# ELECTROMAGNETIC INDUCTION IN THE EKMAN LAYER

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ABSTRACT

Faraday induction in a wind-driven Ekman spiral in the presence of the earth's fixed magnetic field produces electric currents, charge accumulations, and fields. The electric field thus induced may be calculated by analytic methods. The vertical profile of the field predicted by this result is similar to observations by other investigators.

The electric fields induced by the motion of various surface streams are also calculated. The fluid current speeds which would be inferred, by measuring and interpreting these fields by the method of horizontally-towed surface electrodes (GEK), are found to be accurate for broad, shallow streams in deep water, but are not valid in shallow water except under certain highly restrictive assumptions. Furthermore, shallow-water GEK data are not uniquely invertible to fluid current information.

The <u>vertical</u> electric field, which may be useful for measurement of induction in the Ekman spiral, would invariably lead to accurately inferred fluid current speeds in both deep and shallow water. However, the measurement of the vertical field is considerably more difficult than that of the horizontal field.

## INTRODUCTION

One widely used method of measuring ocean currents is the method of towed electrodes (GEK) developed by von Arx (1950) and discussed further by Longuet-Higgins, Stern, and Stommel (1954), Sanford (1967), and several other authors. Essentially, the ocean surface currents are deduced from the measured e.m.f.'s which arise from Faraday induction in ocean currents in the earth's magnetic field. The measurement is made with horizontally towed electrodes.

We present here an examination of the validity of the present methods used for deducing ocean surface currents from towed electrode (GEK) data. Also, theoretical advantages of vertical electric potential gradient measurements, of which there have been relatively few, are demonstrated. The vertical electric field in an Ekman spiral is derived; its existence may be confirmed by such vertical measurements. The feasibility of vertical measurements is discussed briefly.

## EKMAN LAYER

### Derivation

Stommel (1948) derived a fundamental differential equation for the electric potential $\phi$:

$$\nabla^2\phi = \mathbf{B} \cdot \nabla \times \mathbb{V} \qquad (1)$$

where the resistivity, $\rho$, is assumed to be uniform, $\mathbf{B}$ is the uniform and constant magnetic field vector, and $\mathbb{V}$ is the fluid velocity vector. The auxiliary equation for the electric current density, $\mathbf{J}$, is

$$\mathbf{J} = \frac{1}{\rho}(\mathbb{V} \times \mathbf{B} - \nabla\phi) \qquad (2)$$

Stommel solved for the potential using a velocity of the form

$$\begin{Bmatrix} V_x \\ V_y \\ V_z \end{Bmatrix} = \begin{Bmatrix} 0 \\ V_o \cos \frac{\pi}{b} x \\ 0 \end{Bmatrix}$$

for the upper layer above a layer at rest.

We have used the velocity in a wind-driven Ekman current on an infinitely deep and laterally infinite ocean to derive its induced electric potential

$$\mathbb{V} = V_o e^{\pi z/D} \left[ \sin (\theta + \pi/4 - \pi z/D)\, i + \cos (\theta + \pi/4 - \pi z/D)\, j \right] \tag{3}$$

where $z$ is positive upward, $i$ and $j$ are the unit vectors in the $x$- and $y$-directions, and $\theta$ is the azimuth of the wind measured from the $y$- axis. $D$ is the depth of the Ekman layer, and $V_o$ is the surface speed. $D$ and $V_o$ depend on the eddy viscosity, the earth's rotation, latitude, the surface stress, and the water density.

By substituting this Ekman current (3) into the fundamental differential equation (1) and applying the boundary condition at the surface $z = 0$ of zero vertical electrical current,

$$J_z \Big|_{z=0} = \frac{1}{\rho} \left[ (\mathbb{V} \times B)_z - \frac{\partial \phi}{\partial z} \right]_{z=0} = 0 \tag{4}$$

and the condition that $\phi = 0$ as $z \rightarrow -\infty$ we finally obtain the solution

$$\phi(z) = \frac{\sqrt{2}\, V_o D}{2\pi}\, e^{\pi z/D} \left[ B_x \sin(\theta - \pi z/D) + B_y \cos(\theta - \pi z/D) \right] \tag{5}$$

It is a feature of this solution that the vertical potential gradient everywhere balances the vertical induction term, so the vertical electric current density vanishes. There is zero horizontal potential gradient, so the horizontal current density is everywhere equal to the horizontal component of the induction divided by $\rho$.

The solution (5) is plotted in Figure 1 and may be compared with Sanford's data for the Northwest Providence Channel in Figure 2c (Sanford, p. 40, Fig. 21). In plotting Figure 1 we assumed Sanford's values for $V_o$ and B. The constants used are

$V_o = 0.35$ m/sec
$B = 0.25 \times 10^{-4}$, weber/m$^2$ horizontal
$D = 55.6$ m.

