80926878	0.814038405	-0.12618236	129,709	1.297085574
0.62	4.593072968	63,792	459,307	1.388882176	-1.78960287	0.806770206	-0.12522643	128,984	1.289840894
0.63	4.66715479	63,933	466,715	1.369857428	-1.77035988	0.799630647	-0.12428488	128,274	1.282736325
0.64	4.741236612	64,071	474,124	1.351348703	-1.7515263	0.792616344	-0.12335738	127,577	1.275768962
0.65	4.815318434	64,204	481,532	1.333329464	-1.73308921	0.785724029	-0.12244362	126,894	1.268935901
0.66	4.889400256	64,334	488,940	1.315786252	-1.71503622	0.778950546	-0.1215433	126,223	1.262234247
0.67	4.963482079	64,461	496,348	1.298698616	-1.69735546	0.772292849	-0.12065612	125,566	1.25566112
0.68	5.037563901	64,584	503,756	1.282049046	-1.68003552	0.765747995	-0.1197818	124,921	1.249213668
0.69	5.111645723	64,704	511,165	1.265820924	-1.66306548	0.759313138	-0.11892006	124,289	1.242889071
0.7	5.185727545	64,822	518,573	1.249998449	-1.64643484	0.752985529	-0.11807063	123,668	1.236684545
0.71	5.259809367	64,936	525,981	1.234566609	-1.63013352	0.746762508	-0.11723325	123,060	1.230597347
0.72	5.333891189	65,047	533,389	1.219511118	-1.61415182	0.740641504	-0.11640767	122,462	1.22462478
0.73	5.407973011	65,156	540,797	1.20481836	-1.59848045	0.734620028	-0.11559363	121,876	1.218764194
0.74	5.482054833	65,263	548,205	1.190475442	-1.58311046	0.728695673	-0.11479089	121,301	1.213012988
0.75	5.556136655	65,366	555,614	1.176469964	-1.56803322	0.722866108	-0.11399923	120,737	1.207368615
0.76	5.630218477	65,468	563,022	1.162790178	-1.55324048	0.717129076	-0.11321842	120,183	1.201828577
0.77	5.704300299	65,567	570,430	1.149424854	-1.53872419	0.71148239	-0.11244822	119,639	1.196390433
0.78	5.778382121	65,663	577,838	1.136363275	-1.52447675	0.705923934	-0.11168844	119,105	1.191051794
0.79	5.852463943	65,758	585,246	1.123595204	-1.51049073	0.700451665	-0.11093885	118,581	1.185810326
0.8	5.926545765	65,850	592,656	1.111110859	-1.496759	0.695063566	-0.11019926	118,066	1.180663749
0.81	6.000627588	65,941	600,063	1.098900889	-1.48327468	0.689757737	-0.10946946	117,561	1.175609638
0.82	6.07470941	66,029	607,471	1.086956346	-1.47003116	0.6845323	-0.10874927	117,065	1.170646421
0.83	6.148791232	66,116	614,879	1.075268671	-1.45702204	0.67938544	-0.10803849	116,577	1.165771379
0.84	6.222873054	66,201	622,287	1.063829665	-1.44424114	0.6743154	-0.10733694	116,098	1.160982647
0.85	6.296954876	66,284	629,695	1.052631477	-1.43168252	0.669320471	-0.10664444	115,628	1.15627821
0.86	6.371036698	66,365	637,104	1.041666582	-1.41934043	0.664398997	-0.10596083	115,166	1.151656105
0.87	6.44511852	66,445	644,512	1.030927764	-1.40720932	0.659549369	-0.10528592	114,711	1.14711442
0.88	6.519200342	66,522	651,920	1.020408104	-1.39528382	0.654770026	-0.10461955	114,265	1.142651291
0.89	6.593282164	66,599	659,328	1.010100961	-1.38355874	0.65005945	-0.10396156	113,826	1.138264902
0.9	6.667363986	66,674	666,736	0.999999959	-1.37202909	0.645416168	-0.10331181	113,395	1.133953487
0.91	6.741445808	66,747	674,145	0.990098975	-1.36069001	0.640836749	-0.10267012	112,972	1.129715322
0.92	6.81552763	66,819	681,553	0.980392128	-1.34953681	0.6363258	-0.10203635	112,555	1.125548731
0.93	6.889609452	66,889	688,961	0.970873762	-1.33856496	0.631875969	-0.10141036	112,145	1.121452082
0.94	6.963691274	66,959	696,369	0.961538441	-1.32777009	0.627487941	-0.10079201	111,742	1.117423786
0.95	7.037773096	67,026	703,777	0.952380936	-1.31714793	0.623160438	-0.10018114	111,346	1.113462295
0.96	7.111854919	67,093	711,185	0.943396212	-1.30669437	0.618892216	-0.09957764	110,957	1.109566102
0.97	7.185936741	67,158	718,594	0.934579427	-1.29640544	0.614682065	-0.09898137	110,573	1.105733741
0.98	7.260018563	67,222	726,002	0.925925916	-1.28627727	0.610528808	-0.0983922	110,196	1.101963783
0.99	7.334100385	67,285	733,410	0.917431184	-1.27630613	0.606431299	-0.09780999	109,825	1.09825484
1	7.408182207	67,347	740,818	0.909090902	-1.26648839	0.602388424	-0.09723464	109,461	1.094605557
1.01	7.482264029	67,408	748,226	0.900900896	-1.25682054	0.598399097	-0.09666602	109,101	1.091014616
1.02	7.556345851	67,467	755,635	0.892857138	-1.24729917	0.59446226	-0.096104	108,748	1.087480739
1.03	7.630427673	67,526	763,043	0.884955748	-1.23792098	0.590576886	-0.09554849	108,400	1.084002671
1.04	7.704509495	67,583	770,451	0.877192979	-1.22868277	0.586741971	-0.09499936	108,058	1.080579199
1.05	7.778591317	67,640	777,859	0.869565215	-1.21958141	0.582956539	-0.09445651	107,721	1.077209139
1.06	7.852673139	67,695	785,267	0.862068963	-1.2106139	0.579219638	-0.09391982	107,389	1.073891338
1.07	7.926754961	67,750	792,675	0.854700853	-1.20177731	0.575530341	-0.0933892	107,062	1.070624673
1.08	8.000836783	67,804	800,084	0.847457625	-1.19306877	0.571887744	-0.09286454	106,741	1.06740805
1.09	8.074918605	67,856	807,492	0.840336133	-1.18448555	0.568290966	-0.09234575	106,424	1.064240405
1.1	8.149000427	67,908	814,900	0.833333332	-1.17602493	0.564739147	-0.09183272	106,112	1.0611207
1.11	8.22308225	67,959	822,308	0.82644628	-1.16768433	0.561231451	-0.09132535	105,805	1.058047926
1.12	8.297164072	68,010	829,716	0.81967213	-1.1594612	0.557767059	-0.09082356	105,502	1.055021098
1.13	8.371245894	68,059	837,125	0.813008129	-1.15135308	0.554345175	-0.09032726	105,204	1.052039256
1.14	8.445327716	68,107	844,533	0.806451612	-1.14335758	0.550965022	-0.08983635	104,910	1.049101468
1.15	8.519409538	68,155	851,941	0.8	-1.13547235	0.54762584	-0.08935075	104,621	1.046206821
1.16	8.59349136	68,202	859,349	0.793650793	-1.12769514	0.544326889	-0.08887037	104,335	1.04335443
1.17	8.667573182	68,249	866,757	0.787401574	-1.12002375	0.541067447	-0.08839513	104,054	1.04054343
1.18	8.741655004	68,294	874,166	0.78125	-1.11245602	0.537846807	-0.08792494	103,777	1.037772979

1.19	8.815736826	68,339	881,574	0.775193798	-1.10498987	0.534664282	-0.08745973	103,504	1.035042254
1.2	8.889818648	68,383	888,982	0.769230769	-1.09762327	0.531519197	-0.08699941	103,235	1.032350455
1.21	8.96390047	68,427	896,390	0.763358778	-1.09035424	0.528410898	-0.08654392	102,970	1.029696802
1.22	9.037982282	68,470	903,798	0.757575757	-1.08318086	0.525338742	-0.08609317	102,708	1.027080533
1.23	9.112064114	68,512	911,206	0.751879699	-1.07610125	0.522302102	-0.08564709	102,450	1.024500906
1.24	9.186145936	68,553	918,615	0.746268657	-1.06911358	0.519300365	-0.08520561	102,196	1.021957199
1.25	9.260227759	68,594	926,023	0.740740741	-1.06221607	0.516332935	-0.08476866	101,945	1.019448703
1.26	9.334309581	68,635	933,431	0.735294118	-1.05540699	0.513399225	-0.08433617	101,697	1.016974732
1.27	9.408391403	68,674	940,839	0.729927007	-1.04868465	0.510498664	-0.08390807	101,453	1.014534613
1.28	9.482473225	68,714	948,247	0.724637681	-1.04204741	0.507630694	-0.08348429	101,213	1.012127692
1.29	9.556555047	68,752	955,656	0.71942446	-1.03549365	0.504794769	-0.08306477	100,975	1.009753328
1.3	9.630636869	68,790	963,064	0.714285714	-1.02902182	0.501990353	-0.08264944	100,741	1.007410898
1.31	9.704718691	68,828	970,472	0.709219858	-1.02263038	0.499216926	-0.08223825	100,510	1.005099793
1.32	9.778800513	68,865	977,880	0.704225352	-1.01631784	0.496473976	-0.08183113	100,282	1.002819419
1.33	9.852882335	68,901	985,288	0.699300699	-1.01008277	0.493761003	-0.08142802	100,057	1.000569195
1.34	9.926964157	68,937	992,696	0.694444444	-1.00392372	0.491077519	-0.08102887	99,835	0.998348556
1.35	10.00104598	68,973	1,000,105	0.689655172	-0.99783934	0.488423046	-0.0806336	99,616	0.996156949
1.36	10.0751278	69,008	1,007,513	0.684931507	-0.99182826	0.485797116	-0.08024218	99,399	0.993993833
1.37	10.14920962	69,042	1,014,921	0.680272109	-0.98588917	0.48319927	-0.07985454	99,186	0.991858683
1.38	10.22329145	69,076	1,022,329	0.675675676	-0.98002078	0.480629062	-0.07947062	98,975	0.989750982
1.39	10.29737327	69,110	1,029,737	0.67114094	-0.97422184	0.478086051	-0.07909038	98,767	0.987670229
1.4	10.37145509	69,143	1,037,146	0.666666667	-0.96849112	0.475569808	-0.07871376	98,562	0.985615931
1.41	10.44553691	69,176	1,044,554	0.662251656	-0.96282743	0.473079914	-0.07834071	98,359	0.983587611
1.42	10.51961873	69,208	1,051,962	0.657894737	-0.9572296	0.470615956	-0.07797117	98,158	0.981584797
1.43	10.59370056	69,240	1,059,370	0.653594771	-0.95169648	0.468177531	-0.07760511	97,961	0.979607034
1.44	10.66778238	69,271	1,066,778	0.649350649	-0.94622696	0.465764245	-0.07724247	97,765	0.977653873
1.45	10.7418642	69,302	1,074,186	0.64516129	-0.94081995	0.463375711	-0.0768832	97,572	0.975724877
1.46	10.81594602	69,333	1,081,595	0.641025641	-0.93547438	0.461011549	-0.07652726	97,382	0.973819617
1.47	10.89002784	69,363	1,089,003	0.636942675	-0.93018921	0.458671389	-0.0761746	97,194	0.971937678
1.48	10.96410967	69,393	1,096,411	0.632911392	-0.92496343	0.456354867	-0.07582518	97,008	0.970078649
1.49	11.03819149	69,423	1,103,819	0.628930818	-0.91979604	0.454061626	-0.07547894	96,824	0.968242132
1.5	11.11227331	69,452	1,111,227	0.625	-0.91468606	0.451791318	-0.07513586	96,643	0.966427736
1.51	11.18635513	69,480	1,118,636	0.621118012	-0.90963255	0.4495436	-0.07479588	96,464	0.964635079
1.52	11.26043695	69,509	1,126,044	0.617283951	-0.90463456	0.447318137	-0.07445896	96,286	0.962863786
1.53	11.33451878	69,537	1,133,452	0.613496933	-0.89969121	0.445114599	-0.07412506	96,111	0.961113497
1.54	11.4086006	69,565	1,140,860	0.609756098	-0.89480158	0.442932665	-0.07379415	95,938	0.95938385
1.55	11.48268242	69,592	1,148,268	0.606060606	-0.88996482	0.440772017	-0.07346617	95,767	0.957674496
1.56	11.55676424	69,619	1,155,676	0.602409639	-0.88518006	0.438632347	-0.0731411	95,599	0.955985093
1.57	11.63084606	69,646	1,163,085	0.598802395	-0.88044648	0.436513351	-0.07281889	95,432	0.954315308
1.58	11.70492789	69,672	1,170,493	0.595238095	-0.87576325	0.434414729	-0.07249951	95,266	0.952664812
1.59	11.77900971	69,698	1,177,901	0.591715976	-0.87112958	0.432336189	-0.07218292	95,103	0.951033286
1.6	11.85309153	69,724	1,185,309	0.588235294	-0.86654469	0.430277446	-0.07186908	94,942	0.949420415
1.61	11.92717335	69,750	1,192,717	0.584795322	-0.86200781	0.428238216	-0.07155796	94,783	0.947825892
1.62	12.00125518	69,775	1,200,126	0.581395349	-0.85751818	0.426218224	-0.07124952	94,625	0.946249418
1.63	12.075337	69,800	1,207,534	0.578034682	-0.85307506	0.4242172	-0.07094373	94,469	0.944690698
1.64	12.14941882	69,824	1,214,942	0.574712644	-0.84867779	0.422234877	-0.07064055	94,315	0.943149443
1.65	12.22350064	69,849	1,222,350	0.571428571	-0.84432569	0.420270993	-0.07033996	94,163	0.941625373
1.66	12.29758246	69,873	1,229,758	0.568181818	-0.84001781	0.418325294	-0.0700419	94,012	0.94011821
1.67	12.37166429	69,896	1,237,166	0.564971751	-0.83575376	0.416397528	-0.06974637	93,863	0.938627685
1.68	12.44574611	69,920	1,244,575	0.561797753	-0.83153278	0.414487448	-0.06945331	93,715	0.937153531
1.69	12.51982793	69,943	1,251,983	0.558659218	-0.82735423	0.412594811	-0.06916271	93,570	0.935696491
1.7	12.59390975	69,966	1,259,391	0.555555556	-0.82321745	0.41071938	-0.06887454	93,425	0.934253309
1.71	12.66799157	69,989	1,266,799	0.552486188	-0.81912184	0.408860921	-0.06858875	93,283	0.932826736
1.72	12.7420734	70,011	1,274,207	0.549450549	-0.81506679	0.407019205	-0.06830533	93,142	0.931415829
1.73	12.81615522	70,034	1,281,616	0.546448087	-0.81105168	0.405194007	-0.06802423	93,002	0.930019448
1.74	12.89023704	70,056	1,289,024	0.543478261	-0.80707594	0.403385105	-0.06774545	92,864	0.928638259
1.75	12.96431886	70,077	1,296,432	0.540540541	-0.80313898	0.401592283	-0.06746893	92,727	0.927271733
1.76	13.03840068	70,099	1,303,840	0.537634409	-0.79924025	0.399815326	-0.06719467	92,592	0.925919644
1.77	13.11248251	70,120	1,311,248	0.534759358	-0.79537918	0.398054025	-0.06692263	92,458	0.924581771
1.78	13.18656433	70,141	1,318,656	0.531914894	-0.79155524	0.396308174	-0.06665278	92,326	0.923257899
1.79	13.26064615	70,162	1,326,065	0.529100529	-0.7877679	0.39457757	-0.0663851	92,195	0.921947815
1.8	13.33472797	70,183	1,333,473	0.526315789	-0.78401662	0.392862016	-0.06611956	92,065	0.920651311
1.81	13.40880979	70,203	1,340,881	0.523560209	-0.7803009	0.391161314	-0.06585613	91,937	0.919368184
1.82	13.48289162	70,223	1,348,289	0.520833333	-0.77662024	0.389475274	-0.0655948	91,810	0.918098233
1.83	13.55697344	70,243	1,355,697	0.518134715	-0.77297414	0.387803706	-0.06533553	91,684	0.916841263
1.84	13.63105526	70,263	1,363,106	0.515463918	-0.76936211	0.386146426	-0.0650783	91,560	0.915597081
1.85	13.70513708	70,283	1,370,514	0.512820513	-0.76578368	0.384503249	-0.06482309	91,437	0.914365499
1.86	13.7792189	70,302	1,377,922	0.510204082	-0.76223838	0.382873998	-0.06456988	91,315	0.913146331
1.87	13.85330073	70,321	1,385,330	0.507614213	-0.75872576	0.381258496	-0.06431863	91,194	0.911939396
1.88	13.92738255	70,340	1,392,738	0.505050505	-0.75524537	0.37965657	-0.06406934	91,074	0.910744516
1.89	14.00146437	70,359	1,400,146	0.502512563	-0.75179676	0.378068048	-0.06382196	90,956	0.909561517
1.9	14.07554619	70,378	1,407,555	0.5	-0.7483795	0.376492765	-0.0635765	90,839	0.908390226

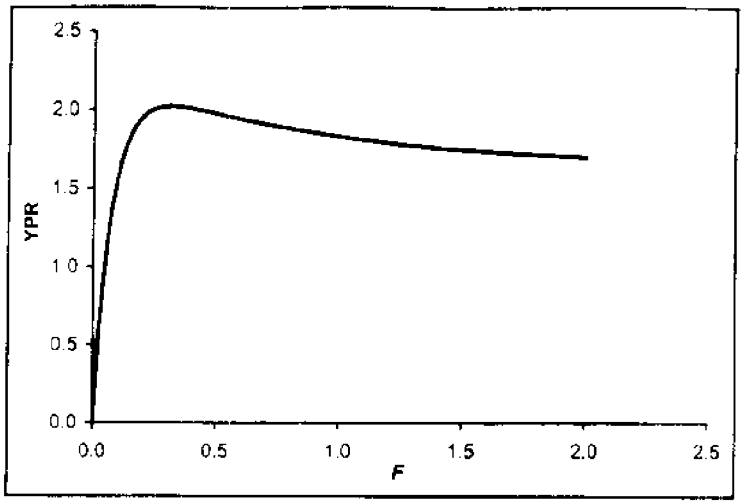
1.91	14.14962802	70,396	1,414,963	0.497512438	-0.74499317	0.374930554	-0.06333291	90,723	**0.907230476**
1.92	14.22370984	70,414	1,422,371	0.495049505	-0.74163735	0.373381254	-0.06309118	90,608	**0.906082101**
1.93	14.29779166	70,432	1,429,779	0.492610837	-0.73831162	0.371844706	-0.06285129	90,494	**0.904944939**
1.94	14.37187348	70,450	1,437,187	0.490196078	-0.73501558	0.370320752	-0.06261322	90,382	**0.903818832**
1.95	14.4459553	70,468	1,444,596	0.487804878	-0.73174885	0.368809239	-0.06237694	90,270	**0.902703623**
1.96	14.52003713	70,486	1,452,004	0.485436893	-0.72851102	0.367310015	-0.06214244	90,160	**0.901599158**
1.97	14.59411895	70,503	1,459,412	0.483091787	-0.72530172	0.365822929	-0.0619097	90,051	**0.900505288**
1.98	14.66820077	70,520	1,466,820	0.480769231	-0.72212057	0.364347837	-0.06167869	89,942	**0.899421864**
1.99	14.74228259	70,537	1,474,228	0.4784689	-0.71896721	0.362884593	-0.0614494	89,835	**0.898348742**
2	14.81636441	70,554	1,481,636	0.476190476	-0.71584126	0.361433054	-0.06122181	89,729	**0.897285779**

Beverton-Holt
Yield Per Recruit

$t_c = 5$

M	0.1
W_∞	10
K	0.2
t_0	0
t_c	5
t_r	0
N_R	100000
t_y	20
M/K	0.50

U	n
1	0
-3	1
3	2
-1	3



F	FW_∞EXP(..)	C	Y1	Y2-0	Y2-1	Y2-2	Y2-3	Yield	YPR
0	0	0	0	7.768698399	-3.6379267	0.811562588	-0.07112242	0	0
0.01	0.060653066	4,455	6,065	7.345000831	-3.52608314	0.795710927	-0.07012097	27,564	0.275638322
0.02	0.121306132	8,438	12,131	6.955842598	-3.42048644	0.780460567	-0.0691473	51,515	0.515147043
0.03	0.181959198	12,005	18,196	6.597891757	-3.3206691	0.765778617	-0.06820027	72,325	0.723251604
0.04	0.242612264	15,207	24,261	6.26816837	-3.22620506	0.751634465	-0.06727681	90,405	0.904050661
0.05	0.30326533	18,087	30,327	5.96400517	-3.1367056	0.737999595	-0.06638169	106,110	1.061100299
0.06	0.363918396	20,682	36,392	5.683012792	-3.0518157	0.724847415	-0.06550857	119,749	1.197486561
0.07	0.424571462	23,025	42,457	5.423049024	-2.97121075	0.712153104	-0.06465791	131,589	1.315888543
0.08	0.485224528	25,145	48,522	5.182191596	-2.89459365	0.699693474	-0.06382905	141,863	1.418632694
0.09	0.545877594	27,069	54,588	4.958714101	-2.8216922	0.68804684	-0.06302116	150,774	1.507739892
0.1	0.60653066	28,817	60,653	4.751064658	-2.75225669	0.676592908	-0.06223345	158,497	1.584966156
0.11	0.667183726	30,409	66,718	4.557847015	-2.68605788	0.665512662	-0.06146519	165,184	1.65183789
0.12	0.727836792	31,863	72,784	4.377803785	-2.62288502	0.654788276	-0.06071566	170,968	1.709682351
0.13	0.788489858	33,194	78,849	4.209801581	-2.56254418	0.644403018	-0.05998418	175,965	1.759654074
0.14	0.849142924	34,414	84,914	4.052817823	-2.50485675	0.634341174	-0.05927012	180,276	1.802757709
0.15	0.90979599	35,536	90,980	3.905929017	-2.44965799	0.624587972	-0.05857285	183,987	1.839867825
0.16	0.970449056	36,569	97,045	3.768300341	-2.39679585	0.615129513	-0.0578918	187,175	1.87174605
0.17	1.031102122	37,524	103,110	3.63917639	-2.34612982	0.605952711	-0.05722639	189,906	1.899055927
0.18	1.091755187	38,407	109,176	3.51787294	-2.29752992	0.597045233	-0.05657611	192,238	1.922375789
0.19	1.152408253	39,225	115,241	3.403769612	-2.25087578	0.58639545	-0.05594044	194,221	1.942209914
0.2	1.213061319	39,986	121,306	3.296303345	-2.20605584	0.579992385	-0.05531689	195,900	1.958996204
0.21	1.273714385	40,696	127,371	3.194962574	-2.16296655	0.571825672	-0.054711	197,312	1.973124573
0.22	1.334367451	41,356	133,437	3.09928204	-2.12151178	0.56388551	-0.05411632	198,492	1.984924224
0.23	1.395020517	41,974	139,502	3.008838155	-2.0816021	0.556162632	-0.05353444	199,469	1.99468997
0.24	1.455673583	42,553	145,567	2.923244863	-2.04315431	0.548648262	-0.05296483	200,268	2.00267771
0.25	1.516326649	43,096	151,633	2.842149947	-2.00609089	0.541334092	-0.05240741	200,911	2.009111194
0.26	1.576979715	43,607	157,698	2.76523172	-1.97033956	0.534212242	-0.0518615	201,419	2.014186153
0.27	1.637632781	44,088	163,763	2.692196062	-1.93583284	0.527275243	-0.05132685	201,807	2.018073895
0.28	1.698285847	44,542	169,829	2.622773775	-1.90250771	0.520516003	-0.05080311	202,092	2.020924419
0.29	1.758938913	44,971	175,894	2.556716207	-1.87030522	0.513927787	-0.05028995	202,287	2.02286914
0.3	1.819591979	45,377	181,959	2.49380312	-1.83917021	0.507504194	-0.04978705	202,402	2.024023236
0.31	1.880245045	45,762	188,025	2.433820776	-1.80905098	0.501239139	-0.04929411	202,449	2.024487715
0.32	1.940898111	46,127	194,090	2.376580227	-1.77989907	0.495126831	-0.04881084	202,435	2.024351191
0.33	2.001551177	46,474	200,155	2.321905762	-1.75166901	0.489161758	-0.04833695	202,369	2.023691448
0.34	2.062204243	46,805	206,220	2.269635527	-1.72431809	0.483338667	-0.04787217	202,258	2.022576792
0.35	2.122857309	47,119	212,286	2.219620268	-1.69780613	0.477652555	-0.04741625	202,107	2.021067233
0.36	2.183510375	47,420	218,351	2.171722206	-1.67209538	0.472098646	-0.04696893	201,922	2.019215517
0.37	2.244163441	47,707	224,416	2.125814023	-1.64715024	0.466672387	-0.04652997	201,707	2.017068022
0.38	2.304816507	47,981	230,482	2.081777946	-1.62293721	0.46136943	-0.04609913	201,467	2.014665544
0.39	2.365469573	48,244	236,547	2.039504914	-1.59942466	0.456185621	-0.04567621	201,204	2.012043982
0.4	2.426122639	48,496	242,612	1.998893831	-1.57658276	0.451116992	-0.04526097	200,923	2.009234935
0.41	2.486775705	48,737	248,678	1.959850894	-1.55438333	0.446159748	-0.04485321	200,627	2.006266223
0.42	2.547428771	48,969	254,743	1.922288971	-1.53279974	0.441310258	-0.04445274	200,316	2.00316235
0.43	2.608081837	49,192	260,808	1.886127053	-1.51180678	0.436565046	-0.04405935	199,994	1.999944897
0.44	2.668734903	49,406	266,873	1.851289742	-1.4913806	0.431920792	-0.04367287	199,663	1.996632879
0.45	2.729387969	49,612	272,939	1.817706803	-1.47149862	0.427374302	-0.0432931	199,324	1.993243044

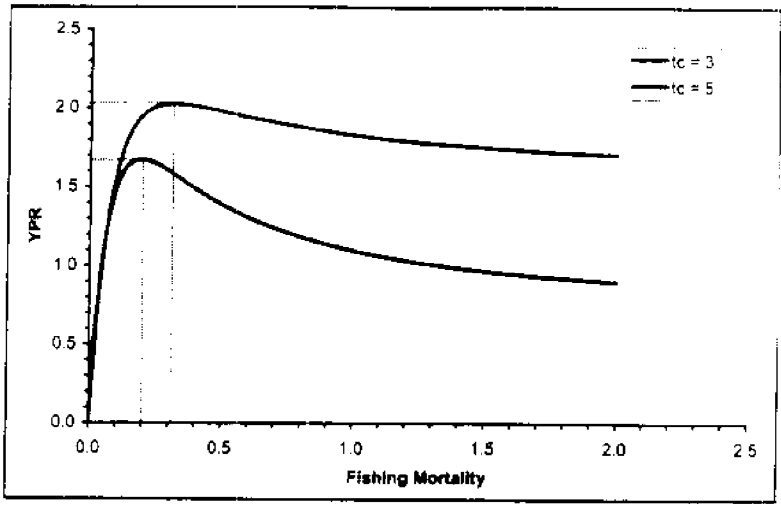
0.47	2.850694101	50,002	285,069	1.754046412	-1.43328271	0.418562531	-0.04255305	198,629	1.986287198
0.48	2.911347167	50,187	291,135	1.723850714	-1.41490919	0.414291512	-0.04219243	198,275	1.982745633
0.49	2.972000233	50,366	297,200	1.694672234	-1.39700056	0.410106773	-0.04183787	197,918	1.979175535
0.5	3.032653299	50,538	303,265	1.666460984	-1.37953943	0.406005726	-0.04148922	197,559	1.975585775
0.51	3.093306365	50,705	309,331	1.639170131	-1.36250924	0.401985884	-0.04114634	197,198	1.971984154
0.52	3.153959431	50,868	315,396	1.612755767	-1.34589427	0.39804486	-0.04080907	196,838	1.968377526
0.53	3.214612496	51,022	321,461	1.587176683	-1.32967952	0.39418036	-0.04047729	196,477	1.964771911
0.54	3.275265562	51,173	327,527	1.562394174	-1.31385072	0.390390175	-0.04015086	196,117	1.961172587
0.55	3.335918628	51,319	333,592	1.538371854	-1.29839426	0.386672182	-0.03982965	195,758	1.957584179
0.56	3.396571694	51,461	339,657	1.515075493	-1.28329717	0.383024339	-0.03951355	195,401	1.954010725
0.57	3.45722476	51,598	345,722	1.492472857	-1.26854707	0.379444679	-0.03920242	195,046	1.950455753
0.58	3.517877826	51,732	351,788	1.470533573	-1.25413214	0.375931308	-0.03889615	194,692	1.94692233
0.59	3.578530892	51,861	357,853	1.449228996	-1.24004108	0.372482401	-0.03859463	194,341	1.943413119
0.6	3.639183958	51,987	363,918	1.428532091	-1.22626312	0.369096202	-0.03829774	193,993	1.939930422
0.61	3.699837024	52,109	369,984	1.408417323	-1.21278794	0.365771014	-0.0380054	193,648	1.936476222
0.62	3.76049009	52,228	376,049	1.388860556	-1.19960566	0.362505205	-0.03771748	193,305	1.933052216
0.63	3.821143156	52,344	382,114	1.369838962	-1.18670684	0.359297196	-0.03743389	192,966	1.92965985
0.64	3.881796222	52,456	388,180	1.351330929	-1.17408244	0.356145469	-0.03715453	192,630	1.926300345
0.65	3.942449288	52,565	394,245	1.33331599	-1.1617238	0.353048554	-0.03687931	192,297	1.922974723
0.66	4.003102354	52,672	400,310	1.315774743	-1.14962261	0.350005033	-0.03660814	191,968	1.919683829
0.67	4.06375542	52,776	406,376	1.298688784	-1.13777092	0.347013538	-0.03634093	191,643	1.91642835
0.68	4.124408486	52,877	412,441	1.282040649	-1.12616109	0.344072747	-0.03607759	191,321	1.913208834
0.69	4.185061552	52,975	418,506	1.265813749	-1.11478579	0.34118138	-0.03581803	191,003	1.910025703
0.7	4.245714618	53,071	424,571	1.24999232	-1.10363799	0.338338203	-0.03556219	190,688	1.906879272
0.71	4.306367684	53,165	430,637	1.234561372	-1.09271092	0.33554202	-0.03530996	190,377	1.903769754
0.72	4.36702075	53,256	436,702	1.219506644	-1.08199811	0.332791676	-0.03506132	190,070	1.90069728
0.73	4.427673816	53,345	442,767	1.204814557	-1.07149331	0.330086053	-0.03481613	189,766	1.8976619
0.74	4.488326882	53,432	448,833	1.190472176	-1.06119052	0.32742407	-0.03457435	189,466	1.894663599
0.75	4.548979948	53,517	454,898	1.176467174	-1.05108397	0.324804677	-0.03433591	189,170	1.891702301
0.76	4.609633014	53,600	460,963	1.162787793	-1.0411681	0.322226863	-0.03410073	188,878	1.888777877
0.77	4.67028608	53,681	467,029	1.149422816	-1.03143758	0.319689644	-0.03386875	188,589	1.885890151
0.78	4.730939146	53,761	473,094	1.136361533	-1.02188724	0.317192069	-0.03363991	188,304	1.883038907
0.79	4.791592212	53,838	479,159	1.123593716	-1.01251214	0.314733216	-0.03341414	188,022	1.880223892
0.8	4.852245278	53,914	485,225	1.111109588	-1.0033075	0.312312191	-0.03319138	187,744	1.877444822
0.81	4.912898344	53,988	491,290	1.098899802	-0.9942687	0.309928129	-0.03297157	187,470	1.874701387
0.82	4.97355141	54,060	497,355	1.086955418	-0.98539131	0.307580188	-0.03275465	187,199	1.871993252
0.83	5.034204476	54,131	503,420	1.075267877	-0.97667104	0.305267556	-0.03254057	186,932	1.869320061
0.84	5.094857542	54,201	509,486	1.063828987	-0.96810376	0.30298944	-0.03232927	186,668	1.866681443
0.85	5.155510608	54,268	515,551	1.052630897	-0.95968547	0.300745073	-0.03212069	186,408	1.864077009
0.86	5.216163674	54,335	521,616	1.041666086	-0.95141232	0.298533713	-0.03191479	186,151	1.861506361
0.87	5.276816739	54,400	527,682	1.03092734	-0.9432806	0.296354634	-0.03171151	185,897	1.858969086
0.88	5.337469805	54,464	533,747	1.020407742	-0.9352867	0.294207137	-0.0315108	185,646	1.856464767
0.89	5.398122871	54,526	539,812	1.010100661	-0.92742715	0.292090539	-0.03131262	185,399	1.853992978
0.9	5.458775937	54,588	545,878	0.999999694	-0.91969859	0.290004178	-0.03111692	185,155	1.851553288
0.91	5.519429003	54,648	551,943	0.990098749	-0.91209778	0.287947411	-0.03092364	184,915	1.849145262
0.92	5.580082069	54,707	558,008	0.980391935	-0.90462157	0.285919612	-0.03073276	184,677	1.846768462
0.93	5.640735135	54,764	564,074	0.970873597	-0.89726692	0.283920174	-0.03054421	184,442	1.844422448
0.94	5.701388201	54,821	570,139	0.9615383	-0.8900309	0.281948507	-0.03035797	184,211	1.842106781
0.95	5.762041267	54,877	576,204	0.952380815	-0.88291065	0.280004034	-0.03017398	183,982	1.839821019
0.96	5.822694333	54,931	582,269	0.943396109	-0.87590343	0.278086198	-0.02999221	183,756	1.837564725
0.97	5.883347399	54,985	588,335	0.934579339	-0.86900655	0.276194456	-0.02981262	183,534	1.835337459
0.98	5.944000465	55,037	594,400	0.925925841	-0.86221744	0.274328277	-0.02963516	183,314	1.833138786
0.99	6.004653531	55,089	600,465	0.91743112	-0.85553358	0.272487147	-0.0294598	183,097	1.830968274
1	6.065306597	55,139	606,531	0.909090847	-0.84895255	0.270670566	-0.02928651	182,883	1.828825493
1.01	6.125959663	55,189	612,596	0.900900848	-0.842472	0.268878046	-0.02911524	182,671	1.826710015
1.02	6.186612729	55,238	618,661	0.892857096	-0.83608964	0.267109112	-0.02894597	182,462	1.824621419
1.03	6.247265795	55,286	624,727	0.884955714	-0.82980325	0.2653633	-0.02877866	182,256	1.822559287
1.04	6.307918861	55,333	630,792	0.87719296	-0.82361069	0.263640162	-0.02861326	182,052	1.820523204
1.05	6.368571927	55,379	636,857	0.869565189	-0.81750987	0.261939258	-0.02844975	181,851	1.818512762
1.06	6.429224993	55,424	642,922	0.862068942	-0.81149877	0.26026016	-0.02828811	181,653	1.816527567
1.07	6.489878059	55,469	648,988	0.854700834	-0.80557542	0.258602452	-0.02812829	181,457	1.814567189
1.08	6.550531125	55,513	655,063	0.84745761	-0.79973791	0.256965728	-0.02797026	181,263	1.812631265
1.09	6.611184191	55,556	661,118	0.84033612	-0.7939644	0.255349591	-0.027814	181,072	1.810719396
1.1	6.671837257	55,599	667,184	0.833333321	-0.78831309	0.253753656	-0.02765948	180,883	1.808831199
1.11	6.732490323	55,640	673,249	0.82644627	-0.78272221	0.252177546	-0.02750667	180,697	1.806966297
1.12	6.793143389	55,682	679,314	0.819672122	-0.77721009	0.250620896	-0.02735553	180,512	1.805124318
1.13	6.853796455	55,722	685,380	0.813008122	-0.77177506	0.249083343	-0.02720605	180,330	1.803304895
1.14	6.914449521	55,762	691,445	0.806451606	-0.7664155	0.247564543	-0.02705819	180,151	1.801507668
1.15	6.975102587	55,801	697,510	0.799999994	-0.76112988	0.246064151	-0.02691193	179,973	1.799732282
1.16	7.035755653	55,839	703,576	0.793650789	-0.75591666	0.244581837	-0.02676724	179,798	1.797978387
1.17	7.096408719	55,877	709,641	0.787401571	-0.75077437	0.243117275	-0.0266241	179,625	1.79624564
1.18	7.157061785	55,915	715,706	0.781249996	-0.74570157	0.241670149	-0.02648246	179,453	1.794533703

1.19	7.217714851	55,951	721,771	0.775193795	-0.74069686	0.240240148	-0.02634236	179,284	1.792842242
1.2	7.278367917	55,987	727,837	0.769230767	-0.73575888	0.23882697	-0.02620372	179,117	1.791170932
1.21	7.339020983	56,023	733,902	0.763358776	-0.73088631	0.237430321	-0.02606653	178,952	1.789519451
1.22	7.399674048	56,058	739,967	0.757575756	-0.72607764	0.236049913	-0.02593076	178,789	1.787887483
1.23	7.460327114	56,093	746,033	0.751879698	-0.72133224	0.234685462	-0.02579641	178,627	1.786274717
1.24	7.52098018	56,127	752,098	0.746268655	-0.71664826	0.233336695	-0.02566344	178,468	1.784680848
1.25	7.581633246	56,160	758,163	0.74074074	-0.71202472	0.232003343	-0.02553183	178,311	1.783105578
1.26	7.642286312	56,193	764,229	0.735294117	-0.70746046	0.230685142	-0.02540157	178,155	1.78154861
1.27	7.702939378	56,226	770,294	0.729927006	-0.70295435	0.229381836	-0.02527262	178,001	1.780009657
1.28	7.763592444	56,258	776,359	0.72463768	-0.69850527	0.228093174	-0.02514498	177,849	1.778488434
1.29	7.82424551	56,290	782,425	0.71942446	-0.69411215	0.22681891	-0.02501863	177,698	1.776984661
1.3	7.884898576	56,321	788,490	0.714285714	-0.68977395	0.225558805	-0.02489353	177,550	1.775498065
1.31	7.945551642	56,351	794,555	0.709219858	-0.68548964	0.224312624	-0.02476969	177,403	1.774028377
1.32	8.006204708	56,382	800,620	0.704225352	-0.68125822	0.223080137	-0.02464706	177,258	1.772575332
1.33	8.066857774	56,412	806,686	0.699300699	-0.67707873	0.22186112	-0.02452565	177,114	1.77113867
1.34	8.12751084	56,441	812,751	0.694444444	-0.6729502	0.220655353	-0.02440543	176,972	1.769718137
1.35	8.188163906	56,470	818,816	0.689655172	-0.66887171	0.219462621	-0.02428637	176,831	1.768313483
1.36	8.248816972	56,499	824,882	0.684931507	-0.66484236	0.218282715	-0.02416848	176,692	1.766924462
1.37	8.309470038	56,527	830,947	0.680272109	-0.66086127	0.217115428	-0.02405172	176,555	1.765550832
1.38	8.370123104	56,555	837,012	0.675675676	-0.65692757	0.215960558	-0.02393609	176,419	1.764192356
1.39	8.43077617	56,582	843,078	0.671140939	-0.65304043	0.21481791	-0.02382156	176,285	1.762848803
1.4	8.491429236	56,610	849,143	0.666666667	-0.64919901	0.213687289	-0.02370813	176,152	1.761519943
1.41	8.552082302	56,636	855,208	0.662251656	-0.64540253	0.212568508	-0.02359577	176,021	1.760205554
1.42	8.612735368	56,663	861,274	0.657894737	-0.64165019	0.21146138	-0.02348447	175,891	1.758905413
1.43	8.673388434	56,689	867,339	0.653594771	-0.63794123	0.210365725	-0.02337421	175,762	1.757619306
1.44	8.7340415	56,715	873,404	0.649350649	-0.6342749	0.209281366	-0.02326499	175,635	1.75634702
1.45	8.794694566	56,740	879,469	0.64516129	-0.63065047	0.208208128	-0.02315678	175,509	1.755088348
1.46	8.855347632	56,765	885,535	0.641025641	-0.62706723	0.207145842	-0.02304957	175,384	1.753843083
1.47	8.916000698	56,790	891,600	0.636942675	-0.62352448	0.20609434	-0.02294335	175,261	1.752611027
1.48	8.976653764	56,814	897,665	0.632911392	-0.62002153	0.205053459	-0.0228381	175,139	1.751391981
1.49	9.03730683	56,838	903,731	0.628930818	-0.61655772	0.20402304	-0.02273382	175,019	1.750185751
1.5	9.097959896	56,862	909,796	0.625	-0.6131324	0.203002925	-0.02263049	174,899	1.748992149
1.51	9.158612962	56,886	915,861	0.621118012	-0.60974493	0.20199296	-0.02252809	174,781	1.747810988
1.52	9.219266028	56,909	921,927	0.617283951	-0.60639468	0.200992995	-0.02242661	174,664	1.746642083
1.53	9.279919094	56,932	927,992	0.613496933	-0.60308105	0.200002882	-0.02232604	174,549	1.745485255
1.54	9.34057216	56,955	934,057	0.609756098	-0.59980344	0.199022475	-0.02222637	174,434	1.744340328
1.55	9.401225226	56,977	940,123	0.606060606	-0.59656126	0.198051634	-0.02212759	174,321	1.743207128
1.56	9.461878292	56,999	946,188	0.602409639	-0.59335394	0.197090218	-0.02202968	174,209	1.742085485
1.57	9.522531357	57,021	952,253	0.598802395	-0.59018092	0.196138092	-0.02193263	174,098	1.740975231
1.58	9.583184423	57,043	958,318	0.596238095	-0.58704166	0.19519512	-0.02183643	173,988	1.739876201
1.59	9.643837489	57,064	964,384	0.591715976	-0.58393562	0.194261172	-0.02174108	173,879	1.738788236
1.6	9.704490555	57,085	970,449	0.588235294	-0.58086228	0.193336119	-0.02164655	173,771	1.737711175
1.61	9.765143621	57,106	976,514	0.584795322	-0.57782111	0.192419834	-0.02155284	173,664	1.736644864
1.62	9.825796687	57,127	982,580	0.581395349	-0.57481163	0.191512193	-0.02145994	173,559	1.735589149
1.63	9.886449753	57,147	988,645	0.578034682	-0.57183333	0.190613075	-0.02136784	173,454	1.73454388
1.64	9.947102819	57,167	994,710	0.574712644	-0.56888573	0.18972236	-0.02127652	173,351	1.73350891
1.65	10.00775589	57,187	1,000,776	0.571428571	-0.56596837	0.18883993	-0.02118599	173,248	1.732484093
1.66	10.06840895	57,207	1,006,841	0.568181818	-0.56308076	0.187965671	-0.02109622	173,147	1.731469288
1.67	10.12906202	57,226	1,012,906	0.564971751	-0.5602225	0.18709947	-0.0210072	173,046	1.730464354
1.68	10.18971508	57,246	1,018,972	0.561797753	-0.55739309	0.186241215	-0.02091894	172,947	1.729469153
1.69	10.25036815	57,265	1,025,037	0.558659218	-0.55459212	0.185390799	-0.02083141	172,848	1.728483552
1.7	10.31102122	57,283	1,031,102	0.555555556	-0.55181916	0.184548114	-0.02074461	172,751	1.727507417
1.71	10.37167428	57,302	1,037,167	0.552486188	-0.54907379	0.183713054	-0.02065853	172,654	1.726540617
1.72	10.43232735	57,320	1,043,233	0.549450549	-0.54635561	0.182885518	-0.02057317	172,558	1.725583025
1.73	10.49298041	57,339	1,049,298	0.546448087	-0.5436642	0.182065403	-0.02048851	172,463	1.724634515
1.74	10.55363348	57,357	1,055,363	0.543478261	-0.54099918	0.181252611	-0.02040454	172,369	1.723694962
1.75	10.61428654	57,375	1,061,429	0.540540541	-0.53836016	0.180447044	-0.02032125	172,276	1.722764246
1.76	10.67493961	57,392	1,067,494	0.537634409	-0.53574676	0.179648606	-0.02023865	172,184	1.721842246
1.77	10.73559268	57,410	1,073,559	0.534759358	-0.53315861	0.178857203	-0.02015671	172,093	1.720928846
1.78	10.79624574	57,427	1,079,625	0.531914894	-0.53059535	0.178072741	-0.02007543	172,002	1.720023929
1.79	10.85689881	57,444	1,085,690	0.529100529	-0.52805661	0.177295131	-0.01999481	171,913	1.719127383
1.8	10.91755187	57,461	1,091,755	0.526315789	-0.52554206	0.176524282	-0.01991483	171,824	1.718239095
1.81	10.97820494	57,478	1,097,820	0.523560209	-0.52305134	0.175760108	-0.01983549	171,736	1.717358956
1.82	11.03885801	57,494	1,103,886	0.520833333	-0.52058411	0.175002521	-0.01975677	171,649	1.716486859
1.83	11.09951107	57,510	1,109,951	0.518134715	-0.51814006	0.174251438	-0.01967868	171,562	1.715622696
1.84	11.16016414	57,527	1,116,016	0.515463918	-0.51571884	0.173506773	-0.01960121	171,477	1.714766364
1.85	11.2208172	57,543	1,122,082	0.512820513	-0.51332015	0.172768447	-0.01952434	171,392	1.71391776
1.86	11.28147027	57,559	1,128,147	0.510204082	-0.51094367	0.172036377	-0.01944807	171,308	1.713076784
1.87	11.34212334	57,574	1,134,212	0.507614213	-0.50858909	0.171310485	-0.0193724	171,224	1.712243336
1.88	11.4027764	57,590	1,140,278	0.505050505	-0.50625611	0.170590693	-0.01929731	171,142	1.711417318
1.89	11.46342947	57,605	1,146,343	0.502512563	-0.50394444	0.169876925	-0.01922281	171,060	1.710598636
1.9	11.52408253	57,620	1,152,408	0.5	-0.50165378	0.169169104	-0.01914887	170,979	1.709787194

1.91	11.5847356	57,636	1,158,474	0.497512438	-0.49938386	0.168467158	-0.01907551	170,898	**1.7089829**
1.92	11.64538867	57,650	1,164,539	0.495049505	-0.49713438	0.167771012	-0.0190027	170,819	**1.708185663**
1.93	11.70604173	57,665	1,170,604	0.492610837	-0.49490508	0.167080597	-0.01893044	170,740	**1.707395392**
1.94	11.7666948	57,680	1,176,669	0.490196078	-0.49269568	0.16639584	-0.01885874	170,661	**1.706612001**
1.95	11.82734786	57,694	1,182,735	0.487804878	-0.49050592	0.165716673	-0.01878757	170,584	**1.7058354**
1.96	11.88800093	57,709	1,188,800	0.485436893	-0.48833554	0.165043028	-0.01871694	170,507	**1.705065506**
1.97	11.948654	57,723	1,194,865	0.483091787	-0.48618428	0.164374838	-0.01864684	170,430	**1.704302235**
1.98	12.00930706	57,737	1,200,931	0.480769231	-0.4840519	0.163712036	-0.01857726	170,355	**1.703545503**
1.99	12.06996013	57,751	1,206,996	0.4784689	-0.48193813	0.163054558	-0.0185082	170,280	**1.702795228**
2	12.13061319	57,765	1,213,061	0.476190476	-0.47984275	0.16240234	-0.01843965	170,205	**1.702051332**

F	YPR (t_c=3)	YPR (t_c=5)
0	0	0
0.01	0.29222576	0.27563832
0.02	0.53854601	0.51514704
0.03	0.74580119	0.7232516
0.04	0.91981043	0.90405068
0.05	1.06552529	1.0611003
0.06	1.18716016	1.19748656
0.07	1.28830298	1.31588854
0.08	1.37200909	1.41863269
0.09	1.44088105	1.50773989
0.1	1.49713621	1.58496616
0.11	1.54266429	1.65183789
0.12	1.57907613	1.70968235
0.13	1.60774519	1.75965407
0.14	1.62984283	1.80275771
0.15	1.64636821	1.83986783
0.16	1.65817381	1.87174605
0.17	1.66598709	1.89905593
0.18	1.6704269	1.92237579
0.19	**1.67202917**	1.94220991
0.2	1.67124028	1.9589982
0.21	1.66844841	1.97312457
0.22	1.66398322	1.98492422
0.23	1.65812611	1.99468997
0.24	1.65111726	2.00267771
0.25	1.64316158	2.00911119
0.26	1.6344339	2.01418615
0.27	1.62508325	2.01807389
0.28	1.61523665	2.02092442
0.29	1.60500225	2.02286914
0.3	1.59447208	2.02402324
0.31	1.58372432	**2.02448772**
0.32	1.57282536	2.02435119
0.33	1.56183141	2.02369145
0.34	1.55079005	2.02257679
0.35	1.53974137	2.02106723
0.36	1.52871911	2.01921552
0.37	1.51775157	2.01706802
0.38	1.50686232	2.01466554
0.39	1.49607097	2.01204398
0.4	1.48539366	2.00923493
0.41	1.47484362	2.00626622
0.42	1.46443154	2.00316235
0.43	1.45416597	1.9999449
0.44	1.44405362	1.99663288
0.45	1.43409961	1.99324304
0.46	1.42430776	1.98979015
0.47	1.4146807	1.9862872
0.48	1.40522013	1.98274563
0.49	1.3959269	1.97917554
0.5	1.38680118	1.97558578
0.51	1.37784255	1.97198415
0.52	1.36905007	1.96837753
0.53	1.36042243	1.96477191
0.54	1.35196794	1.96117259
0.55	1.34366463	1.95758418
0.56	1.33551033	1.95401073
0.57	1.32752263	1.95046575
0.58	1.319689	1.94692233
0.59	1.31200679	1.94341312
0.6	1.30447325	1.93993042
0.61	1.29708557	1.93647622
0.62	1.28984089	1.93305222
0.63	1.28273632	1.92965985
0.64	1.27576896	1.92630034
0.65	1.2689359	1.92297472
0.66	1.26223425	1.91968383
0.67	1.25566112	1.91642835
0.68	1.24921367	1.91320883
0.69	1.24288907	1.9100257
0.7	1.23668454	1.90687927

At a t_c = 3, the maximum yield per recruit is 1.67 kg at F = 0.19. In contrast, if the t_c = 5, the maximum yield per recruit is 2.02 kg at an F = 0.31, an increase of 21%. Interestingly, if t_c is set to 10 years, maximum YPR is achieved at 2.36 kg with F = 2.0. Thus for fish species where M/K is small (0.5), substantially greater yields (about 40%) are realized by delayng entry into the fishery.



0.71	1.23059735	1.90376975
0.72	1.22462478	1.90069728
0.73	1.21876419	1.8976619
0.74	1.21301299	1.8946636
0.75	1.20736861	1.8917023
0.76	1.20182858	1.88877788
0.77	1.19639043	1.88589015
0.78	1.19105179	1.88303891
0.79	1.18581033	1.88022389
0.8	1.18066375	1.87744482
0.81	1.17560984	1.87470139
0.82	1.17064642	1.87199325
0.83	1.16577138	1.86932006
0.84	1.16098265	1.86668144
0.85	1.15627821	1.86407701
0.86	1.15165611	1.86150636
0.87	1.14711442	1.85896909
0.88	1.14265129	1.85646477
0.89	1.1382649	1.85399298
0.9	1.13395349	1.85155329
0.91	1.12971532	1.84914526
0.92	1.12554873	1.84676848
0.93	1.12145208	1.84442245
0.94	1.11742379	1.84210678
0.95	1.11346229	1.83982102
0.96	1.1095661	1.83756472
0.97	1.10573374	1.83533746
0.98	1.10196378	1.83313879
0.99	1.09825484	1.83096827
1	1.09460556	1.82882549
1.01	1.09101462	1.82671002
1.02	1.08748074	1.82462142
1.03	1.08400267	1.82255929
1.04	1.0805792	1.8205232
1.05	1.07720914	1.81851276
1.06	1.07389134	1.81652756
1.07	1.07062467	1.81456719
1.08	1.06740805	1.81263126
1.09	1.0642404	1.8107194
1.1	1.0611207	1.8088312
1.11	1.05804793	1.8069663
1.12	1.0550211	1.80512432
1.13	1.05203926	1.8033049
1.14	1.04910147	1.80150767
1.15	1.04620682	1.79973228
1.16	1.04335443	1.79797839
1.17	1.04054343	1.79624564
1.18	1.03777298	1.7945337
1.19	1.03504225	1.79284224
1.2	1.03235045	1.79117093
1.21	1.0296968	1.78951946
1.22	1.02708053	1.78788748
1.23	1.02450091	1.78627472
1.24	1.0219572	1.78468086
1.25	1.0194487	1.78310558
1.26	1.01697473	1.78154861
1.27	1.01453461	1.78000966
1.28	1.01212769	1.77848843
1.29	1.00975333	1.77698466
1.3	1.0074109	1.77549807
1.31	1.00509979	1.77402838
1.32	1.00281942	1.77257533
1.33	1.0005692	1.77113867
1.34	0.99834856	1.76971814
1.35	0.99615695	1.76831348
1.36	0.99399383	1.76692446
1.37	0.99185868	1.76555083
1.38	0.98975098	1.76419236
1.39	0.98767023	1.7628488
1.4	0.98561593	1.76151994
1.41	0.98358761	1.76020555
1.42	0.9815848	1.75890541

1.43	0.97960703	1.75761931
1.44	0.97765387	1.75634702
1.45	0.97572488	1.75508835
1.46	0.97381962	1.75384308
1.47	0.97193768	1.75261103
1.48	0.97007865	1.75139198
1.49	0.96824213	1.75018575
1.5	0.96642774	1.74899215
1.51	0.96463508	1.74781099
1.52	0.96286379	1.74664208
1.53	0.9611135	1.74548526
1.54	0.95938385	1.74434033
1.55	0.9576745	1.74320713
1.56	0.95598509	1.74208548
1.57	0.95431531	1.74097523
1.58	0.95266481	1.7398762
1.59	0.95103329	1.73878824
1.6	0.94942041	1.73771117
1.61	0.94782589	1.73664486
1.62	0.94624942	1.73558915
1.63	0.9446907	1.73454388
1.64	0.94314944	1.73350891
1.65	0.94162537	1.73248409
1.66	0.94011821	1.73146929
1.67	0.93862768	1.73046435
1.68	0.93715353	1.72946915
1.69	0.93569549	1.72848355
1.7	0.93425331	1.72750742
1.71	0.93282674	1.72654062
1.72	0.93141553	1.72558303
1.73	0.93001945	1.72463451
1.74	0.92863826	1.72369496
1.75	0.92727173	1.72276425
1.76	0.92591964	1.72184225
1.77	0.92458177	1.72092885
1.78	0.9232579	1.72002393
1.79	0.92194781	1.71912738
1.8	0.92065131	1.7182391
1.81	0.91936818	1.71735898
1.82	0.91809823	1.71648686
1.83	0.91684126	1.7156227
1.84	0.91559708	1.71476636
1.85	0.9143655	1.71391776
1.86	0.91314633	1.71307678
1.87	0.9119394	1.71224334
1.88	0.91074452	1.71141732
1.89	0.90956152	1.71059864
1.9	0.90839023	1.70978719
1.91	0.90723048	1.7089829
1.92	0.9060821	1.70818566
1.93	0.90494494	1.70739539
1.94	0.90381883	1.706612
1.95	0.90270362	1.7058354
1.96	0.90159916	1.70506551
1.97	0.90050529	1.70430223
1.98	0.89942186	1.7035455
1.99	0.89834874	1.70279523
2	0.89728578	1.70205133

Evaluate the effects of increasing size at entry from 50 to 100 cm in 10 cm increments on the yield and SSB of an idealized roundfish being harvested by both trawls and gillnets. For trawls, assume the steepness of the LCDF curve is 0.33. For gillnets, assume the standard deviation of the NPDF is 5. The specifications for the idealized roundfish used in this analysis are:

L_∞ =	100
W_∞ =	10
K =	0.2
a =	0.00001
b =	3
$\alpha 1$ =	1
$\beta 1$ =	3
M =	0.2

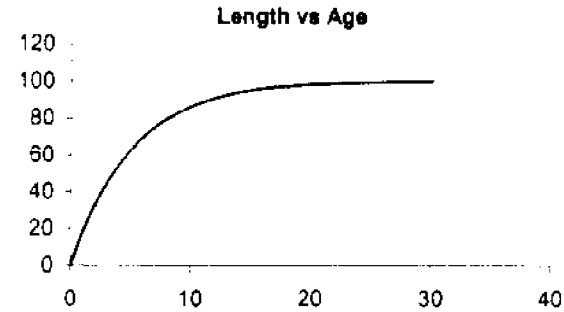
Characteristics of the roundfish can be determined using the following equations.

