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A STOCHASTIC INVESTMENT MODEL
FOR A SURVIVAL CONSCIOUS FISHING FIRM

Prepared by

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The purpose of this study was co deveiop further mathematical aids for invest-ment-financial decision making in shrup fishing. The model developed allows for randon oricos and catuces yossel and, in addition, takes into account all of the information known to the decision maker at each time of decision. Surrivai of the fiscing fivm is regarded as a fundamental factor influencing the firm's investment decisions.

In making each decision, tre fisheman evaluates the firm's net equity position, the worst possible sequance of revenues that might materialize, and all of the firm's forthcoming oflipations. He derives from this information a survivable sot of capacity purnases, and then selects fron this set the investment maximizing fiss net worth th the end of the planning period. After each year's operations and before thenoxt; year hegins, the random revenue variable for the first year has been observei fit is now a part of the information for decision makiny in the second yax. Tro Cisherman repeats the above reasoning process in making his investmert denaion for the second year, and in every year thereafter. Survival must be guaranteed before any investment is undertaken; moreover, investment decisions are alsays conditioned by experience.

In accordance with information obtained from cooperating firms, values for all of the parameters were specified. Initially, the firm was assumed to have had purchased one new 73 foot steel hull vessel, or to have the money equivalent in savings. To reflect inflation, prices were assumed to increase 3 percent per year. For tax purposes, the depreciation period was 11 years, and the income tax rate was 25 percent. The length of the planning period was taken to be 5 years.

Since the shrimp price is highly influenced by the rate of growth in per capita income, expected prices for the years 1970 through 1974 were projected for a modest rate of economic growth (as observed in the late $1950^{\circ} \mathrm{s}$ ) and for a high rate of economic growth ( 8 s observed in the mid $1960^{\circ} \mathrm{s}$ ). Investment solutions were calculated for both growth rates. The marginal value of another vessel was found to be initially larger and to be positive for a longer period of years at the high growth rate than at the lower. Success in shrimp fishing is clearly influenced by the rate of income growth in the economy.

The value of better than average management was also clearly illustrated. Almost six more vessels were purchased than in the case of average management.

In evaluating the rate of return over cost from fishing in relation to the savings alternative, investments in fishing capacity were found to be a better alternative than savings as long as the interest rate was less than 9.5 percent per year. Then a switch occurred in favor of the savings alternative. Thus, given the present borrowing rates, investments in fishing capacity are near the margin of profitability (in a suryival sense), as far as interest rates are concerned.

Solutions were calculated for the case where price was random as wall as landings. Only slight differences were found between the results in the two sets of probiems. Vagrancies in landings per vessel seem to be much more important than unexpected variations in price.

# A Stochastic Investment Model <br> for a Survival Conscious Fishing Fixm 

by
Russell G. Thompson, Richard W. Callen, and Lawrence C. Wolken
(Texas A\&M University)

1. Introduction

In 1969, Thompson and George [2] formulated a stochastic dynamic investment model for the survival conscious firm, derived the optimal decision mules for investment, and computed solutions to several problems. This model takes into account the probability distribution of the yield (catch) and output price, as well as all of the information known to the decision maker at the time of each investment decision. The entrepreneur is assumed to be initially in a financial position so that a feasible invostment solution always exists if the lowest output price and yield occur in every period of the planning horizon. In the model, the objective of the firm is to maximize expected net worth at the end of the planning horizon. Of course, all production expenses, investment outlays, interest costs, and planned cash withdrawals must be paid for as incurred (or scheduled).

Because of the vagrancies of fish prices and catches this model would be expected to be a particularly appropriate decision aid for investments in fishing capacity. There are generally few, if any, alternative uses for specialized fishing equipment. Also, fishermen typically have poor alternative opportunities hy which to am a living Iov prices and small catches would be expectien, as a rosult, to be droaled much more than high prices and large

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catches are desired. A sequence of worse than expected net revenues (even in the case of a very favorable expectation) could terminate the existence of the fishing firm. This could well be an unacceptable risk of failure Hence, survival of the fishing firm would be expected to be fundamental factor influencing the firm's investment decisions.

