STUDY QUESTIONS IN PHYSICAL OCEANOGRAPH

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by

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PREFACE

This book is intended for students with a basic knowledge of physical oceanography. It also may stimulate instructors in other disciplines of the marine sciences to formulate practical exercises for their students. In addition, practical workers in the field may find it useful for review and reference. The book contains a collection of study questions which have their origin in practical exercises accompanying a series of courses given by the authors at the Universities of Hawaii, Kiel, and Miami.

This collection of problems and answers cannot and does not pretend to be complete. However, the selection of the material, and the sequence of its presentation, should lead to an improved understanding of the basic principles of physical oceanography. In a sense, the study questions complete the presently available textbooks. The answers are given either in the form of references to various textbooks or as final results in graphical or numerical form. This leaves the student the freedom to independently solve the problems and to then evaluate his results. Metric units have been used throughout the text. Slide rule accuracy is considered adequate for the solutions.

In addition to the numerous redundant textbook references in Part B (Answers), the user is referred to two generally useful publications: "Handbook of Oceanographic Tables," U. S. Naval Oceanographic Office, Spec. Publ. 68, Washington, 1966, and "Laboratory Manual" on Descriptive Oceanography by R. T. Hodgson from Oregon State University, Department of Oceanography.

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March, 1971

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TABLE OF CONTENTS

TABLE OF CONTENTS

TABLE OF CONTENTS

TABLE OF CONTENTS

TABLE OF CONTENTS

TABLE OF CONTENTS

GLOSSARY OF SYMBOLS

(*) = indicates cross-reference to example stated

x, y, z = Cartesian coordinates in east, north, and downward direction

$\vec{i}, \vec{j}, \vec{k}$ = unit vectors in x, y, z direction

φ = latitude

λ = wave length

$\varkappa, \eta, \gamma$ = wave numbers in x, y, z direction

a = amplitude

ξ = vertical displacement from the state of rest

H = constant water depth for a fixed location

D = constant layer depth for a fixed location

L = constant horizontal distance

Γ = $\frac{1}{\bar{\rho}} \frac{\partial \rho}{\partial z}$ = stability (frequently reported as $E = \Gamma \cdot 10^8$)

t = time

$\vec{\Omega}$ = (o, Ω_y, Ω_z) = earth's angular velocity

$|\vec{\Omega}|$ = $7.29 \cdot 10^5$ [sec^{-1}]

f = $2 \, |\vec{\Omega}| \sin \varphi$ = Coriolis parameter

ω = angular frequency

N = $\sqrt{g\Gamma}$ = Brunt-Väisälä frequency

ζ = relative vorticity

g = local acceleration of gravity

$\vec{K}$ = (K_x, K_y, K_z) = external forces

$\vec{R}$ = (R_x, R_y, R_z) = frictional forces

$\vec{T}$ = (T_x, T_y, T_z) = stress

T = temperature [°C]

S = salinity [‰]

GLOSSARY OF SYMBOLS

p = pressure

ρ = density

α = $\frac{1}{\rho}$ = specific volume

σ_t = conventionalized density for atmospheric pressure

Q = heat content

c_p = specific heat at constant pressure

η = specific entropy

A_x, A_y, A_z = turbulent exchange coefficients in x, y, z direction

PART A - QUESTIONS

CHAPTER 1. PHYSICAL PROPERTIES OF SEAWATER

1.1 Thermodynamics of Seawater

EXAMPLE 1.1.1 The Concept of Salinity

Using the first law of thermodynamics for a multiconstituent system within a gravitational field, the relationship

$$de = T d\eta - p d\alpha + \sum_i \mu_i dm_i - g dz$$

describes seawater as an equilibrium system of unit mass (e = specific total energy = specific internal energy + potential energy; μ_i = specific chemical potential of the i-th constituent of mass fraction m_i). A great simplification is achieved if

$$\sum_i \mu_i dm_i \quad \text{is replaced by} \quad 10^{-3} \mu dS$$

where μ is a combined specific chemical potential and S is the salinity in ‰. Describe the law which allows the replacement of $\sum_i \mu_i dm_i$ and derive the simplified term.

EXAMPLE 1.1.2 The Equation of State for Seawater

T, S, and p are the parameters which are measured by standard hydrographic techniques. In order to use the thermodynamic equation for seawater, given in Example 1.1.1, one of the quantities to be determined is

$$\alpha = \alpha(S,T,p)$$

(a) Using the total differential for α, discuss briefly the set of three fundamental determinations which have been carried out to determine $\alpha = \alpha(S,T,p)$.

(b) Equal volumes of water with the same density ($\sigma_t = 27.50$) are mixed at 1,000 meters.

$S_1 = 35.00$ ‰ $\qquad T_1 = 6.60°C$

$S_2 = 34.47$ ‰ $\qquad T_2 = 2.80°C$

Find the temperature, salinity, and density of the resultant water mass. Discuss the physical significance of this.

(c) Assume the temperature of the world oceans has increased by an average of 2°C since the last ice age. What would be the rise of the sea level due to thermal expansion alone?

Coefficient of thermal expansion $\beta = \dfrac{1}{\alpha_{S,T,p}} \dfrac{\partial \alpha_{S,T,p}}{\partial T} \approx 10^{-4} \, {}^\circ C^{-1}$

Assume that the ocean has vertical sidewalls and an average depth of 3,800 m.

EXAMPLE 1.1.3 Adiabatic Processes

Referring to Examples 1.1.1 and 1.1.2,

$$\eta = \eta(S,T,p)$$

has also to be determined for thermodynamic investigations of seawater. One particular aspect is a process where no changes of η occur (adiabatic processes). Neglecting influences of salinity on entropy changes, the relationship

$$d\eta = \frac{\partial \eta}{\partial T}\, dT + \frac{\partial \eta}{\partial p}\, dp = \frac{c_p}{T}\, dT - \frac{\partial \alpha}{\partial T}\, dp = 0$$

is obtained for adiabatic processes from the thermodynamic equation given in Example 1.1.1.

(a) Find an expression for $\left(\frac{dT}{dp}\right)_{\eta=\mathrm{const}} = \Gamma$

where Γ is the "adiabatic lapse rate." Using Γ, give an expression for the "potential" temperature θ. θ is the temperature which a volume of seawater would attain after it was raised adiabatically from an initial pressure p_1 and an initial temperature T_1 to the sea surface.

(b) Using the data given in TABLE 1.1, plot the TS-diagram on a large-scale TS plotting sheet including a σ_t-grid. For depths greater than 2,000 m, also plot the θS-diagram (TABLE 1.2). Is there any reason to prefer the θS-plot?

(c) In the Cayman Trench (Caribbean Sea), the potential temperature is constant at $\theta = 3.78^\circ$C from 3,000 to 7,000 meters. In the nearby North American Basin, the same potential temperature is found at 1,600 meters. In which direction does the water flow and what is the depth of the sill between the Trench and the Basin?

EXAMPLE 1.1.4 Effects of the Chemical Potential of Seawater

In order to obtain a complete description of seawater as a thermodynamic system at equilibrium

$$\mu = \mu(S,T,p)$$

must also be known.

(a) Which physical properties of seawater are related to the determination of $\mu = \mu(S,T,p)$?

TABLE 1.1

Vertical distribution of parameters in the Philippine Trench ("Willebrord Snellius," $\varphi=9^\circ 41'$N, $\lambda=126^\circ 51'$E, May 16, 1930)

Depth (m)	T°C	S‰	c(m sec^{-1})	Depth (m)	T°C	S‰	c(m sec^{-1})
0	28.80	34.44	1542.5	1,600	3.00	34.59	1488.2
25	28.30	34.35	1541.7	1,800	2.61	34.60	1489.8
50	28.20	34.18	1541.8	2,000	2.25	34.61	1491.6
75	27.50	34.50	1541.0	3,000	1.64	34.66	1505.9
100	25.90	34.71	1538.0	4,000	1.60	34.67	1522.9
200	15.15	34.60	1510.4	5,000	1.72	34.67	1540.7
400	8.50	34.47	1490.4	6,000	1.86	34.67	1558.8
600	6.48	34.52	1485.9	7,000	2.01	34.68	1577.1
800	5.35	34.52	1484.7	8,000	2.15	34.69	1595.4
1,000	4.45	34.55	1484.3	9,000	2.31	34.68	1613.9
1,200	3.80	34.56	1484.9	10,000	2.48	34.67	1632.3
1,400	3.35	34.57	1486.3				

TABLE 1.2

Adiabatic temperature decrease (in 0.01°C) if a water particle of temperature T is raised to the surface.

	T°C			
Depth (m)	-2	0	2	4
1,000	2.6	4.4	6.2	7.8
2,000	7.2	10.7	14.1	17.2
3,000	13.6	18.7	23.6	28.2
4,000	21.7	28.4	34.7	40.6
6,000	42.8	52.2	61.1	69.4
8,000	--	81.5	92.5	102.7
10,000	--	115.7	128.3	140.2

(b) The decrease of the temperature T_f of the freezing point due to increasing salinity is given by

$$\Delta T_f = -0.097 \text{ Cl} \qquad (T_f = 0^\circ\text{C for Cl} = 0 ‰)$$

The decrease of the temperature T_m of the density maximum due to increasing salinity is given by

$$\Delta T_m = -0.39 \text{ Cl} \qquad (T_m = 4^\circ\text{C for Cl} = 0 ‰)$$

(both formulas are approximations for small temperature and salinity ranges. For density/salinity conversion, use S = 1.80655 Cl ‰).

Find temperature and salinity where the lines of maximum density and freezing temperature intersect. Explain the role of this intersection point with respect to vertical convection and ice formation in fresh water lakes and oceanic areas.

