OPTIMAL INVESTMENT AND FINANCIAL DECISIONS FOR A MODEL SHRIMP FISHING FIRM

Prepared by

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ABSTRACT

In this study, the shrimp industry is briefly reviewed historically, and some of the factors influencing the demand for shrimp as well as some of the factors affecting the supply are discussed. The need for better aids for investment planning is pointed out. This is followed by the development of a mathematical investment-financial model for the shrimp fishing firm. Each facet of the model is discussed with a rigorous statement of the complete decision-making model. It is then shown how this first model may be transformed into a form for computational purposes: a sequential linear programming model. An economic interpretation is provided for the possible corner solutions to this problem.

Using this mathematical framework, the model is applied to the case of a relatively small investor who has decided to enter the shrimp fishing business with one 73 foot steel hull vessel. It is assumed that this model investor has borrowed 75 percent of the value of the $100,000 investment and has $5,000 in savings. Values for the remaining parameters in the model are then specified in accordance with information obtained from knowledgeable industry representatives in the Aransas Pass, Texas, area with the expected values for the shrimp price per pound and the annual catch per boat being varied systematically (one at a time) over a range of values observed in the 1960s. Solutions to six feasible problems were computed and discussed. The results indicate how sensitive the investment-financial problem in shrimp fishing is to variations in the expected price and catch.

Summary and Conclusions

In this study, we have attempted to describe the factors stimulating the demand for shrimp and the factors influencing the supply of shrimp. The former indicated how responsive the demand for shrimp is to income and price variations, and why the shrimp price has been relatively favorable in the sixties. The latter served to point out that the shrimp fishery, because of the short life span of each individual in the species, may be different from other fisheries where the fish have longer life spans.

Biological studies to date seem to indicate that the shrimp catch from a given fishery is predominately affected by random factors associated with water properties and weather conditions, and affected relatively little, if any, by fishing effort. Thus, for all practical purposes, the catch from a given fishery may be regarded as a constant with random variation about this parametric value.

With investments in larger more expensive boats, this biological observation means that the catch per boat in a given fishery can be expected to decrease proportionately with the capacity added to the fishery. The size of this decrease may be relatively insignificant if only one firm adds another boat, but quite large when all of the additional boats are taken into account. Thus, what is happening in the fishery at large needs to be monitored by a shrimp fisherman just like what is happening in the economy at large.

Taking these factors into account, the purpose of this study was to make a start toward developing mathematical aids for investment-financial decision making in the shrimp fishery. The model developed in this study still has many limitations, which the user needs to keep clearly in mind. Work is presently underway to overcome some of the most limiting weaknesses. However, while that work is progressing, the shrimp fisherman may be able to obtain useful guidelines for investment decision making from the model developed and applied to six problems in this study.

In the model, the shrimp fisherman maximizes net worth over a finite number of years. This net worth value represents the sum of the terminal money capital position plus the final value of boats, where the latter is adjusted for inflation and obsolesence. In attaining this objective, the model shrimp fisherman is restricted by limitations on borrowings and solvency requirements, in particular.

In accordance with the information obtained from knowledgeable industry representatives and present money market conditions, values of all of the parameters except price and catch per boat were specified first. We assumed that 75 percent of the investment in fishing capacity could be financed on a 10 year basis at an effective borrowing rate of $9\frac{1}{2}$ percent per year: the present prime rate plus one percent for mortgage insurance. The savings rate was taken to be the present 5.5 percent value. Initially, the firm was assumed to have had purchased one new boat and to have used $25,000 of its $30,000 equity to purchase it. This left a savings account balance of $5,000. To reflect expected inflation, purchase prices of new boats were specified to increase 3 percent per year; and annual operating costs per boat were assumed to increase 3 percent per year from the 1969 base of $26,000. Each boat investment was depreciated over an 11 year period for tax purposes; the income tax rate was 20 percent. The boats were assumed to have an effective life of 20 years.

For annual catches varying from 60,000 to 80,000 pounds and for prices varying from $ .80 to $1.00 per pound, six problems were found to have feasible solutions for a 10 year decision-making period. It was not possible for the model fisherman to remain solvent when no investments were made in the following cases: (1) catch of 80,000 pounds and a price of $ .70 per pound or less; or (2) a catch resulting in a total revenue to the owner of less than $56,000 per year.

Solutions to these six problems were computed and discussed. This involved a discussion of the maximum net worth for each of the six problems, and the optimal values of the decision variables as well as the form of the investment pattern.

From the solutions to the six problems, the sensitivity of the shrimp investment problem to price and catch variations is clearly evident. High prices and large catches are conducive, of course, to a favorable investment climate. Here, we have developed quantitative indications of how favorable each variation is. Where the price was one dollar a pound and the catch was 80,000 pounds, the rate of return from fishing was greater than the borrowing rate in the first six years and was less than the savings rate in the 7th, 8th and 9th years. The model fisherman found it profitable to invest all of his savings in each of the first six years supplementing these savings by the maximum possible borrowings. Investment opportunities outside of fishing provided a higher rate of return in years 7 through 9.

Where the price was $.80 per pound for a catch of 80,000 pounds, the investments made in new fishing capacity by the model fisherman were relatively small. The rate of return from fishing exceeded the borrowing rate in the first three years, and was between the two interest rates in the 4^{th} year. In all of the remaining five years, the rate of return from fishing was less than the savings rate. Thus, other alternatives to fishing paid a higher rate of return than fishing in five of the nine years.

Optimal Investment and Financial Decisions

For a Model Shrimp Fishing Firm

by

Russell G. Thompson, Richard W. Callen, and Lawrence C. Wolken
(Texas A&M University)

1. Introduction.

Of the major fisheries in the United States, the shrimp industry has experienced the largest **value** catch of any species in a number of years; and it is one of the few which has had value increases in recent years (see Figure 1). Although the shrimp catch compromises more than 20 percent of the total value of the fish landed domestically in recent years, the total poundage of the species is less than 7.5 percent of the U.S. catch.

In 1967, the shrimp fishery became the first to exceed $100 million in annual landings with the exact value for that year being $103 million. Thus, not only is the shrimp fishery the nation's most valuable one, but it is the only one that has shown any appreciable growth in recent years.

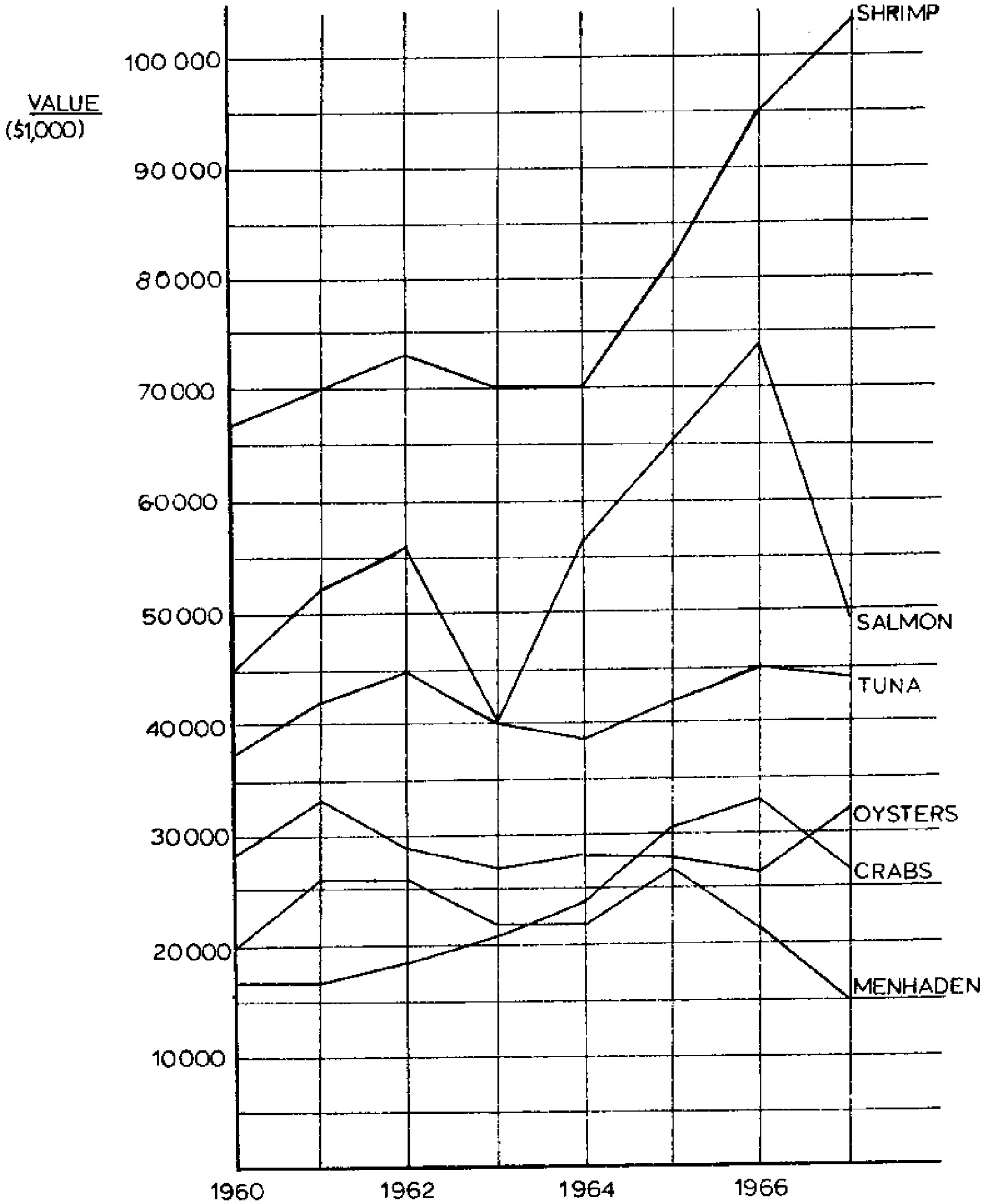


FIGURE 1: VALUE OF LANDINGS OF SIX LEADING SPECIES, [8].

