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PREDICTION DRIFTING FORCE AND MOMENT AN OCEAN PLATFORM FLOATING **OBLIQUE WAVES**

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PREDICTION OF DRIFTING FORCE AND MOMENT ON AN OCEAN PLATFORM FLOATING IN OBLIQUE WAVES

by C. H. Kim and F. Chou

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Approved

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John P. Breslin, Head Department of Ocean Engineering

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ABSTRACT

This paper presents a new procedure for predicting the lateral drifting force and moment on an ocean platform floating in ob1 ique waves. The disturbance of an incident wave caused by the presence of the floating body is represented by the sum of the diffracted and forced wave potentials, which are determined by satisfying the kinematical boundary conditions on the body surface. The scattered waves are determined from the asymptotic expression of the two potentials. Frank's close-fit method. Grim's strip method and Maruo's formula for two-dimensional drifting force are used. Numerical results were compared with experimental results and found to be in good agreement.

KEYWORDS

Drifting Force Scattered Wave Diffracted Wave Potential Forced Wave Potential

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 \Box

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Subscripts

Superscripts

- e suffix designating even function
- (m) suffix designating the number of mode
- o suffix designating odd function

Greek Letters

- slope of a segment α
- damping coefficient δ
- \mathbf{c} phase difference
- heave, or suffix designating heavIng motion, or complex ζ coordinate
- sway, or suffix designating swaying motion, or y-coordinate of 'n. a source point

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- radius of gyrat ion $\pmb{\varkappa}$
- wave length λ
- wave incidence μ
- v wave number
- x-coordinate of a source point 5
- p water density
- velocity potentiai $\pmb{\Phi}$
- roll, or suffix designating roliing motion φ
- X yaw, or suffix designating yawing motion
- pitch, or suffix designating pitching motion Ψ
- w circular frequency

INTRODUCTION

Since the prediction of the motions of an ocean platform in oblique seas can be made quite accurately $^{1\,,2\,,3\,,4}$ it is reasonable to anticipat that the prediction of drifting force on the ocean platform floating In waves will also be reasonably achieved. Maruo⁵ studied analytically how to determine the two-dimensionaI drifting force on a floating cylinder in a beam sea and came to the conclusion that the drifting force is determined by knowing the scattered wave caused by the presence of the floating body in the Incident wave and remarked that his formula for determining the two-dimensional force was the same as that of Haskind. $^{\rm 6}$

Ogawa" applied Maruo's formula to his strip calculation on a fixed body, where the strip method was based on the concept of the snake-type wave generator. He also made an experiment w'Ith a f ixed ship model and the measured drifting force was in good agreement with his calculation. Lalangas⁸ reported experimental results on the lateral drifting force and moment on a fixed model in oblique waves and on a free ship model in beam seas. Newman $^{\circ}$ studied theoretically how to estimate the drifting force and moment. This Is very rigorous but based on the assumption that the ship is a slender body, which does not give realistic answers.

The present study is based on the two-dimensional source method¹⁰ in connection with the strip method^{3,4} and Maruo's formula⁶ for determining the two-dimensional dr if ting force on a cylinder floating in beam seas.

The scattered waves are generated from the disturbance caused by the presence of the oscillating cylinder in beam seas.

This scattered wave system ls represented by the sum of the diffracted wave and forced wave (or radiation) potential, whose amplitudes are determined by satisfying the kinematicai boundary conditions, There are then two components of the scattered wave potential: (l) describing the diffraction of the incident wave from the fixed body and (2) describing the radiation from the forced oscillation of the body in calm water with the

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velocity amplitudes which represent the motion of the body in the given Incident wave. The motion of the body in a given incident wave is predicted according to previously published procedures.^{1,2,3,4} The scattered wa ves, if determined, can then be used to calculate the drifting force on a strIp section by making use of Maruo's formula.

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To begin with, the general outline of the method wIll be described, then: (1) a detailed analysis of how to determine the specific potentials of the diffracted and forced wave will be given, (2) the derivation of the scattered wave from the asymptotic expression for the potentials will be discussed, (3) the scattered waves will be examined in the light of prepublished work¹ and in connection with Haskind-Newman theory,¹² (4) equations for the drifting force and moment will be formulated, and (5) numerical examples of the models which were previously tested and published will be discussed In order to confirm the reliability of the procedure. The comparison of the prediction with the experimental data show excel lent agreements. This work can be extended eas ily to the osci I lating ship with forward **speed** using the techniques of reference 4.

Superior numbers in text matter refer to similarly numbered references listed at the end of this report.

GENERAL OUTLINE OF THE METHOD

Suppose that a ship is oscillating steadily without forward speed in an oblique incident wave and assume that the incident wave amplitude and consequently the resulting motion of the ship are small. Furthermore, the diffracted and forced waves are assumed to be small. Under these conditions, the incident waves encountered by the ship are diffracted from it just as they would be from the fixed ship, while at the same time exciting the ship to undergo oscii lations which in turn emit the forced wave system. In view of these remarks we write the velocity potential for the motion as the following sum:

$$
\Phi = \Phi_{1}(x,y,z,t) + \Phi_{0}(x,y,z,t) + \Phi_{F}(x,y,z,t)
$$
 (1)

where Φ_\parallel = incident wave potent

 $\Phi_{\mathsf{D}} = \text{differential}$ wave potential

 $_{\mathsf{F}}$ = forced wave potential (or radiation potentia The total potential ϕ should satisfy the conditions:¹³

1. The continuity of the liquid in the whole domain.

2. The linearized free-surface condition.

3. The radiation condition.

 $4.$ The deep water condition.