1.2 Acoustical Properties

EXAMPLE 1.2.1 Isothermal and Adiabatic Compressibility

The specific volume, α_T, of a water particle with temperature T and salinity S under atmospheric pressure becomes

$$\alpha_{T,S,p} = \alpha_T - \mu p \alpha_T$$

if the particle is brought slowly to a pressure level p, where μ is the isothermal compressibility. The adiabatic compressibility is

$$\varkappa = -\frac{1}{\alpha_{T,S,p}} \frac{d\alpha_{T,S,p}}{dp}$$

(a) Find a relation between μ and $\varkappa$ by which to compute $\varkappa$ from TABLE 1.3. Compute $\varkappa$ for p = 500, and 7,000 dbar.

(b) Give examples of processes in the ocean which require the knowledge of either μ or $\varkappa$.

(c) What volume would a liter of surface seawater ($\rho_o = 1.02$ g/cm^3) have at a depth of 10,000 m in the Marianas Trench?

Assume an average isothermal compressibility of

$$\mu = 4.9 \times 10^{-11} \frac{\text{cm sec}^2}{\text{g}}$$

and average density of the water column of $\bar{\rho} = 1.04$ g/cm^3.

(d) Compute the sound velocity c in m/sec using Laplaces formula, which is

$$c^2 = \frac{\gamma}{\rho \varkappa}$$

(γ = ratio of specific heat for constant pressure and volume respectively) from the values obtained under (a). Use $\gamma/\rho = 1$.

TABLE 1.3

Isothermal Compressibility μ of Seawater of Salinity 34.85 ‰ and Temperature 5°C

p (dbar)	0	1,000	2,000	4,000	10,000
$\mu\ 10^9$ ($dbar^{-1}$)	4531	4458	4388	4256	3916

EXAMPLE 1.2.2 Vertical Distribution of Sound Velocity

(a) Discuss the vertical distribution of sound velocity given in TABLE 1.1 in terms of the dependence on temperature, salinity, and pressure.

(b) What is meant by the expression "SOFAR CHANNEL"? Explain whether there is any indication of a SOFAR CHANNEL in the data given in TABLE 1.1 and why sonic energy propagates along the axis of this channel.

EXAMPLE 1.2.3 Near Surface Sound Channel

The propagation of sonic energy occurs along rays, which can be described by Snell's Law

$$c_v = \frac{c}{\cos\,\theta}$$

where c_v = the Snell's Law constant for a given ray (cm/sec)
c = the velocity of propagation characteristic of the medium at some point on the ray (cm/sec)
θ = the angle of inclination of the ray at the same point (deg)

For a medium with

$$\frac{dc}{dz} = \text{const.} = g_1$$

a simple analysis yields the following equation for a ray path

$$x^2 + z^2 = \left(\frac{c_v}{g_1}\right)^2$$

with $x = 0$ where $\theta = 0$ (horizontal ray direction) and $z = 0$ where $c = 0$ (provided the region of constant gradient were sufficiently extensive).

(a) The most important constant gradient layer is the isothermal (and isohaline) layer near the ocean surface. Which effect accounts for a positive vertical sound velocity gradient? Give a numerical estimation of the gradient.

(b) Draw qualitatively the ray paths for sonic energy which is emitted by a submerged transducer in a constant gradient surface layer. Assume the reflections of the rays at the surface occur as from a rigid boundary.

(c) A directional sound source is located in the surface of a constant sound velocity gradient layer. The energy is emitted in the x-direction at an angle of $\theta = 30^\circ$ downward from the horizontal. At what distance would a receiver moving in the x-direction along the surface monitor the arrival of sonic energy? Use $g_1 = 0.02\ sec^{-1}$, $c(z = 0m) = 1{,}500m\ sec^{-1}$.

EXAMPLE 1.2.4 Echo Sounding

TABLE 1.4 gives the average sound velocity c for depth intervals.

(a) Determine the time for a sound impulse to be transmitted, reflected and received if the depth of the bottom was 125, 175, 350, 750, and 950 m.

(b) Calculate the absolute (meters) and relative (%) error if your echo-sounder is standardized to a speed of 1,500 m/sec.

TABLE 1.4

Vertical Sound Velocity Distribution

Depth (m)	c (m/sec)	Depth (m)	c (m/sec)
0		300	
	1534.2		1514.2
25		400	
	1534.1		1511.2
50		500	
	1533.8		1508.3
75		600	
	1528.3		1505.8
100		700	
	1523.9		1503.7
150		800	
	1520.3		1500.2
200		900	
	1517.2		1500.5
		1,000	

EXAMPLE 1.2.5 Shadow Zones

For both of the following cases, give a qualitative discussion of the changes in the speed of sound. At what angles is total reflection possible? Would it be possible for a submarine to hide in a shadow zone?

(i) Salinity constant

$T = 25^\circ C$ from 0-50 m

$T = 10^\circ C$ from 50 m down

(ii) Temperature constant

S = 20.0 ‰ from 0-50 m

S = 35.5 ‰ from 50 m down

1.3 Optical and Electrical Properties

EXAMPLE 1.3.1 Attenuation of Light

(a) What causes the attenuation of incident light in pure water and in seawater? Is

$$I = I_o e^{-\kappa z} \qquad (I = \text{intensity},\ \kappa = \text{constant})$$

an adequate general description of the attenuation? Why do you say so?

(b) Figure 1.1 shows the results of some measurements of the spectral attenuation coefficients. Using the results from (a), discuss Fig. 1.1 briefly and determine if the heat gain due to insolation is important at a depth of 5 m in the open ocean (extrapolation of the curves to the right in Fig. 1.1 is possible; the trend does not change throughout the infrared range).

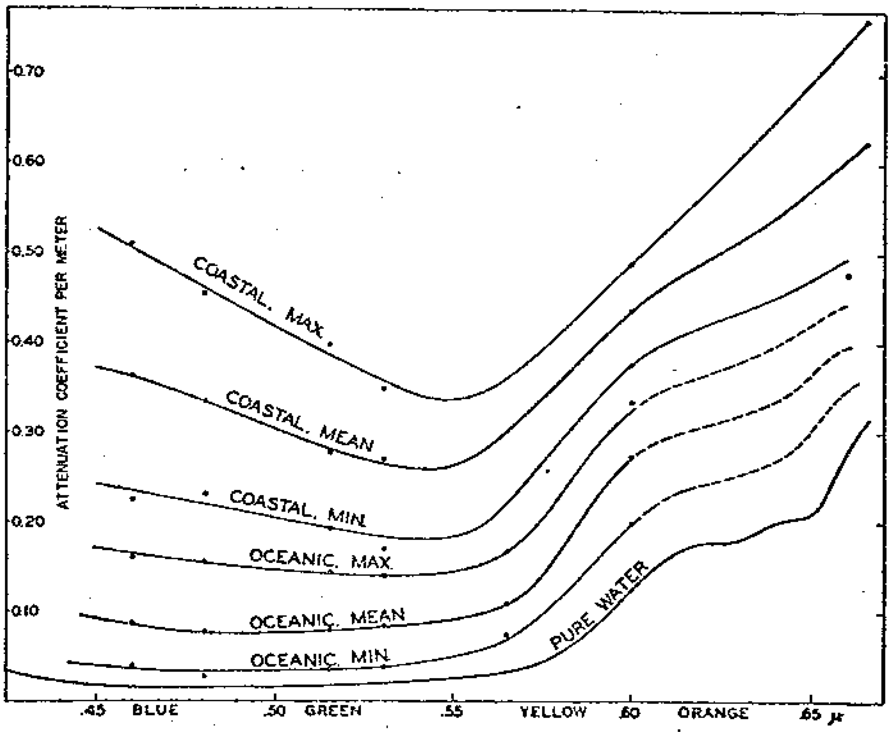


Fig. 1.1: Spectral attenuation coefficients in pure water and different types of seawater. From Sverdrup, et al., (1942).

EXAMPLE 1.3.2 Color of Seawater

The color of oceanic water masses is determined by the transparency D of seawater with

$$D = 100 \cdot e^{-\kappa(\lambda) z}$$

and by molecular scattering, the intensity I of which is characterized by

$$I \sim \frac{1}{\lambda^4}$$

(a) Using Fig. 1.1, describe why clean ocean water has a blue color and why coastal waters tend to appear green.

(b) Which is the most important factor determining the color of seawater?

(c) Why does light scattering by particles only play a minor role in determining the color of the sea (if the particle concentration is not too high)?

(d) How would the color of oceanic and coastal water change if the spectrum of the incident light were cut off for wavelengths $\lambda \leq 0.55\ \mu$?

EXAMPLE 1.3.3 Temperature Measurements from Air and Space

(a) Which physical process offers a possibility for remote sensing of ocean temperatures and which law describes this process? What basic assumptions are involved?

(b) From Wien's displacement law, find the wavelength λ_e at which maximum emission occurs. Using Figure 2.1, decide whether λ_e is suitable for airborne or space borne sensing of ocean temperature. Assume a sea surface temperature of $T = 15°C$.

(c) Which sources of radiative energy contribute to the final signal of a radiometer, which senses radiation in bands from 3 to 4 μ and 9 to 12 μ and which is mounted on an aeroplane or on a satellite so that it faces a small area of the ocean surface? Taking the emissivity of the ocean surface to be 0.98 and the reflectivity of normal incident radiation to be 0.02, make a qualitative statement about the accuracy of the remote temperature sensing techniques.

(d) If the attenuation of infrared light in seawater is 4 orders of magnitude larger than for the visible part of the spectrum, give an order of magnitude estimate of the depth of the surface layer which effects the infrared radiative output from the sea surface (use the results from Example 1.3.2).

