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WAVE-CURRENT FORCE SPECTRA

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INTRODUCTION

When wave propagates into a region of local current, due to interactions between the wave and the current, wave characteristics change. That is, if the current is in the direction of the wave, wave amplitude decreases and its length increases, but if the current opposes the wave, the wave becomes steeper and shorter (Longuet-Higgins and Stewart, 1961). In a random wave field, component amplitude and wave length experience similar changes resulting in modification of wave frequency and wave-number spectra (Huang,et. al.,1972). It is expected therefore that fluid force, being directly related to fluid field kinematics according to the Morrison's formula, would be similarly affected by the presence of current.

In this study, the spectra of fluid force on elements at arbitrary depths of a cylinder, the total force on the cylinder and its statical moment and the same on two cylinders placed in tandem, in a random wave field under the action of current are examined. To demonstrate the influence of wave-current interactions on these quantities, for all cases considered, comparisons are made between the cases when current is simply superimposed on waves and when wave-current interactions are included.

FLUID FORCE ON SINGLE CYLINDER

Consider a cylinder of unit diameter, immersed in a random wave field in deep water, under the action of a steady current uniformly distributed in depth. Examined in this section is the influence of wave-current interactions on the spectra of the fluid force exerted on elements of the cylinder at arbitrary depth, the total force on the cylinder and the statical moment of the total force about a point at mean water level.

In evaluating fluid force, the Morrison's formula is used. That is (Malhotra and Penzien, 1970),

$$F(t) = C_D V(t)|V(t)| + C_M a(t) \tag{1}$$

in which, $F(t)$ is the fluid force on an element of unit length of the cylinder at arbitrary depth z, $C_D = \rho K_D D$ and $C_M = \rho K_M \frac{\pi D^2}{4}$, with ρ, density of water, D, diameter of cylinder and $K_D = 0.5$ and $K_M = 1.4$, the drag and inertia coefficients. In equation 1, $a(t)$ and $V(t)$ are fluid particle acceleration and velocity evaluated at the location of the element with $V(t) = v(t) + U$, $v(t)$ being the oscillatory part of fluid velocity corresponding to wave motion and U is current speed.

In subsequent computation of fluid force spectra, those of fluid field kinematics are required which in turn depend on surface wave spectrum. These are summarized briefly. It was shown (Huang, et. al., 1972) that under the action of a steady current, the frequency spectrum of surface waves of a stationary gravity wave field in deep water is given by

$$\phi(n) = \frac{4\phi^*(n)}{\left[1 + \left(1 + \frac{4Un}{g}\right)^{1/2}\right]\left[\left(1 + \frac{4Un}{g}\right)^{1/2} + \left(1 + \frac{4Un}{g}\right)\right]} \tag{2}$$

in which n is total frequency, g is gravitational acceleration and $\phi(n)$ and $\phi^*(n)$ are respectively the frequency spectrum of surface waves with and without the influence of current and $\phi^*(n)$ is taken to be

$$\phi^*(n) = \frac{\alpha g^2}{n^5} \exp\left(-\beta\left(\frac{n_o}{n}\right)^4\right) \qquad (3)$$

the Kitaigorodskii - Pierson - Moskowitz spectrum, in which α and β are non-dimensional constants equal to 0.81×10^{-2} and 0.74 respectively and $n_o = g/W$ with W the mean wind speed.

The spectra of fluid particle velocity and acceleration at arbitrary depth are

$$\phi_{VV}(n) = n^2\phi(n)e^{-2kz} \qquad (4)$$

and

$$\phi_{aa}(n) = n^4\phi(n)e^{-2kz} \qquad (5)$$

in which k is the wave-number and z is the distance of the element from mean water. In deep water, under the influence of current, the wave-number k is related to wave frequency n, by the generalized dispersive relationship (Huang,et. al.,1972).

$$k = \frac{4n^2/g}{\left[1 + \left(1 + \frac{4Un}{g}\right)^{1/2}\right]^2}. \qquad (6)$$

From equation 6, it is seen that when current is in the direction of (opposite to) wave, that is, when $U > 0$ ($U < 0$), the value of k is less (larger) than that when there is no current, indicating that positive (negative) current lengthens (shortens) the wave.

The spectrum of element force at arbitrary level, in a homogeneous Gaussian sea, was derived by Borgman (1965). With fair degree of accuracy, to the first order of approximation, the spectrum of element fluid force is

$$\phi_{FF}(n) = \{16C_D^2\sigma_v^2[Z(\gamma) + \gamma P(\gamma)]^2 + C_M^2n^2\}n^2\phi(n)e^{-2kz} \qquad (7)$$

in which $\sigma_v = [\int_n \phi_{VV}(n)dn]^{1/2}$ is the standard deviation of the fluid particle velocity at the position of the element, $\gamma = U/\sigma_v$ is a parameter, $Z(\gamma) = \frac{1}{\sqrt{2\pi}}\exp(-\frac{\gamma^2}{2})$ and $P(\gamma) = \int_0^\gamma Z(x)dx$ is the error function (Papoulis, 1965).

In figures 1 and 2 spectra of element forces at z = 0 and z = 30 ft. are presented for wind speed W = 20 mi./hr. for cases when U = 0, U = 2 ft./sec. and U = -2ft./sec. Distinction is further made between the cases when wave-current interactions are considered and ignored. In the former case $\phi_{FF}(n)$ is computed from equation 7 utilizing equations 2 to 6, while in the latter case $\phi(n)$ is replaced by $\phi^*(n)$ and $k = n^2/g$ is used in place of equation 6. Thus, when interactions are neglected, both positive and negative currents give rise to the same force spectrum. It is noted that when interactions are considered, negative current gives rise to an increase in the spectrum but positive current causes the same to decrease as compared with the case when no interactions are present. Comparing figures 1 and 2, it is seen that the spectrum for z = 30 ft. is of narrower band and spectral peak is at lower frequency than that for z = 0. This is because waves of higher frequencies are shorter and do not penetrate deep into the water. The most striking feature in comparing figures 1 and 2 is that interactions have more pronounced effect on force spectrum of the element at the surface $z = 0$ than on that at 30 ft. beneath the surface in which case the effect of interactions is almost nil. This can be explained by referring to figure 3 where the peak of element force spectra is plotted against depth of the element for the various current conditions considered in figures 1 and 2.