5. The kinematical boundary condition on the body surface.

Suppose that we have the potential δ which satisfies conditions I through 4 , then condition 5 is written in the form

$$
\frac{\partial \Phi}{\partial n} = \frac{\partial \Phi}{\partial n} + \frac{\partial \Phi}{\partial n} + \frac{\partial \Phi}{\partial n} = V_n
$$
 (2)

where **V** is the prescribed normal velocity component of a point on the $\frac{1}{n}$ body surface. This velocity is directly obtained from the solution of the equations of motion of the ship **in** the given incident wave. **In** the I Inearized theory this kinematical boundary condition Is sat isf ied by two sepa rate conditions

$$
\frac{\partial \Phi}{\partial n} = -\frac{\partial \Phi}{\partial n}
$$
 (3)

$$
\frac{\partial \Phi_F}{\partial n} = V_n \tag{4}
$$

on the body surface at the mean position of the oscillation. The diffracted wave potential $_{\Phi_{\textbf{D}}}$ in Eq. (3) is nothing but the potential fo estimating the wave-exciting force.³ The forced potential $_{\Phi_F}$ from Eq. (4) is the potential induced by the forced oscillation with the given velocity Vn **In** calm water. From the preceding discussion it is seen that disturbances of the incident waves in the presence of the oscillating body are obtained from the sum of the potentials $\Phi_{\text{D}} + \Phi_{\text{F}}$, and that the scattered waves generated from the disturbances are determined from the asymptotic expressions of $\Phi_{\mathbf{n}} + \Phi_{\mathbf{F}}$ at infinity.

Now we consider a section of the ship oscillating in the strip domain (see section I) where an oblique incident wave is oncoming as if in a beam sea. Suppose we had determined the disturbance potential Φ_D + Φ_F in this domain, then we can estimate the scattered wave in the strip domain and consequently the lateral drifting force on this section of the body according to Maruo.⁵ Then, the summation of the strip forces and moments of the strip force with respect to the longitudinal center of gravity of the ship provide us with the resultant drifting force and moment.

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l. incident Wave Potential in a Strip Domain

Let o-XYZ and o-xyz be the right-handed rectangular space- and body-coordinate systems as illustrated in Figure 1. Coordinate planes o-XZ and o-xz lie on the calm water surface, and the Y- and y-axes point vertically upward.

Let the incident angle of the wave be designated by μ as shown and let the wave progress in the positive X-direction. Then the wave profile Is

$$
h_{\parallel} = a \cos(\nu \times \sin\mu + \nu z \cos\mu - \omega t)
$$
 (5)

where

 $a =$ wave amplitude $v =$ wave number (w^2/q) ω = circular frequency of the wave

Now, suppose two vertical control planes cut the body at z and $z+dz$, and observe the wave motion within the f ict ious iy conf ined domain which is infinitely extended in the lower half domain, i.e., in $\pm x$ and -y directions. This domain is designated the "strip domain." The wave equation, Eq. (5), can be interpreted in this domain by noting that the term $vx\sin\mu$ determines the wave form in the strip domain and that the term $\sqrt{2}\cos\theta$ represents the phase shift of the incident wave at $x=0$ and $z=z$ relative to a crest at the origin. Thus the potential of the incident wave, Eq. (5) , **is** defined in the strip domain in the form

$$
\Phi_{\parallel} = \frac{ga}{\omega} e^{\nu y} \sin(\nu x \sin\mu + \nu z \cos\mu - \omega t)
$$
 (6)

This potential consists of even and odd functions with respect to x and they are expressed in the forms;

$$
\Phi_{\parallel}^{\mathcal{O}} = \frac{g_{\text{a}}}{\omega} e^{\mathsf{V} \mathsf{V}} \sin(\mathsf{v} \mathsf{x} \cdot \mathsf{s} \mathsf{in}_{\parallel}) \cdot \cos(\mathsf{v} \mathsf{z} \cdot \mathsf{cos}_{\parallel} - \mathsf{w} \mathsf{t})
$$
\n
$$
\Phi_{\parallel}^{\mathcal{C}} = \frac{g_{\text{a}}}{\omega} e^{\mathsf{V} \mathsf{V}} \cos(\mathsf{v} \mathsf{x} \cdot \mathsf{s} \mathsf{in}_{\parallel}) \cdot \sin(\mathsf{v} \mathsf{z} \cdot \mathsf{cos}_{\parallel} - \mathsf{w} \mathsf{t})
$$
\n(7)

The odd function is applied to represent the asynmetric f low about the

y-axis, while the even function is applied to the symmetric flow. This functional resolution will be utilized in the kinematical boundary conditions in the following section.

2. Diffracted Wave Potential in a Strip Domain

The incident wave described above will be diffracted from the body section as if it were fixed in the strip domain as described In the preceding section. Since the diffraction is a disturbance, the diffracted wave potential may be represented by the source potential used by Frank.^{10} Referring to the form of the incident wave potential, Eq. (6) , we may write the diffraction potential in the strip domain in the form

$$
\Phi_D^{(m)}(x,y,z;\mu,t) = R_e \left[\int\limits_c^{\infty} Q_D^{(m)}(s) \cdot G(x,y;\xi,\eta) ds e^{i(\nu z \cos \mu - \omega t)} \right] (8)
$$

where c designates the wetted contour of the strip section and m designates the mode of excitation $[m = 2,3,4 = sway,$ heave, roll]. $Q_n^{(m)}(s)$ designates the unknown complex source intensities distributed over the strip surface. These source intensities are determined by satisfying the kinematical boundary condition on the body surface $Eq.(3)$. They depend on the mode of excitation, the geometry of the body and the incident wave. The function $G(x,y;\xi,\eta]^{10}$,¹³ is the two-dimensional pulsating source potential of unit intensity at the point (ξ,η) in the lower half plane (see Figure 2a). The exponential term $e^{i\sqrt{2}cos\mu}$ represents the influence of the location of the strip at z where the disturbance occurs in response to the oblique incident wave of wave number \sqrt{v} and incidence $\frac{1}{2}$.