EXAMPLE 1.3.4 Electrical Conductivity of Seawater

Seawater is known to be a good electrical conductor.

(a) From the structure of the water molecules, find the reason for the high conductivity values. How does conductivity depend on salinity, temperature, and pressure? (Give a qualitative answer.)

(b) Discuss the difficulties involved with salinity determinations using conductivity and induction cells. How are the cells calibrated?

EXAMPLE 1.3.5 Electromagnetic Method of Measuring Currents

The movement of seawater with the velocity $\vec{V}$ through the stationary magnetic field $\vec{H}$ of the earth gives rise to electric potential gradients $\nabla\Phi$

$$\nabla\Phi = (\vec{V} \times \vec{H}) - r\vec{I}$$

(r = resistivity, $\vec{I}$ = electric current density)

(a) Assume a straight canal of width L, a current flowing through the canal with speed v, and two electrodes being mounted in the surface layer at opposite places on the banks of the canal. Derive an expression for the current velocity as a function of the potential difference (in volts) between the electrodes. What further information do you need for computing the current speed, and how could they be obtained?

(b) What else must be considered if the electrodes are not installed at a fixed location but are towed behind a ship? Discuss the shortcomings of current measurements by means of a GEK (geomagnetic electrokinetograph), especially with respect to use in shallow water and in areas with a complicated vertical profile of current velocity.

1.4 Diffusion and Mixing

EXAMPLE 1.4.1 Richardson's Criterion

The Richardson criterion for maintaining turbulence in a stratified medium is given by

$$Rf = \frac{A_d g \frac{d\rho}{dz}}{A_v \rho (\frac{du}{dz})^2} < 1 \quad \text{(Richardson-flux-number)}$$

where A_d and A_v = coefficients of eddy diffusivity and eddy viscosity respectively.

(a) Discuss Rf in terms of potential and kinetic energy rates. Is A_d/A_v generally greater or smaller than 1?

(b) Using finite differences and a maximum Δz value of 40 meters, plot $\frac{A_v}{A_d}$ Rf = R_i = 1 (Richardson-number) and indicate the turbulent regime. Let $\Delta\rho = 10^{-3}$ and 10^{-2}, $\rho = 1.0\text{g cm}^{-3}$.

EXAMPLE 1.4.2 Approximations of Reynolds Stress Terms

Reynolds introduced the assumption $\Psi = \overline{\Psi} + \Psi'$ (with $\Psi = \vec{V}, \rho, p$, $\overline{\Psi}$ = time average, Ψ' = turbulent fluctuation) into the equation of motion. The equation, which results for the averaged field of motion, contains

$$\nabla \cdot \bar{\rho}\overline{\vec{V}'\vec{V}'}$$

This term describes the friction due to turbulent velocity fluctuations in a flow with the average velocity $\vec{V}$ and $\bar{\rho}\overline{\vec{V}\vec{V}} = T$ Reynolds stress tensor. For practical reasons, T is often approximated by

$$T = A\overline{D}$$

where $\overline{D}$ is the deformation tensor of the average velocity and A is a proportionality factor. This factor A is called the coefficient of eddy viscosity and is used either as a scalar, vector, or tensor.

(a) Discuss the significance of using A as a scalar, a vector, or a tensor to describe eddy viscosity.

(b) Why is A most frequently used as a vector?

EXAMPLE 1.4.3 Vertical and Horizontal Eddy Diffusivity

(a) The salinity distribution observed in the transition area between the Baltic and the North Sea was interpreted by Jacobsen as an equilibrium between horizontal advection and vertical turbulent diffusion.

(i) Find the appropriate equation (e.g., refer to Example 2.5.1) and calculate A_z from the data given in TABLE 1.6. The average density of the water column is 1.02g cm^{-3}; the distance between stations A and B was 47 km.

(ii) Plot u, S at both stations and A_z against depth.

(iii) Why do the low-salinity Baltic water and higher-salinity North Sea water essentially maintain their identities?

(b) The horizontal salinity distribution in the Irish Sea (see Fig. 1.2) can be described by horizontal advection due to a current of speed u and eddy diffusion transverse to the current. Using a coordinate system oriented with u parallel to the x-direction and y normal to x, the horizontal eddy diffusivity A_y can be described by

$$A_y = \rho u \frac{\partial s}{\partial x} \Big/ \frac{\partial^2 s}{\partial y^2}$$

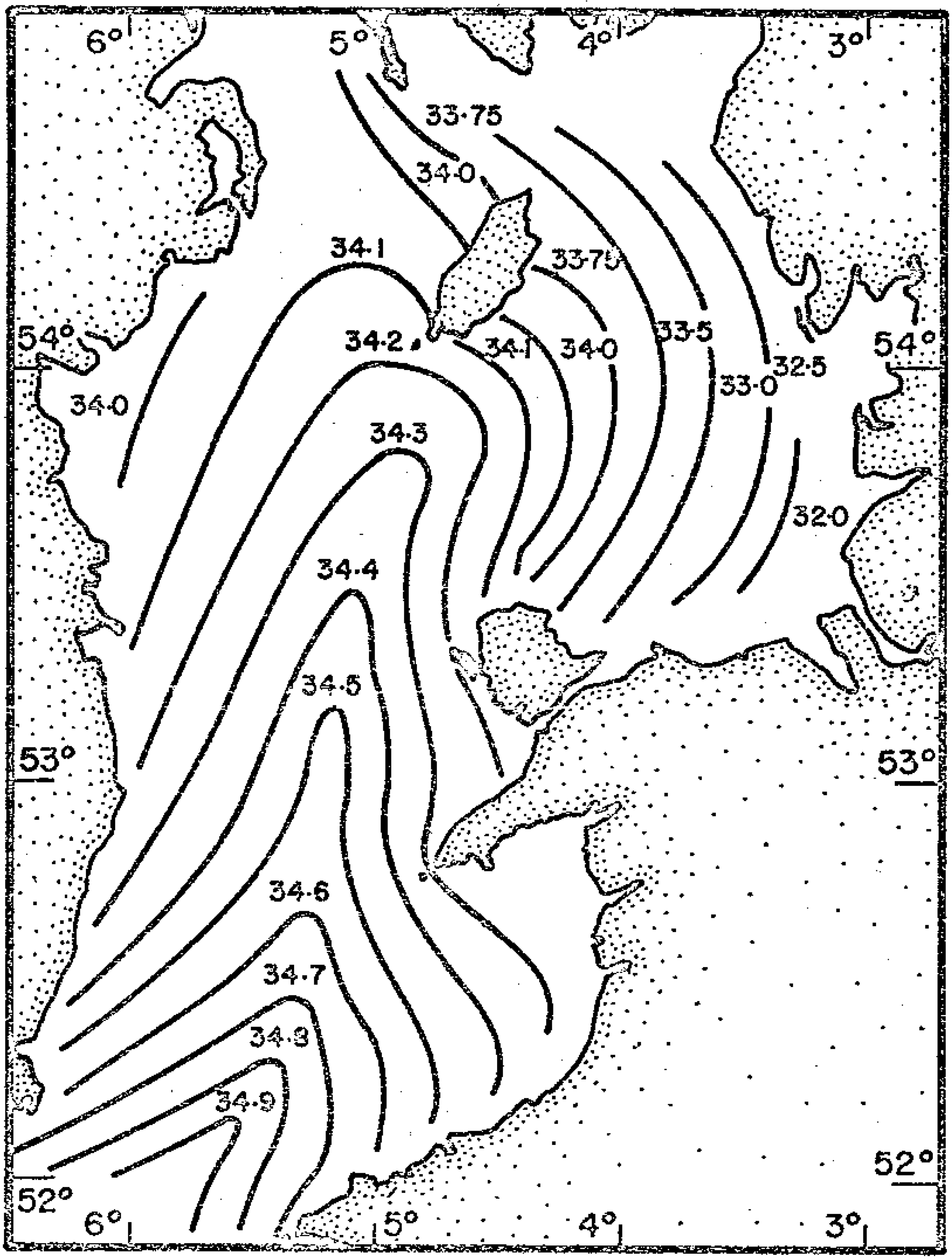


Fig. 1.2: Mean isohalines for the Irish Sea. From Proudman, (1952).

TABLE 1.6

Vertical Distribution of Current, Velocity, and Salinity in the Kattegat

z (m)	u ($cm\ sec^{-1}$)	S ‰ Station A	S ‰ Station B
0.0		18.1	18.2
	7.4		
2.5		18.3	18.7
	5.0		
5.0		18.6	19.1
	0.8		
7.5		19.5	19.5
	- 4.8		
10.0		20.5	20.0
	-10.7		
12.5		23.6	23.4
	-15.5		
15.0		26.7	26.8
	-17.5		
17.5		28.3	29.1
	-17.6		
20.0		-	-

Applying the formula to the vertices of the isohalines shown in Fig. 1.2, A_y can be estimated using

$$A_y = \rho u \frac{1}{4} \frac{(\delta y)^2}{\delta x}$$

for a vertex P, where δx is the distance from the vertex behind P to that in front of P and δy is the intercept of the tangent at P made by the isohaline in front of P.

Using $u = 1\ cm\ sec^{-1}$, find A_y for the Irish Sea from Fig. 1.2 for isohalines 34.4, 34.5, and 34.6 ‰ .

(c) Explain the reason for the order of magnitude differences between A_z and A_y determined in (a) and (b).

EXAMPLE 1.4.4 Spatial Scale of Turbulent Processes

From the statistical theory of turbulence the relation

$$A \sim L^{4/3}$$

where A = eddy coefficient, L = length scale, e.g., diameter of eddy or depth of wind mixed layer, was derived by Weizäcker and Heisenberg. The basic assumptions in this statistical theory are homogeneity (local independence) and isotropy (independence from rotation or reflection) of the statistical parameters.