Figure 3 shows that the peak values of the element force spectra decay with depth but at different rates for the different cases considered. When interactions are taken into account, although negative (positive) current increases (decreases) wave amplitude, it also increases (decreases) the decay rate, as dictated by equation 6, over (below) that when no interactions are considered in which case $k = n^2/g$. Thus, at the surface where no decay is involved, the effect of interactions manifest in wave amplitude, modification are strongly felt whereas below the surface, the effect of interactions on amplitude change is compensated by change in wave length or equivalently, decay rate, resulting in weaker effect of interactions on force spectrum.

The total force, which is the resultant of the element forces on the cylinder, is

$$F_T(t) = \int_0^\infty F(t)\,dz \qquad (8)$$

whose spectrum, to the same order of approximation as $\phi_{FF}(n)$, is shown in the Appendix to be

$$\phi_{F_T F_T}(n) = \{16C_D^2[\int_0^\infty \sigma_v(Z(\gamma) + \gamma P(\gamma))e^{-kz}dz]^2 + C_M^2 n^2/k^2\}n^2\phi(n). \qquad (9)$$

In figure 4, spectra of $F_T(t)$ under wind speed W = 20 mi./hr. and various current conditions are presented with and without interactions considered. It is immediately apparent that the effect of interactions on spectrum of total force is rather weak. While negative current still places the force spectrum higher than that when no interactions are considered, when current speed is positive, the effect of interactions is hardly noticeable. This is due to the fact that when interactions are considered, while positive current reduces wave amplitude, so does it lower decay rate causing the wave to penetrate deeper into the water giving rise to a total force almost equal to that when no interactions are considered. In the case of negative current, the increase in wave amplitude due to interactions overpowers increase in decay rate resulting in a total force still somewhat larger than that when interactions are ignored.

The statical moment of the total force about a point at mean water level, denoted by $M_F(t)$ is

$$M_F(t) = \int_0^\infty zF(t)dz \tag{10}$$

the spectrum of which, to the same order of approximation as $\phi_{FF}(n)$, derived in the Appendix, is

$$\phi_{M_F M_F}(n) = \{ 16C_D^2[\int_0^\infty z\sigma_v(Z(\gamma) + \gamma P(\gamma))e^{-kz}]^2 + C_M^2 n^2/k^4 \} n^2\phi(n). \tag{11}$$

Figure 5 shows the spectra of $M_F(t)$ under wind speed W = 20 mi./hr. and various current conditions with and without interactions considered.

Compared with $\phi_{F_T F_T}(n)$ in figure 4, interactions obviously have more effect on $\phi_{M_F M_F}(n)$. It is noted that instead of taking the moment of the total fluid force about a point on the bottom of the water, which is the usual overturning moment, a point on the mean water level is chosen as the moment center. This is because deep water case is used in this study for convenience. Thus, when interactions are considered, under positive (negative) current, the point of application of the total force is lowered (raised) due to increase (decrease) in wave length as compared with the case when no interactions are considered, rendering an increase (decrease) in the statical moment.

Finally, it is remarked that both $\phi_{F_T F_T}(n)$ and $\phi_{M_F M_F}(n)$, as $\phi_{FF}(n)$ at z = 30 ft., are of narrower band and have their peaks shifted to lower frequencies when compared with $\phi_{FF}(n)$ at z = 0. That the total force $F_T(t)$ and its statical moment $M_F(t)$ embody the contribution of all element forces F(t) along the cylinder, and that the spectra of element forces farther beneath the surface are of narrow band with peaks at lower frequency range, offer an explanation to the phenomenon observed herein.

FLUID FORCE ON TWO CYLINDERS

Consider two cylinders of equal diameter placed at distance S apart in line with the direction of the current and the waves. The element force Q(t), which is the sum of the two element forces on the cylinders at the same level, has its spectrum $\phi_{QQ}(n)$, shown in the Appendix, of the form

$$\phi_{QQ}(n) = \phi_{FF}(n) T(k,S) \qquad (12)$$

in which $T(k,S)$, a function of wave-number k and the spacing between the cylinders S, is termed the transfer function (Borgman, 1966) and is given by

$$T(k,S) = \frac{1 - \cos 2kS}{1 - \cos kS} = 4\cos^2 \frac{kS}{2} \qquad (13)$$

independent of z, the location of the elements below the mean water level. The transfer function, when used to obtain the frequency spectrum of the total element force on the two cylinders, must have its argument k expressed in terms of n, the frequency, by means of the dispersive relationship of equation 6 and is therefore current dependent.

In the Appendix, it is also shown that the spectrum $\phi_{Q_TQ_T}(n)$ of the total force $Q_T(t)$, which is the resultant of all element forces on the two cylinders and the spectrum $\phi_{M_QM_Q}(n)$ of $M_Q(t)$, the statical moment of the total forces of the two cylinders about a point on the mean water level, are respectively given by

$$\phi_{Q_TQ_T}(n) = \phi_{F_TF_T}(n)T(k,S) \qquad (14)$$

and

$$\phi_{M_QM_Q}(n) = \phi_{M_FM_F}(n)T(k,S) \, . \qquad (15)$$

The fact that the quantities $\phi_{QQ}(n)$, $\phi_{Q_TQ_T}(n)$ and $\phi_{M_QM_Q}(n)$ can be derived simply from $\phi_{FF}(n)$, $\phi_{F_TF_T}(n)$ and $\phi_{M_FM_F}(n)$, suggests that in order to study the influence of wave-current interactions on the spectra of $Q(t)$, $Q_T(t)$ and $M_Q(t)$, it suffices to examine the behavior of the transfer function under current.