Since $Q_n^{(m)}$ and G are the complex source intensity and Green's function, respectively, let them be

$$
Q_{D}^{(m)} = Q_{Dr}^{(m)} + iQ_{Di}^{(m)}
$$

$$
G = G_{r} - iG_{i}
$$

where $i = \sqrt{1}$ and where $Q_{\texttt{nr}}^{(\texttt{m})}$ and $Q_{\texttt{nr}}^{(\texttt{m})}$ are real and imaginary parts m) and C and C are real of of Q, "', and G_r and -G_; are real and imaginary parts of G.

6

Equation (8) is then changed to

$$
\Phi_D^{(m)} = \int_C \left(Q_{DT}^{(m)} G_r + Q_{DI}^{(m)} G_i \right) ds \cdot \cos(\sqrt{z} \cos \mu - \omega t)
$$

$$
- \int_C \left(-Q_{DT}^{(m)} G_i + Q_{DI}^{(m)} G_r \right) ds \cdot \sin(\sqrt{z} \cos \mu - \omega t)
$$
 (9)

This potential $\Phi_D^{(m)}$ is determined by satisfying the kinematical boundary condition Eq. (3), which become

$$
\frac{\partial \Phi_{D}^{(2)}}{\partial n} = -\frac{\partial \Phi_{I}^{o}}{\partial n}
$$
\n
$$
\frac{\partial \Phi_{D}^{(3)}}{\partial n} = -\frac{\partial \Phi_{I}^{e}}{\partial n}
$$
\n
$$
\frac{\partial \Phi_{D}^{(4)}}{\partial n} = -\frac{\partial \Phi_{I}^{o}}{\partial n}
$$
\n(3')

on the fixed body surface. This boundary condition is specifically written in the following forms by dropping the suffix D, i.e., for sway $(m=2)$,

$$
\sum_{j=1}^{N} Q_{j}^{(2)} + \sum_{j=1}^{N} Q_{N+j}^{(2)} + \sum_{j=1}^{N} Q_{N+j}^{(2)} = -\omega e^{-\nu y} i_{\left[sin\mu + \cos(\nu x_{1} + \sin\mu)\sin\alpha_{1}\right]} - \sin(\nu x_{1} + \sin\mu)\cos\alpha_{1}
$$
\n
$$
- \sum_{j=1}^{N} Q_{j}^{(2)} + \sum_{i,j=1}^{N} Q_{N+j}^{(2)} + \sum_{j=1}^{N} Q_{N+j}^{(2)} + \sum_{j=1}^{N} Q_{N+j}^{(2)} + \sum_{j=1}^{N} Q_{N+j}^{(3)} + \sum_{j=1}^{N} Q_{N+j}^{(4)} + \sum_{j=1}^{N} Q_{N+j}^{(5)} + \sum_{j=1}^{N} Q_{N+j}^{(6)} + \sum_{j=1}^{N} Q_{N+j}^{(7)} + \sum_{j=1}^{N} Q_{N+j}^{(8)} + \sum_{j=1}^{N} Q_{N+j}^{(9)} + \sum_{j=1}^{N} Q_{N+j}^{(1)} + \sum_{j=1}^{N} Q_{N+j}^{(1)} + \sum_{j=1}^{N} Q_{N+j}^{(1)} + \sum_{j=1}^{N} Q_{N+j}^{(2)} + \sum_{j=1}^{N} Q_{N+j}^{(1)} + \sum_{j=1}^{N} Q_{N+j}^{(2)} + \sum_{j=1}^{N} Q_{N+j}^{(3)} + \sum_{j=1}^{N} Q_{N+j}^{(4)} + \sum_{j=1}^{N} Q_{N+j}^{(5)} + \sum_{j=1}^{N} Q_{N+j}^{(6)} + \sum_{j=1}^{N} Q_{N+j}^{(7)} + \sum_{j=1}^{N} Q_{N+j}^{(8)} + \sum_{j=1}^{N} Q_{N+j}^{(9)} + \sum_{j=1}^{N} Q_{N+j}^{(1)} + \sum_{j=1}^{N} Q_{N+j}^{(1)} + \sum_{j=1}^{N} Q_{N+j}^{(1)} + \sum_{j=1}^{N} Q_{N+j}^{(1)} + \sum_{j=1}^{N} Q_{N+j}^{(2)} + \sum_{j=1}^{N} Q_{N+j}^{(1)} + \sum_{j=1}^{N} Q_{N+j}^{(2)} + \sum_{j=1}^{N}
$$

and, for heave-exciting (m=3),

$$
\sum_{j=1}^{N} Q_j^{(3)} i_{j}^{(3)} + \sum_{j=1}^{N} Q_{N+j}^{(3)} J_{ij}^{(3)} = 0
$$
\n
$$
\sum_{j=1}^{N} Q_j^{(3)} J_{ij}^{(3)} + \sum_{j=1}^{N} Q_{N+j}^{(3)} i_{j}^{(3)} = \omega e^{-\nu y} i_{\sin \mu} \cdot \sin(\nu x_i \cdot \sin \mu) \sin \alpha_i
$$
\n
$$
+ \cos(\nu x_i \cdot \sin \mu) \cos \alpha_i
$$

with specific definitions given ln the Appendix of Reference 10; and $Q_i^{(m)} = Q_r^{(m)}$ and $Q_{N+1} = Q_i^{(m)}$ at the jth segment. For Mode 4, the formula is the same as for mode = 2 . If N sources are distributed over the strip surface, we obtain a (2N x 2N) simultaneous equation system with 2N unknowns, i.e., the real and imaginary parts of the N sources. Here α_I is the angle and both $\prod_{i=1}^{n}$ and $\prod_{i=1}^{n}$ $\frac{S}{\lambda n}$ G ds and c c made between ^{ith} segment and the positive x-axis formally represent the normal derivati G_i ds , and are called the "Influence coefficients"

