# UNIVERSITY OF NORTH CAROLINA

## SEA GRANT PROGRAM



**COMPUTATION OF FLOW THROUGH**

**MASONBORO INLET, N.C.**

MICHEAL AMEIN

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# COMPUTATION OF FLOW THROUGH MASONBORO INLET, N. C.

by

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Messrs. S. G. Wardak and Evan Sun did most of the data processing and program revisions.

## ABSTRACT

The Masonboro Inlet system is a complex arrangement of channels connecting the coastal waterways to the Atlantic Ocean. The flow through the system was computed by means of a numerical simulation model based on the equations of unsteady flow which represent the laws for the conservation of mass and momentum. An implicit method was used to find the solutions for the finite difference equations. The method requires that the basic equations be satisfied at all locations simultaneously. A comparison of the computed results with the field observations demonstrates the accuracy, reliability, and efficiency of the method. It is believed that the simulation model can handle other inlets, which are generally much simpler than the Masonboro Inlet, with little difficulty.

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# I. INTRODUCTION

This report summarizes the results of a study on the dynamics of flow through the Masonboro Inlet System, N. C. It is primarily concerned with the development and testing of a numerical simulation model for the computation of tidal and freshwater flow exchange through coastal inlets. Masonboro Inlet was selected for the following reasons: (1) it is an important component of the North Carolina estuarine system; (2) it is one of the most complex inlet systems, and thus, would serve as a challenging test of modeling techniques and; (3) excellent field data for model testing are available for this inlet.

The simulation model is founded on the equations of unsteady flow in open channels. The equations are also known as the shallow-water equations, the St. Venant equations, the tidal equations and the long-wave equations. The basic assumption made in the derivations of the equations is that the water surface curvature is small so that the wave length is much greater than the water depth. The equations of unsteady flow in open channels constitute a system of partial differential equations. An implicit method (Amein 1968, Amein and Fang 1970, Amein 1972) which was found to be very successful in applications to highly irregular channels was used for simulation.

Masonboro Inlet System is a complex arrangement of waterways. Unlike most coastal inlets which connect an inland body of water to the sea, the Masonboro Inlet connects the Atlantic Ocean to a network of channels. In the development of the inlet model, it was necessary to handle channel junction problems. The junction problem was solved by satisfying the conservation of mass and energy at the confluence of waterways.

A comparison of computed results with the field data indicates that the simulation model can accurately determine the dynamics of flow through Masonboro Inlet. It is believed that modeling of other inlets, which are expected to be much simpler, by the same techniques would produce excellent results.

It shold be mentioned that the simulation model applied to the Masonboro Inlet System is a vigorous and sophisticated technique. Most of the mathematical models in the past had to rely on simplifying assumptions with regard to channel physical parameters, friction losses and boundary conditions in order to obtain analytical solutions. The numerical model proposed in this report has the capability to simulate inlets of irregular shape and can accept any formulation of friction losses.

## II. THEORY

The equations of unsteady flow on which numerical simulation for tidal flow through inlets is based consist of the equations for the conservation of mass and momentum. Their derivation is given in standard reference works (V. T. Chow 1959, Gilcrest 1950, Henderson 1966, Stoker 1957). For the sake of convenience, a brief description of their derivation will be given here.

Consider a short reach of channel of length $\Delta x$ with the flow taking place from section 1-1 to section 2-2 as shown in Figure 1. Let x be the horizontal distance; $\rho$, the water density; g, the acceleration due to gravity; z, the channel bottom elevation; y, the water depth; A, the cross-sectional area; and v, the average velocity. Let Q be the volume flow rate entering at 1-1 and $\{Q + (\partial Q/\partial x) \Delta x\}$ be the volume outflow rate; and, let q be the lateral inflow per unit channel length per unit time. The mass of water entering the channel during a time interval $\Delta t$ will be

$$\rho . Q . \Delta t + \rho q . \Delta t . \Delta x$$

The mass of water leaving the channel reach during the same time interval $\Delta t$ will be

$$\rho \{Q + (\partial Q/\partial x) \Delta x\} \Delta t$$

The net mass outflow during the time $\Delta t$ is, therefore

$$\rho (\partial Q/\partial x) \Delta x \Delta t - \rho q \Delta x \Delta t$$

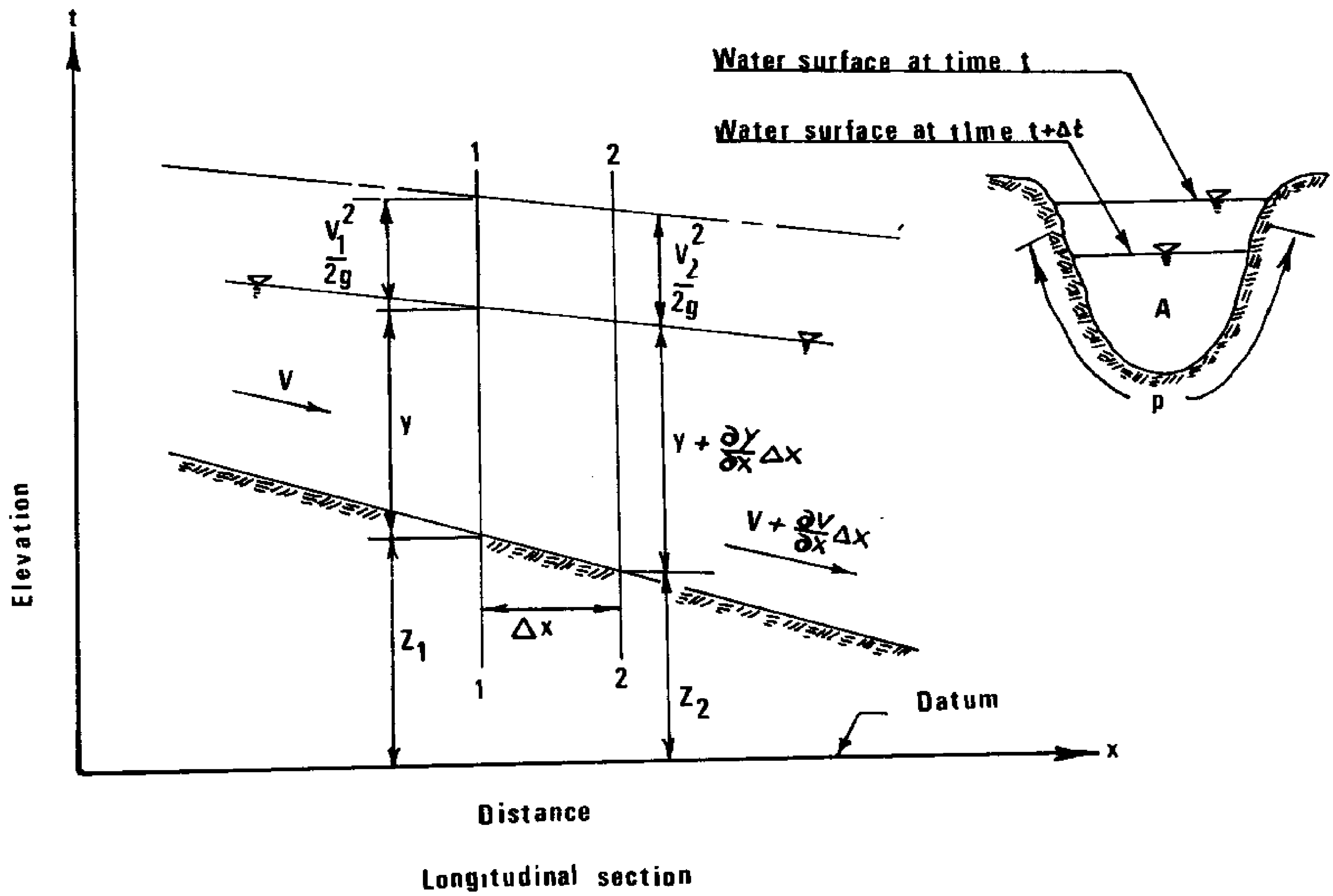



Fig.1. Definition sketch for open channel flow

The rate of decrease of this mass can be expressed as

$$-\frac{\partial}{\partial t}\{\rho\ A\Delta x\} = -\rho\Delta x\ \frac{\partial A}{\partial t}$$

From the conservation of mass, the net mass outflow must be equal to the decrease of mass in the reach. Therefore,

$$\rho\frac{\partial Q}{\partial x}\ \Delta x\Delta t - \rho q\ \Delta x\Delta t = -\rho\Delta x\frac{\partial A}{\partial t}\ \Delta t$$

or

$$\frac{\partial Q}{\partial x} + \frac{\partial A}{\partial t} - q = 0 \qquad (1)$$

Equation (1) is known as the equation of continuity and is the mathematical expression for the law of conservation of mass in unsteady flow in open channels.

The conservation of momentum is given by Newton's second law stating that the rate of change of momentum is equal to the net applied force. In Figure (1), the net applied force on the element of volume in the reach $\Delta x$ is the resultant of the pressure, gravity and frictional forces on the element. The water depth is y at 1-1 and it is $y + \frac{\partial y}{\partial x}\Delta x$ at 2-2. The cross-sectional area is A at 1-1 and it is $A + \frac{\partial A}{\partial x}\Delta x$ at 2-2.

The pressure force at section 1-1 is

$$\rho\ g\ \bar{y}\ A$$

and at section 2-2, it is

$$\rho\ g\ (\bar{y} + \frac{\partial y}{\partial x}\Delta x)\ A$$

Where $\bar{y}$ is the depth to the centroid at 1-1. The component of the gravity force in the direction of motion is

$$- \rho \ g \ A\Delta x \ ( \frac{\Delta z}{\Delta x} )$$

The friction force is a shear force equal to

$$\tau \ P \ \Delta \ x$$

where $\tau$ is the shear stress and P is the wetted perimeter. In terms of the head loss, the shear force is

$$\tau \ P\Delta x = \rho g h_L \quad A = \rho g S_f \Delta x \ A$$

where $h_L$ is the head loss over the distance $\Delta x$, and $S_f$ is the friction slope. The convective (spatial) momentum change is

$$\frac{\partial}{\partial x} (\rho Q v) \Delta x$$

The local (temporal) momentum change is

$$\frac{\partial}{\partial t} (\rho A\Delta x \ . \ v) = \rho \frac{\partial Q}{\partial t} \Delta x$$

Equating the net applied force to the total rate of change of momentum of the fluid moving through the reach $\Delta x$,

$$- \rho g A \ ( \frac{\Delta z}{\Delta x} ) \ \Delta x - \rho g A S_f \ \Delta x - \rho g A \frac{\partial y}{\partial x} \Delta x$$

$$= \frac{\partial}{\partial t} (\rho Q) \ \Delta x + \frac{\partial}{\partial x} (\rho Q v) \ \Delta x$$

Dividing all the terms by $\rho A \Delta x$

$$\frac{1}{A}\frac{\partial Q}{\partial t} + \frac{v}{A}\frac{\partial Q}{\partial x} + v\frac{\partial v}{\partial x} = -g\frac{\partial y}{\partial x} - g\frac{\partial z}{\partial x} - gS_f \qquad (2)$$

By replacing Q by vA and introducing the value of $\partial Q/\partial x$ from equation (1), equation (2) can be reduced to the more well-known form of

$$\frac{\partial v}{\partial t} + v\frac{\partial y}{\partial x} = -g\frac{\partial y}{\partial x} - g\frac{\partial z}{\partial x} - gS_f \qquad (2a)$$

Equations (1) and (2) constitute a system of nonlinear first-order partial differential equations of the hyperbolic type. The equations do not have general analytical solutions.

Numerical methods for the solution of the equation of unsteady flow have been known since the time of Massau (1899). A review of the numerical methods is given in a report by Amein and Fang (1970), and in numerous papers and in several textbooks. For the sake of brevity, a review of the numerical methods will be omitted from this report.

The numerical model for Masonboro Inlet presented in this report is based on the implicit method. Variations of this method have been presented by Abbott and Ionescu (1967), Amein (1968), Lai (1967) and others. It was applied to irregular natural channels by Amein and Fang (1969, 1970). The method was further revised and generalized to accomodate reservoirs and waterway areas in which the changes in channel properties are highly irregular. Details for the revised procedure are given in a report (Amein, 1972) prepared for the U. S. National Weather Services. Essentially the same procedure is used for Masonboro Inlet, except that at Masonboro Inlet, not one but at least four channel flows must be considered at the same time.