Figure 1 was plotted for three different wind directions. Sanford's drift-corrected and temperature- and salinity-corrected data show strong gradients in the first 50 m which indicates the possible influence of a fluid flow field, perhaps an Ekman spiral. Sanford's low resolution makes it difficult to confirm this effect.

The strong electrochemical effects on vertical electrodes unfortunately makes it necessary to make accurate corrections. In Figure 2 Sanford has plotted the contribution to the potential due to temperature and salinity. His data follow the TS effects closely. The effect of motional induction is actually less than the correction for T-S effects. Thus vertical electrode measurements demand very good simultaneous hydrographic measurements enabling temperature and salinity corrections.

Bogorov et al (1969) have also derived an expression for the vertical potential gradient due to an Ekman spiral. These authors also point to the difficulty of resolving such a potential gradient (up to 10μv per meter for a 0.2 m/sec surface current) for they used weakly polarizing chloride--lead electrodes claiming an accuracy of within 1 mv. Bogorov et al also emphasize that in certain oceanic regions, a bioelectric effect (accumulation of phytoplankton and bacteria) and an electrochemical effect (diffusion of ions) in addition to the magnetohydrodynamic effect, contribute to the magnitude of the electric potential.

## THE ELECTRIC FIELD ASSOCIATED WITH SURFACE STREAMS

### Background

In the typical application of the towed electrode method, as discussed by Longuet-Higgins, Stern, and Stommel (1954), the signal is the potential difference between the two GEK electrodes plus the emf induced in the measuring system by its motion through the earth's magnetic field. This signal, divided by the distance separating the electrodes, is the apparent electric field. Under certain conditions, this apparent electric field is the product of the surface current speed, $V$, and the vertical magnetic field. The apparent velocity is therefore the apparent electric field divided by the vertical magnetic field.

More generally, however, the apparent electric field is the product of $V - \bar{V}$ and the vertical magnetic field, where $V$ is the surface current speed and $\bar{V}$ is the vertically-averaged, conductivity-weighted current speed. This basic principle is approximately correct whenever the horizontal scale of the fluid current is larger than its vertical scale. $V - \bar{V}$ may thus be determined under this general condition by dividing the apparent electric field by the vertical magnetic field. This parameter is of intrinsic interest to the oceanographer.

When $V$, itself, is the object of the measurement, it too may be determined under certain conditions. If the vertical profile of current speed is of constant shape throughout a region, $V$ and $V - \bar{V}$ will have a constant ratio. The corrected apparent velocity is therefore determined by multiplying the apparent velocity by this ratio, once the ratio has been measured for a region.

An analytic solution for the electric field associated with surface streams was numerically evaluated for several specific examples to illustrate these principles, and to evaluate the validity of surface current speeds determined by GEK methods. These results are presented and discussed in the ensuing sections.

A simple stream in infinitely deep water

A numerical routine was developed for the rapid computation of the electric field and current density associated with a streamlike flow concentrated near the surface of a half space. The shape of the flow is defined by the following parameter expression:

$$\mathbb{V} = \begin{cases} -2\,V_o(S/W)\,\mathbf{j} & -W < x < -S \\ V_o\,(1 + \cos\,(\pi x/S)\exp \alpha z) - 2S/W)\,\mathbf{j} & -S \le x \le S \\ -2\,V_o\,(S/W)\,\mathbf{j} & S < x < W \end{cases} \tag{6}$$

where $\mathbf{j}$ is the unit vector in the y-direction. S is the half-width of the stream, and 2W is a distance many times greater than S, with the stream repeating itself every 2W in the x-direction.

To solve for the electric potential, $\mathbb{V}$ is first re-expressed as a Fourier sum:

$$\mathbb{V} = \lim_{N \to \infty} \sum_{n=1}^{N} a_n \cos\left(\frac{\pi n x}{W}\right) \exp\,(\alpha z)\,\mathbf{j} \;, \tag{7}$$

where

$$a_n = \frac{V_o W \sin n\pi S/W}{n\pi^2(1-(nS/W)^2)} \tag{8}$$

In summing numerically, N must be substantially larger than W/S. The solution of the boundary value problem obtained by substituting (7) into differential equation (1), and applying the boundary conditions (4) and $\phi(-\infty) = 0$, is readily found to be

$$\phi = \lim_{N\to\infty} \sum_{n=1}^{N} \frac{a_n(\alpha \exp(n\pi z/W) - n\pi/W \exp(\alpha z))\, B_z \sin(n\pi x/W)}{\alpha^2 - (n\pi/W)^2} +$$

$$+ \lim_{N\to\infty} \sum_{n=1}^{N} \frac{a_n((n\pi/W)\exp(n\pi z/W) - \alpha \exp(\alpha z))\, B_x \cos(n\pi x/W)}{\alpha^2 - (n\pi/W)^2} \tag{9}$$

As explained above, in the typical application of the towed electrode method the apparent speed in the y-direction of the surface fluid motion is taken to be the apparent horizontal electric field divided by the vertical component of the magnetic field.