Since the right-hand sides of Eq. (10) and (11) represent the ex-

pression $\frac{1}{a}$ $\frac{\partial \Phi_1}{\partial n}$, the solution $Q_0^{(m)}$ gives the source intensity per unit amplitude of the incident wave and the insertion of this $Q_{n}^{(m)}$ in Eq. (9) then provides the diffraction potential per unit amplitude of the incident wave $\frac{1}{a}$ ϕ [[]

3. Forced Wave potential in the Strip Domain

From the solutions of the coupled heaving and pitching, and the coupled swaying, rolling and yawing motions of a ship in oblique seas, according to the procedure given in Reference 3 we obtain the five motions sway η , heave ζ , roll φ , pitch ψ , and yaw χ , which are expressed in the forms

 $\eta = 1\bar{\eta}$ l cos (wt + $\epsilon_{\eta h}$) $\vert \bar{\zeta} \vert$ cos (wt + $\varepsilon_{\zeta \, \mathsf{h}}$) $\varphi = \log \cos (\omega t + \epsilon_{\varphi h})$ lψl cos (ωt + ε_ψ⊦ cos (wt + $\epsilon_{\chi \rm l}$ (12)

where ϵ desginates the phase difference between the wave maximum at the origin o and the maximum of motion.

8

Now we confine our attention to the motion in a strip domain, 'The

motion consists of three degrees of freedom: sway, heave and roll in**duced by a'll the motions of the ship. The velocities of each strip are written as:**

$$
v^{(2)} = \frac{d}{dt} (zx + \eta)
$$

\n
$$
v^{(3)} = \frac{d}{dt} (-z\psi + \zeta)
$$

\n
$$
v^{(4)} = \frac{d}{dt} \varphi
$$
\n(13)

The normal (x₁,γ₁) o velocity components $V_n^{(m)}$ at the mid-point of the ith segment the **strip body directed into the water are**

$$
V_{n}^{(2)} = V^{(2)} \sin \alpha_{i}
$$

\n
$$
V_{n}^{(3)} = -V^{(3)} \cos \alpha_{i}
$$

\n
$$
V_{n}^{(4)} = -V^{(4)} \{x_{i} \cos \alpha_{i} + (y_{i} - y_{o}) \sin \alpha_{i}\}
$$
 (14)

where y_{0} = chosen roll axis **0**

> **= angle made between the 1 " segment with the positive x-axis as shown in Figure 2b**

Now, let us consider the forced wave potential $\Phi_{\text{r}}^{(m)}$ caused by the oscillation of the strip body with the velocity as given in Eq. (13). This **potential may** be **represented by the source potential as was used in repre**senting the diffraction potential $\lceil \varepsilon q \cdot (8) \rceil$, i.e.,

$$
\Phi_{\Gamma}^{(m)}(x,y,z;\mu,t) = R_{e}\left[\int_{0}^{m} Q_{\Gamma}^{(m)}(s) \cdot G(x,y;\xi,\eta)ds e^{i(\sqrt{z}cos\mu-\omega t)}\right]
$$
(8ⁱ)

where $Q_{\text{r}}^{(m)}$ denote the unknown complex source intensities. These in**tensities are determined by satisfying the kinematical boundary condit'ion, Eq .!,which becomes**

$$
\frac{1}{a} \frac{\partial \Phi_F^{(m)}}{\partial n} = \frac{V_m^{(m)}}{a} \tag{15}
$$

on the oscillating body surface at the mean position of oscillation. Equation (15) is specifically written as follows;

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For Sway:

$$
\sum_{j=1}^{N} Q_{j}^{(2)} \left[1_{ij}^{(2)} \cos(\nu z \cos \mu) + J_{ij}^{(2)} \sin(\nu z \cos \mu) \right]
$$

+
$$
\sum_{j=1}^{N} Q_{N+j}^{(2)} \left[J_{ij}^{(2)} \cos(\nu z \cos \mu) - I_{ij}^{(2)} \sin(\nu z \cos \mu) \right]
$$

=
$$
-\omega \left[\frac{\nu z I \bar{\chi} I}{\nu a} \sin \epsilon_{\chi h} + \frac{I \bar{\eta} I}{a} \sin \epsilon_{\eta h} \right] \sin \alpha_{i}
$$

$$
\sum_{j=1}^{N} Q_{j}^{(2)} \left[I_{ij}^{(2)} \sin(\nu z \cos \mu) - J_{ij}^{(2)} \cos(\nu z \cos \mu) \right]
$$

+
$$
\sum_{j=1}^{N} Q_{N+j}^{(2)} \left[J_{ij}^{(2)} \sin(\nu z \cos \mu) + I_{ij}^{(2)} \cos(\nu z \cos \mu) \right]
$$

=
$$
-\omega \left[\frac{\nu z I \bar{\chi} I}{\nu a} \cos \epsilon_{\chi h} + \frac{I \bar{\eta} I}{a} \cos \epsilon_{\eta h} \right] \sin \alpha_{I}.
$$
 (16)