## III. NUMERICAL PROCEDURE

The one dimensional equations of unsteady flow in open channels are

$$\frac{\partial Q}{\partial x} + \frac{\partial A}{\partial t} - q = 0 \tag{1}$$

$$\frac{1}{A}\frac{\partial Q}{\partial t} + \frac{v}{A}\frac{\partial Q}{\partial x} + v\frac{\partial v}{\partial x} = - g\frac{\partial z}{\partial x} - gS_f - g\frac{\partial y}{\partial x} \tag{2}$$

where

Q = volume flow rate

A = cross-sectional area

q = lateral inflow

v = velocity in the direction of flow

z = elevation of the channel bottom

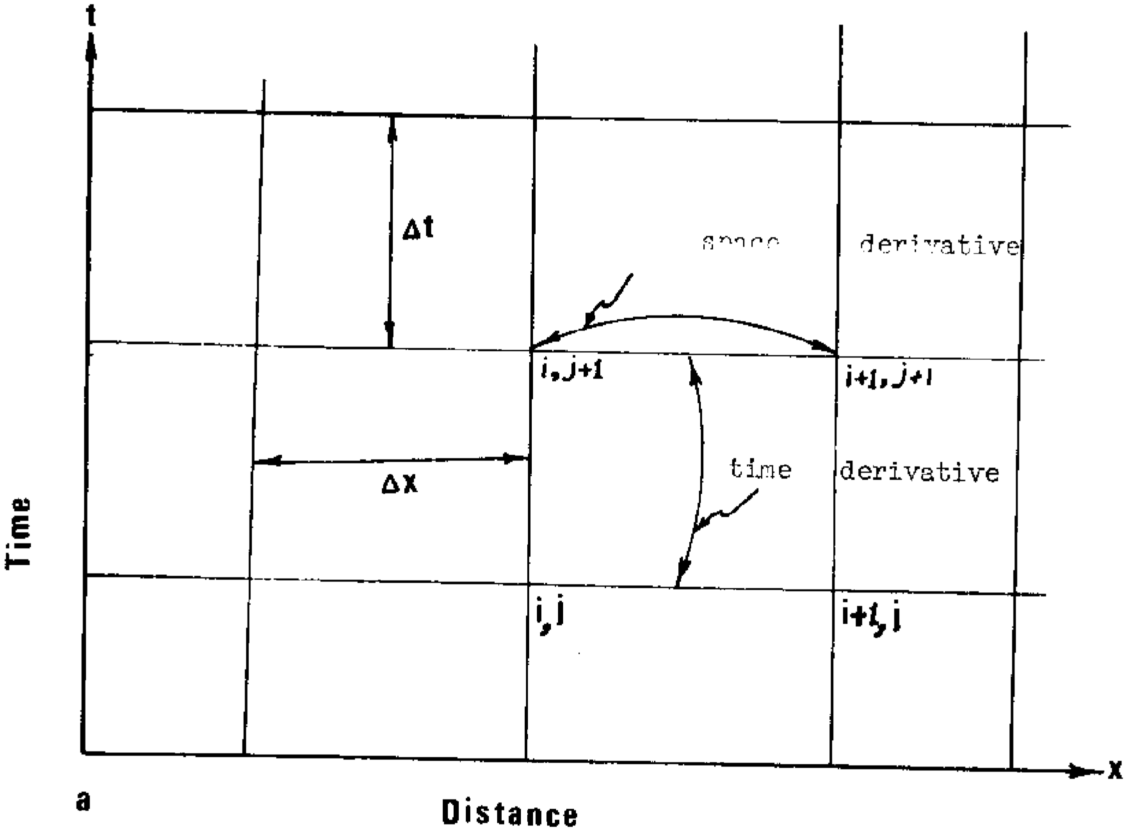
y = water depth

g = acceleration due to gravity

x = distance along the channel

t = time

To develop a simulation model, consider a non-uniform rectangular grid on the x,t plane as shown in Figure 2. Distances along the channel are represented by abscissas and times are represented by ordinates. The differential equations are simulated for a finite distance $\Delta x$ and for a finite time $\Delta t$.



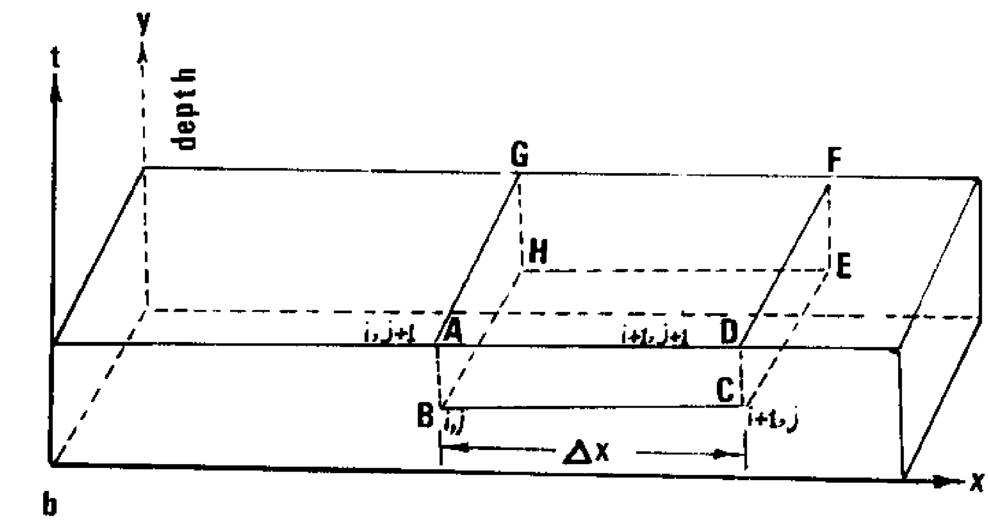




Fig. 2. Definition sketch of network of Points on the x-t Plane and a channel reach of length Δx

It is assumed that all the variables are known at all points $x_i$ at time $t^j$. It is desired to determine the values of the variables for time $t^{j+1}$ at all points $x_i$ with given boundary conditions. The equation of continuity is simulated by

$$\frac{Q_{i+1}^{j+1} - Q_i^{j+1}}{\Delta x} + \frac{\bar{A}_{i+1/2}^{j+1} - \bar{A}_{i+1/2}^{j}}{\Delta t} - q_{i+1/2}^{j+1} = 0 \qquad (3)$$

where

$$\bar{A}_{i+1/2} = \frac{1}{\Delta x} \int_{x_i}^{x_{i+1}} A(x)\Delta x$$

The value of $\bar{A}_{i+1/2}$ depends on the shape of the channel and the manner in which the cross-sectional area varies with distance. A good approximation to $\bar{A}_{i+1/2}$ is $\bar{A}_{i+1/2} \simeq \frac{1}{2}(A_i + A_{i+1})$

If $\frac{\partial A}{\partial x}$ is constant, then

$$\bar{A}_{i+1/2} = \frac{1}{2}(A_i + A_{i+1})$$

The equation for the conservation of momentum (2) is simulated by

$$\frac{1}{A_{i+1/2}^{j+1}} \quad \frac{\bar{Q}_{i+1/2}^{j+1} - \bar{Q}_{i+1/2}^{j}}{\Delta t} + \frac{1}{(\bar{A}_{i+1/2})^2} \times \frac{1}{2\Delta x} \left\{ (Q_{i+1}^{j+1})^2 - (Q_i^{j+1})^2 \right\}$$

$$+ \frac{1}{2\Delta x} \left\{ \left( \frac{Q_{i+1}^{j+1}}{A_{i+1}^{j+1}} \right)^2 - \left( \frac{Q_i^{j+1}}{A_i^{j+1}} \right)^2 \right\} + g \frac{(y_{i+1}^{j+1} + z_{j+1}) - (y_i^{j+1} - z_i)}{\Delta x}$$

$$+ g\, \bar{S}_{f_{i+1/2}}^{j+1} = 0 \qquad (4)$$

In the above equations

$$\overline{Q}^{j+1}_{i+1/2} = \frac{1}{\Delta x} \int_{x_i}^{x_{i-1}} Q(x,t)\ dx$$

and

$$\overline{S}^{j+1}_{f_{i+1/2}} = \frac{1}{\Delta x} \int_{x_i}^{x_{i+1}} S_f\ (x,t)\ dx$$

Again, the values of $\overline{Q}$ and $\overline{S}_f$ depend on the variation of these quantities with distance x. Assuming a linear variation,

$$\overline{S}^{\ j+1}_{f_{i+1/2}} \approx \frac{1}{2} \left\{ S^{j+1}_{f_{i+1}} + S_{f_i}^{\ j+1} \right\}$$

and

$$\overline{Q}^{\ j+1}_{i+1/2} \approx \frac{1}{2} \left\{ Q^{j+1}_{i+1} + Q_i^{\ j+1} \right\}$$

The actual variation of Q and $S_f$ with distance x can be determined from field data. However, if $\Delta x$ is taken sufficiently small, the linear assumption will produce good results.

In equations (3) and (4), all the variables with superscripts (j+1) are unknown. The unknowns consist of the values of Q (volume flow rate), y (depth), v (velocity), A (area), P (wetted perimeter), $S_f$ (friction slope), and B (top width). However, all the unknowns are not independent. If Q and y are chosen as the independent variables, the other variable such as A, B, P, $S_f$ and v can be expressed as functions of Q and y. Equations (3) and (4) contain four independent unknowns, the unknowns being the values of the discharge and stage at grid points (i, j+1) and (i+1, j+1). It should be noted that the distance increment $\Delta x = x_{i+1} - x_i$ and the time interval $\Delta t = t^{j+1} - t^j$

need not be constant. $\Delta x$ can be varied at any $x_i$ and $\Delta t$ can be varied at any $t^j$.

Equations (3) and (4) constitute a system of two nonlinear algebraic equations in four unknowns. By themselves they are not sufficient to evaluate the unknowns at points (i, j+1) and (i+1, j+1). However, the unknowns are common to any two neighboring channel reaches. If the entire system of channels is subdivided into reaches, a number of equations will be obtained that together with additional equations arising from boundary conditions and channel junctions will make the number of equations equal to the number of unknowns.

## IV. MASONBORO INLET FLOW SIMULATION

The Masonboro Inlet system, shown on Figure 3, consists of the Masonboro Inlet, Banks Channel, Masonboro Channel and Shinn Creek. For the purposes of this study, the channel system was represented by 18 stations. At each station, cross-sectional properties were determined from topographic data compiled by the U. S. Army Corps of Engineers. Values of the frictional coefficient and initial values of depth and discharge were also estimated or taken from field observations. Details and input data are given in a subsequent section of this report. This section is concerned with the fundamental aspects of applyin the simulation model to Masonboro Inlet.

The determination of flow dynamics in Masonboro Inlet system by means of numerical simulation of unsteady flow may be interpreted as the evaluation of the values of the discharge Q and the depth y at all locations and at desired times. Therefore, the variables Q and y become dependent and the variables x (positions) and t (time) become independent variables.

The channel junction is represented by stations 7, 10, 11, and 12 (Figure 3). The value of x is taken to be 0 at the junction. Station 7 represents the junction as the boundary for Shinn Creek, Station 12 represents it as the boundary for Masonboro Inlet proper and so on. The value of x increases numerically from the junction into each of the four channels. However, the positive x-axis extends from the junction towards the sea while the extension of x from the junction into each of the other three channels takes place on the negative x-axis.