From (9), the apparent velocity is therefore:

$$v_h^{app} = \lim_{N\to\infty} \sum_{n=1}^{N} \frac{\alpha a_n}{\alpha - n\pi/W} \cos(n\pi x/W) - \sum_{n=1}^{N} \frac{(n\pi/W)B_x a_n}{B_z(\alpha - n\pi/W)} \sin(n\pi x/W) \tag{10}$$

Comparing this expression with that for the actual velocity (7) evaluated at $z = 0$, one observes that the first sum in (10) is identical with the actual surface speed, except that each term is multiplied by a factor which will be close to unity unless the horizontal scale of the flow becomes comparable with the vertical scale. The second, additional sum in (10) also will be significant only when the horizontal scale approaches the vertical, or near the equator. Within the limits of applicability of this infinitely deep ocean model, then, surface currents which are broad compared to their depth may be studied quantitatively by the GEK, in non-equatorial regions far from lateral inhomogeneities such as coast lines. This conclusion is consistent with, and may be arrived at from, the general principal that the apparent electric field is the product of the $V-\overline{V}$ and $B_z$.

If one were to use fixed, vertically spaced electrodes, the apparent velocity would be the vertical electric field divided by the horizontal component of the magnetic field. According to (9), this apparent velocity at the surface is precisely the same as expression (7), the actual velocity. Vertical electric field measurements, within the limits of applicability of the infinitely deep laterally unbounded ocean model, can therefore be transformed directly into fluid current measurements, even in equatorial regions.

The conclusions above are illustrated by Fig. 3. The dashed line in each case represents $V_h^{app}$, while the solid curves represent the actual current speed. It was assumed here that $\alpha = 1/500\,\mathrm{m}$, $W = 1000\,\mathrm{km}$, and $B_x/B_z = 18$. The horizontal measurements in our infinitely deep "ocean" led to very accurate current inferences, with only small errors in current strength and position. The vertical measurements both at the surface and at a depth of 500 m are exactly correct--the dashed and solid curves are indistinguishable. However, if one were to generalize from the conclusions reached by this simple model, serious error would result, as will be shown in the following sections.

A simple stream in shallow water

It is of practical interest to determine how the conclusions of the previous section are modified by introducing into our hypothetical ocean a non-conducting bottom at a finite depth. Mathematically, the only change in the problem is that the boundary condition at $z = -\infty$ is replaced by the condition that the vertical electric current density is zero at a depth H.

As a result of the addition of a non-conducting bottom at $z = -H$, the previous expression for the potential (9) is now modified by the addition of

$$\Delta\phi = \sum_{n=1}^{N} \frac{a_n(\exp(-n\pi H/W)-\exp(-\alpha H))(\alpha B_z \sin(n\pi x/W)+(n\pi/W)B_x \cos(n\pi x/W))\cosh(n\pi z/W)}{(\alpha^2 - (n\pi/W)^2)\sinh(n\pi H/W)} \tag{11}$$

Expression (10), for the apparent velocity from horizontal measurements, is replaced by an expression identical with (10), except for multiplication by a factor

$$1 - \frac{(n\pi/W)(\exp(-n\pi H/W) - \exp(-\alpha H))}{(\alpha - n\pi/W)\sinh(n\pi H/W)}$$

If H (total depth) is large compared with both $1/\alpha$ (current depth) and $W/n\pi$ (current width), this factor will be close to unity, and $V_h^{app}$ will be closely approximated by (10), which is consistent with the nature of the modification which led to (11). However, if H is small compared with $1/\alpha$ and/or $W/nR$, and not very large compared with either, $V_h^{app}$ in the ocean of finite depth may differ considerably from that in an infinitely deep "ocean", and therefore from the actual velocity. This conclusion is consistent with the principle relating the apparent electric field to $V-\overline{V}$.

To illustrate the effect of finite depth, the same surface stream used in the calculations for Fig. 3 was re-used, this time with a non-conducting bottom at a depth of 750 m. The apparent velocity which would be inferred from GEK measurements in this shallow water current is illustrated in Fig. 4. Using horizontally towed electrodes, the inferred current (obviously incorrect) is not only slightly distorted, as in the infinitely deep ocean, but is reduced in magnitude by a factor of 2.0. The current speeds inferred from the vertical electric field, in contrast, are still indistinguishable from the actual current speeds (in reality, there is a very small difference between the two). The generalization that one might be tempted to draw from the results based on the simple current model of the previous section is thus invalid, insofar as horizontal measurements are concerned, except under the additional condition that the current depth is a small fraction of the total depth.

