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A HYDROLOGICAL MODEL FOR ESTIMATING THE INFLOWS TO AND OUTFLOWS FROM GRAND TRAVERSE BAY

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Multidisciplinary Research in the Great Lakes

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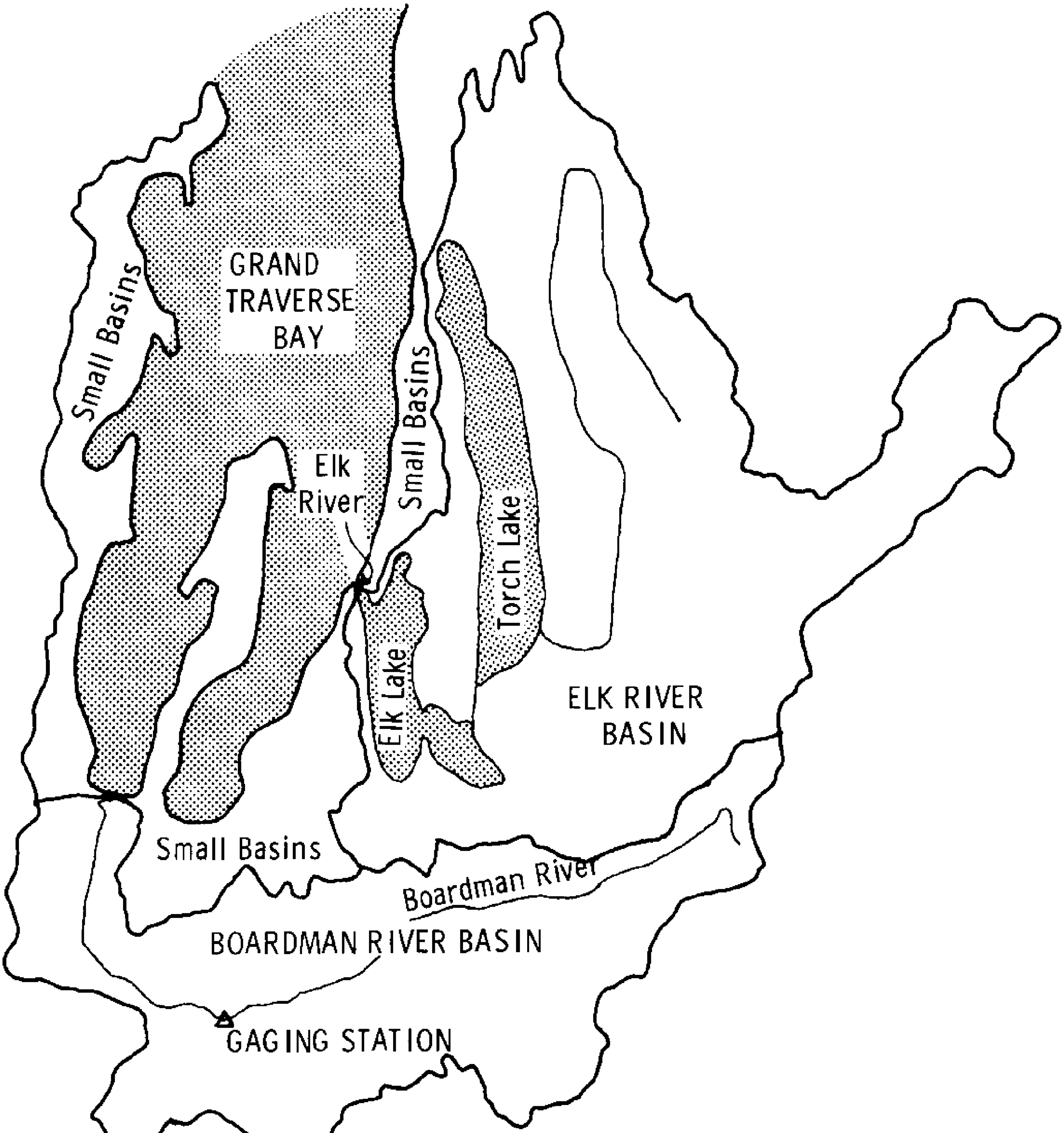
THE UNIVERSITY OF MICHIGAN SEA GRANT PROGRAM

The University of Michigan Sea Grant Program is a part of the National Sea Grant Program, which is maintained by the National Oceanic and Atmospheric Administration of the U.S. Department of Commerce.

The objective of this project is to provide estimates of inflow and outflow for any selected portion of Grand Traverse Bay. The determination is made first for the 17-year period from 1953 to 1971, for which continuous records are available on the upper Boardman River. For this period it was only necessary to find relations between the discharges of the gaged and ungaged portions of the basin. In order to extend inflow and outflow farther into the past or into the future, a deterministic hydrological model was developed for relating runoff to precipitation. This model enables the extension of the estimated inflow and outflow rates with accuracy over as many years as rainfall records are available or can be estimated.

Estimating Inflows from Discharge Records. The inflow to Grand Traverse Bay is the runoff from the drainage area shown in Figure 1. The total drainage area is 973 square miles, of which only that from the Boardman basin above Mayfield (189 square miles) has been gaged. As can be seen from Figure 1, there are two principal river systems, the Boardman and the Elk, and the remaining area is drained by smaller streams. The drainage areas are shown in Table 1. It is estimated that more than 95 percent of the river discharge is groundwater. There is little surface runoff from the land areas compared, for example, with rivers in southeastern Michigan. This is primarily

Figure 1. Grand Traverse Bay Drainage Basin



DRAINAGE AREAS OF GRAND TRAVERSE BAY

Table 1

Drainage Basin	Area on Which Discharges Were Measured (Sq. Mi.)	Total Area (Sq. Mi.)
Boardman		
- Upper Boardman at USGS Station	189*	
- Between USGS Station and Bay	90	
- At Bay		279
Elk**		
- At Bay	491	491
West Bay (Except Old Mission)		
- Northport Creek	4.0	
- Belangers Creek	9.0	
- Wheaton Creek	3.5	
- Cedar Creek	6.2	
- Total		73
East Bay (Except Old Mission)		
- Mitchel Creek	14.0	
- Acme Creek	13.0	
- Yuba Creek	8.2	
- Tobeco Creek	9.5	
- Total		100
Old Mission		30
Summations	837.4	973
Grand Traverse Bay Water Area		277

* Continuous Record of Discharge

** Various tributaries were also measured as a check on measurements at the Bay.

the result of the high infiltration capacity of the soil in the Traverse Bay area. The lack of extensive urbanization also contributes to a greater total infiltration, but this factor is probably counter-balanced by the large number of lakes which act as impermeable surfaces.