Presented in figure 6 is the function $T(k,S)$ plotted against n, the frequency, for a specified value of $S = 30$ ft., for $U = 0$, $U = 2$ ft./sec. and $U = -2$ft./sec. When $U = 0$ and when no interactions are considered, in which case $k = n^2/g$, the transfer functions are the same. The function begins at a value equal to 4.0 at $n = 0$ and decreases to zero at about $n = 1.8$ rad./sec. and oscillates thereafter as n increases. When current speed is positive (negative) the wave number k decreases (increases) and the function, beginning at value of 4.0 at $n = 0$ decreases slower (faster) than the case when $U = 0$ and when no interactions are considered. Thus, in the frequency range of $n = 0$ to $n = 1.6$, it can be said that the influence of wave-current interactions on the spectra of element force and total force on two cylinders is lessened as compared with that on the same when only a single cylinder is involved. The same would be true in the case of total moment had the moment center been chosen at the bottom of the cylinder. Beyond $n = 1.6$ rad./sec., the transfer function rises and falls between 0.0 and 4.0 at varying frequencies depending on current conditions and whether interactions are considered or not. But as was pointed out by Borgman (1966) since most of the energy is contained in the lower frequency range which is therefore of most interest, the behavior of the transfer function and the spectra in higher frequency range is of no important practical significance.

CONCLUDING REMARKS

Based on the above study, the following conclusions can be made.

1. Wave-current interactions have more effect on the spectra of the element forces on a cylinder or cylinders near the surface than on those deeper in the water.

2. Wave-current interactions have more effect on the spectrum of statical moment of force on a cylinder or cylinders than on that of total force.
3. For cylinder spacings current conditions and frequency range of practical interest, as employed in this study, the effect of interactions on the spectra of element force, total force and statical moment of two cylinders in tandem is slightly reduced as compared with that on a single cylinder provided that statical moment is understood to be taken about a point at the bottom of the water.

In conclusion, it can be said that considering the findings of this investigation together with those of the previous study (Tung and Huang, 1973), the phenomenon of wave-current interaction is an important factor to be considered in the evaluation of fluid forces.

ACKNOWLEDGEMENT

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APPENDIX

In this Appendix, the spectra of element force, total force and its statical moment about a point at mean water level, for a single cylinder and an array of cylinders are derived. The procedure used follows closely that of Borgman (1966). The detailed expressions of the spectra obtained here necessarily differ from those of Borgman since no current was involved there. As will become clear subsequently, all the spectra sought here can be deduced from the expressions of the spectra of total force and its statical moment for an array of cylinders. Thus derivation will begin with these quantities.

Consider an array of cylinders, the number of which is denoted by I, equally spaced S units apart arranged in the direction of wave propagation, the x axis. Let C(t) represent either the total force or the statical moment about at point on the mean water level z = 0 with z an axis directed vertically downward. Then

$$C(t) = \sum_{i=1}^{I} \int_{o}^{\infty} q(z)F_i(z,t)dz \qquad \text{(A-1)}$$

in which q(z) = 1.0 if C(t) is the total force and q(z) = z if C(t) is the statical moment. The term $F_i(z,t)$ is used to denote the force per unit length at time t at location z on the i^{th} cylinder whose horizontal coordinate is x_i.

To obtain the spectrum of C(t), first consider its correlation function. Thus form the product of C(t) and C(t + τ) from equation A-1 and take the expected value of the product, giving

$$
\begin{aligned}
R_{CC}(\tau) &= E[C(t)C(t+\tau)] \\
&= E[\sum_{i=1}^{I} \int_{o}^{\infty} q(z)F_i(z,t)dz \sum_{j=1}^{I} \int_{o}^{\infty} q(z')Fj(z',t+\tau)dz'] \\
&= \sum_{i=1}^{I} \sum_{j=1}^{I} \int_{o}^{\infty} \int_{o}^{\infty} q(z)q(z')E[F_i(z,t)F_j(z',t+\tau)]dzdz' \\
&= \sum_{i=1}^{I} \sum_{j=1}^{I} \int_{o}^{\infty} \int_{o}^{\infty} q(z)q(z')R_{F_iF_j}(z,z',\tau)dzdz' \qquad \text{(A-2)}
\end{aligned}
$$

in which $R_{cc}(\tau)$ denotes the correlation function of C(t), E[.] is the expected value of the quantity enclosed in the bracket and $R_{F_iF_j}(z, z',\tau)$ is the cross-correlation function of $F_i(z,t)$ and $F_j(z',t)$.

It was shown by Borgman (1965) that

$$
\begin{aligned}
R_{F_iF_j}(z,z',\tau) &= C_D^2\sigma_v^2(z)\sigma_v^2(z')G\left(\frac{R_{V_iV_j}(z,z',\tau)}{\sigma_v(z)\ \sigma_v(z')}\right) \\
&+ 4C_DC_M\{\sigma_v(z)[Z(\gamma(z)) + \gamma(z)P(\gamma(z))]R_{V_ia_j}(z,z',\tau) \\
&\sigma_v(z')[Z(\gamma(z')) + \gamma(z')P(\gamma(z'))]R_{a_iV_j}(z,z',\tau)\} + \\
&C_M^2R_{a_ia_j}(z,z',\tau) \qquad \text{(A-3)}
\end{aligned}
$$

in which $\sigma_v(z)$ is the standard deviation of fluid particle velocity, which, being a stationary process in x, is only a function of z, and $\gamma(z) = U/\sigma_v(z)$. The R's in equation A-3 are the cross-correlation functions of the quantities in the subscripts. The function G(.) was derived by Borgman (1965). Due to the complicated form of the function and for convenience of performing the Fourier transform subsequently to obtain the spectrum of C(t), it was suggested that the function G(.) be expanded into a series. By taking only the first term of the series expansion and omitting the D.C. term,

$$G\left(\frac{R_{V_iV_j}(z,z',\tau)}{\sigma_v(z)\sigma_v(z')}\right) = \frac{R_{V_iV_j}(z,z',\tau)}{\sigma_v(z)\sigma_v(z')}$$

$$16[Z(\gamma(z)) + \gamma(z)P(\gamma(z))][Z(\gamma(z')) + \gamma(z')P(\gamma(z'))] \qquad \text{(A-4)}$$