For Heave:

$$
\sum_{j=1}^{N} Q_j^{(3)} \left[1 \binom{3}{j} \cos(\sqrt{z} \cos \mu) + J_{ij}^{(3)} \sin(\sqrt{z} \cos \mu) \right]
$$
\n
$$
+ \sum_{j=1}^{N} Q_{N+j}^{(3)} \left[J_{ij}^{(3)} \cos(\sqrt{z} \cos \mu) - J_{ij}^{(3)} \sin(\sqrt{z} \cos \mu) \right]
$$
\n
$$
= \omega \left[-\sqrt{z} \frac{1 \sqrt{y} \ln z}{\sqrt{a}} \sin \epsilon_{\psi h} + \frac{1 \binom{7}{2}}{a} \sin \epsilon_{\zeta h} \right] \cos \alpha_{i} \quad ,
$$
\n
$$
\sum_{j=1}^{N} Q_j^{(3)} \left[I_{ij}^{(3)} \sin(\sqrt{z} \cos \mu) - J_{ij}^{(3)} \cos(\sqrt{z} \cos \mu) \right]
$$
\n
$$
+ \sum_{j=1}^{N} Q_j^{(3)} \left[J_{ij}^{(3)} \sin(\sqrt{z} \cos \mu) + I_{ij}^{(3)} \cos(\sqrt{z} \cos \mu) \right]
$$
\n
$$
= \omega \left[-\sqrt{z} \frac{1 \sqrt{y} \ln z}{\sqrt{a}} \cos \epsilon_{\psi h} + \frac{1 \binom{7}{2}}{a} \cos \epsilon_{\zeta h} \right] \cos \alpha_{i} \quad .
$$
\n(17)

 $\hat{\mathcal{A}}$

 $\pmb{\cdot}$

For Roll:

$$
\sum_{j=1}^{N} Q_{j}^{(4)} \left[I_{ij}^{(4)} \cos(\sqrt{z} \cos \mu) + J_{ij}^{(4)} \sin(\sqrt{z} \cos \mu) \right]
$$

+
$$
\sum_{j=1}^{N} Q_{N+j}^{(4)} \left[J_{ij}^{(4)} \cos(\sqrt{z} \cos \mu) - I_{ij}^{(4)} \sin(\sqrt{z} \cos \mu) \right]
$$

=
$$
\omega \left[-\frac{\sqrt{|\sigma|}}{\sqrt{a}} \sin \epsilon_{\phi h} \right] \left[(\gamma_{1} - \gamma_{0}) \sin \alpha_{1} + \gamma_{1} \cos \alpha_{1} \right]
$$

$$
\sum_{j=1}^{N} Q_{j}^{(4)} \left[I_{ij}^{(4)} \sin(\sqrt{z} \cos \mu) - J_{ij}^{(4)} \cos(\sqrt{z} \cos \mu) \right]
$$

+
$$
\sum_{j=1}^{N} Q_{N+j}^{(4)} \left[J_{ij}^{(4)} \sin(\sqrt{z} \cos \mu) + I_{ij}^{(4)} \cos(\sqrt{z} \cos \mu) \right]
$$

=
$$
\omega \left[\sqrt{-\frac{|\sigma|}{\sqrt{a}}} \cos \epsilon_{\phi h} \right] \left[(\gamma_{1} - \gamma_{0}) \sin \alpha_{1} + \gamma_{1} \cos \alpha_{1} \right]
$$

Solving this system of 2N equations for the 2N unknowns, yields the required source intensities so that ϕ_F is now determined, in terms of the motion amplitudes.

4. The Asymptotic Expression of the Potentials

Since both expressions of the diffracted and forced wave potentials $\frac{1}{a}$ $\Phi_D^{(m)}$ and $\frac{1}{a}$ $\Phi_F^{(m)}$ are identical, we derive the asymptotic expression of the potential $\frac{1}{a} \Phi(m)$ without suffixes D and F. By introducing the symbols \pm for $x \rightarrow \pm \infty$, we write the potential,

$$
\frac{1}{a} \phi_{\pm}^{(m)} (x,y,z;\mu,t) = R_e \left[\int_c \theta^{(m)}(s) G_{\pm} (x,y;\xi,\eta) ds \cdot e^{i(\sqrt{z} \cos \mu - \omega t)} \right] (19)
$$

where $Q^{(m)}(s)$ is the complex source intensity per unit amplitude of the incident wave.

According to Wehausen and Laiton,¹³ the asymptotic expression of the potential G_{\downarrow} is

$$
G_{\pm}(x,y;\xi\eta) = \pm R_{e} \left[i e^{-i\sqrt{(z-\zeta)}} \right] - i R_{e} \left[e^{-i\sqrt{(z-\zeta)}} \right]
$$
 (20)

where

$$
z = x + iy \quad ; \quad \zeta = g + i\eta \quad ; \quad \zeta = \zeta - i\eta \qquad \text{with} \quad \eta < 0
$$

Let

$$
G_{\pm} = G_{r\pm} + i(-G_{i\pm})
$$
 (20¹)

with

$$
G_{r\pm} = \pm R_e \left[i e^{-i \nu (z-\zeta)} \right]
$$

$$
G_{l\pm} = R_e \left[e^{-i \nu (z-\zeta)} \right]
$$

and insert Eq. (20^t) in Eq. (19), then the integration is reduced approximately to,

$$
\frac{1}{a} \Phi_{\pm}^{(m)} = R_{e} \left[\sum_{j=1}^{N} Q_{j}^{(m)}(S_{j}) \left\{ \int_{S_{j}} G_{\pm} ds - (-1)^{m} \cdot \int_{S_{j}} G_{\pm} ds \right\} e^{i (\sqrt{2} \cos \mu - \omega t)} \right]
$$
\n(21)

where S_j and S_{-j} designates the jth and -jth segments, respectively¹⁰ (see Fig. 2b). The integrals in the above equation are put in the form

$$
I_{\pm} = I_{r_{\pm}} + i I_{i_{\pm}}
$$
 (22)

with

$$
\begin{aligned}\nI_{r\pm} &= \int_{S_j} G_{r\pm} ds - (-1)^m \int_{S_{-j}} G_{r\pm} ds \\
I_{i\pm} &= \int_{S_j} G_{i\pm} ds + (-1)^m \int_{S_{-j}} G_{i\pm} ds\n\end{aligned} (23)
$$

and the integrals I_{\pm} are given in the Appendix. Thus, the potential Eq. (21) , is transformed to

$$
\frac{1}{a} \Phi_{\pm}^{(m)} = R_{e} \Big[\sum_{j=1}^{N} (Q_j^{(m)} + i Q_{N+j}^{(m)}) (1_{r\pm} + i 1_{i\pm}) e^{-i (-\nu z \cos \mu + \omega t)} \Big] \qquad (24)
$$

with $Q_i^{(m)} = Q_r^{(m)}$; $Q_{N+1}^{(m)} = Q_i^{(m)}$ on the jth segment.