## 2. Development of tre Survival Model

In the survival model, the fisherman evaluates the worst sequence of ret revenues that could occur in every year of the decision-making period. This sequence, in conjunction with the value of the initial investment in fishing capacity and the position of the money account, determine the survivable set of fishing capacity purchases at the beginning of the first year. The fisherman selects from this set the investment that contributes the most to his teminal net worth. After the first year and before the second fishing year begins, the fish price received and the catch landed in the first year have been observed. This is now a part of the information known to the fisherman for planning in the second year. The fisherman again evaluates the worst sequence of catches and prices that could occur in every remaining year of the decision-making period, This abbreviated sequence is now evaluated in conjunction with the capacity and money position at the end of the first year. It determines the survivable set of capacity purchases for the second year, Again, as in the first year, the f'isherman selert.s from this second set the investment that contributes the most to his terminal not, worth. This procedure is repoated in every yon throurhout the decision-making period, Investment decisions are conditioned bey experience, and are not based solely on expected values.

We simply say that a fishing fjrm survives in a given year if the value of the fishing capacity exceeds the value of the indebtedness. A survivable investment is defined in the followine settine: the fisherman has completed the fishing season in year $k-1$ and is now planning for year $k$. He wants to survive above all else during the remaining $N-(k-1)$ years of the decision period, even if al. future catches; and rrices are the lowest poscible. An investment decision in the th $\quad$ rour: $u_{k}$, is said to be survivable it the value of the fishing capacity in every remaining year is never less than the indebterness owed (with fishing cacreity not being purchased in any of the years after the $k^{\text {th }}$ one and the lowest net revenues being visualized in every year of the yet undisclosed fuiture).

Under these conditions, a survivable capacity purchase in year $k$ is found to be equivalent to the following, one: the product of the capacity units purchased in year $k$ and the reranal value of fishine capacity calculated under the ascumption of the lovost rot rovenue occurring in every forthoming year--the marginal cost of fishing: chnacty visualizing the worst--is never greater than the value of the fishermarem moncy account in year $k-1$ plus the terminal value of the capacity owre ir year k-1 minus the losses from utilizing the present fishing capacity in all of the remaining years (with the lowest prices and smallest catches socurring) minus the fixed cash withdrawals in the rest of the planning period. (All money flows are adjusted for the values of altermative opportmities, income taxes, and depreciation.) This upper-bound would be the value of the fisherman's assets if the worst possible sequence of net revenues occurred--the fisherman's final asset position visualizing the worst.

To reflect the fear of low net revenues when the lowest price and catch occurs, revenue per unit of fishing capacity is assumed to be less than operating cost per unit of fishinf; capacity. It is also assumed that per unit prices of fishing capacity are not increasing so rapidly that operating losser per unit may be covered by value appreciation in fishing capacity. (Speculation is never a sure bet.) Chis implies that the marginal cost of fishing caparity visualising the wowt is positive. Hence, dividing the lower bound for the fisherman's final asset position by this positive marginal cost, the upper-bound for a survivable purchase of fishing capacity in a given year iis obtained, This represent f tie maximum amount of fishing oapacity that the fisherman can purchise and still insure survival of the firm throughout the rest of the decision period. ft depends upon the value of the firm's money arcount, the amount of capacity owner, and the value of that capacity in the previous year, This upper-bound function in year $k$ is denoted by $H_{k}\left(Z_{k-1}\right.$, $\left.y_{k-1} \times x_{k-1}\right)$, where at the end of the $k-1$ year $Z_{k-1}$ is the cash balance, $y_{k-1}$ is the units of fishinf canmity owned and $x_{k-1}$ is the promase vaiue of the firm's napacity. The firm is in debt if $Z_{k-\uparrow}$ is nersere and has savings if $\eta_{k .1}$ is positive.

We will. elso introduce the following notation now: $s_{i}$ is the units of fishing capacity purchased at the beginning of the $i^{\text {th }}$ year and used for the first time in year i); $\tau_{i}$ is the operating costs per unit of fishing capacity in year $i ; \sigma_{i}$ is the per unit rurchase price of fishing capacity before the beginning of the fishing season in year $i$; $\Delta_{i}$ is the fixed cash withdrawal in year i for scheduled expenses independent of fishing operations and investments in fishing capacity per se; $\gamma$ is the interest rate paid (or received on the cash account $Z ;{ }_{i}$ is the unknown revenue per unit of fishing
capacity in the $i^{t h}$ year: $N$ is the number of years in the planning period; $\beta$ is the fraction of the value of the fishing capacity recoverable at the end of the planning period; $\delta$ is the income tax rate; and $\epsilon$ is the straight-line depreciation fraction. Also E will be used to denote the mean of the random variable $\omega_{i}$; and $L$ will be used to denote the smallest possible annual net revenue having a positive probability of occurring.