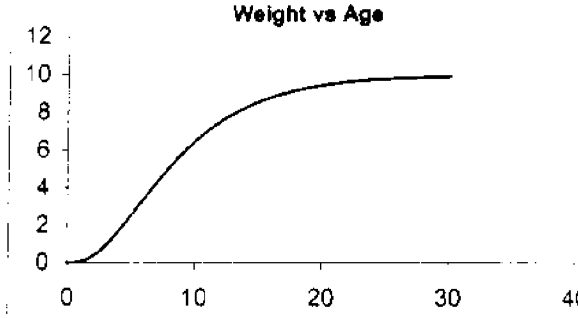
Length	$L_t = L_\infty\left(1 - e^{-Kt}\right)$
Weight	$W_t = aL_t^b$
Maturity	$P_t = \left(1 + e^{-\alpha 1 * (t - \beta 1)}\right)^{-1}$
Number	$N_t = N_0 * e^{-Mt}$

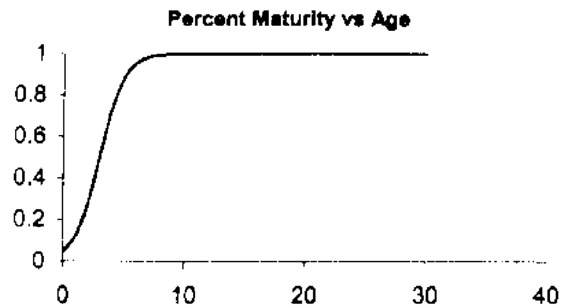
Based on these values, the characteristics of the individuals and the cohort of idealized roundfish are shown in the figures below.

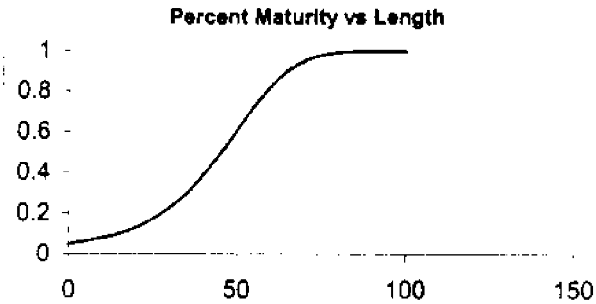
Age	Length	Weight	Maturity	Number (unfished)	Biomass	Spawning biomass
0	0	0	0.047425873	1000	0	0
1	18.12692469	0.059562428	0.119202922	818.7307531	48.7655914	5.813000984
2	32.9679954	0.358325423	0.268941421	670.320046	240.192714	64.59776998
3	45.11883639	0.918488392	0.5	548.8116361	504.077117	252.0385587
4	55.06710359	1.669847083	0.731058579	449.3289641	750.31066	548.5210448
5	63.21205588	2.525804578	0.880797078	367.8794412	929.191577	818.4292257
6	69.88057881	3.412475017	0.952574127	301.1942119	1027.81772	979.0725704
7	75.34030361	4.276437192	0.98201379	246.5969639	1054.55643	1035.588955
8	79.8103482	5.083673109	0.993307149	201.896518	1026.3759	1019.506518
9	83.47011118	5.815579217	0.997527377	165.2988882	961.308779	958.9318246
10	86.46647168	6.464623148	0.999088949	135.3352832	874.891605	874.0945337
11	88.91968416	7.030621766	0.99966465	110.8031584	779.015097	778.7538541
12	90.92820467	7.517887955	0.999876605	90.71795329	682.007408	681.9232523
13	92.57264218	7.933192236	0.999954602	74.27357821	589.226574	589.1998244
14	93.91899374	8.284385359	0.999983299	60.81006263	503.773993	503.7655788
15	95.02129316	8.579516416	0.999993856	49.78706837	427.14897	427.1463459
16	95.9237796	8.826303311	0.99999774	40.76220398	359.779576	359.7787627
17	96.662673	9.031843452	0.999999168	33.37326996	301.42215	301.4218991
18	97.26762776	9.202481906	0.999999694	27.32372245	251.446061	251.4459845
19	97.76292281	9.343778432	0.999999887	22.37077186	209.027536	209.0275121
20	98.16843611	9.46053327	0.999999959	18.31563889	173.275711	173.2757039
21	98.50044232	9.556844995	0.999999985	14.99557682	143.310403	143.3104011
22	98.77226601	9.636183289	0.999999994	12.2773399	118.306698	118.3066969
23	98.99481643	9.701465953	0.999999998	10.05183574	97.5175422	97.51754204
24	99.1770253	9.755133877	0.999999999	8.229747049	80.2822842	80.28228417
25	99.3262053	9.799220529	1	6.737946999	66.0266286	66.02662854
26	99.44834356	9.835414363	1	5.516564421	54.2576969	54.25769693
27	99.54834191	9.865113635	1	4.516580943	44.5565842	44.55658424
28	99.63021363	9.889473809	1	3.697863716	36.5699264	36.56992637
29	99.69724453	9.909448063	1	3.027554745	30.0013965	30.00139651
30	99.75212478	9.925821609	1	2.478752177	24.6036519	24.60365192

An individual idealized roundfish reaches an asymptotic length and weight of 100 cm and 10 kg respectively. Maturation is assumed to occur rapidly, with 50 percent of the cohort mature at an age of 3 years and a length of about 45 cm. Based on an initial cohort of 1000, the number of individuals in the unfished cohort is reduced to about 5 percent of the initial number by the age of 16 years, although the model is extended to an age of 30 years when only a single fish remains. Biomass of the cohort peaks at an age of 6.3 years and an individual fish length of 75 cm.

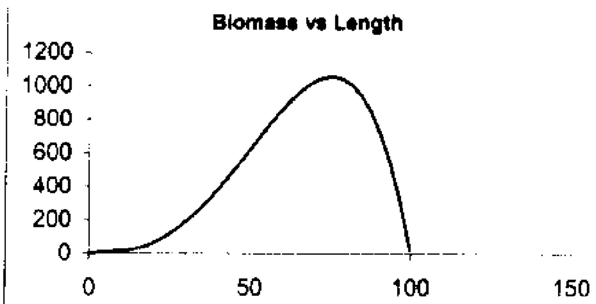

Length vs Age
120
100
80
60
40
20
0
0
10
20
30
40

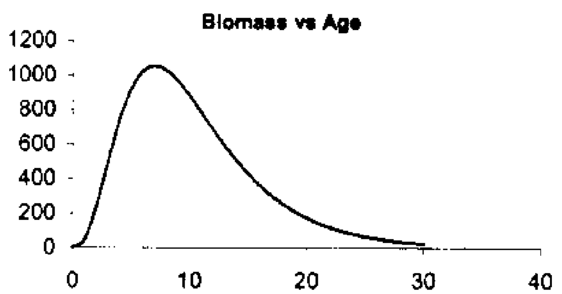

Weight vs Age
12
10
8
6
4
2
0
0
10
20
30
40

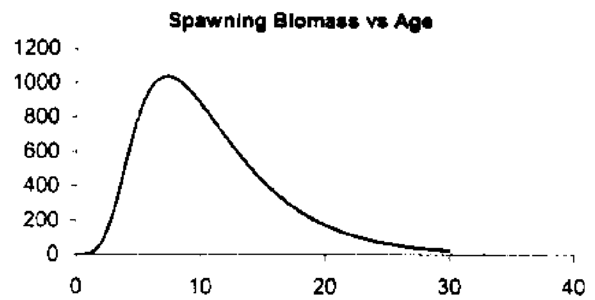

Percent Maturity vs Age
1
0.8
0.6
0.4
0.2
0
0
10
20
30
40


Percent Maturity vs Length
1
0.8
0.6
0.4
0.2
0
0
50
100
150


Numbers vs Age
1200
1000
800
600
400
200
0
0
10
20
30
40

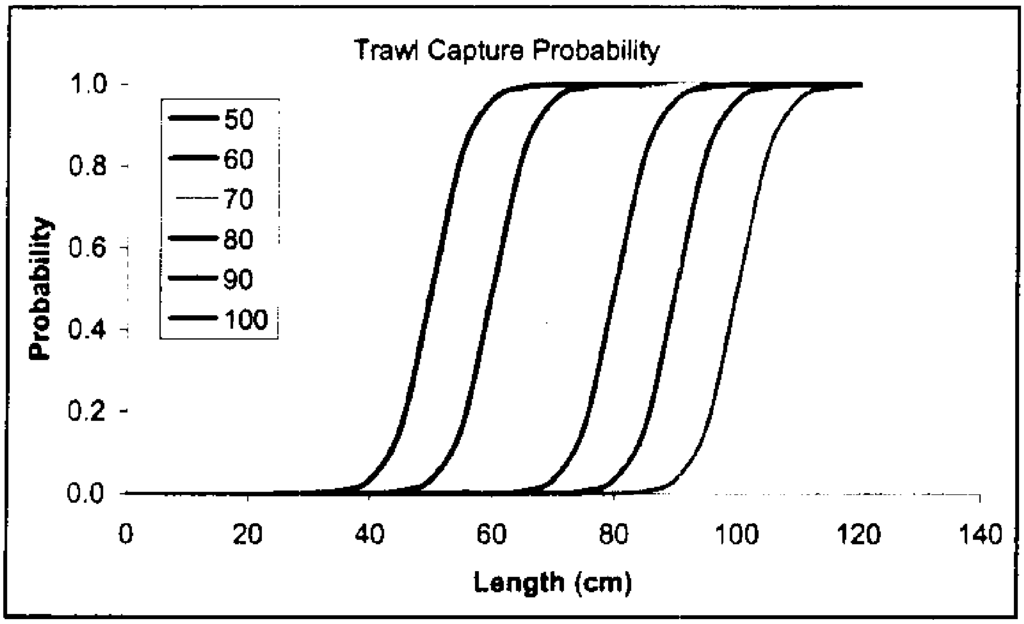

Biomass vs Length
1200
1000
800
600
400
200
0
0
50
100
150


Biomass vs Age
1200
1000
800
600
400
200
0
0
10
20
30
40

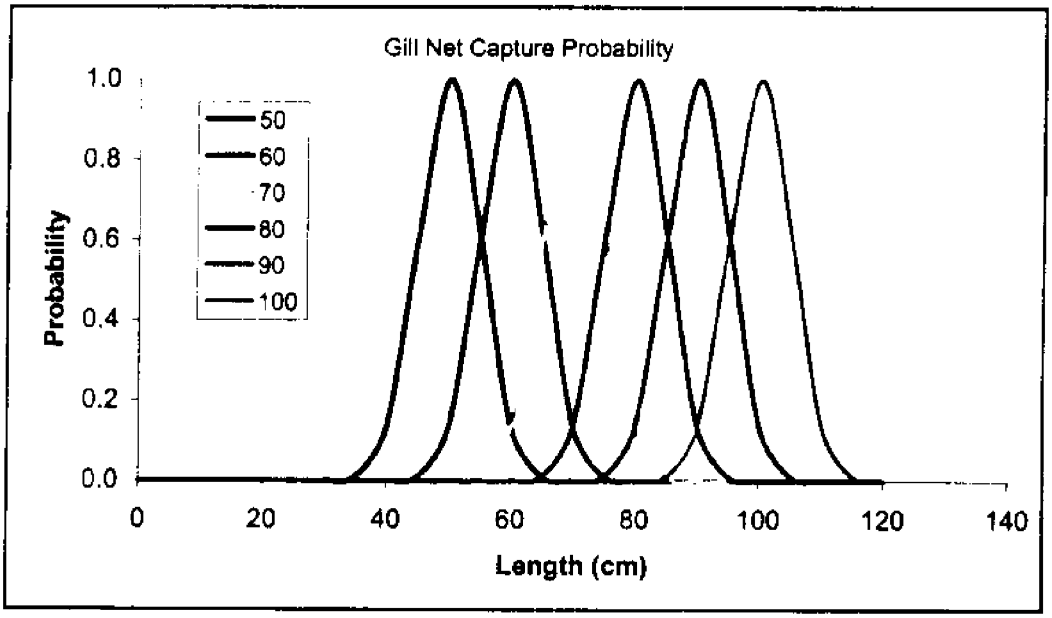

Spawning Biomass vs Age
1200
1000
800
600
400
200
0
0
10
20
30
40

The LCDF and NPDF for size selection are shown in the figures below. The L_{50}s for the LCDF ranged from 50 to 100 cm and a representative steepness of 0.33 is specified. The L_{opt}s ranged from 50 to100 cm, and a representative standard deviation of 5 is specified.

	PL_L for L_{50}s					
Length	50	60	70	80	90	100
0	6.8256E-08	2.5175E-09	9.28533E-11	3.42472E-12	1.2631E-13	4.65889E-15
5	3.55408E-07	1.31086E-08	4.83485E-10	1.78325E-11	6.5772E-13	2.42587E-14
10	1.8506E-06	6.8256E-08	2.5175E-09	9.28533E-11	3.4247E-12	1.26315E-13
15	9.63595E-06	3.55408E-07	1.31086E-08	4.83485E-10	1.7832E-11	6.57718E-13
20	5.01722E-05	1.8506E-06	6.8256E-08	2.5175E-09	9.2853E-11	3.42472E-12
25	0.00026119	9.63595E-06	3.55408E-07	1.31086E-08	4.8349E-10	1.78325E-11
30	0.00135852	5.01722E-05	1.8506E-06	6.8256E-08	2.5175E-09	9.28533E-11
35	0.007033587	0.00026119	9.63595E-06	3.55408E-07	1.3109E-08	4.83485E-10
40	0.035571189	0.00135852	5.01722E-05	1.8506E-06	6.8256E-08	2.5175E-09
45	0.16110895	0.007033587	0.00026119	9.63595E-06	3.5541E-07	1.31086E-08
50	0.5	0.035571189	0.00135852	5.01722E-05	1.8506E-06	6.8256E-08
55	0.83889105	0.16110895	0.007033587	0.00026119	9.636E-06	3.55408E-07
60	0.964428811	0.5	0.035571189	0.00135852	5.0172E-05	1.8506E-06
65	0.992966413	0.83889105	0.16110895	0.007033587	0.00026119	9.63595E-06
70	0.99864148	0.964428811	0.5	0.035571189	0.00135852	5.01722E-05
75	0.99973881	0.992966413	0.83889105	0.16110895	0.00703359	0.00026119
80	0.999949828	0.99864148	0.964428811	0.5	0.03557119	0.00135852
85	0.999990364	0.99973881	0.992966413	0.83889105	0.16110895	0.007033587
90	0.999998149	0.999949828	0.99864148	0.964428811	0.5	0.035571189
95	0.999999645	0.999990364	0.99973881	0.992966413	0.83889105	0.16110895
100	0.999999932	0.999998149	0.999949828	0.99864148	0.96442881	0.5
105	0.999999987	0.999999645	0.999990364	0.99973881	0.99296641	0.83889105
110	0.999999997	0.999999932	0.999998149	0.999949828	0.99864148	0.964428811
115	1	0.999999987	0.999999645	0.999990364	0.99973881	0.992966413
120	1	0.999999997	0.999999932	0.999998149	0.99994983	0.99864148



	PN_L for L_{opt}s					
Length	50	60	70	80	90	100
0	1.92875E-22	5.38019E-32	2.74879E-43	2.57221E-56	4.4085E-71	1.3839E-87
5	2.57676E-18	5.31109E-27	2.00501E-37	1.38634E-49	1.7557E-63	4.07236E-79
10	1.26642E-14	1.92875E-22	5.38019E-32	2.74879E-43	2.5722E-56	4.40863E-71
15	2.28973E-11	2.57676E-18	5.31109E-27	2.00501E-37	1.3863E-49	1.75569E-63
20	1.523E-08	1.26642E-14	1.92875E-22	5.38019E-32	2.7488E-43	2.57221E-56
25	3.72665E-06	2.28973E-11	2.57676E-18	5.31109E-27	2.005E-37	1.38634E-49
30	0.000335463	1.523E-08	1.26642E-14	1.92875E-22	5.3802E-32	2.74879E-43
35	0.011108997	3.72665E-06	2.28973E-11	2.57676E-18	5.3111E-27	2.00501E-37
40	0.135335283	0.000335463	1.523E-08	1.26642E-14	1.9287E-22	5.38019E-32
45	0.60653066	0.011108997	3.72665E-06	2.28973E-11	2.5768E-18	5.31109E-27
50	1	0.135335283	0.000335463	1.523E-08	1.2664E-14	1.92875E-22
55	0.60653066	0.60653066	0.011108997	3.72665E-06	2.2897E-11	2.57676E-18
60	0.135335283	1	0.135335283	0.000335463	1.523E-08	1.26642E-14
65	0.011108997	0.60653066	0.60653066	0.011108997	3.7267E-06	2.28973E-11
70	0.000335463	0.135335283	1	0.135335283	0.00033546	1.523E-08
75	3.72665E-06	0.011108997	0.60653066	0.60653066	0.011109	3.72665E-06
80	1.523E-08	0.000335463	0.135335283	1	0.13533528	0.000335463
85	2.28973E-11	3.72665E-06	0.011108997	0.60653066	0.60653066	0.011108997
90	1.26642E-14	1.523E-08	0.000335463	0.135335283	1	0.135335283
95	2.57676E-18	2.28973E-11	3.72665E-06	0.011108997	0.60653066	0.60653066
100	1.92875E-22	1.26642E-14	1.523E-08	0.000335463	0.13533528	1
105	5.31109E-27	2.57676E-18	2.28973E-11	3.72665E-06	0.011109	0.60653066
110	5.38019E-32	1.92875E-22	1.26642E-14	1.523E-08	0.00033546	0.135335283
115	2.00501E-37	5.31109E-27	2.57676E-18	2.28973E-11	3.7267E-06	0.011108997
120	2.74879E-43	5.38019E-32	1.92875E-22	1.26642E-14	1.523E-08	0.000335463



Use the spreadsheet program in worksheet Example 2 - cont'd.

Insert parameter values for the roundfish in the appropriate places.

Run the program for each value of L_{50} and L_{opt} (50, 60, 70, 80, 90, and 100 cm) using F values of 0.1, 0.2, 0.3, 0.4, 0.5, 1.0, 1.5, 2.0, 2.5, 3.0, 3.5, and 4.0.

For each value of F, copy the YPR/SSBPR row (cells J26 to M26) and Paste Special as Values in the matching F row below (cells D65 to D77 for 50 cm; D81 to D93 for 60 cm; D97 to D109 for 70 cm; K65 to K77 for 80 cm; K81 to K93 for 90 cm; K97 to K109 for 100 cm).

Yield Per Recruit Model

standard deviation = 5
steepness = 0.33

Example: L_{50} and L_{opt} = 100 cm and F = 0.0

Age	L (cm)	W (kg)	P	N_t	Biomass	SSB	Gill	Trawl	Y_{Gill}	SSB_{Gill}	Y_{Trawl}	SSB_{Trawl}	N_{Gill}	N_{Trawl}
0	0.0	0.0	0.05	1000	0	0	0.000	0.000	0	0	0	0	1000	1000
1	18.1	0.1	0.12	819	49	6	0.000	0.000	0	6	0	6	819	819
2	33.0	0.4	0.27	670	240	65	0.000	0.000	0	65	0	65	670	670
3	45.1	0.9	0.50	549	504	252	0.000	0.000	0	252	0	252	549	549
4	55.1	1.7	0.73	449	750	549	0.000	0.000	0	549	0	549	449	449
5	63.2	2.5	0.86	368	929	818	0.000	0.000	0	818	0	818	368	368
6	69.9	3.4	0.95	301	1028	979	0.000	0.000	0	979	0	979	301	301
7	75.3	4.3	0.98	247	1055	1036	0.000	0.000	0	1036	0	1036	247	247
8	79.8	5.1	0.99	202	1026	1020	0.000	0.001	0	1020	0	1020	202	202
9	83.5	5.8	1.00	165	961	959	0.004	0.004	0	959	0	959	165	165
10	86.5	6.5	1.00	135	875	874	0.028	0.011	0	874	0	874	135	135
11	88.9	7.0	1.00	111	779	779	0.086	0.025	0	779	0	779	111	111
12	90.9	7.5	1.00	91	682	682	0.193	0.048	0	682	0	682	91	91
13	92.6	7.9	1.00	74	589	589	0.332	0.079	0	589	0	589	74	74
14	93.9	8.3	1.00	61	504	504	0.477	0.118	0	504	0	504	61	61
15	95.0	8.6	1.00	50	427	427	0.609	0.162	0	427	0	427	50	50
16	95.9	8.8	1.00	41	360	360	0.717	0.207	0	360	0	360	41	41
17	96.7	9.0	1.00	33	301	301	0.800	0.249	0	301	0	301	33	33
18	97.3	9.2	1.00	27	251	251	0.861	0.289	0	251	0	251	27	27
19	97.8	9.3	1.00	22	209	209	0.905	0.323	0	209	0	209	22	22
20	98.2	9.5	1.00	18	173	173	0.935	0.353	0	173	0	173	18	18
							Yield/SSB		**0**	**10832**	**0**	**10832**		
							YPR/SSBPR		**0.000**	**10.832**	**0.000**	**10.832**		

Maturity

$$P_t = (1 + e^{(-\alpha 1 * (t - \beta 1))})^{-1}$$

$\alpha 1$	1
$\beta 1$	3

Operators:

[redacted]	**Fishing Mortality**
100	L_{50} **and** L_{opt}
100	L_∞
0.2	K
0.2	M
1E-05	a
3	b

50 cm

F	Y_{Gill}	SSB_{Gill}	Y_{Trawl}	SSB_{Trawl}
0	0	10.832	0	10.832333
0.1	0.0769	9.628	0.731665	6.510113
0.2	0.1461	8.5634	1.001873	4.451198
0.3	0.2061	7.6222	1.116154	3.3335448
0.4	0.2608	6.7899	1.169429	2.6601963
0.5	0.311	6.054	1.196	2.221
1	0.514	3.468	1.215	1.294
1.5	0.656	2.063	1.202	0.989
2	0.754	1.295	1.187	0.840
2.5	0.821	0.873	1.174	0.751
3	0.868	0.638	1.162	0.691
3.5	0.900	0.507	1.152	0.646
4	0.923	0.432	1.141	0.611

80 cm

F	Y_{Gill}	SSB_{Gill}	Y_{Trawl}	SSB_{Trawl}
0	0	10.832	0	10.83233
0.1	0.31226	9.1979	0.5035511	8.901229
0.2	0.55162	7.9712	0.7646972	7.77362
0.3	0.73743	7.0449	0.917549	7.056743
0.4	0.88237	6.3407	1.0166577	6.566159
0.5	0.996	5.801	1.086	6.210
1	1.293	4.434	1.260	5.274
1.5	1.397	3.946	1.334	4.840
2	1.443	3.709	1.376	4.574
2.5	1.467	3.562	1.404	4.386
3	1.482	3.457	1.424	4.242
3.5	1.492	3.376	1.439	4.127
4	1.499	3.310	1.451	4.031

60 cm

F	Y_{Gill}	SSB_{Gill}	Y_{Trawl}	SSB_{Trawl}
0	0	10.832	0	10.832333
0.1	0.1356	9.4504	0.705569	7.1656401
0.2	0.2491	8.2656	0.999468	5.3247729
0.3	0.3493	7.2492	1.142221	4.2816189
0.4	0.4391	6.3772	1.220702	3.6295501
0.5	0.520	5.629	1.268	3.190
1	0.821	3.198	1.351	2.195
1.5	0.998	2.045	1.369	1.825
2	1.099	1.486	1.372	1.629
2.5	1.159	1.206	1.371	1.504
3	1.194	1.059	1.368	1.415
3.5	1.215	0.977	1.364	1.347
4	1.229	0.929	1.359	1.293

90 cm

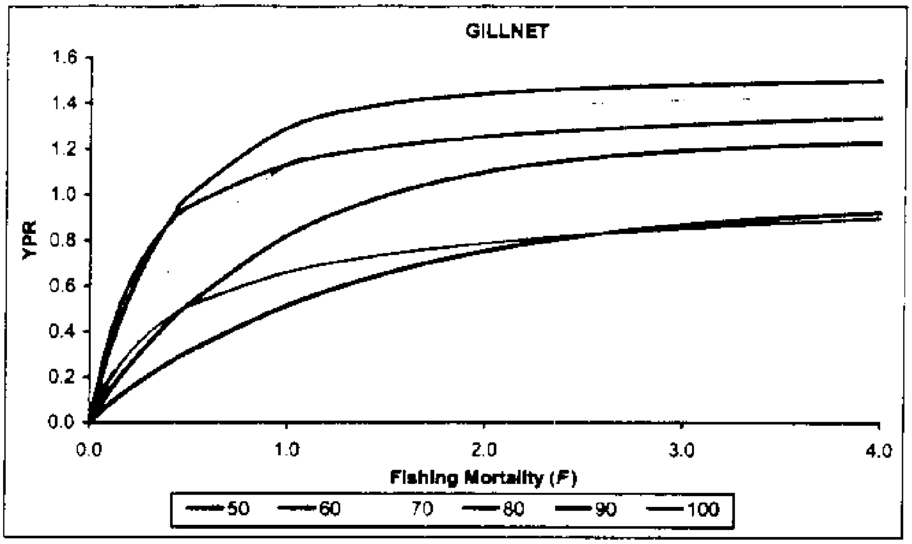
F	Y_{Gill}	SSB_{Gill}	Y_{Trawl}	SSB_{Trawl}
0	0	10.832	0	10.83233
0.1	0.37696	9.4277	0.2937039	9.940451
0.2	0.61377	8.5154	0.4763017	9.32049
0.3	0.7687	7.8999	0.5977309	8.870427
0.4	0.87445	7.4678	0.6835852	8.530316
0.5	0.950	7.152	0.748	8.264
1	1.134	6.345	0.925	7.475
1.5	1.210	5.985	1.015	7.057
2	1.256	5.763	1.072	6.781
2.5	1.285	5.607	1.113	6.577
3	1.307	5.487	1.144	6.416
3.5	1.324	5.391	1.169	6.284
4	1.338	5.311	1.190	6.172

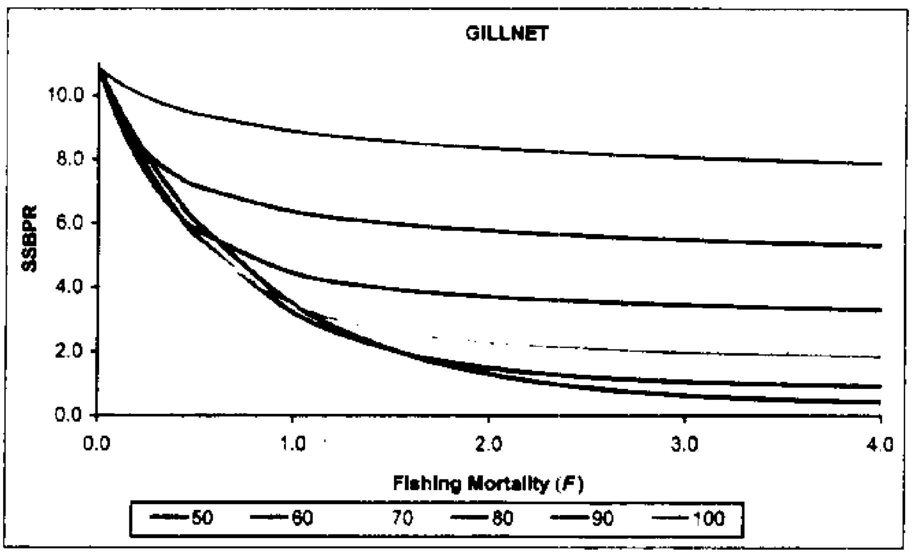
70 cm

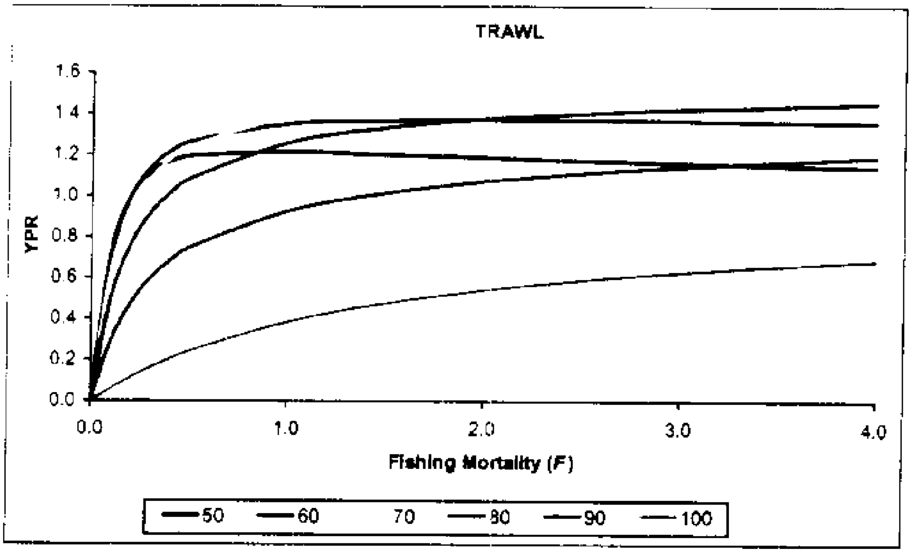
F	Y_{Gill}	SSB_{Gill}	Y_{Trawl}	SSB_{Trawl}
0	0	10.832	0	10.832333
0.1	0.2146	9.2933	0.63542	7.9612165
0.2	0.3923	8.0344	0.930038	6.4274393
0.3	0.5435	7.0033	1.066049	5.5154643
0.4	0.6729	6.1577	1.183494	4.9226876
0.5	0.783	5.463	1.247	4.509
1	1.135	3.439	1.385	3.507
1.5	1.293	2.627	1.433	3.092
2	1.365	2.267	1.456	2.854
2.5	1.400	2.084	1.469	2.694
3	1.417	1.976	1.477	2.577
3.5	1.427	1.902	1.483	2.485
4	1.432	1.848	1.486	2.411

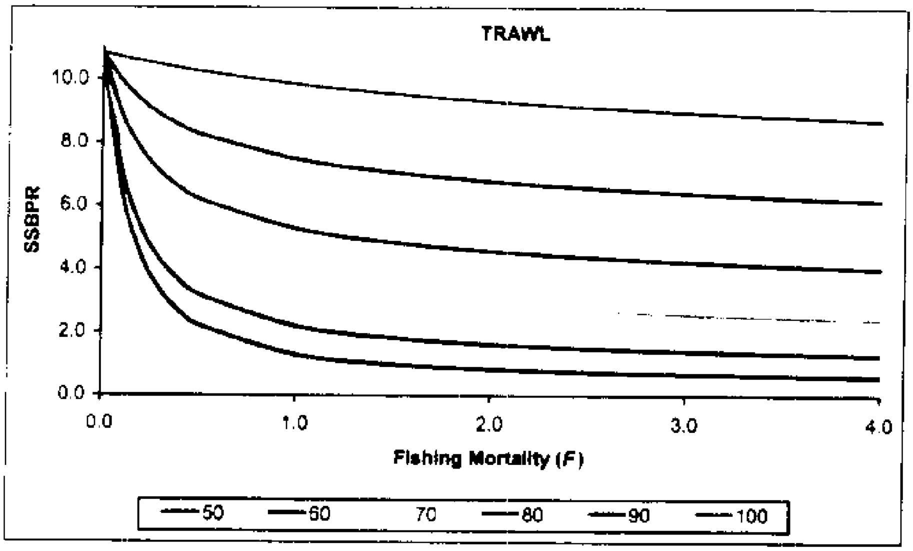
100 cm

F	Y_{Gill}	SSB_{Gill}	Y_{Trawl}	SSB_{Trawl}
0	0	10.832	0	10.83233
0.1	0.18092	10.397	0.061546	10.69679
0.2	0.30559	10.061	0.1156111	10.57169
0.3	0.3952	9.7975	0.1634606	10.45596
0.4	0.46201	9.5852	0.2060649	10.34865
0.5	0.514	9.411	0.244	10.249
1	0.661	8.858	0.386	9.841
1.5	0.737	8.548	0.478	9.540
2	0.787	8.339	0.542	9.308
2.5	0.824	8.184	0.590	9.122
3	0.853	8.061	0.628	8.967
3.5	0.877	7.960	0.659	8.836
4	0.897	7.874	0.686	8.722









The YPR and SSBPR graphs can be transformed into isopleth diagrams by taking the YPRMAX and dividing by any other YPR value which will give a percentage. This will create a grid of data points of percentages that can then be contoured.

Consider the following time series of catch and effort for a trawl fishery. Determine the MEY for this fishery by fitting the Fox model. Recommend a level of effort to achieve MEY.

Year	Catch	Effort
1988	50	623
1989	49	628
1990	47.5	520
1991	45	513
1992	51	661
1993	56	919
1994	66	1158
1995	58	1970
1996	52	1317

FOX MODEL

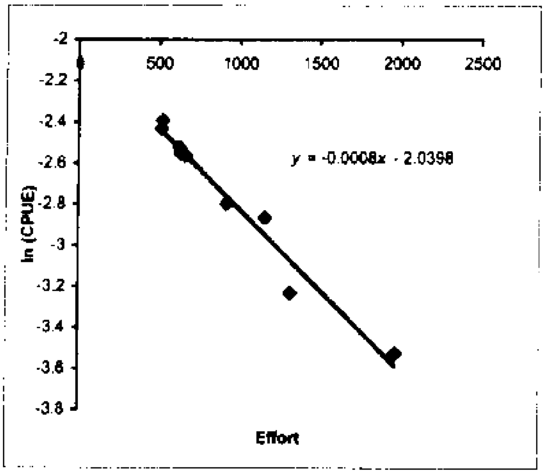
Linear Regression

Year	Catch	Effort	CPUE	ln (CPUE)
1988	50	623	0.0803	-2.52252
1989	49	628	0.078	-2.55072
1990	47.5	520	0.0913	-2.3931
1991	45	513	0.0877	-2.43361
1992	51	661	0.0772	-2.56193
1993	56	919	0.0609	-2.79793
1994	66	1158	0.057	-2.86479
1995	58	1970	0.0294	-3.52535
1996	52	1317	0.0395	-3.23187

There are 2 methods to calculate the slope and *y*-intercept:

(1) Go to **Tools, Data Analysis, Regression**, select the input and output ranges, then click OK. The two numbers that are important are the X variable 1 which is the slope and intercept which is the *y*-intercept.

(2) Select the data series on the graph, right click, and choose **Add Trendline**. Choose "linear" for the **Type**. In the **Options** tab, click "display equation on chart."



From **Tools, Data Analysis, Regression**:

Regression Statistics	
Multiple R	0.98294686
R Square	0.96618452
Adjusted R Square	0.96135373
Standard Error	0.07617603
Observations	9

ANOVA

	df	*SS*	*MS*	*F*	*Significance F*
Regression	1	1.1605909	1.1606	200.0058	2.09715E-06
Residual	7	0.04061951	0.0058		
Total	8	1.20121041			

	Coefficients	*Standard Erro*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*	*Lower 95.0%*	*Upper 95.0%*
Intercept	-2.039817	0.05719768	-35.66	3.54E-09	-2.175067971	-1.90457	-2.175067971	-1.904566141
X Variable 1	-0.000785	5.5515E-05	-14.14	2.1E-06	-0.000916381	-0.00065	-0.000916381	-0.000653838

				Before **Solver**		After **Solver**	
Year	Catch	Effort	CPUE	$Catch_{pred}$	$(Y_{obs}-Y_{pred})^2$	$Catch_{pred}$	$(Y_{obs}-Y_{pred})^2$
1988	50	623	0.0803	49.68019	0.102276255	49.49646	0.253552518
1989	49	628	0.078	49.88271	0.779175349	49.69921	0.488901314
1990	47.5	520	0.0913	44.95918	6.455784376	44.77456	7.428012778
1991	45	513	0.0877	44.59839	0.161293202	44.41402	0.343375463
1992	51	661	0.0772	51.1611	0.025953514	50.97959	0.000416453
1993	56	919	0.0609	58.08768	4.358400546	57.941	3.767498895
1994	66	1158	0.057	60.67162	28.39168623	60.57596	29.42026297
1995	58	1970	0.0294	54.56029	11.8316067	54.65043	11.21963268
1996	52	1317	0.0395	60.90441	79.28855109	60.84684	78.26659198
					131.3947273		131.1882451

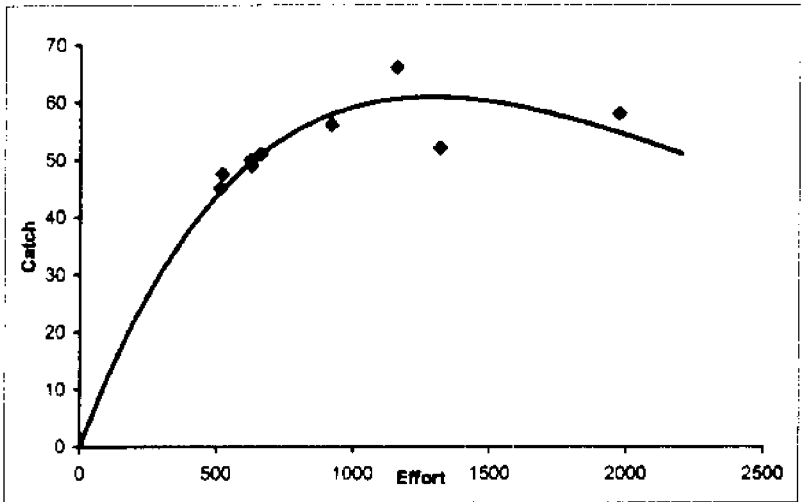
Non-linear regression can be calculated using the values obtained from the linear regression as initial parameter values, and then using **Solver** to adjust those values to

NON-LINEAR REGRESSION

	Before	After
c	-2.039817	-2.0459994
d	0.0007851	0.00078113

To graph the model curve, create a series of *x* values (effort) and calculate *y* (catch) using the generalized Fox

x	*y*
0	0
100	11.953899
200	22.111356
300	30.674851
400	37.826574
500	43.730339
600	48.533309
700	52.367581
800	55.351612
900	57.59153
1000	59.182311
1100	60.208863
1200	60.746995
1300	60.864313
1400	60.621018
1500	60.070641
1600	59.260703
1700	58.233321
1800	57.025744
1900	55.670852
2000	54.197603
2100	52.631433
2200	50.994625



Calculate $f_{MEY} = 1/d$ and $Y_{MEY} = (1/d)*e^{(c-1)}$.

f_{MEY}	**1280.19**
Y_{MEY}	**60.87**

FOX

Microsoft Excel 8.0a Answer Report
Worksheet: [ProductionExamples.xls]Fox (2)
Report Created: 11/23/98 11:32:31 AM

Target Cell (Min)

Cell	Name	Original Value	Final Value
G13	$(Y_{obs}-Y_{pred})^2$	131.3946451	131.188245

Adjustable Cells

Cell	Name	Original Value	Final Value
F19	*c*	-2.039824532	-2.04599982
F20	*d*	0.000785099	0.00078113

Constraints
NONE

Consider the following time series of catch and effort for a trawl fishery. Determine the MEY for this fishery by fitting the Schaefer model. Recommend a level of effort to achieve MEY.

Year	Catch	Effort
1988	50	623
1989	49	628
1990	47.5	520
1991	45	513
1992	51	661
1993	56	919
1994	66	1158
1995	58	1970
1996	52	1317

SCHAEFER MODEL

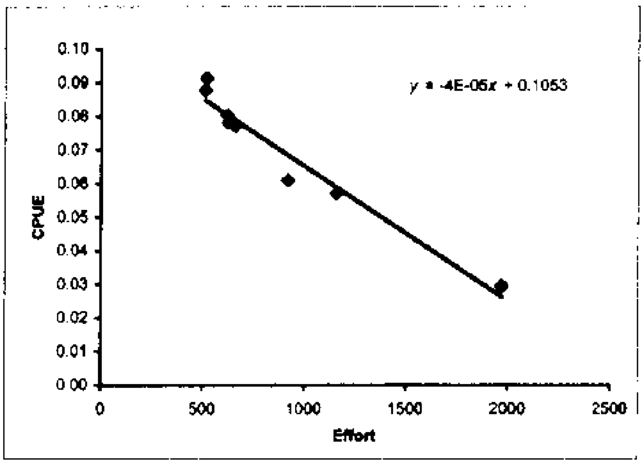
Linear Regression

Year	Catch	Effort	CPUE
1988	50	623	0.08025682
1989	49	628	0.07802548
1990	47.5	520	0.09134615
1991	45	513	0.0877193
1992	51	661	0.07715582
1993	56	919	0.0609358
1994	66	1158	0.05699482
1995	58	1970	0.02944162
1996	52	1317	0.03948368

There are 2 methods to calculate the slope and *y*-intercept:

(1) Go to **Tools, Data Analysis, Regression**, select the input and output ranges, then click OK. The two numbers that are important are the X variable 1 which is the slope and intercept which is the *y*-intercept.

(2) Select the data series on the graph, right click, and choose **Add Trendline**. Choose "linear" for the **Type**. In the **Options** tab, click "display equation on chart."



From **Tools, Data Analysis, Regression**:

Regression Statistics	
Multiple R	0.96323494
R Square	0.92782155
Adjusted R Square	0.91751034
Standard Error	0.00619904
Observations	9

ANOVA

	df	*SS*	*MS*	*F*	*Significance F*
Regression	1	0.00345783	0.00345783	89.9818487	3.0265E-05
Residual	7	0.000269	3.8428E-05		
Total	8	0.00372683			

	Coefficients	*Standard Erro.*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*	*Lower 95.0%*	*Upper 95.0%*
Intercept	0.10638159	0.00465462	22.8550379	7.7788E-08	0.09537516	0.11738802	0.09537516	0.117388019
X Variable 1	-4.285E-05	4.5177E-06	-9.4858763	3.0265E-05	-5.354E-05	-3.217E-05	-5.354E-05	-3.21715E-05

Non-linear Regression $Y = af - bf^2$

				Before Solver		After Solver	
Year	Catch	Effort	CPUE	$Catch_{pred}$	$(Y_{obs}-Y_{pred})^2$	$Catch_{pred}$	$(Y_{obs}-Y_{pred})^2$
1988	50	623	0.08025682	82.9086556	1082.97961	46.8425821	9.96928776
1989	49	628	0.07802548	83.7086161	1204.68803	47.1057779	3.58807753
1990	47.5	520	0.09134615	66.9061798	376.599814	41.0213204	41.9732896
1991	45	513	0.0877193	65.8516305	434.790497	40.5980523	19.3771437
1992	51	661	0.07715582	89.042094	1447.20092	48.7978424	4.84949789
1993	56	919	0.0609358	133.957595	6077.38667	59.3308866	11.0948054
1994	66	1158	0.05699482	180.655706	13145.931	64.8231022	1.38508833
1995	58	1970	0.02944162	375.884264	101050.405	52.8393972	26.6318211
1996	52	1317	0.03948368	214.434544	26384.9811	66.2046537	201.772185
					151204.963		320.641197

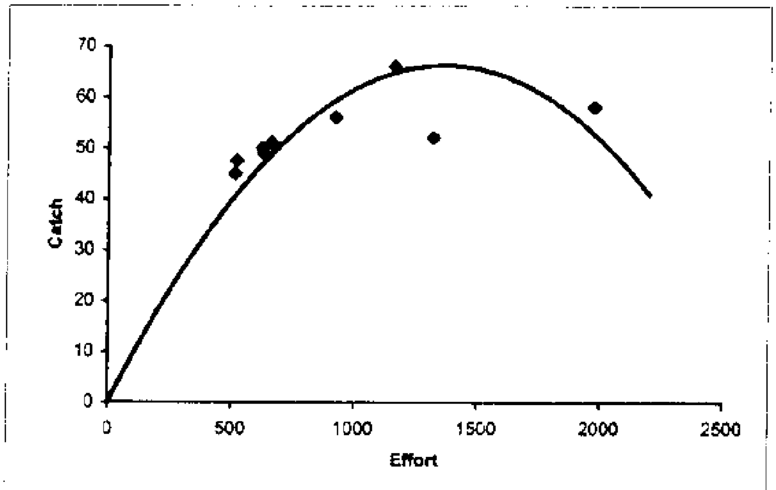
Non-linear regression can be calculated using the values obtained from the linear regression as initial parameter values, and then using **Solver** to adjust those values to provide the best fitting

NON-LINEAR REGRESSION

	Before	After
a	0.10638159	0.09755879
b	-4.285E-05	3.5907E-05

To graph the model curve, create a series of x values (effort) and calculate y (catch) using the generalized Schaefer Model $Y = af$ -

x	y
0	0
100	9.40
200	18.08
300	26.04
400	33.28
500	39.80
600	45.61
700	50.70
800	55.07
900	58.72
1000	61.65
1100	63.87
1200	65.36
1300	66.14
1400	66.20
1500	65.55
1600	64.17
1700	62.08
1800	59.27
1900	55.74
2000	51.49
2100	46.52
2200	40.84



Calculate $f_{MEY} = a/2b$ and $Y_{MEY} = a^2/4b$

f_{MEY}	**1358.49**
Y_{MEY}	**66.27**

SCHAEFER

Microsoft Excel 8.0a Answer Report
Worksheet: [ProductionExamples.xls]Schaefer
Report Created: 2/23/00 10:20:39 AM

Target Cell (Min)

Cell	Name	Original Value	Final Value
H68	$(Y_{obs}-Y_{pred})^2$	151204.9628	320.641197

Adjustable Cells

Cell	Name	Original Value	Final Value
C73	*a*	0.106381591	0.09755879
C74	*b*	-4.28541E-05	3.5907E-05

Constraints
NONE

Given the following stock recruitment data, solve for α and β using the Beverton-Holt model.

Year	Stock	Recruitment
1	8.8	7.1
2	7.4	6.4
3	4.5	6.4
4	13.2	7
5	14.6	7.7
6	7	7
7	3.1	5.4
8	7.7	6.1
9	10.7	6.8
10	8.6	6
11	15.4	6.2
12	2	3.5

Year	Stock	Recruitment	S/R
1	8.8	7.1	1.239
2	7.4	6.4	1.156
3	4.5	6.4	0.703
4	13.2	7	1.886
5	14.6	7.7	1.896
6	7	7	1.000
7	3.1	5.4	0.574
8	7.7	6.1	1.262
9	10.7	6.8	1.574
10	8.6	6	1.433
11	15.4	6.2	2.484
12	2	3.5	0.571

There are 2 methods to calculate the slope and y-intercept:

(1) Go to **Tools, Data Analysis, Regression**, select the input and output ranges, then click OK. The two numbers that are important are the X variable 1 which is the slope and intercept which is the y-intercept.

(2) Select the data series on the graph, right click, and choose **Add Trendline**. Choose "linear" for the **Type**. In the **Options** tab, click "display equation on chart."

LINEAR

From **Tools, Data Analysis, Regression**:

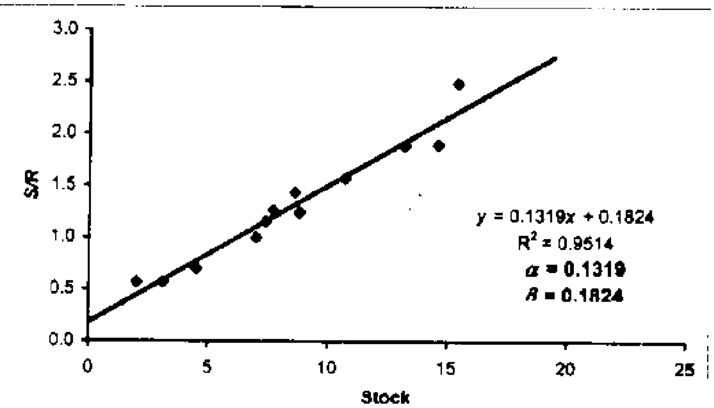
SUMMARY OUTPUT

Regression Statistics	
Multiple R	0.975402447
R Square	0.951409934
Adjusted R Square	0.946550927
Standard Error	0.134439227
Observations	12

ANOVA

	df	*SS*	*MS*	*F*	*Significance F*
Regression	1	3.538931899	3.5389319	195.803383	6.8049E-06
Residual	10	0.180739058	0.0180739		
Total	11	3.719670957			

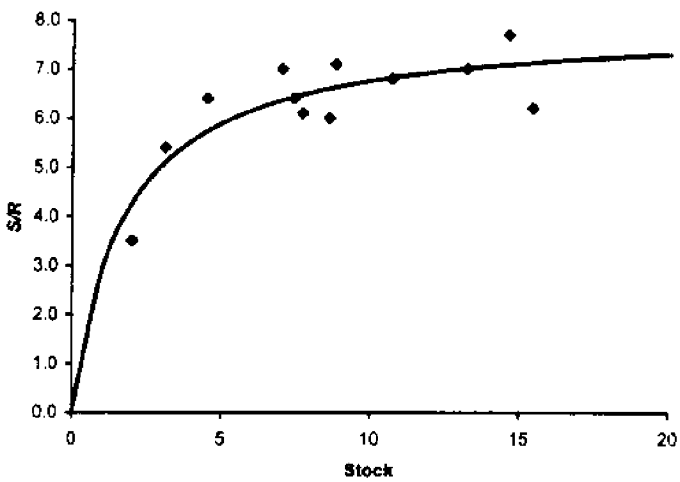
	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*	*Lower 95.0%*	*Upper 95.0%*
Intercept	0.182403751	0.08975906	2.0321486	0.0695544	-0.017591932	0.38239943	-0.0175919	0.382399435
X Variable 1	0.131944818	0.009429361	13.992976	6.8049E-06	0.11093489	0.15295475	0.11093489	0.152954746



NON-LINEAR

			Before Solver		After Solver	
Year	S	R_{obs}	R_{pred}	$(R_{obs}-R_{pred})^2$	R_{pred}	$(R_{obs}-R_{pred})^2$
1	8.8	7.1	6.551909	0.30040376	6.630564668	0.22036953
2	7.4	6.4	6.3877907	0.00014907	6.431922605	0.00101905
3	4.5	6.4	5.7993427	0.36078914	5.735844194	0.44110294
4	13.2	7	6.8625616	0.01888931	7.012124314	0.000147
5	14.6	7.7	6.9255363	0.59979409	7.090374663	0.37164305
6	7	7	6.3308311	0.44778695	6.363447961	0.4051985
7	3.1	5.4	5.2427743	0.02471993	5.099821784	0.09010696
8	7.7	6.1	6.427218	0.10707163	6.479461025	0.14399067
9	10.7	6.8	6.7138097	0.00742876	6.828502767	0.00081241
10	8.6	6	6.5312818	0.28226035	6.605487957	0.36661567
11	15.4	6.2	6.9568046	0.57275315	7.12934205	0.86367665
12	2	3.5	4.4822949	0.96490334	4.263958565	0.58363269
				3.68694949		3.48831511

	Before Solver	After Solver
α	0.1319	0.126196972
β	0.1624	0.216653767



To plot the Beverton-Holt model, create a series of stock (S) and solve for R using the Beverton-Holt equation and the parameter values obtained from **Solver**.

α	0.126196972
β	0.216653767

S	R
0	0
1	2.91672115
2	4.26395656
3	5.03994423
4	5.54445392
5	5.89874016
6	6.16120424
7	6.36344796
8	6.52406401
9	6.6547054
10	6.76304693
11	6.85434933
12	6.93233923
13	6.99973036
14	7.05854581
15	7.11032456
16	7.15625816
17	7.19728347
18	7.23414733
19	7.26745235
20	7.29769015

Microsoft Excel 8.0a Answer Report
Worksheet: [Chapter 10 - Stock Recruitment Examples.xls]B-H
Report Created: 4/4/00 2:55:21 PM

Target Cell (Min)

Cell	Name	Original Value	Final Value
G88	$(R_{obs}-R_{pred})^2$	3.686949491	3.48831511

Adjustable Cells

Cell	Name	Original Value	Final Value
C91	α After **Solver**	0.1319	0.126196972
C92	β After **Solver**	0.1824	0.216653767

Constraints
NONE

Given the following stock recruitment data, solve for α and β using the Ricker model.

Year	Stock	Recruitment
1	8.8	7.1
2	7.4	6.4
3	4.5	6.4
4	13.2	7
5	14.6	7.7
6	7	7
7	3.1	5.4
8	7.7	6.1
9	10.7	6.8
10	8.6	6
11	15.4	6.2
12	2	3.5

Year	Stock	Recruitment	ln(*R/S*)
1	8.8	7.1	-0.215
2	7.4	6.4	-0.145
3	4.5	6.4	0.352
4	13.2	7	-0.634
5	14.6	7.7	-0.640
6	7	7	0.000
7	3.1	5.4	0.555
8	7.7	6.1	-0.233
9	10.7	6.8	-0.453
10	8.6	6	-0.360
11	15.4	6.2	-0.910
12	2	3.5	0.560

There are 2 methods to calculate the slope and *y*-intercept:

(1) Go to **Tools, Data Analysis, Regression**, select the input and output ranges, then click OK. The two numbers that are important are the X variable 1 which is the slope and intercept which is the *y*-intercept.

(2) Select the data series on the graph, right click, and choose **Add Trendline**. Choose "linear" for the **Type**. In the **Options** tab, click "display equation on chart."