2. Some Historical Background

The shrimp fishery first developed in the bays and lagoons of the Gulf Coast and in shallow coastal waters of the South Atlantic waters. Shrimp were caught largely with large drag seines and cast nets, and along the coast of Mexico in fixed traps. The otter trawl was introduced between 1912-1915, freeing the fishermen from dependance on the seasonal abundance of shrimp in shallow water. However, the catch continued to be taken entirely from bays and shallow water. Then in 1938, large scale production of jumbo white shrimp began on the Ship Shoal grounds off Morgan City, Louisiana. The resultant national publicity helped create new outlets for the shrimp catch. The size of the Gulf shrimp fleet increased greatly, and a number of large offshore trawlers were built. Although the addition of new boats to the fleet was stopped during World War II, all white shrimp grounds in the northern Gulf were known and being exploited shortly after the end of the war. The decline of white shrimp production, however, when the demand was high and previous high earnings had attracted many new fishermen (occurring after the end of the war), led to the production of brown shrimp.

Although brown shrimp had been sold for many years, most of them were dried, canned or peeled. Because of the color, the quantity of brown shrimp that could be sold on the fresh market was at first very small. During the first half of 1947, the production of shrimp was very low along the Texas coast and often the boats would bring in mixed catches of white and brown shrimp. The proportion of whites and browns

(50 per cent or more browns) was such that it was impossible to market them in one package. A producer-dealer of Aransas Pass was approached by a group of fishermen who wanted to set up a cooperative marketing agency for their shrimp. He acted as a broker for these vessel owners, who were anxious to push the sale of brown shrimp, until the cooperative was officially established during February 1948. The first carload of brown shrimp was shipped to San Francisco during August 1947. They were carefully graded to uniform size and shipped at cost to develop the market. Most of the brown shrimp that were sold the first year were handled through this brokerage arrangement and the cooperative. After the first six months the market was strong enough to handle all the brown shrimp produced by members of the cooperative. Almost 90 per cent of the production of brown shrimp during 1947 and 1948 was caught along the Texas coast with the greatest production at Aransas Pass. Brown shrimp production developed somewhat later off Mississippi, Alabama, and Louisiana, though the grounds had been known to be there.

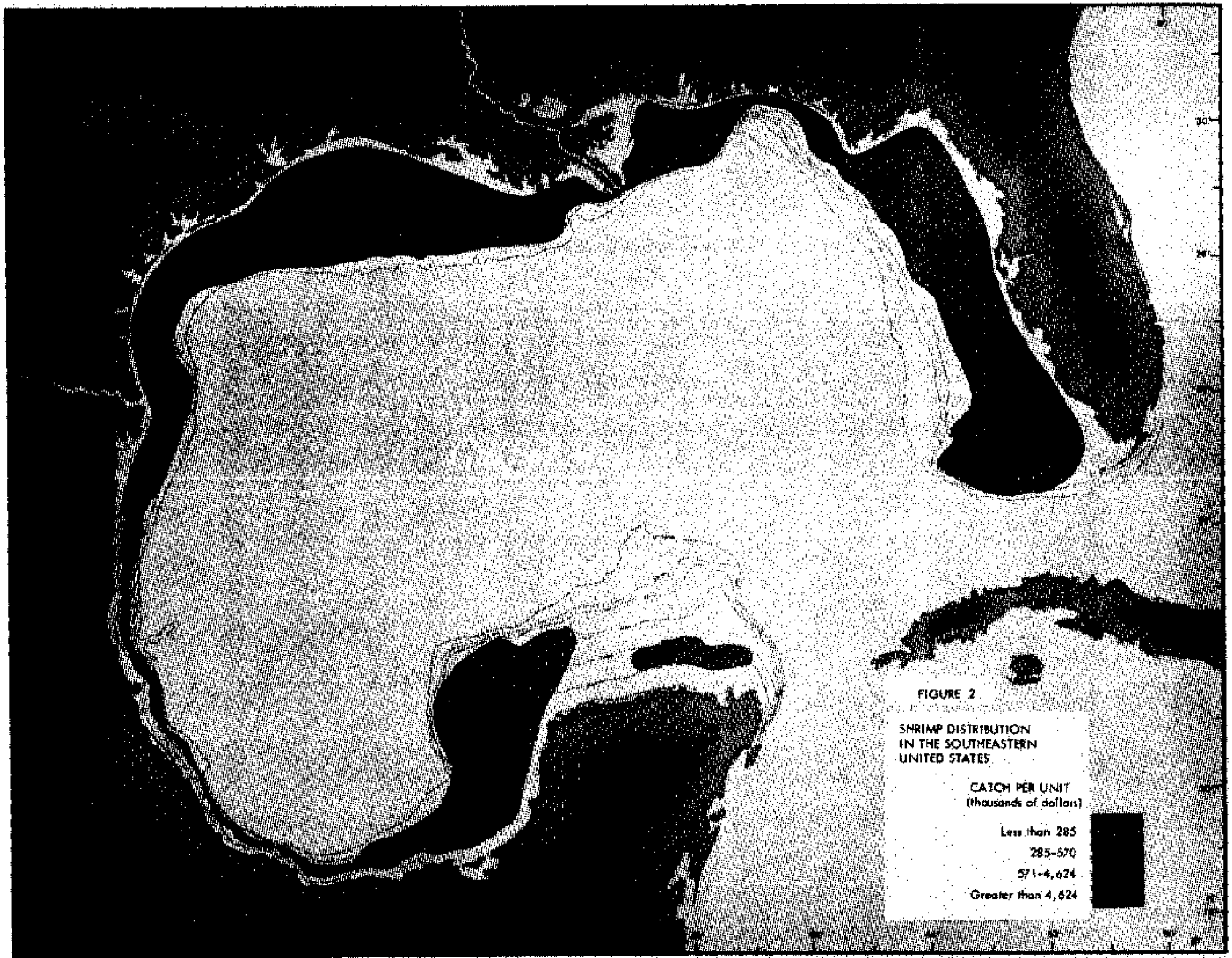
Pink shrimp production in the northern Gulf has always been insignificant (less than 1 per cent of the total catch). The one area where this species is found in abundance is the Tortugas grounds. Exploratory fishing was begun in the Tortugas area during September 1949 following accounts of successful night fishing for grooved shrimp in Texas. Results were encouraging and commercial operations began in January 1950. When the discovery became generally known, rapid expansion followed and during February 1950 an estimated 2,117,000 pounds of shrimp (whole) were landed. Between 250 and 300 boats were fishing there by March 1, 1950.

The fishing grounds are located north of a line drawn from Key West to Loggerhead Key in the Dry Tortugas group. They are approximately 70 miles long and 10-15 miles wide. The bottom is covered by fine calcareous sediments ("coral mud") with some coral obstructions. The large fleet was dispersed by the end of March 1950, and there has been a steady though spectacular production since.

The grounds in the Gulf of Campeche (Mexico) also contain large concentrations of all the species, and these are largely exploited.

At present, Texas is the leading shrimp producing state, with Louisiana second and Florida third, and brown is the leading species taken, comprising over 50 per cent of the total catch. Texas has been the major shrimp producing state since 1951, except for 1963, when Louisiana was first.

The South Atlantic and Gulf Coast of the United States as shown in Figure 2 account for well over 80 percent of the value of the shrimp landed in the United States. The largest concentration of shrimp is along the Gulf coast from the mouth of the Mississippi to the mouth of the Rio Grande. One of the densest populations is between Galveston and Aransas Pass. Between 37-45 per cent of the value of U.S. shrimp landings came from the Texas coast in the ten year period from 1958 through 1967.