Using the above development, the survival model may be stated as follows: Maximize $E\left(Z_{N}+\beta \sigma_{N+1} y_{N}\right)$ over ali n-tuples of functions $s_{i}\left(\omega_{1}, \omega_{2}, \ldots, \omega_{i-1}\right)$, $i=1,2, \ldots, N$, satisfying the difference equations

$$
\begin{aligned}
& x_{i}-x_{i-1}=\sigma_{i} s_{i}, x_{0}=\sigma_{0} y_{0}, \\
& y_{i}-y_{i-1}=s_{i}, y_{0}, v e n \text { and non-negative, } \\
& \left.z_{i}-z_{i-1}=\gamma_{i-1}+y_{i} i_{i}-T_{i}\right)-\sigma_{i} s_{i}-\Delta_{i}-\delta\left[y_{i}\left(\omega_{i}-\tau_{i}\right)\right. \\
& \\
& \left.\quad+\gamma_{i-1}-\varepsilon\left(x_{i-1}+\sigma_{i} s_{i}\right)\right], \quad \varepsilon=.091, \\
& \quad i=1,2, \ldots, N, \text { and satisfying the inequalities } \\
& 0 \leq s_{i} \leq H_{i}\left(z_{i-1}, y_{i-1}, x_{i-1}\right), i=1,2, \ldots, N .
\end{aligned}
$$

In words, the model fisherman desires to maximize expected net worth at the end of the decision period where the purchases of capacity are selected from the survivable set in each year (delineated by the inequality restrictions). Thus, in the maximization process, the model fisherman, who takes into account all of the information known at the time of decision, selects the investment from the survivable set of capacity purchases that maximizes expected net worth at the end of the planning horizon.

## 3. The Decision Kule for Investment

By the use of dynamic programing methods, we extended the method developed by Thompson and George [2] to allow for depreciation and income taxes. The extended rule for optimal investments is summarized in the following theorem.

Theorem: Suppose $H_{1}\left(Z_{o}, y_{0}, x_{o}\right) \geq 0$, i.e. the upper-bound for investmente in the first year is non-negative. Let $R_{k}$ be the expected marginal value of fishing capacity for survival investment decisions--the marginal value of fishing capacity visualizing the worst. Then the decision rule for optimal survivable investment is as follows:

$$
s_{k}^{\circ}=H_{k}\left(Z_{k-1}^{0}, y_{k-1}^{0}, x_{k-1}^{0}\right) \text { if } R_{k}>0 \text {, and } s_{k}^{\circ}=0 \text { if } R_{k}<0 \text { with the }
$$

feasible value of $s_{k}$ being immaterial if $R_{k}=0$.
In other words, the fisherman buys the survivable limit of fishing capacity in year $k$ if the marginal value of fishing capacity visualizing the worst is positive in that year, and he doesr.'t buy anything if this marginal value is negative. It also follows that the optimal purchase is immaterial in any year (because of the linearity of the problem) whenever the decision rule is zero. The upper bound for investments in the first year insures the existence of a feasible investment solution in each year of the planning horizon.

## 4. An Application to Shrimp Fishing

To indicate how the model may be applied to a shrimp fishing firm, parameters were specifited for a relatively small fishing firm operating 73 foot steel hull trawlor: (see Table $1, \mathrm{p} .16$ ). In the specifications, the values of the parameters were specif.ed to reflect prices, costr, and landings per vessel as regortec by the firms cooperating in the study. There is an exception with regard to Problem 4. Average landings per vessel were duecilied to be one standeri deviation above the mean be inoicate the effect of better than averege management.

Since the real price of shrimp -- the price adjusted for the purchasing power of money -- is highly influenced by growth in per capita income (real), and since it appears that the econom may be enterine a poriod of modest growth (possibly much laks the late 1950's), the real price of shrimp was specified to reflect, a L , percent rate of growth in per capita income in Problens 1, 2, and 3 , and to reflect a 3.3 percent rate of frowith (as observed in the mia $1060^{\prime}$ ) in problem 4.

To evaluate the economic attractiveness of shrimp fishing versus the best alternative to $\mathrm{f}^{\prime}$ shing (as reflected by the interest rate on money), the decision marer in Problem 2 initially has the money equivalent of an investment in one vessel. Recsil that the entrepreneur is a profit maximizer, given that he can survive. Thus, the decision maker would opt for the savings aiternative whenever the net rate of return from a dollar invested in fishing capacity is iess than the interest rate on money. That is, the second problem indicates the economic advantage (or disadvantage) on investing in fishing relative to loaning the money to someone else.

Since the model takes into gcoount the information obtained through time as the values of the random variables are revealed, solutions to two sets of problems were computed. In the first set, the landing per vessel is random; wherees in the second set, the price received is random as well, The first set of reaults is presented in Table 2, p. 17, and the second set in Table 3, p. 18.