It might be reasoned that the reduction of $V_h^{app}$ by a factor (2.0, in this case) from the actual velocity does not pose a serious problem. Since the effect is nearly uniform across the width of the stream, a few independent measurements of current speed made concurrently with GEK measurements can be used to evaluate the fudge factor by which all later GEK-inferred speeds may be multiplied to give presumably correct current speeds. This fudge factor is euphemistically referred to in the literature as a k-factor. In general, such a procedure would be required except where the current depth is known to be much less than the total depth. This line of reasoning, suggested by the results based on the relatively simple current model used here, will be scrutinized further after studying an additional current model in the following section, and shown to be valid only under further highly restrictive assumptions.

A complex stream in shallow water

In this section, we use the results obtained above to find the electric field and current densities induced by the motion of two superimposed parallel flows in water of 750 m depth. The first flow is the same as the stream used above–100 km wide and decaying exponentially with depth by a factor e every 500 m. The second flow is a stream with half the maximum speed of the first, and the same shape as the first, but with a width of 25 km and a 1/e depth of 50 m. Its maximum is located on the shoulder of the first stronger, broader stream, halfway between its center and its edge.

In Fig. 5, the solid curve again represents the actual surface current speed, in this case the sum of two parallel streams. The dashed line again shows that obviously incorrect current which would be inferred by horizontal towed electrodes before "correction" by the k-factor. If the experimenter has access to data from which a k-factor may be computed, and makes usual necessary assumptions concerning the spatial invariance of k over large regions, he will multiply the inferred speeds by k to obtain "corrected" inferred speeds. As shown in the previous section, this factor is 2.0 for the stronger, broader stream alone. The resulting "corrected" inferred speed (still obviously incorrect) is now given by the dash-dot curve, which overestimates the speed of the narrow stream by a factor of 2.0, but correctly measures the previously known speed of the broader, stronger current.

Restating these results in terms of the uniqueness problems familiar in not only this branch of geophysics: There is no unique current system which can account for the horizontally-towed surface electrode data in this problem. Indeed, there is an infinity of inversions of the data. Three have been shown: (1) the actual current; (2) a current whose surface speed is given by the dashed curve and which decays exponentially with depth to 1/e in much less than 750 meters <u>at all positions</u>; and (3) a current whose surface speed is given by the dash-dot curve and which decays exponentially with depth to 1/e in 500 meters <u>at all positions</u>.

The tentative conclusions based on the properties of the relatively simple current model studies in the previous section are therefore shown, by counter-example, to be invalid unless the vertical cross-section of relative current speed (relative to the local surface speed) is independent of horizontal position. This is the highly restrictive additional condition under which one must operate in order to validly infer current speed from GEK data. The basic principle relating the apparent electric field to $V-\bar{V}$ remains consistent with this conclusion.

The current that would be inferred by measurment of the vertical electric field is once again indistinguishable from the actual current, both at the surface and at a depth of 500 m.

## SUMMARY AND CONCLUSIONS

1. If the depth of a current is small relative to both its breadth and the total water depth, then the surface speeds which would be inferred from the apparent horizontal electric field at the surface, which is routinely measured by horizontally towed electrodes (GEK), will be valid.
2. If the depth of a current becomes comparable with the total water depth, but small compared with the horizontal scale of the current, then surface speeds can be uniquely inferred from the apparent horizontal electric field at the surface and an independently determined k-factor only if the vertical profile of relative current speed is the same across the entire width of the current.

3. If the highly restrictive condition necessary for the interpreation of GEK measurements in shallow water (Conclusion 2) is not satisfied, the infinite number of highly dissimilar currents which can account for the same surface data will not be resolvable.

4. Conclusions 1-3, above, are consistent with, and may also be derived from, the basic principle that the apparent horizontal electric field at the surface is equal to the product of $\nabla \cdot \overline{V}$ and $B_z$, in any current broad compared with its depth.

5. Current speeds inferred from the vertical electric field are always exactly accurate near the surface, and highly accurate at great depth whenever the horizontal scale of the current is much greater than the total depth. This method may be valuable for observing the Ekman spiral at sea, although existing data are not sufficiently accurate and unambiguous for this purpose. Furthermore, while the easily satisfied conditions for the uniqueness of data inversions are an advantage in interpreting the vertical electric field, the practical problems in its measurement and correction for T-S effects may at this time be a more important contra-indication.