MAP COURTESY OF BUREAU OF COMMERCIAL FISHERIES, DEPARTMENT OF INTERIOR

3. Factors Influencing the Demand for Shrimp

In the ten year period from 1958 through 1967, the per capita consumption of shrimp in the United States rose by 60 percent from around one pound in 1958 to about 1.6 pounds in 1967. The increased per capita consumption was associated with a 30 percent increase in real per capita income--per capita income adjusted for the purchasing power of money. Real per capita income grew relatively steadily at around three per cent per year in this period, while per capita shrimp consumption grew more rapidly, but irregularly, at around six per cent per year (see Figure 3). Variations in consumption about this trend are well-explained by price, as indicated in Figure 4. For example, in 1959 and 1960 when landings were relatively high, the shrimp price fell and per capita consumption exceeded the six percent trend; and in 1962 when landings were relatively low, the price rose and per capita consumption was less than the six percent trend.

To measure more precisely these economic relationships, Waugh and Norton [10], and Doll [1] have recently completed two rigorous analytical studies. Both of these studies have reported (as indicated by the graphs) high positive responsiveness of consumption to income, and high negative responsiveness of consumption to price. This can be more accurately expressed in terms of price and income elasticities of demand. The income elasticity of demand gives the relative increase in consumption typically associated with a one percent increase in income; the price elasticity of demand gives the relative decrease in consumption typically associated with a one percent increase in price.

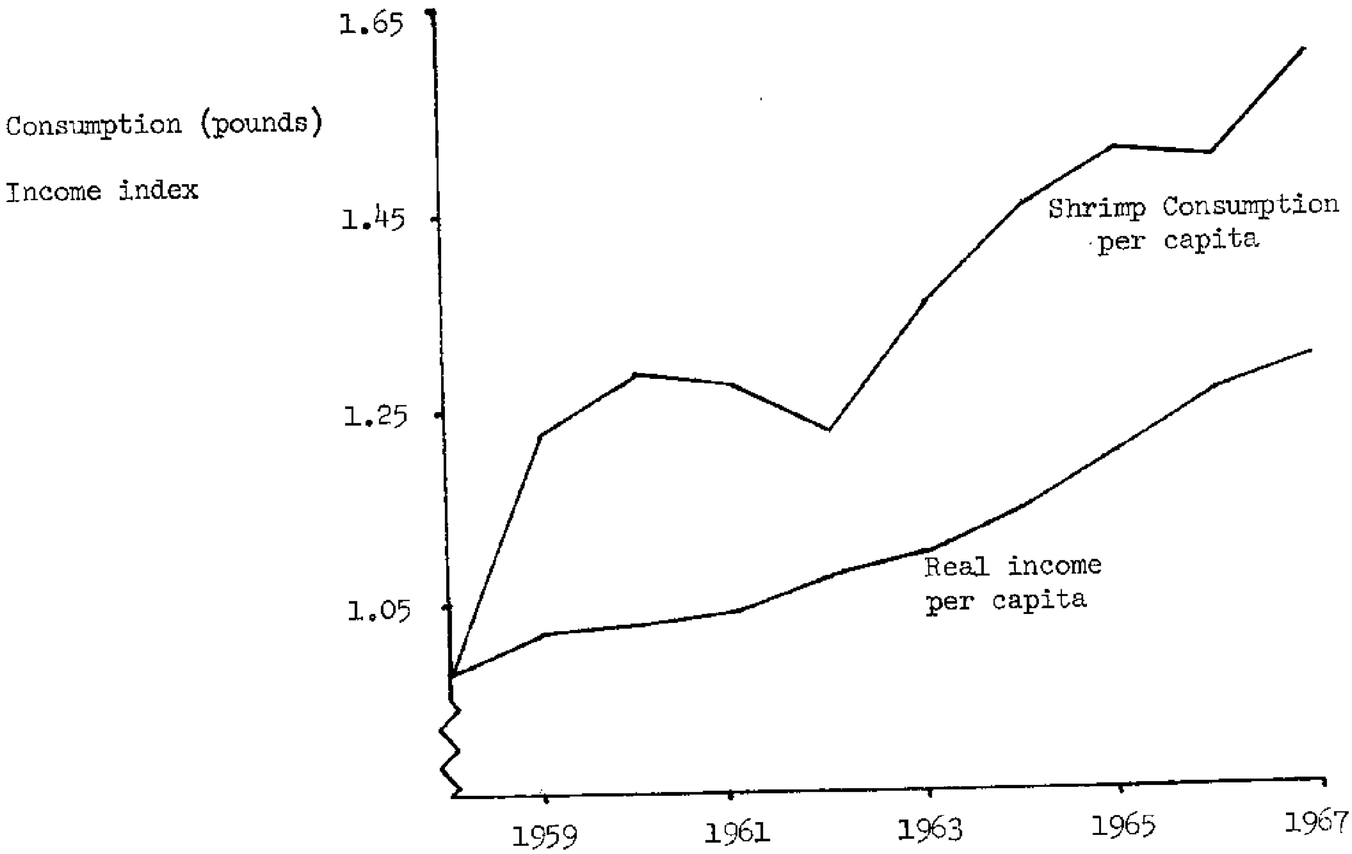


Figure 3: Per capita shrimp consumption and real income versus time, [8], [5]

Figure 4: Exvessel shrimp price per pound versus time

Source: U. S. Department of the Interior Price Records

If either of these elasticities is larger than one in absolute value, the demand is said to be elastic; and if either is less than one in absolute value, the demand is said to be inelastic.

Both of the above studies reported results indicating relatively high income and price elastic demands for shrimp. Waugh's estimates may be interpreted to imply an income elasticity of demand of 2.1 and a price elasticity of -1.2. Doll's estimates in which he reported an income elasticity of 1.4 and a price elasticity of -1.3 indicate similar demand characteristics.

In summary, the demand for shrimp seems to be clearly price and income elastic. Projected growth in real per capita income is one factor every investor in shrimp fishing capacity needs to monitor.

4. Factors Influencing Supply.

Biological considerations.

In a 1962 report by Kutkuhn [2, p. 344], the biological life cycle of common penaeid shrimp was described as follows:

> In general, eggs are fertilized and spawned in the oceanic habitat of the parent shrimp. After a short incubation period, a small larva or nauplius emerges. Rapid growth accompanied by gross morphological changes ensues, the larva, now a component of the zooplankton, being carried shoreward into broad and shallow estuaries. Transformation to adult likeness and habits occurs somewhat before or as the larva enters inshore waters. Here the shrimp, now a postlarva or juvenile, maintains rapid growth for the next 2 or 3 months. As maturation approaches, it departs from the "nursery" grounds, returning to the parental offshore habitat where its life cycle is completed. The average life span of the more important penaeids is thought to approximate 18 months although there are indications that many female shrimp continue to breed to a more advanced age, tending to make this estimate somewhat low.

For the brown shrimp, which prevail in economic importance on the Texas Gulf Coast, the spawning period is estimated to be largely from late August to mid-November; the in-nursery period encompasses most of February and March with the juveniles leaving the nursery in the first part of May. The young shrimp become suspectible to fishing around June 1 with the important part of the season being in the late summer and early fall. Thus in reality shrimp is an annual crop given the life span and the age when the species is subjected to catch.

To date, biologists have not been able to establish a relationship between the rate of change of the fish population over time and fishing effort. Landings per unit of effort seem to be more affected by random factors such as salinity, conditions of the estuaries, and temperature of the water than by previous shrimp catches. Statistics on landings and effort for brown shrimp caught in depths beyond 10 fathoms off the Texas Coast during the months of July, August, and September in the years 1958 through 1967 are presented in Table 1. If landings per unit of effort are plotted against fishing effort as in Figure 5, there is no apparent Schaefer-type relationship, see Schaefer [3] for example.

TABLE 1. Landing statistics for brown shrimp caught in depths beyond 10 fathoms off the Texas coast during July-September, 1958-67.

Year	Landings (millions of lbs.)	Fishing Effort (thousands of hours)	Landings/Effort (lbs./hr.)
1958	14.57	550.8	26.5
1959	19.70	458.5	43.0
1960	18.87	457.2	41.3
1961	9.40	450.5	20.9
1962	9.73	461.5	21.1
1963	15.86	473.6	33.5
1964	12.36	457.0	27.1
1965	16.31	423.3	38.5
1966	14.53	497.2	29.2
1967	22.44	568.2	39.5

Source: Personal correspondence with Richard J. Berry, U. S. Department of Interior, Bureau of Commercial Fisheries, Biological Laboratory, Fort Crockett, Galveston, Texas.