Following the same argument advanced by Borgman (1966), it can be verified that the middle term in equation A-3 does not contribute to $R_{CC}(\tau)$ and therefore can be omitted when substituted into equation A-2. Taking the Fourier transform of both sides of equation A-2, one obtains the spectrum $\phi_{CC}(n)$ of C(t)

$$\phi_{cc}(n) = \sum_{i=1}^{I} \sum_{j=1}^{I} \int_{o}^{\infty} \int_{o}^{\infty} q(z)q(z')\{ C_M^2\phi_{a_ia_j}(z,z',n)$$

$$+ 16C_D^2\sigma_v(z)\sigma_v(z')(Z(\gamma(z)) + \gamma(z)P(\gamma(z)))$$

$$(Z(\gamma(z')) + \gamma(z')P(\gamma(z')))\phi_{V_iV_j}(z,z',n)\}dzdz' \qquad \text{(A-5)}$$

in which

$$\phi_{V_iV_j}(z,z',n) = \phi(n)n^2\exp(-k(z + z'))\cos k(x_i-x_j) \qquad \text{(A-6)}$$

and

$$\phi_{a_ia_j}(z,z',n) = n^2\phi_{V_iV_j}(z,z',n) \qquad \text{(A-7)}$$

Substituting equations A-5 and A-6 into equation A-4 and rearranging, yields

$$\phi_{cc}(n) = \phi(n)\,\{\,[4C_D n \int_o^\infty q(z)\sigma_v(z)\;(Z(\gamma(z)) + \gamma(z)P(\gamma(z)))e^{-kz}dz]^2$$

$$+[C_M n^2 \int_o q(z)e^{-kz}dz]^2\}\sum_{i=1}^{I}\sum_{j=1}^{I} \cos k(x_i - x_j) \qquad \text{(A-8)}$$

in which the double summation was termed the transfer function by Borgman (1966) and was shown to be

$$\sum_{i=1}^{I}\sum_{j=1}^{I} \cos k(x_i - x_j) = \frac{1-\cos IkS}{1-\cos kS} \equiv T(k,S) \qquad \text{(A-9)}$$

for an array of cylinders with equal spacing S.

Equations A-8 and A-9 provide all the information required to determine the spectra considered in this study. Thus, for a single cylinder, $T(k,S) = 1$ and for the spectra of statical moment and total force, $q(z)$ is replaced by $q(z) = z$ and 1.0 respectively in equation A-8 giving the expressions $\phi_{M_F M_F}(n)$ in equation 9 and $\phi_{F_T F_T}(n)$ in equation 8 in the text. For the spectrum of element force, omit the integration operations in equation A-8 and $\phi_{FF}(n)$ of equation 7 of the text is obtained. It is noted that, for brevity, the argument z of $\sigma_v(z)$ and $\gamma(z)$ are omitted in these expressions. With the case of a single cylinder deduced from equation A-8, the expressions for the spectra of element force $\phi_{QQ}(n)$, total force $\phi_{Q_T Q_T}(n)$ and statical moment $\phi_{M_Q M_Q}(n)$ for two cylinders placed in tandem in equation 12, 14, 15 of the text follow immediately.

NOTATION

The following symbols are used in this report:

$a(t)$ = fluid particle acceleration;

$a_i(z,t)$ = fluid particle acceleration at (x_i,z) at time t;

$C(t)$ = generalized force on cylinders (Eq. A-1);

C_D, C_M = coefficients of drag force and inertia force (Eq. 1);

D = diameter of cylinders:

$E[.]$ = expected value of the quantity enclosed in the bracket;

$F(t)$ = fluid force on an element of unit length of a cylinder (Eq. 1);

$F_i(z,t)$ = fluid force on an element of unit length of a cylinder at coordinates (x_i,z) (Eq. A-1);

$F_T(t)$ = total fluid force on a cylinder (Eq. 8);

$G(.)$ = a function given in Eq. A-4;

g = gravitational acceleration;

I = number of cylinders in the array;

i = dummy index;

j = dummy index;

K_D, K_M = drag and inertia coefficients;

k = wave-number (Eq. 6);

$M_F(t)$ = statical moment of total fluid force on a cylinder about a point at mean water level (Eq. 10);

$M_Q(t)$ = Statical moment of total fluid force on two cylinders about a point at mean water level;

n = frequency;

$n_o = g/W$;

$P(\gamma) = \int_o^\gamma Z(x)dx$ the error function;

$Q(t)$ = fluid force on elements of unit length of two cylinders;

$Q_T(t)$ = total fluid force on two cylinders;

$q(z)$ = a function used in Eq. A-1;

$R_{a_i a_j}(z,z',\tau)$ = cross-correlation function of $a_i(z,t)$ and $a_j(z',t)$ (Eq. A-3);

$R_{a_i V_j}(z,z',\tau)$ = cross-correlation function of $a_i(z,t)$ and $V_j(z',t)$ (Eq. A-3);

$R_{cc}(\tau)$ = correlation function of $C(t)$ (Eq. A-2);

$R_{F_i F_i}(z,z',\tau)$ = cross-correlation function of $F_i(z,t)$ and $F_j(z',t)$ (Eq. A-2);

$R_{V_i a_j}(z,z',\tau)$ = cross-correlation functions of $V_i(z,t)$, $a_j(z',t)$ (Eq. A-3);

S = spacing between cylinders;

$T(k,S)$ = transfer function for an array of cylinders with equal spacing and diameter (Eqs. A-9, 12, 13, 14, 15);

t = time;

U = current speed;

$V(t)$ = fluid particle velocity;

$V_i(Z,t)$ = fluid particle velocity at (x_i,z) at time t;

$v(t) = V(t) - U$, oscillatory component of fluid particle velocity;

W = wind speed;

x_i = horizontal coordinate of i^{th} cylinder in an array;

$Z(\gamma) = \frac{1}{2\pi} \exp\left(-\frac{\gamma^2}{2}\right)$;

z,z' = depth of cylinder element;

α,β = constants in surface wave spectrum (Eq. 3);

$\gamma,\gamma(z) = U/\sigma_v(z)$;

ρ = density of water;

$\sigma_v,\sigma_v(z)$ = standard deviation of fluid particle velocity at depth z;

τ = time lag;

$\phi(n)$, $\phi^*(n)$ = frequency spectrum of surface waves with and without the influence of current (Eqs. 2,3);

$\phi_{aa}(n)$ = spectrum of fluid particle acceleration $a(t)$ (Eq. 5);

$\phi_{a_i a_j}(z,z',n)$ = cross spectrum of fluid particle accelerations $a_i(z,t)$ and $a_j(z',t)$ Eq. A-7);