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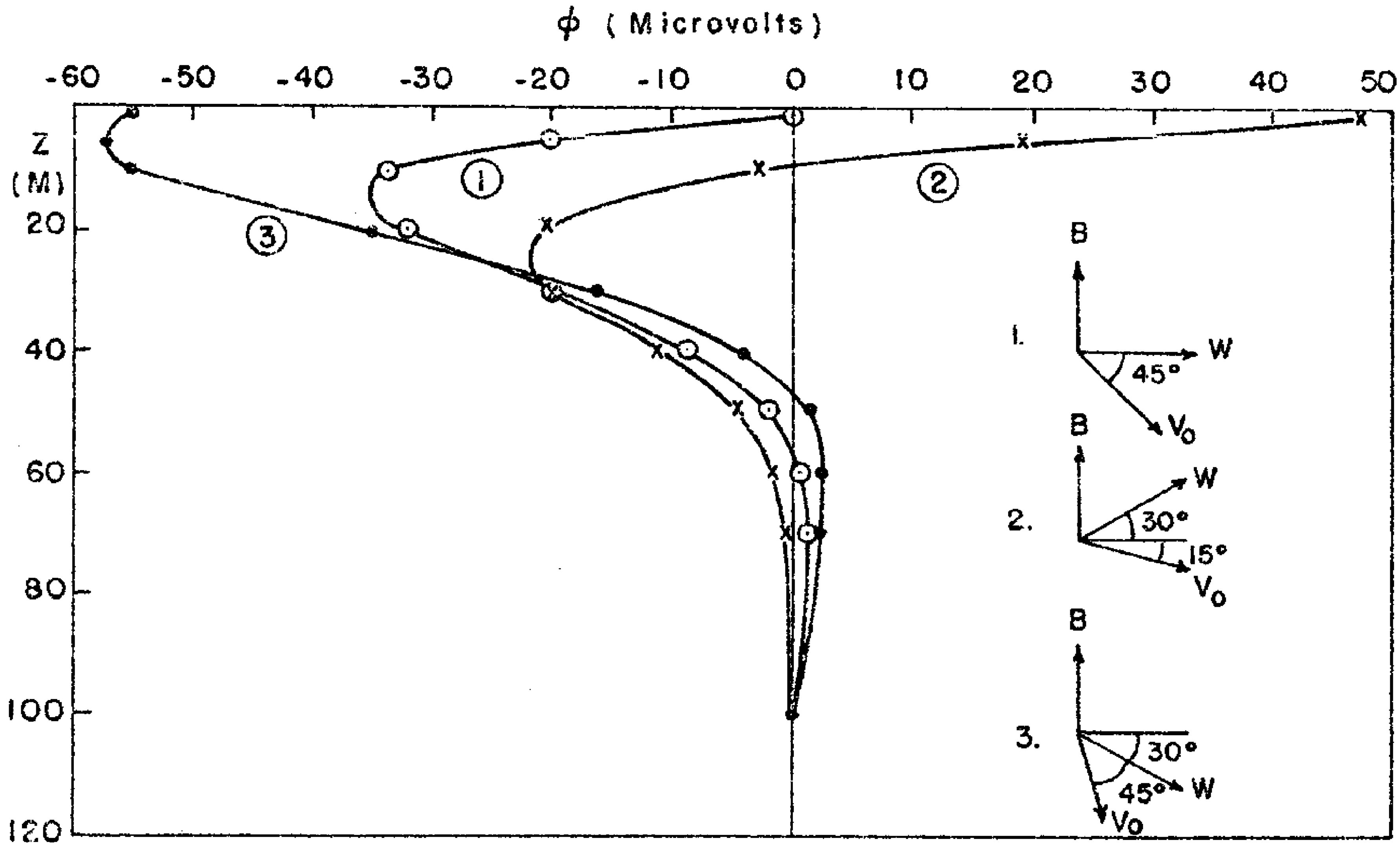


Fig. 1. Electric potential $\phi$ as a function of depth in an Ekman spiral. The three curves correspond to the three insets showing the relative orientation of the horizontal magnetic field (B), the wind speed (W), and the surface current ($V_0$). $V_0 = 0.35$ m/sec; $B = 0.25 \times 10^{-4}$ weber/m$^2$. The depth of the Ekman layer is 55.6 meters.