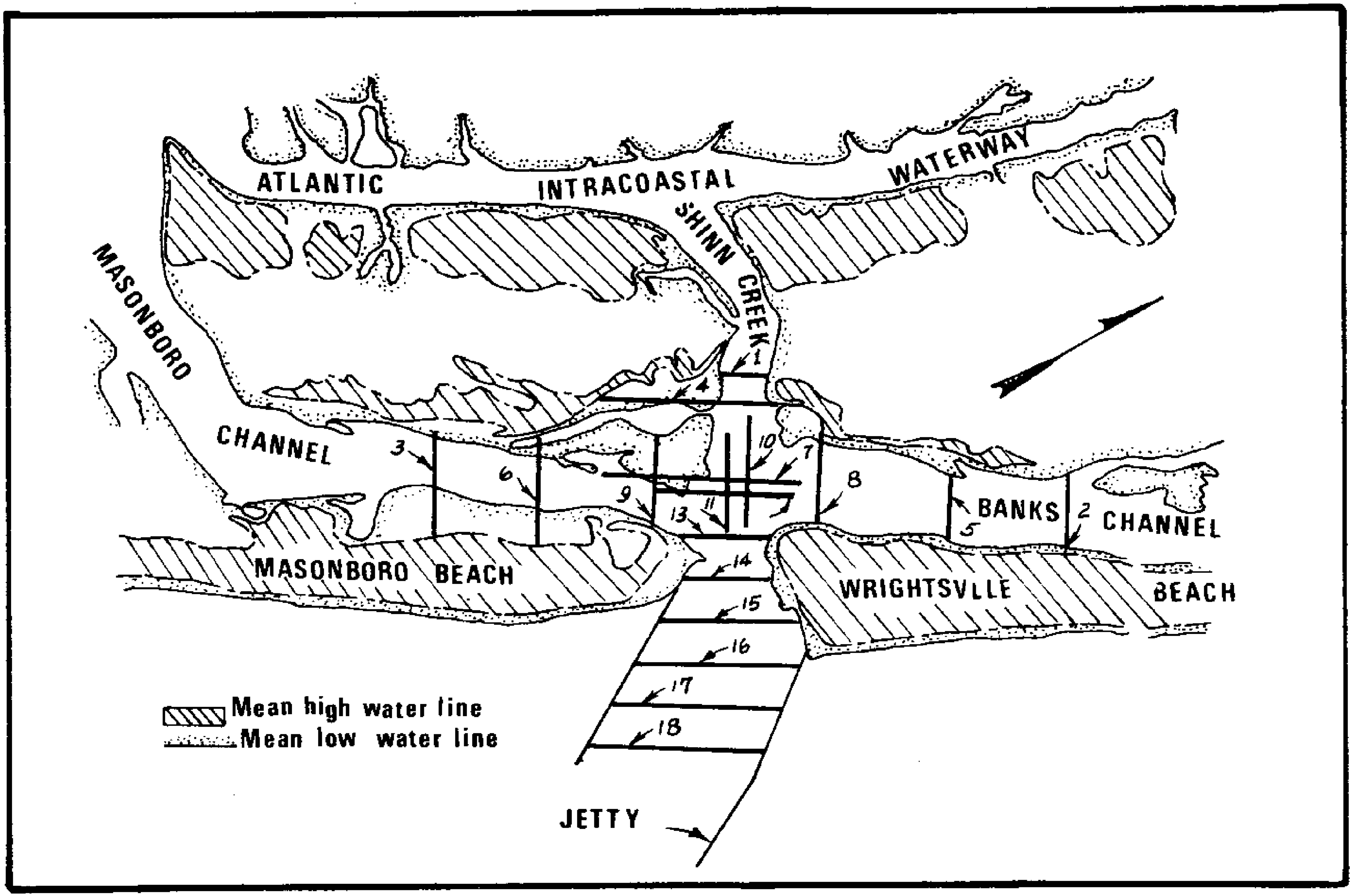



Fig. 3. Stationing for Masonbro Inlet System

## Interior Equations

The procedure for application of equations (3) and (4) and for incorporating boundary conditions are described as follows:

Let equations (3) and (4) be represented by

$$F\ (Q_i,\ Q_{i+1},\ y_i,\ y_{i+1})\ =\ 0 \tag{5}$$

$$G\ (Q_i,\ Q_{i+1},\ y_i,\ y_{i+1})\ =\ 0 \tag{6}$$

where $Q_i$, $Q_{i+1}$, $y_i$, $Y_{i+1}$ are the values of discharge and depth at stations i and i+1 at time $t^{j+1}$. Since it is understood that all the unknowns have superscripts j+1, the superscripts have been dropped in equations (5) and (6). Application of equations (5) and (6) to the Masonboro Inlet system leads to the following equations:

In Banks Channel:

In the reaches between stations (2) and (5), between (5) and (8) and between (8) and (12).

$$F(Q_2,\ Q_5,\ y_2,\ y_5)\ =\ 0 \qquad (S_1)$$

$$G(Q_2,\ Q_5,\ y_1,\ y_5)\ =\ 0 \qquad (S_2)$$

$$F(Q_5,\ Q_8,\ y_5,\ y_8)\ =\ 0 \qquad (S_3)$$

$$G(Q_5,\ Q_8,\ y_5,\ y_8)\ =\ 0 \qquad (S_4)$$

$$F(Q_8,\ Q_{10},\ y_8,\ y_{10})\ =\ 0 \qquad (S_5)$$

$$G(Q_8,\ Q_{10},\ y_8,\ y_{10})\ =\ 0 \qquad (S_6)$$

In Masonboro Channel

$$F(Q_3,\ Q_6,\ y_3,\ y_6)\ =\ 0 \qquad (S_7)$$

$$G(Q_3,\ Q_6,\ y_3,\ y_6)\ =\ 0 \qquad (S_8)$$

$$F(Q_6,\ Q_9,\ y_6,\ y_9)\ =\ 0 \qquad (S_9)$$

$$G(Q_6,\ Q_9,\ y_6,\ y_9)\ =\ 0 \qquad (S_{10})$$

$$F(Q_9,\ Q_{11},\ y_9,\ y_{11})\ =\ 0 \qquad (S_{11})$$

$$G(Q_9,\ Q_{11},\ y_9,\ y_{11})\ =\ 0 \qquad (S_{12})$$

In Shinn Creek

$$F(Q_1, Q_4, y_1, y_4) = 0 \qquad (S_{13})$$

$$G(Q_1, Q_4, y_1, y_4) = 0 \qquad (S_{14})$$

$$F(Q_4, Q_7, y_4, y_7) = 0 \qquad (S_{15})$$

$$G(Q_4, Q_7, y_4, y_7) = 0 \qquad (S_{16})$$

In Masonboro Inlet

$$F(Q_{12}, Q_{13}, y_{12}, y_{13}) = 0 \qquad (S_{17})$$

$$G(Q_{12}, Q_{13}, y_{12}, y_{13}) = 0 \qquad (S_{18})$$

$$F(Q_{13}, Q_{14}, y_{13}, y_{14}) = 0 \qquad (S_{19})$$

.

$$F(Q_{17}, Q_{18}, y_{17}, y_{18}) = 0 \qquad (S_{27})$$

$$G(Q_{17}, Q_{18}, y_{17}, y_{18}) = 0 \qquad (S_{28})$$

The above constitute 28 equations in 36 unknowns. To make the system determinate, eight additional equations are needed. These are provided by the boundary conditions.

## Boundary Conditions

At the channel junction, the energy level is the same since the junction is considered to be a boundary of each joining channel. Thus the energy level at station (7) of Shinn Creek is the same as the energy level at station (12) of Masonboro Inlet. Therefore, the following equations arise from the application of the conservation of energy to the junction.

$$y_7 + v_7^2/2g + z_7 = y_{12} + \frac{v_{12}^2}{2g} + z_{12} \qquad (7)$$

$$y_{10} + v_{10}^2/2g + z_{10} = y_{12} + \frac{v_{12}^2}{2g} + z_{12} \qquad (8)$$

$$y_{11} + v_{11}^{2}/2g + z_{11} = y_{12} + \frac{v_{12}^{2}}{2g} + z_{12} \qquad (9)$$

Equations (7) through (9) can be rewritten as

$$G(Q_7, Q_{12}, y_9, y_{12}) = 0 \qquad (S_{29})$$

$$G(Q_{10}, Q_{12}, y_{10}, y_{12}) = 0 \qquad (S_{30})$$

$$G(Q_{11}, Q_{12}, y_{11}, y_{12}) = 0 \qquad (S_{31})$$

The application of the law of conservation of mass to the junction leads to

$$Q_{12} + Q_7 + Q_{10} + Q_{11} = 0 \qquad (10)$$

Equation (10) states that the algebraic sum of the flows at the junction is zero so that there is no storage in the junction. Equation (10) may be represented as

$$F(Q_7, Q_{10}, Q_{11}, Q_{12}) = 0 \qquad (S_{32})$$

Four additional equations are provided by the boundary conditions in each channel. The boundary conditions may be given as known values of discharge, or depth or a relationship between stage and discharge.

Known values of water level as functions of time were used as the boundary conditions in this report. These values are provided by tidal gages located at selected stations which will serve as the boundaries. Thus

$$y_{18} - k_{18} = 0 \qquad (11)$$

$$y_1 - k_1 = 0 \qquad (12)$$

$$y_3 - k_3 = 0 \qquad (13)$$

$$y_2 - k_2 = 0 \qquad (14)$$

Equations (11) through (14) may be represented as

$$F(y_{18}) = 0 \qquad (S_{33})$$

$$F(y_1) = 0 \qquad (S_{34})$$

$$F(y_3) = 0 \qquad (S_{35})$$

$$F(y_2) = 0 \qquad (S_{36})$$

It is seen that 36 equations identified by subscript S become available for the evaluation of 36 unknowns. The equations, with the exception of five equations arising from the boundary conditions, are nonlinear algebraic equations.

If there are N stations in the channel system, there will be 2N equations, with 2N unknowns, the unknowns being the values of Q (flow rate) and y (depth) at each station. The system of equations may be represented in a general way by

$$
\begin{aligned}
\Phi_1(Q_1, y_1, Q_2, y_2, \qquad & Q_N, y_N) = 0 \\
\Phi_2(Q_1, y_1, Q_2, y_2, \qquad & Q_N, y_N) = 0 \\
\cdots\cdots\cdots\cdots\cdots\cdots & \cdots\cdots \\
\Phi_i(Q_1, Q_2, \qquad & Q_N, y_N) = 0 \\
\Phi_{i+1}(Q_1, Q_2, \qquad & Q_N, y_N) = 0 \\
\cdots\cdots\cdots\cdots\cdots\cdots & \cdots\cdots \\
\Phi_{N-1}(Q_1, Q_2, \qquad & Q_N, y_N) = 0 \\
\Phi_N(Q_1, Q_2, \qquad & Q_N, y_N) = 0
\end{aligned}
\qquad (15)
$$

Routine methods for the solution of the nonlinear system are not known. In this study, the generalized Newton iteration method was used to evaluate the unknowns. Details of the application is described in the report by Amein and Fang (1969) and is summarized in the paper by Amein and Fang (1970) with discussions on several technical points presented by Kamphuis (1971), Contractor and Wiggert (1971), Fread (1971), Frank (1971) and O'Loughlin and Short (1971). A brief outline of the method will be given in the following section.

## V. A SOLUTION METHOD

The value of the water flow rate, water velocity and depth can be determined from the solution of the systems of nonlinear equations represented by (15). A method which was found very successful is the generalized Newton iteration method. In the application of this method, trial values are assigned to the unknowns. When these trial values are substituted into (5), the right sides of the equations may not vanish, but may acquire values known as the residuals. Solutions are obtained by adjusting the trial values in a systematic manner until each residual vanishes or is reduced to a tolerable quantity.