It is important to note that this application of the survival model is not exhuastive of the many that could be made, or to imply that the normative results presented are lkely to occur. This work is only meant to indicate how an investor interosted in shrimp fishing, who has a limited amount of money capital, m: ght obtain bench marks (from the model) for investment planning.

Initial vaiues for the difference squations and values of the parameters.
In this application, the vilue of $y_{0}--$ the initial amount of fishing capacity -- is specified to be : re 73 foot steel hull trawler in Problems 1, 3 and 4. This type of vessel finlly outfitted for shrimp fishing costs $\$ 100,000$ at the beginning of $19 \% \mathrm{~F}$. To reflect inflation, the purchace price of the new vessels was specified to increase at 3 percent per year.

In recent years, there heve ceen steady improvements in technology with newer vessels being powereu by more horsepower. Thus larger trawls could be pulled at a faster rate. Thiri rate of technological improvement was assumed to have increased costs by 2 percent per yeer.

From the cost records of the eoperating firms, the annual cost of operating e 73 foot trawher was sond to be $\$ 30,000$ in 1969. This cost figure includes an allowance for sverhead and insurance costs. Representatives of the firms interviewed *rdicated these costs have increased by 3 percent per year in recent years. Thus, the annual production cost ${ }^{1}$ per vessel, ${ }^{T} t$, was specifised 30 oe $30,000(1.03)^{t}$.

Straight line depreciation methods were used for tax purposes with an 11 year depreciation perfod being used for a fully outfitted vessel. This average was estimated on a value weighted basis from the records of a number of firms. The receporical of this figure, . 091 , was the depreciation fraction used for the value of $\epsilon$.

Income for tax purposes as tha sur of the revenue received by the owner after the "lay" lecs operating costs, interest costs, and depreciation. The ineme tax rate, wh a donoted by 8 , was taken to bo percont of th:s frgure. Tair rato was paid jn the late $1960^{\circ} \mathrm{s}$ by a number of the small fishing firms stuidej.

In chrimp fishing, as $\quad \mathrm{n}$ svery business, there are sundry expenses for a number of factor: :olatel th the firm. Some of these coste, it right be argued, are not absonite"y neessary for the operation of the business; but for the sake of onvonicne (ce acceptance), they are commony incurred. Such costs are difficuit te estimate. Thus, in this study, a base allowance of $\$ 3600$ per yoar wan shertiod for sundry expenses.

In shrimp fishing, the captain and first mate of the vessel are commonly paid on a "lay" basis wherein they receive an agreed upon percentage of the revenue earned oy the vessel. The third crew member, who is called a header, is typically paid ox a per box basis. An allowaree for his wages was included in the value of the production cost per vessel. For vessels of the type being considered, the "lay" for the captain and first mate is commonly 35 percent (who typically pay for $2 l l$ of the groceries).

In interviewing tie cooperating f'rms, the relative resale value of the vessels sold was foura to be fairly well approximated for vessels five to six years old oy summen the accumulated depreciation fractions with an appropriate adjustment for technological improvement. This procedure using the largest possible tecmology factor ${ }^{3}$ gave a 0.65 value for $\beta$ :

## (.091) $\left.5+\left[(1.02)^{5}-\right].\right]$.

In the specification of the owner's expected annual revenue per vessel, $E_{t}$, the log of the real shrimp price received by the cooperating firms, $P_{t}$, was regressed or the log of the index of real per capita income, $y_{t}$, and the $\log$ of the per unst effort landings, $l_{t}$, (caught in depths beyond 10 fathoms off the Texas coast), as reported in the earlier study by Thompson et al.[3, p. 12.. The resulting estimated regres. sion equation waz:

$$
\begin{gathered}
\left.\ln \mathrm{p}_{\mathrm{t}}=-4.571+1.175 \ln y_{t}-.379 \operatorname{In~}_{(\mathrm{t}=3.6)} \cdot \mathrm{I}_{\mathrm{t}}^{2}=.748, \quad \sigma \mathrm{e}=.0888 \mathrm{t}=3.5\right)
\end{gathered}
$$

Variations in landings for a ven fishery (like the one off the Texar coast) are still regarded by bologists as being largely random. Thus to remove the effect of land mes or price, landings were specified to be equal to the mean value obsemed for the Pexas fishery in the period 1958 through 2967 . Hence, the mrirs estimating equation (with base year 1969) was:
$\ln p_{t}=-1.332+1.175 \ln y_{t}$.