Inserting the integral $\frac{1}{4}$ in Eq. (A-6) in Eq. (24) and rearranging the expressions, we obtain the asymptotic expression in the form,

$$
\frac{1}{a} \Phi_{\pm}^{(m)} = A^{(m)} e^{\nu y} \cos \left[\nu x - \omega t + \nu z \cos \mu - \epsilon_{+}^{(m)}\right]
$$
\n
$$
[\nu x + \omega t - \nu z \cos \mu - \epsilon_{-}^{(m)}\right]
$$
\n(25)

re
\n
$$
\epsilon_{+}^{(m)} = \tan^{-1} \left[\frac{p_{+}^{(m)}}{c_{+}^{(m)}} \right]
$$
\n
$$
\epsilon_{-}^{(m)} = \tan^{-1} \left[\frac{p_{-}^{(m)}}{c_{-}^{(m)}} \right]
$$
\n
$$
A^{(m)} = \sqrt{c_{+}^{(m)^{2}} + p_{+}^{(m)^{2}}} = \sqrt{c_{-}^{(m)^{2}} + p_{-}^{(m)^{2}}}
$$
\n
$$
c_{+}^{(m)} = \sum_{j=1}^{N} \{ \pm Q_{j}^{(m)} [1 + (-1)^{m}] K_{j} + Q_{N+j}^{(m)} [1 - (-1)^{m}] L_{j} \}
$$
\n
$$
p_{+}^{(m)} = \sum_{j=1}^{N} \{ \pm Q_{j}^{(m)} [1 - (-1)^{m}] L_{j} - Q_{N+j}^{(m)} [1 + (-1)^{m}] K_{j} \}
$$

The outgoing waves at $x \rightarrow \pm \infty$ are readily derived from the potential $\frac{1}{a}$ $\Phi_{\pm}^{(m)}$ in the form,

$$
\frac{h^{(m)}}{a} = -\frac{1}{ga} \frac{\partial \Phi_{\pm}}{\partial t} \quad \text{on} \quad y=0
$$

or

whe

$$
\frac{h_{\pm}^{(m)}}{a} = \frac{\omega A^{(m)}}{g} \sin \left[-\nu x + \omega t - \nu z \cos \mu + \epsilon_{+}^{(m)} \right]
$$
\n
$$
\frac{1}{a} = \frac{\omega A^{(m)}}{g} \sin \left[-\nu x + \omega t - \nu z \cos \mu - \epsilon_{-}^{(m)} \right]
$$
\n(26)

 m! where $\frac{w_{\text{max}}}{q}$ is the amplitude ratio of the outgoing wave to the incident wav

5. Behavior of the Outgoing Waves

i.e., sway, then it follows that At this stage, It is **desirable** to exami ne the behavior of the waves $h_{\pm}^{(m)}$ for the different modes of forced oscillation in order to confirm the validity of the formula, Eq. (26). If we take the mode = 2 $\,$

$$
c_{\pm}^{(2)} = \pm 2 \sum_{j=1}^{N} K_{j}Q_{j}^{(2)}
$$

\n
$$
D_{\pm}^{(2)} = -2 \sum_{j=1}^{N} K_{j}Q_{N+j}^{(2)}
$$

\n
$$
c_{+}^{(2)} = \tan^{-1}\left[\frac{\sum_{j=1}^{N} K_{j}Q_{N+j}^{(2)}}{\sum_{j=1}^{N} K_{j}Q_{j}^{(2)}}\right]
$$

\n
$$
c_{+}^{(2)} = \tan^{-1}\left[\frac{\sum_{j=1}^{N} K_{j}Q_{N+j}^{(2)}}{\sum_{j=1}^{N} K_{j}Q_{N+j}^{(2)}}\right]
$$

It is seen from the above expressions that $\frac{c}{\varepsilon}$ is related to $\epsilon^{(2)}$ by the equation $\epsilon^{(2)}$ + $\epsilon^{(2)}$ = π . Referring to Eq. (26), it is stated that the outgoing waves $\mathsf{h}^{(2)}_+$ and $\mathsf{h}^{(2)}_-$ have the phase dif + ferences π . For roll, the same relation holds. This is ascribed to the asymmetric motion of the fluid with respect to the y-axis.

In a similar manner, we can examine the outgoing waves for mode = 3 ,

that these characteristics hold also for the diffracted wave i.e., heave, and obtain the relation $\varepsilon^{(3)}_+ + \varepsilon^{(3)}_- = 0$ waves $h_+^{(3)}$ and $h_-^{(3)}$ have the zero phase difference. T course ascribed to the symmetric flow about the y-axis. It is apparentThe outgoing This **is** of

We observe the same characteristic behavior of the outgoing waves derived from the radiation potential used by $Grim¹¹$ This fact assures us that we can readily estimate the wave-exciting forces and moments on a fixed ship in oblique seas by applying the above results in the light of Haskind-Newman theory.¹²