The U.S. Geological Survey gaging station was installed on the Boardman River in 1953, thus providing 17 years of runoff records. During the past two years, as part of the Sea Grant project, spot discharge measurements have been made with current meters at many other locations on the drainage basin. These measurements provide information on discharges on 648.4 square miles of drainage basin which could be related to the continuous discharge records on the upper Boardman. Examples of these correlations for the two principal areas, the lower Boardman and the Elk River basin, are shown in Figures 2 and 3, respectively. The measurements used for the correlations were made during times when preceding precipitation or snow melt conditions did not differ between the basins being measured and the upper Boardman. A problem which needed to be solved before these correlations could be made was the modification of the runoff records on the Boardman to eliminate the effects of changing storage in the dam just above the gaging station. This was done after plotting measured discharge and precipitation as illustrated in Figure 4. The hydrograph was then adjusted to a natural smooth curve for which the total discharge

Figure 2
Discharge Relationship
Boardman River at Bay and Boardman River at Gaging Station

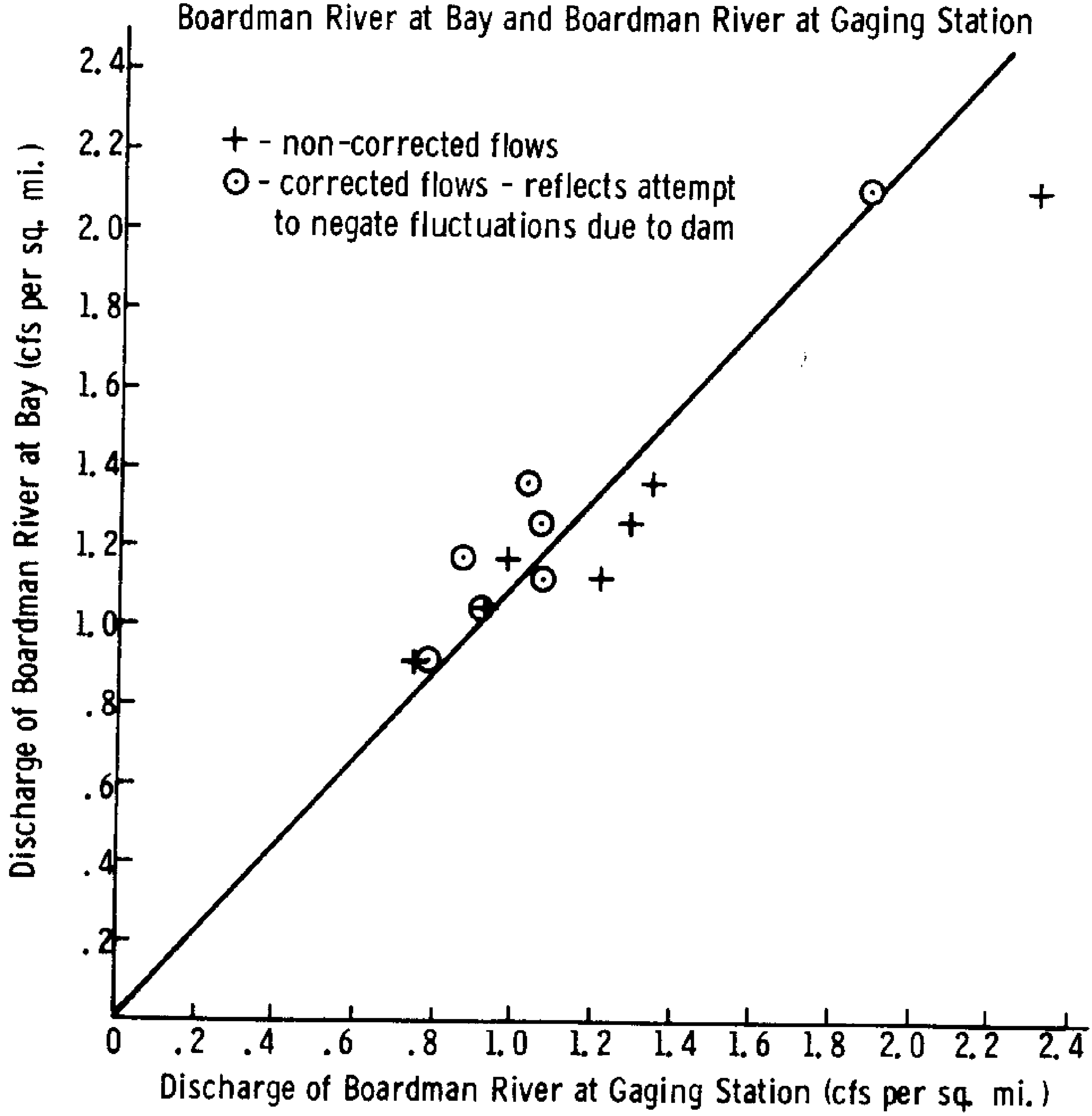


Figure 3
Discharge Relationship
Elk River and Boardman River at Gaging Station

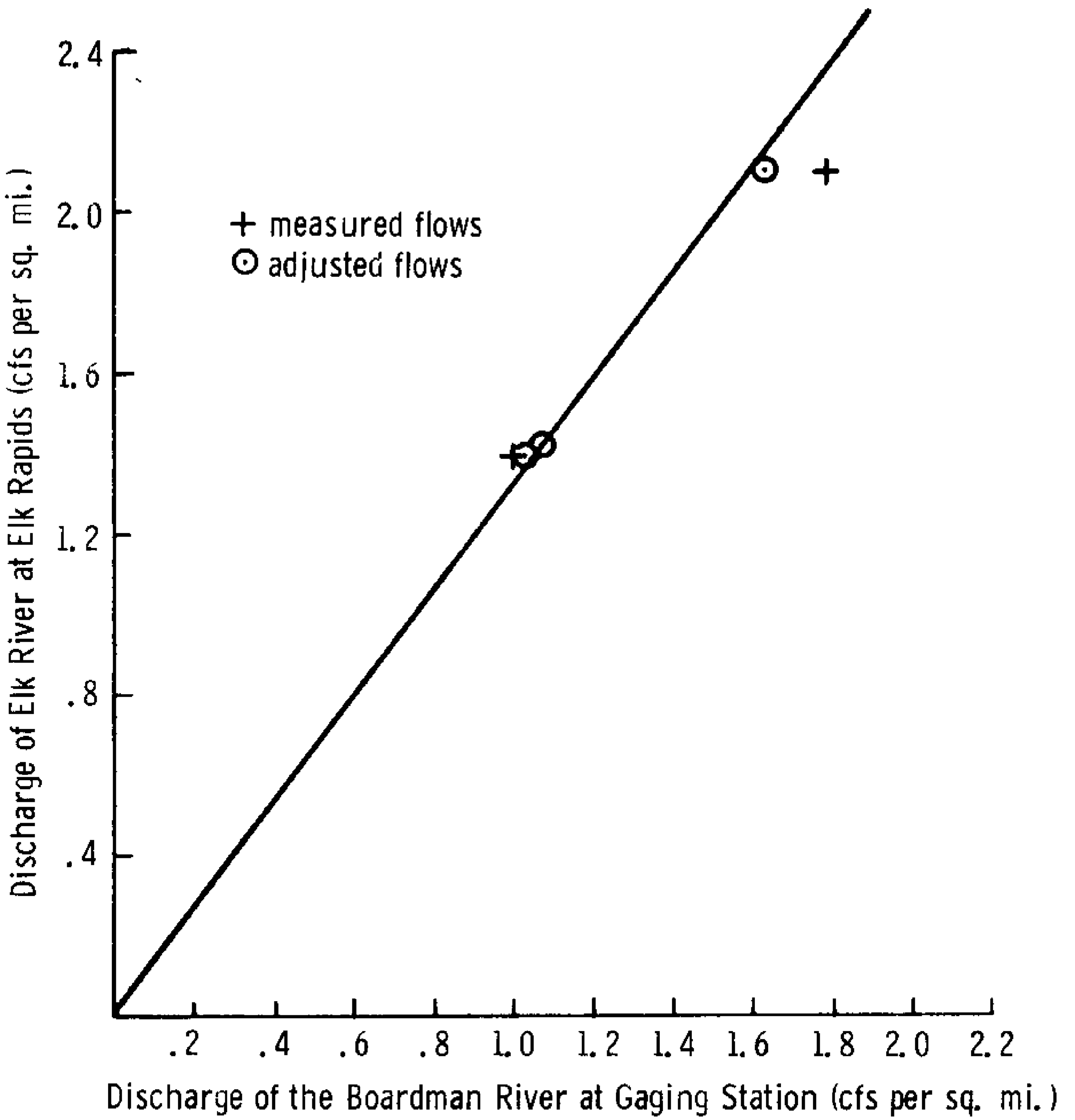
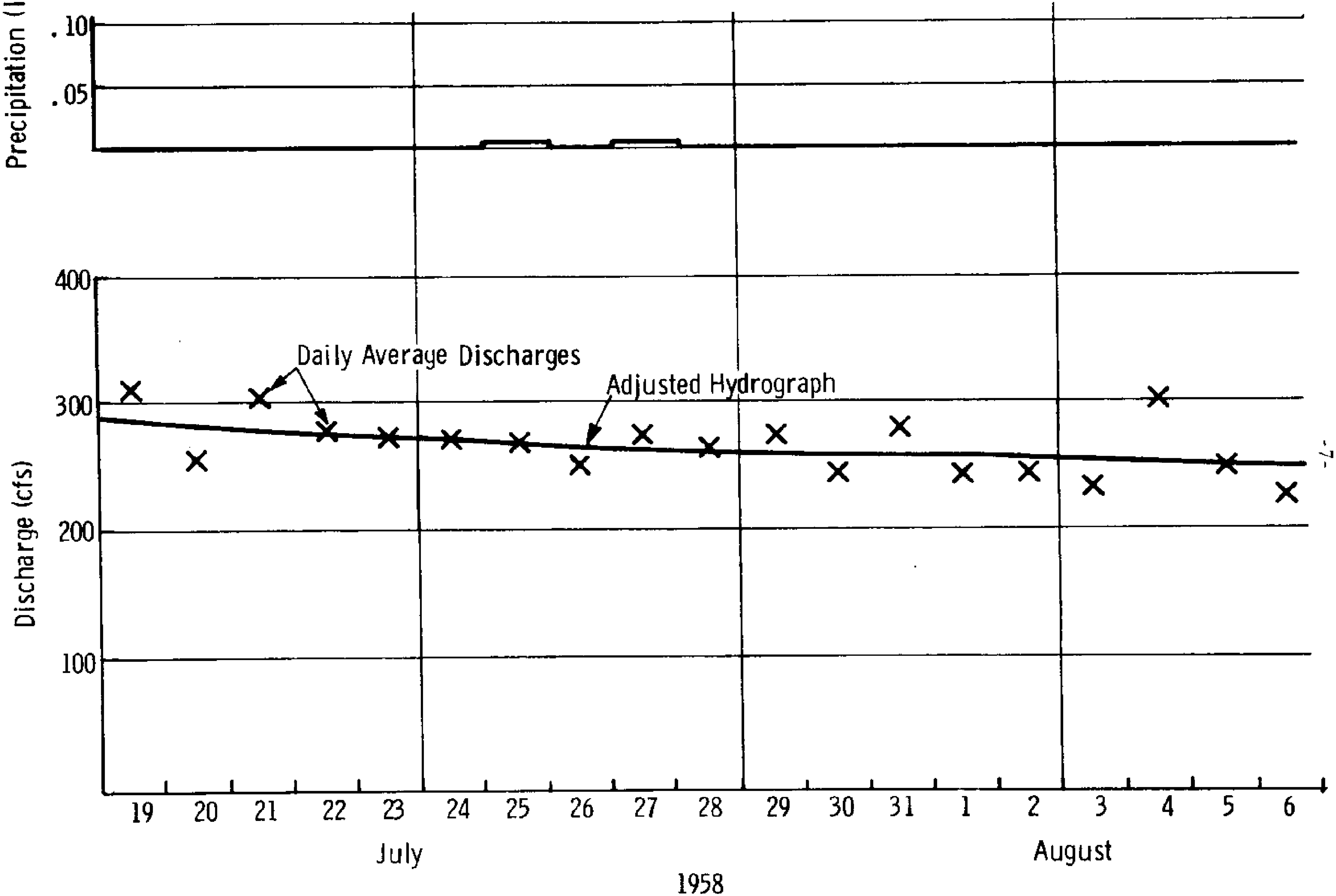