To illustrate the procedure, let it be assumed that the computations have been carried through the k-th iteration cycle so that the values of the unknowns have been approximated through the k-th cycle. Let these values be denoted by the superscript k. It is desired to approximate the values of the variables through the (k+1)th cycle. When the values approximated through the k-th cycle are introduced into equations (15), the right-hand sides become the residuals. Let the residuals be represented by the vector $R_i^k$. Then

$$\phi_1(Q_1^k, y_1^k \qquad Q_N^k, y_N^k) = R_1^k$$
$$\phi_2(Q_1^k, y_1^k \qquad Q_N^k, y_N^k) = R_2^2 \qquad (16)$$
$$\phi_{2N}(Q_1^k, y_1^k \qquad Q_N^k, y_N^k) = R_{2N}^k$$

According to the generalized Newton iteration method, the residuals and the values of the variables at two consecutive iteration cycles, (k) and (k+1) are related by the following equations:

$$\frac{\partial \phi_1}{\partial Q_1} dQ + \frac{\partial \phi_1}{\partial y_1} dy_1 + \cdots \qquad + \frac{\partial \phi_1}{\partial y_N} dy_N = R_1^{\;k}$$

$$\frac{\partial \phi_2}{\partial Q_1} dQ_1 + \frac{\partial \phi_2}{\partial y_2} dy_1 + \cdots \qquad + \frac{\partial \phi_2}{\partial y_N} dy_N = R_2^{\;k}$$

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$$\frac{\partial \phi_{2N}}{\partial Q_1} dy_1 + \frac{\partial \phi_{2N}}{\partial y_1} dy_1 + \cdots + \frac{\partial \phi_{2N}}{\partial Q_N} dQ_N + \frac{\partial \phi_{2N}}{\partial y_N} dy_N = R_{2N}^{\;k} \qquad (17)$$

and all the partial derivatives are evaluated at the k-th iteration cycle. The system of equations (18) is a system of 2N linear equations in 2N unknowns. Any of the standard methods such as the Gaussian elimination or the matrix inversion method can be used for its solution. The solution of the system will provide the values of $Q_1^{k+1}$, $y_1^{k+1}$, $\cdots$, $Q_i^{k+1}$, $y_i^{k+1}$, $\cdots$, $Q_N^{k+1}$, $y_N^{k+1}$ at the (k+1)th iteration cycle. The procedure will be repeated until the difference between the values of any variable in two consecutive iteration cycles falls below a tolerance value. The values found in the terminal iteration cycle will be taken as the values for the time stop (j+1) and the computations will be advanced to time step (j+2).

## VI. INPUT DATA

The numerical simulation model can provide values for the average water velocity, water discharge, and water depth in the Masonboro Inlet system on a continuing basis as the flow conditions keep on changing at the boundaries of the system. The model needs data on channel cross-sectional properties, the friction coefficient and boundary data. To test the model, data obtained for an 18-hour period on September 12, 1969, by the U. S. Army Corps of Engineers were used. During this period, measurements on water velocity, depth and discharge were made at a number of key locations.

Although the data could be used in a number of different ways to test the model, the following approach was considered appropriate in this application. An initial time, in this case 8:53 Eastern Standard Time (EST) was selected. At this time, there was little flow across the inlet proper as tidal flow was being reversed.

Flow rates and water depths were at all channel sections taken from the field data as initial conditions. Values for friction coefficients were estimated on a reasonable basis. The boundary conditions were given as the water depths at the boundary stations as function of time. The simulation model was used to compute values of velocity and discharge. The latter then could be compared to the measured field values.

Details and values of the input data are given as follows:

### Channel Parameters

The cross-sectional areas and wetted parameters at all stations are represented by linear algebraic equations. The equations for area as a function of depth y measured from mean low water is given as

$$A(I) = A_o(I) + A_1(I)\ y$$

where $A_o(I)$ and $A_1(I)$ are coefficients and I is an index identifying each station. Although the area is generally a complex function of depth, however, in the limited range of tidal variations, a linear function is sufficiently accurate to represent the area-depth relationship.

Similarly the wetted parameter can be represented by a linear equation

$$P(I) = P_o(I) + P_1(I) . y$$

where $P_o(I)$ and $P_1(I)$ are coefficients, P(I) is the wetted perimeter and I is an index identifying the station.

The inlet system channels are very wide in relation to their depths, so that the wetted perimeter is not significantly different from the top width. Therefore, the same values are used both for the top width and the wetted perimeter.

The coefficients $A_o(I)$, $A_1(I)$, $P_o(I)$, and $P_1(I)$ at each station are listed in Table 1. The channel cross-sections are shown on Figures 4 through 8.

TABLE 1

Coefficients in Expressions for Area and Wetted Perimeters

| Station | Coefficients for Area | | Coefficients for Wetted Perimeters | |
|---|---|---|---|---|
| | $A_o$ | $A_1$ | $P_o$ | $P_1$ |
| 1 | 6555 | 697 | 581 | 45 |
| 2 | 10400 | 958 | 926 | 19.8 |
| 3 | 2720 | 778 | 600 | 66.2 |
| 4 | 6510 | 1302 | 703 | 180.2 |
| 5 | 8640 | 1064 | 861 | 58 |
| 6 | 2690 | 1234 | 630 | 148.2 |
| 7 | 9500 | 1600 | 1603 | 2 |
| 8 | 5620 | 1110 | 1071 | 16.2 |
| 9 | 2390 | 722 | 481 | 94 |
| 10 | 21000 | 1374 | 1407 | 2 |
| 11 | 18930 | 1280 | 1309 | 2 |
| 12 | 9740 | 1396 | 1145 | 62 |
| 13 | 14400 | 1376 | 1251 | 46.2 |
| 14 | 11070 | 1300 | 983 | 128 |
| 15 | 14910 | 1695 | 1667 | 15 |
| 16 | 13678 | 1840 | 1845 | 2 |
| 17 | 13735 | 1920 | 1934 | 2 |
| 18 | 15280 | 2320 | 2336 | 2 |