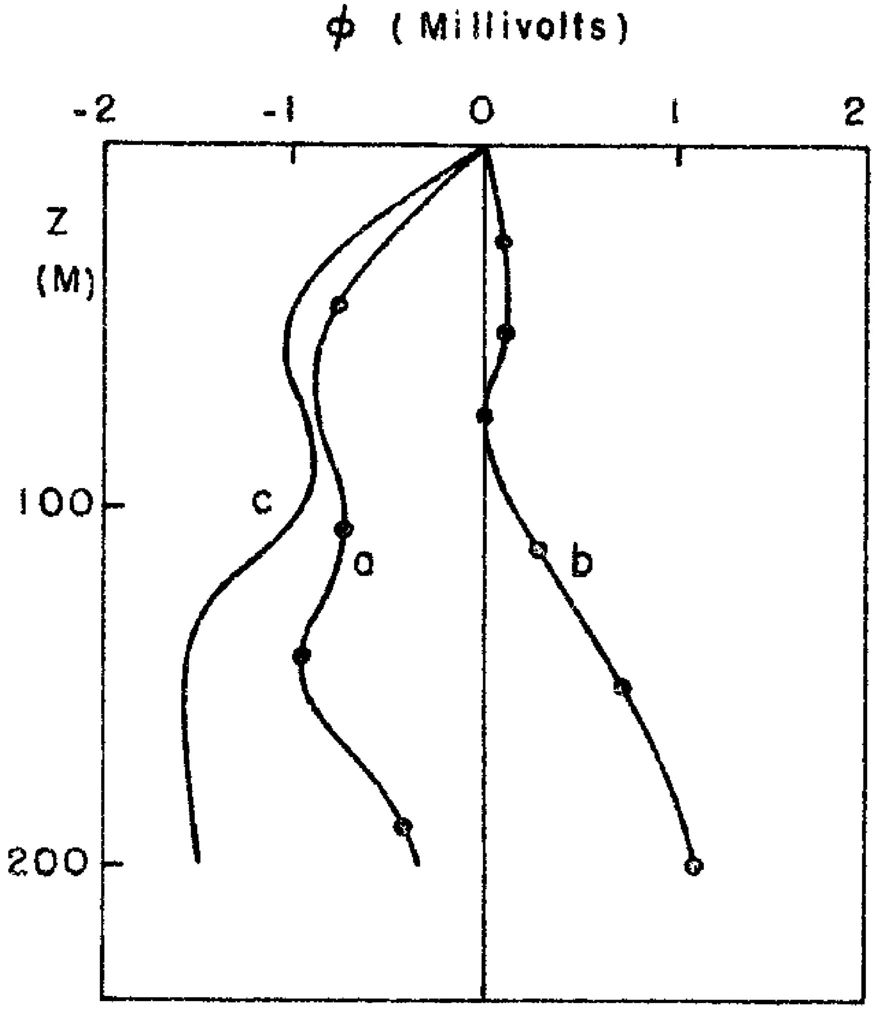


Fig. 2. Observed electric potential as a function of depth.
(a) Drift-corrected potential (fom Sanford, 1967, Fig. 21).
(b) T-S correction (from Sanford, 1967, Fig. 21).
(c) Potential corrected for both drift and T-S effect.