LINEAR

From **Tools, Data Analysis, Regression**:

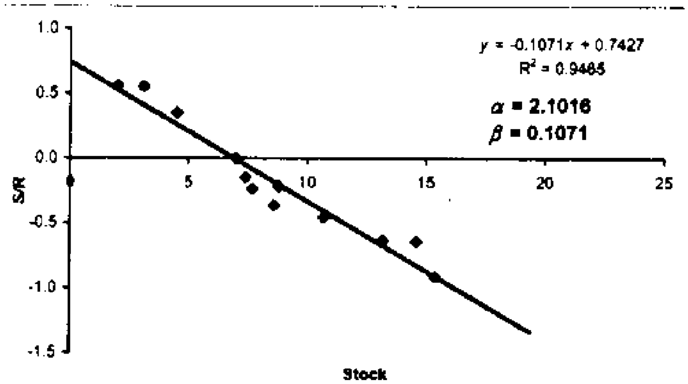
SUMMARY OUTPUT

Regression Statistics	
Multiple R	0.972866801
R Square	0.946469424
Adjusted R Square	0.941116366
Standard Error	0.114883895
Observations	12

ANOVA

	df	*SS*	*MS*	*F*	*Significance F*
Regression	1	2.33358151	2.333582	176.809123	1.1067E-07
Residual	10	0.131983094	0.013198		
Total	11	2.465564604			

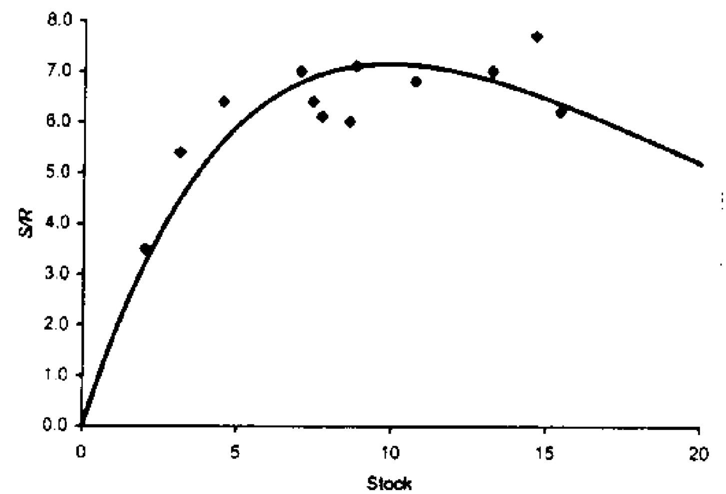
	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*	*Lower 95.0%*	*Upper 95.0%*
Intercept	0.742720114	0.076702839	9.683085	2.1336E-06	0.571815508	0.91362472	0.571815508	0.913624719
X Variable 1	-0.107143967	0.00805778	-13.297	1.1067E-07	-0.125097822	-0.0891901	-0.12509782	-0.08919011



NON-LINEAR

			Before Solver		After Solver	
Year	S	R_{obs}	R_{pred}	$(R_{obs}-R_{pred})^2$	R_{pred}	$(R_{obs}-R_{pred})^2$
1	8.8	7.1	7.206408	0.01132274	7.096062333	1.5505E-05
2	7.4	6.4	7.040217	0.4098775	6.878576576	0.22903554
3	4.5	6.4	5.840558	0.31297525	5.615042153	0.61615682
4	13.2	7	6.74766	0.06367544	6.809159182	0.03642022
5	14.6	7.7	6.424125	1.62785614	6.533415892	1.36091848
6	7	7	6.951164	0.00238497	6.776456294	0.04997179
7	3.1	5.4	4.674354	0.52656144	4.458967512	0.88554214
8	7.7	6.1	7.094	0.98803504	6.942713968	0.71016683
9	10.7	6.8	7.149002	0.12180207	7.114415723	0.09885725
10	8.6	6	7.195107	1.4282799	7.077046893	1.16003001
11	15.4	6.2	6.21973	0.00038927	6.353787411	0.02365057
12	2	3.5	3.392768	0.01149868	3.216669257	0.08027631
				5.50465843		5.25104347

	Before Solver	After Solver
α	2.1016	1.97044872
β	0.1071	0.101531021



To plot the Ricker model, create a series of stock (S) and solve for R using the Ricker equation and the parameter values obtained from **Solver**.

α	1.9704
β	0.1015

S	R
0	0
1	1.78021932
2	3.21678929
3	4.35946268
4	5.25159022
5	5.93089115
6	6.43013824
7	6.77776036
8	6.99837591
9	7.11326354
10	7.14077746
11	7.09671386
12	6.99463354
13	6.8461464
14	6.66116169
15	6.44810809
16	6.21412726
17	5.96524371
18	5.70651398
19	5.44215731
20	5.17567022

Microsoft Excel 8.0a Answer Report
Worksheet: [Chapter 10 - Stock Recruitment Examples.xls]Example 2
Report Created: 4/4/00 3:32:00 PM

Target Cell (Min)

Cell	Name	Original Value	Final Value
G89	$(R_{obs}-R_{pred})^2$	5.504658428	5.251043466

Adjustable Cells

Cell	Name	Original Value	Final Value
C92	α After **Solver**	2.1016	1.97044872
C93	β After **Solver**	0.1071	0.101531021

Constraints
NONE

ANSWERS TO EXERCISES

APPENDIX 3

ANSWERS TO EXERCISES

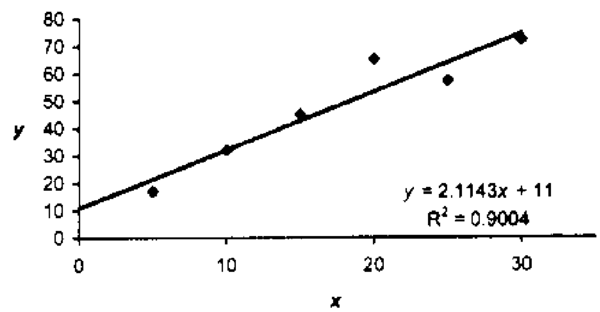
Given the following data points (x, y values), plot the points, fit the linear model and obtain the best estimates of the parameters m and b. On the same graph, plot the predicted model.

x	y
5	17
10	32
15	45
20	65
25	57
30	72

There are 2 methods to calculate the slope and y-intercept:

(1) Go to **Tools, Data Analysis, Regression**, select the input and output ranges, then click OK. The two numbers that are important are the **X variable 1** which is the slope and **intercept** which is the y-intercept.

(2) Select the data series on the graph, right click, and choose **Add Trendline**. Choose "linear" for the **Type**. In the **Options** tab, click "display equation on chart."



From Tools, **Data Analysis, Regression:**

SUMMARY OUTPUT

Regression Statistics	
Multiple R	0.948905128
R Square	0.900420942
Adjusted R Square	0.875526177
Standard Error	7.353327721
Observations	6

ANOVA

	df	*SS*	*MS*	*F*	*Significance F*
Regression	1	1955.714286	1955.714286	36.16908851	0.003849333
Residual	4	216.2857143	54.07142857		
Total	5	2172			

	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*	*Lower 95.0%*	*Upper 95.0%*
Intercept	11	6.845675561	1.606877304	0.183357897	-8.00640413	30.00640413	-8.00640413	30.00640413
X Variable 1	2.114285714	0.351556307	6.014074202	0.003849333	1.138206906	3.090364523	1.138206906	3.090364523

Calculate the following:

$10^3 = 1000$

$10^{-1} = 0.1$

$4^0 = 1$

$8^{2/3} = 4$

$25^{-1/2} = 0.2$

Calculate the following:

$Log_{10}(42.5)$	=	1.628389
ln(2.52)	=	0.924259

Determine the value of *x*:

$0.70=e^{-x}$	$10^4=e^x$
Take the ln of both sides and solve for	Solve the left side of the equation, take the ln of of both sides, and solve for *x*:
$0.70=e^{-x}$ $\ln(0.70) = \ln(e^{-x})$ $-0.35667 = -x$ $x = 0.35667$	$10^4=e^x$ $10000 = e^x$ $\ln(10000) = \ln(e^x)$ $x = 9.21034$

Calculate dy/dx for the following functions:

$y = 3$	$y = e^x$	$y = 4 - 6x$	$y = 5x^2 - 2x$
The derivative of a constant is 0.	The derivative of e^x is e^x	The derivative of a constant is 0 and the derivative of a constant multiplied by x equals the constant.	The derivative of $5x^2 = 5*2x^{2-1}$ and the derivative of a constant multiplied by x equals the constant.
0	e^x	-6	$10x - 2$

Integrate the following:

$$\int_1^2 x^2 dx$$

Follow the integration rule:

$$\int x^2 dx = x^{(2+1)} / (2+1)$$

For $x = 2$, $2^{(2+1)} / (2+1) = 2^3/3 = 8/3$

For $x = 1$, $1^{(2+1)} / (2+1) = 1^3/3 = 1/3$

The integration = 8/3 -1/3 =7/3

$$\int_0^2 e^x dx$$

Follow the integration rule:

$$\int e^x = e^x$$

For $x = 2$, $e^x = e^2 = 7.39$

For $x = 0$, $e^x = e^0 = 1$

The integration = 7.39 - 1 = 6.39

$$\int_2^4 (3+2x)dx$$

Following the rules of integration,

$$\int_2^4 (3+2x)dx = \left(3x + 2x^2/2\right)\Big|_2^4 = \left(3x + x^2\right)\Big|_2^4$$

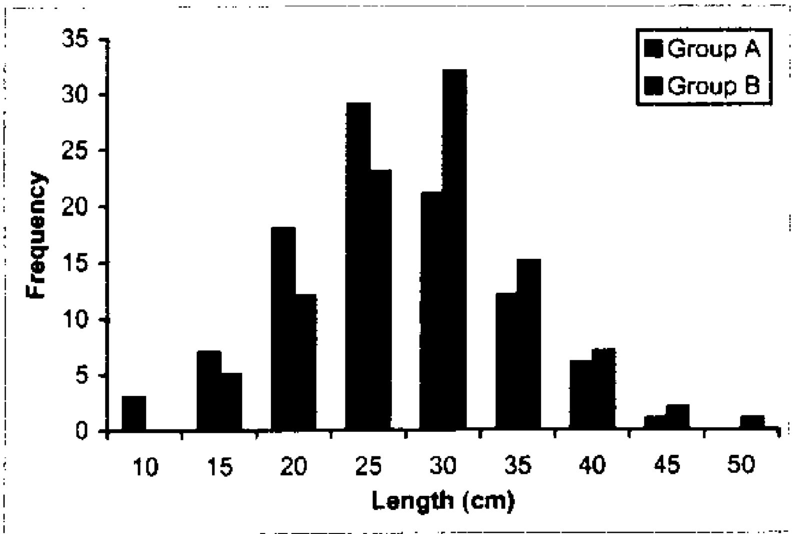
For $x = 4$, $[(3*4) + 4^2] = 12 + 16 = 28$

For $x = 2$, $[(3*2) + 2^2] = 6 + 4 = 10$

The integration = 28 - 10 = 18

Compare the following length-frequency distribution using univariate descriptive statistics for each data set. Plot both L-F distributions as histograms on the same graph. Compare the means and the distributions around the means using confidence intervals.

Length (cm)	10	15	20	25	30	35	40	45	50
Group A	3	7	18	29	21	12	6	1	0
Group B	0	5	12	23	32	15	7	2	1



The mean, standard deviation, and confidence level can be determined directly in *Excel*.

Go to **Tools, Data Analysis, Descriptive Statistics**, select the input and output ranges and check the boxes for **Summary Statistics** and **Confidence Level for Mean**.

To run descriptive statistics on the lengths, each observation needs to be written out for both groups of data (see below).

Group A	Group B
10	15
10	15
10	15
15	15
15	15
15	20
15	20
15	20
15	20
15	20
20	20
20	20
20	20
20	20
20	20
20	20

Group A		*Group B*	
Mean	26.34	Mean	28.814
Standard Error	0.7419	Standard Error	0.7096
Median	25	Median	30
Mode	25	Mode	30
Standard Deviation	7.3071	Standard Deviation	6.9886
Sample Variance	53.393	Sample Variance	48.84
Kurtosis	-0.139	Kurtosis	0.296
Skewness	0.0725	Skewness	0.2715
Range	35	Range	35
Minimum	10	Minimum	15
Maximum	45	Maximum	50
Sum	2555	Sum	2795
Count	97	Count	97
Confidence Level(95.0%)	1.4727	Confidence Level(95.0%)	1.4085

20	20
20	25
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30	30
30	30

To calculate the confidence interval, subtract and add the confidence level from the mean.

Therefore, for Group A, the *C.I.* = 25 ± 1.4727 = 23.5273 ≤ $\bar{X}$ 26.4727
and for Group B, the *C.I.* = 30 ± 1.40851 = 28.5915 ≤ $\bar{X}$ 31.4085

The conclusion that can be made is that the *C.I.* do not overlap so therefore, there is a difference between the means of the two groups.

30	30
30	30
30	30
30	35
30	35
30	35
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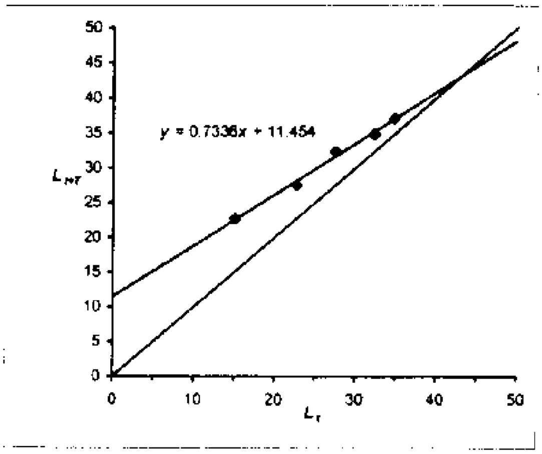
35	35
35	35
35	35
35	35
35	35
35	35
35	40
35	40
35	40
40	40
40	40
40	40
40	40
40	45
40	45
45	50

Given the following age-length data set for mackerel, determine K, L_∞ and t_0 using Ford-Walford and non-linear methods.

Age	Length (cm)
1	15.1
2	22.7
3	27.5
4	32.3
5	34.8
6	37.1

Linear Method

Age	L_t	L_{t+T}
1	15.1	22.7
2	22.7	27.5
3	27.5	32.3
4	32.3	34.8
5	34.8	37.1
6	37.1	



There are 2 methods to calculate the slope and y-intercept:

(1) Go to **Tools, Data Analysis, Regression**, select the input and output ranges, then click OK. The two numbers that are important are the X variable 1 which is the slope and intercept which is the y-intercept.

(2) Select the data series on the graph, right click, and choose **Add Trendline**. Choose "linear" for the **Type**. In the **Options** tab, click "display equation on chart."

To plot the 45° line, simple create a data series with 2 pairs of points that have equal x and y values.

45° Line	
0	0
50	50

From **Tools, Data Analysis, Regression**:

SUMMARY OUTPUT

Regression Statistics	
Multiple R	0.996333704
R Square	0.99268085
Adjusted R Square	0.990241133
Standard Error	0.572640754
Observations	5

ANOVA

	df	*SS*	*MS*	*F*	*Significance F*
Regression	1	133.4242477	133.4242477	406.8836672	0.00026634
Residual	3	0.983752299	0.327917433		
Total	4	134.408			

	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*	*Lower 95.0%*	*Upper 95.0%*
Intercept	**11.45448356**	0.996493924	11.49478515	0.001413384	8.283192179	14.62577494	8.283192179	14.6257749
X Variable 1	**0.733592011**	0.036368004	20.1713576	0.00026634	0.617852683	0.849331339	0.617852683	0.84933134

To solve for L_∞, set the regression line equation and the 45° line equation equal to each other:

Regression Line	45° Line
$y = 11.4544 + 0.7336x$	$y = x$

$x = 11.4544 + 0.7336x$

$.2664x = 11.454$

$x = 11.4544/.2664$

$x = 42.9970$

Solving for t_0

Use the rearranged von Bertalanffy equation to solve for t_0

For $t = 1$ and $L_t = 15.1$ $\quad t_0 = 1 + \frac{1}{0.31}\ln\left(\frac{43.0 - 15.1}{43.0}\right) = -0.395$

For $t = 2$ and $L_t = 22.7$ $\quad t_0 = 2 + \frac{1}{0.31}\ln\left(\frac{43.0 - 22.7}{43.0}\right) = -0.421$

mean = -0.408

To plot the von Bertalanffy curve, create series of ages (t), and solve for L_t.

a (slope)	0.7336
b (y-intercept)	11.4544
$K = -\ln(a)/t$	0.3098
L_∞	42.997
t_0	-0.408

Non-linear Method

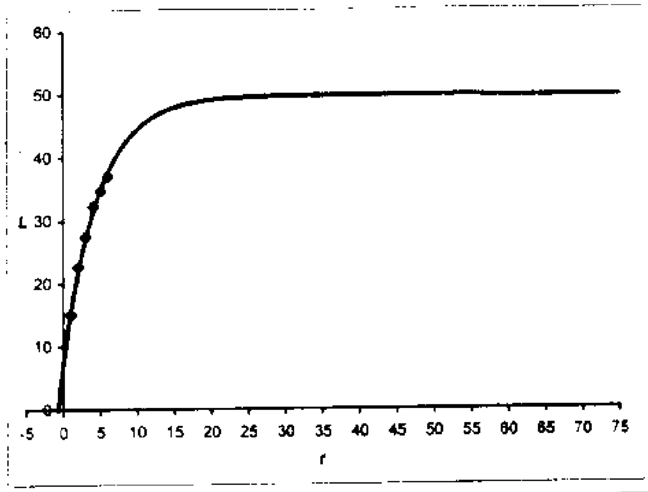
Non-linear regression can be calculated using the values obtained from the linear regression as initial parameter values, and then using **Solver** to adjust those values to provide the best fitting values.

The equation to calculate $L_{predicted}$ is the von Bertalanffy growth equation:

$$L_t = L_\infty\left(1 - e^{-K(t-t_0)}\right)$$

		Before Solver		After Solver	
Age	L_{obs}	L_{pred}	$(L_{obs}-L_{pred})^2$	L_{pred}	$(L_{obs}-L_{pred})^2$
1	15.1	25.13	100.63	15.44	0.12
2	22.7	35.47	163.19	21.91	0.63
3	27.5	42.51	225.30	27.16	0.12
4	32.3	47.30	224.87	31.42	0.78
5	34.8	50.55	248.09	34.88	0.01
6	37.1	52.77	245.40	37.69	0.35
			1207.50		2.00

	Before Solver	After Solver
a	0.7336	0.7336
b	11.4544	11.4544
K	0.3854	0.2082
L_∞	57.47513115	49.82611687
t_0	-0.491991033	-0.78145166



To plot the von Bertalanffy curve, create a series of ages (t) and solve for L_t using the von Bertalanffy equation and the parameter values obtained from **Solver**.

a	0.7336
b	11.4544
K	0.2082
L_∞	49.82811687
t_0	-0.78145166

x	y
-0.78145166	0.00
0	7.48
1	15.44
2	21.91
3	27.16
4	31.42
5	34.88
6	37.69
7	39.97
8	41.82
9	43.33
10	44.55
11	45.54
12	46.35
13	47.00
14	47.53
15	47.96
16	48.32
17	48.60
18	48.83
19	49.02
20	49.17
21	49.29
22	49.39
23	49.46
24	49.54
25	49.60
26	49.64
27	49.67
28	49.70
29	49.73
30	49.7461
31	49.7615
32	49.7741
33	49.7842
34	49.7925
35	49.7992
36	49.8046
37	49.8090
38	49.8126
39	49.8155
40	49.8179
41	49.8198
42	49.8214
43	49.8226
44	49.8237
45	49.8245

46	49.8252
47	49.8257
48	49.8262
49	49.8265
50	49.8268
51	49.8271
52	49.8273
53	49.8274
54	49.8276
55	49.8277
56	49.8278
57	49.8278
58	49.8279
59	49.8279
60	49.8280
61	49.8280
62	49.8280
63	49.8280
64	49.8280
65	49.8281
66	49.8281
67	49.8281
68	49.8281
69	49.8281
70	49.8281
71	49.8281
72	49.8281
73	49.8281
74	49.8281
75	49.8281

Microsoft Excel 8.0a Answer Report
Worksheet: [Chapter3 - Growth Exercises.xls]Exercise 1
Report Created: 4/24/00 1:45:32 PM

Target Cell (Min)

Cell	Name	Original Value	Final Value
F116	$(L_{obs}-L_{pred})^2$	1207.50	2.00

Adjustable Cells

Cell	Name	Original Value	Final Value
D121	K After **Solver**	0.3854	0.2082
D122	L_∞ After **Solver**	57.47513115	49.82811687
D123	t_0 After **Solver**	-0.491991033	-0.78145166

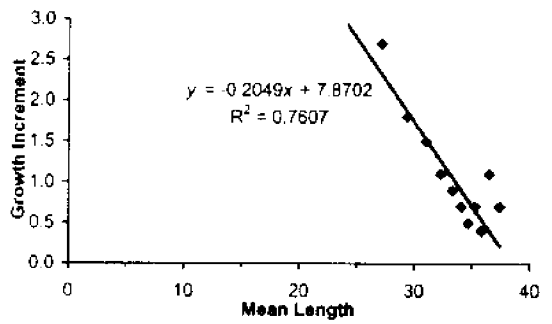
Constraints
NONE

Given the following age-length data set for herring, determine K, L_∞ and t_0 using Gulland-Holt and non-linear methods.

Age	Length (cm)
3	25.7
4	28.4
5	30.2
6	31.7
7	32.8
8	33.7
9	34.4
10	34.9
11	35.6
12	36.0
13	35.9
14	37.0
15	37.7

Linear Method

Age (t)	Length (cm) (L_t)	Growth Increment (cm) $[L_{(t+1)} - L_t]$	Mean Length (cm) $[L_{(t+1)} + L_t]/2$
3	25.7	2.7	27.05
4	28.4	1.8	29.3
5	30.2	1.5	30.95
6	31.7	1.1	32.25
7	32.8	0.9	33.25
8	33.7	0.7	34.05
9	34.4	0.5	34.65
10	34.9	0.7	35.25
11	35.6	0.4	35.8
12	36.0	-0.1	35.95
13	35.9	1.1	36.45
14	37.0	0.7	37.35
15	37.7		



There are 2 methods to calculate the slope and y-intercept:

(1) Go to **Tools, Data Analysis, Regression**, select the input and output ranges, then click OK. The two numbers that are important are the X variable 1 which is the slope and intercept which is the y-intercept.

(2) Select the data series on the graph, right click, and choose **Add Trendline**. Choose "linear" for the **Type**. In the **Options** tab, click "display equation on chart."

From **Tools, Data Analysis, Regression**:

SUMMARY OUTPUT

Regression Statistics	
Multiple R	0.87215823
R Square	0.76065998
Adjusted R Square	0.73672598
Standard Error	0.37578
Observations	12

ANOVA

	df	*SS*	*MS*	*F*	*Significance F*
Regression	1	4.487893887	4.487893887	31.78156264	0.000216153
Residual	10	1.412106113	0.141210611		
Total	11	5.9			

	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*	*Lower 95.0%*	*Upper 95.0%*
Intercept	7.87016633	1.223470359	6.432657949	7.51218E-05	5.144104016	10.59622864	5.144104016	10.59622864
X Variable 1	-0.20492666	0.036350538	-5.63751387	0.000216153	-0.285920721	-0.1239326	-0.285920721	-0.1239326

Solving for t_0

Use the rearranged von Bertalanffy equation to solve for t_0

For t = 3 and L_t = 25.7

$$t_0 = 3 + \frac{1}{0.20} \ln\left(\frac{38.41 - 25.7}{38.41}\right) = -2.53$$

For t = 4 and L_t = 28.4

$$t_0 = 4 + \frac{1}{0.20} \ln\left(\frac{38.41 - 28.4}{38.41}\right) = -2.72$$

mean = -2.625

To plot the von Bertalanffy curve, create series of ages (t), and solve for L_t.

a (slope)	-0.2049
b (y-intercept)	7.8702
K = -slope	0.2049
$L_\infty = b/K$	38.4099561
t_0	-2.625

Non-linear Method

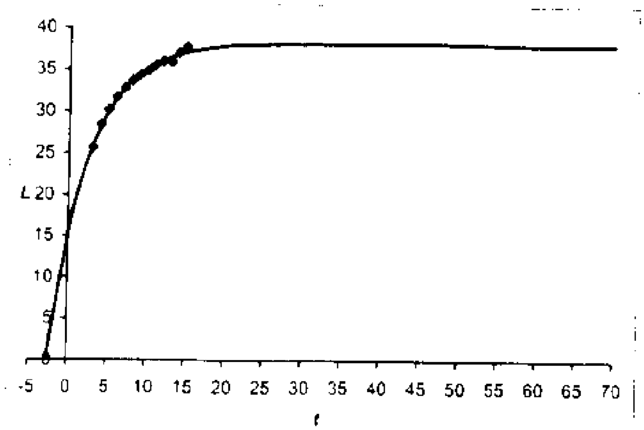
Non-linear regression can be calculated using the values obtained from the linear regression as initial parameter values, and then using **Solver** to adjust those values to provide the best fitting values.

The equation to calculate $L_{predicted}$ is the von Bertalanffy growth equation:

$$L_t = L_\infty\left(1 - e^{-K(t-t_0)}\right)$$

		Before Solver		After Solver	
Age	L_{obs}	L_{pred}	$(L_{obs}-L_{pred})^2$	L_{pred}	$(L_{obs}-L_{pred})^2$
3	25.7	25.54	0.03	25.99	0.08
4	28.4	27.92	0.23	28.21	0.04
5	30.2	29.86	0.11	30.02	0.03
6	31.7	31.45	0.06	31.50	0.04
7	32.8	32.74	0.00	32.71	0.01
8	33.7	33.79	0.01	33.70	0.00
9	34.4	34.64	0.06	34.51	0.01
10	34.9	35.34	0.20	35.17	0.07
11	35.6	35.91	0.10	35.71	0.01
12	36	36.37	0.14	36.15	0.02
13	35.9	36.75	0.72	36.51	0.37
14	37	37.06	0.00	36.80	0.04
15	37.7	37.31	0.15	37.05	0.43
			1.82		1.16

	Before Solver	After Solver
a	-0.2049	-0.2049
b	7.8702	7.8702
K	0.2049	0.2015
L_∞	38.40995608	38.12637658
t_0	-2.625	-2.680111193



To plot the von Bertalanffy curve, create a series of ages (t) and solve for L_t using the von Bertalanffy equation and the parameter values obtained from **Solver**.

a	-0.2049
b	7.8702
K	0.2015
L_∞	38.1263766
t_0	-2.68011119

x	y
-2.680111193	0.000
0	15.912
1	19.967
2	23.281
3	25.991
4	28.208
5	30.017
6	31.497
7	32.707
8	33.696
9	34.505
10	35.166
11	35.706
12	36.148
13	36.509
14	36.804
15	37.046
16	37.243
17	37.404
18	37.536
19	37.644
20	37.732
21	37.804
22	37.863
23	37.911
24	37.950
25	37.982
26	38.009
27	38.030
28	38.048
29	38.062
30	38.074
31	38.083
32	38.091
33	38.098
34	38.103
35	38.107
36	38.111
37	38.114
38	38.116
39	38.118
40	38.119
41	38.121
42	38.122
43	38.123
44	38.123
45	38.124

46	38.124
47	38.125
48	38.125
49	38.125
50	38.125
51	38.126
52	38.126
53	38.126
54	38.126
55	38.126
56	38.126
57	38.126
58	38.126
59	38.126
60	38.126
61	38.126
62	38.126
63	38.126
64	38.126
65	38.126
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67	38.126
68	38.126
69	38.126
70	38.126

Microsoft Excel 8.0a Answer Report
Worksheet: [Growth.xls]Exercise 2
Report Created: 2/24/00 9:23:12 AM

Target Cell (Min)

Cell	Name	Original Value	Final Value
F118	$(L_{obs}-L_{pred})^2$	2.66	1.16

Adjustable Cells

Cell	Name	Original Value	Final Value
D123	*K* After **Solver**	0.2049	0.2015
D124	L_∞ After **Solver**	38.40995608	38.12637658
D125	t_0 After **Solver**	-2.625	-2.680111193

Constraints
NONE

Given an initial population size of N_0 = 25,000 fish, a survival rate of S = 0.47, and a commercial harvest of 10,500 fish, determine N_t, U, Z, F, and M during the first year for both a Type 1 and Type 2 fishery. For the Type 1 fishery, assume fishing occurs only in the second half of the year.

Given data	
N_0 =	25,000
S =	0.47
C =	10,500

Part 1: Type 1 fishery

N_t

$N_t/N_0 = S$

therefore

$N_t = N_0 * S$
=(25,000) * (0.47)
11750

Z

$N_t/N_0 = e^{(-Z)} = S$

therefore, for t = 1

$Z = -\ln(S)$
= -ln(0.47)
0.76

U

$U = 1 - e^{(-Z)}$
$= 1 - e^{(-0.76)}$
0.53

M

Since M occurs only in the first half of the year and F only in the second half of the year, it is necessary to determine population size halfway through the year

Designate the following
N_0 = N at beginning of year
N_1 = N at mid-year
N_2 = N at end of year

Since fishing occurs only in second half of year use the equation
$U = C / N$
to determine N_1

$N_1 = C / U$
= 10,500 / 0.53
19,811
is the number of fish halfway through the year

Now use the decay equation to determine M for the first half of the year

$N_1/N_0 = e^{(-M)}$
$M = -\ln[N_1/N_0]$
= -ln(19811 / 25000)
0.23

F

F only occurs in second half of year
Use the decay equation using N_t at mid year and N_0 at year end

$N_t / N_0 = e^{(-F)}$

therefore

$F = -\ln[N_t / N_0]$
= -ln(11750 / 19811)
0.52

Part 2: **Type 2 fishery**

N_t

Calculated the same way as for a Type 1 fishery

$N_t / N_0 = S$

$N_t = N_0 * S$

$= 25000 * 0.47$

11750

Z

Calculated the same way as for a Type 1 fishery

$Z = -\ln(S)$

$= -\ln(0.47)$

0.76

U

$U = C / N_0$

Fishing occurs throughout the year so

$U = 10500 / 25000$

0.42

F

$U = (F / Z)*(1 - e^{-Z})$

therefore

$F = (Z * U) / (1 - e^{-Z})$

$= (0.76) * (0.42) / (1 - e^{-0.76})$

0.60

M

$Z = M + F$

therefore

$M = Z - F$

$= 0.76 - 0.60$

0.16

Weakfish caught by NEFSC autumn bottom trawl survey were aged by applying annual age-length keys from pooled commercial and research samples to survey caught fish. Catch-at-age (expressed as CPUE) for the 1985 and 1990 year classes is shown below. Estimate total and fishing mortality for the two different years, assuming M = 0.25.

Year	Number at age					
	0	1	2	3	4	5
1985	10.39	4.12	0.93	0.06	0.03	
1990	3.45	0.73	0.13	0.06	0.019	0.013

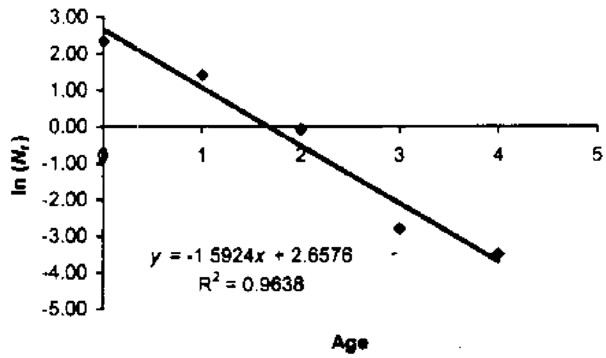
For both years solve for Z using the Catch Curve Analysis equation

$$\ln(N_t) = aX + b$$

where a is the negative of total mortality (Z) and X is time in years.

1985 data

age	N_t	$\ln(N_t)$
0	10.39	2.34
1	4.12	1.42
2	0.93	-0.07
3	0.06	-2.81
4	0.03	-3.51



There are 2 methods to calculate the slope and y-intercept:

(1) Go to **Tools, Data Analysis, Regression**, select the input and output ranges, then click OK. The two numbers that are important are the X variable 1 which is the slope and intercept which is the y-intercept.

(2) Select the data series on the graph, right click, and choose **Add Trendline**. Choose "linear" for the Type. In the **Options** tab, click "display equation on chart."

From Tools, **Data Analysis, Regression**:

SUMMARY OUTPUT

Regression Statistics	
Multiple R	0.981711946
R Square	0.963758344
Adjusted R Square	0.951677792
Standard Error	0.56378474
Observations	5

ANOVA

	df	*SS*	*MS*	*F*	*Significance F*
Regression	1	25.35759189	25.357592	79.777675	0.002960671
Residual	3	0.953559696	0.3178532		
Total	4	26.31115159			

	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*	*Lower 95.0%*	*Upper 95.0%*
Intercept	2.657644989	0.436705781	6.0856648	0.00890971	1.267850965	4.04743899	1.26785096	4.047438994
X Variable 1	-1.59240673	0.178284389	-8.931835	0.00296067	-2.15978775	-1.0250257	-2.1597878	-1.025025702

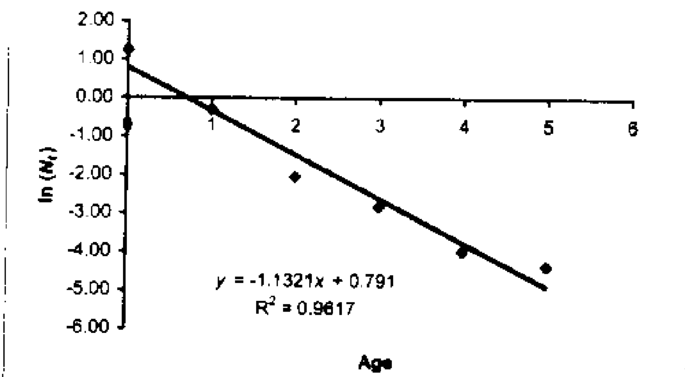
$Y = -1.5924X + 2.6576$
if a = $-Z$ then
Z = 1.59

Given
M = 0.25
then
$Z = F + M$
$F = Z - M$
F = 1.34

1990 data

age	N_t	$\ln(N_t)$
0	3.45	1.24
1	0.73	-0.31
2	0.13	-2.04
3	0.06	-2.81
4	0.019	-3.96
5	0.013	-4.34

Solve same way as 1985
Either plot and **Add Trendline**, or use **Tools, Data Analysis, Regression.**



From **Tools, Data Analysis, Regression:**

SUMMARY OUTPUT

Regression Statistics	
Multiple R	0.980646078
R Square	0.96166673
Adjusted R Square	0.952083413
Standard Error	0.472786296
Observations	6

ANOVA

	df	*SS*	*MS*	*F*	*Significance F*
Regression	1	22.43047543	22.430475	100.348	0.000558237
Residual	4	0.894107526	0.2235269		
Total	5	23.32458295			

	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*	*Lower 95.0%*	*Upper 95.0%*
Intercept	0.791002143	0.342177599	2.3116713	0.08168183	-0.15903715	1.74104143	-0.1590371	1.741041431
X Variable 1	-1.13214021	0.113017541	-10.01738	0.00055824	-1.44592786	-0.8183526	-1.4459279	-0.81835256

$Y = -1.1321X + 0.791$

If a = $-Z$ then
Z = 1.13

Given
M = 0.25
then
F = 0.88

The following data are taken from the Cooperative Striped Bass Tagging Program, conducted by the U.S. Fish and Wildlife Service and the Atlantic States Marine Fisheries Commission. The purpose of the program is to monitor mortality and migration of striped bass for he major producer areas (Hudson River, Chesapeake Bay, and Delaware Bay). This program comprises 4 critical operations: tagging fish, recovering tags, managing records of releases and recoveries, and analyzing recovery data. Total releases of tagged striped bass have exceeded 170,000 fish in ten years, through participation of 10 states. Analysis of these data is performed on an annual basis by the Atlantic States Marine Fisheries Commission tagging group. Data from the Hudson River portion of this program are shown in the following table. Using this data, derive estimates of total and fishing mortality for the years 1990, 1993, and 1996. Natural mortality for striped bass is *M* = 0.15.

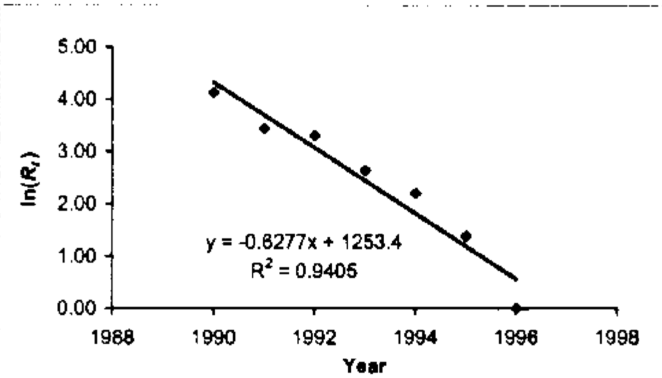
Year	Number tagged	Number recaptured in year 1988	1989	1990	1991	1992	1993	1994	1995	1996
1988	227	25	31	18	11	10	5	4	1	4
1989	387		41	29	17	9	6	8	4	0
1990	446			62	31	27	14	9	4	1
1991	364				38	31	12	10	9	4
1992	699					90	58	35	21	13
1993	537						73	36	24	18
1994	381							43	33	26
1995	462								50	34
1996	683									88

Solve similar to a catch curve: plot natural log of recaptures against time using the equation

$$\ln(R_t) = at + b$$

where a is the negative of total mortality (Z) and *t* is time in years.

Data for 1990		
Time	R_t	$\ln(R_t)$
1990	62	4.13
1991	31	3.43
1992	27	3.30
1993	14	2.64
1994	9	2.20
1995	4	1.39
1996	1	0.00



There are 2 methods to calculate the slope and *y*-intercept:

(1) Go to **Tools, Data Analysis, Regression**, select the input and output ranges, then click OK. The two numbers that are important are the X variable 1 which is the slope and intercept which is the *y*-intercept.

(2) Select the data series on the graph, right click, and choose **Add Trendline.** Choose "linear" for the **Type**. In the **Options** tab, click "display equation on chart."

From **Tools, Data Analysis, Regression**:

SUMMARY OUTPUT

Regression Statistics	
Multiple R	0.969815846
R Square	0.940542775
Adjusted R Square	0.92865133
Standard Error	0.373468752
Observations	7

ANOVA

	df	*SS*	*MS*	*F*	*Significance F*
Regression	1	11.03195446	11.031954	79.094069	0.000299118
Residual	5	0.697394545	0.1394789		
Total	6	11.72934901			

	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*	*Lower 95.0%*	*Upper 95.0%*
Intercept	**1253.4319**	140.6639383	8.910826	0.0002964	891.8443045	1615.019452	891.8443045	1615.019452
X Variable 1	**-0.6276929**	0.07057896	-8.893485	0.0002991	-0.80912159	-0.4462642	-0.80912159	-0.4462642

$Y = -0.6277X + 1253.4$

if $a = -Z$ then

$Z = 0.6277$

Given

$M = 0.15$

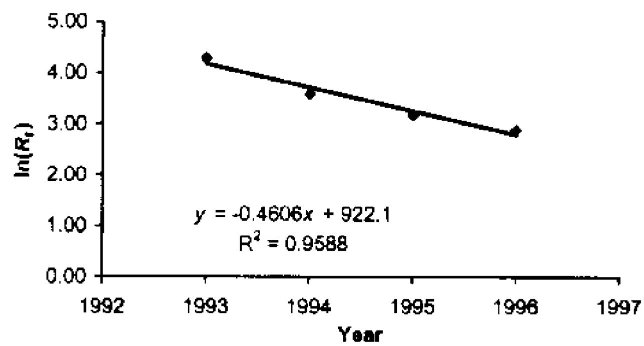
then

$F = 0.6277 - 0.15$

$F = 0.4777$

Data for 1993

Year	R_t	$\ln(R_t)$
1993	73	4.29
1994	36	3.58
1995	24	3.18
1996	18	2.89



From **Tools, Data Analysis, Regression:**

SUMMARY OUTPUT

Regression Statistics	
Multiple R	0.979158729
R Square	0.958751816
Adjusted R Square	0.938127724
Standard Error	0.151046919
Observations	4

ANOVA

	df	*SS*	*MS*	*F*	*Significance F*
Regression	1	1.060636593	1.0606366	46.486983	0.020841271
Residual	2	0.045631552	0.0228158		
Total	3	1.106268145			

	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*	*Lower 95.0%*	*Upper 95.0%*
Intercept	**922.09808**	134.7307505	6.8440061	0.0206889	342.3980476	1501.798117	342.3980476	1501.798117
X Variable 1	**-0.4605728**	0.06755113	-6.818136	0.0208413	-0.75122207	-0.16992356	-0.75122207	-0.16992356

$Y = -0.4606X + 922.1$

if a = -Z then
Z = 0.46

Given
M = 0.15
then
F = 0.31

1996

Since there is only one year of recaptures,
use the single census (Petersen) method.

Number marked (C) = 683
Number recaptured (R) = 88

$U = R / C$
= 88 / 683
U = 0.13
M = 0.15
F = 0.15 << Leave value for F blank in order to use **Solver**.
A value will be returned here.

For a type 2 fishery

$U = (F/Z)*(1-e^{-Z})$
$=(F/(F+M))*(1-e^{-(F+M)})$

Type in equation using cell values for F and M
You should get a zero value at first

0.1300003 Use **Solver** function in **Tools** to set equation equal
to cell B152 (U value) by changing cell B154 (F value)

The value for F magically appears in cell B81
In this case, F = 0.15
$Z = F + M$ = 0.15 + 0.15
Z = 0.30

TRAWL COD-END SELECTION PROBLEM
COVERED COD-END EXPERIMENT
Yellowtail flounder on Georges Bank

Cod-end = 14 cm, diamond mesh; Cover = 5 cm, square mesh

Fish Length (cm)	Number in Cod-end	Number in Cover
10-12	0	0
13-15	0	50
16-18	10	102
19-21	20	90
22-24	33	60
25-27	48	43
28-30	107	21
31-33	95	5
34-36	87	0
37-39	60	0
40-42	60	0
43-45	20	0
46-48	12	0
49-51	2	0
52-54	0	

Part 1: Determine the selection curve by linear regression on natural log transformed data.

Solution:
Step 1: Determine the probability of capture (P) by the cod-end.

Fish Length (cm)	Length midpoint	Number in Cod-end	Number in Cover	Total	Probability (cod end / total)	1-P	ln(P/(1-P))
10-12	11	0	0	0			
13-15	14	0	50	50	0.00	1.00	
16-18	17	10	102	112	0.09	1.00	-2.42
19-21	20	20	90	110	0.18	0.91	-1.61
22-24	23	33	60	93	0.35	0.82	-0.84
25-27	26	48	43	91	0.53	0.65	-0.20
28-30	29	107	21	128	0.84	0.47	0.57
31-33	32	95	5	100	0.95	0.16	1.76
34-36	35	87	0	87	1.00	0.05	3.00
37-39	38	60	0	60	1.00	0.00	
40-42	41	60	0	60	1.00	0.00	
43-45	44	20	0	20	1.00	0.00	
46-48	47	12	0	12	1.00	0.00	
49-51	50	2	0	2	1.00	0.00	
52-54	53	0		0			

Step 2: Solve for α and β by plotting the value of ln(P / (1 - P)) against fish length using the equation

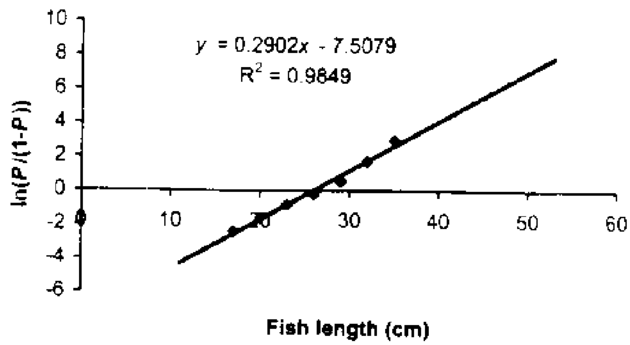
$$\ln(P / (1 - P)) = \alpha + (\beta * L)$$

where α is the y-intercept and β is the slope.

There are 2 methods to calculate the slope and y-intercept:

(1) Go to **Tools, Data Analysis, Regression**, select the input and output ranges, then click OK. The two numbers that are important are the X variable 1 which is the slope and intercept which is the y-intercept.

(2) Select the data series on the graph, right click, and choose **Add Trendline**. Choose "linear" for the **Type**. In the **Options** tab, click "display equation on chart."



From **Tools, Data Analysis, Regression**:

SUMMARY OUTPUT

Regression Statistics	
Multiple R	0.992423654
R Square	0.98490471
Adjusted R Square	0.981885652
Standard Error	0.255044835
Observations	7

ANOVA

	df	*SS*	*MS*	*F*	*Significance F*
Regression	1	21.22051024	21.22051024	326.229144	9.55737E-06
Residual	5	0.325239339	0.065047868		
Total	6	21.54574958			

	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*	*Lower 95.0%*	*Upper 95.0%*
Intercept	-7.507936	0.42870274	-17.5131514	1.1128E-05	-8.60994966	-6.405922309	-8.6099497	-6.40592231
X Variable 1	0.29018679	0.016066314	18.06181452	9.5574E-06	0.248887083	0.3314865	0.24888708	0.3314865

$Y = 0.2902X - 7.5079$

therefore

$\alpha =$ -7.5079
$\beta =$ 0.2902

Part 2: **Based on the selection curve, estimate the L_{50}, SF, SR for yellowtail flounder, using a 14 cm diamond mesh cod-end.**

L_{50}

$\alpha = -(\beta * L_{50})$

therefore

$L_{50} = -\alpha / \beta$
$= -1 * ((-7.5079) / 0.2902)$
25.87

SF

$SF = L_{50} / ml$

where ml = mesh size

for ml = 14 and L_{50} = 25.87

SF = 25.87 / 14
1.85

SR = $L_{75} - L_{25}$
At L_{75}, P = 0.75 and 1-P = 0.25
At L_{25}, P = 0.25 and 1-P = 0.75
Use these values in the equation

$$\ln(P / (1-P)) = \alpha + (\beta * L)$$

Rearrange and solve for L

$$L = [\ln(P / (1 - P)) - \alpha] / \beta$$

L_{75}

P = 0.75
1 - P = 0.25
α = -7.5079
β = 0.2902

$L = [(\ln(P / (1 - P)) - \alpha] / \beta$
$= [\ln(0.75 / 0.25) - (-7.5079)] / 0.2902$
L_{75} = 29.66

L_{25}

P = 0.25
1 - P = 0.75
α = -7.5079
β = 0.2902

$L = [(\ln(P / (1 - P)) - \alpha] / \beta$
$= [\ln(0.25 / 0.75) - (-7.5079)] / 0.2902$
L_{25} = 22.09

SR

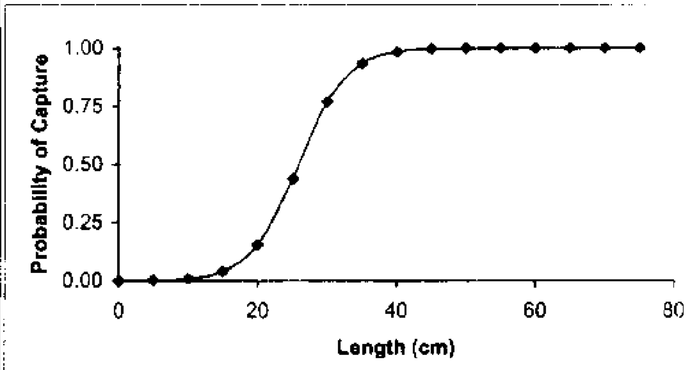
SR = $L_{75} - L_{25}$
= 29.66 - 22.09
7.57

You can then plot the resulting curve using the derived values for α and β and values for L using the equation

$$PL_L = 1 / [1 + e^{-(\alpha + \beta * L)}]$$

Length	Predicted P
0	0.00
5	0.00
10	0.01
15	0.04
20	0.15
25	0.44
30	0.77
35	0.93
40	0.98
45	1.00
50	1.00
55	1.00
60	1.00
65	1.00
70	1.00
75	1.00

α =	-7.5079
β =	0.2902



TRAWL COD-END SELECTION PROBLEM
ALTERNATE TOW EXPERIMENT
Cod on Georges Bank

Exp Trawl = 14 cm, diamond mesh; Lined Trawl= 5 cm

Fish Length (cm)	Number in lined trawl	Number in Exp Trawl
11-15	0	0
16-20	5	0
21-25	7	0
26-30	2	0
31-35	10	0
36-40	30	2
41-45	36	6
46-50	42	8
51-55	50	20
56-60	83	35
61-65	64	42
66-70	53	47
71-75	42	38
76-80	19	20
81-85	11	10
86-90	7	7
91-95	6	5
96-100	4	3
101-105	1	1
106-110	0	0

Part 1: **Determine the selection curve by non-linear regression of PL_L versus L.**

Solution:

Step 1: Determine the probability of capture by the experimental trawl (PL_L)

Step 2: Guess at values for L_{50} and $\alpha 2$ and use the equation

$$PL_L = 1/[1 + e^{-\alpha 2 * (L - L_{50})}]$$

to solve for expected values of PL_L.

Step 3: Find the Sum of Squares for the observed and expected PL_L values. Use **Solver** to minimize the Sum of Squares by changing the guessed values of L_{50} and $\alpha 2$. The new values for $\alpha 2$ and L_{50} are the solutions to the non-linear regression.

Fish Length (cm)	Length Midpoint	Number in lined trawl	Number in Exp Trawl	PL_L (exp / lined)	Before Solver: Expected PL_L	Before Solver: (Observed - Expected)^2	After Solver: Expected PL_L	After Solver: (Observed - Expected)^2
11-15	13	0	0					
16-20	18	5	0	0.0000	0.0001	6.84214E-09	0.0031	9.75622E-06
21-25	23	7	0	0.0000	0.0002	5.05426E-08	0.0065	4.16139E-05
26-30	28	2	0	0.0000	0.0006	3.73174E-07	0.0133	0.000176243
31-35	33	10	0	0.0000	0.0017	2.75162E-06	0.0271	0.000735694
36-40	38	30	2	0.0667	0.0045	0.003865158	0.0546	0.00014519
41-45	43	36	6	0.1667	0.0121	0.023882065	0.1069	0.003570094
46-50	48	42	8	0.1905	0.0323	0.025021142	0.1988	6.87215E-05
51-55	53	50	20	0.4000	0.0832	0.10037954	0.3395	0.003657349
56-60	58	83	35	0.4217	0.1978	0.050118061	0.5158	0.008855943
61-65	63	64	42	0.6563	0.4013	0.064993211	0.6882	0.001021939
66-70	68	53	47	0.8868	0.6457	0.058146641	0.8206	0.004381618
71-75	73	42	38	0.9048	0.8320	0.00529162	0.9046	3.84895E-08
76-80	78	19	20	1.0000	0.9309	0.004780121	0.9516	0.00234711
81-85	83	11	10	1.0000	0.9734	0.0007074	0.9760	0.000575073
86-90	88	7	7	1.0000	0.9900	9.90384E-05	0.9883	0.000137311
91-95	93	6	5	1.0000	0.9963	1.35736E-05	0.9943	3.23689E-05
96-100	98	4	3	1.0000	0.9986	1.84558E-06	0.9972	7.58277E-06
101-105	103	1	1	1.0000	0.9995	2.50201E-07	0.9987	1.77094E-06
106-110	108	0	0					
					Sum of squares	0.337302849		0.025765418

	Before Solver	After Solver
L_{50}	60	52.56635074
α2	0.2	0.145721666

Step 4: Use the new values of L_{50} and α2 to solve for α and β

L_{50} =	52.566
α2 =	0.146
β = α2 =	0.146
α = -β * L_{50} = -1*(0.14 * 52.59)	
α =	-7 674636

Part 2: **Based on the selection curve, estimate the L_{50}, SF, SR for cod using a 14 cm diamond mesh cod-end.**

L_{50}

Solved for above using **Solver**

52.566

SF

SF = L_{50} / ml
= 52.566 / 14
3.75

SR

Use the equation

$$PL_L = 1/[1 + e^{-\alpha 2 * (L - L_{50})}]$$

or

$$PL_L = 1 / [1 + e^{(-\alpha + \beta L)}]$$

and use **Solver** to solve for
L_{75} when PL_L = 0.75 and
L_{25} when PL_L = 0.25

L_{75}

α =	-7.6746
β =	0.14
L_{75} =	62.67
PL_L =	0.75

L_{25}

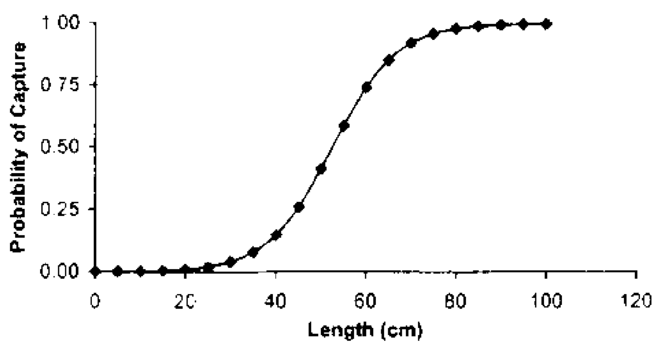
L_{25} =	46.97
PL_L =	0.25

SR = L_{75} - L_{25}
= 60.44 - 44.73
15.69

You can then plot the resulting curve using the derived values for α and β and values for L using the equation

$$PL_L = 1 / [1 + e^{-(\alpha + \beta * L)}]$$

Length	Predicted P		
0	0.00	α =	-7.3626
5	0.00	β =	0.14
10	0.00		
15	0.01		
20	0.01		
25	0.02		
30	0.04		
35	0.08		
40	0.15		
45	0.26		
50	0.41		
55	0.58		
60	0.74		
65	0.85		
70	0.92		
75	0.96		
80	0.98		
85	0.99		
90	0.99		
95	1.00		
100	1.00		



Microsoft Excel 8.0 Answer Report
Worksheet: [Chapter 5 - Selectivity Exercises.xls]Exercise 2
Report Created: 5/17/00 9:09:52 AM

Target Cell (Min)

Cell	Name	Original Value	Final Value
I72	Sum of Squares	0.026148906	0.025765418

Adjustable Cells

Cell	Name	Original Value	Final Value
F76	L_{50} After **Solver**	52.59095727	52.56635074
F77	$\alpha 2$ After **Solver**	0.140893899	0.145721666

Constraints
NONE

Microsoft Excel 8.0 Answer Report
Worksheet: [Chapter 5 - Selectivity Exercises.xls]Exercise 2
Report Created: 5/17/00 9:19:01 AM

Target Cell (Value Of)

Cell	Name	Original Value	Final Value
F105	PL_L	0.81	0.75

Adjustable Cells

Cell	Name	Original Value	Final Value
F104	L_{75}	65.00	62.67

Constraints
NONE

Microsoft Excel 8.0 Answer Report
Worksheet: [Chapter 5 - Selectivity Exercises.xls]Exercise 2
Report Created: 5/17/00 9:20:21 AM

Target Cell (Value Of)

Cell	Name	Original Value	Final Value
I105	PL_L	0.20	0.25

Adjustable Cells

Cell	Name	Original Value	Final Value
I104	L_{25}	45.00	46.97

Constraints
NONE

GILLNET SELECTION PROBLEM FOR COD

Fish Length (cm)	A 13.6 cm	B 14.8 cm	C 16.0 cm
46	0	0	0
48	5	0	0
50	26	0	0
52	52	1	0
54	102	16	4
56	295	131	17
58	309	362	95
60	118	326	199
62	79	191	202
64	27	111	133
66	14	44	52
68	8	14	25
70	7	8	15
72	0	1	5
74	0	0	1
76	0	0	0

Part 1: **Determine parameters *a* and *b* for each paired comparison.**

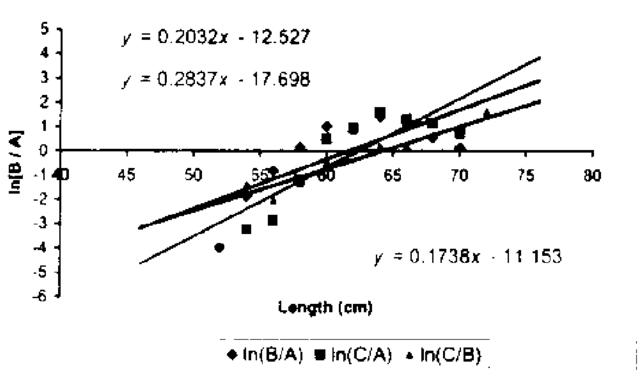
Step 1: Determine ln(B / A) for each comparison

Fish Length (cm)	Webbing 13.6 cm	Webbing 14.8 cm	Webbing 16.0 cm	ln(B/A)	ln(C/A)	ln(C/B)
46	0	0	0			
48	5	0	0			
50	26	0	0			
52	52	1	0	-3.95		
54	102	16	4	-1.85	-3.24	-1.39
56	295	131	17	-0.81	-2.85	-2.04
58	309	362	95	0.16	-1.18	-1.34
60	118	326	199	1.02	0.52	-0.49
62	79	191	202	0.88	0.94	0.06
64	27	111	133	1.41	1.59	0.18
66	14	44	52	1.15	1.31	0.17
68	8	14	25	0.56	1.14	0.58
70	7	8	15	0.13	0.76	0.63
72	0	1	5			1.61
74	0	0	1			
76	0	0	0			

Step 2: Plot ln(B / A) versus length and use the equation

$$\ln(B / A) = a + bL$$

to solve for *a* and *b*



There are 2 methods to calculate the slope and *y*-intercept:

(1) Go to **Tools**, **Data Analysis**, **Regression**, select the input and output ranges, then click OK. The two numbers that are important are the X variable 1 which is the slope and intercept which is the *y*-intercept.