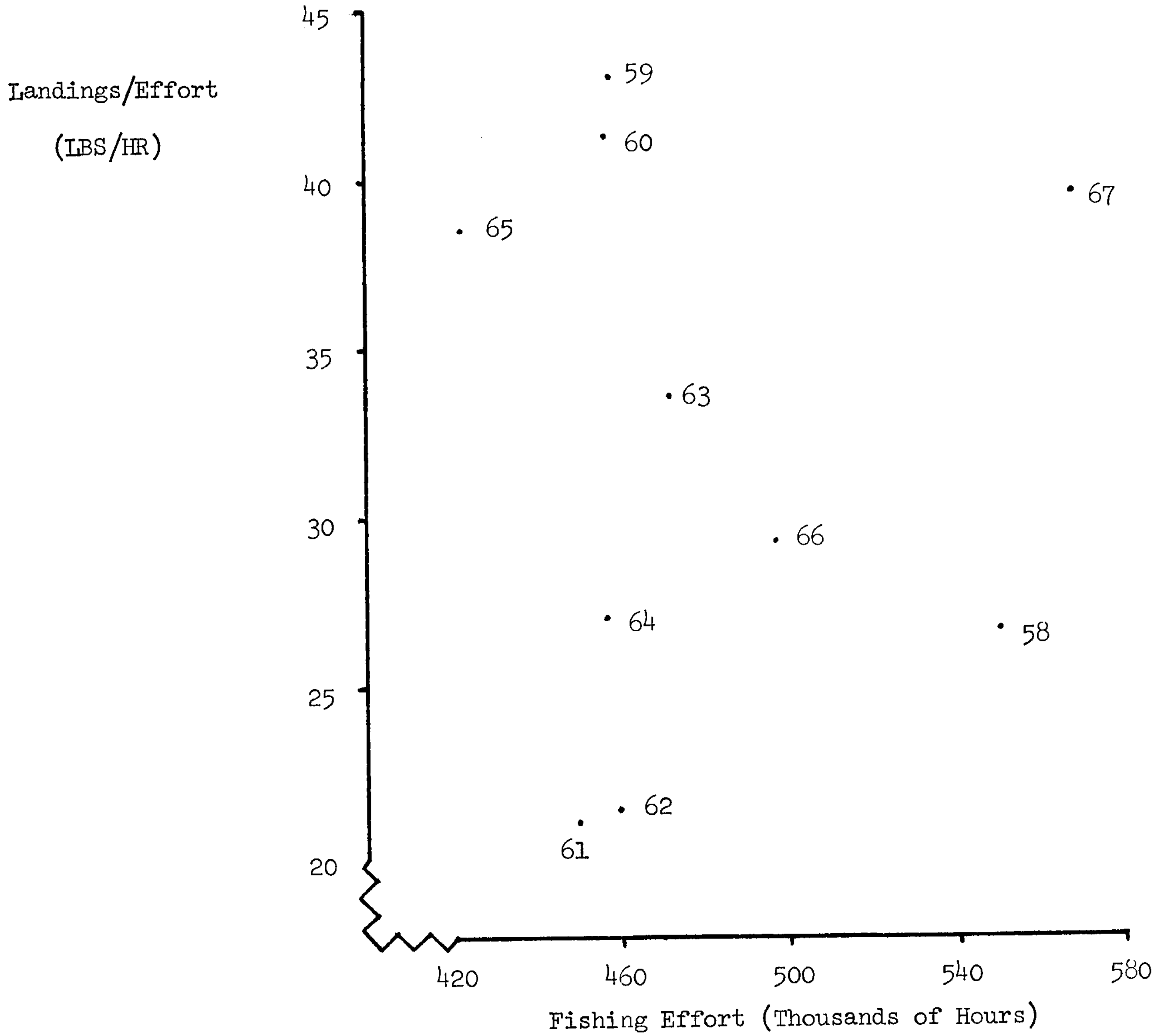


Figure 5: Landings Per Unit of Effort Versus Effort

Vessels in the Gulf fishing fleet.

In the ten year period from 1957 through 1966, the tonnage, length, and horsepower of the average vessel, as documented in the historical fishery statistics of the United States, changed remarkably in power and weight for the Gulf Coast area. In 1957, of the 185 vessels for which information was recorded, the average vessel weighed 51 tons, had 190 horsepower and was 53 feet in length. Vessels documented in 1966 were considerably heavier and more powerful. The average vessel then was only 3 feet longer, but weighed 74 tons and had 280 horsepower, (see [6] for source).

In January 1970, a 73 foot steel hull trawler fully outfitted for fishing as commonly operated on the Texas Gulf Coast cost $100,000, for all practical purposes. This market price, regardless of the technical changes in the vessels since the late 1950's, represents an investment expenditure 30 percent greater than that for a 70-foot steel vessel purchased in 1958 (c.f., [9, Vol. I]).

Since investments in larger vessels represent a larger percentage of fixed committments, and since these committments must still be amortized regardless of prices and landings, investment planning is becoming an increasingly important decision problem for the shrimp fishing firm. More simply, because of the changing cost structure in shrimp fishing, there is less flexibility by which to withstand price and catch adversity today than 10 to 15 years ago.

5. The Purpose of the Study, and Some of Its Limitations

In this study, the purpose was to make a start toward assisting shrimp fisherman in investment planning. This start encompasses the formulation of a straightforward, yet all inclusive, dynamic model for the firm. In the model, the fisherman is visualized as desiring to maximize his net worth over a finite period of time subject to a number of restrictions describing his physical and financial accounts, and also subject to certain limitations on investments and borrowings. The model does not allow for any uncertainty in catch per boat or prices received, since these quantities are assumed to be known for the whole planning horizon at the beginning of the period. Moreover, the model only allows for one type of fishing capacity where in an operating firm boats of several different types might be used. These are examples of the limitations of the study, and some of the reasons why this study serves as a starting point for assisting fisherman in investment planning. Thus, the study is being developed as the first part of what may be a many-part sequel. Work is presently underway to rectify the weaknesses of the model developed in this study. It will be reported stage by stage at later times.

Taking these factors into account, shrimp fisherman may be able to obtain much useful information from the model presented below. It is conceptually simple and straightforward computationally. Also, the results have practical everyday meanings.

Solutions to different problems may be obtained by specifying a relatively small number of parameters. For eight ten-year problems, the computations required about 2.5 minutes on an IBM 360/65 computer. If the time costs $12 per minute, then the computational cost would be $30.

In summary, operating decision-makers should be able to understand the model with limited assistance, program the model as they desire, and interpret the results for their own benefit.

6. The Dynamic Model for the Shrimp Fishing Firm[1].

Description of the model.

In the model, the objective of the fisherman is to maximize the amount of savings held in the last year of the decision-making period, z_T, less the amount of indebtedness outstanding at that time, y_T, plus the value of the boats owned in the last year with an allowance being made for technological depreciation, ψ_t, and inflation in purchase prices, $\sum_{t=0}^{T} \psi_t \tau_t v_t$. There are three sets of difference equations and also three sets of inequality restrictions limiting the size of this objective. Indebtedness, y_t, savings, z_t, and boats owned, x_t, are the state (stock) variables in the model; boat purchases , v_t, and borrowings, w_t, are the control (flow) variables. Initial values of the state variables--number of boats, indebtedness, and savings--are taken as given; final values to the state variables are determined as a part of the solution to the problem.

In each yeat t, the shrimp fisherman in the model must repay a specified percentage of the indebtedness outstanding at the end of the previous year. In case the fisherman chooses to borrow in year t, he cannot borrow more than a fraction of the value of the boat investment in that year. That is, the fishing firm can only borrow money for the purchase of new boats; and in every case, the fisherman must have enough savings in the bank to cover the difference between the maximum loan value and the investment in boats. Letting κ denote the

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[1] This model is an application with some minor extensions of a formulation reported by Thompson and George [4].

fraction (maximum) of the boat investment that can be borrowed, the upper-limit for borrowings in yeat t is $\kappa\tau_t v_t$, where τ_t is the purchase price (per boat) and v_t is the number of boats bought. We may now state the inequality restrictions on w_t as follows:

$$(2.1) \qquad 0 \leq w_t \leq \kappa\tau_t v_t, \; t = 1, 2, \ldots, T - 1.$$

These restrictions mean that in any year t borrowings, which must clearly be non-negative, may occur only if new boats are purchased, and then they cannot exceed the fraction κ of the investment $\tau_t v_t$.