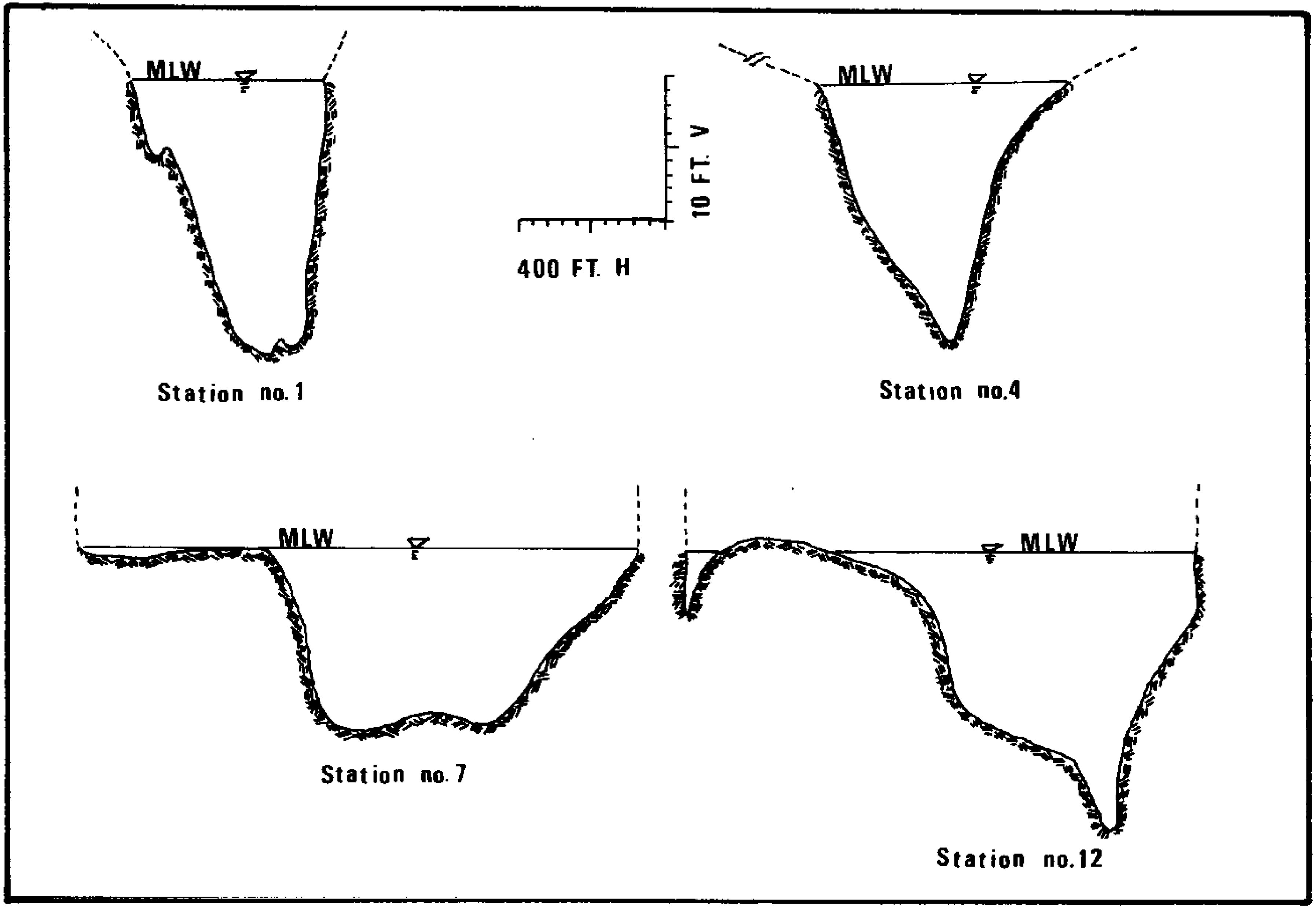


Fig. 4. Channel Cross-sections for Stations 1, 4, 7 and 12

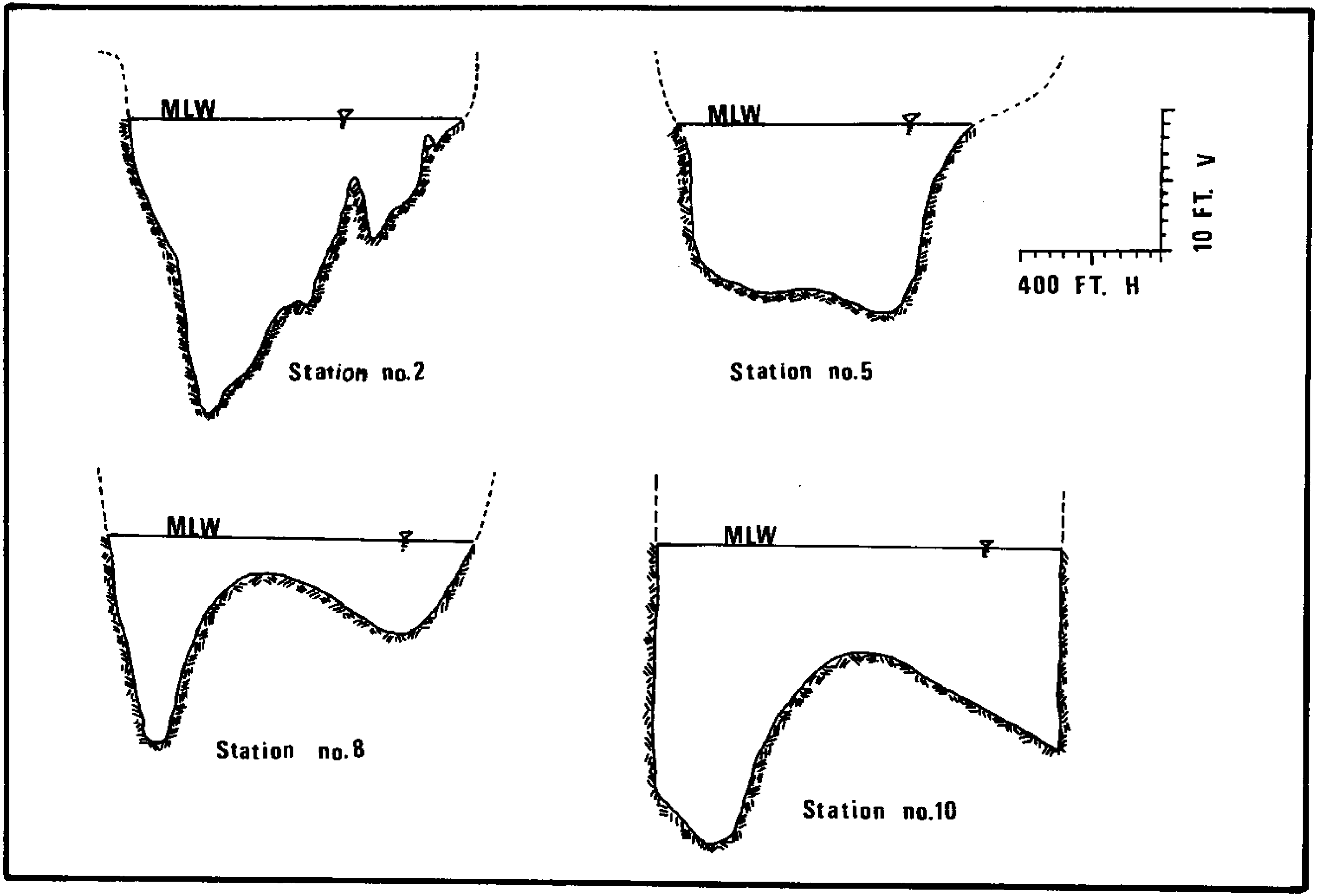


Fig.5 Channel Cross sections for Stations 2, 5, 8 and 10

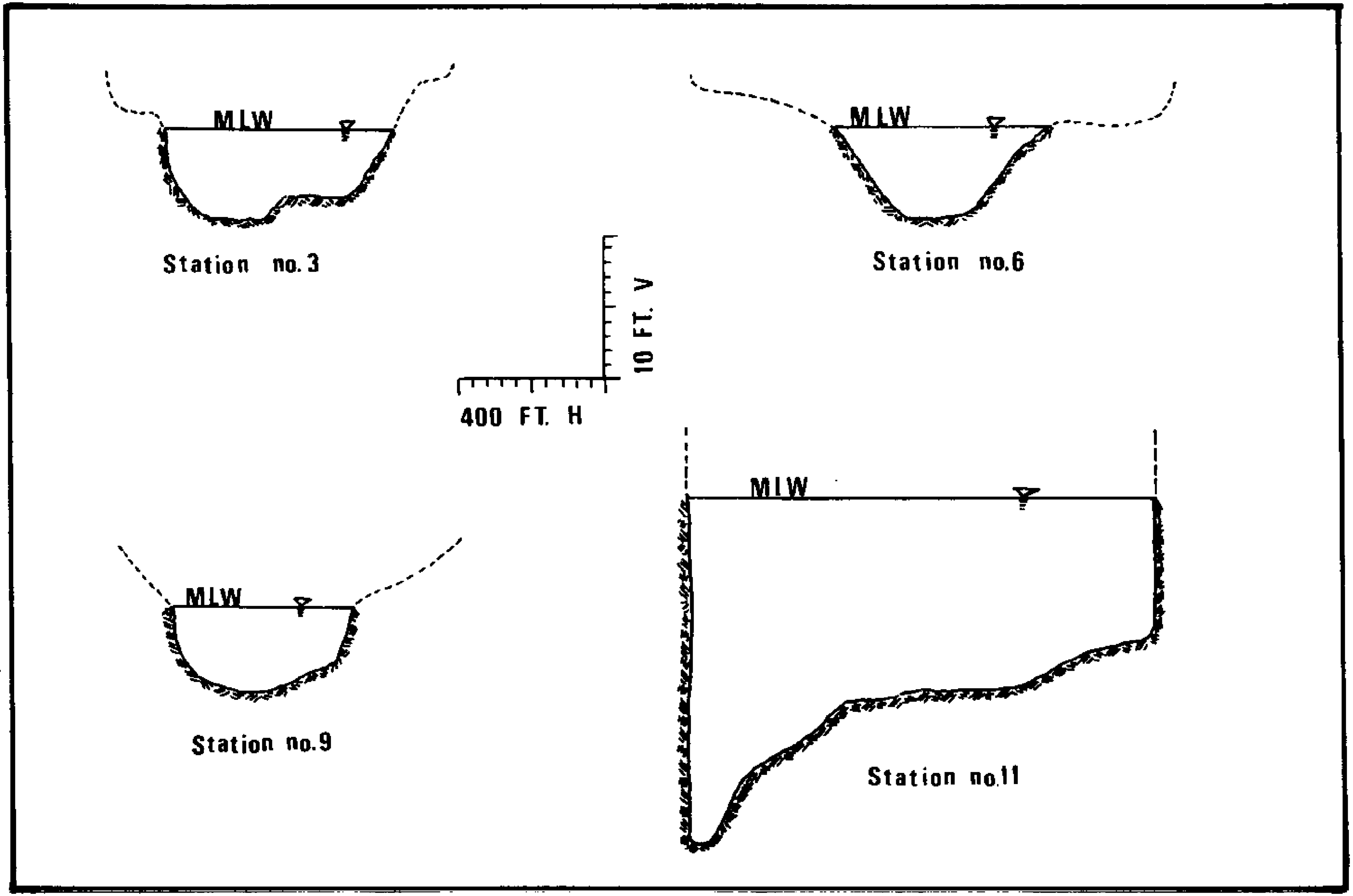


Fig. 6. Channel Cross-sections for Stations 3,6,9 and 11

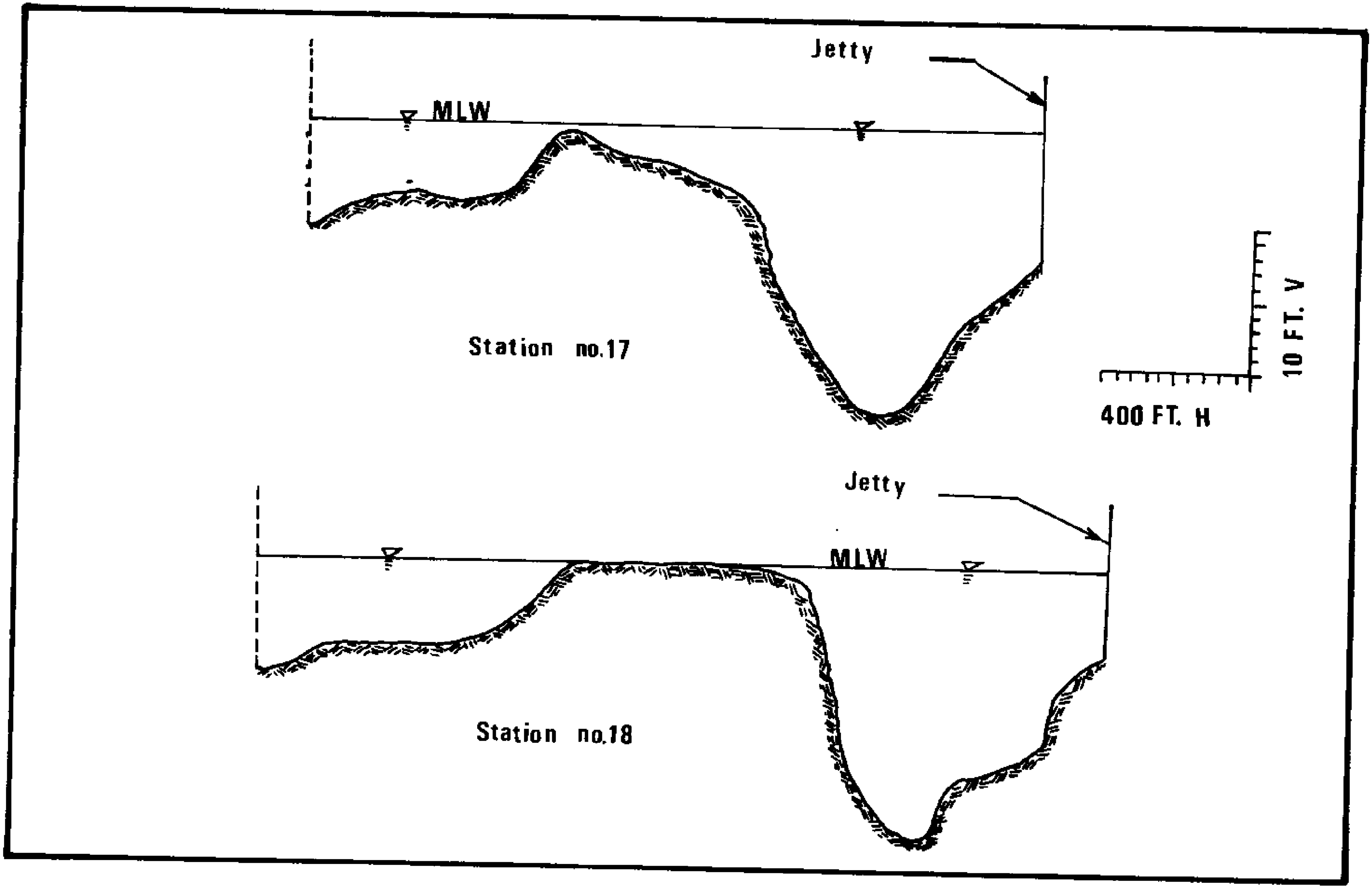


Fig. 7. Channel Cross-sections for Stations 17 and 18

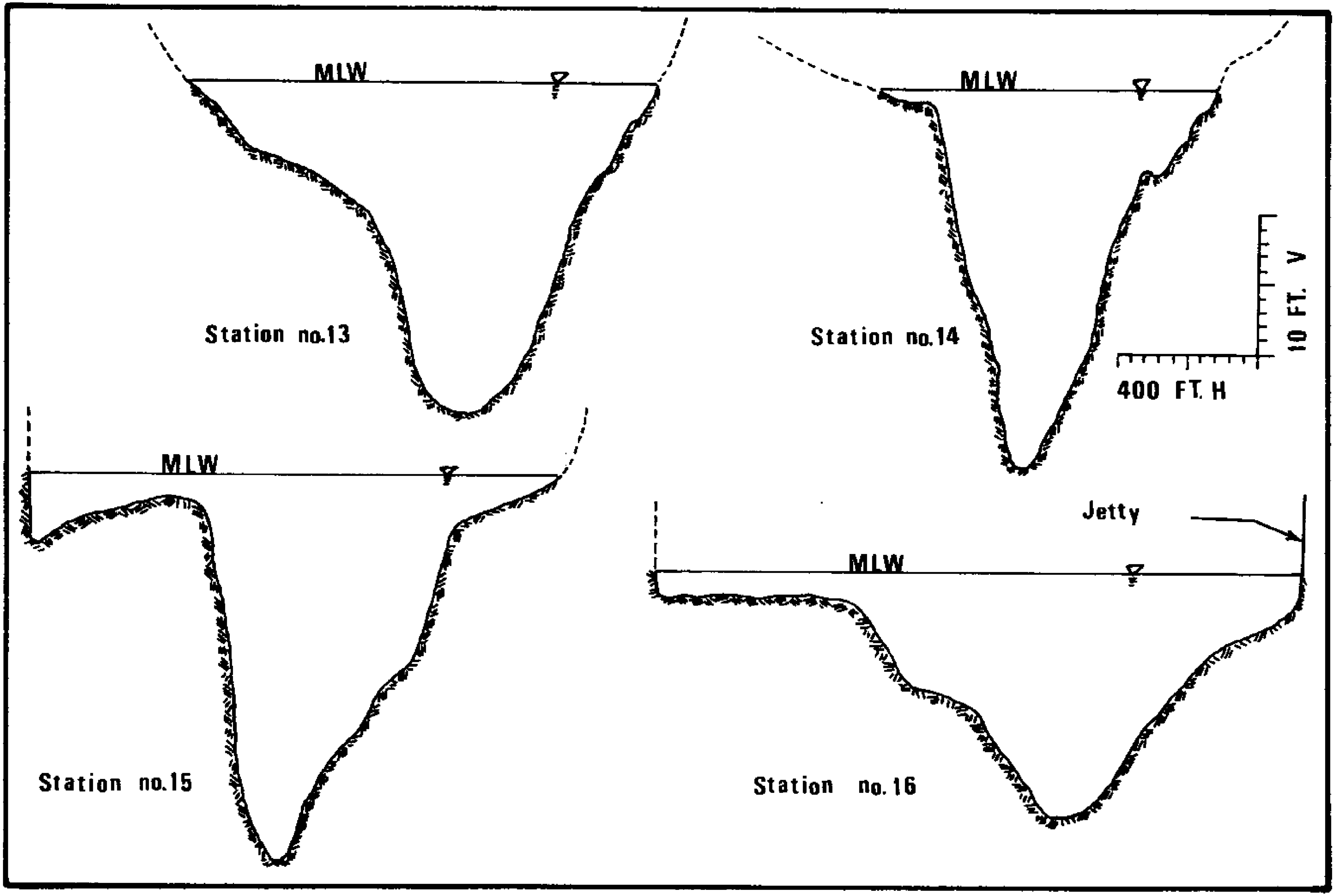


Fig. 8. Channel Cross-sections for Stations 13,14,15 and 16

## Friction Coefficients

For purposes of computing energy losses due to friction, Manning's formula was used. Although any other reasonable formula is acceptable, however, Manning's is widely used in practice. On the basis of past experience, a value of n = .020 was used for the inlet proper. This is the value associated with sandy channels. In the inland channels, a value of n = .040 was used to account for the higher losses due to vegetation at the banks and rougher boundaries. Although all computational results are sensitive to some extent to the choice of friction coefficients, small variations from the selected values would not significantly influence the results. Evaluation of "n" from field data would be very difficult because no steady state flow condition could be established.