(2) Select the data series on the graph, right click, and choose **Add Trendline**. Choose "linear" for the **Type**. In the **Options** tab, click "display equation on chart."

	Equation	*a*	*b*
A vs. B	$Y = 0.20X - 12.53$	-12.53	0.2
A vs. C	$Y = 0.28X - 17.70$	-17.7	0.28
B vs. C	$Y = 0.17X - 11.15$	-11.15	0.17

Part 2: Determine SD for each paired comparison.

Use the equation

$$SD = [-2*a*(ML_B - ML_A) / (b^2*(ML_A+ML_B))]^{0.5}$$

for each comparison.

A vs. B

ML_A = 13.6
ML_B = 14.8
a = -12.53
b = 0.2

SD = [-2*(-12.53)*(14.8-13.6)/((0.2^2)*(13.6+14.8))]^0.5
SD = 4.83

A vs. C

ML_A = 13.6
ML_C = 16
a = -17.7
b = 0.28

SD = [-2*(-17.7)*(16.0-13.6)/((0.28^2)*(13.6+16.0))]^0.5
SD = 3.97

B vs. C

ML_B = 14.8
ML_C = 16
a = -11.15
b = 0.17

SD = [-2*(-11.15)*(16.0-14.8)/((0.17^2)*(14.8+16.0))]^0.5
SD = 5.81

Part 3: Determine L_{opt} for 13.6, 14.8, and 16.0 webbing

For each comparison, determine L_{opt}s for the two gears using

$$L_{opt}A = -2 * [(a * ml_A) / (b * (ml_A + ml_B))]$$

and

$$L_{opt}B = -2 * [(a * ml_B) / (b * (ml_A + ml_B))]$$

A vs. B

a = -12.53
b = 0.2
ml_A = 13.6
ml_B = 14.8

$L_{opt}A$ = -2*[(-12.53*13.6) / (0.2*(13.6+14.8))]
$L_{opt}A$ = 60.0028169

$L_{opt}B$ = -2*[(-12.53*14.8) / (0.2*(13.6+14.8))]
$L_{opt}B$ = 65.2971831

A vs. C

a = -17.7
b = 0.28
ml_A = 13.6
ml_C = 16

$L_{opt}A$ = -2*[(-17.7*13.6) / (0.28*(13.6+16))]
$L_{opt}A$ = 58.088803

$L_{opt}C$ = -2*[(-17.7*16) / (0.28*(13.6+16))]
$L_{opt}C$ = 68.339768

B vs. C

a = -11.15
b = 0.17
ml_B = 14.8
ml_C = 16

$L_{opt}B$ = -2*[(-11.15*14.8) / (0.17*(14.8+16))]
$L_{opt}B$ = 63.0328495

$L_{opt}C$ = -2*[(-11.15*16) / (0.17*(14.8+16))]
$L_{opt}C$ = 68.14362108

Part 4: **Average the parameters *a*, *b*, SD and L_{opt}s resulting from the two paired comparisons.**

	a	b	SD	$L_{opt}A$
A vs. B	-12.53	0.2	4.83	60.00
A vs. C	-17.7	0.28	3.97	58.09
Average	-15.115	0.24	4.4	59.05

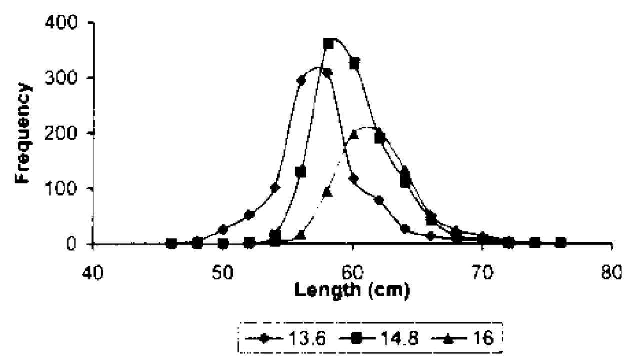
	a	b	SD	$L_{opt}B$
A vs. B	-12.53	0.2	4.83	65.30
B vs. C	-11.15	0.17	5.81	63.03
Average	-11.84	0.185	5.32	64.17

	a	b	SD	$L_{opt}C$
A vs. C	-17.7	0.28	3.97	68.34
B vs. C	-11.15	0.17	5.81	68.14
Average	-14.425	0.225	4.89	68.24

Part 5. **Plot L-F and selectivity curve for each webbing.**

Use the original data to plot L-F

Fish Length (cm)	Webbing 13.6 cm	Webbing 14.8 cm	Webbing 16.0 cm
46	0	0	0
48	5	0	0
50	26	0	0
52	52	1	0
54	102	16	4
56	295	131	17
58	309	362	95
60	118	326	199
62	79	191	202
64	27	111	133
66	14	44	52
68	8	14	25
70	7	8	15
72	0	1	5
74	0	0	1
76	0	0	0

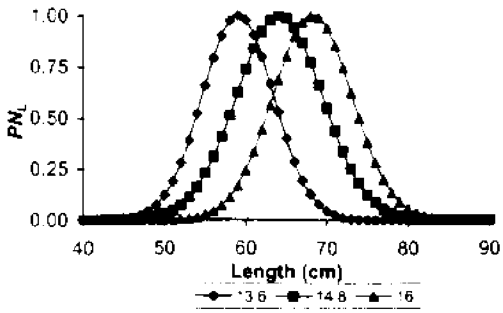


Determine PN_L over a range of length values using the equation

$$PN_L = e^{-[((L-L_{opt})^2 / 2*(SD^2))]}$$

Plot PN_L against L

Length	PN_L A	PN_L B	PN_L C
40	0.00	0.00	0.00
41	0.00	0.00	0.00
42	0.00	0.00	0.00
43	0.00	0.00	0.00
44	0.00	0.00	0.00
45	0.01	0.00	0.00
46	0.01	0.00	0.00
47	0.02	0.01	0.00
48	0.04	0.01	0.00
49	0.07	0.02	0.00
50	0.12	0.03	0.00
51	0.19	0.05	0.00
52	0.28	0.07	0.00
53	0.39	0.11	0.01
54	0.52	0.16	0.01
55	0.66	0.23	0.03
56	0.79	0.31	0.04
57	0.90	0.40	0.07
58	0.97	0.51	0.11
59	1.00	0.62	0.17
60	0.98	0.74	0.24
61	0.91	0.84	0.33
62	0.80	0.92	0.44
63	0.67	0.98	0.56
64	0.53	1.00	0.69
65	0.40	0.99	0.80
66	0.29	0.94	0.90
67	0.20	0.87	0.97
68	0.13	0.77	1.00
69	0.08	0.66	0.99
70	0.05	0.55	0.94
71	0.02	0.44	0.85
72	0.01	0.34	0.74
73	0.01	0.25	0.62
74	0.00	0.18	0.50
75	0.00	0.13	0.38
76	0.00	0.08	0.28
77	0.00	0.05	0.20
78	0.00	0.03	0.14
79	0.00	0.02	0.09
80	0.00	0.01	0.06
81	0.00	0.01	0.03
82	0.00	0.00	0.02
83	0.00	0.00	0.01
84	0.00	0.00	0.01
85	0.00	0.00	0.00
86	0.00	0.00	0.00
87	0.00	0.00	0.00
88	0.00	0.00	0.00
89	0.00	0.00	0.00
90	0.00	0.00	0.00



Northwest Atlantic groundfish species have markedly different growth and mortality rates as indicated in the following table.

Species	K	M	M/K	W_∞	L_∞
*cod	0.12	0.2	1.7	33.7	148
haddock	0.38	0.2	0.5	4.4	74
*silver hake	0.18	0.4	2.2	2.0	65
winter flounder	0.37	0.2	0.5	3.5	63
*yellowtail flounder	0.63	0.2	0.3	0.9	46
plaice	0.17	0.2	1.2	2.4	65
summer flounder	0.21	0.2	1.0	7.6	84

Given this variability in M/K ratios and L_∞, there is the need to have different harvesting strategies in terms of age at entry into the fishery and target fishery mortality levels to maximize yield. The implementation of these strategies requires differing mesh size regulations for a trawl fishery so as to control age at entry, or retention by the gear.

1. Using the Beverton-Holt analytical solution to the yield per recruit problem, compare the harvesting strategies (age at entry to the fishery, and fishing mortality) to maximize yield for cod, silver hake and yellowtail flounder. Note that silver hake and cod have a M/K ratio of about 2.0, while yellowtail flounder has a M/K ratio of less than 0.5. Assuming that the selection factor for diamond mesh trawl codends are 3.7, 3.5, and 2.6 for cod, silver hake, and yellowtail flounder, respectively, and that management seeks to match trawl selection (L_{50}) to YPRMAX, targets, determine the appropriate mesh size for each species. Recall that the simplified von Bertalanffy age-length relationship is $L_t = L_\infty(1 - e^{-Kt})$

2. Using the discrete YPR and SSB model for summer flounder, compare the yield and spawning stock biomass curves for gillnets and trawls if gear regulations are set so as to achieve L_{50}s and L_{opts}, of 35 cm as in the present regulations, and 55 cm as may be a future target.

Notes

A. Summer flounder maturity parameters are $\alpha1 = 5$ and $\beta1 = 2.37$ and length-width relationship parameters are $a = 0.000$, and $b = 3.07$.

B. NPDF SD is 5, and the LCDF steepness 0.33.

C. Develop the discrete time YPR and SSB per recruit curves at F intervals of 0.1 in the range of 0.0 to 0.5, and intervals of 0.5 in the range of 0.5 to 3.0.

D. When evaluating the SSB curve, note the 20% of virgin SSB line.

Use the spreadsheet program in each worksheet to solve for YPR.

Insert the parameter values for each species and use ranges of t_c from 1 until a noticeable maximum yield is reached.

Copy and Paste Special as Values the YPR column (cells K12 to K262) for each t_c value.

Graph all t_c series versus F.

From the graph, select the t_c that gives the highest yield. To obtain exact values, look at the data series.

Next, use the von Bertalanffy growth equation to solve for L_t using the t_c that gives the highest yield.

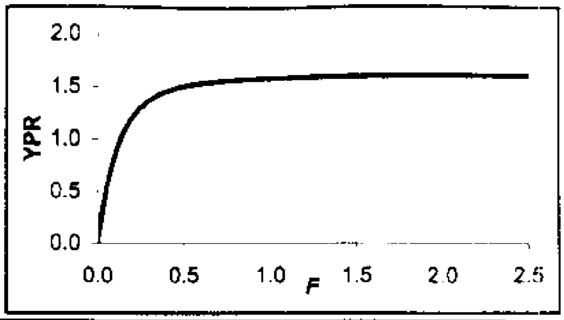
Using that L_t, solve for mesh length by using the equation $ml = L_{50}$/SF.

Beverton-Holt
Yield Per Recruit

COD
Example: $t_c = 8$

Parameter	Value
M	0.2
W_∞	33.7
K	0.12
t_0	0
t_c	8
t_r	0
N_{tr}	1E+05
t_y	20
M/K	1.67

U	n
1	0
-3	1
3	2
-1	3



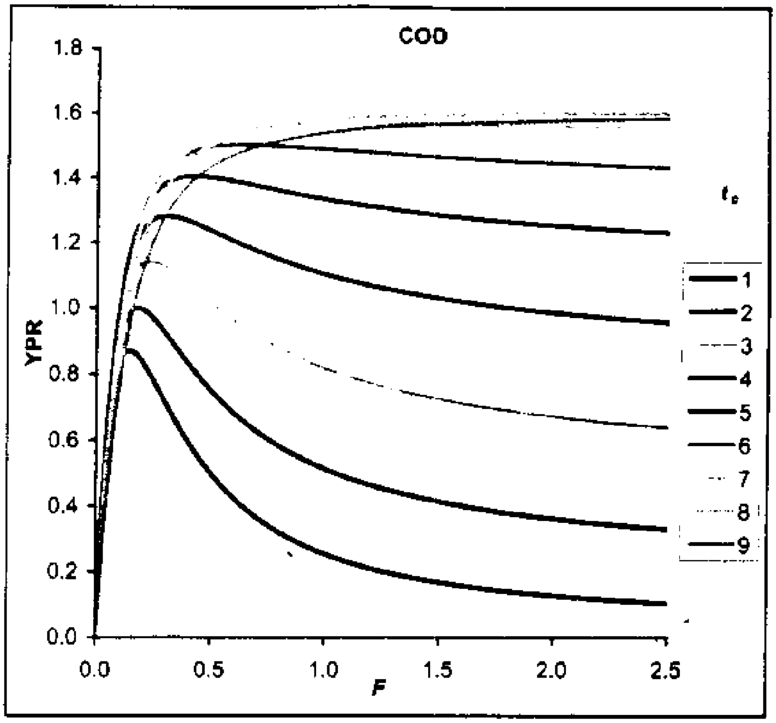
F	FW_∞EXP(..)	C	Y1	Y2-0	Y2-1	Y2-2	Y2-3	Yield	YPR
0	0	0	0	4.5464	-3.512	0.9945	-0.1	0.0000	0
0.01	0.068039127	884	6,804	4.3788	-3.414	0.973	-0.098	12511.4685	0.125115
0.02	0.136078253	1,704	13,608	4.2211	-3.321	0.9523	-0.097	23886.4939	0.238865
0.03	0.20411738	2,467	20,412	4.0726	-3.233	0.9325	-0.095	34237.0335	0.34237
0.04	0.272156506	3,176	27,216	3.9328	-3.148	0.9134	-0.093	43663.5046	0.436635
0.05	0.340195633	3,837	34,020	3.8009	-3.068	0.8951	-0.092	52256.0066	0.52256
0.06	0.408234759	4,453	40,823	3.6763	-2.991	0.8775	-0.09	60095.4109	0.600954
0.07	0.476273886	5,029	47,627	3.5587	-2.918	0.8605	-0.089	67254.3344	0.672543
0.08	0.544313013	5,568	54,431	3.4474	-2.848	0.8442	-0.088	73798.0078	0.73798
0.09	0.612352139	6,073	61,235	3.342	-2.781	0.8284	-0.086	79785.0515	0.797851
0.1	0.680391266	6,546	68,039	3.2423	-2.717	0.8132	-0.085	85268.1672	0.852682
0.11	0.748430392	6,990	74,843	3.1476	-2.656	0.7986	-0.084	90294.7569	0.902948
0.12	0.816469519	7,408	81,647	3.0578	-2.597	0.7844	-0.083	94907.4750	0.949075
0.13	0.884508645	7,802	88,451	2.9725	-2.541	0.7708	-0.081	99144.7216	0.991447
0.14	0.952547772	8,173	95,255	2.8914	-2.487	0.7576	-0.08	103041.0839	1.030411
0.15	1.020586898	8,523	102,059	2.8143	-2.435	0.7448	-0.079	106627.7300	1.066277
0.16	1.088626025	8,854	108,863	2.7408	-2.386	0.7325	-0.078	109932.7614	1.099328
0.17	1.156665152	9,167	115,667	2.6708	-2.338	0.7205	-0.077	112981.5283	1.129815
0.18	1.224704278	9,463	122,470	2.604	-2.292	0.709	-0.076	115796.9110	1.157969
0.19	1.292743405	9,745	129,274	2.5403	-2.247	0.6978	-0.075	118399.5726	1.183996
0.2	1.360782531	10,012	136,078	2.4794	-2.205	0.6869	-0.074	120808.1843	1.208082
0.21	1.428821658	10,268	142,882	2.4212	-2.164	0.6764	-0.073	123039.6274	1.230396
0.22	1.496860784	10,507	149,686	2.3655	-2.124	0.6662	-0.072	125109.1743	1.251092
0.23	1.564899911	10,737	156,490	2.3122	-2.086	0.6562	-0.071	127030.6503	1.270307
0.24	1.632939038	10,956	163,294	2.2612	-2.049	0.6466	-0.07	128816.5789	1.288166
0.25	1.700978164	11,166	170,098	2.2122	-2.013	0.6373	-0.069	130478.3117	1.304783
0.26	1.769017291	11,366	176,902	2.1652	-1.979	0.6282	-0.068	132026.1448	1.320261
0.27	1.837056417	11,557	183,706	2.1201	-1.945	0.6193	-0.068	133469.4239	1.334694
0.28	1.905095544	11,740	190,510	2.0768	-1.913	0.6108	-0.067	134816.6371	1.348166
0.29	1.97313467	11,916	197,313	2.0351	-1.882	0.6024	-0.066	136075.5002	1.360755
0.3	2.041173797	12,084	204,117	1.995	-1.852	0.5943	-0.065	137253.0312	1.37253
0.31	2.109212923	12,245	210,921	1.9565	-1.822	0.5864	-0.065	138355.6185	1.383556
0.32	2.17725205	12,400	217,725	1.9193	-1.794	0.5786	-0.064	139389.0819	1.393891
0.33	2.245291177	12,549	224,529	1.8835	-1.766	0.5711	-0.063	140358.7274	1.403587
0.34	2.313330303	12,693	231,333	1.849	-1.74	0.5638	-0.062	141269.3958	1.412694
0.35	2.38136943	12,830	238,137	1.8157	-1.714	0.5567	-0.062	142125.5078	1.421255
0.36	2.449408556	12,963	244,941	1.7836	-1.689	0.5497	-0.061	142931.1031	1.429311
0.37	2.517447683	13,092	251,745	1.7525	-1.664	0.543	-0.06	143689.8767	1.436899
0.38	2.585486809	13,215	258,549	1.7225	-1.641	0.5363	-0.06	144405.2107	1.444052
0.39	2.653525936	13,334	265,353	1.6935	-1.618	0.5299	-0.059	145080.2039	1.450802
0.4	2.721565063	13,450	272,157	1.6654	-1.595	0.5236	-0.058	145717.6975	1.457177
0.41	2.789604189	13,561	278,960	1.6383	-1.573	0.5174	-0.058	146320.2992	1.463203
0.42	2.857643316	13,669	285,764	1.612	-1.552	0.5114	-0.057	146890.4039	1.468904
0.43	2.925682442	13,773	292,568	1.5865	-1.531	0.5055	-0.057	147430.2136	1.474302
0.44	2.993721569	13,874	299,372	1.5618	-1.511	0.4998	-0.056	147941.7543	1.479418
0.45	3.061760695	13,972	306,176	1.5378	-1.492	0.4942	-0.056	148426.8918	1.484269
0.46	3.129799822	14,066	312,980	1.5146	-1.473	0.4887	-0.055	148887.3460	1.488873

0.47	3.197838949	14,158	319,784	1.4921	-1.454	0.4833	-0.054	149324.7038	1.493247
0.48	3.265878075	14,247	326,588	1.4702	-1.436	0.4781	-0.054	149740.4302	1.497404
0.49	3.333917202	14,334	333,392	1.4489	-1.418	0.4729	-0.053	150135.8795	1.501359
0.5	3.401956328	14,416	340,196	1.4283	-1.401	0.4679	-0.053	150512.3044	1.505123
0.51	3.469995455	14,500	347,000	1.4082	-1.384	0.463	-0.052	150870.8649	1.508709
0.52	3.538034581	14,579	353,803	1.3886	-1.367	0.4581	-0.052	151212.6358	1.512126
0.53	3.606073708	14,656	360,607	1.3696	-1.351	0.4534	-0.051	151538.6140	1.515386
0.54	3.674112834	14,731	367,411	1.3512	-1.336	0.4488	-0.051	151849.7252	1.518497
0.55	3.742151961	14,804	374,215	1.3332	-1.32	0.4443	-0.051	152146.8293	1.521468
0.56	3.810191088	14,875	381,019	1.3156	-1.305	0.4398	-0.05	152430.7261	1.524307
0.57	3.878230214	14,944	387,823	1.2986	-1.291	0.4355	-0.05	152702.1601	1.527022
0.58	3.946269341	15,012	394,627	1.2819	-1.276	0.4312	-0.049	152961.8247	1.529618
0.59	4.014308467	15,077	401,431	1.2657	-1.262	0.427	-0.049	153210.3665	1.532104
0.6	4.082347594	15,141	408,235	1.2499	-1.249	0.4229	-0.048	153448.3890	1.534484
0.61	4.15038672	15,204	415,039	1.2345	-1.235	0.4189	-0.048	153676.4556	1.536765
0.62	4.218425847	15,265	421,843	1.2194	-1.222	0.4149	-0.048	153895.0931	1.538951
0.63	4.286464974	15,324	428,646	1.2048	-1.209	0.411	-0.047	154104.7943	1.541048
0.64	4.3545041	15,382	435,450	1.1904	-1.197	0.4072	-0.047	154306.0205	1.54306
0.65	4.422543227	15,439	442,254	1.1764	-1.184	0.4035	-0.046	154499.2040	1.544992
0.66	4.490582353	15,494	449,058	1.1628	-1.172	0.3998	-0.046	154684.7501	1.546848
0.67	4.55862148	15,548	455,862	1.1494	-1.16	0.3962	-0.046	154863.0391	1.54863
0.68	4.626660606	15,601	462,666	1.1363	-1.149	0.3927	-0.045	155034.4283	1.550344
0.69	4.694699733	15,652	469,470	1.1236	-1.137	0.3892	-0.045	155199.2535	1.551993
0.7	4.762738859	15,703	476,274	1.1111	-1.126	0.3858	-0.045	155357.8303	1.553578
0.71	4.830777986	15,752	483,078	1.0989	-1.115	0.3825	-0.044	155510.4560	1.555105
0.72	4.898817113	15,800	489,882	1.0869	-1.104	0.3792	-0.044	155657.4105	1.556574
0.73	4.966856239	15,848	496,686	1.0753	-1.094	0.3759	-0.044	155798.9577	1.55799
0.74	5.034895366	15,894	503,490	1.0638	-1.084	0.3727	-0.043	155935.3463	1.559353
0.75	5.102934492	15,939	510,293	1.0526	-1.074	0.3696	-0.043	156066.8112	1.560668
0.76	5.170973619	15,983	517,097	1.0417	-1.064	0.3665	-0.043	156193.5743	1.561936
0.77	5.239012745	16,027	523,901	1.0309	-1.054	0.3635	-0.042	156315.8450	1.563158
0.78	5.307051872	16,069	530,705	1.0204	-1.044	0.3605	-0.042	156433.8217	1.564338
0.79	5.375090999	16,111	537,509	1.0101	-1.035	0.3576	-0.042	156547.6917	1.565477
0.8	5.443130125	16,152	544,313	1	-1.026	0.3547	-0.041	156657.6327	1.566576
0.81	5.511169252	16,192	551,117	0.9901	-1.017	0.3519	-0.041	156763.8129	1.567638
0.82	5.579208378	16,231	557,921	0.9804	-1.008	0.3491	-0.041	156866.3916	1.568664
0.83	5.647247505	16,269	564,725	0.9709	-0.999	0.3463	-0.04	156965.5201	1.569655
0.84	5.715286631	16,307	571,529	0.9615	-0.99	0.3436	-0.04	157061.3418	1.570613
0.85	5.783325758	16,344	578,333	0.9524	-0.982	0.3409	-0.04	157153.9930	1.57154
0.86	5.851364885	16,380	585,136	0.9434	-0.973	0.3383	-0.04	157243.6030	1.572436
0.87	5.919404011	16,416	591,940	0.9346	-0.965	0.3357	-0.039	157330.2948	1.573303
0.88	5.987443138	16,451	598,744	0.9259	-0.957	0.3332	-0.039	157414.1853	1.574142
0.89	6.055482264	16,485	605,548	0.9174	-0.949	0.3307	-0.039	157495.3859	1.574954
0.9	6.123521391	16,519	612,352	0.9091	-0.942	0.3282	-0.038	157574.0024	1.57574
0.91	6.191560517	16,552	619,156	0.9009	-0.934	0.3258	-0.038	157650.1356	1.576501
0.92	6.259599644	16,584	625,960	0.8929	-0.926	0.3234	-0.038	157723.8817	1.577239
0.93	6.32763877	16,616	632,764	0.885	-0.919	0.321	-0.038	157795.3322	1.577953
0.94	6.395677897	16,646	639,568	0.8772	-0.912	0.3187	-0.037	157864.5744	1.578646
0.95	6.463717024	16,678	646,372	0.8696	-0.904	0.3164	-0.037	157931.6917	1.579317
0.96	6.53175615	16,709	653,176	0.8621	-0.897	0.3142	-0.037	157996.7635	1.579968
0.97	6.599795277	16,738	659,980	0.8547	-0.89	0.3119	-0.037	158059.8659	1.580599
0.98	6.667834403	16,768	666,783	0.8475	-0.884	0.3097	-0.036	158121.0712	1.581211
0.99	6.73587353	16,796	673,587	0.8403	-0.877	0.3076	-0.036	158180.4489	1.581804
1	6.803912656	16,825	680,391	0.8333	-0.87	0.3054	-0.036	158238.0651	1.582381
1.01	6.871951783	16,853	687,195	0.8264	-0.864	0.3033	-0.036	158293.9832	1.58294
1.02	6.93999091	16,880	693,999	0.8197	-0.857	0.3012	-0.036	158348.2639	1.583483
1.03	7.008030036	16,907	700,803	0.813	-0.851	0.2992	-0.035	158400.9651	1.58401
1.04	7.076069163	16,933	707,607	0.8065	-0.845	0.2972	-0.035	158452.1423	1.584521
1.05	7.144108289	16,959	714,411	0.8	-0.838	0.2952	-0.035	158501.8486	1.585018
1.06	7.212147416	16,985	721,215	0.7937	-0.832	0.2932	-0.035	158550.1355	1.585501

1.07	7.280186542	17,010	728,019	0.7874	-0.826	0.2913	-0.034	158597.0513	1.585971
1.08	7.348225669	17,035	734,823	0.7812	-0.82	0.2894	-0.034	158642.6429	1.586426
1.09	7.416264795	17,059	741,628	0.7752	-0.815	0.2875	-0.034	158686.9553	1.58687
1.1	7.484303922	17,084	748,430	0.7692	-0.809	0.2856	-0.034	158730.0314	1.5873
1.11	7.552343049	17,107	755,234	0.7634	-0.803	0.2838	-0.034	158771.9128	1.587719
1.12	7.620382175	17,131	762,038	0.7576	-0.798	0.2819	-0.033	158812.6390	1.588126
1.13	7.688421302	17,154	768,842	0.7519	-0.792	0.2801	-0.033	158852.2481	1.588522
1.14	7.756460428	17,176	775,646	0.7463	-0.787	0.2784	-0.033	158890.7768	1.588908
1.15	7.824499555	17,199	782,450	0.7407	-0.781	0.2766	-0.033	158928.2602	1.589283
1.16	7.892538681	17,221	789,254	0.7353	-0.776	0.2749	-0.033	158964.7321	1.589647
1.17	7.960577808	17,242	796,058	0.7299	-0.771	0.2732	-0.032	159000.2249	1.590002
1.18	8.028616935	17,264	802,862	0.7246	-0.766	0.2715	-0.032	159034.7700	1.590348
1.19	8.096656061	17,285	809,666	0.7194	-0.761	0.2698	-0.032	159068.3973	1.590684
1.2	8.164695188	17,305	816,470	0.7143	-0.756	0.2682	-0.032	159101.1359	1.591011
1.21	8.232734314	17,326	823,273	0.7092	-0.751	0.2666	-0.032	159133.0134	1.59133
1.22	8.300773441	17,346	830,077	0.7042	-0.746	0.265	-0.032	159164.0567	1.591641
1.23	8.368812567	17,366	836,881	0.6993	-0.741	0.2634	-0.031	159194.2916	1.591943
1.24	8.436851694	17,386	843,685	0.6944	-0.736	0.2618	-0.031	159223.7428	1.592237
1.25	8.504890821	17,405	850,489	0.6897	-0.732	0.2602	-0.031	159252.4345	1.592524
1.26	8.572929947	17,424	857,293	0.6849	-0.727	0.2587	-0.031	159280.3895	1.592804
1.27	8.640969074	17,443	864,097	0.6803	-0.722	0.2572	-0.031	159307.6302	1.593076
1.28	8.7090082	17,461	870,901	0.6757	-0.718	0.2557	-0.031	159334.1780	1.593342
1.29	8.777047327	17,480	877,705	0.6711	-0.713	0.2542	-0.03	159360.0534	1.593601
1.3	8.845086453	17,498	884,509	0.6667	-0.709	0.2528	-0.03	159385.2765	1.593853
1.31	8.91312558	17,516	891,313	0.6623	-0.705	0.2513	-0.03	159409.8665	1.594099
1.32	8.981164706	17,533	898,116	0.6579	-0.7	0.2499	-0.03	159433.8420	1.594338
1.33	9.049203833	17,550	904,920	0.6536	-0.696	0.2485	-0.03	159457.2207	1.594572
1.34	9.11724296	17,568	911,724	0.6494	-0.692	0.2471	-0.03	159480.0202	1.5948
1.35	9.185282086	17,585	918,528	0.6452	-0.688	0.2457	-0.029	159502.2569	1.595023
1.36	9.253321213	17,601	925,332	0.641	-0.684	0.2443	-0.029	159523.9471	1.595239
1.37	9.321360339	17,618	932,136	0.6369	-0.68	0.243	-0.029	159545.1063	1.595451
1.38	9.389399466	17,634	938,940	0.6329	-0.676	0.2417	-0.029	159565.7496	1.595657
1.39	9.457438592	17,650	945,744	0.6289	-0.672	0.2403	-0.029	159585.8915	1.595859
1.4	9.525477719	17,666	952,548	0.625	-0.668	0.239	-0.029	159605.5461	1.596055
1.41	9.593516846	17,682	959,352	0.6211	-0.664	0.2377	-0.028	159624.7269	1.596247
1.42	9.661555972	17,697	966,156	0.6173	-0.66	0.2365	-0.028	159643.4471	1.596434
1.43	9.729595099	17,712	972,960	0.6135	-0.656	0.2352	-0.028	159661.7195	1.596617
1.44	9.797634225	17,727	979,763	0.6098	-0.653	0.2339	-0.028	159679.5563	1.596796
1.45	9.865673352	17,742	986,567	0.6061	-0.649	0.2327	-0.028	159696.9694	1.59697
1.46	9.933712478	17,757	993,371	0.6024	-0.645	0.2315	-0.028	159713.9703	1.59714
1.47	10.0017516	17,772	1,000,175	0.5988	-0.642	0.2303	-0.028	159730.5701	1.597306
1.48	10.06979073	17,786	1,006,979	0.5952	-0.638	0.2291	-0.028	159746.7797	1.597468
1.49	10.13782986	17,800	1,013,783	0.5917	-0.635	0.2279	-0.027	159762.6095	1.597626
1.5	10.20586898	17,814	1,020,587	0.5882	-0.631	0.2267	-0.027	159778.0697	1.597781
1.51	10.27390811	17,828	1,027,391	0.5848	-0.628	0.2255	-0.027	159793.1699	1.597932
1.52	10.34194724	17,842	1,034,195	0.5814	-0.624	0.2244	-0.027	159807.9197	1.598079
1.53	10.40998636	17,856	1,040,999	0.578	-0.621	0.2233	-0.027	159822.3283	1.598223
1.54	10.47802549	17,869	1,047,803	0.5747	-0.618	0.2221	-0.027	159836.4046	1.598364
1.55	10.54606462	17,882	1,054,606	0.5714	-0.614	0.221	-0.027	159850.1572	1.598502
1.56	10.61410374	17,895	1,061,410	0.5682	-0.611	0.2199	-0.026	159863.5946	1.598636
1.57	10.68214287	17,908	1,068,214	0.565	-0.608	0.2188	-0.026	159876.7248	1.598767
1.58	10.750182	17,921	1,075,018	0.5618	-0.605	0.2177	-0.026	159889.5557	1.598896
1.59	10.81822112	17,934	1,081,822	0.5587	-0.601	0.2167	-0.026	159902.0949	1.599021
1.6	10.88626025	17,946	1,088,626	0.5556	-0.598	0.2156	-0.026	159914.3499	1.599143
1.61	10.95429938	17,959	1,095,430	0.5525	-0.595	0.2145	-0.026	159926.3278	1.599263
1.62	11.0223385	17,971	1,102,234	0.5495	-0.592	0.2135	-0.026	159938.0355	1.59938
1.63	11.09037763	17,983	1,109,038	0.5464	-0.589	0.2125	-0.026	159949.4800	1.599495
1.64	11.15841676	17,995	1,115,842	0.5435	-0.586	0.2115	-0.026	159960.6677	1.599607
1.65	11.22645588	18,007	1,122,646	0.5405	-0.583	0.2104	-0.025	159971.6050	1.599716
1.66	11.29449501	18,019	1,129,450	0.5376	-0.58	0.2094	-0.025	159982.2980	1.599823

1.67	11.36253414	18,030	1,136,253	0.5348	-0.577	0.2084	-0.025	159992.7529	1.599928
1.68	11.43057326	18,042	1,143,057	0.5319	-0.574	0.2075	-0.025	160002.9754	1.60003
1.69	11.49861239	18,053	1,149,861	0.5291	-0.571	0.2065	-0.025	160012.9713	1.60013
1.7	11.56665152	18,064	1,156,665	0.5263	-0.569	0.2055	-0.025	160022.7459	1.600227
1.71	11.63469064	18,076	1,163,469	0.5236	-0.566	0.2046	-0.025	160032.3047	1.600323
1.72	11.70272977	18,087	1,170,273	0.5208	-0.563	0.2036	-0.025	160041.6529	1.600417
1.73	11.7707689	18,097	1,177,077	0.5181	-0.56	0.2027	-0.025	160050.7955	1.600508
1.74	11.83880802	18,108	1,183,881	0.5155	-0.558	0.2018	-0.024	160059.7374	1.600597
1.75	11.90684715	18,119	1,190,685	0.5128	-0.555	0.2008	-0.024	160068.4835	1.600685
1.76	11.97488628	18,129	1,197,489	0.5102	-0.552	0.1999	-0.024	160077.0382	1.60077
1.77	12.0429254	18,140	1,204,293	0.5076	-0.55	0.199	-0.024	160085.4063	1.600854
1.78	12.11096453	18,150	1,211,096	0.5051	-0.547	0.1981	-0.024	160093.5920	1.600936
1.79	12.17900365	18,161	1,217,900	0.5025	-0.544	0.1972	-0.024	160101.5996	1.601016
1.8	12.24704278	18,171	1,224,704	0.5	-0.542	0.1963	-0.024	160109.4333	1.601094
1.81	12.31508191	18,181	1,231,508	0.4975	-0.539	0.1955	-0.024	160117.0971	1.601171
1.82	12.38312103	18,191	1,238,312	0.495	-0.537	0.1946	-0.024	160124.5949	1.601246
1.83	12.45116016	18,201	1,245,116	0.4926	-0.534	0.1938	-0.023	160131.9306	1.601319
1.84	12.51919929	18,210	1,251,920	0.4902	-0.532	0.1929	-0.023	160139.1079	1.601391
1.85	12.58723841	18,220	1,258,724	0.4878	-0.529	0.1921	-0.023	160146.1305	1.601461
1.86	12.65527754	18,229	1,265,528	0.4854	-0.527	0.1912	-0.023	160153.0017	1.60153
1.87	12.72331667	18,239	1,272,332	0.4831	-0.525	0.1904	-0.023	160159.7251	1.601597
1.88	12.79135579	18,248	1,279,136	0.4808	-0.522	0.1896	-0.023	160166.3041	1.601663
1.89	12.85939492	18,258	1,285,939	0.4785	-0.52	0.1888	-0.023	160172.7418	1.601727
1.9	12.92743405	18,267	1,292,743	0.4762	-0.517	0.188	-0.023	160179.0415	1.60179
1.91	12.99547317	18,276	1,299,547	0.4739	-0.515	0.1872	-0.023	160185.2062	1.601852
1.92	13.0635123	18,285	1,306,351	0.4717	-0.513	0.1864	-0.023	160191.2389	1.601912
1.93	13.13155143	18,294	1,313,155	0.4695	-0.511	0.1856	-0.023	160197.1427	1.601971
1.94	13.19959055	18,303	1,319,959	0.4673	-0.508	0.1848	-0.022	160202.9202	1.602029
1.95	13.26762968	18,312	1,326,763	0.4651	-0.506	0.184	-0.022	160208.5744	1.602086
1.96	13.33566881	18,320	1,333,567	0.463	-0.504	0.1833	-0.022	160214.1078	1.602141
1.97	13.40370793	18,329	1,340,371	0.4608	-0.502	0.1825	-0.022	160219.5233	1.602195
1.98	13.47174706	18,337	1,347,175	0.4587	-0.499	0.1817	-0.022	160224.8232	1.602248
1.99	13.53978619	18,346	1,353,979	0.4566	-0.497	0.181	-0.022	160230.0103	1.6023
2	13.60782531	18,354	1,360,783	0.4545	-0.495	0.1803	-0.022	160235.0868	1.602351
2.01	13.67586444	18,363	1,367,586	0.4525	-0.493	0.1795	-0.022	160240.0552	1.602401
2.02	13.74390357	18,371	1,374,390	0.4505	-0.491	0.1788	-0.022	160244.9178	1.602449
2.03	13.81194269	18,379	1,381,194	0.4484	-0.489	0.1781	-0.022	160249.6770	1.602497
2.04	13.87998182	18,387	1,387,998	0.4464	-0.487	0.1773	-0.022	160254.3348	1.602543
2.05	13.94802095	18,395	1,394,802	0.4444	-0.485	0.1766	-0.022	160258.8934	1.602589
2.06	14.01606007	18,403	1,401,606	0.4425	-0.483	0.1759	-0.021	160263.3551	1.602634
2.07	14.0840992	18,411	1,408,410	0.4405	-0.481	0.1752	-0.021	160267.7217	1.602677
2.08	14.15213833	18,419	1,415,214	0.4386	-0.479	0.1745	-0.021	160271.9954	1.60272
2.09	14.22017745	18,426	1,422,018	0.4367	-0.477	0.1738	-0.021	160276.1781	1.602762
2.1	14.28821658	18,434	1,428,822	0.4348	-0.475	0.1732	-0.021	160280.2716	1.602803
2.11	14.35625571	18,442	1,435,626	0.4329	-0.473	0.1725	-0.021	160284.2779	1.602843
2.12	14.42429483	18,449	1,442,429	0.431	-0.471	0.1718	-0.021	160288.1988	1.602882
2.13	14.49233396	18,457	1,449,233	0.4292	-0.469	0.1711	-0.021	160292.0361	1.60292
2.14	14.56037308	18,464	1,456,037	0.4274	-0.467	0.1705	-0.021	160295.7914	1.602958
2.15	14.62841221	18,471	1,462,841	0.4255	-0.465	0.1698	-0.021	160299.4665	1.602995
2.16	14.69645134	18,479	1,469,645	0.4237	-0.463	0.1692	-0.021	160303.0630	1.603031
2.17	14.76449046	18,486	1,476,449	0.4219	-0.461	0.1685	-0.021	160306.5826	1.603066
2.18	14.83252959	18,493	1,483,253	0.4202	-0.459	0.1679	-0.02	160310.0268	1.6031
2.19	14.90056872	18,500	1,490,057	0.4184	-0.458	0.1672	-0.02	160313.3971	1.603134
2.2	14.96860784	18,507	1,496,861	0.4167	-0.456	0.1666	-0.02	160316.6952	1.603167
2.21	15.03664697	18,514	1,503,665	0.4149	-0.454	0.166	-0.02	160319.9223	1.603199
2.22	15.1046861	18,521	1,510,469	0.4132	-0.452	0.1653	-0.02	160323.0800	1.603231
2.23	15.17272522	18,528	1,517,273	0.4115	-0.45	0.1647	-0.02	160326.1697	1.603262
2.24	15.24076435	18,535	1,524,076	0.4098	-0.449	0.1641	-0.02	160329.1928	1.603292
2.25	15.30880348	18,542	1,530,880	0.4082	-0.447	0.1635	-0.02	160332.1505	1.603322
2.26	15.3768426	18,548	1,537,684	0.4065	-0.445	0.1629	-0.02	160335.0442	1.60335

2.27	15.44488173	18,555	1,544,488	0.4049	-0.444	0.1623	-0.02	160337.8751	**1.603379**
2.28	15.51292086	18,561	1,551,292	0.4032	-0.442	0.1617	-0.02	160340.6446	**1.603406**
2.29	15.58095998	18,568	1,558,096	0.4016	-0.44	0.1611	-0.02	160343.3538	**1.603434**
2.3	15.64899911	18,574	1,564,900	0.4	-0.438	0.1605	-0.02	160346.0039	**1.60346**
2.31	15.71703824	18,581	1,571,704	0.3984	-0.437	0.1599	-0.02	160348.5962	**1.603486**
2.32	15.78507736	18,587	1,578,508	0.3968	-0.435	0.1594	-0.019	160351.1316	**1.603511**
2.33	15.85311649	18,594	1,585,312	0.3953	-0.433	0.1588	-0.019	160353.6115	**1.603536**
2.34	15.92115562	18,600	1,592,116	0.3937	-0.432	0.1582	-0.019	160356.0367	**1.60356**
2.35	15.98919474	18,606	1,598,919	0.3922	-0.43	0.1576	-0.019	160358.4085	**1.603584**
2.36	16.05723387	18,612	1,605,723	0.3906	-0.429	0.1571	-0.019	160360.7278	**1.603607**
2.37	16.125273	18,618	1,612,527	0.3891	-0.427	0.1565	-0.019	160362.9957	**1.60363**
2.38	16.19331212	18,625	1,619,331	0.3876	-0.425	0.156	-0.019	160365.2132	**1.603652**
2.39	16.26135125	18,631	1,626,135	0.3861	-0.424	0.1554	-0.019	160367.3813	**1.603674**
2.4	16.32939038	18,637	1,632,939	0.3846	-0.422	0.1549	-0.019	160369.5008	**1.603695**
2.41	16.3974295	18,643	1,639,743	0.3831	-0.421	0.1543	-0.019	160371.5728	**1.603716**
2.42	16.46546863	18,648	1,646,547	0.3817	-0.419	0.1538	-0.019	160373.5981	**1.603736**
2.43	16.53350776	18,654	1,653,351	0.3802	-0.418	0.1532	-0.019	160375.5777	**1.603756**
2.44	16.60154688	18,660	1,660,155	0.3788	-0.416	0.1527	-0.019	160377.5124	**1.603775**
2.45	16.66958601	18,666	1,666,959	0.3774	-0.415	0.1522	-0.019	160379.4031	**1.603794**
2.46	16.73762513	18,672	1,673,763	0.3759	-0.413	0.1517	-0.019	160381.2506	**1.603813**
2.47	16.80566426	18,677	1,680,566	0.3745	-0.412	0.1511	-0.019	160383.0558	**1.603831**
2.48	16.87370339	18,683	1,687,370	0.3731	-0.41	0.1506	-0.018	160384.8194	**1.603848**
2.49	16.94174251	18,689	1,694,174	0.3717	-0.409	0.1501	-0.018	160386.5422	**1.603865**
2.5	17.00978164	18,694	1,700,978	0.3704	-0.407	0.1496	-0.018	160388.2251	**1.603882**



F	$t_c = 1$	$t_c = 2$	$t_c = 3$	$t_c = 4$	$t_c = 5$	$t_c = 6$	$t_c = 7$	$t_c = 8$	$t_c = 9$
0	0	0	0	0	0	0	0	0	0
0.01	0.1781	0.1787	0.1769	0.1719	0.1636	0.152569	0.1395	0.1251	0.1102
0.02	0.325	0.3292	0.3286	0.3216	0.308	0.288754	0.2652	0.2389	0.2112
0.03	0.4456	0.4556	0.4586	0.452	0.4356	0.410416	0.3786	0.3424	0.3039
0.04	0.5442	0.5616	0.5699	0.5657	0.5483	0.519195	0.481	0.4366	0.3889
0.05	0.6242	0.6502	0.6652	0.6647	0.648	0.616535	0.5735	0.5226	0.467
0.06	0.6885	0.7239	0.7465	0.751	0.7362	0.703715	0.6573	0.601	0.5388
0.07	0.7397	0.7848	0.8159	0.8262	0.8142	0.781862	0.7331	0.6725	0.6048
0.08	0.7798	0.835	0.8748	0.8917	0.8834	0.851972	0.8018	0.738	0.6656
0.09	0.8106	0.876	0.9249	0.9486	0.9446	0.914927	0.8641	0.7979	0.7217
0.1	0.8336	0.909	0.9672	0.9982	0.999	0.971505	0.9207	0.8527	0.7734
0.11	0.8501	0.9354	1.0028	1.0412	1.0471	1.022397	0.9722	0.9029	0.8211
0.12	0.8611	0.9561	1.0326	1.0786	1.0899	1.068213	1.019	0.9491	0.8652
0.13	0.8675	0.9719	1.0574	1.111	1.1278	1.109494	1.0617	0.9914	0.906
0.14	0.8701	0.9835	1.0779	1.139	1.1614	1.146722	1.1006	1.0304	0.9438
0.15	0.8696	0.9916	1.0946	1.1632	1.1913	1.180321	1.1361	1.0663	0.9788
0.16	0.8664	0.9967	1.1081	1.184	1.2178	1.210672	1.1686	1.0993	1.0113
0.17	0.8611	0.9993	1.1188	1.2019	1.2413	1.238108	1.1982	1.1298	1.0414
0.18	0.854	0.9998	1.127	1.2171	1.2622	1.262931	1.2254	1.158	1.0695
0.19	0.8455	0.9984	1.1332	1.2301	1.2807	1.285406	1.2503	1.184	1.0955
0.2	0.8358	0.9955	1.1375	1.2411	1.2971	1.30577	1.2732	1.2081	1.1198
0.21	0.8252	0.9913	1.1403	1.2503	1.3117	1.324234	1.2942	1.2304	1.1425
0.22	0.8139	0.9861	1.1417	1.258	1.3245	1.340987	1.3135	1.2511	1.1636
0.23	0.802	0.9799	1.1419	1.2643	1.3359	1.356198	1.3312	1.2703	1.1833
0.24	0.7897	0.973	1.1412	1.2694	1.346	1.370016	1.3476	1.2882	1.2018
0.25	0.7772	0.9655	1.1395	1.2734	1.3548	1.382576	1.3627	1.3048	1.219
0.26	0.7644	0.9576	1.1372	1.2765	1.3626	1.393996	1.3767	1.3203	1.2352
0.27	0.7515	0.9492	1.1342	1.2788	1.3694	1.404391	1.3896	1.3347	1.2504
0.28	0.7385	0.9405	1.1306	1.2803	1.3754	1.413851	1.4015	1.3482	1.2646
0.29	0.7256	0.9316	1.1266	1.2813	1.3806	1.422465	1.4126	1.3608	1.278
0.3	0.7128	0.9225	1.1223	1.2816	1.3852	1.430311	1.4228	1.3725	1.2906
0.31	0.7	0.9133	1.1175	1.2815	1.3891	1.437458	1.4323	1.3836	1.3025

0.32	0.6874	0.9041	1.1126	1.2809	1.3924	1.443971	1.4411	1.3939	1.3136
0.33	0.6749	0.8948	1.1074	1.2799	1.3953	1.449905	1.4493	1.4036	1.3242
0.34	0.6627	0.8854	1.102	1.2786	1.3977	1.455313	1.4569	1.4127	1.3341
0.35	0.6506	0.8761	1.0964	1.277	1.3997	1.46024	1.464	1.4213	1.3436
0.36	0.6387	0.8669	1.0908	1.2752	1.4013	1.464728	1.4707	1.4293	1.3525
0.37	0.6271	0.8577	1.085	1.2731	1.4026	1.468815	1.4768	1.4369	1.3609
0.38	0.6157	0.8486	1.0792	1.2708	1.4036	1.472535	1.4826	1.4441	1.3689
0.39	0.6046	0.8396	1.0733	1.2684	1.4044	1.47592	1.488	1.4508	1.3765
0.4	0.5936	0.8307	1.0674	1.2658	1.4049	1.478997	1.493	1.4572	1.3837
0.41	0.583	0.8219	1.0615	1.2631	1.4052	1.481793	1.4977	1.4632	1.3905
0.42	0.5725	0.8132	1.0556	1.2603	1.4053	1.484329	1.5021	1.4689	1.397
0.43	0.5623	0.8047	1.0498	1.2574	1.4052	1.486627	1.5062	1.4743	1.4032
0.44	0.5524	0.7963	1.0439	1.2544	1.4049	1.488708	1.5101	1.4794	1.4091
0.45	0.5427	0.788	1.0381	1.2514	1.4046	1.490587	1.5137	1.4843	1.4147
0.46	0.5332	0.7799	1.0323	1.2483	1.404	1.492281	1.5171	1.4889	1.4201
0.47	0.524	0.7719	1.0265	1.2452	1.4034	1.493805	1.5203	1.4932	1.4252
0.48	0.5149	0.7641	1.0209	1.242	1.4027	1.495172	1.5233	1.4974	1.4301
0.49	0.5062	0.7564	1.0152	1.2388	1.4019	1.496394	1.5261	1.5014	1.4348
0.5	0.4976	0.7489	1.0096	1.2356	1.401	1.497483	1.5288	1.5051	1.4393
0.51	0.4892	0.7415	1.0041	1.2324	1.4	1.498448	1.5313	1.5087	1.4436
0.52	0.4811	0.7342	0.9987	1.2291	1.399	1.4993	1.5336	1.5121	1.4477
0.53	0.4731	0.7271	0.9933	1.2259	1.3978	1.500046	1.5358	1.5154	1.4517
0.54	0.4654	0.7201	0.988	1.2227	1.3967	1.500695	1.5379	1.5185	1.4554
0.55	0.4578	0.7133	0.9828	1.2195	1.3955	1.501254	1.5399	1.5215	1.4591
0.56	0.4505	0.7065	0.9776	1.2163	1.3942	1.501729	1.5417	1.5243	1.4626
0.57	0.4433	0.7	0.9725	1.2131	1.393	1.502128	1.5434	1.527	1.4659
0.58	0.4363	0.6935	0.9675	1.2099	1.3916	1.502455	1.5451	1.5296	1.4692
0.59	0.4295	0.6872	0.9626	1.2067	1.3903	1.502716	1.5466	1.5321	1.4723
0.6	0.4228	0.681	0.9577	1.2036	1.3889	1.502916	1.5481	1.5345	1.4753
0.61	0.4164	0.675	0.9529	1.2005	1.3875	1.503059	1.5495	1.5368	1.4781
0.62	0.41	0.669	0.9482	1.1974	1.3861	1.50315	1.5508	1.539	1.4809
0.63	0.4039	0.6632	0.9436	1.1943	1.3847	1.503191	1.552	1.541	1.4836
0.64	0.3978	0.6575	0.939	1.1913	1.3833	1.503186	1.5531	1.5431	1.4862
0.65	0.392	0.6519	0.9345	1.1883	1.3818	1.503143	1.5542	1.545	1.4887
0.66	0.3863	0.6465	0.9301	1.1853	1.3804	1.503059	1.5553	1.5468	1.4911
0.67	0.3807	0.6411	0.9257	1.1824	1.3789	1.502939	1.5562	1.5486	1.4934
0.68	0.3752	0.6358	0.9215	1.1795	1.3774	1.502786	1.5571	1.5503	1.4957
0.69	0.3699	0.6307	0.9172	1.1766	1.376	1.502602	1.558	1.552	1.4978
0.7	0.3647	0.6256	0.9131	1.1737	1.3745	1.502389	1.5588	1.5536	1.5
0.71	0.3596	0.6207	0.909	1.1709	1.373	1.502151	1.5596	1.5551	1.502
0.72	0.3547	0.6158	0.905	1.1681	1.3716	1.501887	1.5603	1.5566	1.504
0.73	0.3499	0.6111	0.9011	1.1654	1.3701	1.501601	1.561	1.558	1.5059
0.74	0.3452	0.6064	0.8972	1.1627	1.3686	1.501294	1.5617	1.5594	1.5077
0.75	0.3405	0.6019	0.8933	1.16	1.3672	1.500968	1.5623	1.5607	1.5095
0.76	0.336	0.5974	0.8896	1.1573	1.3657	1.500623	1.5628	1.5619	1.5113
0.77	0.3317	0.593	0.8859	1.1547	1.3643	1.500263	1.5634	1.5632	1.513
0.78	0.3274	0.5887	0.8823	1.1521	1.3628	1.499887	1.5639	1.5643	1.5146
0.79	0.3232	0.5845	0.8787	1.1496	1.3614	1.499497	1.5644	1.5655	1.5162
0.8	0.3191	0.5803	0.8752	1.147	1.36	1.499095	1.5648	1.5666	1.5177
0.81	0.3151	0.5762	0.8717	1.1445	1.3586	1.49868	1.5653	1.5676	1.5192
0.82	0.3111	0.5723	0.8683	1.1421	1.3571	1.498255	1.5656	1.5687	1.5207
0.83	0.3073	0.5683	0.8649	1.1396	1.3557	1.49782	1.566	1.5697	1.5221
0.84	0.3036	0.5645	0.8616	1.1372	1.3543	1.497376	1.5664	1.5706	1.5235
0.85	0.2999	0.5607	0.8584	1.1349	1.353	1.496923	1.5667	1.5715	1.5248
0.86	0.2963	0.557	0.8552	1.1325	1.3516	1.496464	1.567	1.5724	1.5262
0.87	0.2928	0.5534	0.852	1.1302	1.3502	1.495997	1.5673	1.5733	1.5274
0.88	0.2894	0.5498	0.8489	1.1279	1.3489	1.495524	1.5676	1.5741	1.5287
0.89	0.286	0.5464	0.8459	1.1257	1.3475	1.495046	1.5678	1.575	1.5299
0.9	0.2827	0.5429	0.8429	1.1234	1.3462	1.494563	1.5681	1.5757	1.531
0.91	0.2795	0.5395	0.8399	1.1213	1.3449	1.494075	1.5683	1.5765	1.5322