(a) Discuss the order of magnitude range of the eddy coefficient in the oceans in terms of the proportionality relation given above.

(b) Discuss the validity of the assumptions for oceanic conditions.

1.5 Properties of Sea Ice

EXAMPLE 1.5.1 The Anomalous Density of Water

(a) Why does ice float? (Consider the molecular arrangements in water and ice.) What would happen to the climate if ice would behave like a normal solid phase?

(b) Discuss the effect of an ice cover on the vertical distribution of temperature in a water column below it. Distinguish between a freshwater lake and the ocean..

(c) Is it possible to hinder the formation of ice by pumping bottom water to the surface in the (i) North Sea; in the (ii) Baltic Sea?

EXAMPLE 1.5.2 Freezing and Aging of Sea Ice

(a) Discuss the formation of sea ice under different environmental conditions as characterized by the data given in TABLES 1.7, 1.8, 1.9.

(b) Why does the porosity of sea ice increase with increasing age of the ice? How can this effect be used to explain the fact that the air content of sea ice from high saline ocean areas is larger than that from waters with lower salinities?

TABLE 1.7

Salinity of Young Sea Ice, Formed at Different Air Temperatures

Air temperature (°C)	-16	-28	-30	-40
Salinity of sea ice (‰)	5.64	8.01	8.77	10.16

TABLE 1.8

Vertical Salinity Distribution in a Sheet of Young Sea Ice

Vertical distance from ice surface (cm)	0	6	13	26	45	95
S(‰) of the ice	6.74	5.28	5.31	3.84	4.37	3.17

TABLE 1.9

Specific Heat of Sea Ice

Salinity (S ‰) \ Temperature (°C)	-2°	-4°	-8°	-14°	-20°
2	2.57	1.00	0.63	0.54	0.52
4	4.63	1.50	0.76	0.57	0.55
8	8.76	2.49	1.01	0.64	0.60
10	10.84	2.99	1.14	0.68	0.62
15	16.01	4.24	1.46	0.77	0.68

NOTE: Tables 1.7, 1.8, 1.9 from Neumann and Pierson (1966).

EXAMPLE 1.5.3 Growth Rate of an Ice Sheet

Assume an ice sheet of thickness h. If it grows at a rate dh during a time interval dt, the released heat of fusion per unit volume, W, where

$$W = \rho\lambda\frac{dh}{dt}$$

must be transported upward by the heat flux through the ice, F, given by

$$F = -\ell\frac{dT}{dz}$$

where λ is the latent heat of fusion for ice of density ρ and thermal conductivity ℓ.

(a) Find an approximate formula which enables you to calculate the thickness h of an ice sheet from the time history of the surface temperature of the ice. Assume the temperature gradient in the ice to be linear and the lower boundary of the ice sheet be kept at the freezing temperature, 0°C.

(b) What assumptions about the water underlying the ice sheet are necessary? Would a layer of snow on the ice affect the calculations?

(c) Calculate the thickness of an ice sheet which has grown for a period of 30 days at surface temperatures of -15°C. Use $\rho = 0.917 g\ cm^{-3}$, $\lambda = 80\ cal\ g^{-1}$, $\ell = 0.006\ cal\ cm^{-1}\ sec^{-1}\ degr^{-1}$.

CHAPTER 2. BUDGET OF HEAT AND MASS

2.1 Heat Transfer Ocean <-> Atmosphere

EXAMPLE 2.1.1 Equation of Heat Balance

Derive the heat balance equation for a water column of 1 m^2 cross section, which stretches from the surface to the bottom of the ocean. Arrange the heat gain terms at the left side of the equation.

EXAMPLE 2.1.2 Radiation Balance

The short wave radiation from the sun has an intensity of 1.9 cal cm^{-2} min^{-1} at the distance of the earth's orbit. The earth emits 0.49 cal cm^{-2} min^{-1} in long wave radiation. Explain quantitatively whether the earth's temperature can remain constant for these radiation fluxes. (Acc. to v. Arx, 1962.)

EXAMPLE 2.1.3 Influence of Atmospheric Gases (*1.3.3)

Figure 2.1 gives the spectral absorption of atmospheric gases.

(a) Using Wien's displacement law, indicate on Fig. 2.1 the wavelength bands of maximum solar radiation input ($T_{sun} \sim 6{,}000^\circ$ K) and maximum earth's emission ($T_{earth} \sim 287^\circ$ K).

(b) Explain the expressions "radiation window" and "green house effect."

(c) How would temperatures on earth change due to a slight decrease in solar radiation intensity?

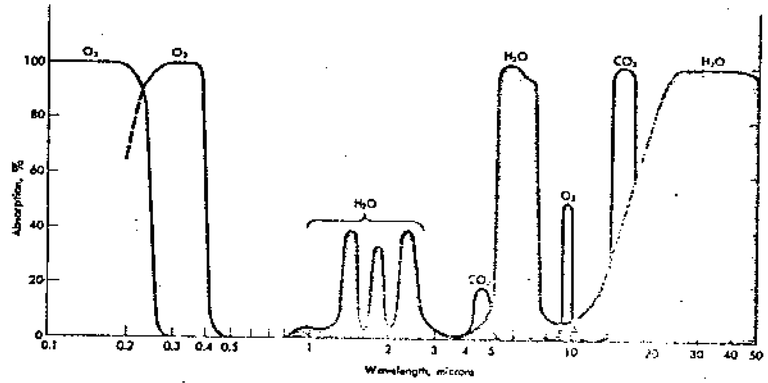


Fig. 2.1: Spectral absorptivity of water vapor and other atmospheric gases. From v. Arx (1962).

EXAMPLE 2.1.4 Absorption of Radiation Energy

Assume the surface of an oceanic area to receive 295 cal/cm^2 during the day and to emit 295 cal/cm^2 during the night.

(a) Using $\varkappa = \frac{2.3}{D}$ (log Q_z - log Q_{z+D}) and TABLE 2.1, compute the diurnal temperature variations for depth intervals given in TABLE 2.1. For the specific heat of seawater use 1 cal g^{-1} °C^{-1}. Q_z = amount of heat available at depth z in cal cm^{-2}, D = depth interval.

(b) Note the assumption involved.

TABLE 2.1

Extinction Coefficient $\varkappa$ Per Meter for Oceanic Water Masses (After Sverdrup, et al., 1942)

Depth Interval (m)	$\varkappa$
0 - 1	1.080
1 - 2	0.230
2 - 5	0.159
5 - 10	0.120
10 - 20	0.094
20 - 50	0.083

EXAMPLE 2.1.5 Heat Storage Capacity (*2.2.1)

(a) The global annual temperature fluctuations can be approximated by a 2°C amplitude for a layer of 100 m depth in the ocean and a 7°C amplitude for a layer of 10 m depth of the land surface.

Find the annual heat storage capacity of ocean and land masses and compare it to the daily incoming radiation of 0.21 cal cm^{-2} min^{-1} at the earth's surface. How do you explain the different capacities?

(b) By what amount could the temperature of the "homogeneous" atmosphere (height 8,000 m) be increased by using the heat which is annually stored in the oceans and in the land masses?

(c) Explain the following observation: During January the earth passes the perigee of her solar orbit and thus the heat gain of the earth's surface increases by 7%. Despite this fact, the earth's surface has average temperatures of 12.5°C in January and 16.1°C in July.

(d) Figure 2.2 shows isopleths of the Atlantic Ocean surface temperatures.

(i) Why is the amplitude of seasonal temperature variation in the Atlantic greater in the Northern Hemisphere than in the Southern?

(ii) Why is the "thermal equator" (dotted line in Fig. 2.2) located in the Northern Hemisphere?

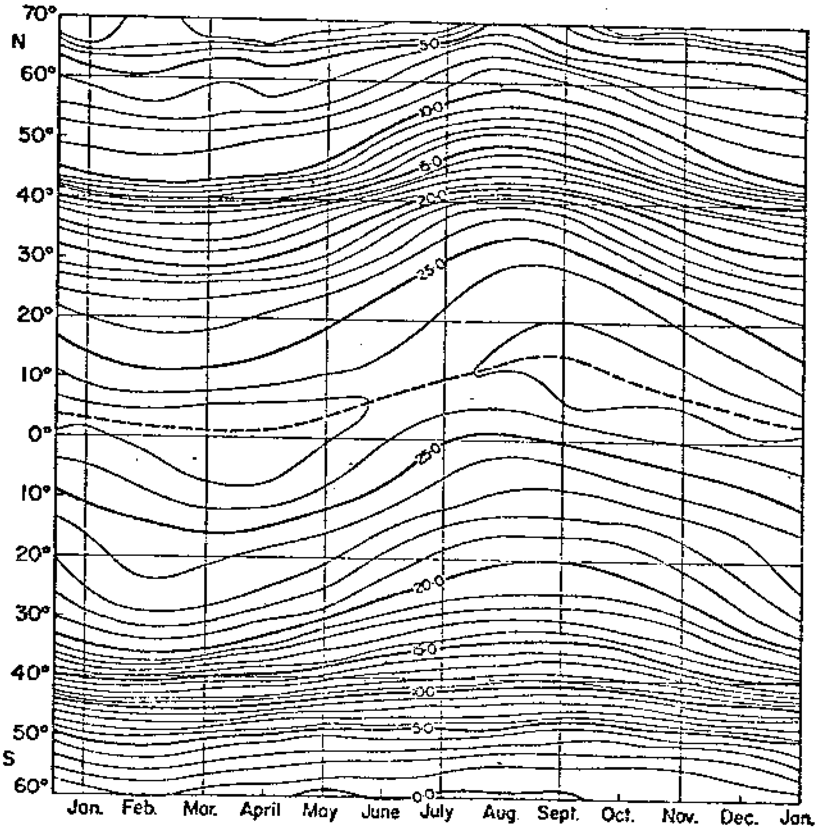


Fig. 2.2: Isopleths of surface temperature in the Atlantic Ocean. From Defant (1961).