Figure 4. Discharge Fluctuation Due to Dam, Boardman River at Gaging Station



over a period of time was the same as that of the measured artificially controlled hydrograph. The other constraints were that the hydrograph could only rise due to rain or snow melt, must recede at other times, and must agree with steady conditions at the beginning and ends of rainfall periods. Within these constraints there is surprisingly little margin for variations in the location of the adjusted hydrograph.

As a result of these correlations, good estimates of discharge can be provided for the past 17 years on 837.4 square miles of drainage area, or 86 percent of the total area. Runoff from the remaining 14 percent of the area was estimated on the basis of runoff per unit area from the nearest contiguous measured area.

Estimating Inflows from Precipitation. A deterministic hydrological model was developed for computing runoff from precipitation based on the 17 years of records on the upper Boardman. The equation of continuity for a drainage basin is written for a time interval such as one year as follows:

$$S_1 + P - ET - RO \pm G = S_2 \quad (1)$$

Equation (1) may be rearranged to solve for the water losses as in Equation (2) or to solve for runoff as in Equation (3):

$$ET = S_1 - S_2 + P - RO \pm G \quad (2)$$

$$RO = S_1 - S_2 + P - ET \pm G . \qquad (3)$$

In these equations S_1 and S_2 are groundwater storages at the beginning and end of the year, respectively, P is the weighted average precipitation during the year, ET is the evapotranspiration, RO is the runoff and G is groundwater inflow or outflow. The units for all terms are inches depth on the drainage basin.

The storage terms should include soil moisture storages as well as groundwater storages. However, without extensive field tests or an elaborate set of computations the soil moisture content cannot be obtained. Because water years were selected to end at the end of the growing season (October 1) when soil moisture tends to be low, the differences between the beginning and end of a year are usually quite small. The groundwater storage term is evaluated from a groundwater depletion curve, which is the hydrograph of river discharge during periods of no groundwater accretion. It can be derived graphically by selecting sections of the hydrograph which occurred during periods of no or very little rainfall or snow melt and fitting them together. A more objective method is now commonly used[1] in which a differential equation in the form

$$\frac{dQ}{dt} = CQ + b \qquad (4)$$

is obtained from appropriate sections of the hydrograph. In this equation Q is discharge and t is time. Solution of this equation

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[1]Brater, E. F., "Steps Toward a Better Understanding of Urban Runoff Processes," Water Resources Res., April 1968.

yields the equation for the groundwater depletion curve. The groundwater depletion curve for the Boardman River at the gaging station is shown in Figure 5, and its equation is

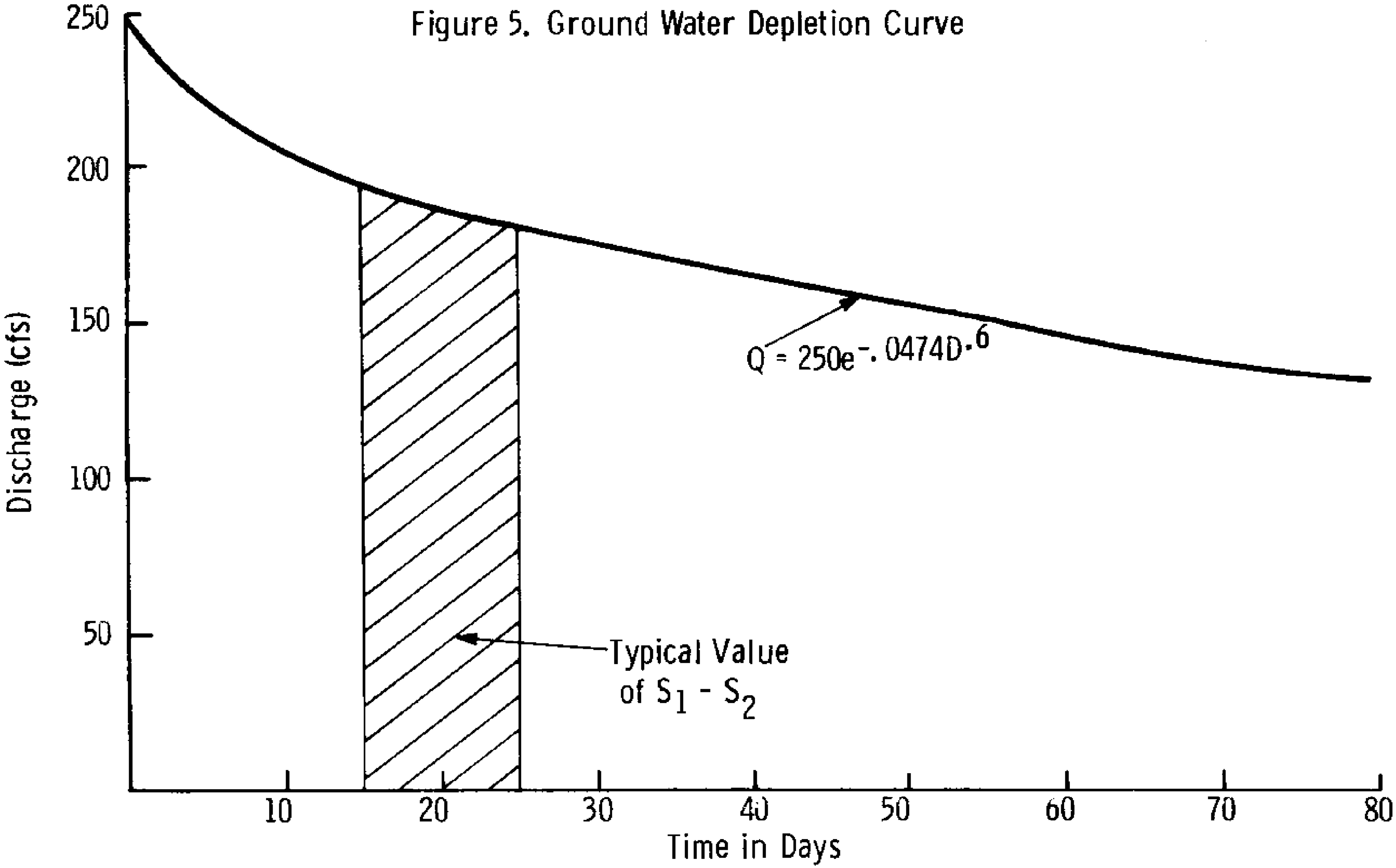
$$Q = 250e^{-.0474t^{.6}} , \qquad (5)$$