0.92	0.2764	0.5362	0.837	1.1191	1.3436	1.493584	1.5685	1.5772	1.5333
0.93	0.2733	0.533	0.8342	1.117	1.3423	1.493089	1.5687	1.578	1.5344
0.94	0.2703	0.5298	0.8313	1.1148	1.341	1.492591	1.5688	1.5786	1.5354
0.9[illegible]	0.2673	0.5266	0.8286	1.1128	1.3397	1.49209	1.569	1.5793	1.5365
0.9[illegible]	0.2644	0.5236	0.8258	1.1107	1.3384	1.491588	1.5691	1.58	1.5375
0.9[illegible]	0.2616	0.5205	0.8231	1.1087	1.3372	1.491083	1.5693	1.5806	1.5385
0.9[illegible]	0.2588	0.5176	0.8205	1.1067	1.3359	1.490577	1.5694	1.5812	1.5394
0.99	0.2561	0.5146	0.8179	1.1047	1.3347	1.49007	1.5695	1.5818	1.5404
1	0.2534	0.5118	0.8153	1.1027	1.3335	1.489562	1.5696	1.5824	1.5413
1.01	0.2508	0.5089	0.8128	1.1008	1.3323	1.489053	1.5697	1.5829	1.5422
1.02	0.2482	0.5061	0.8103	1.0989	1.3311	1.488544	1.5698	1.5835	1.543
1.03	0.2457	0.5034	0.8078	1.097	1.3299	1.488035	1.5699	1.584	1.5439
1.04	0.2433	0.5007	0.8054	1.0952	1.3287	1.487526	1.5699	1.5845	1.5447
1.05	0.2409	0.4981	0.803	1.0933	1.3276	1.487017	1.57	1.585	1.5455
1.06	0.2385	0.4955	0.8007	1.0915	1.3264	1.486509	1.57	1.5855	1.5463
1.07	0.2362	0.4929	0.7984	1.0897	1.3253	1.486001	1.5701	1.586	1.5471
1.08	0.2339	0.4904	0.7961	1.088	1.3241	1.485495	1.5701	1.5864	1.5479
1.09	0.2316	0.4879	0.7938	1.0862	1.323	1.484989	1.5701	1.5869	1.5486
1.1	0.2295	0.4855	0.7916	1.0845	1.3219	1.484484	1.5701	1.5873	1.5493
1.11	0.2273	0.4831	0.7894	1.0828	1.3208	1.483981	1.5702	1.5877	1.55
1.12	0.2252	0.4808	0.7873	1.0811	1.3197	1.48348	1.5702	1.5881	1.5507
1.13	0.2231	0.4785	0.7851	1.0795	1.3187	1.48298	1.5702	1.5885	1.5514
1.14	0.2211	0.4762	0.783	1.0778	1.3176	1.482481	1.5702	1.5889	1.5521
1.15	0.2191	0.4739	0.781	1.0762	1.3165	1.481985	1.5701	1.5893	1.5527
1.16	0.2171	0.4717	0.7789	1.0746	1.3155	1.48149	1.5701	1.5896	1.5534
1.17	0.2152	0.4695	0.7769	1.073	1.3145	1.480997	1.5701	1.59	1.554
1.18	0.2133	0.4674	0.7749	1.0715	1.3134	1.480506	1.5701	1.5903	1.5546
1.19	0.2114	0.4653	0.773	1.0699	1.3124	1.480018	1.5701	1.5907	1.5552
1.2	0.2096	0.4632	0.771	1.0684	1.3114	1.479531	1.57	1.591	1.5558
1.21	0.2078	0.4612	0.7691	1.0669	1.3104	1.479047	1.57	1.5913	1.5564
1.22	0.2061	0.4592	0.7673	1.0654	1.3094	1.478566	1.57	1.5916	1.5569
1.23	0.2043	0.4572	0.7654	1.064	1.3085	1.478086	1.5699	1.5919	1.5575
1.24	0.2026	0.4552	0.7636	1.0625	1.3075	1.477609	1.5699	1.5922	1.558
1.25	0.201	0.4533	0.7618	1.0611	1.3065	1.477135	1.5698	1.5925	1.5586
1.26	0.1993	0.4514	0.76	1.0597	1.3056	1.476663	1.5698	1.5928	1.5591
1.27	0.1977	0.4495	0.7583	1.0583	1.3047	1.476193	1.5697	1.5931	1.5596
1.28	0.1961	0.4477	0.7565	1.0569	1.3037	1.475727	1.5696	1.5933	1.5601
1.29	0.1946	0.4459	0.7548	1.0555	1.3028	1.475262	1.5696	1.5936	1.5606
1.3	0.193	0.4441	0.7531	1.0542	1.3019	1.474801	1.5695	1.5939	1.5611
1.31	0.1915	0.4423	0.7515	1.0528	1.301	1.474342	1.5694	1.5941	1.5616
1.32	0.19	0.4406	0.7498	1.0515	1.3001	1.473886	1.5694	1.5943	1.562
1.33	0.1886	0.4389	0.7482	1.0502	1.2992	1.473433	1.5693	1.5946	1.5625
1.34	0.1871	0.4372	0.7466	1.0489	1.2983	1.472982	1.5692	1.5948	1.5629
1.35	0.1857	0.4356	0.745	1.0476	1.2975	1.472534	1.5692	1.595	1.5634
1.36	0.1843	0.4339	0.7434	1.0464	1.2966	1.472089	1.5691	1.5952	1.5638
1.37	0.183	0.4322	0.7419	1.0451	1.2958	1.471646	1.569	1.5955	1.5642
1.38	0.1816	0.4306	0.7404	1.0439	1.2949	1.471207	1.5689	1.5957	1.5646
1.39	0.1803	0.4291	0.7389	1.0427	1.2941	1.47077	1.5688	1.5959	1.5651
1.4	0.179	0.4275	0.7374	1.0415	1.2933	1.470336	1.5687	1.5961	1.5655
1.4[illegible]	0.1777	0.426	0.7359	1.0403	1.2924	1.469905	1.5687	1.5962	1.5659
1.42	0.1764	0.4245	0.7344	1.0391	1.2916	1.469476	1.5686	1.5964	1.5662
1.43	0.1752	0.423	0.733	1.0379	1.2908	1.469051	1.5685	1.5966	1.5666
1.44	0.174	0.4215	0.7316	1.0368	1.29	1.468628	1.5684	1.5968	1.567
1.45	0.1728	0.42	0.7302	1.0356	1.2893	1.468208	1.5683	1.597	1.5674
1.46	0.1716	0.4186	0.7288	1.0345	1.2885	1.46779	1.5682	1.5971	1.5677
1.47	0.1704	0.4172	0.7274	1.0334	1.2877	1.467376	1.5681	1.5973	1.5681
1.48	0.1693	0.4158	0.7261	1.0323	1.2869	1.466964	1.568	1.5975	1.5684
1.49	0.1681	0.4144	0.7248	1.0312	1.2862	1.466555	1.5679	1.5976	1.5688
1.5	0.167	0.4131	0.7234	1.0301	1.2854	1.466149	1.5678	1.5978	1.5691
1.51	0.1659	0.4117	0.7221	1.029	1.2847	1.465746	1.5677	1.5979	1.5695

1.52	0.1648	0.4104	0.7208	1.028	1.2839	1.465345	1.5676	1.5981	1.5698
1.53	0.1638	0.4091	0.7196	1.0269	1.2832	1.464947	1.5675	1.5982	1.5701
1.54	0.1627	0.4078	0.7183	1.0259	1.2825	1.464552	1.5674	1.5984	1.5704
1.55	0.1617	0.4065	0.7171	1.0249	1.2818	1.46416	1.5673	1.5985	1.5708
1.56	0.1607	0.4052	0.7158	1.0238	1.2811	1.46377	1.5672	1.5986	1.5711
1.57	0.1596	0.404	0.7146	1.0228	1.2804	1.463383	1.5671	1.5988	1.5714
1.58	0.1587	0.4028	0.7134	1.0218	1.2797	1.462998	1.567	1.5989	1.5717
1.59	0.1577	0.4016	0.7122	1.0209	1.279	1.462616	1.5669	1.599	1.572
1.6	0.1567	0.4004	0.711	1.0199	1.2783	1.462237	1.5668	1.5991	1.5723
1.61	0.1558	0.3992	0.7099	1.0189	1.2776	1.461861	1.5667	1.5993	1.5725
1.62	0.1548	0.398	0.7087	1.018	1.2769	1.461487	1.5666	1.5994	1.5728
1.63	0.1539	0.3968	0.7076	1.017	1.2763	1.461116	1.5665	1.5995	1.5731
1.64	0.153	0.3957	0.7065	1.0161	1.2756	1.460747	1.5664	1.5996	1.5734
1.65	0.1521	0.3946	0.7054	1.0152	1.275	1.460381	1.5663	1.5997	1.5736
1.66	0.1512	0.3935	0.7042	1.0142	1.2743	1.460017	1.5662	1.5998	1.5739
1.67	0.1503	0.3924	0.7032	1.0133	1.2737	1.459656	1.5661	1.5999	1.5742
1.68	0.1495	0.3913	0.7021	1.0124	1.273	1.459298	1.566	1.6	1.5744
1.69	0.1486	0.3902	0.701	1.0115	1.2724	1.458942	1.5659	1.6001	1.5747
1.7	0.1478	0.3891	0.7	1.0107	1.2718	1.458588	1.5658	1.6002	1.5749
1.71	0.1469	0.3881	0.6989	1.0098	1.2712	1.458237	1.5657	1.6003	1.5752
1.72	0.1461	0.387	0.6979	1.0089	1.2705	1.457889	1.5656	1.6004	1.5754
1.73	0.1453	0.386	0.6969	1.0081	1.2699	1.457543	1.5655	1.6005	1.5757
1.74	0.1445	0.385	0.6958	1.0072	1.2693	1.457199	1.5654	1.6006	1.5759
1.75	0.1437	0.384	0.6948	1.0064	1.2687	1.456857	1.5653	1.6007	1.5761
1.76	0.1429	0.383	0.6939	1.0055	1.2681	1.456519	1.5651	1.6008	1.5764
1.77	0.1422	0.382	0.6929	1.0047	1.2675	1.456182	1.565	1.6009	1.5766
1.78	0.1414	0.381	0.6919	1.0039	1.267	1.455848	1.5649	1.6009	1.5768
1.79	0.1407	0.3801	0.6909	1.0031	1.2664	1.455516	1.5648	1.601	1.577
1.8	0.1399	0.3791	0.69	1.0023	1.2658	1.455186	1.5647	1.6011	1.5772
1.81	0.1392	0.3782	0.689	1.0015	1.2652	1.454859	1.5646	1.6012	1.5775
1.82	0.1385	0.3773	0.6881	1.0007	1.2647	1.454534	1.5645	1.6012	1.5777
1.83	0.1378	0.3764	0.6872	0.9999	1.2641	1.454211	1.5644	1.6013	1.5779
1.84	0.1371	0.3754	0.6863	0.9992	1.2636	1.453891	1.5643	1.6014	1.5781
1.85	0.1364	0.3745	0.6854	0.9984	1.263	1.453572	1.5642	1.6015	1.5783
1.86	0.1357	0.3737	0.6845	0.9976	1.2625	1.453256	1.5641	1.6015	1.5785
1.87	0.135	0.3728	0.6836	0.9969	1.2619	1.452942	1.564	1.6016	1.5787
1.88	0.1344	0.3719	0.6827	0.9961	1.2614	1.452631	1.5639	1.6017	1.5789
1.89	0.1337	0.371	0.6818	0.9954	1.2609	1.452321	1.5638	1.6017	1.5791
1.9	0.1331	0.3702	0.681	0.9947	1.2603	1.452014	1.5637	1.6018	1.5793
1.91	0.1324	0.3694	0.6801	0.994	1.2598	1.451708	1.5636	1.6019	1.5794
1.92	0.1318	0.3685	0.6793	0.9932	1.2593	1.451405	1.5635	1.6019	1.5796
1.93	0.1312	0.3677	0.6785	0.9925	1.2588	1.451104	1.5634	1.602	1.5798
1.94	0.1305	0.3669	0.6776	0.9918	1.2583	1.450805	1.5633	1.602	1.58
1.95	0.1299	0.3661	0.6768	0.9911	1.2578	1.450508	1.5632	1.6021	1.5802
1.96	0.1293	0.3653	0.676	0.9904	1.2572	1.450213	1.563	1.6021	1.5803
1.97	0.1287	0.3645	0.6752	0.9897	1.2567	1.44992	1.5629	1.6022	1.5805
1.98	0.1281	0.3637	0.6744	0.9891	1.2563	1.449629	1.5628	1.6022	1.5807
1.99	0.1276	0.3629	0.6736	0.9884	1.2558	1.44934	1.5627	1.6023	1.5809
2	0.127	0.3622	0.6728	0.9877	1.2553	1.449053	1.5626	1.6024	1.581
2.01	0.1264	0.3614	0.672	0.9871	1.2548	1.448768	1.5625	1.6024	1.5812
2.02	0.1259	0.3606	0.6713	0.9864	1.2543	1.448484	1.5624	1.6024	1.5813
2.03	0.1253	0.3599	0.6705	0.9858	1.2538	1.448203	1.5623	1.6025	1.5815
2.04	0.1247	0.3592	0.6698	0.9851	1.2534	1.447924	1.5622	1.6025	1.5817
2.05	0.1242	0.3584	0.669	0.9845	1.2529	1.447646	1.5621	1.6026	1.5818
2.06	0.1237	0.3577	0.6683	0.9838	1.2524	1.447371	1.562	1.6026	1.582
2.07	0.1231	0.357	0.6675	0.9832	1.252	1.447097	1.5619	1.6027	1.5821
2.08	0.1226	0.3563	0.6668	0.9826	1.2515	1.446825	1.5618	1.6027	1.5823
2.09	0.1221	0.3556	0.6661	0.982	1.2511	1.446555	1.5617	1.6028	1.5824
2.1	0.1216	0.3549	0.6654	0.9813	1.2506	1.446286	1.5616	1.6028	1.5826
2.11	0.1211	0.3542	0.6647	0.9807	1.2502	1.44602	1.5615	1.6028	1.5827

2.12	0.1206	0.3535	0.664	0.9801	1.2497	1.445755	1.5614	1.6029	1.5829
2.13	0.1201	0.3528	0.6633	0.9795	1.2493	1.445492	1.5613	1.6029	1.583
2.14	0.1196	0.3522	0.6626	0.9789	1.2489	1.445231	1.5612	1.603	1.5832
2.15	0.1191	0.3515	0.6619	0.9784	1.2484	1.444971	1.5611	1.603	1.5833
2.16	0.1186	0.3509	0.6612	0.9778	1.248	1.444713	1.561	1.603	1.5834
2.17	0.1181	0.3502	0.6605	0.9772	1.2476	1.444457	1.5609	1.6031	1.5836
2.18	0.1177	0.3496	0.6599	0.9766	1.2471	1.444203	1.5608	1.6031	1.5837
2.19	0.1172	0.3489	0.6592	0.976	1.2467	1.44395	1.5607	1.6031	1.5838
2.2	0.1167	0.3483	0.6586	0.9755	1.2463	1.443699	1.5607	1.6032	1.584
2.21	0.1163	0.3477	0.6579	0.9749	1.2459	1.443449	1.5606	1.6032	1.5841
2.22	0.1158	0.3471	0.6573	0.9744	1.2455	1.443201	1.5605	1.6032	1.5842
2.23	0.1154	0.3464	0.6566	0.9738	1.2451	1.442955	1.5604	1.6033	1.5844
2.24	0.1149	0.3458	0.656	0.9733	1.2447	1.44271	1.5603	1.6033	1.5845
2.25	0.1145	0.3452	0.6554	0.9727	1.2443	1.442467	1.5602	1.6033	1.5846
2.26	0.1141	0.3446	0.6548	0.9722	1.2439	1.442226	1.5601	1.6034	1.5847
2.27	0.1136	0.344	0.6541	0.9716	1.2435	1.441986	1.56	1.6034	1.5848
2.28	0.1132	0.3434	0.6535	0.9711	1.2431	1.441747	1.5599	1.6034	1.585
2.29	0.1128	0.3429	0.6529	0.9706	1.2427	1.441511	1.5598	1.6034	1.5851
2.3	0.1124	0.3423	0.6523	0.9701	1.2423	1.441275	1.5597	1.6035	1.5852
2.31	0.112	0.3417	0.6517	0.9695	1.2419	1.441041	1.5596	1.6035	1.5853
2.32	0.1116	0.3412	0.6511	0.969	1.2415	1.440809	1.5595	1.6035	1.5854
2.33	0.1112	0.3406	0.6505	0.9685	1.2412	1.440578	1.5594	1.6035	1.5855
2.34	0.1108	0.34	0.65	0.968	1.2408	1.440348	1.5593	1.6036	1.5857
2.35	0.1104	0.3395	0.6494	0.9675	1.2404	1.44012	1.5592	1.6036	1.5858
2.36	0.11	0.3389	0.6488	0.967	1.24	1.439894	1.5592	1.6036	1.5859
2.37	0.1096	0.3384	0.6482	0.9665	1.2397	1.439669	1.5591	1.6036	1.586
2.38	0.1092	0.3379	0.6477	0.966	1.2393	1.439445	1.559	1.6037	1.5861
2.39	0.1088	0.3373	0.6471	0.9655	1.2389	1.439223	1.5589	1.6037	1.5862
2.4	0.1085	0.3368	0.6466	0.965	1.2386	1.439002	1.5588	1.6037	1.5863
2.41	0.1081	0.3363	0.646	0.9646	1.2382	1.438782	1.5587	1.6037	1.5864
2.42	0.1077	0.3358	0.6455	0.9641	1.2379	1.438564	1.5586	1.6037	1.5865
2.43	0.1073	0.3352	0.6449	0.9636	1.2375	1.438347	1.5585	1.6038	1.5866
2.44	0.107	0.3347	0.6444	0.9631	1.2372	1.438131	1.5584	1.6038	1.5867
2.45	0.1066	0.3342	0.6439	0.9627	1.2368	1.437917	1.5584	1.6038	1.5868
2.46	0.1063	0.3337	0.6433	0.9622	1.2365	1.437704	1.5583	1.6038	1.5869
2.47	0.1059	0.3332	0.6428	0.9618	1.2361	1.437493	1.5582	1.6038	1.587
2.48	0.1056	0.3327	0.6423	0.9613	1.2358	1.437283	1.5581	1.6038	1.5871
2.49	0.1052	0.3322	0.6418	0.9608	1.2354	1.437074	1.558	1.6039	1.5872
2.5	0.1049	0.3318	0.6412	0.9604	1.2351	1.436866	1.5579	1.6039	1.5873

For cod, the t_c that gives the highest yield is age 8.

Solving for L_t:

$L_t = L_\infty * (1-e^{(-Kt)})$

$L_t = 148 * (1-e^{(-0.12*8)})$

91.332

Then, solve for the mesh length:

$ml = L_{50}/SF$

ml = 91.33/3.7

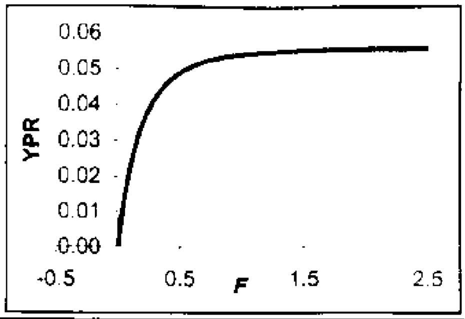
24.684 cm

Beverton-Holt
Yield Per Recruit

SILVER HAKE
Example: $t_c = 4$

Parameter	Value
M	0.4
W_∞	2
K	0.18
t_0	0
t_c	4
t_r	0
N_r	1E+05
t_y	20
M/K	2.22

U	n
1	0
-3	1
3	2
-1	3



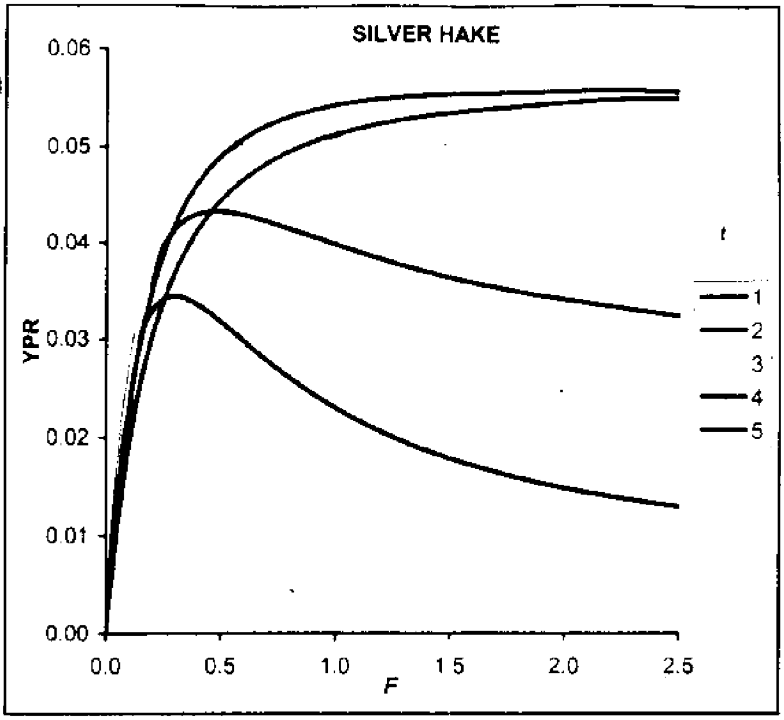
F	FW_∞EXP(..)	C	Y1	Y2-0	Y2-1	Y2-2	Y2-3	Yield	YPR
0	0	0	0	2.4958	-2.517	0.9352	-0.123	0.0000	**0**
0.01	0.00403793	492	404	2.4356	-2.475	0.9231	-0.121	307.8730	**0.003079**
0.02	0.008075861	960	808	2.3781	-2.434	0.9113	-0.12	594.0684	**0.005941**
0.03	0.012113791	1,407	1,211	2.3232	-2.394	0.8997	-0.119	860.4376	**0.008604**
0.04	0.016151721	1,834	1,615	2.2707	-2.355	0.8885	-0.118	1108.6494	**0.011086**
0.05	0.020189652	2,242	2,019	2.2206	-2.318	0.8775	-0.116	1340.2104	**0.013402**
0.06	0.024227582	2,632	2,423	2.1725	-2.282	0.8668	-0.115	1556.4824	**0.015565**
0.07	0.028265513	3,005	2,827	2.1265	-2.246	0.8564	-0.114	1758.6978	**0.017587**
0.08	0.032303443	3,363	3,230	2.0824	-2.212	0.8462	-0.113	1947.9735	**0.01948**
0.09	0.036341373	3,707	3,634	2.04	-2.179	0.8362	-0.112	2125.3228	**0.021253**
0.1	0.040379304	4,037	4,038	1.9993	-2.147	0.8265	-0.111	2291.6663	**0.022917**
0.11	0.044417234	4,353	4,442	1.9602	-2.116	0.817	-0.11	2447.8413	**0.024478**
0.12	0.048455164	4,658	4,846	1.9226	-2.086	0.8077	-0.109	2594.6106	**0.025946**
0.13	0.052493095	4,951	5,249	1.8864	-2.057	0.7986	-0.108	2732.6697	**0.027327**
0.14	0.056531025	5,233	5,653	1.8515	-2.028	0.7898	-0.107	2862.6534	**0.028627**
0.15	0.060568955	5,505	6,057	1.8179	-2	0.7811	-0.106	2985.1424	**0.029851**
0.16	0.064606886	5,768	6,461	1.7855	-1.973	0.7726	-0.105	3100.6678	**0.031007**
0.17	0.068644816	6,021	6,864	1.7542	-1.947	0.7643	-0.104	3209.7163	**0.032097**
0.18	0.072682746	6,265	7,268	1.724	-1.921	0.7562	-0.103	3312.7345	**0.033127**
0.19	0.076720677	6,501	7,672	1.6948	-1.896	0.7482	-0.102	3410.1324	**0.034101**
0.2	0.080758607	6,729	8,076	1.6666	-1.872	0.7404	-0.101	3502.2871	**0.035023**
0.21	0.084796538	6,950	8,480	1.6392	-1.848	0.7328	-0.1	3589.5457	**0.035895**
0.22	0.088834468	7,164	8,883	1.6128	-1.825	0.7253	-0.099	3672.2279	**0.036722**
0.23	0.092872398	7,371	9,287	1.5872	-1.803	0.718	-0.099	3750.6286	**0.037506**
0.24	0.096910329	7,571	9,691	1.5624	-1.781	0.7108	-0.098	3825.0204	**0.03825**
0.25	0.100948259	7,765	10,095	1.5384	-1.759	0.7037	-0.097	3895.6550	**0.038957**
0.26	0.104986189	7,953	10,499	1.5151	-1.738	0.6968	-0.096	3962.7653	**0.039628**
0.27	0.10902412	8,136	10,902	1.4925	-1.718	0.6901	-0.095	4026.5672	**0.040266**
0.28	0.11306205	8,313	11,306	1.4706	-1.698	0.6834	-0.095	4087.2607	**0.040873**
0.29	0.11709998	8,485	11,710	1.4493	-1.678	0.6769	-0.094	4145.0316	**0.04145**
0.3	0.121137911	8,653	12,114	1.4286	-1.659	0.6706	-0.093	4200.0522	**0.042001**
0.31	0.125175841	8,815	12,518	1.4084	-1.641	0.6643	-0.092	4252.4829	**0.042525**
0.32	0.129213772	8,973	12,921	1.3889	-1.623	0.6581	-0.092	4302.4730	**0.043025**
0.33	0.133251702	9,127	13,325	1.3699	-1.605	0.6521	-0.091	4350.1613	**0.043502**
0.34	0.137289632	9,276	13,729	1.3513	-1.587	0.6462	-0.09	4395.6774	**0.043957**
0.35	0.141327563	9,422	14,133	1.3333	-1.57	0.6403	-0.089	4439.1422	**0.044391**
0.36	0.145365493	9,563	14,537	1.3158	-1.553	0.6346	-0.089	4480.6686	**0.044807**
0.37	0.149403423	9,701	14,940	1.2987	-1.537	0.629	-0.088	4520.3619	**0.045204**
0.38	0.153441354	9,836	15,344	1.282	-1.521	0.6235	-0.087	4558.3208	**0.045583**
0.39	0.157479284	9,967	15,748	1.2658	-1.505	0.6181	-0.087	4594.6376	**0.045946**
0.4	0.161517214	10,095	16,152	1.25	-1.49	0.6127	-0.086	4629.3988	**0.046294**
0.41	0.165555145	10,219	16,556	1.2346	-1.475	0.6075	-0.085	4662.6853	**0.046627**
0.42	0.169593075	10,341	16,959	1.2195	-1.46	0.6024	-0.085	4694.5732	**0.046946**
0.43	0.173631005	10,460	17,363	1.2048	-1.446	0.5973	-0.084	4725.1338	**0.047251**
0.44	0.177668936	10,576	17,767	1.1905	-1.432	0.5923	-0.084	4754.4342	**0.047544**
0.45	0.181706866	10,689	18,171	1.1765	-1.418	0.5874	-0.083	4782.5373	**0.047825**
0.46	0.185744797	10,799	18,574	1.1628	-1.404	0.5826	-0.082	4809.5023	**0.048095**

0.47	0.189782727	10,907	18,978	1.1494	-1.391	0.5779	-0.082	4835.3850	**0.048354**
0.48	0.193820657	11,013	19,382	1.1364	-1.378	0.5732	-0.081	4860.2379	**0.048602**
0.49	0.197858588	11,116	19,786	1.1236	-1.365	0.5686	-0.081	4884.1106	**0.048841**
0.5	0.201896518	11,216	20,190	1.1111	-1.352	0.5641	-0.08	4907.0495	**0.04907**
0.51	0.205934448	11,315	20,593	1.0989	-1.34	0.5597	-0.08	4929.0989	**0.049291**
0.52	0.209972379	11,412	20,997	1.087	-1.328	0.5553	-0.079	4950.3001	**0.049503**
0.53	0.214010309	11,506	21,401	1.0753	-1.316	0.551	-0.078	4970.6924	**0.049707**
0.54	0.218048239	11,598	21,805	1.0638	-1.304	0.5468	-0.078	4990.3130	**0.049903**
0.55	0.22208617	11,689	22,209	1.0526	-1.292	0.5426	-0.077	5009.1969	**0.050092**
0.56	0.2261241	11,777	22,612	1.0417	-1.281	0.5385	-0.077	5027.3772	**0.050274**
0.57	0.230162031	11,864	23,016	1.0309	-1.27	0.5344	-0.076	5044.8854	**0.050449**
0.58	0.234199961	11,949	23,420	1.0204	-1.259	0.5304	-0.076	5061.7512	**0.050618**
0.59	0.238237891	12,032	23,824	1.0101	-1.248	0.5265	-0.075	5078.0028	**0.05078**
0.6	0.242275822	12,114	24,228	1	-1.238	0.5226	-0.075	5093.6669	**0.050937**
0.61	0.246313752	12,194	24,631	0.9901	-1.227	0.5188	-0.074	5108.7689	**0.051088**
0.62	0.250351682	12,272	25,035	0.9804	-1.217	0.5151	-0.074	5123.3327	**0.051233**
0.63	0.254389613	12,349	25,439	0.9709	-1.207	0.5114	-0.073	5137.3812	**0.051374**
0.64	0.258427543	12,424	25,843	0.9615	-1.197	0.5077	-0.073	5150.9361	**0.051509**
0.65	0.262465473	12,498	26,247	0.9524	-1.187	0.5041	-0.073	5164.0180	**0.05164**
0.66	0.266503404	12,571	26,650	0.9434	-1.178	0.5006	-0.072	5176.6465	**0.051766**
0.67	0.270541334	12,642	27,054	0.9346	-1.168	0.4971	-0.072	5188.8401	**0.051888**
0.68	0.274579264	12,712	27,458	0.9259	-1.159	0.4936	-0.071	5200.6166	**0.052006**
0.69	0.278617195	12,781	27,862	0.9174	-1.15	0.4902	-0.071	5211.9928	**0.05212**
0.7	0.282655125	12,848	28,266	0.9091	-1.141	0.4868	-0.07	5222.9849	**0.05223**
0.71	0.286693056	12,914	28,669	0.9009	-1.132	0.4835	-0.07	5233.6081	**0.052336**
0.72	0.290730986	12,979	29,073	0.8929	-1.123	0.4803	-0.069	5243,8769	**0.052439**
0.73	0.294768916	13,043	29,477	0.885	-1.115	0.477	-0.069	5253.8054	**0.052538**
0.74	0.298806847	13,106	29,881	0.8772	-1.106	0.4739	-0.069	5263.4066	**0.052634**
0.75	0.302844777	13,167	30,284	0.8696	-1.098	0.4707	-0.068	5272.6934	**0.052727**
0.76	0.306882707	13,228	30,688	0.8621	-1.09	0.4676	-0.068	5281.6776	**0.052817**
0.77	0.310920638	13,287	31,092	0.8547	-1.082	0.4646	-0.067	5290.3709	**0.052904**
0.78	0.314958568	13,346	31,496	0.8475	-1.074	0.4615	-0.067	5298.7843	**0.052988**
0.79	0.318996498	13,403	31,900	0.8403	-1.066	0.4586	-0.067	5306.9283	**0.053069**
0.8	0.323034429	13,460	32,303	0.8333	-1.058	0.4556	-0.066	5314.8128	**0.053148**
0.81	0.327072359	13,515	32,707	0.8264	-1.051	0.4527	-0.066	5322.4476	**0.053224**
0.82	0.33111029	13,570	33,111	0.8197	-1.043	0.4499	-0.066	5329.8418	**0.053298**
0.83	0.33514822	13,624	33,515	0.813	-1.036	0.447	-0.065	5337.0042	**0.05337**
0.84	0.33918615	13,677	33,919	0.8065	-1.028	0.4442	-0.065	5343.9432	**0.053439**
0.85	0.343224081	13,729	34,322	0.8	-1.021	0.4415	-0.064	5350.6669	**0.053507**
0.86	0.347262011	13,780	34,726	0.7937	-1.014	0.4388	-0.064	5357.1829	**0.053572**
0.87	0.351299941	13,831	35,130	0.7874	-1.007	0.4361	-0.064	5363.4987	**0.053635**
0.88	0.355337872	13,880	35,534	0.7812	-1	0.4334	-0.063	5369.6213	**0.053696**
0.89	0.359375802	13,929	35,938	0.7752	-0.993	0.4308	-0.063	5375.5575	**0.053756**
0.9	0.363413732	13,977	36,341	0.7692	-0.987	0.4282	-0.063	5381.3139	**0.053813**
0.91	0.367451663	14,025	36,745	0.7634	-0.98	0.4256	-0.062	5386.8966	**0.053869**
0.92	0.371489593	14,072	37,149	0.7576	-0.974	0.4231	-0.062	5392.3117	**0.053923**
0.93	0.375527523	14,118	37,553	0.7519	-0.967	0.4206	-0.062	5397.5648	**0.053976**
0.94	0.379565454	14,163	37,957	0.7463	-0.961	0.4181	-0.061	5402.6616	**0.054027**
0.95	0.383603384	14,208	38,360	0.7407	-0.954	0.4157	-0.061	5407.6072	**0.054076**
0.96	0.387641315	14,252	38,764	0.7353	-0.948	0.4132	-0.061	5412.4068	**0.054124**
0.97	0.391679245	14,295	39,168	0.7299	-0.942	0.4109	-0.06	5417.0652	**0.054171**
0.98	0.395717175	14,338	39,572	0.7246	-0.936	0.4085	-0.06	5421.5872	**0.054216**
0.99	0.399755106	14,380	39,976	0.7194	-0.93	0.4062	-0.06	5425.9772	**0.05426**
1	0.403793036	14,421	40,379	0.7143	-0.924	0.4039	-0.059	5430.2397	**0.054302**
1.01	0.407830966	14,462	40,783	0.7092	-0.918	0.4016	-0.059	5434.3786	**0.054344**
1.02	0.411868897	14,502	41,187	0.7042	-0.913	0.3993	-0.059	5438.3981	**0.054384**
1.03	0.415906827	14,542	41,591	0.6993	-0.907	0.3971	-0.059	5442.3021	**0.054423**
1.04	0.419944757	14,581	41,994	0.6944	-0.901	0.3949	-0.058	5446.0941	**0.054461**
1.05	0.423982688	14,620	42,398	0.6897	-0.896	0.3927	-0.058	5449.7779	**0.054498**
1.06	0.428020618	14,658	42,802	0.6849	-0.89	0.3905	-0.058	5453.3567	**0.054534**

1.07	0.432058549	14,696	43,206	0.6803	-0.885	0.3884	-0.057	5456.8341	0.054568
1.08	0.436096479	14,733	43,610	0.6757	-0.88	0.3863	-0.057	5460.2131	0.054602
1.09	0.440134409	14,770	44,013	0.6711	-0.874	0.3842	-0.057	5463.4967	0.054635
1.1	0.44417234	14,806	44,417	0.6667	-0.869	0.3821	-0.057	5466.6881	0.054667
1.11	0.44821027	14,841	44,821	0.6623	-0.864	0.3801	-0.056	5469.7900	0.054698
1.12	0.4522482	14,877	45,225	0.6579	-0.859	0.3781	-0.056	5472.8052	0.054728
1.13	0.456286131	14,911	45,629	0.6536	-0.854	0.3761	-0.056	5475.7362	0.054757
1.14	0.460324061	14,946	46,032	0.6494	-0.849	0.3741	-0.055	5478.5858	0.054786
1.15	0.464361991	14,979	46,436	0.6452	-0.844	0.3721	-0.055	5481.3564	0.054814
1.16	0.468399922	15,013	46,840	0.641	-0.839	0.3702	-0.055	5484.0503	0.054841
1.17	0.472437852	15,046	47,244	0.6369	-0.834	0.3683	-0.055	5486.6698	0.054867
1.18	0.476475782	15,078	47,648	0.6329	-0.83	0.3664	-0.054	5489.2172	0.054892
1.19	0.480513713	15,110	48,051	0.6289	-0.825	0.3645	-0.054	5491.6946	0.054917
1.2	0.484551643	15,142	48,455	0.625	-0.82	0.3626	-0.054	5494.1040	0.054941
1.21	0.488589574	15,174	48,859	0.6211	-0.816	0.3608	-0.054	5496.4476	0.054964
1.22	0.492627504	15,205	49,263	0.6173	-0.811	0.359	-0.053	5498.7271	0.054987
1.23	0.496665434	15,235	49,667	0.6135	-0.807	0.3572	-0.053	5500.9445	0.055009
1.24	0.500703365	15,265	50,070	0.6098	-0.802	0.3554	-0.053	5503.1015	0.055031
1.25	0.504741295	15,295	50,474	0.6061	-0.798	0.3536	-0.053	5505.2000	0.055052
1.26	0.508779225	15,325	50,878	0.6024	-0.794	0.3519	-0.052	5507.2415	0.055072
1.27	0.512817156	15,354	51,282	0.5988	-0.789	0.3501	-0.052	5509.2278	0.055092
1.28	0.516855086	15,383	51,686	0.5952	-0.785	0.3484	-0.052	5511.1603	0.055112
1.29	0.520893016	15,411	52,089	0.5917	-0.781	0.3467	-0.052	5513.0407	0.05513
1.3	0.524930947	15,439	52,493	0.5882	-0.777	0.345	-0.051	5514.8703	0.055149
1.31	0.528968877	15,467	52,897	0.5848	-0.773	0.3434	-0.051	5516.6506	0.055167
1.32	0.533006808	15,494	53,301	0.5814	-0.769	0.3417	-0.051	5518.3831	0.055184
1.33	0.537044738	15,522	53,704	0.578	-0.765	0.3401	-0.051	5520.0689	0.055201
1.34	0.541082668	15,548	54,108	0.5747	-0.761	0.3385	-0.051	5521.7095	0.055217
1.35	0.545120599	15,575	54,512	0.5714	-0.757	0.3369	-0.05	5523.3060	0.055233
1.36	0.549158529	15,601	54,916	0.5682	-0.753	0.3353	-0.05	5524.8598	0.055249
1.37	0.553196459	15,627	55,320	0.565	-0.749	0.3337	-0.05	5526.3718	0.055264
1.38	0.55723439	15,653	55,723	0.5618	-0.745	0.3321	-0.05	5527.8434	0.055278
1.39	0.56127232	15,678	56,127	0.5587	-0.741	0.3306	-0.049	5529.2755	0.055293
1.4	0.56531025	15,703	56,531	0.5556	-0.738	0.3291	-0.049	5530.6693	0.055307
1.41	0.569348181	15,728	56,935	0.5525	-0.734	0.3275	-0.049	5532.0257	0.05532
1.42	0.573386111	15,752	57,339	0.5495	-0.73	0.326	-0.049	5533.3459	0.055333
1.43	0.577424041	15,777	57,742	0.5464	-0.726	0.3246	-0.049	5534.6307	0.055346
1.44	0.581461972	15,801	58,146	0.5435	-0.723	0.3231	-0.048	5535.8811	0.055359
1.45	0.585499902	15,824	58,550	0.5405	-0.719	0.3216	-0.048	5537.0980	0.055371
1.46	0.589537833	15,848	58,954	0.5376	-0.716	0.3202	-0.048	5538.2823	0.055383
1.47	0.593575763	15,871	59,358	0.5348	-0.712	0.3187	-0.048	5539.4349	0.055394
1.48	0.597613693	15,894	59,761	0.5319	-0.709	0.3173	-0.048	5540.5566	0.055406
1.49	0.601651624	15,917	60,165	0.5291	-0.705	0.3159	-0.047	5541.6481	0.055416
1.5	0.605689554	15,939	60,569	0.5263	-0.702	0.3145	-0.047	5542.7104	0.055427
1.51	0.609727484	15,961	60,973	0.5236	-0.699	0.3131	-0.047	5543.7440	0.055437
1.52	0.613765415	15,983	61,377	0.5208	-0.695	0.3117	-0.047	5544.7499	0.055447
1.53	0.617803345	16,005	61,780	0.5181	-0.692	0.3104	-0.047	5545.7286	0.055457
1.54	0.621841275	16,027	62,184	0.5155	-0.689	0.309	-0.047	5546.6810	0.055467
1.55	0.625879206	16,048	62,588	0.5128	-0.686	0.3077	-0.046	5547.6076	0.055476
1.56	0.629917136	16,069	62,992	0.5102	-0.682	0.3064	-0.046	5548.5091	0.055485
1.57	0.633955067	16,090	63,396	0.5076	-0.679	0.3051	-0.046	5549.3862	0.055494
1.58	0.637992997	16,111	63,799	0.5051	-0.676	0.3038	-0.046	5550.2395	0.055502
1.59	0.642030927	16,131	64,203	0.5025	-0.673	0.3025	-0.046	5551.0696	0.055511
1.6	0.646068858	16,152	64,607	0.5	-0.67	0.3012	-0.045	5551.8770	0.055519
1.61	0.650106788	16,172	65,011	0.4975	-0.667	0.2999	-0.045	5552.6624	0.055527
1.62	0.654144718	16,192	65,414	0.495	-0.664	0.2986	-0.045	5553.4263	0.055534
1.63	0.658182649	16,211	65,818	0.4926	-0.661	0.2974	-0.045	5554.1691	0.055542
1.64	0.662220579	16,231	66,222	0.4902	-0.658	0.2962	-0.045	5554.8915	0.055549
1.65	0.666258509	16,250	66,626	0.4878	-0.655	0.2949	-0.045	5555.5940	0.055556
1.66	0.67029644	16,269	67,030	0.4854	-0.652	0.2937	-0.044	5556.2770	0.055563

1.67	0.67433437	16,288	67,433	0.4831	-0.649	0.2925	-0.044	5556.9411	**0.055569**
1.68	0.6783723	16,307	67,837	0.4808	-0.646	0.2913	-0.044	5557.5866	**0.055576**
1.69	0.682410231	16,326	68,241	0.4785	-0.643	0.2901	-0.044	5558.2140	**0.055582**
1.7	0.686448161	16,344	68,645	0.4762	-0.64	0.2889	-0.044	5558.8239	**0.055588**
1.71	0.690486092	16,362	69,049	0.4739	-0.638	0.2878	-0.044	5559.4165	**0.055594**
1.72	0.694524022	16,380	69,452	0.4717	-0.635	0.2866	-0.043	5559.9924	**0.0556**
1.73	0.698561952	16,398	69,856	0.4695	-0.632	0.2855	-0.043	5560.5519	**0.055606**
1.74	0.702599883	16,416	70,260	0.4673	-0.629	0.2843	-0.043	5561.0955	**0.055611**
1.75	0.706637813	16,433	70,664	0.4651	-0.627	0.2832	-0.043	5561.6234	**0.055616**
1.76	0.710675743	16,451	71,068	0.463	-0.624	0.2821	-0.043	5562.1362	**0.055621**
1.77	0.714713674	16,468	71,471	0.4608	-0.621	0.2809	-0.043	5562.6341	**0.055626**
1.78	0.718751604	16,485	71,875	0.4587	-0.619	0.2798	-0.042	5563.1175	**0.055631**
1.79	0.722789534	16,502	72,279	0.4566	-0.616	0.2787	-0.042	5563.5867	**0.055636**
1.8	0.726827465	16,519	72,683	0.4545	-0.614	0.2776	-0.042	5564.0421	**0.05564**
1.81	0.730865395	16,535	73,087	0.4525	-0.611	0.2766	-0.042	5564.4841	**0.055645**
1.82	0.734903326	16,552	73,490	0.4505	-0.608	0.2755	-0.042	5564.9129	**0.055649**
1.83	0.738941256	16,568	73,894	0.4484	-0.606	0.2744	-0.042	5565.3288	**0.055653**
1.84	0.742979186	16,584	74,298	0.4464	-0.603	0.2734	-0.041	5565.7322	**0.055657**
1.85	0.747017117	16,600	74,702	0.4444	-0.601	0.2723	-0.041	5566.1233	**0.055661**
1.86	0.751055047	16,616	75,106	0.4425	-0.598	0.2713	-0.041	5566.5024	**0.055665**
1.87	0.755092977	16,632	75,509	0.4405	-0.596	0.2703	-0.041	5566.8698	**0.055669**
1.88	0.759130908	16,648	75,913	0.4386	-0.594	0.2692	-0.041	5567.2258	**0.055672**
1.89	0.763168838	16,663	76,317	0.4367	-0.591	0.2682	-0.041	5567.5707	**0.055676**
1.9	0.767206768	16,678	76,721	0.4348	-0.589	0.2672	-0.041	5567.9047	**0.055679**
1.91	0.771244699	16,694	77,124	0.4329	-0.586	0.2662	-0.04	5568.2280	**0.055682**
1.92	0.775282629	16,709	77,528	0.431	-0.584	0.2652	-0.04	5568.5409	**0.055685**
1.93	0.779320559	16,724	77,932	0.4292	-0.582	0.2642	-0.04	5568.8437	**0.055688**
1.94	0.78335849	16,738	78,336	0.4274	-0.579	0.2633	-0.04	5569.1365	**0.055691**
1.95	0.78739642	16,753	78,740	0.4255	-0.577	0.2623	-0.04	5569.4196	**0.055694**
1.96	0.791434351	16,768	79,143	0.4237	-0.575	0.2613	-0.04	5569.6933	**0.055697**
1.97	0.795472281	16,782	79,547	0.4219	-0.573	0.2604	-0.04	5569.9577	**0.0557**
1.98	0.799510211	16,796	79,951	0.4202	-0.57	0.2594	-0.039	5570.2130	**0.055702**
1.99	0.803548142	16,811	80,355	0.4184	-0.568	0.2585	-0.039	5570.4595	**0.055705**
2	0.807586072	16,825	80,759	0.4167	-0.566	0.2575	-0.039	5570.6974	**0.055707**
2.01	0.811624002	16,839	81,162	0.4149	-0.564	0.2566	-0.039	5570.9268	**0.055709**
2.02	0.815661933	16,853	81,566	0.4132	-0.562	0.2557	-0.039	5571.1480	**0.055711**
2.03	0.819699863	16,866	81,970	0.4115	-0.559	0.2548	-0.039	5571.3611	**0.055714**
2.04	0.823737793	16,880	82,374	0.4098	-0.557	0.2539	-0.039	5571.5663	**0.055716**
2.05	0.827775724	16,893	82,778	0.4082	-0.555	0.2529	-0.039	5571.7638	**0.055718**
2.06	0.831813654	16,907	83,181	0.4065	-0.553	0.2521	-0.038	5571.9538	**0.05572**
2.07	0.835851584	16,920	83,585	0.4049	-0.551	0.2512	-0.038	5572.1364	**0.055721**
2.08	0.839889515	16,933	83,989	0.4032	-0.549	0.2503	-0.038	5572.3118	**0.055723**
2.09	0.843927445	16,946	84,393	0.4016	-0.547	0.2494	-0.038	5572.4802	**0.055725**
2.1	0.847965376	16,959	84,797	0.4	-0.545	0.2485	-0.038	5572.6417	**0.055726**
2.11	0.852003306	16,972	85,200	0.3984	-0.543	0.2477	-0.038	5572.7965	**0.055728**
2.12	0.856041236	16,985	85,604	0.3968	-0.541	0.2468	-0.038	5572.9447	**0.055729**
2.13	0.860079167	16,998	86,008	0.3953	-0.539	0.2459	-0.038	5573.0864	**0.055731**
2.14	0.864117097	17,010	86,412	0.3937	-0.537	0.2451	-0.037	5573.2219	**0.055732**
2.15	0.868155027	17,023	86,816	0.3922	-0.535	0.2443	-0.037	5573.3512	**0.055734**
2.16	0.872192958	17,035	87,219	0.3906	-0.533	0.2434	-0.037	5573.4745	**0.055735**
2.17	0.876230888	17,047	87,623	0.3891	-0.531	0.2426	-0.037	5573.5920	**0.055736**
2.18	0.880268818	17,059	88,027	0.3876	-0.529	0.2418	-0.037	5573.7037	**0.055737**
2.19	0.884306749	17,072	88,431	0.3861	-0.527	0.2409	-0.037	5573.8097	**0.055738**
2.2	0.888344679	17,084	88,834	0.3846	-0.525	0.2401	-0.037	5573.9103	**0.055739**
2.21	0.89238261	17,095	89,238	0.3831	-0.523	0.2393	-0.037	5574.0055	**0.05574**
2.22	0.89642054	17,107	89,642	0.3817	-0.522	0.2385	-0.036	5574.0954	**0.055741**
2.23	0.90045847	17,119	90,046	0.3802	-0.52	0.2377	-0.036	5574.1802	**0.055742**
2.24	0.904496401	17,131	90,450	0.3788	-0.518	0.2369	-0.036	5574.2600	**0.055743**
2.25	0.908534331	17,142	90,853	0.3774	-0.516	0.2361	-0.036	5574.3349	**0.055743**
2.26	0.912572261	17,154	91,257	0.3759	-0.514	0.2354	-0.036	5574.4049	**0.055744**

2.27	0.916610192	17,165	91,661	0.3745	-0.512	0.2346	-0.036	5574.4703	**0.055745**
2.28	0.920648122	17,176	92,065	0.3731	-0.511	0.2338	-0.036	5574.5310	**0.055745**
2.29	0.924686052	17,187	92,469	0.3717	-0.509	0.233	-0.036	5574.5873	**0.055746**
2.3	0.928723983	17,199	92,872	0.3704	-0.507	0.2323	-0.036	5574.6391	**0.055746**
2.31	0.932761913	17,210	93,276	0.369	-0.505	0.2315	-0.035	5574.6867	**0.055747**
2.32	0.936799843	17,221	93,680	0.3676	-0.504	0.2308	-0.035	5574.7300	**0.055747**
2.33	0.940837774	17,231	94,084	0.3663	-0.502	0.23	-0.035	5574.7692	**0.055748**
2.34	0.944875704	17,242	94,488	0.365	-0.5	0.2293	-0.035	5574.8044	**0.055748**
2.35	0.948913635	17,253	94,891	0.3636	-0.498	0.2285	-0.035	5574.8356	**0.055748**
2.36	0.952951565	17,264	95,295	0.3623	-0.497	0.2278	-0.035	5574.8630	**0.055749**
2.37	0.956989495	17,274	95,699	0.361	-0.495	0.2271	-0.035	5574.8867	**0.055749**
2.38	0.961027426	17,285	96,103	0.3597	-0.493	0.2264	-0.035	5574.9066	**0.055749**
2.39	0.965065356	17,295	96,507	0.3584	-0.492	0.2256	-0.035	5574.9229	**0.055749**
2.4	0.969103286	17,305	96,910	0.3571	-0.49	0.2249	-0.035	5574.9357	**0.055749**
2.41	0.973141217	17,316	97,314	0.3559	-0.488	0.2242	-0.034	5574.9451	**0.055749**
2.42	0.977179147	17,326	97,718	0.3546	-0.487	0.2235	-0.034	5574.9511	**0.05575**
2.43	0.981217077	17,336	98,122	0.3534	-0.485	0.2228	-0.034	5574.9537	**0.05575**
2.44	0.985255008	17,346	98,526	0.3521	-0.484	0.2221	-0.034	5574.9532	**0.05575**
2.45	0.989292938	17,356	98,929	0.3509	-0.482	0.2214	-0.034	5574.9495	**0.055749**
2.46	0.993330869	17,366	99,333	0.3497	-0.48	0.2207	-0.034	5574.9427	**0.055749**
2.47	0.997368799	17,376	99,737	0.3484	-0.479	0.2201	-0.034	5574.9329	**0.055749**
2.48	1.001406729	17,386	100,141	0.3472	-0.477	0.2194	-0.034	5574.9201	**0.055749**
2.49	1.00544466	17,395	100,544	0.346	-0.476	0.2187	-0.034	5574.9044	**0.055749**
2.5	1.00948259	17,405	100,948	0.3448	-0.474	0.218	-0.034	5574.8859	**0.055749**



F	$t_c = 1$	$t_c = 2$	$t_c = 3$	$t_c = 4$	$t_c = 5$
0	0	0	0	0	0
0.01	0.00396	0.00386	0.00355	0.00308	0.00254
0.02	0.00751	0.00737	0.00681	0.00594	0.00492
0.03	0.01069	0.01056	0.00983	0.0086	0.00715
0.04	0.01353	0.01348	0.01261	0.01109	0.00924
0.05	0.01607	0.01613	0.01518	0.0134	0.01121
0.06	0.01834	0.01856	0.01756	0.01556	0.01305
0.07	0.02037	0.02078	0.01977	0.01759	0.01479
0.08	0.02218	0.0228	0.02182	0.01948	0.01642
0.09	0.0238	0.02466	0.02372	0.02125	0.01796
0.1	0.02524	0.02635	0.02548	0.02292	0.01941
0.11	0.02652	0.0279	0.02712	0.02448	0.02079
0.12	0.02766	0.02932	0.02865	0.02595	0.02209
0.13	0.02867	0.03062	0.03007	0.02733	0.02332
0.14	0.02956	0.03181	0.0314	0.02863	0.02448
0.15	0.03035	0.0329	0.03263	0.02985	0.02559
0.16	0.03104	0.0339	0.03379	0.03101	0.02663
0.17	0.03164	0.03481	0.03487	0.0321	0.02763
0.18	0.03217	0.03564	0.03587	0.03313	0.02858
0.19	0.03262	0.03641	0.03682	0.0341	0.02948
0.2	0.03301	0.03711	0.0377	0.03502	0.03033
0.21	0.03335	0.03775	0.03852	0.0359	0.03115
0.22	0.03362	0.03833	0.0393	0.03672	0.03193
0.23	0.03385	0.03887	0.04002	0.03751	0.03267
0.24	0.03404	0.03936	0.0407	0.03825	0.03338
0.25	0.03419	0.0398	0.04134	0.03896	0.03406
0.26	0.0343	0.0402	0.04194	0.03963	0.03471
0.27	0.03438	0.04057	0.0425	0.04027	0.03533
0.28	0.03443	0.0409	0.04303	0.04087	0.03592
0.29	0.03445	0.04121	0.04353	0.04145	0.03649
0.3	0.03445	0.04148	0.04399	0.042	0.03704
0.31	0.03443	0.04173	0.04443	0.04252	0.03756

0.32	0.03438	0.04195	0.04485	0.04302	0.03806
0.33	0.03432	0.04215	0.04523	0.0435	0.03855
0.34	0.03424	0.04232	0.0456	0.04396	0.03901
0.35	0.03415	0.04248	0.04594	0.04439	0.03945
0.36	0.03405	0.04262	0.04627	0.04481	0.03988
0.37	0.03393	0.04274	0.04657	0.0452	0.04029
0.38	0.0338	0.04285	0.04686	0.04558	0.04069
0.39	0.03366	0.04294	0.04713	0.04595	0.04107
0.4	0.03352	0.04302	0.04738	0.04629	0.04144
0.41	0.03336	0.04308	0.04762	0.04663	0.04179
0.42	0.0332	0.04314	0.04785	0.04695	0.04213
0.43	0.03303	0.04318	0.04806	0.04725	0.04246
0.44	0.03286	0.04321	0.04826	0.04754	0.04278
0.45	0.03268	0.04324	0.04845	0.04783	0.04309
0.46	0.0325	0.04325	0.04863	0.0481	0.04339
0.47	0.03232	0.04326	0.0488	0.04835	0.04367
0.48	0.03213	0.04326	0.04896	0.0486	0.04395
0.49	0.03194	0.04325	0.0491	0.04884	0.04422
0.5	0.03175	0.04324	0.04924	0.04907	0.04448
0.51	0.03155	0.04322	0.04938	0.04929	0.04473
0.52	0.03136	0.04319	0.0495	0.0495	0.04497
0.53	0.03116	0.04316	0.04962	0.04971	0.04521
0.54	0.03096	0.04313	0.04972	0.0499	0.04543
0.55	0.03077	0.04309	0.04983	0.05009	0.04566
0.56	0.03057	0.04305	0.04992	0.05027	0.04587
0.57	0.03037	0.043	0.05001	0.05045	0.04608
0.58	0.03017	0.04295	0.0501	0.05062	0.04628
0.59	0.02997	0.04289	0.05018	0.05078	0.04647
0.6	0.02977	0.04284	0.05025	0.05094	0.04666
0.61	0.02958	0.04278	0.05032	0.05109	0.04685
0.62	0.02938	0.04272	0.05039	0.05123	0.04703
0.63	0.02919	0.04265	0.05045	0.05137	0.0472
0.64	0.02899	0.04259	0.05051	0.05151	0.04737
0.65	0.0288	0.04252	0.05056	0.05164	0.04753
0.66	0.0286	0.04245	0.05061	0.05177	0.04769
0.67	0.02841	0.04238	0.05065	0.05189	0.04785
0.68	0.02822	0.04231	0.0507	0.05201	0.048
0.69	0.02803	0.04223	0.05073	0.05212	0.04815
0.7	0.02785	0.04216	0.05077	0.05223	0.04829
0.71	0.02766	0.04208	0.0508	0.05234	0.04843
0.72	0.02748	0.042	0.05083	0.05244	0.04856
0.73	0.02729	0.04193	0.05086	0.05254	0.0487
0.74	0.02711	0.04185	0.05089	0.05263	0.04883
0.75	0.02693	0.04177	0.05091	0.05273	0.04895
0.76	0.02676	0.04169	0.05093	0.05282	0.04907
0.77	0.02658	0.04161	0.05095	0.0529	0.04919
0.78	0.02641	0.04153	0.05097	0.05299	0.04931
0.79	0.02623	0.04145	0.05098	0.05307	0.04942
0.8	0.02606	0.04137	0.051	0.05315	0.04953
0.81	0.02589	0.04128	0.05101	0.05322	0.04964
0.82	0.02573	0.0412	0.05102	0.0533	0.04974
0.83	0.02556	0.04112	0.05103	0.05337	0.04985
0.84	0.0254	0.04104	0.05103	0.05344	0.04995
0.85	0.02524	0.04096	0.05104	0.05351	0.05005
0.86	0.02508	0.04087	0.05104	0.05357	0.05014
0.87	0.02492	0.04079	0.05105	0.05363	0.05023
0.88	0.02476	0.04071	0.05105	0.0537	0.05033
0.89	0.0246	0.04063	0.05105	0.05376	0.05041
0.9	0.02445	0.04055	0.05105	0.05381	0.0505
0.91	0.0243	0.04046	0.05105	0.05387	0.05059