Use the following values:

	Ocean	Land	Atmosphere
Area (km^2)	360.8×10^6	149.3×10^6	
Density ($g \cdot cm^{-3}$)	1.0	1.6	1.29×10^{-3}
Specific Heat ($cal \cdot g^{-1} \cdot °C^{-1}$)	1.0	0.2	0.24

EXAMPLE 2.1.6 Temperature Difference Water-Air

(a) Why is the sea normally warmer than the air above it?

(b) Give examples where the converse is true.

(c) Areas in which the atmosphere is warmer than the sea are often hazardous to shipping. Why?

2.2 Heat Transfer in the Ocean

EXAMPLE 2.2.1 Energy of Oceanic and Atmospheric Circulation (*2.1.5)

(a) Calculate the heat energy which would be released if all oceanic and atmospheric circulation came to a standstill (consider only the released kinetic energy).

(b) Compare these results with the heat which the ocean and atmosphere receive per day by radiation.

(c) Compare the kinetic energy contained in the oceans and in the atmosphere with the latent heat contained by water vapor in the air.

(d) Although the kinetic energy of oceanic circulation is low compared to atmospheric circulation, why does the former have a pronounced effect on the latter?

	Ocean	Atmosphere
Average speed (V)	10 cm/sec	10 m/sec
Mass (m)	1.4×10^{24} g	10^{22} g
Area (A)	360×10^6 km^2	510×10^6 km^2
Radiation (Qi)	295 $cal/cm^2/day$	700 $cal/cm^2/day$
Water vapor content (M)		3 g/cm^2 of earth's surface
Heat of evaporation (L)	590 cal/g	

EXAMPLE 2.2.2 Meridional Net Heat Transfer

Sverdrup gives the following estimates of mass transport across the equator in the Atlantic Ocean.

Water Mass	Direction (towards)	Transport ($10^6 m^3/sec$)	Average Temperature (°C)
Surface Water	N	6	25
Central Water	N	2	5
North Atlantic Deep Water	S	9	3
Antarctic Bottom Water	N	1	2

Calculate the net heat transport and its direction.

EXAMPLE 2.2.3 Advective Heat Transfer of the Gulf Stream

Assume that the Gulf Stream is $L = 100$ km wide, $D = 500$ m deep and that its average speed if $V = 150$ cm/sec. The temperature of the Gulf Stream at 40°N is $\Delta T = 4°C$ above the average for this latitude.

(a) Calculate the excess heat flux across 40°N by the stream per cm^2 per second. Take the specific heat of water $c_p = 1$ cal g^{-1} $°C^{-1}$. From this heat flux, calculate the power in kilowatts of the Gulf Stream.

(b) Compare the kinetic energy of the stream with its excess heat content per unit mass.

EXAMPLE 2.2.4 Vertical Heat Conduction

The theory of heat conduction shows if the surface change in temperature has the simple form

$$T_o = a_o \cos \frac{2\pi t}{\tau}$$

at depth z the temperature change is given by:

$$T\ (t,z) = a_o \cdot e^{-\alpha z} \cdot \cos\ (\frac{2\pi t}{\tau} - \alpha z)$$

$\alpha = \sqrt{\frac{\pi}{A_z \tau}}$, a_o = amplitude of the temperature change at the surface, τ = period of the temperature change.

(a) Let $\cos\ (\frac{2\pi t}{\tau} - \alpha z) = 1$ and determine the depths at which the temperature change is $e^{-\pi}$ of its surface value: if $\tau = 1$ day; if $\tau =$ 1 year.

In both cases use $A_z = 1.3 \times 10^{-3}$ cm^2/sec (molecular thermal diffusivity); $A_z = 30$ cm^2/sec (eddy thermal diffusivity).

Derive the general formulas first and then calculate the numerical answer. Work in the cgs system but give the answers in meters.

(b) At what depths is the phase lag 180°, 360° compared to the surface change? Use $A_z = 30$ cm^2/sec; $\tau = 1$ day, 1 year.

(c) At what speed does the surface disturbance penetrate downwards? i.e., what is the phase velocity? Use $A_z = 30$ cm^2/sec; $\tau = 1$ day, 1 year.

EXAMPLE 2.2.5 Vertical Heat Transfer Due to Tidal Mixing

The vertical distribution of maximum tidal currents in shallow water can be described by

(1) $$u(z) = u_o \left(\frac{H - z}{H}\right)^{1/5}$$ ($z = o$: surface; $z = H$: bottom; u_o = velocity at $z = o$)

Active vertical mixing in the presence of a density stratification takes place if

(2) $$Ri = \frac{g}{\bar{\rho}} \frac{\frac{d\rho}{dz}}{\left(\frac{du}{dz}\right)^2} < 0.5$$ (Ri = "Richardson" number)

The maximum vertical temperature gradient, which is created by downward penetration of heat from diurnal insolation into an initially homogeneous water mass $(\bar{T},\bar{S})$, can be obtained from the heat conduction equation as

(3) $$\frac{dT}{dz} = \sqrt{2}\ \beta \cdot T_o \cdot e^{-\beta z}$$ $$\beta = \sqrt{\frac{\bar{\rho}\pi}{A_z \tau}}, \quad \tau = \frac{2\pi}{1\ \text{day}},$$

T_o = amplitude of diurnal temperature changes

(a) Determine the depth where the vertical density gradient created by diurnal insolation prevents vertical mixing due to a shearing tidal current. (Derive an expression for the critical vertical density gradient from (1) and (2). Convert the temperature gradients obtained from (3) into density gradients assuming constant salinity. Use 10 m - steps for computation.)

(b) Compute the current velocity which would maintain homogeneity of the entire water column.

(c) Explain the remarkable low amplitudes of the seasonal temperature variations in the English Channel area.

Use the following values (North Sea):

$H = 60$ m	$A_z = 100$ g cm^{-1}sec^{-1}	$\bar{T} = 12°C$
$u_o = 0.7$ m·sec^{-1}	$T_o = 0.2°C$	$\bar{S} = 35$ ‰

2.3 Water Transport Ocean <-> Atmosphere

EXAMPLE 2.3.1 Meridional Distribution of Evaporation and Precipitation

Figure 2.3 gives the meridional distribution of surface salinity (S) and evaporation minus precipitation (E - P) for the world oceans.

(a) Discuss the close similarity of the S and E - P curves and the significance of the differences.

(b) Why is the salinity of the Atlantic Ocean higher than in the other oceans?

In answering, consider the complete water-budget equation:

$$W = (E - P) + (F - M) + (G - L) + R$$

where

E = Evaporation
P = Precipitation
F = Formation of ice
M = Melting of ice
G = Gain by currents
L = Loss by currents
R = River run-off

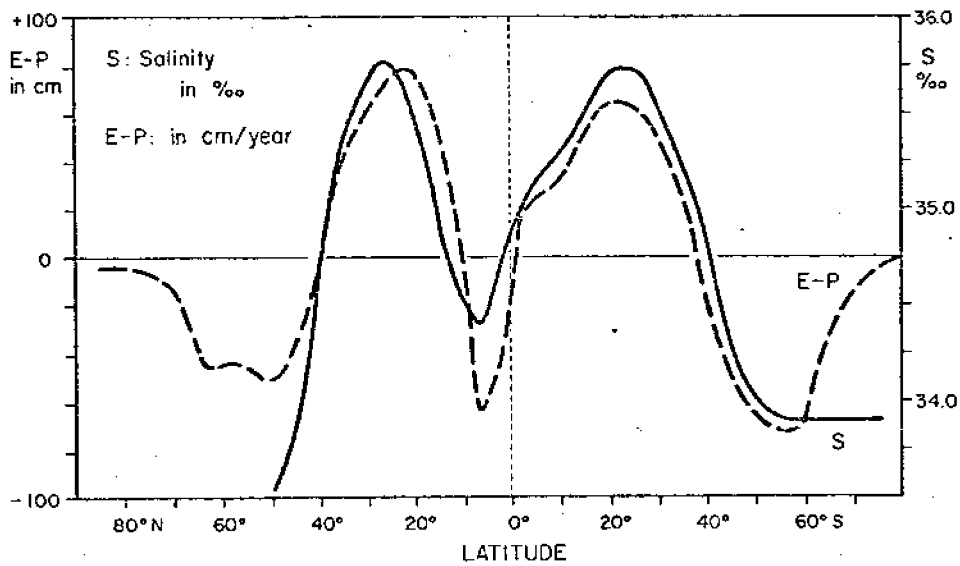


Fig. 2.3: Mean meridional distribution of evaporation minus precipitation and surface salinity for the world ocean. After Wüst (1954).

EXAMPLE 2.3.2 Surface Salinity, Evaporation, and Precipitation

Plot the values given in TABLE 2.2 in a S vs. (E - P) diagram.

(a) Is a linear relationship between S and (E - P) justified? Base your explanation on the graph and on the processes determining S.