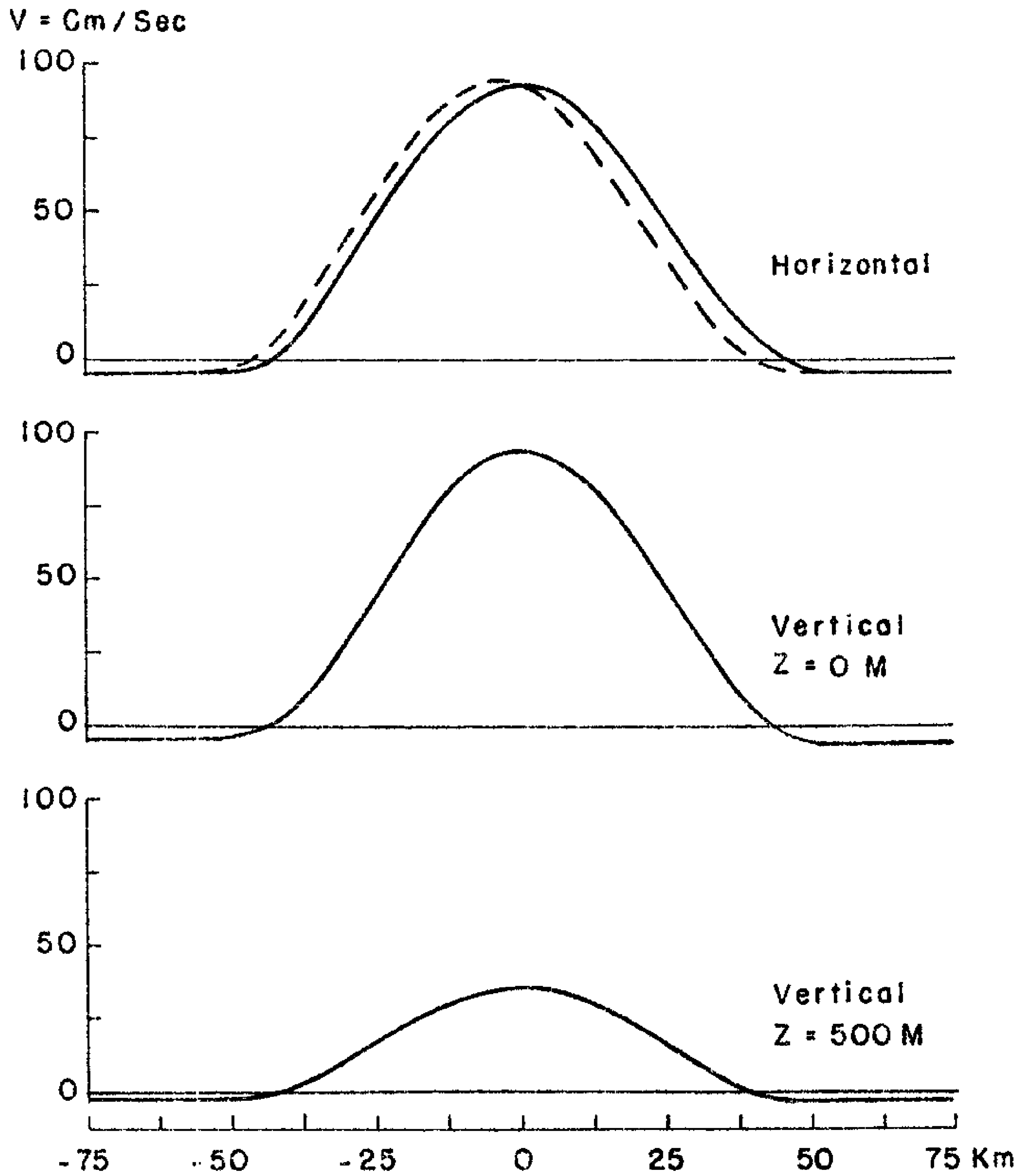


Fig. 3. The actual (solid curve) and apparent (dashed curve) current speeds for a broad shallow stream in deep water. The apparent current speed has been inferred from horizontally towed surface electrodes (upper figure), fixed vertically-spaced electrodes at the surface (center figure) and fixed vertically-spaced electrodes at a depth of 500 m (lower figure).