0.92	0.02415	0.04038	0.05104	0.05392	0.05067
0.93	0.024	0.0403	0.05104	0.05398	0.05075
0.94	0.02386	0.04022	0.05103	0.05403	0.05083
0.95	0.02371	0.04014	0.05103	0.05408	0.05091
0.96	0.02357	0.04006	0.05102	0.05412	0.05099
0.97	0.02343	0.03998	0.05102	0.05417	0.05106
0.98	0.02329	0.0399	0.05101	0.05422	0.05113
0.99	0.02315	0.03982	0.051	0.05426	0.0512
1	0.02301	0.03974	0.05099	0.0543	0.05127
1.01	0.02288	0.03967	0.05098	0.05434	0.05134
1.02	0.02274	0.03959	0.05097	0.05438	0.05141
1.03	0.02261	0.03951	0.05096	0.05442	0.05148
1.04	0.02248	0.03943	0.05095	0.05446	0.05154
1.05	0.02235	0.03936	0.05094	0.0545	0.0516
1.06	0.02223	0.03928	0.05092	0.05453	0.05167
1.07	0.0221	0.0392	0.05091	0.05457	0.05173
1.08	0.02198	0.03913	0.0509	0.0546	0.05179
1.09	0.02185	0.03905	0.05088	0.05463	0.05184
1.1	0.02173	0.03898	0.05087	0.05467	0.0519
1.11	0.02161	0.03891	0.05086	0.0547	0.05196
1.12	0.02149	0.03883	0.05084	0.05473	0.05201
1.13	0.02138	0.03876	0.05083	0.05476	0.05207
1.14	0.02126	0.03869	0.05081	0.05479	0.05212
1.15	0.02115	0.03861	0.05079	0.05481	0.05217
1.16	0.02103	0.03854	0.05078	0.05484	0.05222
1.17	0.02092	0.03847	0.05076	0.05487	0.05227
1.18	0.02081	0.0384	0.05075	0.05489	0.05232
1.19	0.0207	0.03833	0.05073	0.05492	0.05237
1.2	0.02059	0.03826	0.05071	0.05494	0.05242
1.21	0.02049	0.03819	0.05069	0.05496	0.05246
1.22	0.02038	0.03812	0.05068	0.05499	0.05251
1.23	0.02028	0.03806	0.05066	0.05501	0.05255
1.24	0.02017	0.03799	0.05064	0.05503	0.0526
1.25	0.02007	0.03792	0.05062	0.05505	0.05264
1.26	0.01997	0.03785	0.05061	0.05507	0.05268
1.27	0.01987	0.03779	0.05059	0.05509	0.05272
1.28	0.01977	0.03772	0.05057	0.05511	0.05276
1.29	0.01968	0.03766	0.05055	0.05513	0.0528
1.3	0.01958	0.03759	0.05053	0.05515	0.05284
1.31	0.01949	0.03753	0.05051	0.05517	0.05288
1.32	0.01939	0.03747	0.05049	0.05518	0.05292
1.33	0.0193	0.0374	0.05047	0.0552	0.05296
1.34	0.01921	0.03734	0.05046	0.05522	0.05299
1.35	0.01912	0.03728	0.05044	0.05523	0.05303
1.36	0.01903	0.03722	0.05042	0.05525	0.05307
1.37	0.01894	0.03716	0.0504	0.05526	0.0531
1.38	0.01885	0.03709	0.05038	0.05528	0.05314
1.39	0.01876	0.03703	0.05036	0.05529	0.05317
1.4	0.01868	0.03697	0.05034	0.05531	0.0532
1.41	0.01859	0.03692	0.05032	0.05532	0.05324
1.42	0.01851	0.03686	0.0503	0.05533	0.05327
1.43	0.01843	0.0368	0.05028	0.05535	0.0533
1.44	0.01834	0.03674	0.05026	0.05536	0.05333
1.45	0.01826	0.03668	0.05024	0.05537	0.05336
1.46	0.01818	0.03663	0.05022	0.05538	0.05339
1.47	0.0181	0.03657	0.05021	0.05539	0.05342
1.48	0.01803	0.03651	0.05019	0.05541	0.05345
1.49	0.01795	0.03646	0.05017	0.05542	0.05348
1.5	0.01787	0.0364	0.05015	0.05543	0.05351
1.51	0.0178	0.03635	0.05013	0.05544	0.05354

1.52	0.01772	0.03629	0.05011	0.05545	0.05357
1.53	0.01765	0.03624	0.05009	0.05546	0.05359
1.54	0.01757	0.03619	0.05007	0.05547	0.05362
1.55	0.0175	0.03613	0.05005	0.05548	0.05365
1.56	0.01743	0.03608	0.05003	0.05549	0.05367
1.57	0.01736	0.03603	0.05001	0.05549	0.0537
1.58	0.01729	0.03598	0.04999	0.0555	0.05372
1.59	0.01722	0.03592	0.04997	0.05551	0.05375
1.6	0.01715	0.03587	0.04995	0.05552	0.05377
1.61	0.01708	0.03582	0.04993	0.05553	0.0538
1.62	0.01701	0.03577	0.04992	0.05553	0.05382
1.63	0.01694	0.03572	0.0499	0.05554	0.05385
1.64	0.01688	0.03567	0.04988	0.05555	0.05387
1.65	0.01681	0.03562	0.04986	0.05556	0.05389
1.66	0.01675	0.03557	0.04984	0.05556	0.05391
1.67	0.01668	0.03553	0.04982	0.05557	0.05394
1.68	0.01662	0.03548	0.0498	0.05558	0.05396
1.69	0.01656	0.03543	0.04978	0.05558	0.05398
1.7	0.01649	0.03538	0.04976	0.05559	0.054
1.71	0.01643	0.03534	0.04975	0.05559	0.05402
1.72	0.01637	0.03529	0.04973	0.0556	0.05404
1.73	0.01631	0.03524	0.04971	0.05561	0.05406
1.74	0.01625	0.0352	0.04969	0.05561	0.05408
1.75	0.01619	0.03515	0.04967	0.05562	0.0541
1.76	0.01613	0.03511	0.04965	0.05562	0.05412
1.77	0.01608	0.03506	0.04964	0.05563	0.05414
1.78	0.01602	0.03502	0.04962	0.05563	0.05416
1.79	0.01596	0.03497	0.0496	0.05564	0.05418
1.8	0.01591	0.03493	0.04958	0.05564	0.0542
1.81	0.01585	0.03489	0.04956	0.05564	0.05422
1.82	0.0158	0.03484	0.04955	0.05565	0.05424
1.83	0.01574	0.0348	0.04953	0.05565	0.05425
1.84	0.01569	0.03476	0.04951	0.05566	0.05427
1.85	0.01563	0.03472	0.04949	0.05566	0.05429
1.86	0.01558	0.03468	0.04947	0.05567	0.05431
1.87	0.01553	0.03463	0.04946	0.05567	0.05432
1.88	0.01548	0.03459	0.04944	0.05567	0.05434
1.89	0.01542	0.03455	0.04942	0.05568	0.05436
1.9	0.01537	0.03451	0.0494	0.05568	0.05437
1.91	0.01532	0.03447	0.04939	0.05568	0.05439
1.92	0.01527	0.03443	0.04937	0.05569	0.05441
1.93	0.01522	0.03439	0.04935	0.05569	0.05442
1.94	0.01517	0.03435	0.04934	0.05569	0.05444
1.95	0.01512	0.03431	0.04932	0.05569	0.05445
1.96	0.01508	0.03427	0.0493	0.0557	0.05447
1.97	0.01503	0.03424	0.04928	0.0557	0.05448
1.98	0.01498	0.0342	0.04927	0.0557	0.0545
1.99	0.01493	0.03416	0.04925	0.0557	0.05451
2	0.01489	0.03412	0.04923	0.05571	0.05453
2.01	0.01484	0.03408	0.04922	0.05571	0.05454
2.02	0.0148	0.03405	0.0492	0.05571	0.05456
2.03	0.01475	0.03401	0.04918	0.05571	0.05457
2.04	0.01471	0.03397	0.04917	0.05572	0.05458
2.05	0.01466	0.03394	0.04915	0.05572	0.0546
2.06	0.01462	0.0339	0.04914	0.05572	0.05461
2.07	0.01457	0.03387	0.04912	0.05572	0.05462
2.08	0.01453	0.03383	0.0491	0.05572	0.05464
2.09	0.01449	0.0338	0.04909	0.05572	0.05465
2.1	0.01445	0.03376	0.04907	0.05573	0.05466
2.11	0.0144	0.03373	0.04905	0.05573	0.05468

2.12	0.01436	0.03369	0.04904	0.05573	0.05469
2.13	0.01432	0.03366	0.04902	0.05573	0.0547
2.14	0.01428	0.03362	0.04901	0.05573	0.05471
2.15	0.01424	0.03359	0.04899	0.05573	0.05473
2.16	0.0142	0.03356	0.04898	0.05573	0.05474
2.17	0.01416	0.03352	0.04896	0.05574	0.05475
2.18	0.01412	0.03349	0.04895	0.05574	0.05476
2.19	0.01408	0.03346	0.04893	0.05574	0.05477
2.2	0.01404	0.03342	0.04892	0.05574	0.05478
2.21	0.014	0.03339	0.0489	0.05574	0.0548
2.22	0.01396	0.03336	0.04888	0.05574	0.05481
2.23	0.01393	0.03333	0.04887	0.05574	0.05482
2.24	0.01389	0.0333	0.04885	0.05574	0.05483
2.25	0.01385	0.03326	0.04884	0.05574	0.05484
2.26	0.01381	0.03323	0.04883	0.05574	0.05485
2.27	0.01378	0.0332	0.04881	0.05574	0.05486
2.28	0.01374	0.03317	0.0488	0.05575	0.05487
2.29	0.0137	0.03314	0.04878	0.05575	0.05488
2.3	0.01367	0.03311	0.04877	0.05575	0.05489
2.31	0.01363	0.03308	0.04875	0.05575	0.0549
2.32	0.0136	0.03305	0.04874	0.05575	0.05491
2.33	0.01356	0.03302	0.04872	0.05575	0.05492
2.34	0.01353	0.03299	0.04871	0.05575	0.05493
2.35	0.01349	0.03296	0.04869	0.05575	0.05494
2.36	0.01346	0.03293	0.04868	0.05575	0.05495
2.37	0.01343	0.0329	0.04867	0.05575	0.05496
2.38	0.01339	0.03287	0.04865	0.05575	0.05497
2.39	0.01336	0.03284	0.04864	0.05575	0.05498
2.4	0.01333	0.03282	0.04862	0.05575	0.05499
2.41	0.0133	0.03279	0.04861	0.05575	0.055
2.42	0.01326	0.03276	0.0486	0.05575	0.05501
2.43	0.01323	0.03273	0.04858	0.05575	0.05502
2.44	0.0132	0.0327	0.04857	0.05575	0.05503
2.45	0.01317	0.03268	0.04856	0.05575	0.05504
2.46	0.01314	0.03265	0.04854	0.05575	0.05505
2.47	0.0131	0.03262	0.04853	0.05575	0.05505
2.48	0.01307	0.0326	0.04852	0.05575	0.05506
2.49	0.01304	0.03257	0.0485	0.05575	0.05507
2.5	0.01301	0.03254	0.04849	0.05575	0.05508

For silver hake, the t_c that gives the highest yield is age 4.

Solving for L_t:

$L_t = L_\infty*(1-e^{(-Kt)})$

$L_t = 65*(1-e^{(-0.0.18*4)})$

33.3611

Then, solve for the mesh length:

$ml = L_{50}/SF$

ml = 33.36/3.5

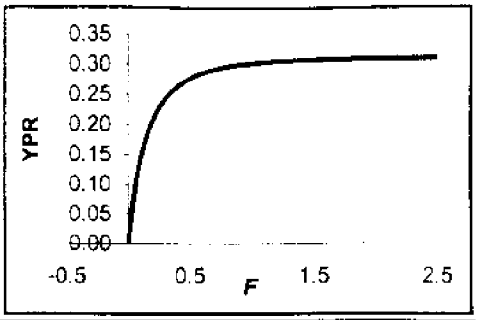
9.53174

Beverton-Holt
Yield Per Recruit

YELLOWTAIL
FLOUNDER
Example: $t_c = 3$

M	0.2
W_∞	0.9
K	0.63
t_0	0
t_c	3
t_r	0
N_{tr}	1E+05
t_y	20
M/K	0.32

U	n
1	0
-3	1
3	2
-1	3



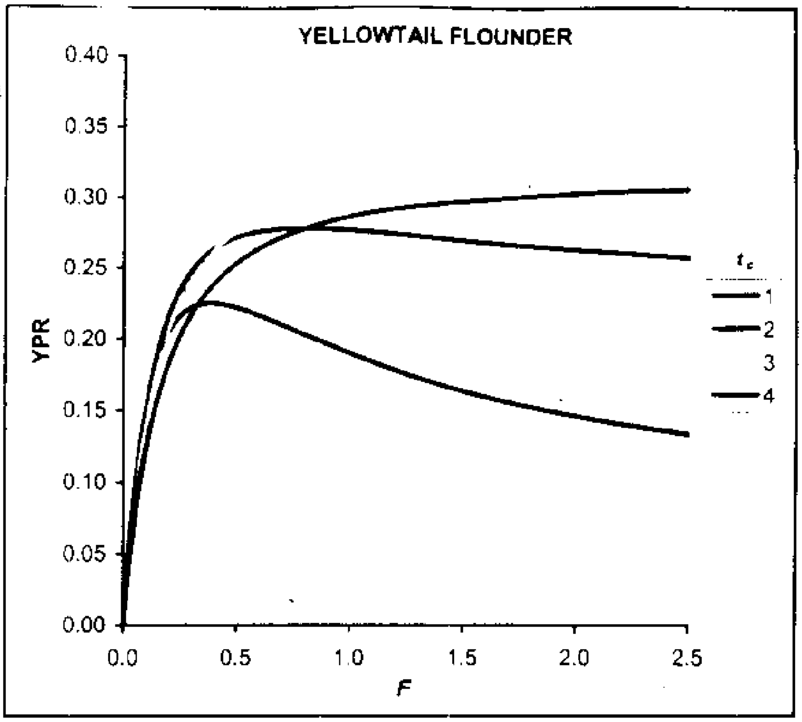
F	FW_EXP(..)	C	Y1	Y2-0	Y2-1	Y2-2	Y2-3	Yield	YPR
0	0	0	0	4.8331	-0.546	0.0469	-0.002	0.0000	**0**
0.01	0.0049393	2,540	494	4.6278	-0.54	0.0466	-0.002	2041.5245	**0.020415**
0.02	0.00987861	4,871	988	4.4375	-0.533	0.0463	-0.002	3900.9791	**0.03901**
0.03	0.01481791	7,015	1,482	4.2607	-0.527	0.046	-0.002	5598.2441	**0.055982**
0.04	0.01975722	8,992	1,976	4.0962	-0.521	0.0456	-0.002	7150.7470	**0.071507**
0.05	0.02469652	10,820	2,470	3.9429	-0.515	0.0453	-0.002	8573.7875	**0.085738**
0.06	0.02963583	12,512	2,964	3.7999	-0.509	0.045	-0.002	9880.8187	**0.098808**
0.07	0.03457513	14,084	3,458	3.6661	-0.504	0.0448	-0.002	11083.6885	**0.110837**
0.08	0.03951444	15,546	3,951	3.5408	-0.498	0.0445	-0.002	12192.8483	**0.121928**
0.09	0.04445374	16,909	4,445	3.4234	-0.493	0.0442	-0.002	13217.5329	**0.132175**
0.1	0.04939305	18,182	4,939	3.313	-0.487	0.0439	-0.002	14165.9157	**0.141659**
0.11	0.05433235	19,374	5,433	3.2092	-0.482	0.0436	-0.002	15045.2431	**0.150452**
0.12	0.05927166	20,491	5,927	3.1114	-0.477	0.0433	-0.002	15861.9512	**0.15862**
0.13	0.06421096	21,541	6,421	3.0192	-0.472	0.0431	-0.002	16621.7657	**0.166218**
0.14	0.06915027	22,528	6,915	2.9321	-0.467	0.0428	-0.002	17329.7901	**0.173298**
0.15	0.07408957	23,459	7,409	2.8497	-0.462	0.0425	-0.002	17990.5810	**0.179906**
0.16	0.07902888	24,338	7,903	2.7717	-0.458	0.0423	-0.002	18608.2139	**0.186082**
0.17	0.08396818	25,169	8,397	2.6977	-0.453	0.042	-0.002	19186.3411	**0.191863**
0.18	0.08890749	25,956	8,891	2.6275	-0.449	0.0417	-0.002	19728.2409	**0.197282**
0.19	0.09384679	26,702	9,385	2.5607	-0.444	0.0415	-0.002	20236.8616	**0.202369**
0.2	0.09878609	27,410	9,879	2.4972	-0.44	0.0412	-0.002	20714.8593	**0.207149**
0.21	0.1037254	28,083	10,373	2.4367	-0.436	0.041	-0.001	21164.6312	**0.211646**
0.22	0.1086647	28,724	10,866	2.3791	-0.432	0.0408	-0.001	21588.3447	**0.215883**
0.23	0.11360401	29,335	11,360	2.324	-0.428	0.0405	-0.001	21987.9627	**0.21988**
0.24	0.11854331	29,918	11,854	2.2714	-0.424	0.0403	-0.001	22365.2661	**0.223653**
0.25	0.12348262	30,475	12,348	2.2212	-0.42	0.04	-0.001	22721.8736	**0.227219**
0.26	0.12842192	31,007	12,842	2.173	-0.416	0.0398	-0.001	23059.2585	**0.230593**
0.27	0.13336123	31,517	13,336	2.1269	-0.412	0.0396	-0.001	23378.7645	**0.233788**
0.28	0.13830053	32,005	13,830	2.0827	-0.408	0.0393	-0.001	23681.6169	**0.236816**
0.29	0.14323984	32,473	14,324	2.0403	-0.405	0.0391	-0.001	23968.9442	**0.239689**
0.3	0.14817914	32,922	14,818	1.9996	-0.401	0.0389	-0.001	24241.7691	**0.242418**
0.31	0.15311845	33,353	15,312	1.9604	-0.398	0.0387	-0.001	24501.0377	**0.24501**
0.32	0.15805775	33,768	15,806	1.9228	-0.394	0.0385	-0.001	24747.6175	**0.247476**
0.33	0.16299706	34,167	16,300	1.8866	-0.391	0.0383	-0.001	24982.3070	**0.249823**
0.34	0.16793636	34,551	16,794	1.8517	-0.387	0.038	-0.001	25205.8420	**0.252058**
0.35	0.17287567	34,921	17,288	1.818	-0.384	0.0378	-0.001	25418.9020	**0.254189**
0.36	0.17781497	35,278	17,781	1.7856	-0.381	0.0376	-0.001	25622.1147	**0.256221**
0.37	0.18275427	35,622	18,275	1.7543	-0.378	0.0374	-0.001	25816.0612	**0.258161**
0.38	0.18769358	35,955	18,769	1.724	-0.375	0.0372	-0.001	26001.2798	**0.260013**
0.39	0.19263288	36,276	19,263	1.6948	-0.371	0.037	-0.001	26178.2701	**0.261783**
0.4	0.19757219	36,586	19,757	1.6666	-0.368	0.0368	-0.001	26347.4961	**0.263475**
0.41	0.20251149	36,886	20,251	1.6393	-0.365	0.0366	-0.001	26509.3892	**0.265094**
0.42	0.2074508	37,177	20,745	1.6129	-0.363	0.0364	-0.001	26664.3513	**0.266644**
0.43	0.2123901	37,458	21,239	1.5873	-0.36	0.0362	-0.001	26812.7568	**0.268128**
0.44	0.21732941	37,730	21,733	1.5625	-0.357	0.036	-0.001	26954.9551	**0.26955**
0.45	0.22226871	37,994	22,227	1.5384	-0.354	0.0358	-0.001	27091.2726	**0.270913**
0.46	0.22720802	38,250	22,721	1.5151	-0.351	0.0357	-0.001	27222.0145	**0.27222**

0.47	0.23214732	38,498	23,215	1.4925	-0.349	0.0355	-0.001	27347.4664	**0.273475**
0.48	0.23708663	38,739	23,709	1.4706	-0.346	0.0353	-0.001	27467.8957	**0.274679**
0.49	0.24202593	38,973	24,203	1.4493	-0.343	0.0351	-0.001	27583.5533	**0.275836**
0.5	0.24696524	39,201	24,697	1.4286	-0.341	0.0349	-0.001	27694.6746	**0.276947**
0.51	0.25190454	39,421	25,190	1.4084	-0.338	0.0348	-0.001	27801.4808	**0.278015**
0.52	0.25684385	39,636	25,684	1.3889	-0.336	0.0346	-0.001	27904.1796	**0.279042**
0.53	0.26178315	39,845	26,178	1.3699	-0.333	0.0344	-0.001	28002.9666	**0.28003**
0.54	0.26672246	40,048	26,672	1.3513	-0.331	0.0342	-0.001	28098.0260	**0.28098**
0.55	0.27166176	40,246	27,166	1.3333	-0.328	0.0341	-0.001	28189.5312	**0.281895**
0.56	0.27660106	40,439	27,660	1.3158	-0.326	0.0339	-0.001	28277.6458	**0.282776**
0.57	0.28154037	40,626	28,154	1.2987	-0.324	0.0337	-0.001	28362.5242	**0.283625**
0.58	0.28647967	40,809	28,648	1.282	-0.321	0.0336	-0.001	28444.3123	**0.284443**
0.59	0.29141898	40,987	29,142	1.2658	-0.319	0.0334	-0.001	28523.1476	**0.285231**
0.6	0.29635828	41,161	29,636	1.25	-0.317	0.0332	-0.001	28599.1605	**0.285992**
0.61	0.30129759	41,330	30,130	1.2346	-0.315	0.0331	-0.001	28672.4743	**0.286725**
0.62	0.30623689	41,495	30,624	1.2195	-0.313	0.0329	-0.001	28743.2056	**0.287432**
0.63	0.3111762	41,657	31,118	1.2048	-0.31	0.0328	-0.001	28811.4650	**0.288115**
0.64	0.3161155	41,814	31,612	1.1905	-0.308	0.0326	-0.001	28877.3572	**0.288774**
0.65	0.32105481	41,968	32,105	1.1765	-0.306	0.0324	-0.001	28940.9817	**0.28941**
0.66	0.32599411	42,118	32,599	1.1628	-0.304	0.0323	-0.001	29002.4327	**0.290024**
0.67	0.33093342	42,265	33,093	1.1494	-0.302	0.0321	-0.001	29061.7998	**0.290618**
0.68	0.33587272	42,408	33,587	1.1364	-0.3	0.032	-0.001	29119.1680	**0.291192**
0.69	0.34081203	42,548	34,081	1.1236	-0.298	0.0318	-0.001	29174.6183	**0.291746**
0.7	0.34575133	42,685	34,575	1.1111	-0.296	0.0317	-0.001	29228.2276	**0.292282**
0.71	0.35069064	42,819	35,069	1.0989	-0.294	0.0316	-0.001	29280.0689	**0.292801**
0.72	0.35562994	42,950	35,563	1.087	-0.292	0.0314	-0.001	29330.2120	**0.293302**
0.73	0.36056924	43,079	36,057	1.0753	-0.291	0.0313	-0.001	29378.7233	**0.293787**
0.74	0.36550855	43,204	36,551	1.0638	-0.289	0.0311	-0.001	29425.6658	**0.294257**
0.75	0.37044785	43,327	37,045	1.0526	-0.287	0.031	-0.001	29471.0999	**0.294711**
0.76	0.37538716	43,448	37,539	1.0417	-0.285	0.0308	-0.001	29515.0830	**0.295151**
0.77	0.38032646	43,565	38,033	1.0309	-0.283	0.0307	-0.001	29557.6701	**0.295577**
0.78	0.38526577	43,681	38,527	1.0204	-0.282	0.0306	-0.001	29598.9132	**0.295989**
0.79	0.39020507	43,794	39,021	1.0101	-0.28	0.0304	-0.001	29638.8626	**0.296389**
0.8	0.39514438	43,905	39,514	1	-0.278	0.0303	-0.001	29677.5659	**0.296776**
0.81	0.40008368	44,014	40,008	0.9901	-0.276	0.0302	-0.001	29715.0686	**0.297151**
0.82	0.40502299	44,120	40,502	0.9804	-0.275	0.03	-0.001	29751.4146	**0.297514**
0.83	0.40996229	44,225	40,996	0.9709	-0.273	0.0299	-0.001	29786.6453	**0.297866**
0.84	0.4149016	44,327	41,490	0.9615	-0.271	0.0298	-0.001	29820.8009	**0.298208**
0.85	0.4198409	44,428	41,984	0.9524	-0.27	0.0296	-0.001	29853.9194	**0.298539**
0.86	0.42478021	44,526	42,478	0.9434	-0.268	0.0295	-0.001	29886.0374	**0.29886**
0.87	0.42971951	44,623	42,972	0.9346	-0.267	0.0294	-0.001	29917.1900	**0.299172**
0.88	0.43465882	44,718	43,466	0.9259	-0.265	0.0293	-0.001	29947.4108	**0.299474**
0.89	0.43959812	44,811	43,960	0.9174	-0.263	0.0291	-0.001	29976.7318	**0.299767**
0.9	0.44453743	44,903	44,454	0.9091	-0.262	0.029	-0.001	30005.1840	**0.300052**
0.91	0.44947673	44,993	44,948	0.9009	-0.26	0.0289	-0.001	30032.7970	**0.300328**
0.92	0.45441603	45,081	45,442	0.8929	-0.259	0.0288	-0.001	30059.5990	**0.300596**
0.93	0.45935534	45,168	45,936	0.885	-0.258	0.0286	-0.001	30085.6174	**0.300856**
0.94	0.46429464	45,253	46,429	0.8772	-0.256	0.0285	-0.001	30110.8783	**0.301109**
0.95	0.46923395	45,337	46,923	0.8696	-0.255	0.0284	-0.001	30135.4068	**0.301354**
0.96	0.47417325	45,419	47,417	0.8621	-0.253	0.0283	-0.001	30159.2271	**0.301592**
0.97	0.47911256	45,500	47,911	0.8547	-0.252	0.0282	-0.001	30182.3622	**0.301824**
0.98	0.48405186	45,579	48,405	0.8475	-0.25	0.0281	-0.001	30204.8346	**0.302048**
0.99	0.48899117	45,657	48,899	0.8403	-0.249	0.0279	-0.001	30226.6655	**0.302267**
1	0.49393047	45,734	49,393	0.8333	-0.248	0.0278	-0.001	30247.8757	**0.302479**
1.01	0.49886978	45,810	49,887	0.8264	-0.246	0.0277	-0.001	30268.4848	**0.302685**
1.02	0.50380908	45,884	50,381	0.8197	-0.245	0.0276	-0.001	30288.5120	**0.302885**
1.03	0.50874839	45,957	50,875	0.813	-0.244	0.0275	-0.001	30307.9757	**0.30308**
1.04	0.51368769	46,029	51,369	0.8065	-0.242	0.0274	-0.001	30326.8934	**0.303269**
1.05	0.518627	46,100	51,863	0.8	-0.241	0.0273	-0.001	30345.2823	**0.303453**
1.06	0.5235663	46,170	52,357	0.7937	-0.24	0.0272	-0.001	30363.1586	**0.303632**

1.07	0.52850561	46,238	52,851	0.7874	-0.239	0.0271	-0.001	30380.5382	**0.303805**
1.08	0.53344491	46,306	53,344	0.7812	-0.237	0.027	-0.001	30397.4362	**0.303974**
1.09	0.53838422	46,372	53,838	0.7752	-0.236	0.0269	-0.001	30413.8674	**0.304139**
1.1	0.54332352	46,438	54,332	0.7692	-0.235	0.0267	-0.001	30429.8459	**0.304298**
1.11	0.54826282	46,502	54,826	0.7634	-0.234	0.0266	-0.001	30445.3853	**0.304454**
1.12	0.55320213	46,566	55,320	0.7576	-0.232	0.0265	-0.001	30460.4987	**0.304605**
1.13	0.55814143	46,628	55,814	0.7519	-0.231	0.0264	-0.001	30475.1989	**0.304752**
1.14	0.56308074	46,690	56,308	0.7463	-0.23	0.0263	-0.001	30489.4981	**0.304895**
1.15	0.56802004	46,751	56,802	0.7407	-0.229	0.0262	-0.001	30503.4081	**0.305034**
1.16	0.57295935	46,810	57,296	0.7353	-0.228	0.0261	-0.001	30516.9404	**0.305169**
1.17	0.57789865	46,869	57,790	0.7299	-0.227	0.026	-0.001	30530.1058	**0.305301**
1.18	0.58283796	46,927	58,284	0.7246	-0.225	0.0259	-0.001	30542.9152	**0.305429**
1.19	0.58777726	46,985	58,778	0.7194	-0.224	0.0258	-0.001	30555.3787	**0.305554**
1.2	0.59271657	47,041	59,272	0.7143	-0.223	0.0257	-0.001	30567.5063	**0.305675**
1.21	0.59765587	47,097	59,766	0.7092	-0.222	0.0256	-0.001	30579.3077	**0.305793**
1.22	0.60259518	47,151	60,260	0.7042	-0.221	0.0255	-0.001	30590.7920	**0.305908**
1.23	0.60753448	47,205	60,753	0.6993	-0.22	0.0255	-0.001	30601.9682	**0.30602**
1.24	0.61247379	47,259	61,247	0.6944	-0.219	0.0254	-0.001	30612.8451	**0.306128**
1.25	0.61741309	47,311	61,741	0.6897	-0.218	0.0253	-0.001	30623.4311	**0.306234**
1.26	0.6223524	47,363	62,235	0.6849	-0.217	0.0252	-0.001	30633.7341	**0.306337**
1.27	0.6272917	47,414	62,729	0.6803	-0.216	0.0251	-0.001	30643.7622	**0.306438**
1.28	0.632231	47,465	63,223	0.6757	-0.215	0.025	-0.001	30653.5229	**0.306535**
1.29	0.63717031	47,515	63,717	0.6711	-0.214	0.0249	-0.001	30663.0235	**0.30663**
1.3	0.64210961	47,564	64,211	0.6667	-0.213	0.0248	-0.001	30672.2712	**0.306723**
1.31	0.64704892	47,612	64,705	0.6623	-0.212	0.0247	-0.001	30681.2729	**0.306813**
1.32	0.65198822	47,660	65,199	0.6579	-0.211	0.0246	-0.001	30690.0352	**0.3069**
1.33	0.65692753	47,707	65,693	0.6536	-0.21	0.0245	-0.001	30698.5647	**0.306986**
1.34	0.66186683	47,754	66,187	0.6494	-0.209	0.0245	-0.001	30706.8676	**0.307069**
1.35	0.66680614	47,800	66,681	0.6452	-0.208	0.0244	-0.001	30714.9499	**0.307149**
1.36	0.67174544	47,845	67,175	0.641	-0.207	0.0243	-1E-03	30722.8176	**0.307228**
1.37	0.67668475	47,890	67,668	0.6369	-0.206	0.0242	-1E-03	30730.4763	**0.307305**
1.38	0.68162405	47,934	68,162	0.6329	-0.205	0.0241	-1E-03	30737.9317	**0.307379**
1.39	0.68656336	47,978	68,656	0.6289	-0.204	0.024	-1E-03	30745.1890	**0.307452**
1.4	0.69150266	48,021	69,150	0.625	-0.203	0.0239	-1E-03	30752.2534	**0.307523**
1.41	0.69644197	48,064	69,644	0.6211	-0.202	0.0239	-1E-03	30759.1300	**0.307591**
1.42	0.70138127	48,106	70,138	0.6173	-0.201	0.0238	-1E-03	30765.8237	**0.307658**
1.43	0.70632058	48,147	70,632	0.6135	-0.201	0.0237	-1E-03	30772.3392	**0.307723**
1.44	0.71125988	48,188	71,126	0.6098	-0.2	0.0236	-1E-03	30778.6812	**0.307787**
1.45	0.71619919	48,229	71,620	0.6061	-0.199	0.0235	-1E-03	30784.8540	**0.307849**
1.46	0.72113849	48,269	72,114	0.6024	-0.198	0.0234	-1E-03	30790.8621	**0.307909**
1.47	0.72607779	48,309	72,608	0.5988	-0.197	0.0234	-1E-03	30796.7096	**0.307967**
1.48	0.7310171	48,348	73,102	0.5952	-0.196	0.0233	-1E-03	30802.4006	**0.308024**
1.49	0.7359564	48,386	73,596	0.5917	-0.195	0.0232	-1E-03	30807.9391	**0.308079**
1.5	0.74089571	48,425	74,090	0.5882	-0.195	0.0231	-1E-03	30813.3290	**0.308133**
1.51	0.74583501	48,462	74,584	0.5848	-0.194	0.0231	-1E-03	30818.5739	**0.308186**
1.52	0.75077432	48,500	75,077	0.5814	-0.193	0.023	-1E-03	30823.6776	**0.308237**
1.53	0.75571362	48,537	75,571	0.578	-0.192	0.0229	-1E-03	30828.6435	**0.308286**
1.54	0.76065293	48,573	76,065	0.5747	-0.191	0.0228	-9E-04	30833.4751	**0.308335**
1.55	0.76559223	48,609	76,559	0.5714	-0.19	0.0227	-9E-04	30838.1757	**0.308382**
1.56	0.77053154	48,645	77,053	0.5682	-0.19	0.0227	-9E-04	30842.7485	**0.308427**
1.57	0.77547084	48,680	77,547	0.565	-0.189	0.0226	-9E-04	30847.1968	**0.308472**
1.58	0.78041015	48,715	78,041	0.5618	-0.188	0.0225	-9E-04	30851.5235	**0.308515**
1.59	0.78534945	48,749	78,535	0.5587	-0.187	0.0224	-9E-04	30855.7316	**0.308557**
1.6	0.79028876	48,783	79,029	0.5556	-0.187	0.0224	-9E-04	30859.8241	**0.308598**
1.61	0.79522806	48,817	79,523	0.5525	-0.186	0.0223	-9E-04	30863.8038	**0.308638**
1.62	0.80016737	48,850	80,017	0.5495	-0.185	0.0222	-9E-04	30867.6733	**0.308677**
1.63	0.80510667	48,883	80,511	0.5464	-0.184	0.0222	-9E-04	30871.4354	**0.308714**
1.64	0.81004597	48,916	81,005	0.5435	-0.183	0.0221	-9E-04	30875.0926	**0.308751**
1.65	0.81498528	48,948	81,499	0.5405	-0.183	0.022	-9E-04	30878.6475	**0.308786**
1.66	0.81992458	48,980	81,992	0.5376	-0.182	0.0219	-9E-04	30882.1025	**0.308821**

1.67	0.82486389	49,012	82,486	0.5348	-0.181	0.0219	-9E-04	30885.4600	**0.308855**
1.68	0.82980319	49,043	82,980	0.5319	-0.181	0.0218	-9E-04	30888.7223	**0.308887**
1.69	0.8347425	49,074	83,474	0.5291	-0.18	0.0217	-9E-04	30891.8917	**0.308919**
1.7	0.8396818	49,104	83,968	0.5263	-0.179	0.0217	-9E-04	30894.9703	**0.30895**
1.71	0.84462111	49,134	84,462	0.5236	-0.178	0.0216	-9E-04	30897.9604	**0.30898**
1.72	0.84956041	49,164	84,956	0.5208	-0.178	0.0215	-9E-04	30900.8640	**0.309009**
1.73	0.85449972	49,194	85,450	0.5181	-0.177	0.0215	-9E-04	30903.6832	**0.309037**
1.74	0.85943902	49,223	85,944	0.5155	-0.176	0.0214	-9E-04	30906.4199	**0.309064**
1.75	0.86437833	49,252	86,438	0.5128	-0.176	0.0213	-9E-04	30909.0760	**0.309091**
1.76	0.86931763	49,281	86,932	0.5102	-0.175	0.0213	-9E-04	30911.6534	**0.309117**
1.77	0.87425694	49,309	87,426	0.5076	-0.174	0.0212	-9E-04	30914.1540	**0.309142**
1.78	0.87919624	49,338	87,920	0.5051	-0.174	0.0211	-9E-04	30916.5796	**0.309166**
1.79	0.88413555	49,365	88,414	0.5025	-0.173	0.0211	-9E-04	30918.9318	**0.309189**
1.8	0.88907485	49,393	88,907	0.5	-0.172	0.021	-9E-04	30921.2124	**0.309212**
1.81	0.89401416	49,420	89,401	0.4975	-0.172	0.0209	-9E-04	30923.4229	**0.309234**
1.82	0.89895346	49,447	89,895	0.495	-0.171	0.0209	-9E-04	30925.5651	**0.309256**
1.83	0.90389276	49,474	90,389	0.4926	-0.17	0.0208	-9E-04	30927.6405	**0.309276**
1.84	0.90883207	49,501	90,883	0.4902	-0.17	0.0207	-9E-04	30929.6506	**0.309297**
1.85	0.91377137	49,527	91,377	0.4878	-0.169	0.0207	-9E-04	30931.5969	**0.309316**
1.86	0.91871068	49,553	91,871	0.4854	-0.168	0.0206	-9E-04	30933.4809	**0.309335**
1.87	0.92364998	49,579	92,365	0.4831	-0.168	0.0206	-9E-04	30935.3039	**0.309353**
1.88	0.92858929	49,604	92,859	0.4808	-0.167	0.0205	-9E-04	30937.0673	**0.309371**
1.89	0.93352859	49,629	93,353	0.4785	-0.167	0.0204	-9E-04	30938.7726	**0.309388**
1.9	0.9384679	49,654	93,847	0.4762	-0.166	0.0204	-9E-04	30940.4209	**0.309404**
1.91	0.9434072	49,679	94,341	0.4739	-0.165	0.0203	-9E-04	30942.0137	**0.30942**
1.92	0.94834651	49,704	94,835	0.4717	-0.165	0.0203	-9E-04	30943.5520	**0.309436**
1.93	0.95328581	49,728	95,329	0.4695	-0.164	0.0202	-9E-04	30945.0373	**0.30945**
1.94	0.95822512	49,752	95,823	0.4673	-0.164	0.0201	-9E-04	30946.4705	**0.309465**
1.95	0.96316442	49,776	96,316	0.4651	-0.163	0.0201	-9E-04	30947.8530	**0.309479**
1.96	0.96810373	49,800	96,810	0.463	-0.162	0.02	-9E-04	30949.1858	**0.309492**
1.97	0.97304303	49,823	97,304	0.4608	-0.162	0.02	-8E-04	30950.4701	**0.309505**
1.98	0.97798234	49,846	97,798	0.4587	-0.161	0.0199	-8E-04	30951.7068	**0.309517**
1.99	0.98292164	49,869	98,292	0.4566	-0.161	0.0198	-8E-04	30952.8972	**0.309529**
2	0.98786094	49,892	98,786	0.4545	-0.16	0.0198	-8E-04	30954.0421	**0.30954**
2.01	0.99280025	49,915	99,280	0.4525	-0.16	0.0197	-8E-04	30955.1427	**0.309551**
2.02	0.99773955	49,937	99,774	0.4505	-0.159	0.0197	-8E-04	30956.1998	**0.309562**
2.03	1.00267886	49,959	100,268	0.4484	-0.158	0.0196	-8E-04	30957.2144	**0.309572**
2.04	1.00761816	49,981	100,762	0.4464	-0.158	0.0196	-8E-04	30958.1876	**0.309582**
2.05	1.01255747	50,003	101,256	0.4444	-0.157	0.0195	-8E-04	30959.1201	**0.309591**
2.06	1.01749677	50,024	101,750	0.4425	-0.157	0.0195	-8E-04	30960.0129	**0.3096**
2.07	1.02243608	50,046	102,244	0.4405	-0.156	0.0194	-8E-04	30960.8668	**0.309609**
2.08	1.02737538	50,067	102,738	0.4386	-0.156	0.0193	-8E-04	30961.6828	**0.309617**
2.09	1.03231469	50,088	103,231	0.4367	-0.155	0.0193	-8E-04	30962.4615	**0.309625**
2.1	1.03725399	50,109	103,725	0.4348	-0.155	0.0192	-8E-04	30963.2039	**0.309632**
2.11	1.0421933	50,130	104,219	0.4329	-0.154	0.0192	-8E-04	30963.9107	**0.309639**
2.12	1.0471326	50,150	104,713	0.431	-0.154	0.0191	-8E-04	30964.5828	**0.309646**
2.13	1.05207191	50,170	105,207	0.4292	-0.153	0.0191	-8E-04	30965.2207	**0.309652**
2.14	1.05701121	50,190	105,701	0.4274	-0.153	0.019	-8E-04	30965.8254	**0.309658**
2.15	1.06195052	50,210	106,195	0.4255	-0.152	0.019	-8E-04	30966.3975	**0.309664**
2.16	1.06688982	50,230	106,689	0.4237	-0.152	0.0189	-8E-04	30966.9377	**0.309669**
2.17	1.07182913	50,250	107,183	0.4219	-0.151	0.0189	-8E-04	30967.4468	**0.309674**
2.18	1.07676843	50,269	107,677	0.4202	-0.151	0.0188	-8E-04	30967.9253	**0.309679**
2.19	1.08170773	50,289	108,171	0.4184	-0.15	0.0188	-8E-04	30968.3740	**0.309684**
2.2	1.08664704	50,308	108,665	0.4167	-0.15	0.0187	-8E-04	30968.7934	**0.309688**
2.21	1.09158634	50,327	109,159	0.4149	-0.149	0.0187	-8E-04	30969.1843	**0.309692**
2.22	1.09652565	50,346	109,653	0.4132	-0.149	0.0186	-8E-04	30969.5473	**0.309695**
2.23	1.10146495	50,364	110,146	0.4115	-0.148	0.0186	-8E-04	30969.8828	**0.309699**
2.24	1.10640426	50,383	110,640	0.4098	-0.148	0.0185	-8E-04	30970.1916	**0.309702**
2.25	1.11134356	50,401	111,134	0.4082	-0.147	0.0185	-8E-04	30970.4742	**0.309705**
2.26	1.11628287	50,419	111,628	0.4065	-0.147	0.0184	-8E-04	30970.7312	**0.309707**

2.27	1.12122217	50,437	112,122	0.4049	-0.146	0.0184	-8E-04	30970.9631	**0.30971**
2.28	1.12616148	50,455	112,616	0.4032	-0.146	0.0183	-8E-04	30971.1704	**0.309712**
2.29	1.13110078	50,473	113,110	0.4016	-0.145	0.0183	-8E-04	30971.3537	**0.309714**
2.3	1.13604009	50,491	113,604	0.4	-0.145	0.0182	-8E-04	30971.5136	**0.309715**
2.31	1.14097939	50,508	114,098	0.3984	-0.144	0.0182	-8E-04	30971.6504	**0.309717**
2.32	1.1459187	50,526	114,592	0.3968	-0.144	0.0181	-8E-04	30971.7647	**0.309718**
2.33	1.150858	50,543	115,086	0.3953	-0.143	0.0181	-8E-04	30971.8570	**0.309719**
2.34	1.15579731	50,560	115,580	0.3937	-0.143	0.018	-8E-04	30971.9277	**0.309719**
2.35	1.16073661	50,577	116,074	0.3922	-0.143	0.018	-8E-04	30971.9774	**0.30972**
2.36	1.16567592	50,594	116,568	0.3906	-0.142	0.0179	-8E-04	30972.0064	**0.30972**
2.37	1.17061522	50,610	117,062	0.3891	-0.142	0.0179	-8E-04	30972.0152	**0.30972**
2.38	1.17555452	50,627	117,555	0.3876	-0.141	0.0178	-8E-04	30972.0043	**0.30972**
2.39	1.18049383	50,643	118,049	0.3861	-0.141	0.0178	-8E-04	30971.9740	**0.30972**
2.4	1.18543313	50,660	118,543	0.3846	-0.14	0.0177	-8E-04	30971.9248	**0.309719**
2.41	1.19037244	50,676	119,037	0.3831	-0.14	0.0177	-8E-04	30971.8570	**0.309719**
2.42	1.19531174	50,692	119,531	0.3817	-0.139	0.0176	-8E-04	30971.7712	**0.309718**
2.43	1.20025105	50,708	120,025	0.3802	-0.139	0.0176	-8E-04	30971.6676	**0.309717**
2.44	1.20519035	50,723	120,519	0.3788	-0.139	0.0176	-8E-04	30971.5466	**0.309715**
2.45	1.21012966	50,739	121,013	0.3774	-0.138	0.0175	-8E-04	30971.4087	**0.309714**
2.46	1.21506896	50,755	121,507	0.3759	-0.138	0.0175	-8E-04	30971.2541	**0.309713**
2.47	1.22000827	50,770	122,001	0.3745	-0.137	0.0174	-8E-04	30971.0833	**0.309711**
2.48	1.22494757	50,786	122,495	0.3731	-0.137	0.0174	-8E-04	30970.8966	**0.309709**
2.49	1.22988688	50,801	122,989	0.3717	-0.137	0.0173	-8E-04	30970.6943	**0.309707**
2.5	1.23482618	50,816	123,483	0.3704	-0.136	0.0173	-8E-04	30970.4767	**0.309705**



F	$t_c = 1$	$t_c = 2$	$t_c = 3$	$t_c = 4$
0	0	0	0	0
0.01	0.02416	0.02289	0.020415	0.01748
0.02	0.04568	0.04356	0.03901	0.03348
0.03	0.06488	0.06227	0.055982	0.04817
0.04	0.08203	0.07923	0.071507	0.06167
0.05	0.09736	0.09465	0.085738	0.07411
0.06	0.1111	0.10868	0.098808	0.08559
0.07	0.12342	0.12148	0.110837	0.09621
0.08	0.13447	0.13318	0.121928	0.10605
0.09	0.14439	0.14388	0.132175	0.11519
0.1	0.15331	0.15369	0.141659	0.12369
0.11	0.16134	0.1627	0.150452	0.13161
0.12	0.16855	0.17098	0.15862	0.139
0.13	0.17505	0.17861	0.166218	0.14591
0.14	0.18089	0.18565	0.173298	0.15238
0.15	0.18615	0.19214	0.179906	0.15845
0.16	0.19088	0.19815	0.186082	0.16415
0.17	0.19513	0.20371	0.191863	0.16951
0.18	0.19895	0.20886	0.197282	0.17457
0.19	0.20239	0.21364	0.202369	0.17933
0.2	0.20546	0.21808	0.207149	0.18384
0.21	0.20821	0.2222	0.211646	0.18809
0.22	0.21067	0.22604	0.215883	0.19213
0.23	0.21286	0.22961	0.21988	0.19595
0.24	0.21481	0.23294	0.223653	0.19958
0.25	0.21653	0.23604	0.227219	0.20303
0.26	0.21806	0.23894	0.230593	0.20631
0.27	0.21939	0.24164	0.233788	0.20943
0.28	0.22055	0.24416	0.236816	0.2124
0.29	0.22156	0.24652	0.239689	0.21524
0.3	0.22242	0.24873	0.242418	0.21795
0.31	0.22316	0.25079	0.24501	0.22054

0.32	0.22377	0.25272	0.247476	0.22302
0.33	0.22427	0.25453	0.249823	0.22539
0.34	0.22467	0.25622	0.252058	0.22766
0.35	0.22497	0.2578	0.254189	0.22984
0.36	0.22519	0.25929	0.256221	0.23193
0.37	0.22533	0.26068	0.258161	0.23393
0.38	0.22539	0.26198	0.260013	0.23586
0.39	0.22539	0.2632	0.261783	0.23771
0.4	0.22532	0.26434	0.263475	0.23949
0.41	0.2252	0.26541	0.265094	0.2412
0.42	0.22503	0.26641	0.266644	0.24285
0.43	0.22481	0.26734	0.268128	0.24444
0.44	0.22454	0.26822	0.26955	0.24598
0.45	0.22423	0.26904	0.270913	0.24745
0.46	0.22389	0.26981	0.27222	0.24888
0.47	0.22351	0.27053	0.273475	0.25026
0.48	0.2231	0.2712	0.274679	0.25159
0.49	0.22266	0.27183	0.275836	0.25287
0.5	0.22219	0.27241	0.276947	0.25412
0.51	0.2217	0.27296	0.278015	0.25532
0.52	0.22118	0.27346	0.279042	0.25648
0.53	0.22065	0.27394	0.28003	0.25761
0.54	0.22009	0.27437	0.28098	0.2587
0.55	0.21952	0.27478	0.281895	0.25976
0.56	0.21893	0.27516	0.282776	0.26078
0.57	0.21833	0.27551	0.283625	0.26177
0.58	0.21772	0.27583	0.284443	0.26274
0.59	0.2171	0.27613	0.285231	0.26367
0.6	0.21646	0.27641	0.285992	0.26458
0.61	0.21582	0.27666	0.286725	0.26546
0.62	0.21516	0.27689	0.287432	0.26632
0.63	0.2145	0.2771	0.288115	0.26715
0.64	0.21384	0.27729	0.288774	0.26796
0.65	0.21316	0.27746	0.28941	0.26874
0.66	0.21249	0.27762	0.290024	0.26951
0.67	0.2118	0.27776	0.290618	0.27025
0.68	0.21112	0.27788	0.291192	0.27097
0.69	0.21043	0.27799	0.291746	0.27168
0.7	0.20974	0.27808	0.292282	0.27236
0.71	0.20905	0.27817	0.292801	0.27303
0.72	0.20835	0.27823	0.293302	0.27368
0.73	0.20766	0.27829	0.293787	0.27431
0.74	0.20696	0.27834	0.294257	0.27493
0.75	0.20627	0.27837	0.294711	0.27553
0.76	0.20557	0.2784	0.295151	0.27612
0.77	0.20488	0.27841	0.295577	0.27669
0.78	0.20418	0.27842	0.295989	0.27725
0.79	0.20349	0.27841	0.296389	0.2778
0.8	0.2028	0.2784	0.296776	0.27833
0.81	0.20211	0.27838	0.297151	0.27885
0.82	0.20142	0.27836	0.297514	0.27936
0.83	0.20074	0.27832	0.297866	0.27985
0.84	0.20005	0.27828	0.298208	0.28034
0.85	0.19937	0.27824	0.298539	0.28081
0.86	0.1987	0.27818	0.29886	0.28127
0.87	0.19802	0.27812	0.299172	0.28172
0.88	0.19735	0.27806	0.299474	0.28217
0.89	0.19668	0.27799	0.299767	0.2826
0.9	0.19602	0.27791	0.300052	0.28302
0.91	0.19535	0.27784	0.300328	0.28343

0.92	0.1947	0.27775	0.300596	0.28384
0.93	0.19404	0.27766	0.300856	0.28423
0.94	0.19339	0.27757	0.301109	0.28462
0.95	0.19274	0.27748	0.301354	0.285
0.96	0.1921	0.27738	0.301592	0.28537
0.97	0.19146	0.27727	0.301824	0.28573
0.98	0.19082	0.27717	0.302048	0.28609
0.99	0.19019	0.27706	0.302267	0.28644
1	0.18956	0.27695	0.302479	0.28678
1.01	0.18894	0.27683	0.302685	0.28712
1.02	0.18832	0.27671	0.302885	0.28744
1.03	0.18771	0.2766	0.30308	0.28777
1.04	0.1871	0.27647	0.303269	0.28808
1.05	0.18649	0.27635	0.303453	0.28839
1.06	0.18589	0.27622	0.303632	0.28869
1.07	0.18529	0.2761	0.303805	0.28899
1.08	0.1847	0.27597	0.303974	0.28928
1.09	0.18411	0.27583	0.304139	0.28957
1.1	0.18352	0.2757	0.304298	0.28985
1.11	0.18294	0.27557	0.304454	0.29012
1.12	0.18236	0.27543	0.304605	0.2904
1.13	0.18179	0.27529	0.304752	0.29066
1.14	0.18122	0.27515	0.304895	0.29092
1.15	0.18066	0.27502	0.305034	0.29118
1.16	0.1801	0.27487	0.305169	0.29143
1.17	0.17954	0.27473	0.305301	0.29168
1.18	0.17899	0.27459	0.305429	0.29192
1.19	0.17845	0.27445	0.305554	0.29216
1.2	0.1779	0.2743	0.305675	0.29239
1.21	0.17737	0.27416	0.305793	0.29262
1.22	0.17683	0.27401	0.305908	0.29285
1.23	0.1763	0.27387	0.30602	0.29307
1.24	0.17578	0.27372	0.306128	0.29329
1.25	0.17525	0.27357	0.306234	0.2935
1.26	0.17474	0.27342	0.306337	0.29371
1.27	0.17422	0.27328	0.306438	0.29392
1.28	0.17371	0.27313	0.306535	0.29412
1.29	0.17321	0.27298	0.30663	0.29432
1.3	0.17271	0.27283	0.306723	0.29452
1.31	0.17221	0.27268	0.306813	0.29471
1.32	0.17171	0.27253	0.3069	0.2949
1.33	0.17122	0.27238	0.306986	0.29509
1.34	0.17074	0.27223	0.307069	0.29528
1.35	0.17026	0.27209	0.307149	0.29546
1.36	0.16978	0.27194	0.307228	0.29564
1.37	0.1693	0.27179	0.307305	0.29581
1.38	0.16883	0.27164	0.307379	0.29599
1.39	0.16837	0.27149	0.307452	0.29616
1.4	0.1679	0.27134	0.307523	0.29633
1.41	0.16744	0.27119	0.307591	0.29649
1.42	0.16699	0.27104	0.307658	0.29665
1.43	0.16654	0.27089	0.307723	0.29681
1.44	0.16609	0.27074	0.307787	0.29697
1.45	0.16564	0.27059	0.307849	0.29713
1.46	0.1652	0.27045	0.307909	0.29728
1.47	0.16477	0.2703	0.307967	0.29743
1.48	0.16433	0.27015	0.308024	0.29758
1.49	0.1639	0.27	0.308079	0.29773
1.5	0.16347	0.26986	0.308133	0.29787
1.51	0.16305	0.26971	0.308186	0.29801

1.52	0.16263	0.26956	0.308237	0.29815
1.53	0.16221	0.26942	0.308286	0.29829
1.54	0.1618	0.26927	0.308335	0.29843
1.55	0.16139	0.26912	0.308382	0.29856
1.56	0.16098	0.26898	0.308427	0.2987
1.57	0.16057	0.26883	0.308472	0.29883
1.58	0.16017	0.26869	0.308515	0.29895
1.59	0.15978	0.26855	0.308557	0.29908
1.6	0.15938	0.2684	0.308598	0.29921
1.61	0.15899	0.26826	0.308638	0.29933
1.62	0.1586	0.26812	0.308677	0.29945
1.63	0.15821	0.26797	0.308714	0.29957
1.64	0.15783	0.26783	0.308751	0.29969
1.65	0.15745	0.26769	0.308786	0.29981
1.66	0.15708	0.26755	0.308821	0.29992
1.67	0.1567	0.26741	0.308855	0.30004
1.68	0.15633	0.26727	0.308887	0.30015
1.69	0.15596	0.26713	0.308919	0.30026
1.7	0.1556	0.26699	0.30895	0.30037
1.71	0.15524	0.26685	0.30898	0.30048
1.72	0.15488	0.26672	0.309009	0.30058
1.73	0.15452	0.26658	0.309037	0.30069
1.74	0.15417	0.26644	0.309064	0.30079
1.75	0.15382	0.26631	0.309091	0.3009
1.76	0.15347	0.26617	0.309117	0.301
1.77	0.15312	0.26604	0.309142	0.3011
1.78	0.15278	0.2659	0.309166	0.30119
1.79	0.15244	0.26577	0.309189	0.30129
1.8	0.1521	0.26563	0.309212	0.30139
1.81	0.15176	0.2655	0.309234	0.30148
1.82	0.15143	0.26537	0.309256	0.30158
1.83	0.1511	0.26524	0.309276	0.30167
1.84	0.15077	0.2651	0.309297	0.30176
1.85	0.15045	0.26497	0.309316	0.30185
1.86	0.15013	0.26484	0.309335	0.30194
1.87	0.1498	0.26471	0.309353	0.30203
1.88	0.14949	0.26458	0.309371	0.30212
1.89	0.14917	0.26445	0.309388	0.3022
1.9	0.14886	0.26433	0.309404	0.30229
1.91	0.14855	0.2642	0.30942	0.30237
1.92	0.14824	0.26407	0.309436	0.30245
1.93	0.14793	0.26395	0.30945	0.30254
1.94	0.14763	0.26382	0.309465	0.30262
1.95	0.14733	0.26369	0.309479	0.3027
1.96	0.14703	0.26357	0.309492	0.30278
1.97	0.14673	0.26344	0.309505	0.30286
1.98	0.14643	0.26332	0.309517	0.30293
1.99	0.14614	0.2632	0.309529	0.30301
2	0.14585	0.26308	0.30954	0.30309
2.01	0.14556	0.26295	0.309551	0.30316
2.02	0.14527	0.26283	0.309562	0.30323
2.03	0.14499	0.26271	0.309572	0.30331
2.04	0.1447	0.26259	0.309582	0.30338
2.05	0.14442	0.26247	0.309591	0.30345
2.06	0.14415	0.26235	0.3096	0.30352
2.07	0.14387	0.26223	0.309609	0.30359
2.08	0.14359	0.26211	0.309617	0.30366
2.09	0.14332	0.262	0.309625	0.30373
2.1	0.14305	0.26188	0.309632	0.3038
2.11	0.14278	0.26176	0.309639	0.30386

2.12	0.14251	0.26165	0.309646	0.30393
2.13	0.14225	0.26153	0.309652	0.304
2.14	0.14199	0.26142	0.309658	0.30406
2.15	0.14173	0.2613	0.309664	0.30412
2.16	0.14147	0.26119	0.309669	0.30419
2.17	0.14121	0.26107	0.309674	0.30425
2.18	0.14095	0.26096	0.309679	0.30431
2.19	0.1407	0.26085	0.309684	0.30437
2.2	0.14045	0.26074	0.309688	0.30444
2.21	0.14019	0.26063	0.309692	0.3045
2.22	0.13995	0.26052	0.309695	0.30456
2.23	0.1397	0.2604	0.309699	0.30461
2.24	0.13945	0.2603	0.309702	0.30467
2.25	0.13921	0.26019	0.309705	0.30473
2.26	0.13897	0.26008	0.309707	0.30479
2.27	0.13873	0.25997	0.30971	0.30484
2.28	0.13849	0.25986	0.309712	0.3049
2.29	0.13825	0.25975	0.309714	0.30496
2.3	0.13801	0.25965	0.309715	0.30501
2.31	0.13778	0.25954	0.309717	0.30506
2.32	0.13755	0.25944	0.309718	0.30512
2.33	0.13732	0.25933	0.309719	0.30517
2.34	0.13709	0.25923	0.309719	0.30522
2.35	0.13686	0.25912	0.30972	0.30528
2.36	0.13663	0.25902	0.30972	0.30533
2.37	0.13641	0.25891	0.30972	0.30538
2.38	0.13619	0.25881	0.30972	0.30543
2.39	0.13596	0.25871	0.30972	0.30548
2.4	0.13574	0.25861	0.309719	0.30553
2.41	0.13552	0.25851	0.309719	0.30558
2.42	0.13531	0.25841	0.309718	0.30563
2.43	0.13509	0.25831	0.309717	0.30568
2.44	0.13488	0.25821	0.309715	0.30572
2.45	0.13466	0.25811	0.309714	0.30577
2.46	0.13445	0.25801	0.309713	0.30582
2.47	0.13424	0.25791	0.309711	0.30586
2.48	0.13403	0.25781	0.309709	0.30591
2.49	0.13382	0.25771	0.309707	0.30596
2.5	0.13362	0.25762	0.309705	0.306

For yellowtail flounder, the t_c that gives the highest yield is age 3.