## Stationing

For purposes of computation, it was assumed that flow from the inland channels arrive at the junction and flow out to sea. With this approach, the values of the distance variable x increases toward the sea. For convenience, x = 0 was taken at the junction point (Stations 7, 10, 11 and 12). Then x would be increasing positively toward the sea and would have increasing negative values going upstream into the inland channels. Whenever flows are opposite to the assumed direction, velocities will become negative. The values of x at each station together with initial values are given in Table 2.

## Initial Values

The computations are started at initial time t = 0 which is 8:53 EST. At this time, the values of the depth and discharge rate must be known. These values are listed in Table 2.

TABLE 2

INITIAL VALUES

| Station | Value of x (Miles) | Channel Bottom Elevation Below MLW (Ft.) | Initial Depth Above MLW (Ft.) | Initial Flow Rate (cfs) |
|---|---|---|---|---|
| 1 | -0.240 | -19.50 | 3.980 | +6171.0 |
| 2 | -0.710 | -22.00 | 4.34 | +8214.0 |
| 3 | -0.640 | - 6.40 | 4.13 | + 557.0 |
| 4 | -0.170 | -19.00 | 4.02 | +6171.0 |
| 5 | -0.430 | -18.00 | 4.05 | +8332.0 |
| 6 | -0.400 | - 6.40 | 4.02 | + 563.0 |
| 7 | -0.000 | -13.00 | 4.02 | +6171.0 |
| 8 | -0.140 | -14.00 | 4.02 | +8214.0 |
| 9 | -0.170 | - 6.00 | 4.02 | + 557.0 |
| 10 | -0.000 | -24.00 | 4.02 | +8214.0 |
| 11 | 0.000 | -24.00 | 4.02 | + 563.0 |
| 12 | 0.000 | -20.00 | 4.01 | +14948.0 |
| 13 | 0.090 | -24.00 | 4.01 | +14948.0 |
| 14 | 0.190 | -28.00 | 4.00 | +14948.0 |
| 15 | 0.280 | -28.00 | 4.00 | +14948.0 |
| 16 | 0.380 | -18.00 | 4.00 | +14948.0 |
| 17 | 0.470 | -19.00 | 4.00 | +14948.0 |
| 18 | 0.570 | -20.00 | 4.00 | +14948.0 |

Boundary Data

The boundary data are given as values of water surface elevation at the boundary stations as function of time. These values are listed in Table 3.

TABLE 3

FIELD BOUNDARY DATA

| Time (EST) | Water Surface Elevation Above MLW In (Ft.) Station (1) | Station (2) | Station (3) | Station (4) |
|---|---|---|---|---|
| 8.880 | 4.000 | 4.300 | 4.100 | 4.000 |
| 8.970 | 3.950 | 4.250 | 4.000 | 3.920 |
| 9.470 | 3.650 | 3.900 | 3.770 | 3.490 |
| 9.970 | 3.250 | 3.500 | 3.440 | 2.950 |
| 10.470 | 2.800 | 3.150 | 3.240 | 2.450 |
| 10.770 | 2.500 | 2.800 | 2.850 | 2.150 |
| 11.000 | 2.250 | 2.650 | 2.680 | 1.630 |
| 11.500 | 1.800 | 2.250 | 2.250 | 1.200 |
| 11.900 | 1.520 | 1.900 | 2.140 | 0.800 |
| 12.010 | 1.450 | 1.850 | 1.990 | 0.730 |
| 12.400 | 1.150 | 1.550 | 1.580 | 0.480 |
| 12.500 | 1.1000 | 1.470 | 1.480 | 0.450 |
| 13.000 | 0.800 | 1.070 | 1.230 | 0.310 |
| 13.500 | 0.600 | 0.780 | 0.900 | 0.160 |
| 14.000 | 0.500 | 0.600 | 0.600 | 0.260 |
| 14.330 | 0.500 | 0.700 | 0.100 | 0.380 |
| 14.500 | 0.500 | 0.730 | 0.250 | 0.500 |
| 15.000 | 0.6500 | 0.970 | 0.700 | 0.850 |
| 15.500 | 1.000 | 1.250 | 1.000 | 1.240 |
| 16.000 | 1.500 | 1.750 | 1.400 | 1.900 |
| 16.500 | 2.050 | 2.150 | 1.800 | 2.330 |
| 17.000 | 2.500 | 2.630 | 2.300 | 2.760 |
| 17.500 | 2.900 | 3.080 | 2.700 | 3.290 |
| 18.000 | 3.300 | 3.450 | 3.000 | 3.550 |
| 18.500 | 3.600 | 3.750 | 3.300 | 3.850 |
| 19.000 | 3.900 | 4.040 | 3.700 | 4.100 |
| 19.500 | 4.000 | 4.220 | 3.900 | 4.300 |
| 20.000 | 4.100 | 4.430 | 4.080 | 4.400 |
| 20.500 | 4.050 | 4.400 | 4.100 | 4.150 |
| 21.000 | 3.900 | 4.200 | 3.850 | 3.900 |

## VII. RESULTS AND CONCLUSIONS

A numerical simulation model is used to predict flow in the Masonboro Inlet system. Channel geometrical data, and initial and boundary dary must be introduced as input to the model. The model was tested with a set of field data consisting of the measured values of the stage and discharge at the boundaries of the system. The values of the stage were given as input to the model and the model produced values for the discharge which then could be compared to the measured values.

The following are the principal factors which introduce discrepancies between the measured and computed values. (1) There is no definite boundary for the inlet channel as it diverges into the sea. In this study, the boundaries were delineated along what appeared to be reasonable limits. It was expected that flow taking place outside the delineated boundaries did not constitute a significant proportion of the flow. (2) There is considerable uncertainty in the selection of initial values because the water is in unsteady motion. It would be desirable to introduce instantaneous values at all locations as initial values. However, this would be very difficult to implement in the field. (3) The values of the friction coefficients are not known with precision. Some alterations to the computed results could be made by changing these values.

The values of the computed results are plotted on Figures 9 through 14. The computed values of discharge are in very good agreement with the measured values at most of the stations. Under the circumstances, it is believed that better agreement between the measured and computed values could not be expected.

This study indicates that the simulation model used is a very powerful tool for the analysis of very complex flow situations. The results obtained could not have been obtained by analytical methods of classical mathematics.

The model can be used to predict the changes in flow regimes of inlets brought about by natural or man-made causes, and could serve as a very useful management tool for estuarine waters.