In the model, we do not allow the fisherman to sell boats. He can only purchase boats during the decision-making period:

$$(2.2) \qquad v_t \geq 0 \;, \; t = 1, 2, \ldots, T - 1.$$

Since some time is generally necessary between the time when the decision is made to buy a boat and the boat is operational, the number of boats operated in year t was specified to be the number owned at the end of year t - 1; and accordingly boat purchases in the last year of the planning period were specified to be zero. Thus, the change in the number of boats owned is described as follows:

$$(2.3) \qquad x_t - x_{t-1} = v_t, \; x_o \text{ given}, \; t = 1, 2, \ldots, T - 1,$$

$$x_T - x_{T-1} = 0.$$

In accordance with the final purchase assumption above, borrowings in the last year are also specified to be zero. Moreover, since the fisherman must always repay in year t a fraction β of the indebtedness owed at the end of the previous year, the change in indebtedness is as follows:

(2.4) $$y_t - y_{t-1} = w_t - \beta y_{t-1},\ y_o \text{ given},\ t = 1, 2, \ldots, T-1,$$

$$y_T - y_{T-1} = -\beta y_{t-1}\ .$$

To describe the fishing firm's cash flow, it is helpful to have the following symbols: γ is the exvessel price received by the owner in year t after the lay is paid; λ is the expected catch per boat in pounds of shrimp; ζ is the interest rate paid on debt; ξ is the interest rate earned in savings; σ is the income tax rate; θ_t is the cost of operating a fishing boat in year t; and $g_t(v_i)$ is the depreciation allowed in year t on the boats purchased in year i. Then the difference equations describing the firm's cash flow are:

(2.5) $$z_t - z_{t-1} = w_t - \beta y_{t-1} - \tau_t v_t + (\gamma\lambda - \theta_t)x_{t-1} - \zeta y_{t-1} + \xi z_{t-1}$$

$$- \sigma[(\gamma\lambda - \theta_t)x_{t-1} - \zeta y_{t-1} + \xi z_{t-1} - \sum_{i=0}^{t-1} g_t(v_i)]\ ,$$

$$z_o \text{ given},\ t = 1, 2, \ldots, T-1,$$

$$z_T - z_{T-1} = -\beta y_{T-1} + (\gamma\lambda - \theta_T)x_{T-1} - \zeta y_{T-1} + \xi z_{T-1}$$

$$- \sigma[(\gamma\lambda - \theta_T)x_{T-1} - \zeta y_{T-1} + \xi z_{T-1} - \sum_{i=0}^{T-1} g_t(v_i)].$$

In every year except the last one, the change in savings is equal to the change in indebtedness less the boat investment plus the earnings retained after taxes. Before tax earnings equal net revenues to the boat owner and interest earnings on savings less interest payments on debt.

Initially, the fishing firm is regarded as having a given amount of fishing capacity, $x_o > 0$, with possibly some indebtedness, $y_o \geq 0$. It may or may not have any savings at the beginning of the period, $z_o \geq 0$.

The parameters in the model, which are denoted by Greek letters, are all positive with σ, ζ, ξ, β, and κ being less than unity. It is also assumed that $\zeta > \xi$.

Mathematical statement of the decision-making model.

In this section, the model described above is formally stated as a discrete-time control problem. This model is called Problem I.

Problem I:

$$\text{Maximize } I = z_T - y_T + \sum_{i=0}^{T} \psi_i \tau_i v_i$$

satisfying the difference equations

(I.1) $x_t - x_{t-1} = v_t$, x_o given and positive,

$x_T - x_{T-1} = 0$

(I.2) $y_t - y_{t-1} = w_t - \beta y_{t-1}$, y_o given and non-negative,

$y_T - y_{T-1} = -\beta y_{T-1}$,

$$(I.3)\quad z_t - z_{t-1} = w_t - \beta y_{t-1} - \tau_t v_t + (\gamma\lambda-\theta_t)x_{t-1}$$

$$- \zeta y_{t-1} + \xi z_{t-1} - \sigma[(\gamma\lambda-\theta_t)x_{t-1} - \zeta y_{t-1} + \xi z_{t-1} - \sum_{i=0}^{t-1} g_t(v_i)] ,$$

z_o given and non-negative,

$$z_T - z_{T-1} = -\beta y_{T-1} + (\gamma\lambda-\theta_T)x_{T-1} - \zeta y_{T-1} + \xi z_{T-1}$$

$$- \sigma[(\gamma\lambda-\theta_T)x_{T-1} - \sum_{i=0}^{T-1} g_{T-1}(v_i) - \zeta y_{T-1} + \xi z_{T-1}],$$

and satisfying the inequalities

$$(I.4)\quad w_t \geq 0 \quad , t = 1, 2,\ldots, T-1,$$

$$(I.5)\quad w_t \leq \kappa \tau_t v_t \quad , t = 1, 2,\ldots, T-1,$$

$$(I.6)\quad z_t \geq 0 \quad , t = 1, 2,\ldots, T,$$

$$(I.7)\quad v_t \geq 0 \quad , t = 1, 2,\ldots, T-1,$$

Solving the difference equations in I.1, I.2, and I.3 for their respective "closed-form" solutions, the state variables can be stated in terms of their initial values and the unknown control variables:

$$(2.6)\qquad x_t = x_o + \sum_{i=1}^{t} v_i ,$$

$$(2.7)\qquad y_t = y_o(1-\beta)^t + \sum_{i=1}^{t} w_i(1-\beta)^{t-i} ,$$

$$(2.8)\qquad z_t = z_1 Q_{t1} + \sum_{i=2}^{t} [w_i - \tau_i v_i + \Delta_i x_{i-1} + \pi y_{i-1} + .091\sigma \sum_{j=0}^{i-1} \tau_j v_j] ,$$

$$\text{where } v_T = 0 = w_T,\ g_t(v_i) = .091\ \tau_i v_i\ ,$$

$$\Delta_i = (\gamma\lambda)(1-\sigma) - (1-\sigma)\theta_i\ ,$$

$$\pi = \zeta(\sigma-1) - \beta\ ,$$

$$Q_{ti} = (1+\Gamma)^{t-i}\ ,$$

$$\Gamma = \xi(1-\sigma),\ i = 1, 2,\ldots, t \text{ and } t = 1, 2,\ldots, T.$$

Substituting the closed-form solution for x_t and also y_t from (2.6) and (2.7) into (2.8), we obtain the following solution for z_t in terms of the initial values for the states, the unknown controls, and the parameters:

$$(2.9) \qquad z_t = C_t + \sum_{i=1}^{t} w_i P_{ti} + \sum_{i=1}^{t} v_i D_{ti},$$

where

$$C_t = \sum_{i=1}^{t} Q_{ti}(\Delta_i + .091\sigma\,\tau_o)x_o$$

$$+ \pi y_o \sum_{i=1}^{t} Q_{ti}\,\chi^{i-1}$$

$$+ (1+\Gamma)z_o Q_{t1}\ ,\ t = 1, 2,\ldots, T-1,$$

$$\chi = 1 - \beta\ ,$$

$$P_{tt} = Q_{tt}\ ,\ t = 1, 2,\ldots, T-1\ ,$$

$$P_{ti} = Q_{ti} + \pi \sum_{j=i+1}^{t} Q_{tj}R_{j-1,i},$$
$$i = 1, 2,\ldots, t-1 \text{ and } t = 2,\ldots, T-1$$

$$D_{tt} = -\tau_t Q_{tt} \ , \ t = 1, 2, \ldots, T-1,$$

$$D_{ti} = \sum_{j=i+1}^{t} \Delta_j Q_{tj} + .091\sigma\tau_i \sum_{j=i+1}^{t} Q_{tj} - \tau_i Q_{ti}$$

$$i = 1, 2, \ldots, t-1 \text{ and } t = 2, 3, \ldots, T-1;$$

$$(2.9)' \qquad z_T = \sum_{i=1}^{T-1} w_i \rho_{Ti} + \sum_{i=1}^{T-1} v_i D_{Ti} + C_T \ ,$$

where

$$C_T = (1+\Gamma)C_{T-1} + .091\ \tau_o x_o \sigma$$

$$+ \Delta_T x_o + \pi y_o X^{T-1} \ ,$$

$$P_{Ti} = \pi R_{T-1,i} + (1+\Gamma)P_{T-1,i},\ i=1, 2, \ldots, T-1,$$

$$D_{Ti} = \Delta_T + (1+\Gamma)D_{T-1,i} + .091\sigma\tau_i,\ i=1, 2, \ldots, T-1$$

$$R_{ti} = (1-\beta)^{t-i},\ i = 1, 2, \ldots, t \text{ and } t = 1, 2, \ldots, T.$$

The Sequential Linear Programming Model.

Substituting the solutions above for the state variables--x_t, y_t, z_t--into the objective function and the inequality restrictions of the control model in Problem I, the state variables (and the difference equations describing them) are removed from the problem. The resulting problem is the following linear programming model:

Problem II:

$$\text{Maximize } I = a + \sum_{t=1}^{T-1} B_t v_t + \sum_{t=1}^{T-1} A_t w_t$$

subject to the inequality restrictions

(II.1) $w_t \geq 0, \quad t = 1, 2, \ldots, T - 1,$

(II.2) $\kappa \tau_t v_t - w_t \geq 0, \quad t = 1, 2, \ldots, T - 1,$

(II.3) $\sum_{i=1}^{t} P_{ti} w_i + \sum_{i=1}^{t} D_{ti} v_i \geq -C_t, \quad t = 1, 2, \ldots, T,$

(II.4) $v_t \geq 0, \quad t = 1, 2, \ldots, T - 1,$

where

$$A_t = P_{Tt} - R_{T-1,t}(1-\beta), \quad t = 1, 2, \ldots, T,$$

$$B_t = P_{Tt} + \psi_t \tau_t, \quad t = 1, 2, \ldots, T, \text{ and}$$

$$a = C_T + \psi_o \tau_o x_o - y_o X^T.$$

Letting

$$h_t \equiv h_t(w_1^o, \ldots, w_{t-1}^o, v_1^o, \ldots, v_{t-1}^o) \equiv C_t + \sum_{i=1}^{t-1} P_{ti} w_i + \sum_{i=1}^{t-1} D_{ti} v_i,$$

$$t = 1, 2, \ldots, T - 1,$$

inequality II.3 may be expressed as follows in terms of the non-negative function h_t:

(II.3)' $w_t - \tau_t v_t + h_t \geq 0, \quad t = 1, 2, \ldots, T - 1.$

The graph of the non-trivial constraint set for Problem II is shown in Figure 6. The optimal solution for investment, v_t^o, and borrowing, w_t^o, may be any point in the shaded area or on the boundary of that area.