in which Q is in cubic feet per second and t is in days. The area under the curve beyond any Q ($\int_t^\infty Qdt$) is the storage associated with that discharge. It is never necessary to evaluate the total storage since these terms always appear as a difference as shown in Equations (2) and (3). The value of $S_1 - S_2$ is then an area such as shown in Figure 4 which can be obtained either graphically or analytically.

The average precipitations (P) are obtained by developing a Thiessen diagram to provide weighting factors. The rain gages which contributed to the weighted average on the upper Boardman basin were Kalkaska, Traverse City and Fife Lake. The runoff was obtained from the U.S. Geological Survey records.

The term G is usually very near to zero. However, on some watersheds the topographic divide does not coincide with the phreatic divide and thus the groundwater is not divided between adjacent basins in proportion to the drainage basin areas. Another situation in which G may not be zero sometimes occurs in limestone regions where underground passages may exist. In the case of the upper Boardman there is no evidence to indicate that G is other than near zero.

Figure 5. Ground Water Depletion Curve



Equation (3) shows the variables that must be considered in developing a hydrological model for estimating runoff. Some watersheds have a good relationship between runoff and precipitation alone. This is most likely to be the situation where the soil has a low infiltration capacity so that years of large precipitation produce large amounts of surface runoff and the terms $(S_1 - S_2)$ and (ET) become less important. For example, on Mill Creek near Ann Arbor, the annual runoff can be predicted with an average error of about 14 percent from annual precipitation or with somewhat less error by including the effect of the previous year of precipitation. Such a simple relationship does not exist in the Grand Traverse Bay regions. On the Boardman basin all of the terms in Equation (3) except G must be included in the model.

The most important term is the water loss (ET). Values of ET were computed for the 17 years of records using Equation (2). The values used in the computations are shown in Table 2. As shown in Figure 6, the water losses are closely related to the precipitation. The linear relation given as Equation (6) fits the data very well.

$$ET = .671\ P - 3.55 \qquad (6)$$

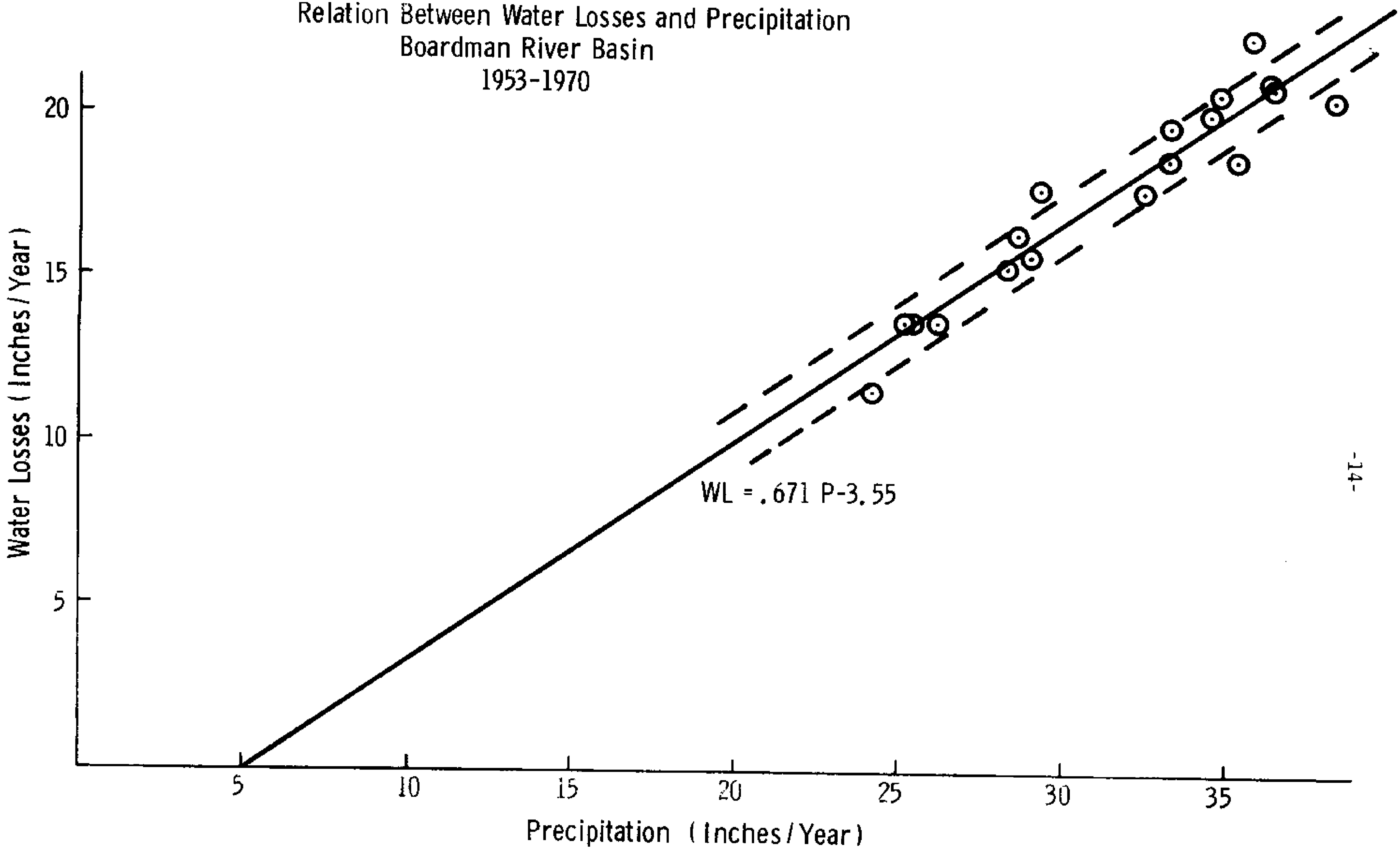
The linear correlation coefficient is 0.95 whereas a value as small as 0.61 would be significant at the 1 percent level for 15 degrees of freedom. The standard deviation from the regression is 0.93 inches.