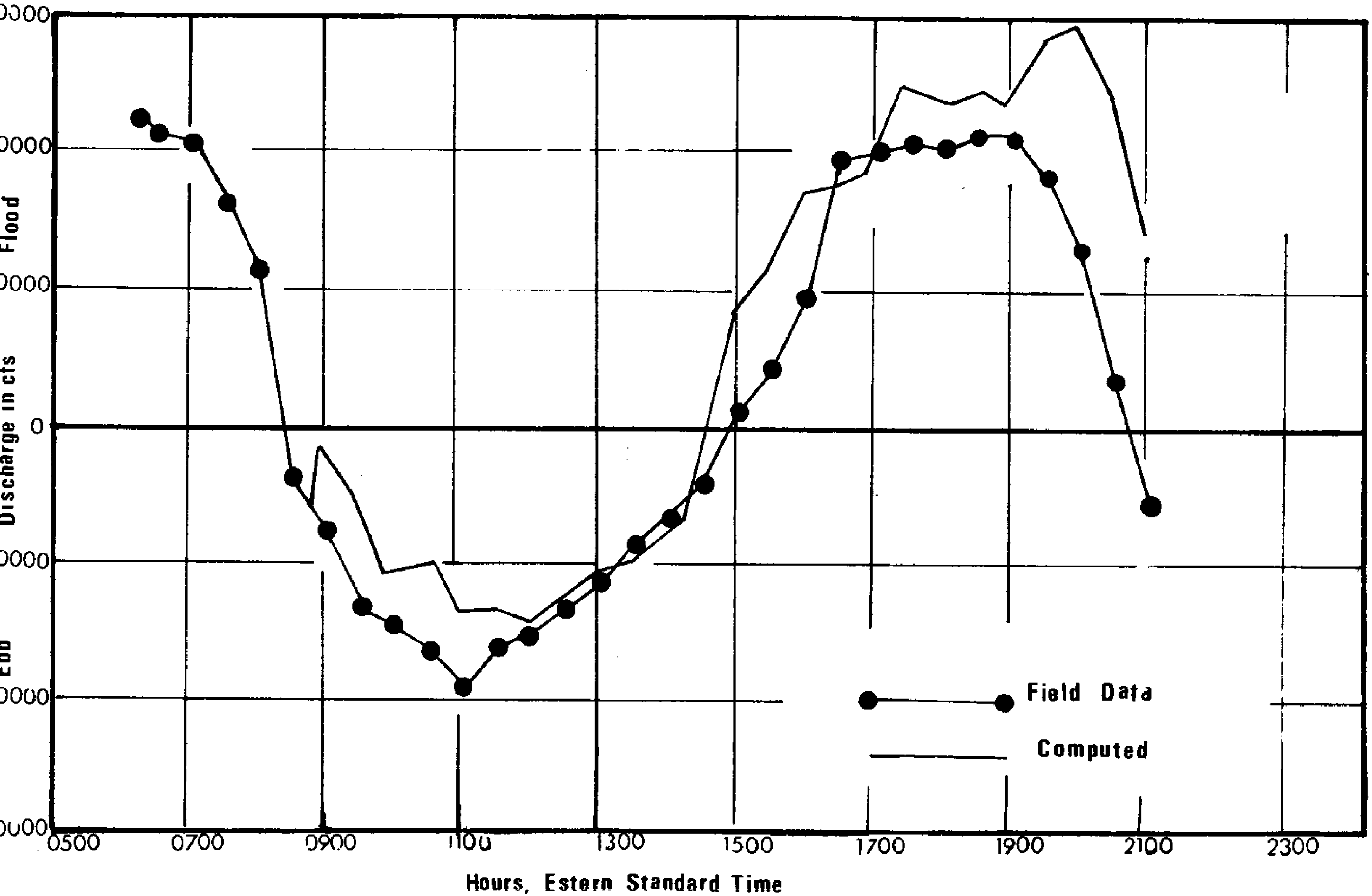


Fig. 9. Time-discharge graph for Station No.1

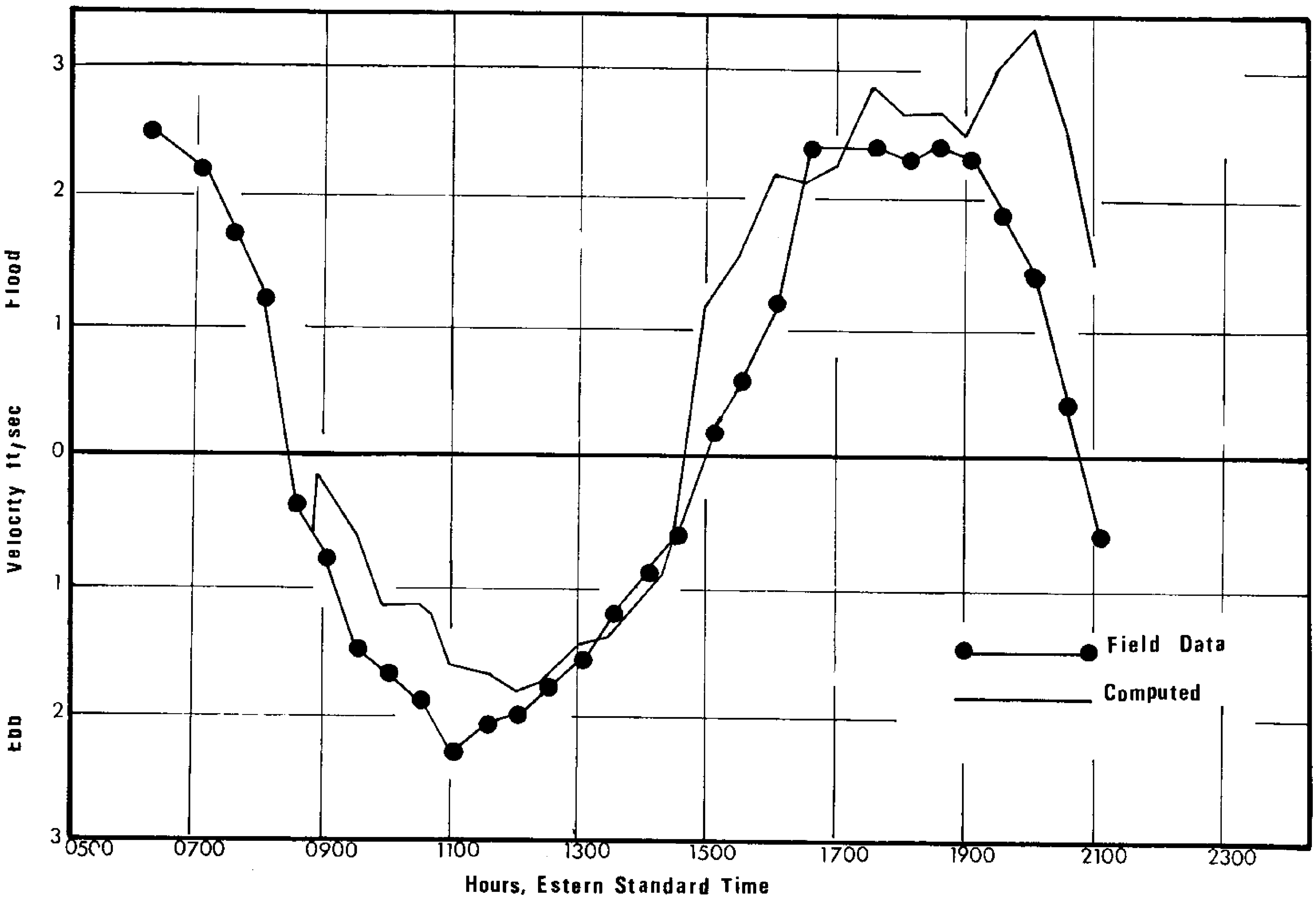


Fig.10. Time-Velocity graph for Station No.1

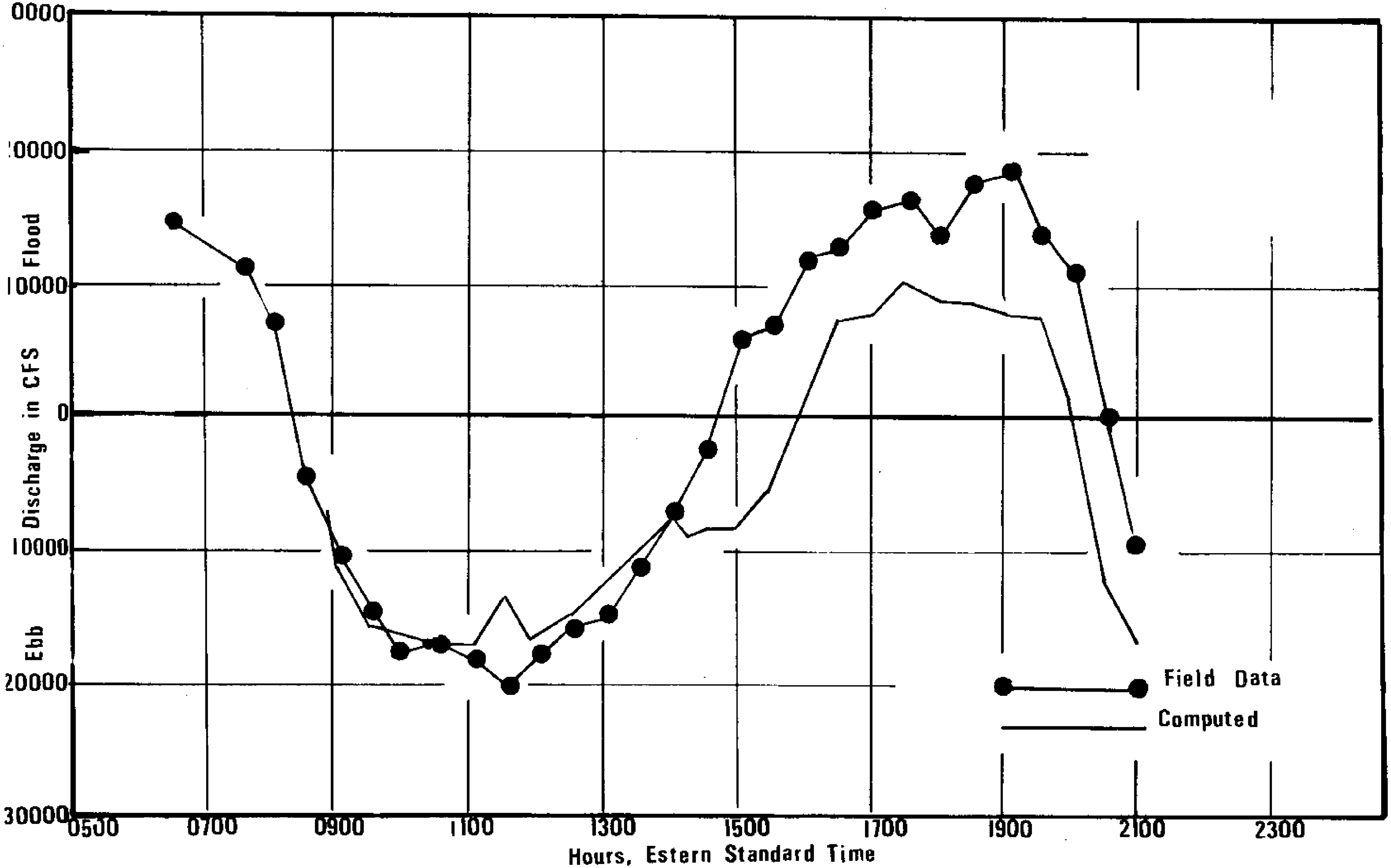


Fig. 11. Time-discharge graph for Station No.2

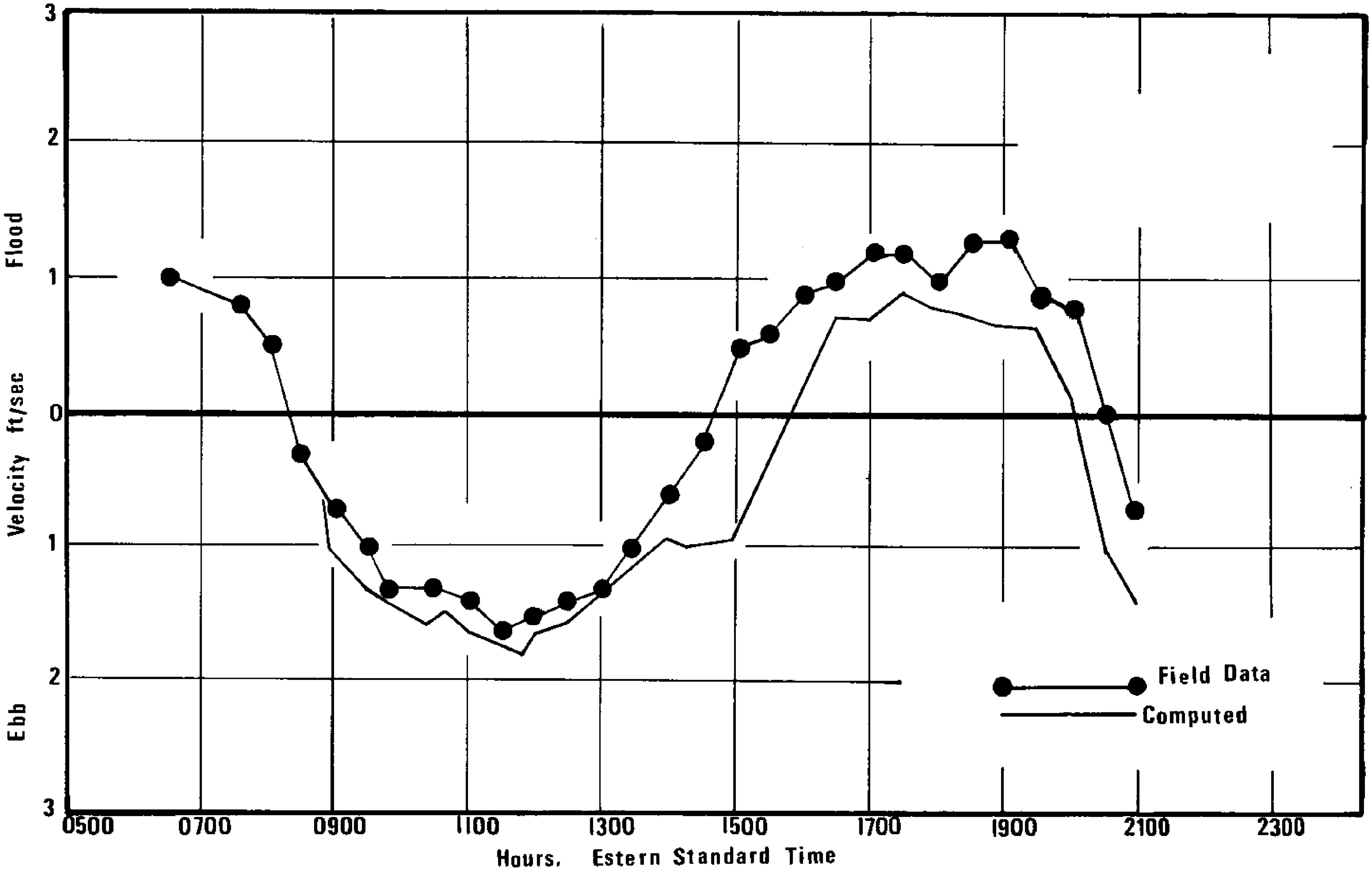


Fig. 12. Time-velocity graph for Station No.2

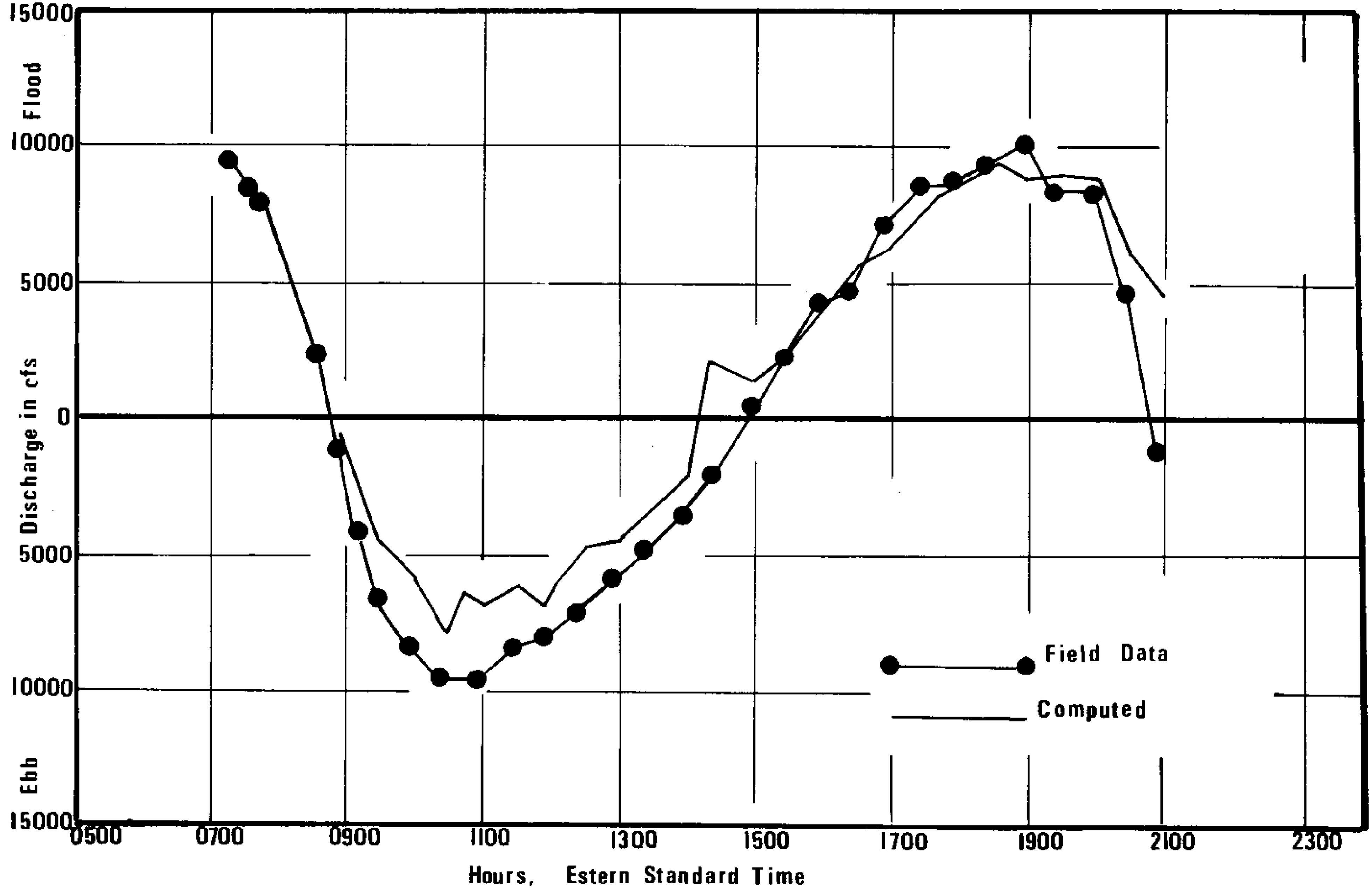


Fig. 13. Time-discharge graph for Station No.3

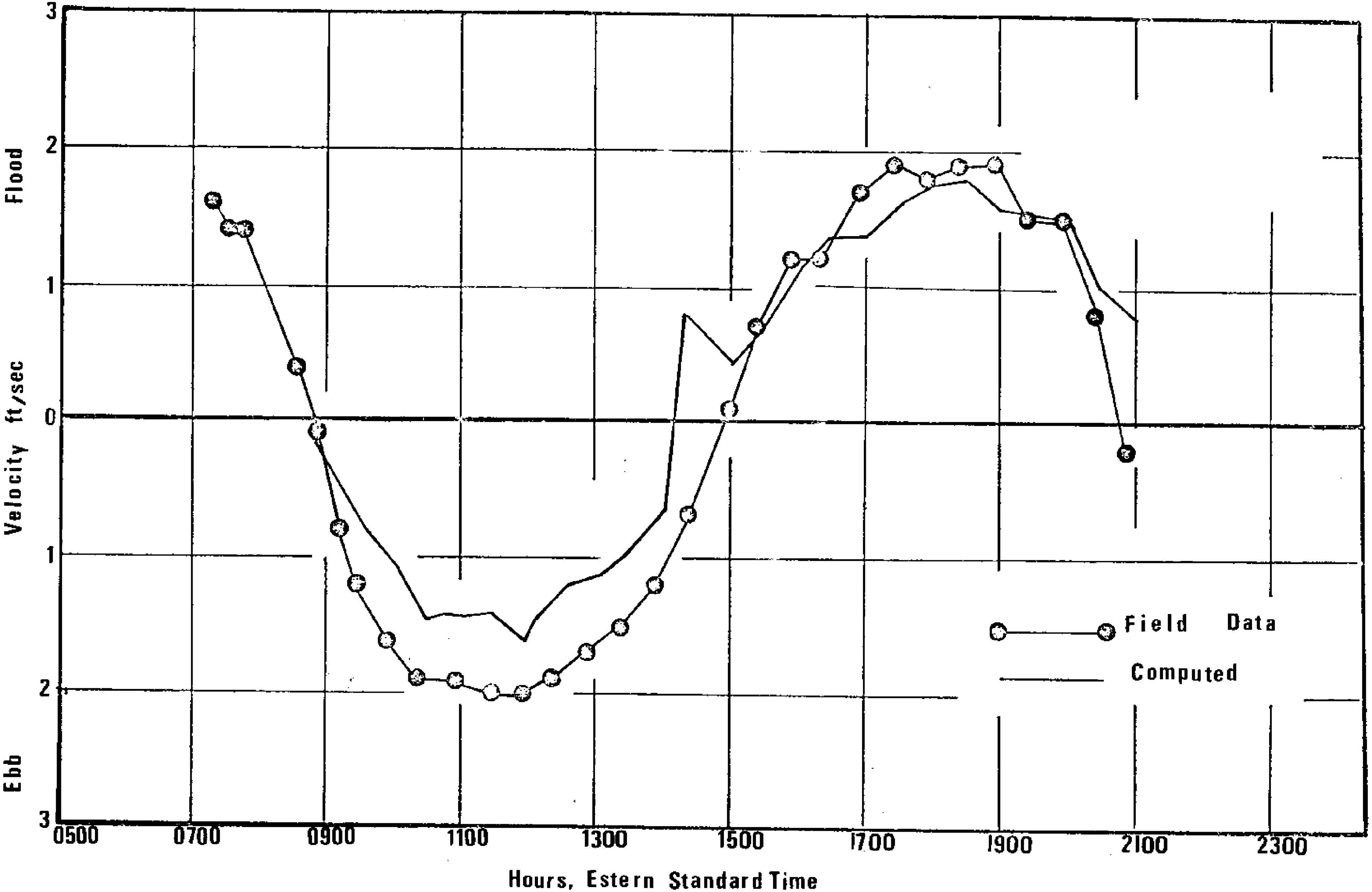


Fig. 14. Time – Velocity graph for Station No.3

## REFERENCES


1. Abbott, M. B. and Ionescu, F., "On the Numerical Computation of Nearly Horizontal Flows," Journal of Hydraulic Research, Netherlands, Vol. 5, 1967, pp. 98-117.

2. Amein, M., "An Implicit Method for Numerical Flood Routing," Water Resources Research, Vol. 4, No. 4, 1968, pp. 719-726.

3. Amein, M. and Fang, C. S., "Implicit Flood Routing in Natural Channels," Journal of the Hydraulics Division, Proc. ASCE, Vol. 96, No. HY12, 1970, pp. 2481-2500.

4. Amein, M., "Numerical Simulation of Unsteady Flows in Rivers and Reservoirs," Technical Report to U. S. National Weather Services, Office of Hydrology, NOAA, 1972.

5. Chow, V. T., "Open Channel Hydraulics," McGraw-Hill Book Company, Inc., New York, 1959.

6. Gilcrest, B. R., "Flood Routing," in Engineering Hydraulics, H. Rouse, Ed., John Wiley, New York, 1950, pp. 635-710.

7. Henderson, F. M., Open Channel Flow, McMillan Company, New York, 1966.

8. Isaacson, E., "Fluid Dynamical Calculations," Numerical Solution of Partial Differential Equations, Academic Press, New York, 1966, pp. 35-49.

9. Massau, J., "Memoire sur l'integration graphique des equations sux derivations partielles," Ann. Ingenieurs Sortis Gand, 12, 1889, pp. 185-444.

10. Kamphuis, J. W., Discussion on "Implicit Flood Routing in Natural Channels," by Amein and Fang, Journal of the Hydraulics Division, Proc. ASCE, Vol. 97, No. HY5, 1971, pp. 761.

11. Contractors, D. N. and Wiggert, J. W., Discussion on "Implicit Flood Routing in Natural Channels," by Amein and Fang, Journal of the Hydraulics Division, Proc. ASCE, Vol. 97, No. HY5, 1971, pp. 762.

12. Franz, D. D., Discussion on "Implicit Flood Routing in Natural Channels," by Amein and Fang, Journal of the Hydraulics Division, Proc. ASCE, Vol. 97, No. HY9, 1971, pp. 1552-1555.

13. Fread, D. L., Discussion on "Implicit Flood Routing in Natural Channels," by Amein and Fang, Journal of the Hydraulics Division, Proc. ASCE, Vol. 97, No. HY7, 1971, pp. 1156-1159.

14. Liggett, J. A. and Woolhiser, D. A., "Difference Solution of the Shallow-Water Equation," Journal of the Engineering Mechanics Division, ASCE, Vol. 98, No. EM2, 1967, pp. 39-71.