$\phi_{cc}(n)$ = spectrum of generalized force $C(t)$ (Eq. A-5);

$\phi_{FF}(n)$, $\phi_{F_T F_T}(n)$, $\phi_{M_F M_F}(n)$ = spectra of element fluid force $F(t)$, total fluid force $F_T(t)$ and its statical moment $M_F(t)$ about a point at $z = 0$, of a single cylinder (Eqs. 7,9,11);

$\phi_{QQ}(n)$, $\phi_{Q_T Q_T}(n)$, $\phi_{M_T M_T}(n)$ = spectra of element fluid force $Q(t)$, total fluid force $Q_T(t)$ and its statical moment $M_Q(t)$ about a point at $z = 0$, of two cylinders (Eqs. 12, 14, 15);

$\phi_{VV}(n)$ = spectrum of fluid particle velocity $V(t)$ (Eq. 4); and

$\phi_{V_i V_j}(z,z',n)$ = cross-spectrum of fluid particle velocities $V_i(z,t)$ and $V_j(z',t)$ (Eq. A-6).

CAPTION OF FIGURES

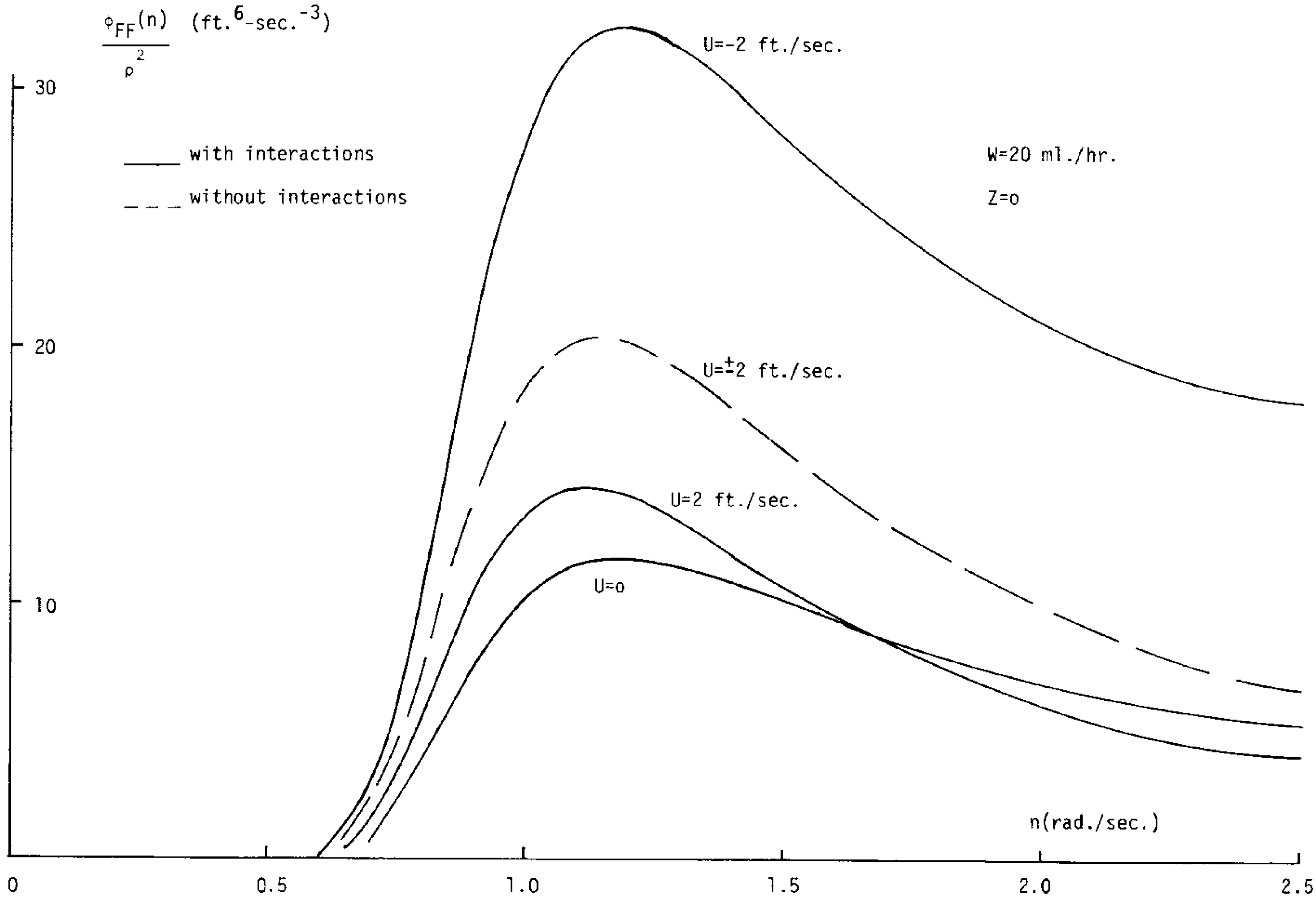

FIG. 1 SPECTRUM OF ELEMENT FORCE (z = 0)