Solving for L_t:

$L_t = L_\infty * (1-e^{(-Kt)})$

$L_t = 46 * (1-e^{(-0.63*3)})$

39.0507

Then, solve for the mesh length:

$ml = L_{50}/SF$

ml = 39.05/2.6

15.0195

If *M/K* is large, then natural mortality exceeds growth indicating many fish will die before completing their potential growth. Management should allow heavy fishing at a small size relative to maximum size. Cod and silver hake reach maximum biomass at only 61% and 51% respectively of their L_∞.

If *M/K* is small then growth is high relative to natural mortality, and the cohort will reach maximum biomass at a larger size relative to maximum size. Management should maximize the size or age at entry. Yellowtail flounder with *M/K* = 0.3 reaches maximum YPR at 85% of its maximum size.

The management dilemma is that to maximize YPR, cod requires a mesh size of 24.6 cm, silver hake requires a mesh size of 9.5 cm, and yellowtail flounder requires a mesh size of 15.0 cm. In a multi-species fishery, using a mesh size appropriate to maximize the yield of cod will result in escape of almost all whiting and most yellowtail flounder. Conversely, using a mesh size appropriate for whiting will result in the capture of juvenile cod and severely growth and recruitment overfish the cod stocks.

Use the spreadsheet program in worksheet Exercise 2.

Insert parameter values for summer flounder in the appropriate places.

For each value of L_{50} and L_{opt} (35 and 55 cm), use F values of 0.0, 0.1, 0.2, 0.3, 0.4, 0.5, 1.0, 1.5, 2.0, 2.5, and 3.0.

For each value of F, copy the YPR/SSBPR row (cells J26 to M26) and Paste Special as Values in the matching F row below (cells C40 to C50 for 35 cm and J40 to J50 for 55 cm).

Yield Per Recruit Model

standard deviation = 5
steepness = 0.33

Example: L_{50} and L_{opt} = 55 cm and F = 0.0

Age	L(cm)	W(kg)	P	N_t	Biomass	SSB	Gill	Trawl	Y_{Gill}	SSB_{Gill}	Y_{Trawl}	SSB_{Trawl}	N_{Gill}	N_{Trawl}
0	0.0	0.0	0.00	1000	0	0	0.000	0.000	0	0	0	0	1000	1000
1	15.9	0.5	0.00	819	400	0	0.000	0.000	0	0	0	0	819	819
2	28.8	3.0	0.14	670	2028	276	0.000	0.000	0	276	2	275	670	670
3	39.3	7.8	0.96	549	4295	4118	0.007	0.006	118	4118	95	4115	549	548
4	47.7	14.3	1.00	449	6407	6405	0.348	0.083	1396	6226	974	6260	437	439
5	54.6	21.5	1.00	368	7925	7925	0.997	0.467	7138	1914	3534	5549	89	258
6	60.2	29.0	1.00	301	8742	8742	0.586	0.846	2340	39	6168	943	1	33
7	64.7	36.2	1.00	247	8937	8937	0.153	0.961	34	4	1089	33	0	[illegible]
8	68.3	42.9	1.00	202	8663	8663	0.028	0.988	1	2	36	1	0	0
9	71.3	48.9	1.00	165	8081	8081	0.005	0.995	0	2	1	0	0	0
10	73.7	54.1	1.00	135	7325	7325	0.001	0.998	0	1	0	0	0	0
11	75.7	58.6	1.00	111	6497	6497	0.000	0.999	0	1	0	0	0	0
12	77.2	62.5	1.00	91	5668	5668	0.000	0.999	0	1	0	0	0	0
13	78.5	65.7	1.00	74	4880	4880	0.000	1.000	0	1	0	0	0	0
14	79.6	68.4	1.00	61	4160	4160	0.000	1.000	0	1	0	0	0	0
15	80.4	70.7	1.00	50	3518	3518	0.000	1.000	0	1	0	0	0	0
16	81.1	72.5	1.00	41	2956	2956	0.000	1.000	0	1	0	0	0	0
17	81.6	74.0	1.00	33	2471	2471	0.000	1.000	0	1	0	0	0	0
18	82.1	75.3	1.00	27	2057	2057	0.000	1.000	0	0	0	0	0	0
19	82.4	76.3	1.00	22	1707	1707	0.000	1.000	0	0	0	0	0	0
20	82.7	77.2	1.00	18	1413	1413	0.000	1.000	0	0	0	0	0	0

	Y_{Gill}	SSB_{Gill}	Y_{Trawl}	SSB_{Trawl}
Yield/SSB	**11027**	**12590**	**11899**	**17178**
YPR/SSBPR	**11.027**	**12.590**	**11.899**	**17.178**

Maturity

$$P_l = (1 + e^{(-\alpha 1 * (l - \beta 1))})^{-1}$$

$\alpha 1$	5
$\beta 1$	2.37

Operators:

[redacted]	**Fishing Mortality**
55	L_{50} and L_{opt}
84	L_∞
0.21	K
0.2	M
0.0006	a
3.07	b

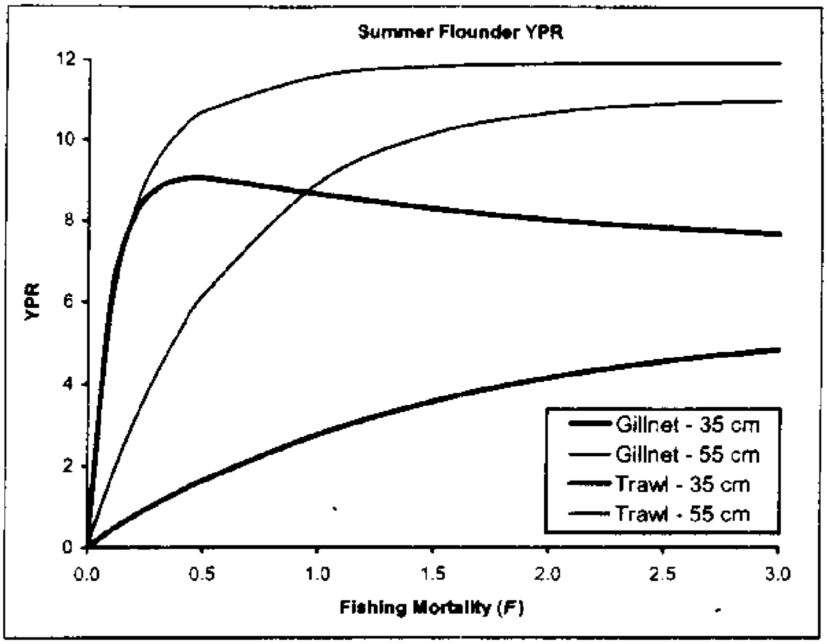
35 cm

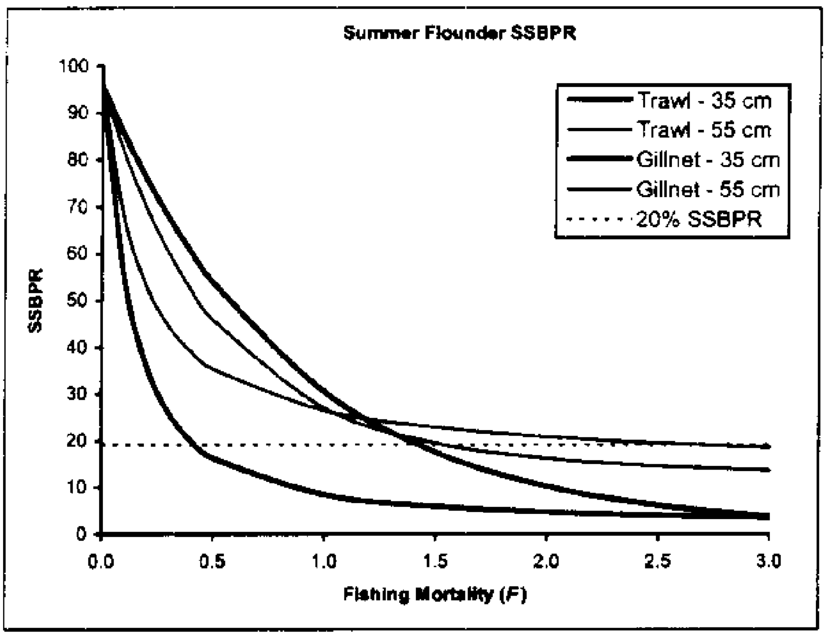
F	Y_{Gill}	SSB_{Gill}	Y_{Trawl}	SSB_{Trawl}
0	0.000	95.798	0.000	95.798
0.1	0.414	85.3	6.14	54.8387
0.2	0.766	75.97	8.12	35.9611
0.3	1.082	67.68	8.81	25.9584
0.4	1.373	60.32	9.02	20.0429
0.5	1.644	53.774	9.051	16.240
1	2.767	30.472	8.657	8.430
1.5	3.583	17.500	8.290	5.944
2	4.162	10.240	8.031	4.760
2.5	4.566	6.147	7.838	4.069
3	4.846	3.815	7.68	3.61053

55 cm

F	Y_{Gill}	SSB_{Gill}	Y_{Trawl}	SSB_{Trawl}
0	0.000	95.798	0.000	95.798
0.1	1.6745	81.495	5.7364	67.619862
0.2	3.0736	69.784	8.2507	53.028843
0.3	4.2709	60.186	9.5313	44.536563
0.4	5.2979	52.31	10.267	39.100139
0.5	6.177	45.837	10.728	35.353
1	8.964	26.995	11.618	26.457
1.5	10.181	19.496	11.846	22.876
2	10.702	16.232	11.921	20.857
2.5	10.919	14.613	11.940	19.517
3.000	11.003	13.676	11.936	18.541

$SSBPR_{20\%}$ =	19.16	
$SSBPR_{20\%}$ line =	0	19.16
	3	19.16





Based on the YPR plots, yield of summer flounder with an L_{50} of 55 cm in the trawl fishery asymptotes at about 11.9 and reaches 97% of that level at F = 1.0.

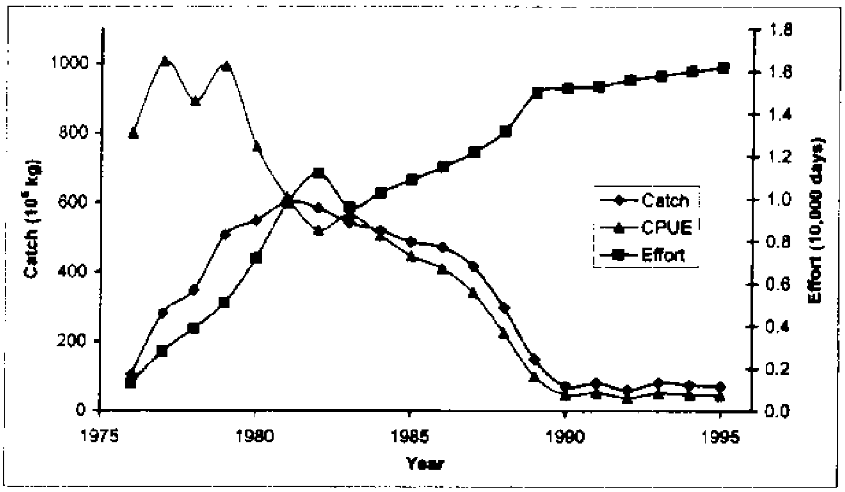
Given the maturity characteristics of summer flounder, the SSBPR curve for trawls with an L_{50} of 55 cm remains above the 20% minimum recruitment overfishing level beyond Fs of 2.5.

Given the following catch and effort data for the trawl fishery on this pelagic fish species for the period 1976 to 1995:

Year	Catch (10^6 kg)	Effort (10,000 days)
1976	104	0.13
1977	282	0.28
1978	348	0.39
1979	507	0.51
1980	548	0.72
1981	602	0.96
1982	584	1.12
1983	542	0.96
1984	521	1.03
1985	487	1.09
1986	472	1.16
1987	416	1.22
1988	298	1.32
1989	150	1.50
1990	72	1.52
1991	81	1.53
1992	60	1.58
1993	82	1.58
1994	75	1.60
1995	71	1.62

A. Plot the trajectories of catch and effort. Describe the time history of the fishery.
B. Estimate the parameters of Schaefer and Fox Surplus Production models for the data using linear regression.
C. Use Solver to improve the parameter estimates for the Schaefer and Fox models.
D. Estimate Y_{MSY} and f_{MSY} for both models, compare graphic and empirical estimates.

(A)



History of Fishery
- Steady increase in effort over 20 years
- Sharp increase in landings early in history of fishery, peak in 1981, then steady decline to collapse.
- Peak in CPUE in late 1970s, then steady decline of abundance due to overfishing.

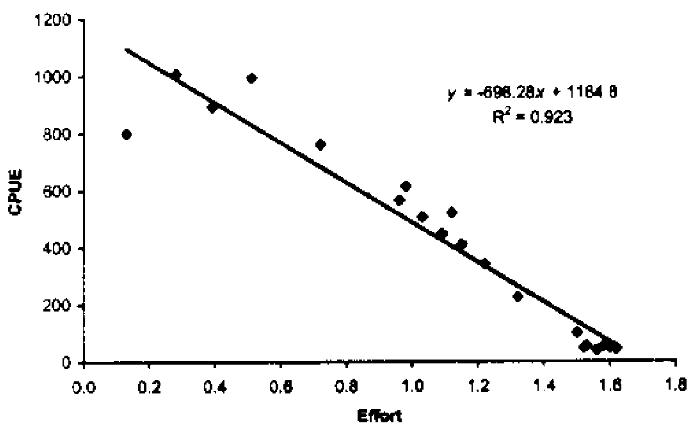
(B) SCHAEFER MODEL - Linear Regression

Year	Catch (10^6 kg)	Effort (10,000 days)	CPUE
1976	104	0.13	800.00
1977	282	0.28	1007.14
1978	348	0.39	892.31
1979	507	0.51	994.12
1980	548	0.72	761.11
1981	602	0.98	614.29
1982	584	1.12	521.43
1983	542	0.96	564.58
1984	521	1.03	505.83
1985	487	1.09	446.79
1986	472	1.15	410.43
1987	416	1.22	340.98
1988	298	1.32	225.76
1989	150	1.50	100.00
1990	72	1.52	47.37
1991	81	1.53	52.94
1992	60	1.56	38.46
1993	82	1.58	51.90
1994	75	1.60	46.88
1995	71	1.62	43.83

There are 2 methods to calculate the slope and *y*-intercept:

(1) Go to **Tools, Data Analysis, Regression**, select the input and output ranges, then click OK. The two numbers that are important are the X variable 1 which is the slope and intercept which is the *y*-intercept.

(2) Select the data series on the graph, right click, and choose **Add Trendline**. Choose "linear" for the **Type**. In the **Options** tab, click "display equation on chart."



For the linearized Schaefer model: $Y/f = a - bf$

the *a* parameter equals the *y*-intercept and the *b* parameter equals the negative of the slope.

So, $a = 1184.8$
and $b = 698.28$

From **Tools, Data Analysis, Regression**:

SUMMARY OUTPUT

Regression Statistics	
Multiple R	0.96074
R Square	0.9230214
Adjusted R Square	0.9187448
Standard Error	97.36776
Observations	20

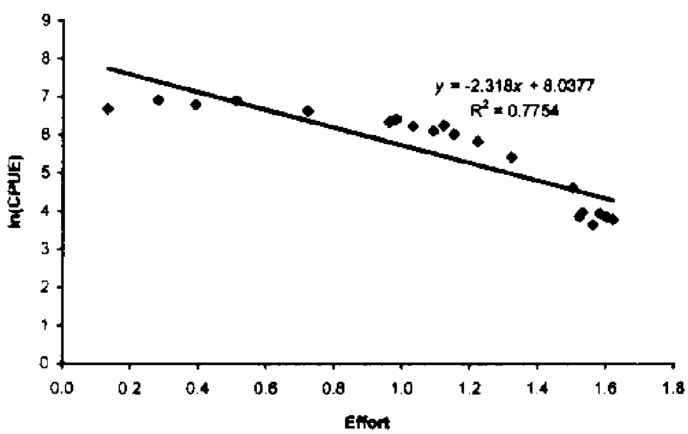
ANOVA

	df	*SS*	*MS*	*F*	*Significance F*
Regression	1	2046184.527	2046184.5	215.8313079	1.82475E-11
Residual	18	170648.6508	9480.4806		
Total	19	2216833.178			

	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*	*Lower 95.0%*	*Upper 95.0%*
Intercept	1184.7803	56.2189859	21.074381	3.89588E-14	1066.668518	1302.8921	1066.66852	1302.89211
X Variable 1	-698.2791	47.53043861	-14.691198	1.82475E-11	-798.1369228	-598.42128	-798.13692	-598.421276

FOX MODEL - Linear Regression

Year	Catch (10^6 kg)	Effort (10,000 days)	ln(CPUE)
1976	104	0.13	6.68
1977	282	0.28	6.91
1978	348	0.39	6.79
1979	507	0.51	6.90
1980	548	0.72	6.63
1981	602	0.98	6.42
1982	584	1.12	6.26
1983	542	0.96	6.34
1984	521	1.03	6.23
1985	487	1.09	6.10
1986	472	1.15	6.02
1987	416	1.22	5.83
1988	298	1.32	5.42
1989	150	1.50	4.61
1990	72	1.52	3.86
1991	81	1.53	3.97
1992	60	1.56	3.65
1993	62	1.58	3.95
1994	75	1.60	3.86
1995	71	1.62	3.76



From **Tools, Data Analysis, Regression**:

SUMMARY OUTPUT

Regression Statistics	
Multiple R	0.8805578
R Square	0.775382
Adjusted R Square	0.7629032
Standard Error	0.6023971
Observations	20

ANOVA

	df	*SS*	*MS*	*F*	*Significance F*
Regression	1	22.54807076	22.548071	62.1360553	3.02287E-07
Residual	18	6.531880271	0.3628822		
Total	19	29.07996103			

	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*	*Lower 95.0%*	*Upper 95.0%*
Intercept	8.0377088	0.347816905	23.109023	7.8443E-15	7.30697208	8.7684456	7.30697208	8.76844561
X Variable 1	-2.317989	0.294062402	-7.8826427	3.02287E-07	-2.935791493	-1.7001862	-2.9357915	-1.70018617

For the linearized Fox model: $\ln\left(\frac{Y}{f}\right) = c - df$

the *c* parameter equals the *y*-intercept and the *d* parameter equals the negative of the slope.

So, *c* = 8.0377
and *d* = 2.318

(C)

			SCHAEFER	Before Solver		After Solver	
Year	Catch (10^4 kg)	Effort (10,000 days)	CPUE	$Catch_{pred}$	$(C_{pred}-C_{obs})^2$	$Catch_{pred}$	$(C_{pred}-C_{obs})^2$
1976	104	0.13	800.00	165.82	3822.01	165.36	3765.53
1977	282	0.28	1007.14	386.48	10916.83	320.51	1482.69
1978	348	0.39	892.31	568.27	48520.07	409.99	3842.99
1979	507	0.51	994.12	785.86	77763.23	484.18	520.86
1980	548	0.72	761.11	1215.03	444929.26	555.16	51.25
1981	602	0.98	614.29	1831.71	1512193.62	539.28	3933.85
1982	584	1.12	521.43	2202.88	2620760.75	483.18	10164.90
1983	542	0.96	564.58	1780.92	1534932.56	544.58	6.64
1984	521	1.03	505.83	1961.13	2073971.47	523.06	4.26
1985	487	1.09	446.79	2121.04	2670076.96	498.00	121.06
1986	472	1.15	410.43	2285.97	3290496.82	466.83	26.76
1987	416	1.22	340.98	2484.75	4279734.58	422.73	45.27
1988	298	1.32	225.76	2780.59	6163268.44	345.30	2236.85
1989	150	1.50	100.00	3348.30	10229125.92	163.12	172.16
1990	72	1.52	47.37	3414.17	11170114.96	139.48	4553.95
1991	81	1.53	52.94	3447.32	11332093.75	127.41	2153.81
1992	60	1.56	38.46	3547.59	12163294.47	90.17	910.17
1993	82	1.58	51.90	3815.14	12483071.84	64.49	306.49
1994	75	1.60	46.88	3883.25	13019434.16	38.14	1358.82
1995	71	1.62	43.83	3751.91	13549099.48	11.10	3587.61
					108677621.20		39245.92

			FOX	Before Solver		After Solver	
Year	Catch (10^6 kg)	Effort (10,000 days)	ln(CPUE)	$Catch_{pred}$	$(C_{pred}-C_{obs})^2$	$Catch_{pred}$	$(C_{pred}-C_{obs})^2$
1976	104	0.13	6.68	297.72	37525.79	257.39	23527.19
1977	282	0.28	6.91	452.91	29210.37	422.07	19620.98
1978	348	0.39	6.79	488.86	19840.89	481.35	17782.59
1979	507	0.51	6.90	484.04	526.99	506.10	0.80
1980	548	0.72	6.63	419.99	16386.02	487.79	3625.81
1981	602	0.98	6.42	312.89	83583.56	413.88	35387.60
1982	584	1.12	6.26	258.49	105954.80	366.74	47203.14
1983	542	0.96	6.34	321.05	48818.74	420.45	14774.99
1984	521	1.03	6.23	292.87	52044.64	397.21	15324.01
1985	487	1.09	6.10	269.69	47225.45	376.92	12118.34
1986	472	1.15	6.02	247.59	50361.20	356.58	13322.57
1987	416	1.22	5.83	223.32	37126.86	333.09	6874.87
1988	298	1.32	5.42	191.63	11314.36	300.49	6.20
1989	150	1.50	4.61	143.48	42.57	246.18	9251.03
1990	72	1.52	3.86	138.80	4462.54	240.56	28412.05
1991	81	1.53	3.97	136.51	3081.81	237.78	24579.92
1992	60	1.56	3.65	129.84	4877.68	229.58	28755.98
1993	82	1.58	3.95	125.55	1896.39	224.22	20226.00
1994	75	1.60	3.85	121.38	2150.85	218.95	20721.71
1995	71	1.62	3.78	117.33	2146.20	213.77	20384.12
					558577.73		361899.71

The "Before **Solver**" values are the linear regression parameters values.

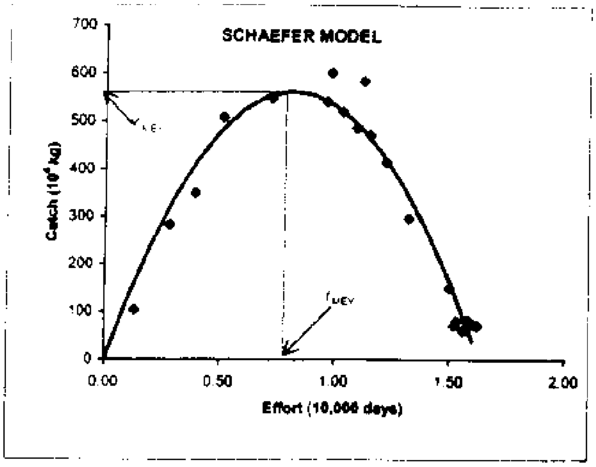
In **Solver**, the Set Target Cell is the Sum of the Squares and the By Changing Cells are the cells that contain the parameter values that were plugged into the equation in the $Catch_{pred}$ column.

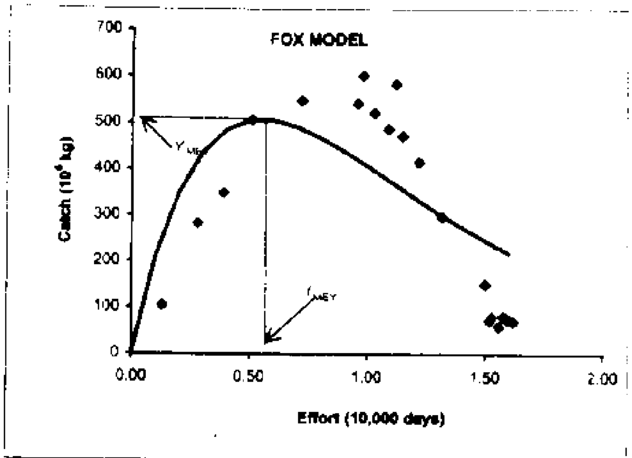
SCHAEFER MODEL		
	Before	After
a	1184.7803	1382.414604
b	-698.28	849.1115245

FOX MODEL		
	Before	After
c	8.0377	7.827092179
d	2.318	1.817856878

To graph the non-linear regression curves to the original catch and effort data, create a series of effort values and solve for catch using the appropriate parameter values from **Solver** for each model.

SCHAEFER MODEL		FOX MODEL	
Effort	Catch	Effort	Catch
0.0	0.0000	0.0	0.0000
0.1	129.7503	0.1	209.0851
0.2	242.5185	0.2	348.6690
0.3	338.3043	0.3	436.0786
0.4	417.1080	0.4	484.8011
0.5	478.9294	0.5	505.2820
0.6	523.7686	0.6	505.5629
0.7	551.6256	0.7	491.7928
0.8	562.5003	0.8	468.6346
0.9	556.3928	0.9	439.5892
1.0	533.3031	1.0	407.2534
1.1	493.2311	1.1	373.5232
1.2	436.1769	1.2	339.7563
1.3	362.1405	1.3	306.8941
1.4	271.1219	1.4	275.5709
1.5	163.1210	1.5	246.1823
1.6	38.1379	1.6	218.9604





Looking at the sum of square residuals, we can see that the Schaefer model fits the data much better than the Fox model, with much lower values. This can also be seen by looking at the graphs with the model curves, it is obvious that the Schaefer curves fits the data better.

(D) For the SCHAEFER MODEL:

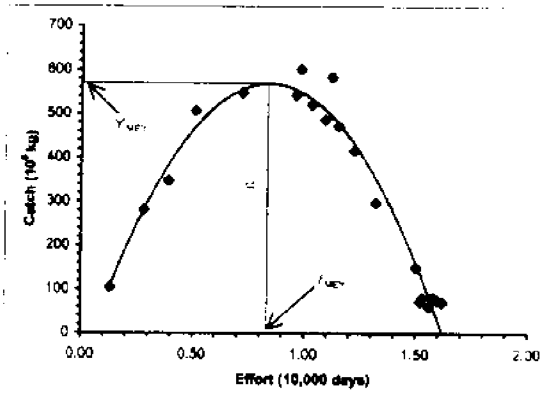
$$Y_{MEY} = a^2/4b$$

$$f_{MEY} = a/2b$$

Therefore,

$$Y_{MEY} = 1382.41^2/4(849.11) = 1911057.41/3396.44 = 562.66$$

$$f_{MEY} = 1382.41/2(849.11) = 1382.41/1698.22 = 0.814$$



For the FOX MODEL:

$$Y_{MEY} = \left(1/d\right)e^{(c-1)}$$

$$f_{MEY} = 1/d$$

Therefore,

$$Y_{MEY} = \left(1/1.82\right)e^{(7.83-1)} = (0.549)e^{6.83} = (0.549)(925.19) = 508.35$$

$$f_{MEY} = 1/1.82 = 0.55$$

If we add a trendline to the catch versus effort data and consider it the equivalent to hand drawing in a curve, we can see that the curve resembles the Schaefer model curve much more than the Fox model curve. Values of f_{MEY} and Y_{MEY} from the "hand drawn curve" would be approximately 570 and 0.84, reespectively, slightly higher than the Schaefer model values and very different from the Fox model values.

SCHAEFER MODEL

Microsoft Excel 8.0a Answer Report
Worksheet: [Chapter 9 - Production Exercises.xls]Example 1
Report Created: 3/27/00 4:43:01 PM

Target Cell (Min)

Cell	Name	Original Value	Final Value
I180	$(C_{pred}-C_{obs})^2$	108677621.20	39245.92

Adjustable Cells

Cell	Name	Original Value	Final Value
D186	*a* After	1184.780316	1382.414604
D187	*b* After	-698.28	849.1115245

Constraints
NONE

FOX MODEL

Microsoft Excel 8.0a Answer Report
Worksheet: [Chapter 9 - Production Exercises.xls]Example 1
Report Created: 3/27/00 4:43:14 PM

Target Cell (Min)

Cell	Name	Original Value	Final Value
N180	$(C_{pred}-C_{obs})^2$	558577.73	361899.71

Adjustable Cells

Cell	Name	Original Value	Final Value
D192	*c* After	8.0377	7.827092179
D193	*d* After	2.318	1.817656676

Constraints
NONE

Given the following stock-recruitment data, for each species:

(a) Plot the time history of spawning stock size and recruitment.

(b) Estimate the parameters of a Beverton-Holt stock-recruitment model using both the linear and non-linear regression methods.

(c) Estimate the parameters of a Ricker stock-recruitment model using both the linear and non-linear regression methods.

(d) Describe and interpret the models for each species over their time history.

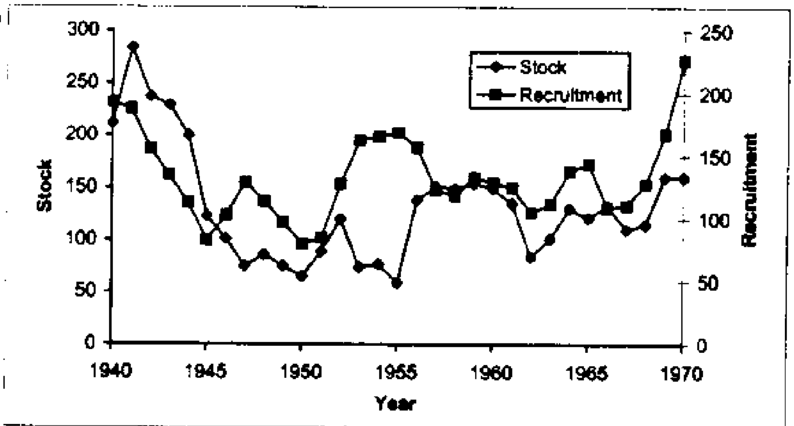
	Shad		Salmon	
Year	*S*	*R*	*S*	*R*
1921			150	449
1922			40	228
1923			70	199
1924			106	81
1925			162	161
1926			253	146
1927			87	162
1928			109	263
1929			90	159
1930			109	117
1931			87	258
1932			74	254
1933			97	219
1934			145	126
1935			88	125
1936			137	135
1937			126	133
1938			123	159
1939			71	183
1940	212	193	88	86
1941	284	188	93	57
1942	237	156	63	69
1943	229	135	92	150
1944	200	113	77	114
1945	123	83	66	126
1946	101	103	44	82
1947	75	129	48	77
1948	86	114	75	81
1949	75	97		
1950	65	80		
1951	89	88		
1952	120	128		
1953	74	163		
1954	77	166		
1955	59	169		
1956	138	157		
1957	151	123		
1958	149	119		
1959	154	133		
1960	149	129		
1961	135	126		
1962	84	105		
1963	101	112		
1964	130	138		
1965	121	144		
1966	133	109		
1967	110	111		
1968	115	128		
1969	160	168		
1970	160	227		

	Yellowtail Flounder		Blue Crab	
Year	*S*	*R*	*S*	*R*
1956			1.6	2.8
1957			0.5	4.8
1958			0.2	0.3
1959			0	4
1960			0.3	0.5
1961			0.1	0
1962			0.2	1
1963			0.5	0.4
1964			0.2	8.6
1965			0.4	3.7
1966			0.8	3.1
1967			1.6	0.5
1968			3	15.2
1969			1.5	1.5
1970			4.7	17.8
1971			3	7.1
1972			2.1	7.5
1973			0.1	4
1974			0.1	0.7
1975			0	0.7
1976			0.1	5.7
1977			0.2	6.3
1978	11.9	20.1	0.2	6.4
1979	13.5	14.1	0.4	0.7
1980	9.2	50.5	0.4	12.3
1981	6.7	26.8	0.6	9.3
1982	11.6	23.8	0.8	5
1983	12.9	56.2	0.4	10.5
1984	12.4	20.4	0.3	4.5
1985	16.4	7.4	0.2	8.7
1986	11.4	9.4	0.6	8.5
1987	3.3	17.3	0.4	12.7
1988	2.7	6.3	3.8	17.1
1989	3.8	6.2	4.5	24.3
1990	2.6	17.2	7.1	11.3
1991	2.2	6.6	4	16.8
1992	5.1	6.2	1.5	16.2
1993	4.3	16.8	1.2	19.4
1994	3.5	4.9	0.6	12.5
1995	3.7	5.2		

SHAD

Year	S	R
1940	212	193
1941	284	188
1942	237	156
1943	229	136
1944	200	113
1945	123	83
1946	101	103
1947	75	129
1948	86	114
1949	75	97
1950	66	80
1951	89	85
1952	120	128
1953	74	163
1954	77	166
1955	59	169
1956	138	157
1957	151	123
1958	149	119
1959	154	133
1960	149	129
1961	135	128
1962	84	106
1963	101	112
1964	130	138
1965	121	144
1966	133	109
1967	110	111
1968	116	128
1969	160	168
1970	160	227

(a)



(b) Beverton-Holt Model

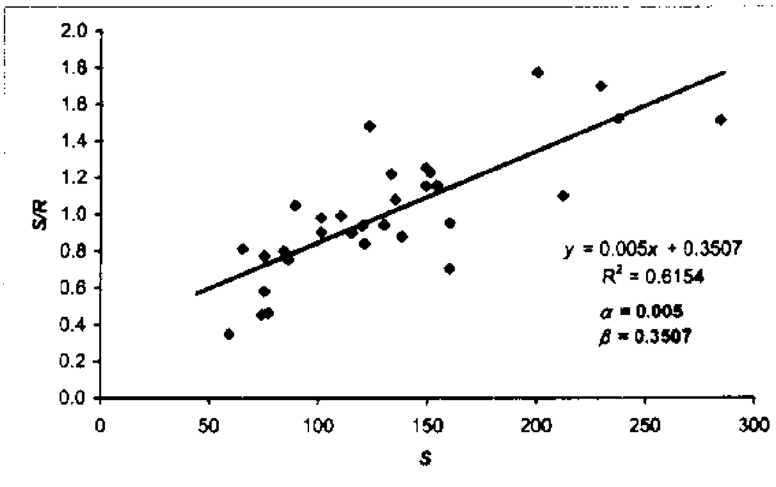
Linear

Year	S	R	(S/R)
1940	212	193	1.098
1941	284	188	1.511
1942	237	156	1.519
1943	229	135	1.696
1944	200	113	1.770
1945	123	83	1.482
1946	101	103	0.981
1947	75	129	0.581
1948	86	114	0.754
1949	75	97	0.773
1950	65	80	0.813
1951	89	85	1.047
1952	120	128	0.938
1953	74	163	0.454
1954	77	166	0.464
1955	59	169	0.349
1956	138	157	0.879
1957	151	123	1.228
1958	149	119	1.252
1959	154	133	1.158
1960	149	129	1.155
1961	135	125	1.080
1962	84	105	0.800
1963	101	112	0.902
1964	130	138	0.942
1965	121	144	0.840
1966	133	109	1.220
1967	110	111	0.991
1968	115	128	0.898
1969	160	168	0.952
1970	160	227	0.705

There are 2 methods to calculate the slope and *y*-intercept

(1) Go to **Tools, Data Analysis, Regression**, select the input and output ranges, then click OK. The two numbers that are important are the X variable 1 which is the slope and Intercept which is the *y*-intercept.

(2) Select the data series on the graph, right click, and choose **Add Trendline.** Choose "linear" for the **Type**. In the **Options** tab, click "display equation on chart."



From **Tools**, **Data Analysis**, **Regression**:

SUMMARY OUTPUT

Regression Statistics	
Multiple R	0.784506046
R Square	0.615449739
Adjusted R Square	0.602189385
Standard Error	0.218433911
Observations	31

ANOVA

	df	*SS*	*MS*	*F*	*Significance F*
Regression	1	2.214509774	2.214509774	46.412769	1.7613E-07
Residual	29	1.383687826	0.047713373		
Total	30	3.5981976			

	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*	*Lower 95.0%*	*Upper 95.:%*
Intercept	0.350692246	0.104086614	3.369234852	0.00214567	0.1378111	0.56367339	0.137811103	0.58357:39
X Variable 1	0.004970985	0.000729665	6.812691759	1.7613E-07	0.00347866	0.006463319	0.003478651	0.00646:19

Non-linear

			Before Solver		After Solver	
Year	S	R_{obs}	R_{pred}	$(R_{obs}-R_{pred})^2$	R_{pred}	$(R_{obs}-R_{pred})^2$
1940	212	193	150.260	1824.998	150.400	1814.743
1941	284	188	160.389	762.392	156.873	968.901
1942	237	156	154.327	2.799	153.027	8.837
1943	229	135	153.106	327.812	152.240	297.202
1944	200	113	148.071	1230.001	148.946	1292.097
1945	123	83	127.369	1968.586	134.555	2657.899
1946	101	103	118.032	225.962	127.581	604.212
1947	75	129	103.348	658.000	115.934	170.713
1948	88	114	110.158	14.764	121.443	55.393
1949	75	97	103.348	40.303	115.934	358.507
1950	65	80	96.197	262.328	109.937	896.198
1951	89	85	111.861	720.987	122.783	1427.584
1952	120	128	126.223	3.158	133.716	32.671
1953	74	163	102.678	3638.749	115.381	2267.535
1954	77	166	104.662	3762.322	117.012	2399.820
1955	59	169	91.374	6025.842	105.763	3998.873
1956	138	157	132.603	595.211	138.328	348.639
1957	151	123	136.565	184.011	141.122	328.405
1958	149	119	135.986	288.529	140.717	471.627
1959	154	133	137.414	19.484	141.714	75.931
1960	149	129	135.986	48.806	140.717	137.288
1961	135	125	131.617	43.790	137.625	159.387
1962	84	105	108.992	15.935	120.513	240.656
1963	101	112	118.032	36.385	127.581	242.759
1964	130	138	129.909	65.463	136.398	2.566
1965	121	144	126.609	302.455	133.999	100.020
1966	133	109	130.944	481.547	137.143	792.003
1967	110	111	122.127	123.815	130.679	387.265
1968	115	128	124.230	14.211	132.246	18.029
1969	160	168	139.046	838.346	142.846	632.792
1970	160	227	139.046	7735.942	142.846	7082.123
			SSR:	32262.934	SSR:	30270.676

To run **Solver**, go to **Tools**, **Solver.**

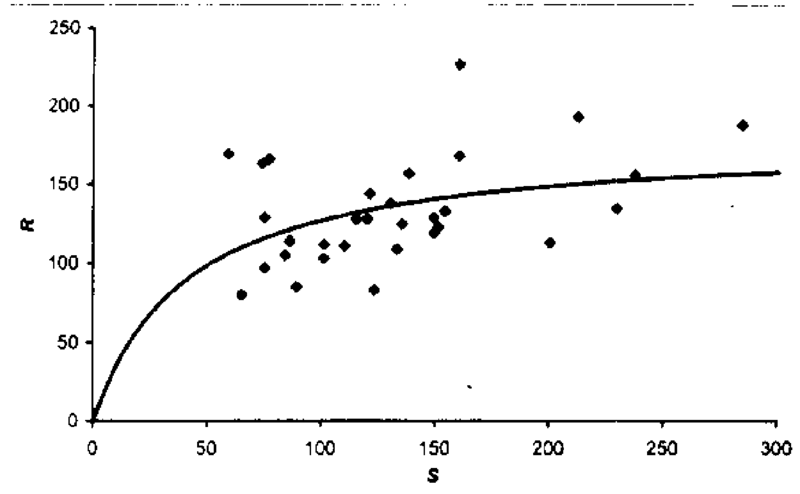
As the target cell, select the sum **of square residuals** (SSR)

Select "Min" for the **Equal** To:

Select the α **and** β as the By Changing Cells

It is important to note that the α **and** β **to be** changed must be **the same** α and β that are in the **Beverton**-Holt equation in the R_{pred} column and the SSR **selected** must also be related to **the same** R_{pred} column.

	Before Solver	After Solver
α	0.005	0.00556682
β	0.3507	0.229406803



To plot the Beverton-Holt model, create a series of stock (S) and solve for R using the Beverton-Holt equation and the parameter values obtained from **Solver**.

α	0.00556682
β	0.229406803

S	R
0	0
10	35.0784882
20	58.6952293
30	75.6789563
40	88.4799957
50	98.474088
60	106.493252
70	113.070246
80	118.562014
90	123.216683
100	127.212097
110	130.67906
120	133.715886
130	136.397971
140	138.784037
150	140.920526
160	142.844651
170	144.586576
180	146.171009
190	147.618387
200	148.945755
210	150.167445
220	151.295594
230	152.340548
240	153.311185
250	154.215159
260	155.05911
270	155.848824
280	156.589367
290	157.285193
300	157.940233

(c) Ricker Model

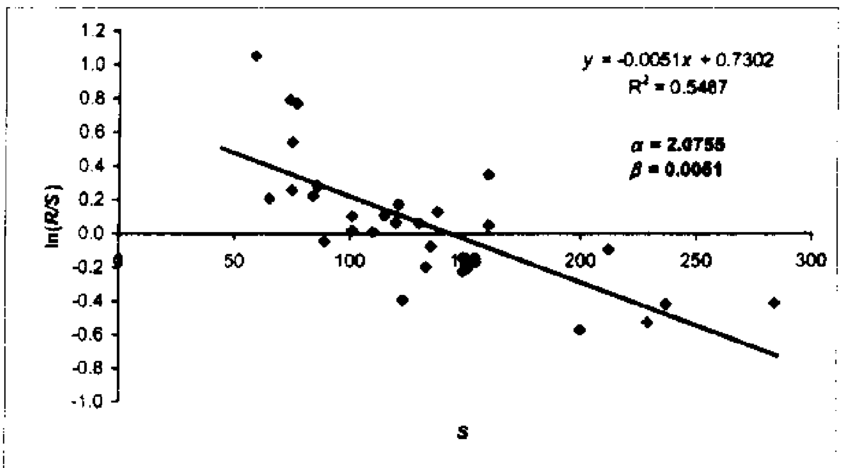
Linear

Year	S	R	ln(R/S)
1940	212	193	-0.094
1941	284	188	-0.413
1942	237	156	-0.418
1943	229	135	-0.528
1944	200	113	-0.571
1945	123	83	-0.393
1946	101	103	0.020
1947	75	129	0.542
1948	86	114	0.282
1949	75	97	0.257
1950	65	80	0.208
1951	89	85	-0.046
1952	120	128	0.065
1953	74	163	0.790
1954	77	166	0.768
1955	59	169	1.052
1956	138	157	0.129
1957	151	123	-0.205
1958	149	119	-0.225
1959	154	133	-0.147
1960	149	129	-0.144
1961	135	125	-0.077
1962	84	105	0.223
1963	101	112	0.103
1964	130	138	0.060
1965	121	144	0.174
1966	133	109	-0.199
1967	110	111	0.009
1968	115	128	0.107
1969	160	168	0.049
1970	160	227	0.350

There are 2 methods to calculate the slope and y-intercept.

(1) Go to **Tools, Data Analysis, Regression**, select the input and output ranges, then click OK. The two numbers that are important are the X variable 1 which is the slope and intercept which is the y-intercept.

(2) Select the data series on the graph, right click, and choose **Add Trendline.** Choose "linear" for the **Type.** In the **Options** tab, click "display equation on chart."



From **Tools, Data Analysis, Regression**:

SUMMARY OUTPUT

Regression Statistics	
Multiple R	0.740770583
R Square	0.548741056
Adjusted R Square	0.533180403
Standard Error	0.257340987
Observations	31

ANOVA

	df	*SS*	*MS*	*F*	*Significance F*
Regression	1	2.335379992	2.335379992	35.2646542	1.8863E-06
Residual	29	1.920507128	0.066224384		
Total	30	4.25588712			

	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*	*Lower 95.0%*	*Upper 95.0%*
Intercept	0.730221461	0.122626344	5.9548498	1.803E-06	0.47942229	0.981020632	0.47942229	0.981020632
X Variable 1	-0.005104843	0.000859632	-5.93840502	1.8863E-06	-0.006863	-0.0033467	-0.006862989	-0.003346697

Non-linear

			Before Solver		After Solver	
Year	S	R_{obs}	R_{pred}	$(R_{obs}-R_{pred})^2$	R_{pred}	$(R_{obs}-R_{pred})^2$
1940	212	193	149.095	1927.684	155.610	1397.993
1941	284	188	138.299	2470.159	149.008	1520.374
1942	237	156	146.706	86.370	154.818	1.397
1943	229	135	147.663	160.358	155.278	411.202
1944	200	113	149.541	1335.240	155.251	1785.140
1945	123	83	136.253	2835.833	136.724	2886.312
1946	101	103	125.180	491.953	124.399	457.902
1947	75	129	106.149	522.156	104.282	610.977
1948	86	114	115.071	1.148	113.598	0.162
1949	75	97	106.149	83.709	104.282	53.028
1950	65	80	96.814	282.718	94.692	215.854
1951	89	85	117.276	1041.715	115.927	956.505
1952	120	128	134.981	48.731	135.269	52.836
1953	74	163	105.270	3332.757	103.373	3555.435
1954	77	166	107.873	3378.756	106.069	3591.719
1955	59	169	90.611	6144.870	88.390	6497.986
1956	138	157	141.600	237.157	143.035	195.017
1957	151	123	144.991	483.593	147.304	590.669
1958	149	119	144.539	652.217	146.715	768.097
1959	154	133	145.624	159.366	148.143	229.319
1960	149	129	144.539	241.448	146.715	313.806
1961	135	125	140.660	245.221	141.897	285.505
1962	84	106	113.549	73.080	111.996	48.939
1963	101	112	125.180	173.713	124.399	153.727
1964	130	138	138.952	0.906	139.865	3.477
1965	121	144	135.413	73.744	135.761	67.873
1966	133	109	139.996	960.861	141.105	1030.703
1967	110	111	130.213	369.127	129.915	357.792
1968	115	128	132.701	22.097	132.691	22.001
1969	160	168	146.734	452.249	149.668	336.046
1970	160	227	146.734	6442.656	149.668	5980.168
			SSR:	34731.591	SSR:	34377.959

To run **Solver**, go to **Tools**, **Solver.**

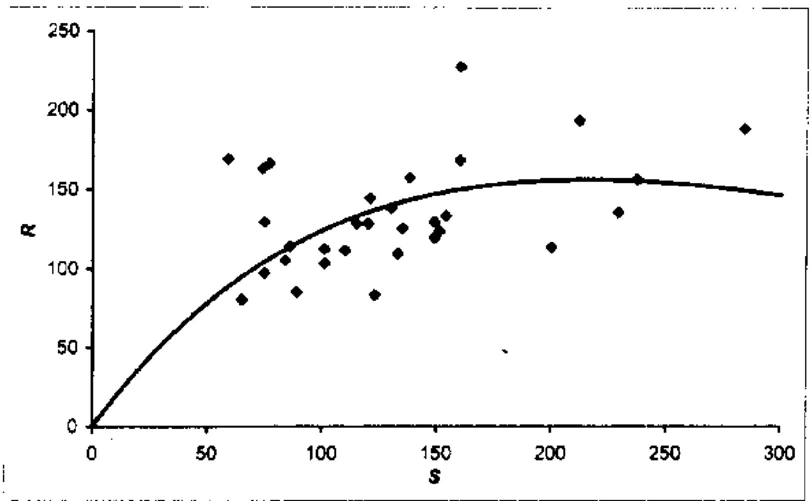
As the target cell, select the sum of square residuals (SSR)

Select "Min" for the Equal To:

Select the α and β as the By Changing Cells

It is important to note that the α and β to be changed must be the same α and β that are in the Ricker equation in the R_{pred} column and the SSR selected must also be related to the same R_{pred} column.

	Before Solver	After Solver
α	2.075540209	1.972580449
β	0.005104843	0.004663087



To plot the Ricker model, create a series of stock (S) and solve for R using the Ricker equation and the parameter values obtained from **Solver**.