TABLE 2.2

Five Degree Zonal Averages of Precipitation, Evaporation, and Salinity (After Wüst)

No.	Latitude Zone	P (cm/yr)	E (cm/yr)	S ‰
1	N 55-50	105	55	33.41
2	50-45	112	66	33.69
3	45-40	102	84	34.14
4	40-35	86	108	35.41
5	35-30	74	125	35.50
6	30-25	63	132	35.76
7	25-20	57	137	35.64
8	20-15	70	135	35.14
9	15-10	103	132	34.78
10	S 10-15	94	139	35.42
11	15-20	76	137	35.62
12	20-25	68	133	35.74
13	25-30	65	123	35.68
14	30-35	70	110	35.46
15	35-40	90	96	35.04
16	40-45	110	78	34.54
17	45-50	117	56	34.14
18	50-55	109	39	33.96
19	55-60	84	12	33.94

EXAMPLE 2.3.3 Diurnal Salinity Variations

TABLE 2.3 gives hourly surface salinities averaged over several days at a Meteor station and an Altair station.

(a) What is the average daily variation of salinity at each station?

(b) What causes the maximum salinity in each case? Consider the difference in position of the stations.

(c) The decrease with depth of a salinity perturbation follows an exponential law similar to that of temperature (see Example 2.2.4). At what depth does the perturbation reach 0.002 ‰ at each station? Use an eddy diffusivity $A_z = 30\ cm^2/sec$.

(d) Assume the period to be one year instead of one day. Calculate the depth at which the amplitude reaches 0.002 ‰. Why is such a depth not reached when a thermocline is present?

TABLE 2.3

Variation of Salinity with Time of Day
(After Defant)

Local Time (Hours)	METEOR STATION 2.1° S, 4° W	ALTAIR STATION 44.5° N, 34.0° W
1	35.468	35.860
3	35.466	35.866
5	35.464	35.887
7	35.464	35.876
9	35.466	35.882
11	35.470	35.889
13	35.480	35.885
15	35.490	35.893
17	35.504	35.913
19	35.486	35.900
21	35.474	35.883
23	35.466	35.879

EXAMPLE 2.3.4 Evaporation from the Mediterranean Sea

In the arid climate of the Mediterranean Sea, the heat-loss of the water to the air is Q_e = 65 cal/cm^2/day greater than in the equivalent latitudes of the Atlantic.

(a) If a dam were built across the Straits of Gibraltar, by how much would the level between the Mediterranean and the Atlantic differ in a year? (Neglect river discharge.)

(b) If a 100% efficient hydroelectric power station was built at the dam, how many kilowatt-hours of electricity could it produce per year? Area of Mediterranean, $A = 2.97 \times 10^6$ km^2.

(c) How long would it be before the salinity in the Mediterranean increased to 38‰? Let the present average salinity, S, be 37‰ and average depth, H, be 1,500 m.

2.4 Mass Transport in the Ocean

EXAMPLE 2.4.1 Propagation of Surface Salinity Disturbances

The downward propagation of a positive or negative salinity anomaly spreading from the surface is approximated by

$$\frac{A_z}{\bar{\rho}} \cdot \frac{\partial S}{\partial z} = (E - P) \cdot S_o$$

Given $A_z = 1\ g\ cm^{-1} sec^{-1}$, $S_0 = 35.00$ ‰ (salinity at the surface), $\bar{\rho} = 1.02\ g/cm^3$, and assuming reasonable values for (E - P), calculate the salinity at a depth of 10 m at the end of one year in:

(a) an arid climate

(b) a humid climate.

EXAMPLE 2.4.2 Horizontal Salinity Distribution in an Estuarine Area

The horizontal salinity distribution in the German Bight (North Sea) is strongly influenced by the fresh water discharge of the Elbe and Weser rivers (see Fig. 2.4). A possible two-dimensional description is given by

$$S(x,y) = S_0 - \frac{M}{\varkappa\sqrt{A_x A_y}} \cos \varkappa y \; e^{-\varkappa x\sqrt{\frac{A_x}{A_y}}}$$

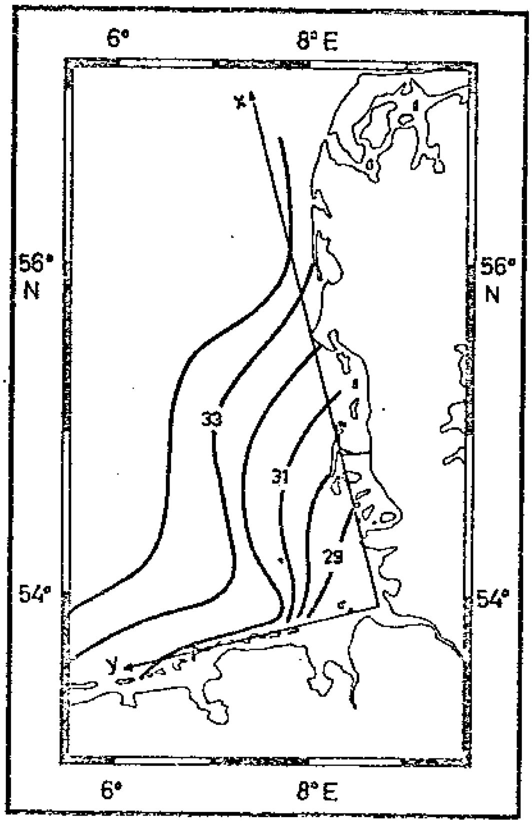


Fig. 2.4: Surface salinity distribution in the German Bight. From Schott (1966).

where the fresh water run-off at x = 0, $\Sigma(y)$, is

$\Sigma(y) = M \cos \varkappa y$ at $0 \leq y \leq y_0$,

$\Sigma(y) = 0$ at $y > y_0$

and $S_0 = 34$ ‰.

$M = 6.5 \times 10^{-3}\ g\ cm^{-2} sec^{-1}$ = amount of salt to increase the salinity of the water masses with salinity S_0

$\varkappa = \frac{\pi}{2y_0}$, $y_0 = 80$ km

$A_x = 2 \cdot 10^6\ g\ cm^{-1} sec^{-1}$

$A_y = 6 \cdot 10^6\ g\ cm^{-1} sec^{-1}$

(A_x and A_y are exchange coefficients in x and y direction (see Fig.).

Discuss and compute S(x,y) for a grid $\Delta y = 20$ km, $\Delta x = 50$ km and draw the isohalines from 29 -33 ‰ S.

2.5 Water Mass Identification and Formation

EXAMPLE 2.5.1 Distribution of Various Properties

The distribution of a scalar property s (e.g., temperature, salinity, oxygen, etc.) may be described by a functional relationship

$$s = f(x,y,z,t)$$

(a) Using a Taylor series expansion, derive

$$\frac{ds}{dt} = \frac{\partial s}{\partial t} + u\frac{\partial s}{\partial x} + v\frac{\partial s}{\partial y} + w\frac{\partial s}{\partial z}$$

and identify the various kinds of terms as well as the assumptions involved.

(b) Individual changes in s are mainly caused by mixing processes

$$\frac{\partial}{\partial x}\left(\frac{A_x}{\rho}\frac{\partial s}{\partial x}\right) + \frac{\partial}{\partial y}\left(\frac{A_y}{\rho}\frac{\partial s}{\partial y}\right) + \frac{\partial}{\partial z}\left(\frac{A_z}{\rho}\frac{\partial s}{\partial z}\right)$$

or by biochemical processes ΣB.

(i) Include the expressions for mixing and biochemical processes in the formula derived under (a).

(ii) Explain the expressions "conservative" and "nonconservative" properties of seawater.

(iii) Which processes can affect the field of the scalar property s in the following cases:

$$\frac{\partial s}{\partial t} = 0,\ \frac{ds}{dt} = 0,\ A_x = A_y = A_z = 0,\ \Sigma B = 0,\ u = v = w = 0,$$

$$\frac{\partial s}{\partial t} = \frac{\partial s}{\partial y} = \frac{\partial s}{\partial z} = 0$$

EXAMPLE 2.5.2 T-S Relationship

Local changes of a property s (e.g., temperature, salinity) with time due to mixing processes may be described by

$$\frac{\partial s(x,t)}{\partial t} = A_x \frac{\partial^2 s(x,t)}{\partial x^2}$$

Using the initial condition

$$s(x,t=0) = \begin{cases} s_1 & \text{for } -\infty < x < 0 \\ s_0 & \text{for } 0 < x < \infty \end{cases}$$

the solution of the differential equation is

$$s(x,t) = \frac{s_1 - s_o}{2} [1 - \Phi (\frac{x}{2\sqrt{A_x t}})] + s_o$$

where

$$\Phi(x) = \frac{2}{\sqrt{\pi}} \int_o^x e^{-t^2} dt \qquad \text{(error function, erfc)}$$

(a) Two water masses of equal densities are initially separated by a vertical wall. Their temperatures and salinities are T_o, S_o, and T_1, S_1, respectively. At time $t = 0$ the separating wall is removed (treat the problem 1-dimensional with the wall placed at $x = 0$, $-\infty < x < +\infty$). Find temperature and salinity of the mixed water mass at $x = \pm 0$, $\pm 10^2$, $\pm 2 \cdot 10^2$, $\pm 5 \cdot 10^2$, $\pm 10^3$ cm at the times $t = 0$, 10^2, $9 \cdot 10^2$, 10^4, and ∞ seconds. Present the results graphically in an x, T-diagram and an x, S-diagram with t as the parameter.

Let $T_o = 0^\circ C$ $\quad S_o = 28$ ‰ $\quad A_x = 10^2$ $cm^2 sec^{-1}$

$T_1 = 20^\circ C$ $\quad S_1 = 32$ ‰

(b) Plot TS-diagrams for each instant by reading T and S values from $x = 0$ to $x = \pm 10^3$ in 10^2 cm intervals from the graph obtained under (a).

(c) Considering the preceding exercises, what is the practical use of a TS-diagram?