WATER LOSSES FROM BOARDMAN BASIN

Table 2

Year	Storage (S) (Inches)	S_1-S_2	Precipitation (Inches)	Runoff (RO) (Inches)	Evapo-transpiration (Inches)
1953	4.98	0	34.42	14.44	19.98
1954	4.98	-1.01	33.68	14.52	18.15
1955	5.99	2.06	26.19	14.58	13.67
1956	3.93	-1.40	32.52	13.46	17.66
1957	5.33	.53	28.34	13.61	15.26
1958	4.80	.14	25.12	11.73	13.53
1959	4.66	.73	29.28	12.31	17.70
1960	3.93	- .99	36.36	14.64	20.73
1961	4.92	-1.07	34.81	13.11	20.63
1962	5.99	1.26	29.03	14.73	15.56
1963	4.73	.73	25.48	12.59	13.62
1964	4.00	- .66	28.71	11.72	16.33
1965	4.66	- .89	35.72	12.43	22.40
1966	5.55	1.50	24.19	14.16	11.53
1967	4.05	-1.36	38.15	16.41	20.38
1968	5.41	.89	33.29	14.55	19.63
1969	4.52	-1.38	35.27	15.28	18.61
1970	5.90	.12	36.07	15.23	20.96
	5.78				

Figure 6
Relation Between Water Losses and Precipitation
Boardman River Basin
1953-1970



Equation (6) can be used to estimate the water losses for any year for which precipitation is known.

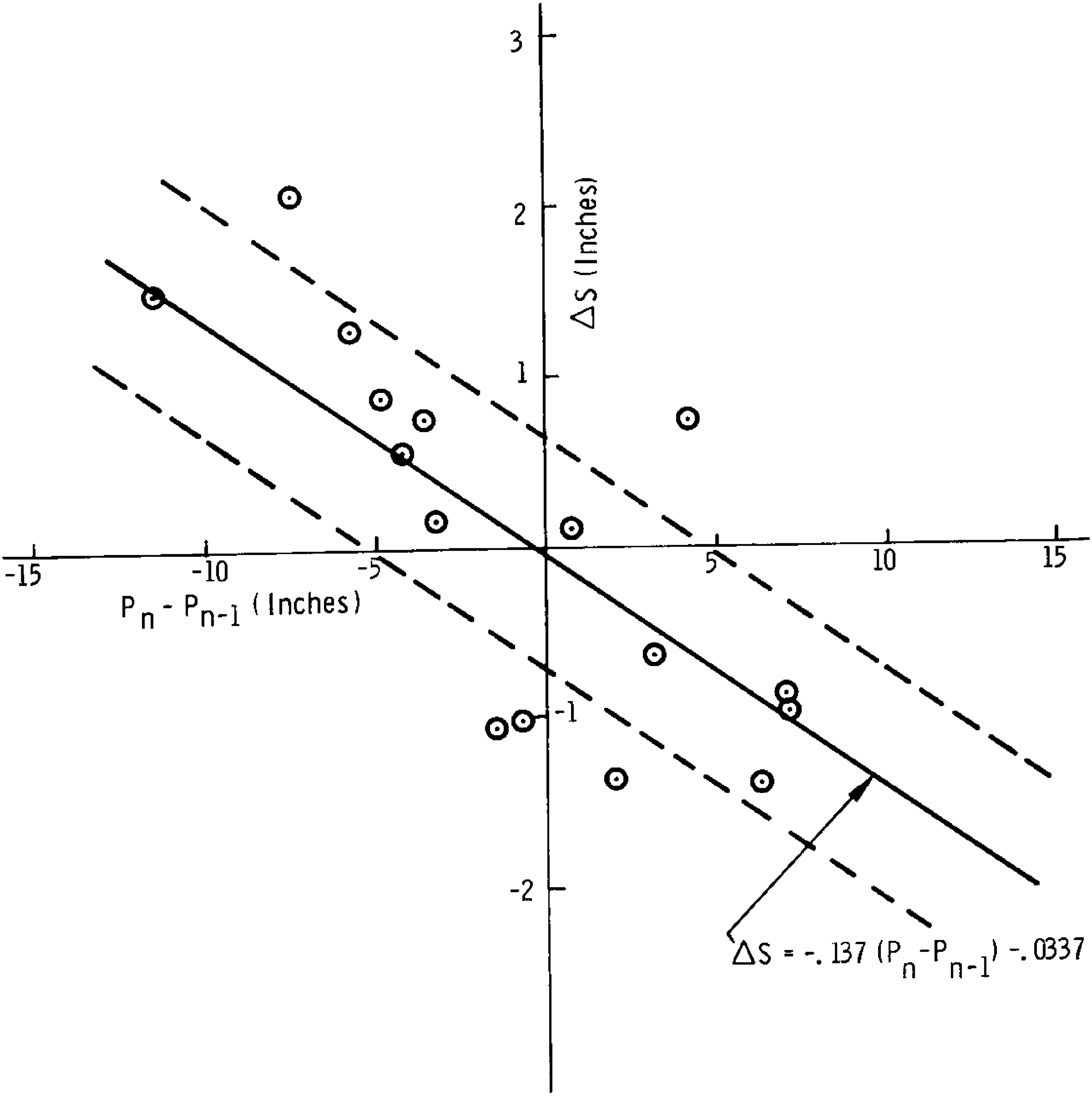
The change in storage ($S_1 - S_2$) is much smaller than ET. Its value fluctuates from positive to negative and over a long period of time its average is near zero. It could be neglected in computing a series of values of runoff which are not related to specific dates. However, including this variable in the computations provides more accurate estimates for each individual year. This term is related to the difference between the precipitation in any year (P_n) and the precipitation during the previous year (P_{n-1}) as shown for the Boardman in Figure 7. The linear regression determined by a least squares analysis is given by Equation (7):