To use this equation, it was recessary to project the index of real income per capita for the five years 1970 through 1974. This was done by regressing $\ln y_{t}$ on tine, $t$, for the years 1953 through 1960 , and also for the years 1961 through 1968 . The following two income projection
equations were developed for the period $t=1970, \ldots, 1974$.
Specification $I:$ L. 5 rate of growth in real per capita income

$$
\ln y_{t}=4.94+.015 t
$$

Specification II: 3.0 rate of growth in real per capita income

$$
\ln y_{t}=4.94+.033 t
$$

To convert to money temn, the prosected prices from these equations were multiplied by the vaine tife onnumor proe index (with base $1957 / 59=$
 each year thereafter. ratine mornot or the prosected price and the expected anmal land ne por wos with an adjusment for the lay fraction, the owner's expectec arnal revenus per vessel was obtained. The expected anmal landeng mor vacol used in this otudy was the average of the landings per vesso stained by the cooperating firms in the period 1958 through $1969(57,560$ pound of heads off shrimp). There was, of course, a steady rate of teura: Egical improvement in that period so trat this averabe is likely to be qu manerestimate of a 73 foot vessel.'s annual catch potential. Tha, tis vazue of the oxpocted annual omer's revenue per vessel for eani stiphaten economic growth rate, E ${ }_{t}$, is a more conservative estimete thar posibly is the case, it might have been fur her inoroased for expered tecinclogical improvements.

For the first set cf four problems, the estimete of the owner's lowest annual revenue per vessel, $\mathcal{I}_{t}$, was found by taking the lay residual of the product of tre jof chrerp price and the projected lower bound for landing per vessel. This fower bound was taken to be 3.4 standard deviations (i.n $t$ units for 11 doreces ef freedom) below the mean landing per vessel of 57,560 pounds w'. 'rn tne ample standard reviation being 5,731 pounds. Thus, the probab: $1:-y$ the 1 anding per vessrig being greater than this lower bound (ascuming thia to be e valid probability basis) is greater than 99. Moreover, since the growth rate in real per capita income was implicitly taken to be zero, the probability of revenue per vessel falling below the implied estimate of the owner's lowest annual revenue per
vessel (where the price is projected under either specification) decreases steadily as the plannire perion minolds. In other words, the estimate of $\mathbf{L}_{t}$ is very conservative for the jear 1970 and becomes increasingly conservative thereafter in the planning pericd.

For the second set, tif: problers in which the shrimp price is random as well as the landing per vessel, the same value was used for the owrer's lowest ammal rovenue ner vesaf. . This resulted in a slightily smaller probability of survival than i: the first four problems (because of the additional randomess in the prime), bit one still greater than .99 . Thus, in the interest of simplisity, the same value of $I_{t}$ was used in both sets of problems.

Knowledgeable indietry representatives (who were consulted with regard to the above specificatione) indinated a five year survival period would be esoecially meaningful for fros oferating the 73 foot trawlers. Accoriingly, two five year sequencos of runom revenues per vessel were developed with only the landing per vessel being random ir the first sequence. Landings per vessel were regarded as independent, af' price, since the fishery has a relatively competitive structure; moreover, Aor the period studied, per vessel landings for the cooperating firms were not h: Ehly correlated with landings per unit of effort in the mexas fishery ${ }^{5}\left(r^{2}=.76\right)$. Using the regression estimate for price in each year 1970 through $19^{\circ}$ and the estimated standard error of the regresm sion, and also using sample mean and standard deviation for landings per ves. sel of the cooperating firms, the random prices and landings per vessels were calculated as follows: (1) By use of the Box-Muller [1] method, normal random deviates for prices and landings per vessel were independently generated; and (2) the products of these two random variables were adjusted for the lay and changes in the purchasing power cf money. The following random sequences were accordingly obtained and used in the analysis.

Random Sequences of Revenues per Vessel
Sequences No. 1

| Problems 1\& 2 | Problem 3 | Problem 4 |
| :---: | :---: | :---: |
| $\$ 30,741$ | $\$ 36,141$ | $\$ 31,413$ |
| 42,572 | 48,233 | 44,457 |
| 39,859 | 45,795 | 42,531 |
| 39,797 | 46,020 | 43,393 |
| 50,784 | 57,308 | 56,583 |
|  | Sequences No. 2 |  |
| Problem1 | Eroblem 3 |  |
| 25,450 | 29,920 |  |
| 47,261 | 53,546 |  |
| 38,810 | 44,589 |  |
| 36,077 | 42,719 |  |
| 44,747 | 50,495 |  |

It may be helpfui to recall that the decision maker is regarded as being a better than average manager in Problen 3. The 2.5 percent rate of real economic growth per capita is used in Proolems 2, 2 and 3; and the 3.3 percent rate of economic growth is used in Problem 4.