Since the problem is linear, the optimal solution to the problem in each year t is known to be on the boundary of the constraint set in Figure 6 (or at the origin in the trivial case). Moreover, the solutions at the vertices of this constraint set have an immediate economic interpretation. Clearly, the marginal rate of return from fishing must be greater than the interest rate on debt at point B, less than the interest rate on savings at point A, and between these two interest rates at point C.

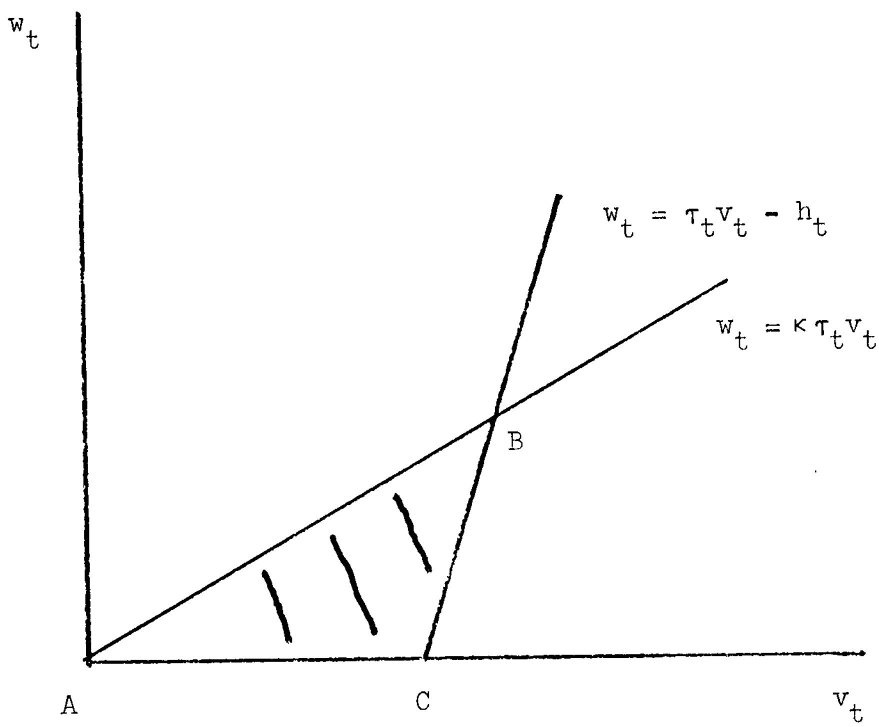

Figure 6: The Nontrivial Constraint Set for Problem II.

7. An Application of the Model.

In this section, the model developed above is applied in particular to a relatively small shrimp fishing firm operating 73 foot steel hull trawlers. This application is not meant to be exhaustive of the many that could be made, or to imply that this is the most important type of vessel in the shrimping fleet. Instead, it is meant to indicate how a shrimp fisherman having a given amount of physical and money capital might use the model to obtain guidelines for investment and financial decision-making.

<u>Initial values for the difference equations and values of the parameters considered.</u>

In this application, the value of x_o -- the initial value to the first difference equation -- is specified to be one boat. That is, the model firm is initially operating just one 73 foot steel hull trawler. It is further visualized that this boat was purchased at the end of 1969 for a price of $100,000 being completely outfitted for shrimp fishing. The model fisherman had $30,000 in cash with the minimum downpayment of $25,000 being made on the boat: $\kappa = .75$, $y_o = \$75,000$, and $z_o = \$5,000$. This loan contract requires the indebtedness to be repaid in ten equal payments starting at the end of the 1^{st} year with interest (including mortgage insurance) being $9\frac{1}{2}$ percent annually: $\beta = .10$ and $\zeta = .095$. In accordance with this borrowing rate, which reflects current conditions, the interest rate on savings is specified to be $5\frac{1}{2}$ percent annually--the present saving rate.

Since it is quite common for owners of vessels like this one to obtain 65 percent of the gross revenues with the captain and first mate (who pay for all of the groceries) receiving the other 35 percent, the net price per pound of shrimp landed is specified to be 65 percent of the exvessel price in year t, ϵ_t. That is, $\gamma_t = .65\ \epsilon_t$. To determine the effect of variations in the price of shrimp on the profitability of investments in the model firm, the price, ϵ_t, is varied by 10 cent increments from 60 cents to $1.00 per pound. Similarly, to determine the effect of variations in landings, the catch, λ , is varied from 50,000 pounds per year to 80,000 pounds. This range of catch variation has recently been obtained by boat owners of 73 foot boats in the Aransas Pass area.

In year t, the cost of operating a 73 foot shrimp trawler is initially specified to be $26,000 per year with a 3 percent annual increase to reflect price increases: $\theta_t = 26{,}000\ (1.03)^t$. This cost figure includes an allowance for overhead and all insurance costs.

To reflect inflation, the per boat purchase price is regarded as increasing at three per cent per year. This figure is conservative in light of happenings in 1968 and 1969 when these prices increased by around five percent per year.

Similarly to reflect technological depreciation in value, the 73 foot steel hull trawlers were assumed to have a competitive life of 20 years. This meant that the value of each boat purchased would decrease in value due to technological obsolescense by five per cent per year: $\psi_t = 1/(1.05)^t$.

Straight-line depreciation methods were used for tax purposes with the average rate for all of the boat's equipment averaging 11 years. Using the reciprocal of this figure, the depreciation function for tax purposes was specified as follows for investments in year i:

$$g(v_i) = .091\ \tau_i v_i\ .$$

The rate of income withdrawal for income taxes was specified to be 20 percent of the revenues received by the owner after interest earnings and payments were taken into account. An allowance for depreciation was also made, as indicated above.

In summary, the initial values for the difference equations and the values of all but two of the parameters used in the analysis are given in Table 2. The remaining two parameters are the shrimp price received per pound and the annual catch per boat. Both, price and catch are singlely varied over previously observed ranges to determine the effect of that variation on the investment and borrowing decisions.

TABLE 2. Values of parameters and initial values used in the applications.

Parameter	Value
β -- debt repayment rate	.10
τ_t -- purchase price per boat	\$100,000 $(1.03)^t$
ψ_t -- technological depreciation	$1/(1.05)^t$
ζ -- interest rate on debt	.095
ξ -- interest rate on savings	.055
κ -- financeable fraction of investment	.75
θ_t -- operating costs per boat	\$26,000 $(1.03)^t$
σ -- rate of withdrawal from income for taxes	.20
$\sum_{i=0}^{t-1} g(v_i)$ -- the depreciation function for taxes	$.091 \sum_{i=0}^{t-1} \tau_i v_i$
x_o -- initial fishing capacity	1 boat
y_o -- initial indebtedness	\$75,000
z_o -- initial savings	\$5,000

Problems studied.

To determine the effect, as mentioned above of varying the shrimp price and the catch, a number of problems were analyzed in which these two parameters were varied systematically (one at a time) over a range of catches and prices.[2] In the first set of problems, the catch was specified to be 80,000 pounds with the price being varied downward by 10 cent decrements from $1.00 per pound; in the second set, catch was decreased by 10,000 pounds to 70,000 pounds with the same price variation being considered. This process was continued in a third set of problems for a catch of 60,000 pounds. Only six of these problems had feasible solutions (in which all of the inequality constraints in Problem II were satisfied). Those problems are identified by number in Table 3. Other problems for annual catches below 60,000 pounds were also considered; but everyone of them was infeasible too. The source of the infeasibility was the non-negative savings requirement. With the rising operating costs of three per cent per year, the model shrimp fisherman could not stay solvent. He simply went broke, in everyday parlance.

Since a shrimp price close to $1.00 per pound has only been received most recently by fishermen on the Texas Gulf Coast, and since 70,000 pounds seems to be a typical catch for a 73 foot trawler with catches of 80,000 pounds (or more) being regarded as favorable (and more likely exceptional), the lack of more feasible solutions in Table 3 for lower prices and smaller catches indicates the sensitivity of the investment problem being considered to price and catch variations.