15. Lai, C., "Computation of Transient Flows in Rivers and Estuaries by the Multiple Reach Implicit Method," Geol. Survey Prof. Paper 575-B, U. S. Government Printing Office, Washington, D. C., 1967.

16. O'Loughlin, E. M. and Short, D. L., Discussion on "Implicit Flood Routing In Natural Channels" by M. Amein and C. S. Fang, Journal of the Hydraulics Division, ASCE, Vol. 97, No. HY10, 1971, pp. 1771-1774.

17. Stoker, J. J., "Water Waves," Intersciences Publishers, New York, 1957.

University of North Carolina Sea Grant Publications*

UNC-SG-72-01. Lyman, J. and W. Rickards. University of North Carolina Sea Grant Prog Annual Report, 1 July 1970 - 30 June 1971.

UNC-SG-72-02. Wurfel, S.W., ed. Attitudes regarding a law of the sea convention to establish an international seabed regime.

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UNC-SG-72-09. Porter, H.J. and J. Tyler. Sea shells common to North Carolina.

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UNC-SG-72-13. Schoenbaum, T.J. Public rights and coastal zone management.

UNC-SG-72-14. Tung, C.C. and N.E. Huang. Some statistical properties of wave-current force on objects.

UNC-SG-72-15. Tung, C.C. and N.E. Huang. Wave-current force spectra.

UNC-SG-73-01. Wurfel, S.W. The surge of sea law.

UNC-SG-73-02. Williams, A.B., G.S. Posner, W.J. Woods, and E.E. Deubler, Jr. A hydrographic atlas of larger North Carolina sounds.

UNC-SG-73-03. Gerhardt, R.R., J.C. Dukes, J.M. Falter, and R.C. Axtell. Public opinion on insect pest management in coastal North Carolina.

UNC-SG-73-04. Rickards, W.L. A bibliography of artificial reefs and other man-made fish attractants.

UNC-SG-73-05. Tyler, J., M. McKenzie, and D. King. To catch a million fish.

UNC-SG-73-06. Graetz, K. Seacoast plants of the Carolinas for conservation and beautification.

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UNC-SG-73-09. Rhyne, C.F. Field and ecological studies of the systematics and ecology of Ulva curvata and Ulva rotundata.

UNC-SG-73-10. Kuenzler, E.J., A.F. Chestnut and C.M. Weiss. The structure and functioning of brackish water ecosystems receiving treated sewage effluent III, 1971-1972.

UNC-SG-73-11. Dahle, E. The continental shelf lands of the United States: mineral resources and the laws affecting their development, exploitation and investment potential.

UNC-SG-73-12. Schwartz, F.J. and A.F. Chestnut. Hydrographic atlas of North Carolina estuarine and sound waters, 1972.

UNC-SG-73-13. Lyman, J. and W. Rickards. University of North Carolina Sea Grant Program, Annual Report, 1 July 1971 - 31 December 1972.

UNC-SG-73-14. Broome, S.W., W.W. Woodhouse, and E.D. Seneca. An investigation of propagation and the mineral nutrition of Spartina alterniflora.

UNC-SG-73-15. Amein, M. Computation of flow through Masonboro Inlet, North Carolina.

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