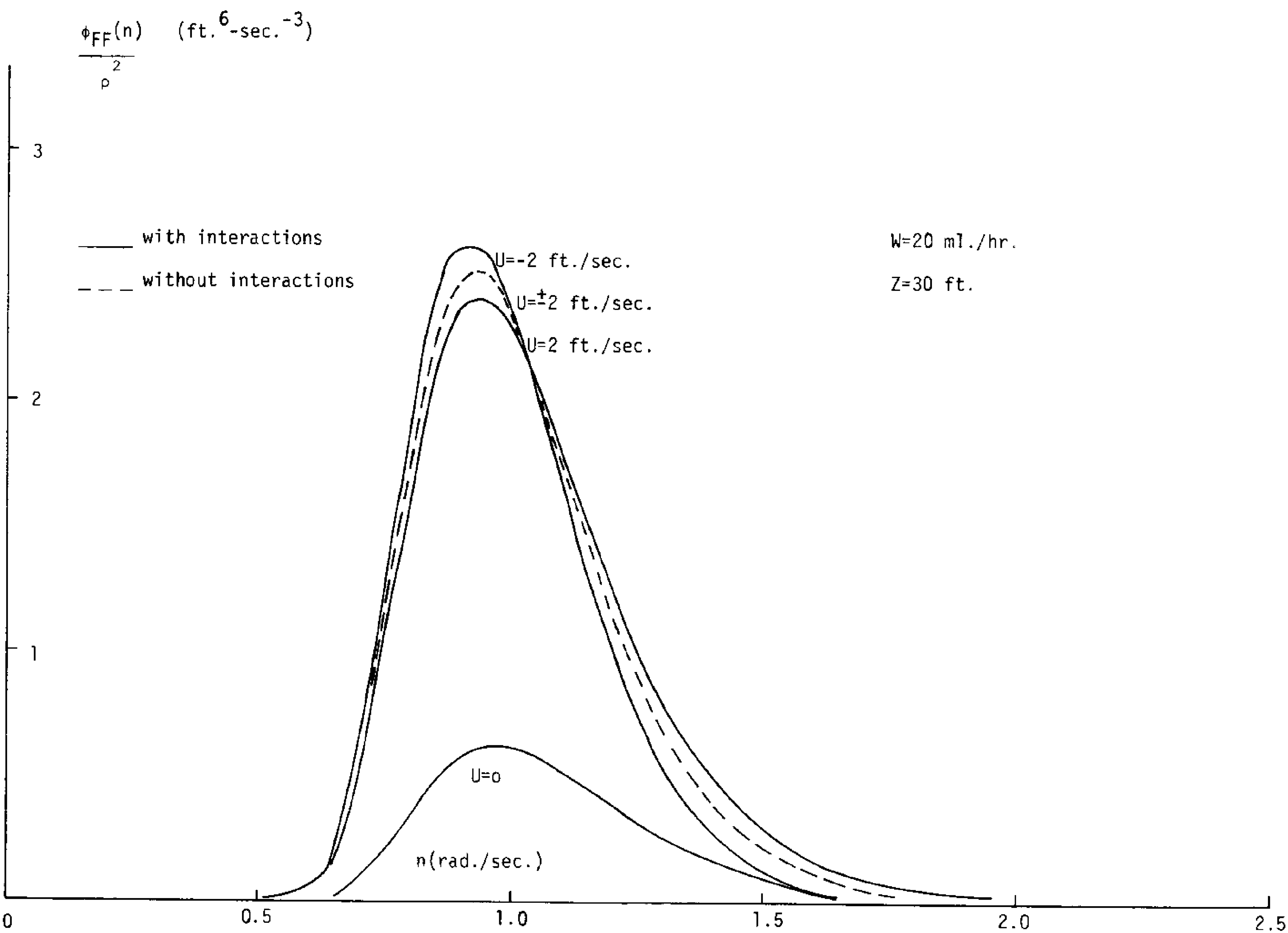

FIG. 2 SPECTRUM OF ELEMENT FORCE (z = 30 ft.)