(d) How many water masses can you distinguish in the TS-diagram for a station in the western subtropical South Atlantic given in Figure 2.5? Identify them and give approximate temperature and salinity core values.

EXAMPLE 2.5.3 Application of the TS-Relationship

The TS-relationship is used to trace Atlantic water masses spreading into the Norwegian Sea. The two original water masses are defined as Atlantic: $T = 10,2^\circ C$, $S = 35.45$ ‰ ; Norw. Sea: $T = 2.5^\circ C$, $S = 34.90$ ‰ . Plot these two TS-pairs in a TS-diagram and connect both points by a straight line. Divide this line into ten equal portions so that they mark fractions of 1/10 + 9/10, 2/10 + 8/10, etc., of each of the original water masses. If the TS-pairs from TABLE 2.4 are also plotted, for each depth one can read the fraction of Atlantic water observed at the corresponding station. The product of layer depth (difference in depth between two TS-observations) and average fraction of the inflowing water mass is called the "equivalent layer depth." Subsequent summation for all depth intervals yields the total equivalent layer depth for one station.

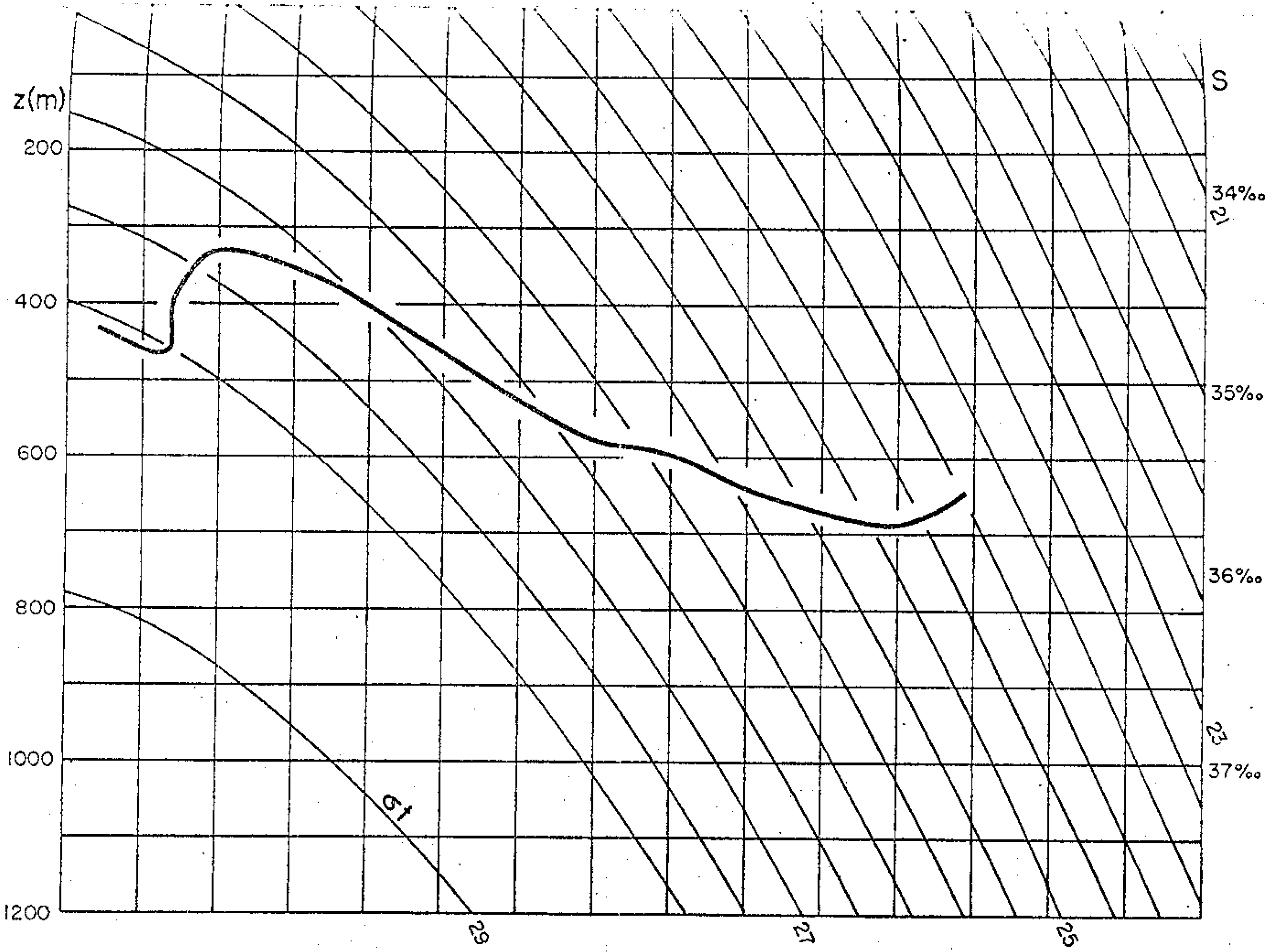


Fig. 2.5: TS-diagram for a station in the western subtropical South Atlantic.

(a) Sketch the distribution of the equivalent layer depths along the section given in TABLE 2.4.

(b) State the assumptions involved.

TABLE 2.4

Temperature and Salinity for Norwegian Sea Stations

	1		2		3		4	
φ	63° 02' N		67° 41' N		70° 10' N		72° 55' N	
λ	03° 40' E		08° 30' E		10° 10' E		15° 50' E	
Depth (m)	T(°C)	S(‰)	T	S	T	S	T	S
0	7.1	35.00	6.4	35.14	5.6	35.08	4.3	35.05
30	.7	.18	.98	.17	.63	.09	.29	.05
50	.66	.18	.68	.12	.48	.10	.27	.06
75	.60	.34	.63	.16	.73	.10	.25	.06
100	.71	.18	.15	.15	.73	.10	.22	.06
150	.65	.14	5.57	.13	.33	.10	3.93	.06
200	.38	.15	.30	.16	.65	.09	.60	.05
300	6.90	.14	4.92	.15	.61	.08	2.98	.04
400	5.62	.14	4.12	.08	.49	.12	1.73	34.99
500	2.95	.00	2.23	.00	.21	.09	-	-
600	0.31	34.92	0.47	34.94	4.06	.07	-	-

EXAMPLE 2.5.4 Radioactive Tracers

Oceanic water mass circulation can be investigated by using radioactive substances with an appropriate decay time.

(a) State the assumptions which have to be fulfilled by a radioactive tracer in order to obtain information on the path and velocity of water mass spreading.

(b) Figure 2.6 shows the meridional distribution of ^{14}C-concentrations in the western Atlantic Ocean. The numbers represent the average ^{14}C-concentrations given as 10%-difference from the age-corrected concentration of ^{14}C in 19th-century atmospheric CO_2. The numbers at z = 0 represent the ^{14}C-concentrations of the corresponding water masses in the area of their formation. Determine the age of the different water masses by using the assumption that within 80 years the concentration of ^{14}C decreases by 1% of the amount available.

(c) Does ^{14}C fulfill the assumptions stated under (a)? If the answer is yes, compute the velocities of spreading for the different water masses and compare the results to "classical" computations.

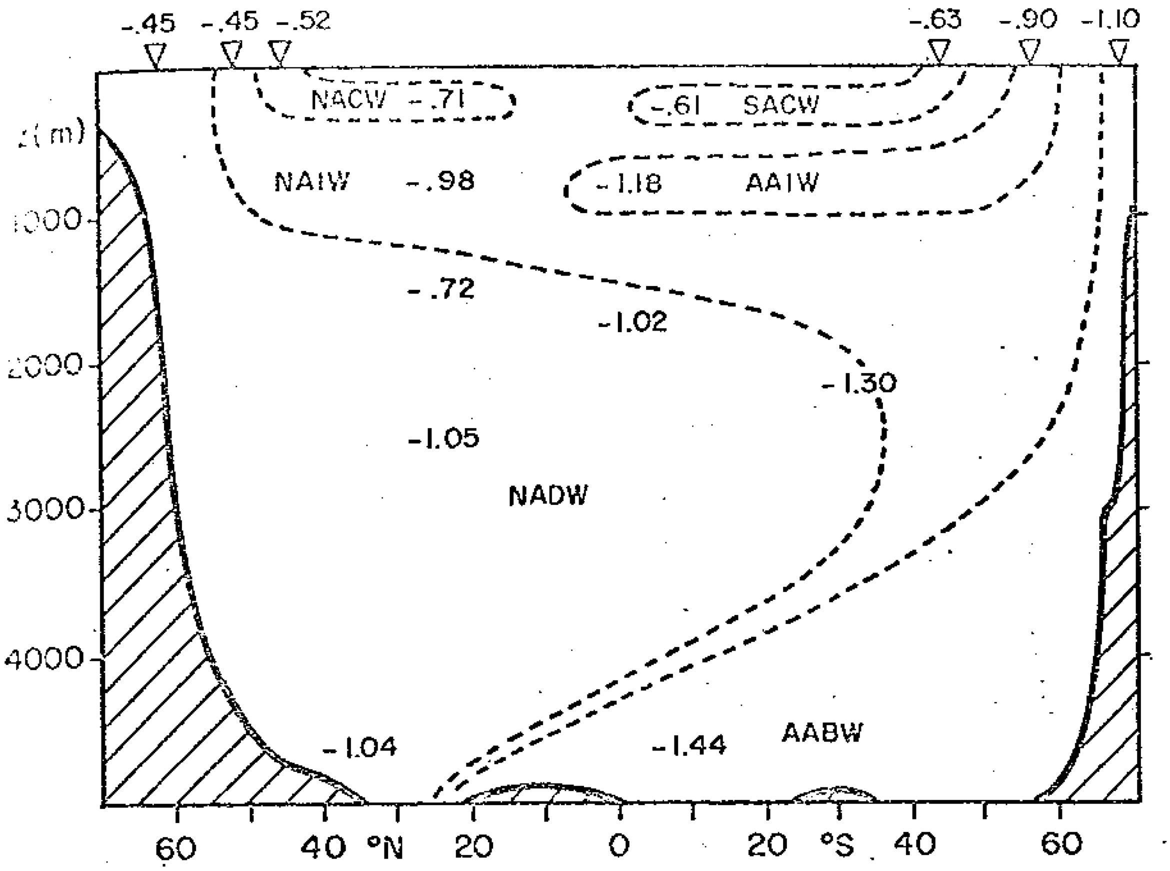


Fig. 2.6: Idealized distribution of water masses and average ^{14}C concentration in the western Atlantic Ocean. After Broecker (1963).