6. The Lateral Drifting Force and Moment

The scattered waves caused by the presence of the f loat ing strip section in the strip domain are the vectorial sum of the outgoing waves Eq. (26) due to diffraction and radiation, Since

$$
\frac{h_{-}^{(m)}}{a} = R_{e} \left\{ \frac{\omega}{g} A^{(m)} e^{-i \left[e_{-}^{(m)} + \frac{\pi}{2} + \sqrt{2} \cos \mu \right]} e^{i \left(\sqrt{x} + \omega t \right)} \right\}
$$

the complex outgoing wave at $x \rightarrow -\infty$ is

 $\mathbf{z}=\mathbf{z}$

$$
\frac{\overline{h}_{-}^{(m)}}{a} = \overline{A}_{-}^{(m)} \quad e^{i(\sqrt{x} + \omega t)}
$$
 (27)

with

$$
\bar{A}^{(m)}_{-} = \frac{\omega}{g} A^{(m)} e^{-i\left[\epsilon \frac{(m)}{2} + \frac{\pi}{2} + \sqrt{2} \cos \mu\right]}
$$
 (28)

 $\mathsf{A}^{\mathsf{new}}_{\mathsf{a}}$ is designated as the complex amplitude ratio. The vector sum of ; (m) the amplitude ratio $\bar{A}^{(m)}$ is given in the form

$$
\bar{A}_{-} = \left[\sum_{m=2,3} \bar{A}^{(m)}_{-}\right]_{\text{diffracted}} + \left[\sum_{m=2,3,4} \bar{A}^{(m)}_{-}\right]_{\text{radiated}}
$$
(29)

When the body is fixed, the second term on the right-hand side vanishes and the mode m=4 does not contr ibute to the scatter.

According to Maruo,⁶ the lateral drifting force on the strip body is represented in the form

$$
dK = \frac{1}{2} \rho g a^2 l \bar{A}_{\mu} l^2 dz
$$
 (30)

Thus, the resultant lateral drifting force and moment about the center of gravity are

$$
K = \frac{1}{2} \rho g a^{2} \int_{-\ell_{1}}^{\ell_{2}} |\bar{A}_{-}|^{2} dz
$$
\n
$$
M = \frac{1}{2} \rho g a^{2} \int_{-\ell_{1}}^{\ell_{2}} i \bar{A}_{-} l^{2} z dz
$$
\n(31)

or in dimensionless form

$$
\frac{K}{\frac{1}{2} \rho g a^{2}L} = \frac{1}{L} \int_{-\ell_{1}}^{\ell_{2}} I \bar{A}_{1}^{2} dz
$$
\n
$$
\frac{M}{\frac{1}{2} \rho g a^{2}L^{2}} = \frac{1}{L^{3}} \int_{-\ell_{1}}^{\ell_{2}} I \bar{A}_{2}^{2} dz
$$
\n(32)

where

 $L = ship length$

DISCUSSION

R-2

In order to conf irm the applicability of the theoretical procedure for the prediction of the drifting forces and moments, we chose the two important experimental results by Lalangas⁸ and Ogawa.⁷ Lalangas⁸ gives extensive measured data of the lateral forces and moments on the f ixed model in oblique seas and additionally the lateral forces on the "free model" in beam seas which is the most valuable data for our comparison. Ogawa⁷ gives the experimentally measured values of the lateral forces and moments on a f ixed model In oblique seas. The lateral forces on the fixed model measured by Lalangas appear to be negative in low frequency range, which is not acceptable both in the physical and theoretica **I** sense.

This is an error which is seemingly caused by the enormous difficulty of instrumental techniques to measure such a small dc-component of an oscillating force, especially In low frequency ranges. **In** order to clear up this suspicious matter, we first compared our prediction with Qgawa's results and it was seen that the agreement between the prediction and experiments were excellent. Since the lateral forces on **a** free body oscillating in oblique waves are of primary interest in this study, we present the model particulars tested by Lalangas and the damping characteristics of the model in tabulated forms (see Table I and II). The model was f itted with rudder, but in the calculations it was assumed that the rudder was not present.

The natural period, as well as the damping characters in Table II, are utilized in solving the coupled-swaying-rolling-yawing motions according to the procedure described in Ref. 3. The computation of the drifting forces and moments of the model were carried out for the range of wave-ship length ratio $\sqrt{L} = 0.2 - 1.6$, and for the headings $\mu = 30^{\circ}$, 60° , 90° , 120° and 150⁰ for both fixed and free models. The calculated results are illustrated in Figures 3 to 9. First of all, we are interested in Fig. 5 which shows the experimental results of the lateral forces on both f ixed

 i see Ref. 8 , page 8

and free models in beam seas. Referring to this figure, we see that first the agreement between measured and predicted values for the free body is excellent ln the Important high f requency range, while the discrepancy Is quite large in the low f requency range. As once mentioned **in** the begfnning of this section, the measurements by Lalangas may not be quite accurate. We observe also that the discrepancy is quite large between the measured and predicted forces on the fixed body. however, the accuracy of the present calculations is confirmed by comparison with the recent experimental results by Ogawa fn the Delft Shipbuilding Laboratory.⁷

Secondly, in Fig. 5, the forces in the very high frequency range are fluctuating signif fcantly. This fluctuation might be ascribed to the numerical procedure and could be avoided with a different computational scheme. However, the effort on this problem was abandoned **in** this study.

Thirdly, the drifting forces on the free body are generally less than the forces on the fixed body in both experiments and theory.

With respect to Figures 3, 4, 6 and 7, the discussions are similar to the above. **In** referring to Fig. 8, we see that the predicted lateral peak forces on the f ixed body always occur In beam seas, while the maximum forces on the free body occur not in beam seas but in quartering seas. The lateral moments about the vertical axis through the center of gravity are illustrated in Fig. 9. The maximum amplItude of the moments both on the fixed and free models occur in beam and quarterfng seas.

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CONCLUSIONS

From the preceding comparison and discussion, the following concl us ions are obta ined.