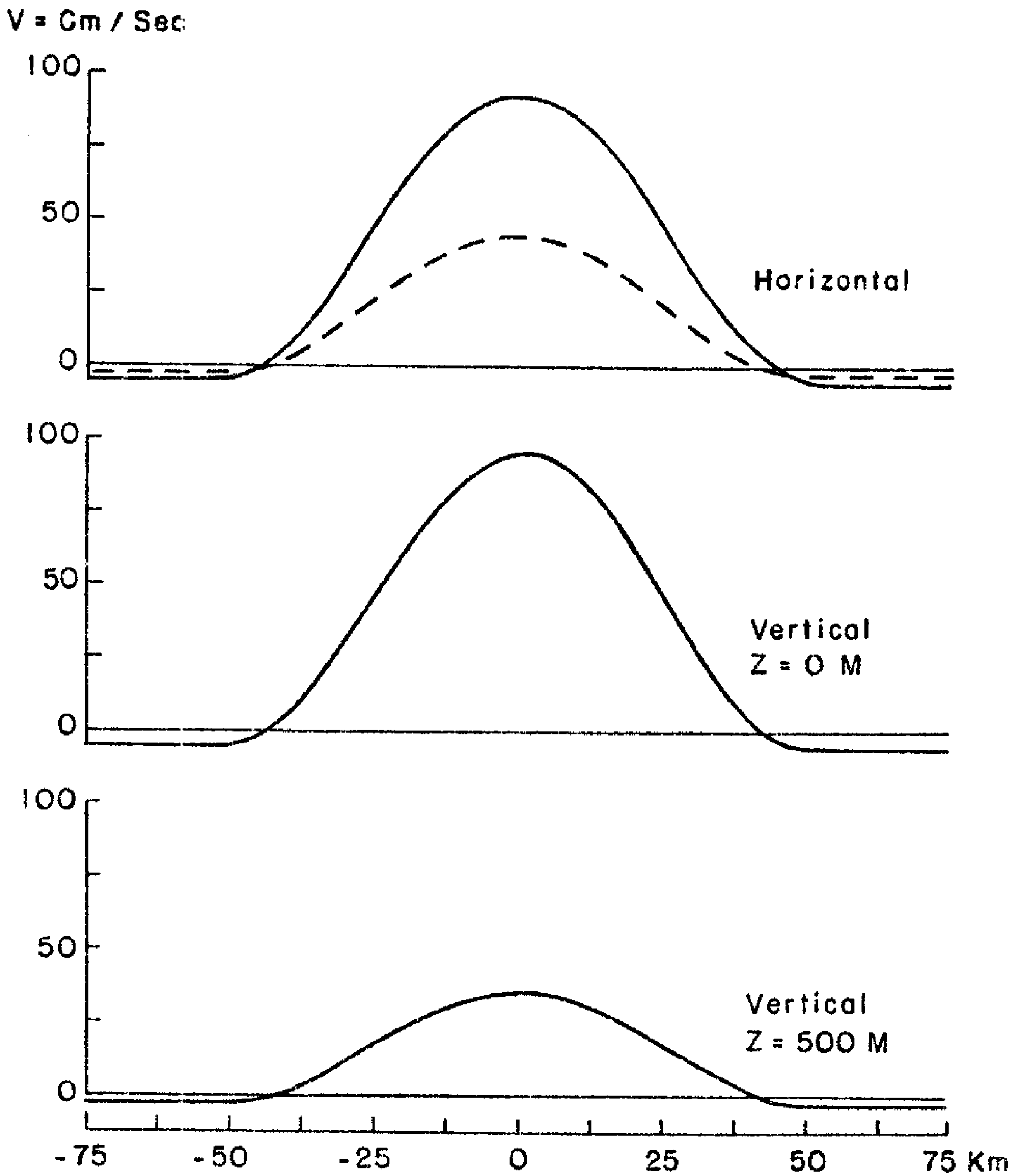


Fig. 4. Same as Fig. 3 for a broad stream in shallow water.

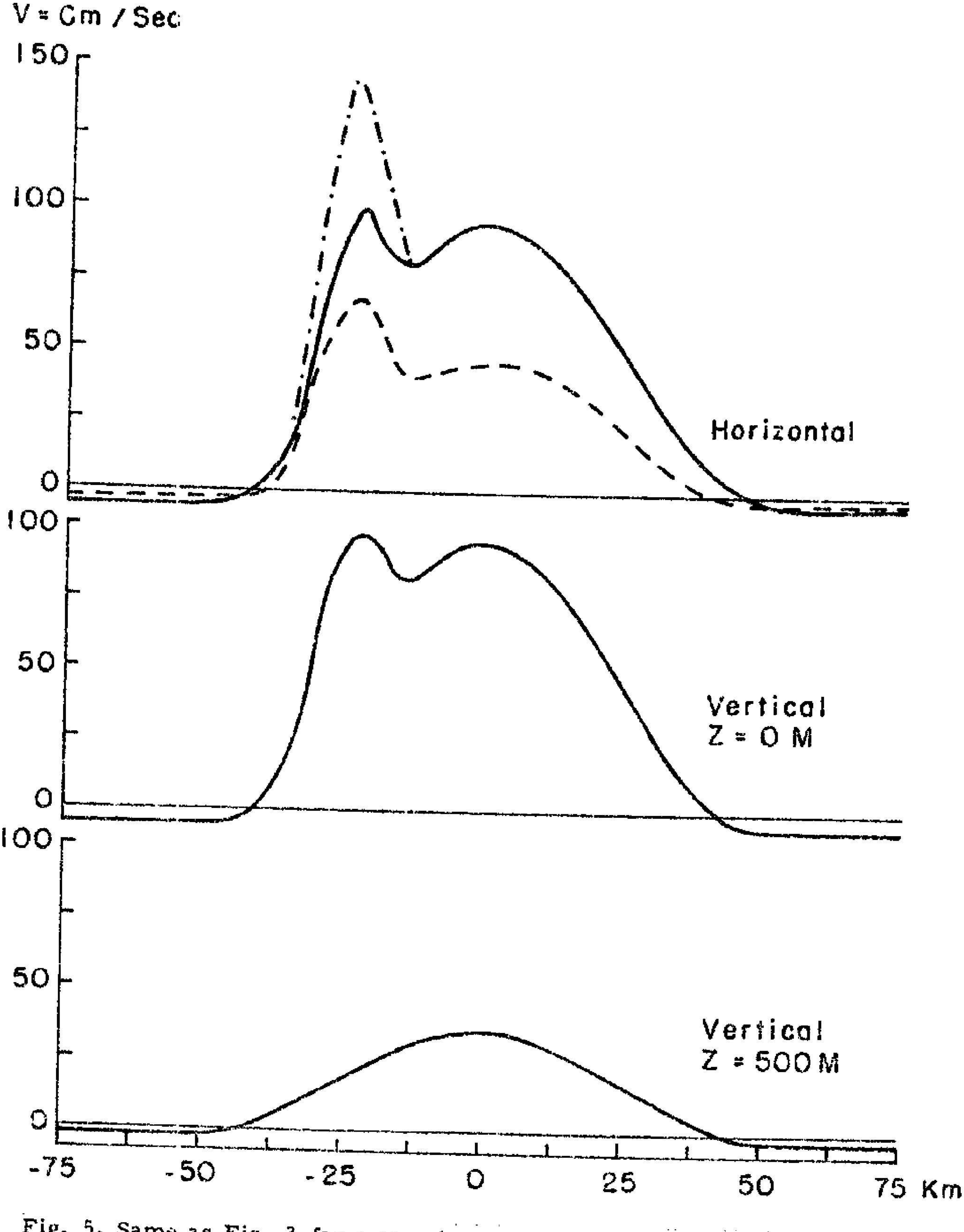


Fig. 5. Same as Fig. 3 for a complex stream in shallow water. The dash-dot curve in the upper figure represents the inferred speed after "correction" by a k-factor.