[2] In the model, the exvessel price received per pound is ϵ with the price obtained by the owner being $.65\epsilon$. This implies that the boat owner gets 65 percent of the revenues received.

The shrimp price must be very good to insure continued operations by the model fisherman if a catch of 60,000 pounds is obtained. Some variation in the expected price is possible for catches of 70,000 pounds. The model fisherman can then stay in business if the expected price is $ .90 per pound. He has still more leeway if it is possible for him to land an average catch of 80,000 pounds. There, he can stay in business and make money if the average shrimp price is $ .80 per pound.

Solutions to Six Problems.

Maximum values for net worth in Six problems.--- In Table 4, the maximum values of the objective function--net worth at the end of ten years--are presented for each of the six problems studied. To evaluate these results, it is helpful to keep in mind that the initial equity of the model fishing firm was $30,000. Thus, in all of the problems,net worth increased by at least 3.5 times in the ten year period.

The importance of price is vividly illustrated in Problems 1 through 3 where the catch was 80,000 pounds. In the ten year period, net worth almost doubled with a per pound price increase from $.80 to $.90, and didn't quite double again with a further price increase from $.90 to $1.00. Thus, the improved economic attractiveness of shrimp fishing with higher expected prices is clearly evident.

TABLE 3. Variations in exvessel shrimp price received and annual catch per boat considered.

Problem	Exvessel shrimp price received before lay, ϵ (dollars)	Exvessel shrimp price received after lay, γ (dollars)	Annual catch per boat (pounds)
1	1.00	.650	80,000
2	.90	.585	80,000
3	.80	.520	80,000
4	1.00	.650	70,000
5	.90	.585	70,000
6	1.00	.650	60,000

Table 4. Maximum Net Worth for the Eight 10 Year Shrimp Fishing Problems

Problem	Maximum Net Worth (dollars)
1	601,613
2	301,381
3	155,001
4	254,770
5	142,500
6	107,354

This represents the maximum feasible value in each problem for $I = \sum_{i=0}^{T} \psi_i \tau_i v_i + z_T - y_T$, where $\sum_{i=0}^{T} \psi_i \tau_i v_i$ is the terminal value of the boats owned, z_T is the terminal savings, and y_T is the terminal indebtedness.

The effect of catch may be seen by comparing the net worth for Problems 1 and 4, and 2 and 5 (in which the price for each pair is the same). At $.90 per pound, the model fisherman who can land an average catch of 80,000 pounds rather than 70,000 pounds can make more than twice as much money in the 10 year period: $301,381 for an 80,000 pound catch versus $142,500 for a 70,000 pound catch. The difference is even more dramatic for the high price of $1.00 per pound, which may become more commonplace with higher real per capita incomes in the future. There, the model fisherman who lands 80,000 pounds per year makes $601,613 in ten years, while his counterpart who lands 70,000 pounds per year ends up with only about 42 percent of that much wealth: $254,770.

In summary, the effects of price and catch on the economic fortunes of shrimp fisherman is clearly evident. With a price of $.90 per pound and a catch of 70,000 pounds, the model fisherman's net worth increases $112,500 in 10 years. This represents an average annual rate of return of about 38 percent on the initial equity of $30,000 in the 10 year period: (112,500/30,000)/10 = .376. Of course, anyone investing his money in shrimp fishing must remember that these are normative results: what would happen if the model is reflective and the values of the papameters would materialize. Values actually materializing for some of the parameters might be less favorable than those analyzed above. Instead of operating costs, for example, increasing by 3 percent yearly, they might increase by 5 percent (or more) per year. This type of unforseeable factor could have a significant effect on the attractiveness of the investment. In fact, some of the problems analyzed might be infeasible for such a rate of increase in operating costs. Also, it must be borne in mind that 90 cents per pound is a very favorable price, and that the problem is infeasible for a price of 80 cents per pound.

Optimal values of decision variables and magnitude of the investment incentive.

In Tables 5 through 10, the optimal values of the state variables--boats owned, indebtedness, and savings--and the optimal values of the control variables--boat purchases and borrowings--are given for Problems 1 through 6. In each year, the investment-borrowing solution is identified with respect to the constraint set for Problem II, as presented graphically in Figure 6. This identification is especially revealing for the corner solutions at points B, A, and C because it immediately implies whether the rate of return from fishing[3] is greater than the borrowing rate, or less than the savings rate, or between these two rates.

In evaluating these results for Problems 1 and 2, where the catch is 80,000 pounds with a per pound price of $1.00 in the first problem and $.90 in the second one, the model fisherman finds the rate of return from fishing to be initially greater than the borrowing rate; and accordingly, he uses his savings to negotiate the largest possible loan and to buy the maximum possible number of boats in the first six years of Problem 1 and in the first five years of Problem 2.

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[3]Note that the marginal rate of return from fishing capacity equals the average rate since the model is linear.

Table 5. Optimal Solution to Problem 1 in Table 3

Year	States			Controls		Location of Solution in Figure 6
	Boats Owned x_t^o (number)	Indebtedness y_t^o (dollars)	Savings z_t^o (dollars)	Boats Purchased v_t^o (number)	Borrowings w_t^o (dollars)	
0	1.00	75,000	5,000	-	-	-
1	2.057	149,148	0.0	1.057	81,648	B
2	2.725	187,423	0.0	0.668	53,190	B
3	3.588	239,346	0.0	0.862	70,665	B
4	4.651	305,178	0.0	1.063	89,767	B
5	5.915	384,531	0.0	1.264	109,870	B
6	7.359	475,433	0.0	1.444	129,355	B
7	7.359	427,889	49,022	0.0	0.0	A
8	7.359	385,100	102,878	0.0	0.0	A
9	7.359	346,590	160,856	0.0	0.0	A
10	7.359	311,931	222,148	-	-	-

Table 6. Optimal Solution to Problem 2 in Table 3

Year	States			Controls		Location of Solution in Figure 6
	Boats Owned x_t^o (number)	Indebtedness y_t^o (dollars)	Savings z_t^o (dollars)	Boats Purchased v_t^o (number)	Borrowings w_t^o (dollars)	
0	1.000	75,000	5,000	-	-	-
1	1.895	136,668	0	0.895	69,168	B
2	2.219	148,751	0	0.324	25,750	B
3	2.607	165,682	0	0.388	31,806	B
4	3.044	185,991	0	0.437	36,877	B
5	3.514	208,302	0	0.471	40,910	B
6	3.635	187,472	0	0.121	0	C
7	3.635	168,725	17,177	0.0	0	A
8	3.635	151,852	35,569	0.0	0	A
9	3.635	136,667	54,947	0.0	0	A
10	3.635	123,000	74,808	-	-	-

His savings are the limiting factor in each case; and, as a result, increasingly larger investments were possible in the early years in Problem 1 than in Problem 2. In the 6th year of Problem 2 the rate of return from fishing falls between the two interest rates. Starting in the 7th year of both problems, the model fisherman can make more money by leaving his savings in the bank at 5.5 percent interest: other investment alternatives, as reflected by the savings account, are economically the most attractive.

In Problems 3, 4, 5 and 6, the investment incentive pattern and the form of the decision-making over the period are like that in Problems 1 and 2. The rate of return from fishing at first exceeds the borrowing rate with the model fisherman using all of his savings in Problems 3 and 4 to negotiate the largest possible loan and then using all of these monies to make the largest possible investment. Savings in Problems 3 and 4 are the factor limiting the size of the investment with differences in investments reflecting the profitableness of the problem. There is just one year in Problems 3 and 4, and two years in Problem 5 where the rate of return from fishing is between the two interest rates. In each case, the model fisherman cannot afford to borrow money at 9.5 percent interest; but he does find investing all of his savings in more boats to be a better alternative than leaving those

Table 7. Optimal Solution to Problem 3 in Table 3

Year	States			Controls		Location of Solution in Figure 6
	Boats Owned x_t^o (number)	Indebtedness y_t^o (dollars)	Savings z_t^o (dollars)	Boats Purchased v_t^o (number)	Borrowings w_t^o (dollars)	
0	1.00	75,000	5,000	-	-	-
1	1.734	124,188	0	0.734	56,688	B
2	1.763	114,111	0	0.029	2,342	B
3	1.829	108,056	0	0.065	5,356	B
4	1.850	97,250	0	0.021	0	C
5	1.850	87,525	3,281	0.0	0	A
6	1.850	78,773	7,055	0.0	0	A
7	1.850	70,896	11,187	0.0	0	A
8	1.850	63,806	15,427	0.0	0	A
9	1.850	57,425	19,701	0.0	0	A
10	1.850	51,683	23,705	-	-	-