α	1.972560449
β	0.004663087

S	R
0	0.000
10	18.827
20	35.939
30	51.452
40	65.477
50	78.117
60	89.470
70	99.626
80	108.671
90	116.685
100	123.743
110	129.915
120	135.269
130	139.865
140	143.761
150	147.012
160	149.668
170	151.778
180	153.384
190	154.529
200	155.251
210	155.587
220	155.569
230	155.231
240	154.600
250	153.704
260	152.570
270	151.219
280	149.675
290	147.968
300	146.087

BEVERTON-HOLT

Microsoft Excel 8.0a Answer Report
Worksheet: [Chapter 10 - Stock Recruitment Exercises.xls]SHAD
Report Created: 4/5/00 11:52:01 AM

Target Cell (Min)

Cell	Name	Original Value	Final Value
H145	SSR	32262.934	30270.676

Adjustable Cells

Cell	Name	Original Value	Final Value
D148	α After **Solver**	0.005	0.00556682
D149	β After **Solver**	0.3507	0.229406803

Constraints
NONE

RICKER

Microsoft Excel 8.0a Answer Report
Worksheet: [Chapter 10 - Stock Recruitment Exercises.xls]SHAD
Report Created: 4/5/00 11:52:01 AM

Target Cell (Min)

Cell	Name	Original Value	Final Value
H299	SSR	34731.591	34377.959

Adjustable Cells

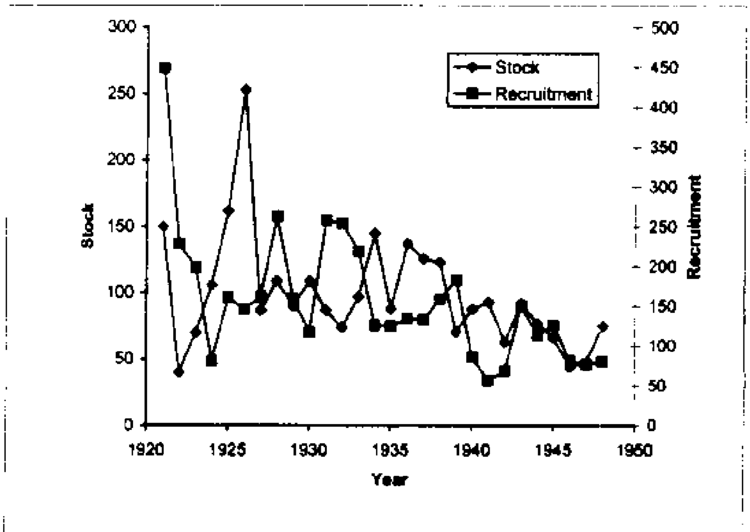
Cell	Name	Original Value	Final Value
D302	α After **Solver**	2.075540209	1.972580449
D303	β After **Solver**	0.005104843	0.004663087

Constraints
NONE

SALMON

Year	*S*	*R*
1921	150	449
1922	40	228
1923	70	199
1924	106	81
1925	162	161
1926	253	146
1927	87	162
1928	109	263
1929	90	159
1930	109	117
1931	87	258
1932	74	254
1933	97	219
1934	145	128
1935	88	125
1936	137	135
1937	128	133
1938	123	159
1939	71	183
1940	88	86
1941	93	57
1942	63	69
1943	92	150
1944	77	114
1945	66	126
1946	44	82
1947	48	77
1948	75	81

(a)



(b) Beverton-Holt Model

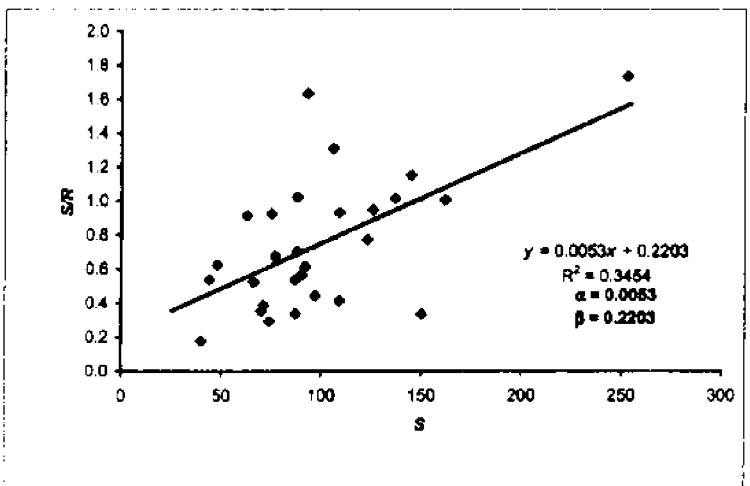
Linear

Year	*S*	*R*	(*S/R*)
1921	150	449	0.334
1922	40	228	0.175
1923	70	199	0.352
1924	106	81	1.309
1925	162	161	1.006
1926	253	146	1.733
1927	87	162	0.537
1928	109	263	0.414
1929	90	159	0.566
1930	109	117	0.932
1931	87	258	0.337
1932	74	254	0.291
1933	97	219	0.443
1934	145	126	1.151
1935	88	125	0.704
1936	137	135	1.015
1937	126	133	0.947
1938	123	159	0.774
1939	71	183	0.388
1940	88	86	1.023
1941	93	57	1.632
1942	63	69	0.913
1943	92	150	0.613
1944	77	114	0.675
1945	66	126	0.524
1946	44	82	0.537
1947	48	77	0.623
1948	75	81	0.926

There are 2 methods to calculate the slope and *y*-intercept:

(1) Go to **Tools, Data Analysis, Regression, select** the input and output ranges, then click OK. The two numbers that are important are the X variable 1 which is the slope and intercept which is the *y*-intercept.

(2) Select the data series on the graph, right click, and choose **Add Trendline.** Choose "linear" for the **Type.** In the **Options** tab, click "display equation on chart."



From **Tools, Data Analysis, Regression**:

SUMMARY OUTPUT

Regression Statistics	
Multiple R	0.587683666
R Square	0.345372091
Adjusted R Square	0.320194094
Standard Error	0.323425036
Observations	28

ANOVA

	df	*SS*	*MS*	*F*	*Significance F*
Regression	1	1.434872592	1.43487259	13.7172189	0.001007561
Residual	26	2.719697596	0.10460375		
Total	27	4.154570189			

	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*	*Lower 95.0%*	*Upper 95.0%*
Intercept	0.220273551	0.154427764	1.42638568	0.16565527	-0.097157472	0.537704575	-0.097157472	0.537704575
X Variable 1	0.005309332	0.00143353	3.70367641	0.00100758	0.002362666	0.008255997	0.002362666	0.008255997

Non-linear

			Before Solver		After Solver	
Year	*S*	R_{obs}	R_{pred}	$(R_{obs}-R_{pred})^2$	R_{pred}	$(R_{obs}-R_{pred})^2$
1921	150	449	150.078	89354.372	178.087	73393.911
1922	40	228	92.454	18372.674	116.396	12455.470
1923	70	199	118.258	6519.291	145.941	2815.225
1924	106	81	135.366	2955.653	164.896	7038.379
1925	162	161	149.947	122.180	180.666	386.757
1926	253	146	161.813	250.047	193.250	2232.560
1927	87	162	127.531	1188.090	156.276	32.760
1928	109	263	136.422	16021.961	166.049	9399.467
1929	90	159	128.919	904.874	157.810	1.415
1930	109	117	136.422	377.218	166.049	2405.819
1931	87	258	127.531	17022.079	156.276	10347.694
1932	74	254	120.685	17772.763	148.661	11096.310
1933	97	219	131.923	7582.446	161.120	3350.104
1934	145	126	146.446	418.035	176.911	2591.946
1935	88	125	128.001	9.006	156.796	1010.982
1936	137	135	144.568	91.544	174.888	1591.091
1937	126	133	141.693	75.560	171.781	1503.944
1938	123	159	140.842	329.726	170.858	140.622
1939	71	183	118.881	4111.251	146.640	1322.026
1940	88	86	128.001	1764.082	156.796	5012.066
1941	93	57	130.246	5364.764	159.273	10459.715
1942	63	69	113.562	1985.801	140.652	5134.017
1943	92	150	129.809	407.665	158.793	77.315
1944	77	114	122.399	70.537	150.574	1337.669
1945	66	126	115.650	107.131	143.006	289.270
1946	44	82	96.941	223.235	121.619	1569.636
1947	48	77	101.027	577.287	126.343	2434.714
1948	75	81	121.266	1621.377	149.310	4666.286
			SSR:	197124.40	SSR:	174097.170

To run **Solver**, go to **Tools, Solver.**

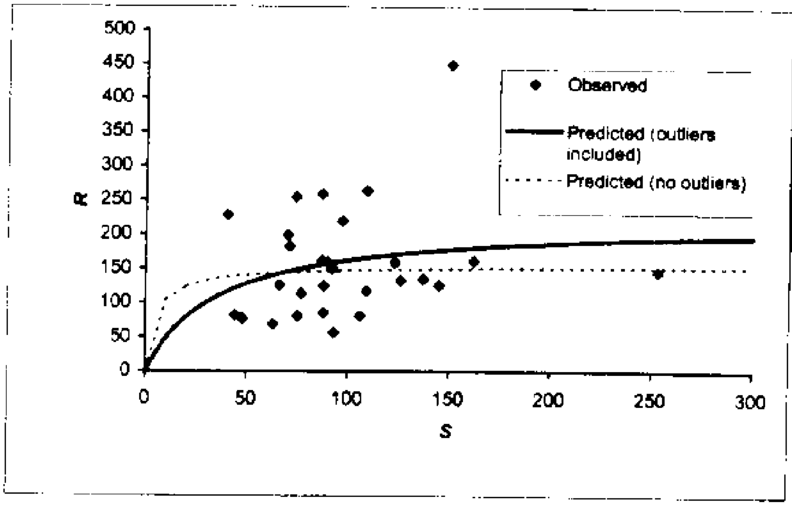
As the target cell, select the sum of square residuals (SSR)

Select "Min" for the **Equal To**

Select the α **and** β **as the By Changing Cells**

It is important to note that the α **and** β **to be changed must be the same** α **and** β **that are in the Beverton-Holt equation in the** R_{pred} **column and the SSR selected must also be related to the same** R_{pred} **column.**

	Before Solver	After Solver
α	0.005309332	0.004533007
β	0.220273551	0.162334359



NOTE: On this graph, the year 1921 (150, 449) appears to be an outlier. If this data point is not used in the analysis, the results of the B-H solution would be:

	Before Solver	After Solver
α	0.00605943	0.006444458
β	0.17276777	0.032804199
R^2 (linear)	0.4447	

To plot the Beverton-Holt model, create a series of stock (S) and solve for R using the Beverton-Holt equation and the parameter values obtained from Solver.

α	0.004533007
β	0.162334359

S	R	R (no outliers)
0	0	0
10	48.1546114	102.8290546
20	79.0531006	123.6909179
30	100.561611	132.6623924
40	116.395923	137.6545297
50	128.539753	140.8343198
60	138.148644	143.0370702
70	145.941309	144.6531269
80	152.388226	145.8893384
90	157.810291	146.865542
100	162.433887	147.6559618
110	166.423296	148.3090252
120	169.900623	148.8576738
130	172.95652	149.3250956
140	175.668556	149.7280854
150	178.086895	150.0791068
160	180.258227	150.3876035
170	182.218567	150.6608616
180	183.997217	150.9045927
190	185.618342	151.1233373
200	187.101974	151.3207503
210	188.464892	151.4998071
220	189.721256	151.6629541
230	190.883087	151.8122215
240	191.960671	151.9493084
250	192.96285	152.0756471
260	193.89727	152.1924541
270	194.77058	152.3007689
280	195.588584	152.401486
290	196.356375	152.495375
300	197.078439	152.5831101

(c) Ricker Model

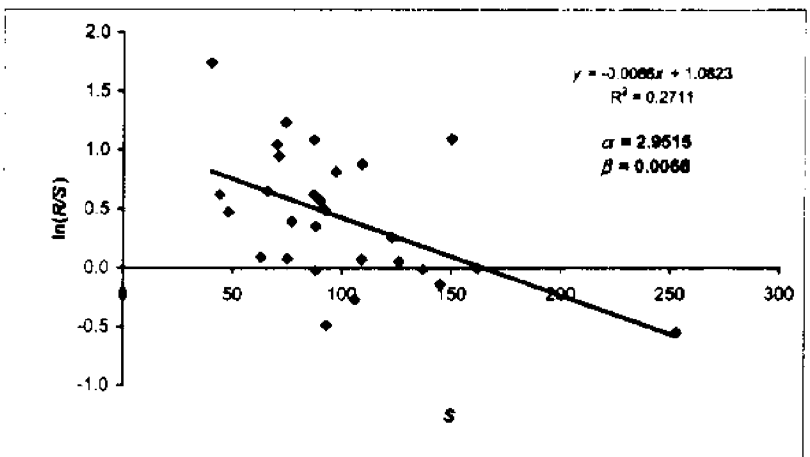
Linear

Year	*S*	*R*	ln(*R/S*)
1921	150	449	1.096
1922	40	228	1.740
1923	70	199	1.045
1924	106	81	-0.269
1925	162	161	-0.006
1926	253	146	-0.550
1927	87	162	0.622
1928	109	263	0.881
1929	90	159	0.569
1930	109	117	0.071
1931	87	258	1.087
1932	74	254	1.233
1933	97	219	0.814
1934	145	126	-0.140
1935	88	125	0.351
1936	137	135	-0.015
1937	126	133	0.054
1938	123	159	0.257
1939	71	183	0.947
1940	88	86	-0.023
1941	93	57	-0.490
1942	63	69	0.091
1943	92	150	0.489
1944	77	114	0.392
1945	66	126	0.647
1946	44	82	0.623
1947	48	77	0.473
1948	75	81	0.077

There are 2 methods to calculate the slope and *y*-intercept:

(1) Go to **Tools, Data Analysis, Regression**, select the input and output ranges, then click OK. The two numbers that are important are the X variable 1 which is the slope and intercept which is the *y*-intercept.

(2) Select the data series on the graph, right click, and choose **Add Trendline**. Choose "linear" for the **Type**. In the **Options** tab, click "display equation on chart."



From **Tools**, **Data Analysis**, **Regression**:

SUMMARY OUTPUT

Regression Statistics	
Multiple R	0.520674597
R Square	0.271102036
Adjusted R Square	0.243067498
Standard Error	0.477706877
Observations	28

ANOVA

	df	*SS*	*MS*	*F*	*Significance F*
Regression	1	2.206796957	2.20679696	9.67028757	0.004502014
Residual	26	5.933300378	0.22820386		
Total	27	8.140097335			

	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*	*Lower 95.0%*	*Upper 95.0%*
Intercept	1.082296938	0.228093675	4.7449669	6.5891E-05	0.613443367	1.551150609	0.613443367	1.551150509
X Variable 1	-0.006584372	0.00211736	-3.1097086	0.00450201	-0.010936671	-0.00223207	-0.010936671	-0.002232074

Non-linear

			Before Solver		**After Solver**	
Year	S	R_{obs}	R_{pred}	$(R_{obs}-R_{pred})^2$	R_{pred}	$(R_{obs}-R_{pred})^2$
1921	150	449	164.899	80713.466	185.525	69419.229
1922	40	228	90.723	18844.838	96.706	17238.069
1923	70	199	130.309	4718.389	140.963	3368.269
1924	106	81	155.685	5577.786	171.415	8174.946
1925	162	161	164.562	12.685	186.239	637.014
1926	253	146	141.166	23.370	167.059	443.497
1927	87	162	144.806	295.623	157.958	16.339
1928	109	263	156.960	11244.564	173.074	8086.685
1929	90	159	146.870	147.141	160.445	2.088
1930	109	117	156.960	1596.774	173.074	3144.294
1931	87	258	144.806	12812.811	157.958	10008.423
1932	74	254	134.175	14357.995	145.430	11787.401
1933	97	219	151.163	4601.834	165.703	2840.518
1934	145	126	164.737	1500.561	184.888	3467.840
1935	88	125	145.510	420.642	158.803	1142.639
1936	137	135	164.066	844.843	183.414	2343.890
1937	126	133	162.227	854.201	180.381	2244.937
1938	123	159	161.523	6.367	179.334	413.486
1939	71	183	131.304	2672.509	142.106	1672.120
1940	88	86	145.510	3541.387	158.803	5300.266
1941	93	57	148.797	8426.729	162.790	11191.525
1942	63	69	122.810	2895.532	132.396	4018.936
1943	92	150	148.170	3.350	162.024	144.572
1944	77	114	136.884	523.681	148.586	1196.122
1945	66	126	126.142	0.020	136.187	103.781
1946	44	82	97.202	231.099	103.815	475.914
1947	48	77	103.282	690.759	110.528	1124.014
1948	75	81	135.096	2926.369	146.500	4290.264
			SSR:	180485.316	SSR:	174297.074

To run **Solver**, go to **Tools**, **Solver**.

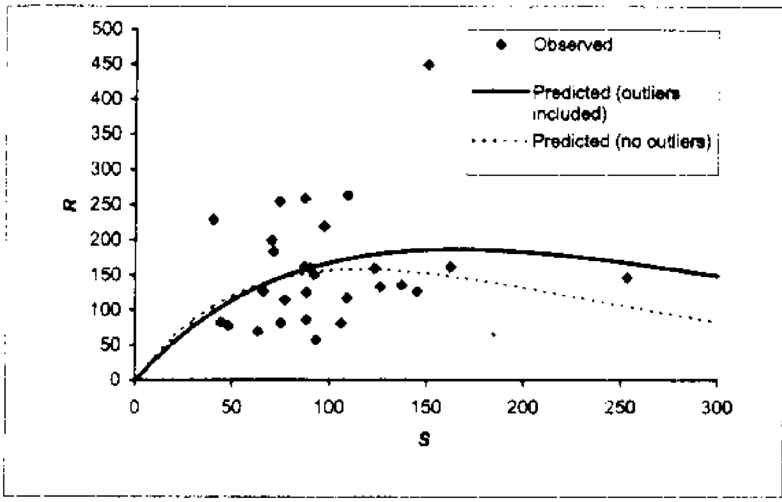
As the target cell, select the sum of square residuals (SSR)

Select "Min" for the Equal To:

Select the α and β as the By Changing Cells

It is important to note that the α and β to be changed must be the same α and β that are in the Ricker equation in the R_{pred} column and the SSR selected must also be related to the same R_{pred} column.

	Before Solver	After Solver
α	2.951451073	3.084912383
β	0.006584	0.006093136



NOTE: On this graph, the year 1921 (150, 449) appears to be an outlier. If this data point is not used in the analysis, the results of the Ricker solution would be:

	Before Solver	After Solver
α	3.16456018	3.728290857
β	0.00768518	0.008672664
R^2 (linear)	0.3706	

To plot the Ricker model, create a series of stock (S) and solve for R using the Ricker equation and the parameter values obtained from **Solver**.

α	3.084912383
β	0.006093136

S	R	R (no outliers)
0	0.000	0.000
10	29.026	34.186
20	54.620	62.692
30	77.086	86.226
40	96.706	105.417
50	113.737	120.825
60	128.417	132.945
70	140.963	142.218
80	151.576	149.032
90	160.445	153.733
100	167.734	156.625
110	173.601	157.975
120	178.186	158.020
130	181.626	156.967
140	184.036	154.999
150	185.525	152.275
160	186.195	148.933
170	186.136	145.096
180	185.437	140.869
190	184.169	136.342
200	182.402	131.596
210	180.201	126.697
220	177.623	121.704
230	174.719	116.666
240	171.539	111.625
250	168.124	106.617
260	164.513	101.671
270	160.742	96.810
280	156.841	92.056
290	152.840	87.423
300	148.765	82.925

Microsoft Excel 8.0 Answer Report
Worksheet: [Chapter 8 - Stock Recruitment Exercises.xls]SALMON
Report Created: 5/12/00 1:11:30 PM

Target Cell (Min)

Cell	Name	Original Value	Final Value
H220	SSR	197124.395	174097.170

Adjustable Cells

Cell	Name	Original Value	Final Value
D223	α After **Solver**	0.005309332	0.00453301
D224	β After **Solver**	0.220273551	0.16233436

Constraints
NONE

Microsoft Excel 8.0 Answer Report
Worksheet: [Chapter 8 - Stock Recruitment Exercises.xls]SALMON
Report Created: 5/12/00 1:34:22 PM

Target Cell (Min)

Cell	Name	Original Value	Final Value
H406	SSR	180485.316	174297.074

Adjustable Cells

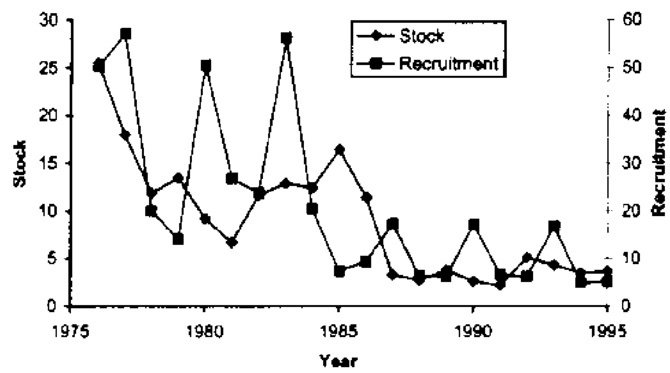
Cell	Name	Original Value	Final Value
D409	α After **Solver**	2.951451073	3.084912383
D410	β After **Solver**	0.006584	0.006093136

Constraints
NONE

YELLOWTAIL FLOUNDER

Year	S	R
1976	25.5	50.3
1977	18.0	57.1
1978	11.9	20.1
1979	13.5	14.1
1980	9.2	50.5
1981	6.7	26.8
1982	11.6	23.8
1983	12.9	56.2
1984	12.4	20.4
1985	16.4	7.4
1986	11.4	9.4
1987	3.3	17.3
1988	2.7	6.3
1989	3.8	6.2
1990	2.6	17.2
1991	2.2	6.6
1992	5.1	6.2
1993	4.3	16.8
1994	3.5	4.9
1995	3.7	5.2

(a)



(b) **Beverton-Holt Model**

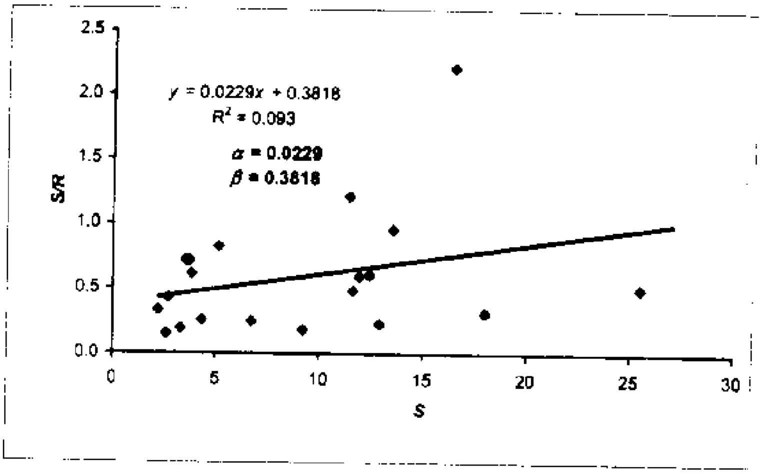
Linear

Year	S	R	(S/R)
1976	25.5	50.3	0.507
1977	18.0	57.1	0.315
1978	11.9	20.1	0.592
1979	13.5	14.1	0.957
1980	9.2	50.5	0.182
1981	6.7	26.8	0.250
1982	11.6	23.8	0.487
1983	12.9	56.2	0.230
1984	12.4	20.4	0.608
1985	16.4	7.4	2.216
1986	11.4	9.4	1.213
1987	3.3	17.3	0.191
1988	2.7	6.3	0.429
1989	3.8	6.2	0.613
1990	2.6	17.2	0.151
1991	2.2	6.6	0.333
1992	5.1	6.2	0.823
1993	4.3	16.8	0.256
1994	3.5	4.9	0.714
1995	3.7	5.2	0.712

There are 2 methods to calculate the slope and y-intercept:

(1) Go to **Tools, Data Analysis, Regression**, select the input and output ranges, then click OK. The two numbers that are important are the X variable 1 which is the slope and intercept which is the y-intercept.

(2) Select the data series on the graph, right click, and choose **Add Trendline.** Choose "linear" for the **Type.** In the **Options** tab, click "display equation on chart."



From **Tools, Data Analysis, Regression**:

SUMMARY OUTPUT

Regression Statistics	
Multiple R	0.30494909
R Square	0.092993948
Adjusted R Square	0.042604722
Standard Error	0.466176831
Observations	20

ANOVA

	df	*SS*	*MS*	*F*	*Significance F*
Regression	1	0.40106833	0.401068334	1.84551255	0.191089215
Residual	18	3.91177508	0.217320838		
Total	19	4.31284342			

	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*	*Lower 95.0%*	*Upper 95.0%*
Intercept	0.381833703	0.18467992	2.067543149	0.05336892	-0.006164711	0.769832117	-0.006164711	0.769832117
X Variable 1	0.022922095	0.01687314	1.358496431	0.19108922	-0.012527076	0.058371266	-0.012527076	0.058371265

Year	S	R_{obs}	Before Solver R_{pred}	Before Solver $(R_{obs}-R_{pred})^2$	After Solver R_{pred}	After Solver $(R_{obs}-R_{pred})^2$
1976	25.5	50.3	29.139	447.776	45.881	19.528
1977	18.0	57.1	26.792	918.554	37.041	402.348
1978	11.9	20.1	23.494	11.522	27.730	58.218
1979	13.5	14.1	24.551	109.230	30.403	265.785
1980	9.2	50.5	21.231	856.665	22.773	768.808
1981	6.7	26.8	18.328	71.780	17.599	84.666
1982	11.6	23.8	23.276	0.275	27.208	11.615
1983	12.9	56.2	24.175	1025.595	29.422	717.067
1984	12.4	20.4	23.844	11.858	28.585	66.995
1985	16.4	7.4	26.096	349.510	34.816	751.659
1986	11.4	9.4	23.126	188.402	26.856	304.725
1987	3.3	17.3	12.069	27.358	9.454	61.556
1988	2.7	6.3	10.500	17.636	7.861	2.437
1989	3.8	6.2	13.242	49.588	10.743	20.642
1990	2.6	17.2	10.218	48.743	7.591	92.341
1991	2.2	6.6	9.031	5.910	6.493	0.011
1992	5.1	6.2	15.829	92.709	13.942	59.932
1993	4.3	16.8	14.309	6.207	11.999	23.049
1994	3.5	4.9	12.552	58.556	9.974	25.746
1995	3.7	5.2	13.016	61.095	10.488	27.966
			SSR:	4358.970	SSR:	3765.090

To run **Solver**, go to **Tools, Solver**

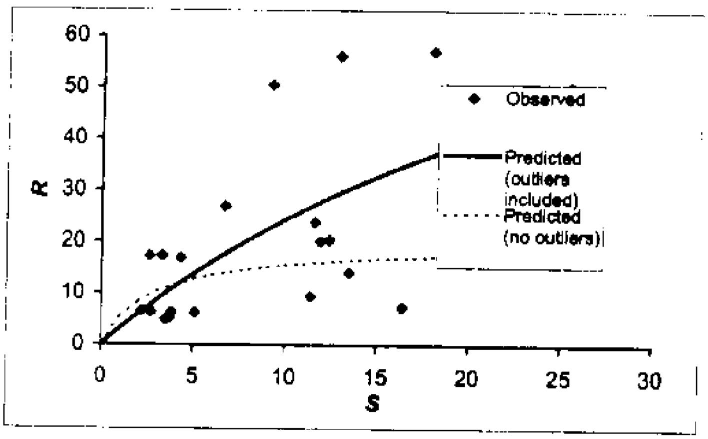
As the target cell, select the sum of square residuals (SSR)

Select "Min" for the Equal To:

Select the α and β as the y Changing Cells

It is important to note that the α and β to be changed must be the same α and β that are in the Beverton-Holt equation in the R_{pred} column and the SSR selected must also be related to the same R_{pred} column.

	Before Solver	After Solver
α	0.022922096	0.00931243
β	0.381833703	0.3183186



NOTE: On this graph, the years 1976, 1977, 1980, and 1983 appear to be outliers. If these points are not used in the analysis, the results of the B-H solution would be:

	Before Solver	After Solver
α	0.0724	0.05075
β	0.1379	0.13843
R^2 (linear)	0.4661	

To plot the Beverton-Holt model, create a series of stock (S) and solve for R using the Beverton-Holt equation and the parameter values obtained from Solver.

α	0.009312433
β	0.3183186

S	R	R (no outliers)
0	0	0
1	3.05221392	5.28597103
2	5.93571386	8.335784598
3	8.66411233	10.32062749
4	11.2495956	11.71543215
5	13.7031066	12.74924779
6	16.0344978	13.54615854
7	18.2526656	14.17922541
8	20.3656823	14.6942674
9	22.3807946	15.12147586
10	24.3047074	15.48156373
11	26.1434577	15.7891715
12	27.9025776	16.05501519
13	29.5871302	16.28705305
14	31.2017584	16.49134793
15	32.7507275	16.6725947
16	34.2379626	16.83448544
17	35.6670823	16.97996364
18	37.0414277	17.11140475
19	38.3640886	17.23074691
20	39.6379267	17.33958714

(c) Ricker Model

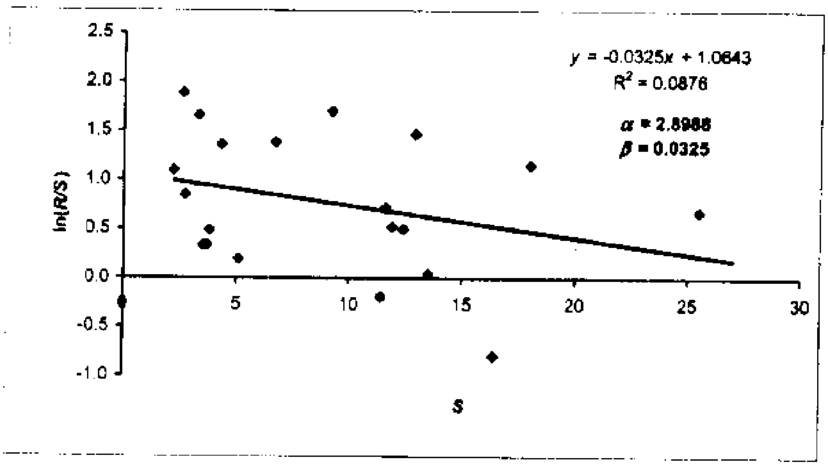
Linear

Year	*S*	*R*	ln(*R/S*)
1976	25.5	50.3	0.679
1977	18.0	57.1	1.154
1978	11.9	20.1	0.524
1979	13.5	14.1	0.043
1980	9.2	50.5	1.703
1981	6.7	26.8	1.386
1982	11.6	23.8	0.719
1983	12.9	56.2	1.472
1984	12.4	20.4	0.498
1985	16.4	7.4	-0.796
1986	11.4	9.4	-0.193
1987	3.3	17.3	1.657
1988	2.7	6.3	0.847
1989	3.8	6.2	0.490
1990	2.6	17.2	1.889
1991	2.2	6.6	1.099
1992	5.1	6.2	0.195
1993	4.3	16.8	1.363
1994	3.5	4.9	0.336
1995	3.7	5.2	0.340

There are 2 methods to calculate the slope and *y*-intercept:

(1) Go to **Tools, Data Analysis, Regression**, select the input and output ranges, then click OK. The two numbers that are important are the X variable 1 which is the slope and intercept which is the *y*-intercept.

(2) Select the data series on the graph, right click, and choose **Add Trendline**. Choose "linear" for the **Type**. In the **Options** tab, click "display equation on chart."



From **Tools, Data Analysis, Regression**:

SUMMARY OUTPUT

Regression Statistics	
Multiple R	0.295897137
R Square	0.087555116
Adjusted R Square	0.036863733
Standard Error	0.683958762
Observations	20

ANOVA

	df	*SS*	*MS*	*F*	*Significance F*
Regression	1	0.80799231	0.807992307	1.72721894	0.205268719
Residual	18	8.42039258	0.467799588		
Total	19	9.22838489			

	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*	*Lower 95.0%*	*Upper 95.0%*
Intercept	1.064277519	0.27095609	3.927859812	0.00098625	0.495019487	1.633535572	0.495019487	1.633535572
X Variable 1	-0.032534844	0.02475569	-1.314237018	0.20526872	-0.084544658	0.019474971	-0.084544658	0.019474971

Non-linear

			Before Solver		After Solver	
Year	*S*	R_{obs}	R_{pred}	$(R_{obs}-R_{pred})^2$	R_{pred}	$(R_{obs}-R_{pred})^2$
1976	25.5	50.3	14.872	1255.141	46.570	13.912
1977	18.0	57.1	18.060	1524.109	37.533	382.858
1978	11.9	20.1	18.582	2.384	27.639	56.836
1979	13.5	14.1	18.757	21.684	30.481	268.326
1980	9.2	50.5	17.446	1092.566	22.412	788.911
1981	6.7	26.8	15.224	134.008	17.060	94.876
1982	11.6	23.8	18.491	28.182	27.085	10.794
1983	12.9	56.2	18.718	1404.899	29.437	716.284
1984	12.4	20.4	18.656	3.044	28.547	66.370
1985	16.4	7.4	18.474	122.631	35.178	771.618
1986	11.4	9.4	18.437	81.674	26.713	299.728
1987	3.3	17.3	9.589	59.458	8.923	70.175
1988	2.7	6.3	8.194	3.586	7.378	1.163
1989	3.8	6.2	10.650	19.800	10.186	15.876
1990	2.6	17.2	7.947	85.610	7.118	101.652
1991	2.2	6.6	6.922	0.104	6.068	0.286
1992	5.1	6.2	13.010	46.379	13.358	51.239
1993	4.3	16.8	11.623	26.802	11.423	28.910
1994	3.5	4.9	10.024	26.257	9.430	20.524
1995	3.7	5.2	10.445	27.507	9.934	22.411
			SSR:	5965.806	SSR:	3782.744

To run **Solver**, go to **Tools, Solver.**

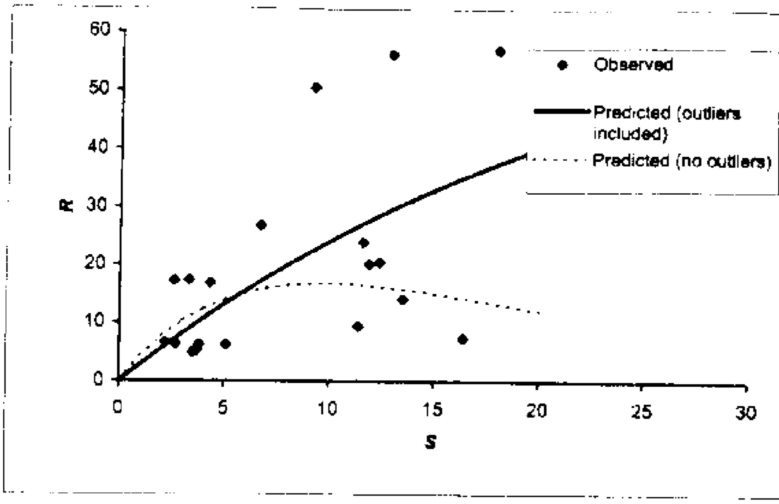
As the target cell, select the sum of square residuals (SSR)

Select "Min" for the Equal To:

Select the α and β as the y Changing Cells

It is important to note that the α and β to be changed must be the same α and β that are in the Ricker equation in the R_{pred} column and the SSR selected must also be related to the same R_{pred} column.

	Before Solver	After Solver
α	3.889222124	2.86834855
β	0.072337728	0.01767672



NOTE: On this graph, the years 1976, 1977, 1980, and 1983 appear to be outliers. If these points are not used in the analysis, the results of the B-H solution would be:

	Before Solver	After Solver
α	3.849	4.66206
β	0.0969	0.10256
R^2 (linear)	0.4299	

To plot the Ricker model, create a series of stock (S) and solve for R using the Ricker equation and the parameter values obtained from **Solver**.

α	2.866346553
β	0.017678719

S	R	R (no outliers)
0	0.000	0.000
1	2.816	4.208
2	5.534	7.595
3	8.155	10.282
4	10.683	12.373
5	13.119	13.959
6	15.467	15.118
7	17.729	15.918
8	19.907	16.419
9	22.003	16.671
10	24.019	16.717
11	25.958	16.597
12	27.822	16.340
13	29.612	15.977
14	31.331	15.528
15	32.981	15.016
16	34.564	14.456
17	36.080	13.862
18	37.533	13.247
19	38.924	12.620
20	40.255	11.989

Microsoft Excel 8.0 Answer Report
Worksheet: [Chapter 8 - Stock Recruitment Exercises.xls]YELLOWTAIL
Report Created: 5/12/00 2:35:15 PM

Target Cell (Min)

Cell	Name	Original Value	Final Value
H146	SSR	5237.757	3765.090

Adjustable Cells

Cell	Name	Original Value	Final Value
D149	α After **Solver**	0.022922095	0.009312433
D150	β After **Solver**	0.381833703	0.3183186

Constraints
NONE

Microsoft Excel 8.0 Answer Report
Worksheet: [Chapter 8 - Stock Recruitment Exercises.xls]YELLOWTAIL
Report Created: 5/15/00 10:20:25 AM

Target Cell (Min)

Cell	Name	Original Value	Final Value
H293	SSR	5965.806	3782.744

Adjustable Cells

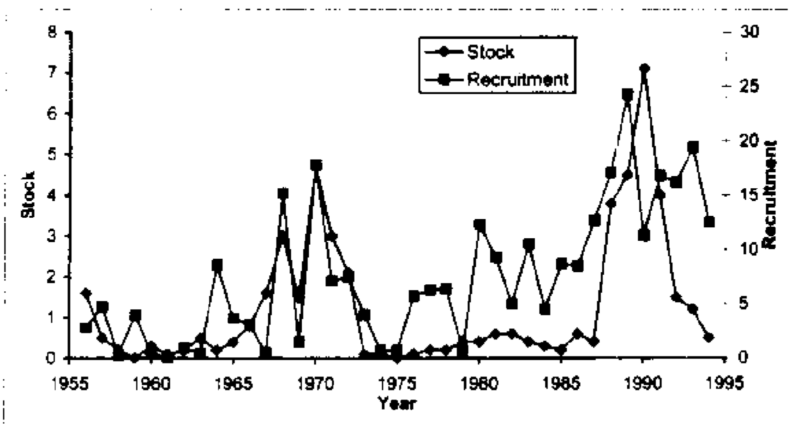
Cell	Name	Original Value	Final Value
D296	α After **Solver**	3.689222124	2.866346553
D297	β After **Solver**	0.072337728	0.017676719

Constraints
NONE

BLUE CRAB

Year	S	R
1956	1.6	2.8
1957	0.5	4.8
1958	0.2	0.3
1959	0	4
1960	0.3	0.5
1961	0.1	0
1962	0.2	1
1963	0.5	0.4
1964	0.2	8.6
1965	0.4	3.7
1966	0.8	3.1
1967	1.6	0.5
1968	3	15.2
1969	1.5	1.5
1970	4.7	17.8
1971	3	7.1
1972	2.1	7.5
1973	0.1	4
1974	0.1	0.7
1975	0	0.7
1976	0.1	5.7
1977	0.2	6.3
1978	0.2	6.4
1979	0.4	0.7
1980	0.4	12.3
1981	0.6	9.3
1982	0.6	5
1983	0.4	10.5
1984	0.3	4.5
1985	0.2	8.7
1986	0.6	8.5
1987	0.4	12.7
1988	3.8	17.1
1989	4.5	24.3
1990	7.1	11.3
1991	4	16.8
1992	1.5	16.2
1993	1.2	19.4
1994	0.5	12.5

(a)



(b) **Beverton-Holt Model**

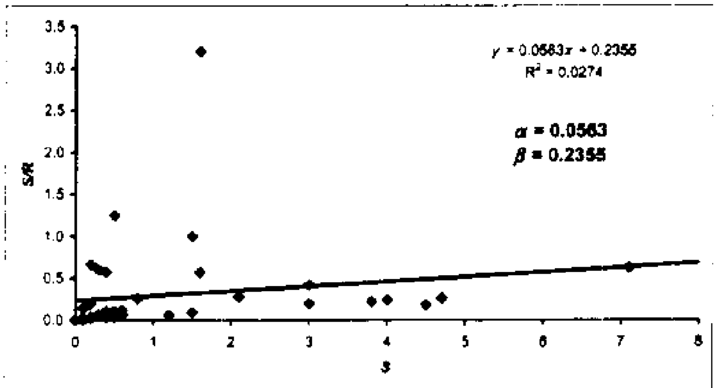
Linear

Year	S	R	(S/R)
1956	1.6	2.8	0.571
1957	0.5	4.8	0.104
1958	0.2	0.3	0.667
1959	0	4	0.000
1960	0.3	0.5	0.600
1961	0.1	0	0.000
1962	0.2	1	0.200
1963	0.5	0.4	1.250
1964	0.2	8.6	0.023
1965	0.4	3.7	0.108
1966	0.8	3.1	0.258
1967	1.6	0.5	3.200
1968	3	15.2	0.197
1969	1.5	1.5	1.000
1970	4.7	17.8	0.264
1971	3	7.1	0.423
1972	2.1	7.5	0.280
1973	0.1	4	0.025
1974	0.1	0.7	0.143
1975	0	0.7	0.000
1976	0.1	5.7	0.018
1977	0.2	6.3	0.032
1978	0.2	6.4	0.031
1979	0.4	0.7	0.571
1980	0.4	12.3	0.033
1981	0.6	9.3	0.065
1982	0.6	5	0.120
1983	0.4	10.5	0.038
1984	0.3	4.5	0.067
1985	0.2	8.7	0.023
1986	0.6	8.5	0.071
1987	0.4	12.7	0.031
1988	3.8	17.1	0.222
1989	4.5	24.3	0.185
1990	7.1	11.3	0.628
1991	4	16.8	0.238
1992	1.5	16.2	0.093
1993	1.2	19.4	0.062
1994	0.5	12.5	0.040

There are 2 methods to calculate the slope and *y*-intercept:

(1) Go to **Tools, Data Analysis, Regression**, select the input and output ranges, then click OK. The two numbers that are important are the X variable 1 which is the slope and intercept which is the *y*-intercept.

(2) Select the data series on the graph, right click, and choose **Add Trendline**. Choose "linear" for the **Type**. In the **Options** tab, click "display equation on chart."



From **Tools, Data Analysis, Regression:**

SUMMARY OUTPUT

Regression Statistics	
Multiple R	0.16560684
R Square	0.027425625
Adjusted R Square	0.001139831
Standard Error	0.555527335
Observations	39

ANOVA

	df	*SS*	*MS*	*F*	*Significance F*
Regression	1	0.32199291	0.321993	1.04336302	0.31367316
Residual	37	11.41859296	0.308611		
Total	38	11.74058587			

	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*	*Lower 95.0%*	*Upper 95.0%*
Intercept	0.235474099	0.111789914	2.106398	0.04201249	0.00896644	0.461981759	0.00896644	0.46198175[illegible]
X Variable 1	0.056307209	0.055124705	1.021451	0.31367316	-0.05538594	0.168000362	-0.055385944	0.16800036[illegible]

Non-linear

			Before **Solver**		After **Solver**	
Year	*S*	R_{obs}	R_{pred}	$(R_{obs}-R_{pred})^2$	R_{pred}	$(R_{obs}-R_{pred})^2$
1956	1.6	2.8	4.915	4.471	11.145	69.645
1957	0.5	4.8	1.897	8.430	6.598	3.232
1958	0.2	0.3	0.811	0.261	3.490	10.178
1959	0	4	0.000	16.000	0.000	16.000
1960	0.3	0.5	1.189	0.474	4.727	17.870
1961	0.1	0	0.415	0.172	1.956	3.824
1962	0.2	1	0.811	0.036	3.490	6.202
1963	0.5	0.4	1.897	2.240	6.598	38.411
1964	0.2	8.6	0.811	60.675	3.490	26.109
1965	0.4	3.7	1.550	4.621	5.745	4.183
1966	0.8	3.1	2.852	0.062	8.486	29.014
1967	1.6	0.5	4.915	19.488	11.145	113.324
1968	3	15.2	7.418	60.552	13.054	4.605
1969	1.5	1.5	4.688	10.166	10.917	88.686
1970	4.7	17.8	9.398	70.597	14.049	14.073
1971	3	7.1	7.418	0.101	13.054	35.451
1972	2.1	7.5	5.937	2.443	12.044	20.646
1973	0.1	4	0.415	12.854	1.955	4.180
1974	0.1	0.7	0.415	0.081	1.955	1.576
1975	0	0.7	0.000	0.490	0.000	0.490
1976	0.1	5.7	0.415	27.934	1.955	14.022
1977	0.2	6.3	0.811	30.134	3.490	7.894
1978	0.2	6.4	0.811	31.242	3.490	8.466
1979	0.4	0.7	1.550	0.723	5.745	25.454
1980	0.4	12.3	1.550	115.564	5.745	42.965
1981	0.6	9.3	2.228	50.006	7.322	3.913
1982	0.6	5	2.228	7.682	7.322	5.391
1983	0.4	10.5	1.550	80.096	5.745	22.608
1984	0.3	4.5	1.189	10.964	4.727	0.052
1985	0.2	8.7	0.811	62.243	3.490	27.140
1986	0.6	8.5	2.228	39.334	7.322	1.388
1987	0.4	12.7	1.550	124.313	5.745	48.369
1988	3.8	17.1	8.455	74.737	13.615	12.145
1989	4.5	24.3	9.206	227.854	13.966	106.809
1990	7.1	11.3	11.177	0.015	14.717	11.679
1991	4	16.8	8.682	65.896	13.726	9.452
1992	1.5	16.2	4.688	132.516	10.917	27.906
1993	1.2	19.4	3.960	238.399	10.091	86.649
1994	0.5	12.5	1.897	112.432	6.598	34.837
			SSR:	1706.289	SSR:	1004.840

To run **Solver**, go to **Tools, Solver.**

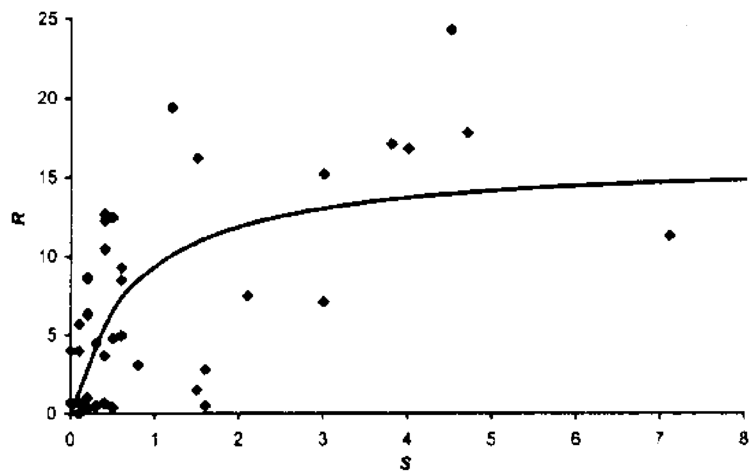
As the target cell, select the sum of square residuals (SSR)

Select "Min" for the Equal To:

Select the α **and** β **as the y Changing Cells**

It is important to note that the α and β to be changed must be the same α and β that are in the Beverton-Holt equation in the R_{pred} column and the SSR selected must also be related to the same R_{pred} column.

	Before Solver	After Solver
α	0.056307209	0.061811656
β	0.235474099	0.0449785



To plot the Beverton-Holt model, create a series of stock (S) and solve for R using the Beverton-Holt equation and the parameter values obtained from **Solver**.

α	0.061611655
β	0.0449785

S	R
0	0
0.5	6.59767023
1	9.38172951
1.5	10.9173496
2	11.8904785
2.5	12.562332
3	13.0540654
3.5	13.4295507
4	13.7256528
4.5	13.9651391
5	14.1628305
5.5	14.3287899
6	14.4700896
6.5	14.5918462
7	14.6978515
7.5	14.7909768
8	14.8734344

(c) **Ricker Model**

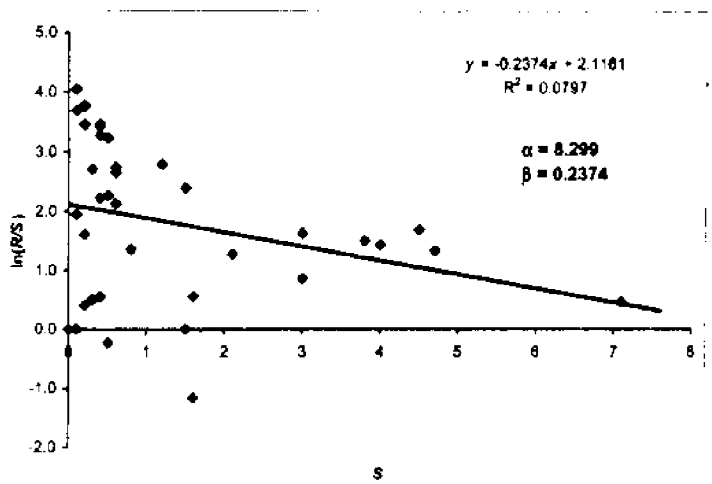
Linear

Year	$\bar{S}$	$\bar{R}$	$\ln(\bar{R}/\bar{S})$
1956	1.6	2.8	0.560
1957	0.5	4.8	2.262
1958	0.2	0.3	0.405
1959	0	4	0.000
1960	0.3	0.5	0.511
1961	0.1	0	0.000
1962	0.2	1	1.609
1963	0.5	0.4	-0.223
1964	0.2	8.6	3.761
1965	0.4	3.7	2.225
1966	0.8	3.1	1.355
1967	1.6	0.5	-1.163
1968	3	15.2	1.623
1969	1.5	1.5	0.000
1970	4.7	17.8	1.332
1971	3	7.1	0.861
1972	2.1	7.5	1.273
1973	0.1	4	3.689
1974	0.1	0.7	1.946
1975	0	0.7	0.000
1976	0.1	5.7	4.043
1977	0.2	6.3	3.450
1978	0.2	6.4	3.466
1979	0.4	0.7	0.560
1980	0.4	12.3	3.426
1981	0.6	9.3	2.741
1982	0.6	5	2.120
1983	0.4	10.5	3.268
1984	0.3	4.5	2.708
1985	0.2	8.7	3.773
1986	0.6	8.5	2.651
1987	0.4	12.7	3.458
1988	3.8	17.1	1.504
1989	4.5	24.3	1.686
1990	7.1	11.3	0.466
1991	4	16.8	1.435
1992	1.5	16.2	2.380
1993	1.2	19.4	2.783
1994	0.5	12.5	3.219

There are 2 methods to calculate the slope and *y*-intercept:

(1) Go to **Tools, Data Analysis, Regression**, select the input and output ranges, then click OK. The two numbers that are important are the X variable 1 which is the slope and intercept which is the *y*-intercept.

(2) Select the data series on the graph, right click, and choose **Add Trendline**. Choose "linear" for the **Type**. In the **Options** tab, click "display equation on chart."



From **Tools, Data Analysis, Regression:**

SUMMARY OUTPUT

Regression Statistics	
Multiple R	0.282292646
R Square	0.079689138
Adjusted R Square	0.054815672
Standard Error	1.33638823
Observations	39

ANOVA

	df	*SS*	*MS*	*F*	*Significance F*
Regression	1	5.721786729	5.721786	3.20380671	0.08165397
Residual	37	66.07953951	1.785934		
Total	38	71.80132524			

	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*	*Lower 95.0%*	*Upper 95.0%*
Intercept	2.116116616	0.268924165	7.868823	2.0385E-09	1.57122503	2.661008201	1.571225031	2.661008201
X Variable 1	-0.237359513	0.132609149	-1.78992	0.08165397	-0.50605091	0.031331684	-0.506050909	0.031331684

Non-linear

Year	S	R_{obs}	Before Solver R_{pred}	Before Solver $(R_{obs}-R_{pred})^2$	After Solver R_{pred}	After Solver $(R_{obs}-R_{pred})^2$
1956	1.6	2.8	9.082	39.469	11.038	67.862
1957	0.5	4.8	3.686	1.243	4.463	0.113
1958	0.2	0.3	1.583	1.646	1.915	2.609
1959	0	4	0.000	16.000	0.000	16.000
1960	0.3	0.5	2.319	3.307	2.806	5.319
1961	0.1	0	0.810	0.657	0.980	0.961
1962	0.2	1	1.583	0.340	1.915	0.838
1963	0.5	0.4	3.685	10.792	4.463	16.509
1964	0.2	8.6	1.583	49.241	1.915	44.686
1965	0.4	3.7	3.019	0.464	3.655	0.002
1966	0.8	3.1	5.491	5.716	6.656	12.648
1967	1.6	0.5	9.082	73.659	11.038	111.046
1968	3	15.2	12.215	8.911	14.909	0.086
1969	1.5	1.5	8.719	52.119	10.593	82.687
1970	4.7	17.8	12.783	25.174	15.685	4.474
1971	3	7.1	12.215	26.161	14.909	60.984
1972	2.1	7.5	10.587	9.528	12.886	29.008
1973	0.1	4	0.810	10.173	0.980	9.118
1974	0.1	0.7	0.810	0.012	0.980	0.079
1975	0	0.7	0.000	0.490	0.000	0.490
1976	0.1	5.7	0.810	23.908	0.980	22.275
1977	0.2	6.3	1.583	22.252	1.915	19.226
1978	0.2	6.4	1.583	23.206	1.915	20.113
1979	0.4	0.7	3.019	5.377	3.656	8.733
1980	0.4	12.3	3.019	86.139	3.656	74.733
1981	0.6	9.3	4.318	24.817	5.232	16.550
1982	0.6	5	4.318	0.465	5.232	0.054
1983	0.4	10.5	3.019	56.967	3.656	46.852
1984	0.3	4.5	2.319	4.759	2.806	2.868
1985	0.2	8.7	1.583	50.654	1.915	46.033
1986	0.6	8.5	4.318	17.486	5.232	10.681
1987	0.4	12.7	3.019	93.724	3.656	81.809
1988	3.8	17.1	12.796	18.522	15.658	2.080
1989	4.5	24.3	12.834	131.476	15.738	73.313
1990	7.1	11.3	10.924	0.141	13.504	4.859
1991	4	16.8	12.846	15.640	15.727	1.160
1992	1.5	16.2	8.719	55.960	10.593	31.436
1993	1.2	19.4	7.490	141.841	9.092	106.263
1994	0.5	12.5	3.686	77.703	4.463	64.590
			SSR:	1186.139	SSR:	1099.138

To run **Solver**, go to **Tools, Solver.**

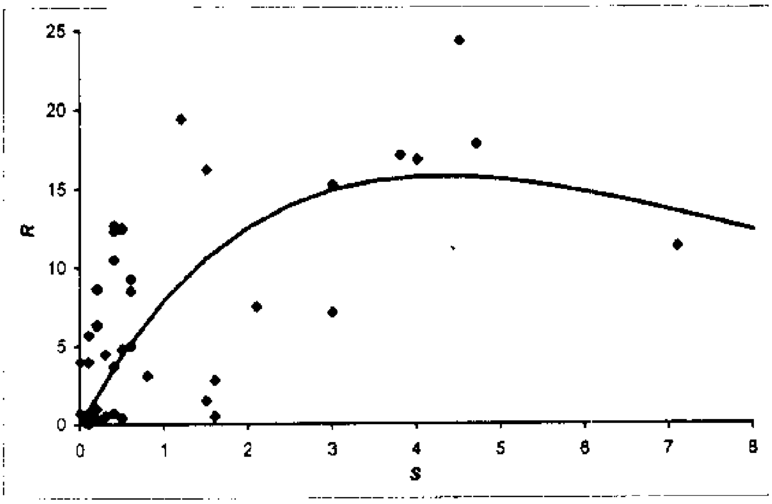
As the target cell, select the sum of square residuals (SSR)

Select "Min" for the **Equal To:**

Select the α and β as the **By Changing Cells**

It is important to note that the α and β to be changed must be the same α and β that are in the Ricker equation in the R_{pred} column and the SSR selected must also be related to the same R_{pred} column.

	Before Solver	After Solver
α	8.298847217	10.03559483
β	0.237369613	0.234257061



To plot the Ricker model, create a series of stock (S) and solve for R using the Ricker equation and the parameter values obtained from **Solver**.

α	10.03559483
β	0.234257051

S	R
0	0.000
0.5	4.463
1	7.940
1.5	10.593
2	12.563
2.5	13.968
3	14.909
3.5	15.472
4	15.727
4.5	15.738
5	15.554
5.5	15.218
6	14.766
6.5	14.229
7	13.630
7.5	12.969
8	12.324

BEVERTON-HOLT

Microsoft Excel 8.0a Answer Report
Worksheet: [Chapter 10 - Stock Recruitment Exercises.xls]BLUECRAB
Report Created: 4/5/00 3:07:10 PM

Target Cell (Min)

Cell	Name	Original Value	Final Value
H170	SSR	1706.289	1004.840

Adjustable Cells

Cell	Name	Original Value	Final Value
D173	α After **Solver**	0.056307209	0.061611655
D174	β After **Solver**	0.235474099	0.0449785

Constraints
NONE

RICKER

Microsoft Excel 8.0a Answer Report
Worksheet: [Chapter 10 - Stock Recruitment Exercises.xls]BLUECRAB
Report Created: 4/5/00 3:07:10 PM

Target Cell (Min)

Cell	Name	Original Value	Final Value
H325	SSR	1185.139	1099.138

Adjustable Cells

Cell	Name	Original Value	Final Value
D328	α After **Solver**	8.298847217	10.03559483
D329	β After **Solver**	0.237359513	0.234257051

Constraints
NONE