In evaluating the solutions to the first set of four problems in Table 2, the results indicate the profitability of investing in shrimp fishing capacity during the 5 year planning period. The model fisherman opted for investing in fishing capacity in Problem 2, even though he had the option to leave his money in savings at 8.5 percent interest. Thus, the rate of return over cost from shrimp fishing was greater than 8.5 percent. In further analysis, it was found to continue to be until the rate of interest reached 9.5 percent; then the rate
of return over cost switched in favor of savings.
The value of better than average management is indicated by the results in Problem 3. There, the aycrape landing per vessel was taken to be one standard deviation (5,73.1 pounds) preateen than in Problem . . The same amount was invested in the first year; but in the reond and third years there were striking differences. "he model f'isheman bought 5.8 vessels in Problem 2, while he did not buy any in Problem. in fonce $=0$ pay off debt in the first problem atter the initial Envestment, sinca that represented a more profitable use of his money. It may be noticed that the invesjment upper bound limited the size of the purchases in the first three years of Froblem 3 (and the first year of Problem 1). The marginal value of anotrer vessel was positive; however, the money was not available for investment (given the desire to survive).

Success in shrimp fishing is clearly influanced by the rate of incone growth In the economy, compare Problems 1 and 4. In Froblem 4, the marginal value of another vessel is almost twice as large in the first year as in Problem 1 , and remeins large in the second year when the value in the first problem goes negative. This increased erowth in pe: capita income results in an incressed ability to invest in the second year in Problem 4 and still further increased ability, at a lower marefor ircentive, in the third year. The model fisherman carrys a considerably larger neot load, as a result of the increased profitableress, in Probiem 4 thar i. Problem 1.

In evaluating the econd set rf results given in Table 3 and comparing these sclutions to the cren in Tabic 2 , only slight differences between the results may ba neticed. Gombiat ees is srvested over the planning pertod in Problem 3 in the soconi sase than in the first. Also, a slightly larger aebt loak wa generally carried in most of the planaing period.

The marginal investment incentives were, of course, the same in both sets oi problems, they are based on expected values. Vagrancjes in landings seem to be much more important than unexpeted variations in price.

TABLE 1. Values of the Parameters for the Survivel Problems

Problems

|  | 1 | 2 | 3 | 4 |
| :---: | :---: | :---: | :---: | :---: |
| $\mathrm{N}-$-number of years in planning period | 5 | 5 | 5 | 5 |
| $Z_{0}$--initial cash balance <br> in dollars | 0 | 96,145 | 0 | 0 |
| $y_{0}$--initis. $n$ number of boats in fleet | 1 | 0 | 1 | 1. |
| $\begin{aligned} & x_{0}-\text {-initial investment } \\ & \text { in dollars } \end{aligned}$ | 100,000 | 0 | 100,000 | 100,000 |
| $\gamma$-manual interest rate per dollar | . 085 | . 085 | . 085 | . 035 |
| ${ }^{T} t^{--a n n u a l}$ production cost per vessel in dollars | $\begin{aligned} & 30,000 \mathrm{x} \\ & (1.03)^{\mathrm{t}} \end{aligned}$ | $\begin{aligned} & 30,000 \mathrm{x} \\ & (1.03)^{\mathrm{t}} \end{aligned}$ | $\begin{aligned} & 30,000 \mathrm{x} \\ & (1.03)^{\mathrm{t}} \end{aligned}$ | $\begin{aligned} & 30,000 \mathrm{x} \\ & (1.03)^{t} \end{aligned}$ |
| $\sigma_{t}$--per vessel purchase price in dodlars | $\begin{gathered} 1.00,000 x^{t} \\ (1.03)^{t} \end{gathered}$ | $\begin{gathered} 100,000 x^{x} \\ (1.03)^{t} \end{gathered}$ | $\begin{gathered} 100,000 x^{x} \\ (1.03)^{t} \end{gathered}$ | $\begin{gathered} 100,000 \mathrm{x} \\ (1.03)^{t} \end{gathered}$ |
| e--annual depreciation fraction per dollar invested | . 091 | . 091 | . 091 | .091 |
| $\zeta$--annual income tax rate per dollar of taxable income | . 25 | . 25 | . 25 | . 25 |
| $\beta-$ recoverable fraction of the investment in fishing capacity | . 65 | . 65 | . 65 | . 6 |
| $\Delta$--annual cash withdrawal for sundry expenses in dollars | $\begin{gathered} 3,600 x \\ (1.03)^{t} \end{gathered}$ | $\begin{gathered} 3,600 \mathrm{x} \\ (1.03)^{\mathrm{t}} \end{gathered}$ | $\begin{gathered} 3,600 \mathrm{x} \\ (1.03)^{\mathrm{t}} \end{gathered}$ | $\begin{aligned} & 3,600 \mathrm{x} \\ & (1.03)^{\mathrm{t}} \end{aligned}$ |
| $\mathrm{E}_{\mathrm{t}}$--owner's expected annual revenue per vessel in dollars* | $\begin{aligned} & 49,790 \mathrm{x} \\ & \hat{p}_{t}(1.03)^{t} \end{aligned}$ | $\begin{aligned} & 49,790 x \\ & \hat{p}_{t}(1.03)^{t} \end{aligned}$ | $\begin{aligned} & 54,400 \mathrm{x} \\ & \hat{\mathrm{p}}_{\mathrm{t}}(1.03)^{\mathrm{t}} \end{aligned}$ | $\begin{aligned} & 49,790 \mathrm{x} \\ & \mathrm{P}_{t}(1.03)^{\mathrm{t}} \end{aligned}$ |
| $L_{t}$--owner's lowest anmual revenue per vessel in dollars | $\begin{aligned} & 22,500 \mathrm{x} \\ & (1.03)^{t} \end{aligned}$ | $\begin{aligned} & 22,500 \mathrm{x} \\ & (1.03)^{t} \end{aligned}$ | $\begin{aligned} & 22,500 \mathrm{x} \\ & (1.03)^{t} \end{aligned}$ | $\begin{aligned} & 22,500 \mathrm{x} \\ & (1.03)^{t} \end{aligned}$ |