$$\Delta S = S_1 - S_2 = -.137(P_n - P_{n-1}) - .0337 . \qquad (7)$$

The linear correlation coefficient is 0.78 as compared with a value 0.61 at a 1 percent level of significance. The standard deviation from the regression is 0.69. Equation (7) provides a method of estimating ΔS for any year for which precipitation records are available.

With these two relationships, the runoff can be computed from precipitation alone by means of Equation (3). This was done for the 17 years of records on the Boardman. Pertinent data used in the computations are shown in Table 3. Both the measured and

Figure 7.
Relation Between Changes in Storage and Differences in Annual Precipitation
Boardman River Basin



COMPUTED RUNOFF

(All Values in Inches)

Table 3

Year	Precip-itation (P)	P_n-P_{n-1}	S_1-S_2 from Eq. (7)	ET from Eq. (6)	RO from Eq. (3)	RO Measured
1954	33.68	- .74	.07	19.05	14.70	14.52
1955	26.19	- 7.49	.99	14.02	13.16	14.58
1956	32.52	6.33	- .90	18.27	13.35	13.46
1957	28.34	- 4.18	.54	15.47	13.41	13.61
1958	25.12	- 3.22	.41	13.31	12.22	11.73
1959	29.28	4.16	- .60	16.10	12.58	12.31
1960	36.36	7.08	-1.00	20.85	14.51	14.64
1961	34.81	- 1.55	.18	19.81	15.18	13.11
1962	29.03	- 5.78	.76	15.93	13.86	14.73
1963	25.48	- 3.55	.45	13.55	12.38	12.59
1964	28.71	3.23	.06	15.71	13.06	11.72
1965	35.72	7.01	- .99	20.42	14.31	12.43
1966	24.19	-11.53	1.55	12.68	13.06	14.16
1967	38.15	13.96	-1.95	22.05	14.15	16.41
1968	33.29	- 4.86	.63	18.79	15.13	14.55
1969	35.27	1.98	- .30	20.12	14.85	15.28
1970	36.07	.80	- .14	20.65	15.28	15.23

computed values of runoff are plotted in Figure 8. Also shown in Figure 8 are the annual average precipitation, the evapotranspiration (ET) computed by means of Equation (2), as well as the value of ET estimated from Equation (6). The average error in estimating actual discharges for the 17 years was 5.5 percent.

<u>Outflow to Lake Michigan</u>. The annual outflow from Grand Traverse Bay to Lake Michigan was computed from the equation for continuity written for the Bay itself as follows:

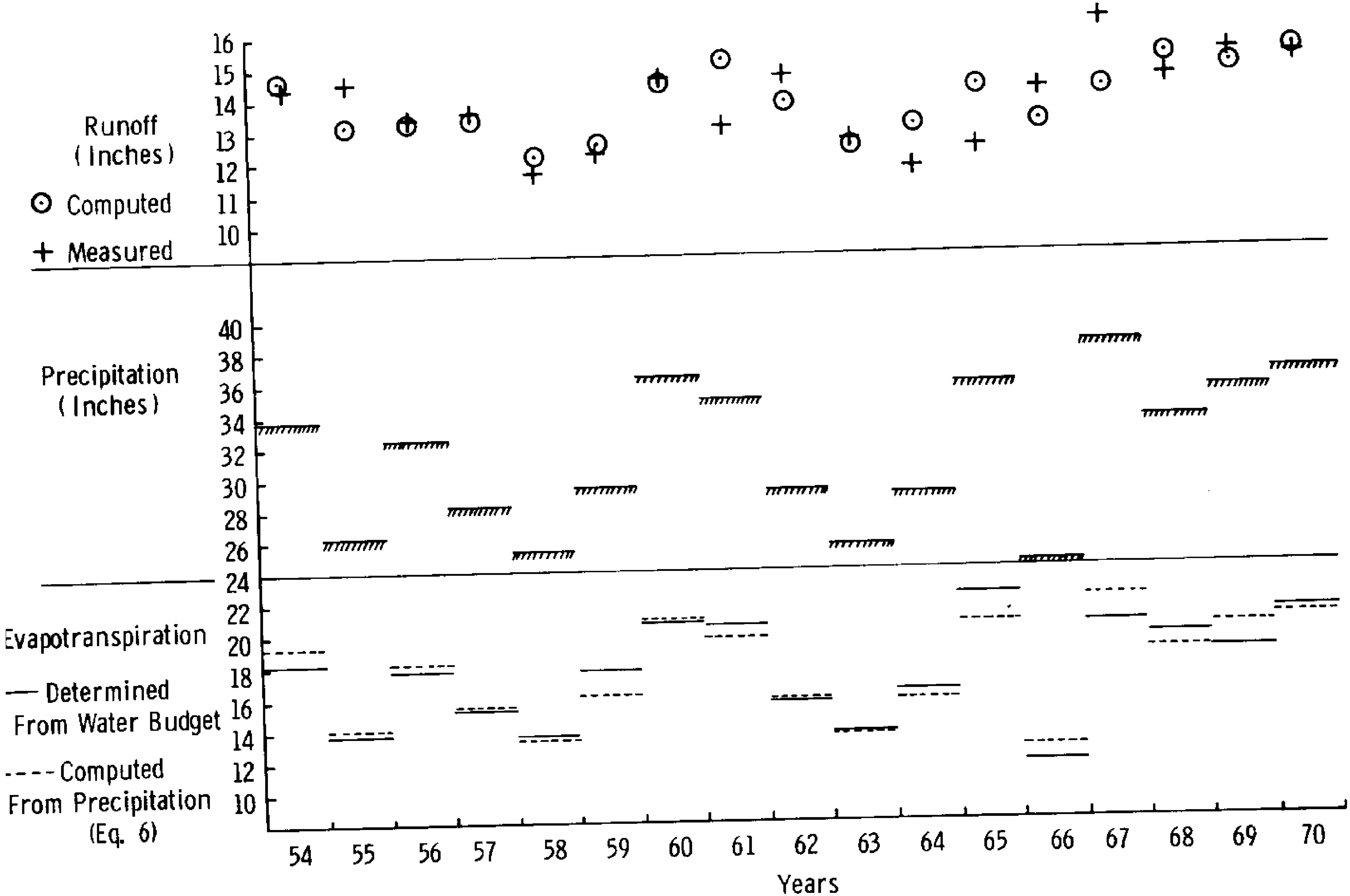
$$S_1 + P + RO - E - O \pm DS = S_2 \qquad (8)$$

or

$$O = S_1 - S_2 + P + RO - E \pm DS \ . \qquad (9)$$

In these equations, O is the outflow, $(S_1 - S_2)$ is the change in storage, P is the weighted average precipitation on the Bay, RO is the runoff from the land areas and E is the evaporation. The term DS represents "deep seepage" which might enter or leave the Bay. Any such water would probably have distant sources or destinations. One could, however, visualize artesian conditions which might produce inflow to the Bay either from remote portions of the drainage area or from other basins such as from Lake Superior. Another possibility would be underground flow from Torch Lake or Elk Lake to the Bay. If this were occurring, the discharge from the Elk River basin would be reduced. However, as shown by Figure 3, the discharge

Figure 8
Runoff Precipitation and Evapotranspiration
Boardman River Basin



per square mile from the Elk River basin is higher than that of the Boardman, thus indicating that any such flow must be small. In the absence of any specific information, this term is taken at zero. There seems to be little doubt that its value would be small compared with other terms in Equation (9).

The change in storage is obtained from the U.S. Lake Survey records of the levels of Lake Michigan. The precipitation is averaged by the Thiessen method using five applicable rain gages. The runoff was computed using the records from the gaging station together with the correlations for the ungaged areas discussed earlier in this report. The evaporation was determined from a comprehensive model[2] of Lakes Michigan and Huron in which in each case Equation (9) was applied to the entire lake. Table 4 shows the values used to estimate O as well as the computed values of O in terms of inches per year and cubic feet per second.

The methods used in obtaining values of outflow for the entire Bay can also be applied to any smaller portions of the Bay. It is necessary in each case to determine the drainage areas which contribute to the study area in order to estimate the inflow.

The inflows to the Bay as well as the outflows to Lake Michigan can be computed by the methods developed in this report

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[2] Quinn, Frank H., "Quantitative Dynamic Models for Great Lakes Research," Ph.D. Thesis, Department of Civil Engineering, University of Michigan, Ann Arbor, Michigan, 1971.

OUTFLOW TO LAKE MICHIGAN FROM GRAND TRAVERSE BAY

Table 4

Year	Elev. of Lake Michigan	S_1-S_2 (Inches)	Average Precipitation on Bay (Inches)	Inflow to Bay (RO) (Inches)	Evapo-transpiration (Inches)	Outflow to Lk. Michigan (Inches)	(cfs)
1953	580.55	7.2	31.61	59.90	28.92	69.79	1424
1954	579.95	1.8	35.06	60.24	26.52	70.58	1440
1955	579.80	13.8	22.04	60.48	29.64	66.68	1361
1956	578.65	3.0	31.02	55.84	25.32	64.54	1317
1957	578.40	8.4	26.55	56.46	25.80	65.61	1339
1958	577.70	1.02	25.83	48.66	27.60	47.91	978
1959	576.85	- 2.4	32.11	51.07	25.80	54.98	1122
1960	577.05	-23.4	36.53	60.73	22.68	51.18	1044
1961	579.00	11.4	28.91	54.39	26.40	68.30	1392
1962	578.05	6.6	29.66	61.11	24.84	72.53	1480
1963	577.50	12.0	24.68	52.23	24.12	64.79	1322
1964	576.50	7.8	26.11	48.62	27.60	54.93	1121
1965	575.85	-14.4	26.74	51.57	28.68	35.23	719
1966	577.05	- 1.2	21.89	58.74	26.	53.43	1090
1967	577.15	- 9.0	33.29	68.07	26.	66.36	1354
1968	577.90	- 8.4	28.05	60.36	26.	54.01	1102
1969	578.60	- 9.6	31.28	63.39	26.	59.07	1205
1970	579.40	1.8	33.87	63.18	26.	72.85	1487
	579.25						

for the period starting in 1889, when the first rain gage was established in the area. In this manner there could be provided more than 80 years of hydrologic data which would supply a dependable basis for predicting future conditions.