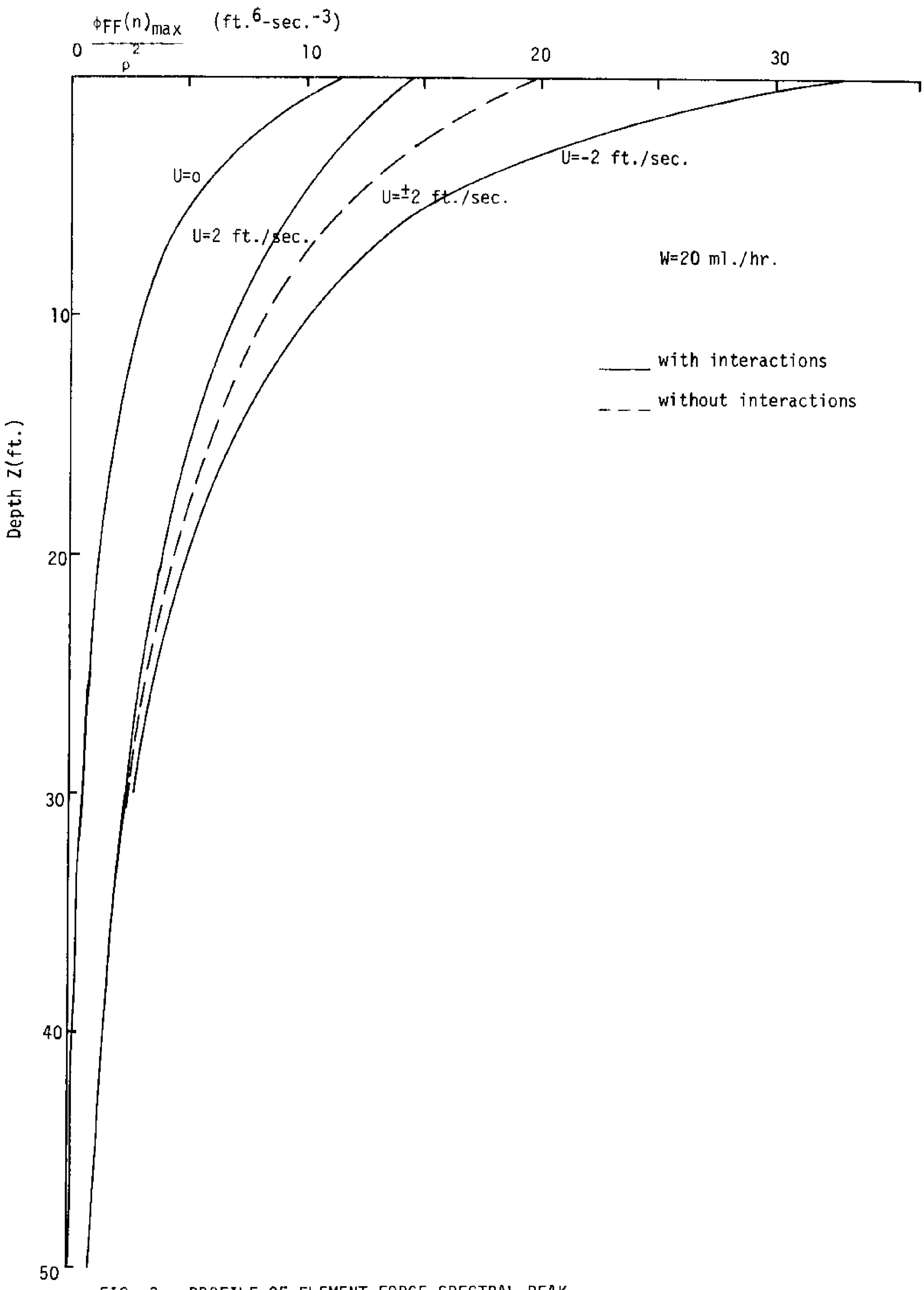

FIG. 3 PROFILE OF ELEMENT FORCE SPECTRAL PEAK

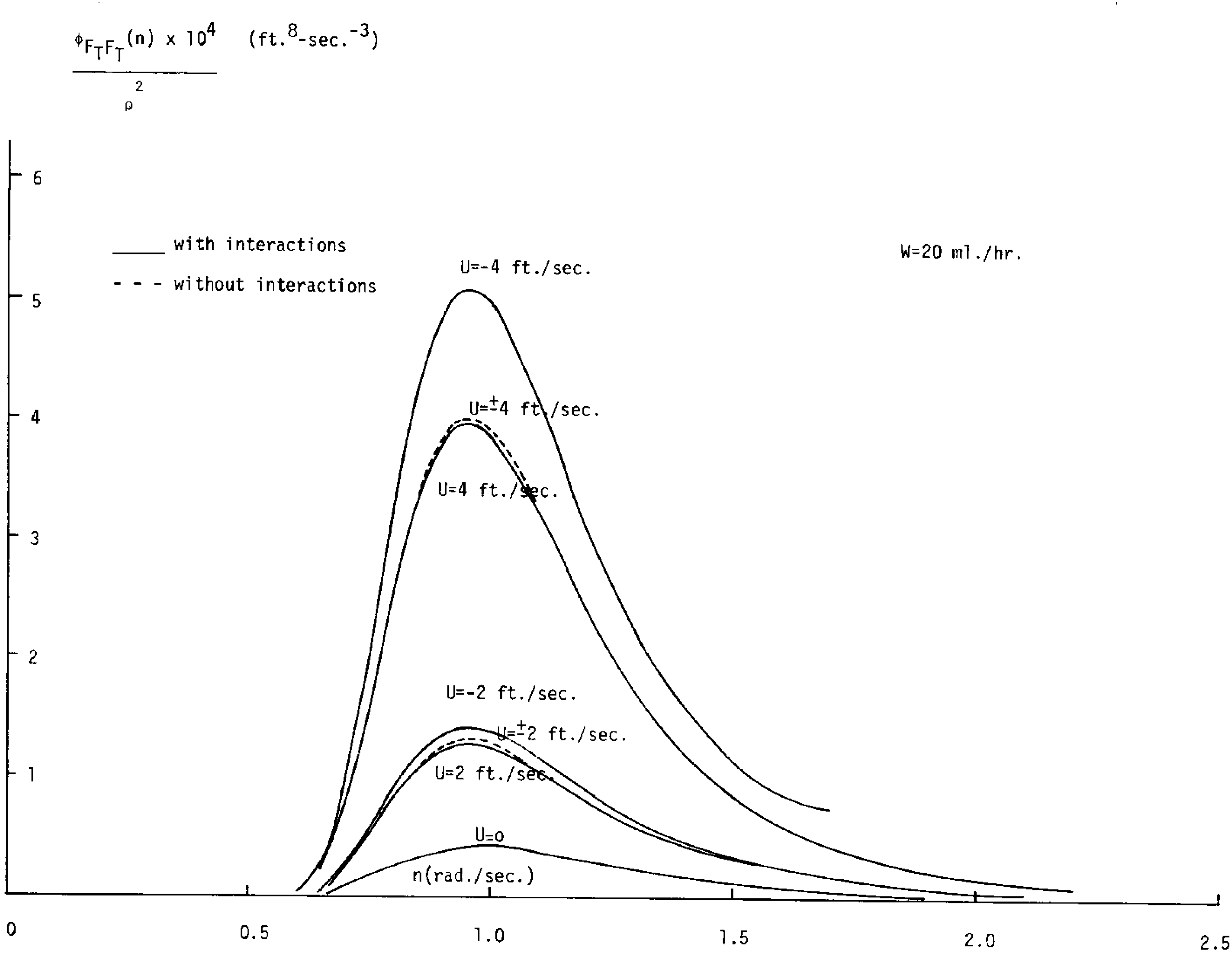

FIG. 4 SPECTRUM OF TOTAL FORCE