[^0]TABIE 2. Solutions to Four Survival Problens in Table l, Landings Per Vessel are Random.

| Probiem | Year | Marginal Value of Another Vessel (dollars) | Inves tment in toats (number) | Boats Owned (number) | Cash Balance (dollars) | Debt to Gross Asset Ratio |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 0 | - | - | 1.00 | 0 | - |
|  | . 1 | 5,843 | 1,44 | 2.44 | -146,356 | . 5 |
|  | 2 | -784 | 0 | 2.44 | -127,678 | . 48 |
|  | 3 | -7,896 | $\bigcirc$ | 2.44 | -116,862 | . 43 |
|  | if | - $1.5,471$ | 0 | 2.44 | -108,022 | . 38 |
|  | 5 | -23,490 | 9 | 2.44 | $-74,436$ | . 26 |
| 2 | 0 | - | $\cdots$ | 0 | 96,145 | - |
|  | 1 | 5,843 | 2.42 | 2.42 | -1145,083 | . 57 |
|  | 2 | -781 | 9 | 2.42 | -126,507 | . 48 |
|  | 3 | -7,896 | 0 | 2.42 | -115,728 | . 43 |
|  | it | $-1.5,47$. | 0 | 2.42 | -106,908 | . 38 |
|  | 5 | -23,490 | 9 | 2.42 | -73,534 | .26 |
| 3 | 0 | - | "* | 1.00 | 0 | - |
|  | 1 | 21,419 | 1.44 | 2.44 | -136,487 | . 53 |
|  | 2 | 16,198 | 1.13 | 3.57 | -216,534 | . 56 |
|  | 3 | 7,080 | 4.03 | 7.59 | -581,958 | . 68 |
|  | 4 | -5,56\% | 0 | 7.59 | -511,662 | . 58 |
|  | 5 | -18,570 | 0 | 7.59 | -353,977 | . 40 |
| 4 | 0 | - ${ }^{\text {- }}$ | - | 1.00 | 0 | - |
|  | 1 | 10,655 | 1.44 | 2.4 .4 | -145,128 | . 56 |
|  | 2 | 9,942 | . 80 | 3.23 | -240,502 | . 58 |
|  | 3 | 2,624 | 3.15 | 6.38 | -503,596 | . 70 |
|  | 4 | -7,595 | 0 | 6.38 | -462,898 | . 63 |
|  | 5 | -19,119 | 0 | 6.38 | -341,999 | . 45 |

TABLE 3. Solations to Two Survival Problems in Table 1, Shrimp Prices anc Landings per Vessel are Rendon.