- I. The lateral drifting forces are significant in high frequencies, while negligible in low frequencies.
- 2. The maximum lateral forces on the f ixed body occur in beam seas only, while the maximum lateral forces on the free body occur in quartering seas. As the frequency is reduced, both of these maxima are reduced and become less pronounced.
- 3. The lateral forces on the free body are generally less than those on the f ixed body.
- 4. The lateral forces predicted and measured are general ly in good agreement.

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 \mathcal{L}_{max} and \mathcal{L}_{max}

 $\frac{1}{\sqrt{2}}$

TABLE 1 MODEL PARTICULARS

LBP	(L)	5.00 ft	
Breadth	(B)	0.667 ft	
Draft (level trim)	(T)	0.267 ft	
Displacement (FW)	Δ	33.27 lb	
LCG (abaft midsection)		0.075 ft	
VCG (below water line)	(00)	0.022 ft	
Rudder area		0.030 ft^2	
Water-plane area	(A_{ω})	2.355 ft^2	
Load water-line coefficient (c_{ω})		0.706	
Pitch gyradius	(κ_{ψ})	1.275 ft	
Yaw gyradius	(κ_{χ})	1.275 ft	
Roll metacentric height	$\sqrt{G}M$	0.025 ft	

 Δ

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L.

TABLE II ROLLING NATURAL PERIOD AND DAMPING COEFFICIENT

 $\mathcal{L}^{\text{max}}_{\text{max}}$ and $\mathcal{L}^{\text{max}}_{\text{max}}$

 $\ddot{}$

APPENOIX

Referring to Eqs. (22) and (23) and Reference 10, we consider the $\,$ integra l

$$
\int_{S_{j}} G_{r\pm} ds = \pm \operatorname{Re} \int_{S_{j}} i e^{-i \sqrt{z-\zeta}} ds
$$
 (A-1)

8y denoting the angle of the Jth segment with the positive x-axis by α_1 we write the differential arc (see Fig. 2b),

$$
ds = e^{\frac{1}{2}\alpha} J d\bar{\zeta}
$$

and

$$
\int_{S_j} i e^{-i\sqrt{c}} e^{i\sqrt{c}} ds = i e^{-i\sqrt{c}} e^{i\sqrt{c}} \int_{S_j} e^{i\sqrt{c}} d\zeta
$$

$$
= \frac{e^{-i\sqrt{c}}}{\sqrt{c}} e^{i\sqrt{c}} \left[e^{i\sqrt{c}} \right]_{S_j}^{\zeta_{j+1}} \eta_j
$$

and thus we obtain

$$
\int_{S_j} G_{r\pm} ds = \pm \operatorname{Re} \left\{ \frac{e^{-i\sqrt{2}}}{\nu} \left[e^{\sqrt{11}} j + 1 \right] e^{-i(\sqrt{5}j + 1 + \alpha_j)} \right\} e^{\sqrt{11}} j e^{i(\sqrt{5}j + \alpha_j)} \right\} (A-2)
$$

Next, we consider the integral,

$$
\int_{S_{-j}} G_{r\pm} ds = \pm \text{ Re } \int_{S_{-j}} i e^{-i\sqrt{z-\zeta}} ds
$$

where $\overline{\zeta} = -(\overline{\xi} + i\eta)$ and $d\overline{\zeta} = -dse^{i\alpha}j$.

In a similar way we obtain the result,

$$
\int_{S_{-j}} G_{r\pm} ds = \pm \operatorname{Re} \left\{ -\frac{e^{-i\sqrt{2}}}{\nu} \left[e^{-i(\sqrt{5}j + 1 + \alpha_j)} - e^{-i(\sqrt{5}j + \alpha_j)} \right] \right\}
$$
\n
$$
\tag{A-3}
$$

The rest of the component integrals are also calculated in the similar manner to the above and they are given in the form,

$$
\int_{S} G_{\mathbf{i} \pm} ds = Re \left\{ \frac{e^{-i\sqrt{z}}}{i\sqrt{z}} \left[e^{-i\sqrt{z}} \right]_{j+1} e^{-i(\sqrt{z}} j + i^{\alpha} \Delta j) - e^{-i\sqrt{z}} \right\} e^{-i(\sqrt{z}} j + \alpha j) \right\} \qquad (A-4)
$$

$$
\int_{S} G_{\mathbf{i} \pm} ds = Re \left\{ \frac{ie^{-i\sqrt{z}}}{\sqrt{z}} \left[e^{-i(\sqrt{z}} j + i^{\alpha} \Delta j) - e^{-i(\sqrt{z}} j + i^{\alpha} \Delta j) \right] e^{-i(\sqrt{z}} j + \alpha j) \right\} \qquad (A-5)
$$

Inserting the component integrals in Eq. (23) and rearranging, we obtain:

$$
I_{r\pm} = \pm \left[[1 + (-1)^{m}] K_{j} \cos w + [1 - (-1)^{m}] L_{j} \sin w k \right] e^{y}
$$

\n
$$
I_{i\pm} = - \left\{ [-(-1)^{m}] L_{j} \cos w + [1 + (-1)^{m}] K_{j} \sin w k \right\} e^{y}
$$
 (A-6)

where,

$$
K_{j} = \frac{1}{\nu} \left[e^{\nu T_{j+1}} \cos(\nu \xi_{j+1} + \alpha_{j}) - e^{\nu T_{j}} \cos(\nu \xi_{j} + \alpha_{j}) \right]
$$

\n
$$
L_{j} = \frac{1}{\nu} \left[e^{\nu T_{j+1}} \sin(\nu \xi_{j+1} + \alpha_{j}) - e^{\nu T_{j}} \sin(\nu \xi_{j} + \alpha_{j}) \right]
$$
\n(A-7)

FIGURE 1 COORDINATE SYSTEMS

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FIGURE 5 LATERAL FORCE ON FREE AND FIXED SERIES 60 MODEL IN BEAM SEAS

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