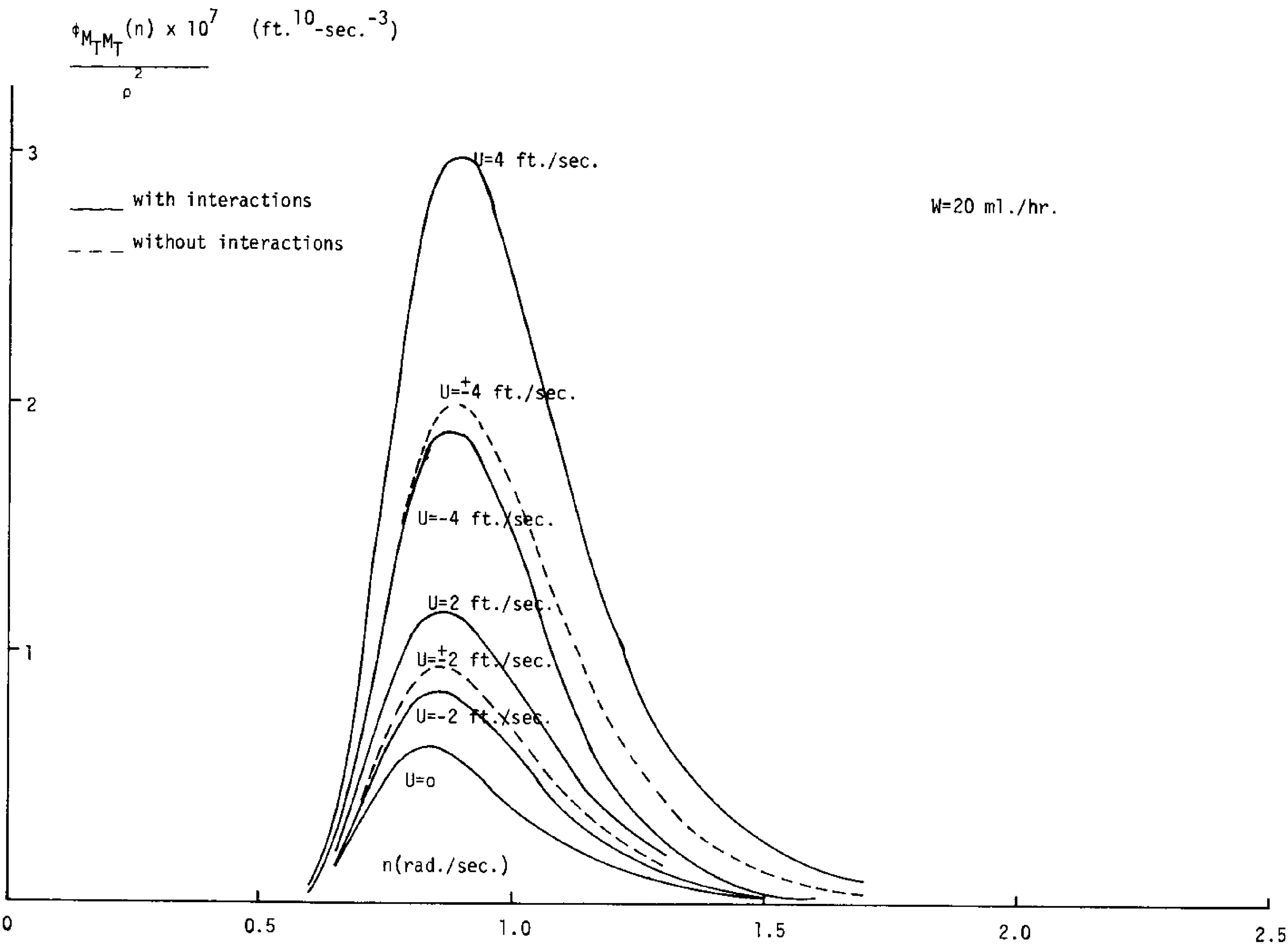

FIG. 5 SPECTRUM OF STATICAL MOMENT OF TOTAL FORCE

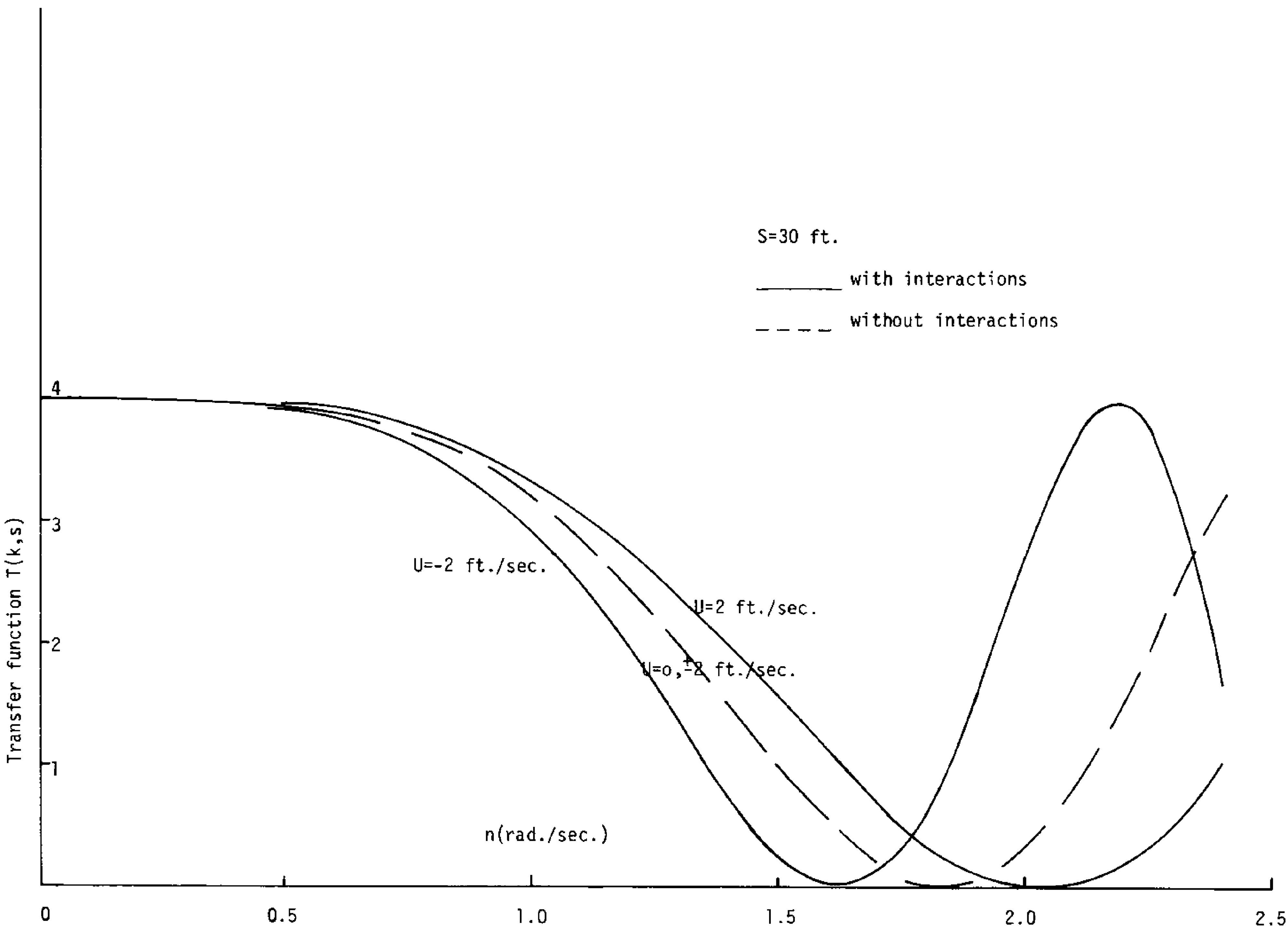

FIG. 6 TRANSFER FUNCTION