| Problem | Year | Marginal Value of Another verse]. (doliars) | rivestment in boats (number) | Boats Owner (number) | Cash Balance (Collars) | Iebt to Gross Asse $\ddagger$ Ratis |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | $\bigcirc$ | - | $\cdots$ | 1.00 | 0 | 0 |
|  | 1 | 5.843 | 1.94 | 2.44 | -156,026 | . 60 |
|  | 2 | -781 | 0 | 2.44 | -126,538 | .48 |
|  | 3 | $-7,896$ | a | 2.14 | -118,206 | . 43 |
|  | 1 | -15, 12.4 | a) | 2.44 | -118,517 | . 42 |
|  | 5 | -23, 4 4, | 0 | 2.44 | -10,311. | . 34 |
| 3 | 0 | - | $\cdots$ | 1.00 | 0 | . |
|  | 1 | 21,497 | 1. $.^{1} 4+$ | 2.44 | $-1+7,856$ | . 57 |
|  | 2 | 16,19 | . 159 | 3.13 | -176,192 | - 2 ? |
|  | 3 | 7,080 | ! \% ? | 7.31 | -569,170 | . 69 |
|  | + | - 5,56 | 0 | 7.34 | -533,307 | . 63 |
|  | 5 | -. 8 , 5770 | $\cdots$ | 7.34 | -4:3,263 | . 50 |

## Footnotes

1. This oost figure is $\$ 1,000$ higher in the base year than the one usec. by Thompsor et ale [3].
?. [n efoct, the owner only gets 65 percent of the exvessel pricen
2. Jhis was done since the vintage was not kept track of in the model.
3. To havo a probability support at $I_{t}$, it is being implicitly assumed that this mal. probability of non-survival is insurable. This point ras p.intod at by Roverts R. W....an.

50 Landinps per unt effort in the Texas Fishery were highly correlated with lamdites per urit eifort for the Gulf and South Atlantic.

## References

1. Box, G.E.P., and Mervin E. Muller, "A Note on the Generation of Random Normal Deviates," The Annals of Mnthematical Statistics, Vol. 29, (June, 1958), pp. 610-611.
2. Thomp:on, Russeli. G., and Melvin D. George, "A Stochastic Investment Model. for a Survival Conscious F'imn," submitted to Management Science for review.
3. Thompson, Russell G., Richerd W. Callen, and Lawrence C. Wolken, Optimal Tnvestment and Financial Dectsions for a Model Fishing Firm, Texas ABM iniversity, See Grant Frogram, TAMU-SG-70-205, College Station, April 3970.

## Appendix

Appendix Table 1. Values of Projected Index of Real Per Capita Income

| Vear | Specification I | Specificaticn II |
| :---: | :---: | :---: |
| 1 | 136.98 | 1.39 .52 |
| 0 | 139.06 | 144.27 |
| 3 | 141.17 | 249.19 |
| 4 | 143.32 | 154.27 |
| 5 | 145.50 | 159.53 |

Appendix Table 2. Values of Projected Real Shrimp Prices

| Year | Specification $I_{,}, \hat{p}_{t}$ <br> (cents per pound) | Specification IT, <br> (cents per pound) |
| :--- | :---: | :---: |
| 1 | 85.68 | 87.56 |
| 2 | 87.72 | 91.07 |
| 3 | 88.78 | 94.73 |
| 4 | 90.37 | 98.53 |
| 5 | 91.99 | 102.49 |

Appendix Toble 3. Values of Landings per Vessel for Random Sequences l. and a

| Year | Problems 1, $2 \& 4$ <br> (pounds) | Problem 3 <br> (pounds) |
| :---: | :---: | :---: |
| 1 | 41,965 | 49,336 |
| 2 | 55,435 | 62,806 |
| 3 | 49,501 | 56,872 |
| 4 | 47,140 | 54,511 |
| 5 | 57,375 | 64,746 |

Appendix Table 4. Values of Real Shrimp Prices for Random Sequence 2

| Year | Problems 1 \& 3 <br> (cents per pound) |
| :---: | :---: |
| 1 | 70.93 |
| 2 | 96.82 |
| 3 | 86.44 |
| 4 | 81.92 |
| 5 | 81.05 |

Appendix Table 5. Values for Expected Revenues per Vessel

| Year | Specif'ication I <br> (dollars per vessel <br> per year) | Specification II <br> (dollars per vessel <br> per year) |
| :---: | :---: | :---: |
| 1 | 42,231 | 43,154 |
| 2 | 41,277 | 46,234 |
| 3 | 46,121 | 49,533 |
| 4 | 43,670 | 53,068 |
| 5 | 51,028 | 56,355 |


[^0]:    Fee Appendix Table ; for numbers used.