Table 8. Optimal Solution to Problem 4 in Table 3

Year	States			Controls		Location of Solution in Figure 6
	Boats Owned x_t^o (number)	Indebtedness y_t^o (dollars)	Savings z_t^o (dollars)	Boats Purchased v_t^o (number)	Borrowings w_t^o (dollars)	
0	1.0000	75,000	5,000	-	-	-
1	1.855	133,548	0	0.855	66,048	B
2	2.100	139,713	0	0.245	19,520	B
3	2.394	149,846	0	0.294	24,104	B
4	2.721	162,437	0	0.327	27,575	B
5	2.807	146,193	0	0.086	0	C
6	2.807	131,574	12,056	0.0	0	A
7	2.807	118,416	25,148	0.0	0	A
8	2.807	106,575	38,955	0.0	0	A
9	2.807	95,917	53,274	0.0	0	A
10	2.807	86,326	67,755	-	-	-

Table 9. Optimal Solution to Problem 5 in Table 3

Year	States			Controls		Location of Solution in Figure 6
	Boats Owned x_t^o (number)	Indebtedness y_t^o (dollars)	Savings z_t^o (dollars)	Boats Purchased v_t^o (number)	Borrowings w_t^o (dollars)	
0	1.000	75,000	5,000	-	-	-
1	1.710	122,350	92	0.710	54,850	Arc AB
2	1.710	110,115	0.0	0.0	0	Origin (trivial solution)
3	1.719	99,104	0.0	0.009	0	C
4	1.734	89,194	0.0	0.016	0	C
5	1.734	80,274	2,513	0.0	0	A
6	1.734	72,247	5,427	0.0	0	A
7	1.734	65,022	8,618	0.0	0	A
8	1.734	58,520	11,852	0.0	0	A
9	1.734	52,668	15,060	0.0	0	A
10	1.734	47,401	17,957	-	-	-

Table 10. Optimal Solution to Problem 6 in Table 3

Year	States			Controls		Location of Solution in Figure 6
	Boats Owned x_t^o (number)	Indebtedness y_t^o (dollars)	Savings z_t^o (dollars)	Boats Purchased v_t^o (number)	Borrowings w_t^o (dollars)	
0	1.00	75,000	5,000	-	-	-
1	1.490	105,110	3,963	0.490	37,610	Arc AB
2	1.490	94,599	1,985	0.0	0	A
3	1.490	85,139	791	0.0	0	A
4	1.490	76,625	172	0.0	0	A
5	1.490	68,963	0.0	0.0	0	A
6	1.490	62,066	75	0.0	0	A
7	1.490	55,860	276	0.0	0	A
8	1.490	50,274	409	0.0	0	A
9	1.490	45,246	393	0.0	0	A
10	1.490	40,722	0	-	-	-

monies in the bank at 5.5 percent interest. No investments in fishing capacity are made in years five through nine in Problems 3 and 5, year 6 through 9 in Problem 4, and years 2 through 9 in Problem 6, since the savings rate of 5.5 percent is greater than the rate of return from fishing.

In Problems 5 and 6, where the model fisherman finds fishing to be the least attractive of the six cases considered, there is initially a different investment solution than in the other five problems. In year 1, the optimal investment-borrowing solution is on the arc AB with the maximum possible amount being borrowed on the investment, but not all of the savings being utilized. This result means that investing in fishing capacity earns a higher rate of return than the borrowing rate; yet because of future liquidity needs, the model fisherman cannot invest all of his savings in year 1. Instead, the model fisherman finds it to his advantage to borrow as much as he can on the investment made and to retain as much of his savings as possible to meet committments later in the 10 year period. For example in Problem 6, the model fisherman purchases $50,470 of new fishing capacity in the 1st year and borrows $37,610 on that investment leaving $3,963 in savings; thereafter the rate of return from fishing is less than the 5.5 percent savings rate.

Computed boat prices. —— Using the reduced net worth values associated with the non-negativity restriction on boat purchases and the specified boat purchase price in Table 2, τ_t, it is possible to determine how much the model fisherman would have paid for a boat in all of those years when he didn't invest in fishing capacity. This may be accomplished in every year (where the rate of return from more boats is less than the savings rate) by subtracting the reduced net worth value per boat (given as an output in the linear programming analysis and identified in Appendix Tables 1 and 2 by s_t^o) from the respective value for the purchase price of a new boat, τ_t. The reduced net worth value in year t indicates how much the model fisherman would have decreased his net worth over the ten year period if he had purchased another boat in that year. Boat prices computed in this way as well as the associated reduced net worth figures and the prices used in the programming are given in Table 11 for Problems 1 and 3.

In Problem 1, the model fisherman found boat investments to be an economically profitable venture in the 1st six years. However, in the 7th, 8th and 9th years, this was not the case. Before he would have purchased another boat in the 8th year, the price would have had to have been reduced by \$18,872 to \$107,804. A even larger reduction would have been necessary in the 9th year where it would have taken a \$36,089 decrease in the purchase price to interest him in further investment.

In Problem 3, where there is investment incentive only in the 1st four years, sizeable decreases in the boat price would have been necessary in years 6 through 9 to have induced investment. It would have required a decrease varying from \$15,409 in the 5^{th} year to \$44,409 in the 9^{th} year.

This analysis may also be carried out for the other problems by using the reduced net worth values, s_t^o, presented in the Appendix. In the interest of brevity, this is left to the reader.

Table 11. Per boat reduced net worth values for an additional boat, boat price used in programming, and computed boat prices for Problems 1 and 3 when boats were not purchased.

Year		Problem 1		Problem 3	
	Boat price used in programming (dollars)	Reduced net worth of an additional boat (dollars for one more boat)	Computed boat price when investments were zero (dollars per boat)	Reduced net worth of an additional boat (dollars for one more boat)	Computed boat price when investments were zero (dollars per boat)
0	100,000	-	-	-	-
1	103,000	0	-	0	-
2	106,090	0	-	0	-
3	109,272	0	-	0	-
4	112,550	0	-	0	-
5	115,927	0	-	3,340	112,587
6	119,405	0	-	15,409	103,996
7	122,987	159	122,828	26,233	96,754
8	126,676	18,872	107,804	35,878	90,798
9	130,476	36,089	94,387	44,409	86,067

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Appendix Table 1.

Shadow prices for Problems 1, 2, and 3 in Table 3*

Year	Problem 1				Problem 2				Problem 3			
	l_t^o	m_t^o	n_t^o	s_t^o	l_t^o	m_t^o	n_t^o	s_t^o	l_t^o	m_t^o	n_t^o	s_t^o
1	0.0	6.905	3.189	0.0	0.0	3.045	1.381	0.0	0.0	1.092	0.638	0.0
2	0.0	4.383	2.055	0.0	0.0	1.994	0.979	0.0	0.0	0.585	0.548	0.0
3	0.0	2.761	1.374	0.0	0.0	1.233	0.736	0.0	0.0	0.108	0.225	0.0
4	0.0	1.659	0.960	0.0	0.0	0.628	0.596	0.0	0.081	0.0	0.089	0.0
5	0.0	0.859	0.725	0.0	0.0	0.089	0.215	0.0	0.144	0.0	0.0	3,340
6	0.0	0.202	0.320	0.0	0.098	0.0	0.020	0.0	0.118	0.0	0.0	15,409
7	0.091	0.0	0.0	159	0.091	0.0	0.0	13,196	0.091	0.0	0.0	26,233
8	0.062	0.0	0.0	18,872	0.062	0.0	0.0	27,375	0.062	0.0	0.0	35,878
9	0.032	0.0	0.0	36,089	0.032	0.0	0.0	40,249	0.032	0.0	0.0	44,409

Appendix Table 2.

Shadow prices for Problems 4, 5, and 6 in Table 3*

Year	Problem 4				Problem 5				Problem 6			
	l_t^o	m_t^o	n_t^o	s_t^o	l_t^o	m_t^o	n_t^o	s_t^o	l_t^o	m_t^o	n_t^o	s_t^o
1	0.0	2.436	1.123	0.0	0.0	0.795	0.0	0.0	0.0	0.0	0.0	0.0
2	0.0	1.584	0.821	0.0	0.182	1.159	1.035	0.0	0.181	0.249	0.0	0.0
3	0.0	0.941	0.644	0.0	0.009	0.0	0.134	0.0	0.318	0.461	0.0	0.0
4	0.0	0.397	0.464	0.0	0.112	0.0	0.058	0.0	0.0	0.221	0.0	35,142
5	0.026	0.0	0.118	0.0	0.144	0.0	0.0	6,179	0.0	0.308	0.442	41,428
6	0.118	0.0	0.0	2,081	0.118	0.0	0.0	17,630	0.107	0.0	0.0	26,662
7	0.091	0.0	0.0	16,455	0.091	0.0	0.0	27,863	0.078	0.0	0.0	35,260
8	0.062	0.0	0.0	29,501	0.062	0.795	0.0	36,941	0.046	0.0	0.0	42,757
9	0.032	0.0	0.0	41,289	0.032	1.159	0.0	44,929	0.014	0.0	0.0	49,215

*l_t^o, m_t^o, n_t^o and s_t^o are the multipliers associated with constraints II.1, II.2, II.3, and II.4, respectively, in Problem II.