NACW:	N. Atlantic Central Water	SACW:	S. Atlantic Central Water
NAIW:	N. Atlantic Intermediate Water	AAIW:	Antarctic Intermediate Water
NADW:	N. Atlantic Deep Water	AABW:	Antarctic Bottom Water

EXAMPLE 2.5.5 Water Mass Formation in the Gulf of Naples

(a) From TABLES 2.5, 2.6, and 2.8, given below, draw isopleth diagrams of temperature, salinity, and density (months on the abscissa, depth on the ordinate; draw isopleths separated by one-tenth of $1^\circ C$, 1 ‰ S, and 1 σ_t-unit).

(b) When does spring warming begin? When does the thermocline begin to form? How do you explain the intermediate temperature minimum during the summer?

(c) Explain the coincidence of maximum salinity in December with maximum precipitation (TABLE 2.7) and the coincidence of minimum salinity in July with minimum precipitation. Hint: In the following budget relationship, consider only the first two terms:

$S = f(E - P, C, M)$

E = Evaporation   P = Precipitation   C = Horizontal Advection

M = Vertical Mixing

(d) State whether temperature or salinity has a more important effect on the density structure in the Gulf. To what depth does the vertical convection reach in winter? Show the region of homogeneity by cross-hatching in the density diagram.

(e) On what factors does the maximum depth of vertical convection generally depend?

(i) In the Gulf of Naples?

(ii) South of Greenland?

(iii) In the Central Baltic?

(f) Sketch an isopleth diagram of the temperature between 0-200 m for:

(i) A tropical sea whose temperature distribution remains constant during the year. A temperature difference of 10°C is found between 80 and 100 m. The surface temperature is 27°C.

(ii) A sea which is completely mixed by tidal turbulence, e.g., the Irish Sea, for which surface temperatures (°C) are given in TABLE 2.9.

(g) What factors are decisive for the formation of:

(i) a thermocline

(ii) a halocline

(iii) a pycnocline

In each case give a characteristic example.

TABLE 2.5

Average Monthly Temperatures (^{o}C) for the Gulf of Naples

	1957												1958	
z (m)	Jan	Feb	Mar	Apr	May	June	July	Aug	Sept	Oct	Nov	Dec	Jan	Feb
0	14.1	14.1	13.8	15.2	16.0	20.1	23.2	24.1	24.3	23.1	17.8	15.8	14.8	14.1
20	14.1	14.1	13.8	14.5	15.0	16.8	18.3	19.8	20.8	19.2	16.8	15.2	14.7	14.1
40	14.1	14.1	13.8	13.9	14.4	14.7	15.7	16.3	16.7	17.2	15.9	15.1	14.6	14.1
60	14.1	14.1	13.9	13.8	13.9	14.4	14.9	15.0	15.5	15.2	15.4	15.0	14.6	14.1
80	14.1	14.1	13.9	13.8	13.8	13.9	14.0	14.2	14.3	14.7	14.7	14.8	14.5	14.1
100	14.1	14.1	13.9	13.8	13.8	13.8	13.9	13.9	13.9	14.0	14.3	14.3	14.5	14.1
120	14.1	14.1	13.9	13.8	13.8	13.8	13.8	13.8	13.8	13.9	14.0	14.0	14.3	14.5
140	14.1	14.1	13.9	13.8	13.8	13.8	13.8	13.8	13.8	13.8	13.9	13.9	14.0	14.1
160	14.1	14.1	13.9	13.9	13.9	13.9	14.0	14.0	13.9	13.9	13.9	13.9	14.0	14.1
180	14.1	14.1	13.9	13.9	13.9	14.0	14.1	14.1	14.1	14.1	14.1	14.1	14.1	14.1
200	14.1	14.0	13.9	13.9	14.0	14.1	14.1	14.1	14.1	14.1	14.1	14.1	14.1	14.1

TABLE 2.6

Average Monthly Salinities (‰) for the Gulf of Naples

	1957												1958	
z(m)	Jan	Feb	Mar	Apr	May	June	July	Aug	Sept	Oct	Nov	Dec	Jan	Feb
0	37.7	37.6	37.6	37.7	37.7	37.3	37.1	37.4	37.8	37.9	37.9	38.0	37.9	37.8
20	37.7	37.6	37.6	37.9	37.8	37.7	37.5	37.6	37.8	37.9	37.9	38.0	37.9	37.8
40	37.7	37.6	37.6	37.9	37.8	37.8	37.7	37.7	37.8	37.9	38.0	38.0	37.9	37.8
60	37.7	37.6	37.7	38.0	37.9	37.8	37.8	37.8	37.8	37.9	38.0	38.0	38.0	37.8
80	37.7	37.7	37.8	38.0	37.9	37.9	37.9	37.8	37.9	38.0	38.1	38.0	38.0	37.9
100	37.7	37.7	37.9	38.0	37.9	37.9	37.9	37.9	38.0	38.2	38.2	38.1	38.0	37.9
120	37.7	37.7	37.9	38.0	38.0	38.0	38.1	38.1	38.2	38.3	38.2	38.1	38.0	38.0
140	37.7	37.8	38.0	38.0	38.0	38.1	38.1	38.1	38.3	38.4	38.3	38.2	38.1	38.1
160	37.7	37.8	38.0	38.1	38.2	38.2	38.2	38.2	38.4	38.4	38.4	38.3	38.2	38.2
180	37.8	37.9	38.0	38.1	38.2	38.4	38.4	38.4	38.5	38.5	38.5	38.4	38.3	38.3
200	37.8	37.9	38.1	38.2	38.4	38.4	38.4	38.4	38.5	38.5	38.5	38.5	38.4	38.4

TABLE 2.7

Average Monthly Precipitation for the Gulf of Naples

	1957											
	Jan	Feb	Mar	Apr	May	June	July	Aug	Sept	Oct	Nov.	Dec
Average rainfall in mm	94	71	74	74	51	38	18	23	74	135	114	119

TABLE 2.8

Average Monthly Densities (σ_t) for the Gulf of Naples

	1957												1958	
z (m)	Jan	Feb	Mar	Apr	May	June	July	Aug	Sept	Oct	Nov	Dec	Jan	Feb
0	28.3	28.2	28.3	28.0	27.8	26.5	25.5	25.4	25.7	26.1	27.6	28.1	28.3	28.3
20	28.3	28.2	28.3	28.3	28.1	27.7	27.1	26.8	26.7	27.2	27.8	28.3	28.3	28.3
40	28.3	28.2	28.3	28.5	28.3	28.2	27.9	27.8	27.8	27.7	28.1	28.3	28.3	28.3
60	28.3	28.2	28.3	28.6	28.5	28.3	28.2	28.1	28.0	28.2	28.2	28.3	28.4	28.3
80	28.3	28.3	28.4	28.6	28.5	28.5	28.4	28.3	28.4	28.4	28.4	28.3	28.4	28.4
100	28.3	28.3	28.5	28.6	28.5	28.5	28.5	28.5	28.5	28.7	28.6	28.5	28.4	28.4
120	28.3	28.3	28.5	28.6	28.6	28.6	28.6	28.6	28.7	28.8	28.7	28.6	28.5	28.4
140	28.3	28.3	28.5	28.6	28.6	28.6	28.6	28.6	28.8	28.9	28.8	28.7	28.6	28.6
160	28.3	28.3	28.5	28.6	28.7	28.7	28.7	28.7	28.9	28.9	28.9	28.8	28.7	28.7
180	28.3	28.4	28.5	28.6	28.7	28.8	28.8	28.8	28.9	28.9	28.9	28.8	28.7	28.7
200	28.3	28.4	28.6	28.7	28.8	28.8	28.8	28.8	28.9	28.9	28.9	28.9	28.8	28.8

TABLE 2.9

Mean Temperatures for the Irish Sea

Jan	Mar	May	July	Sept	Nov	Jan
8°	7°	8°	11°	12°	11°	8°

2.6 Thermohaline Circulation

EXAMPLE 2.6.1 Thermohaline Surface Layer Circulation (*3.5.4)

(a) Name the three driving forces of oceanic surface layer circulation caused by meridional temperature differences.

(b) Sketch qualitatively the distribution of the meridional components of the oceanic surface layer mass transport between 60°N and 60°S, which is caused by the forces mentioned under (a).

(c) Add to the schematic diagram obtained under (b) the meridional distribution of the surface height which is due to the prevailing (geostrophic) surface current systems. From comparison of the distributions obtained in (b) and (c), state qualitatively to what extent the thermohaline circulation contributes to the observed current system. Give a thermodynamic reason.

EXAMPLE 2.6.2 Circulation Patterns of Adjacent Seas

Sketch the pattern of vertical and horizontal circulation between the ocean and an adjacent sea connected over a sill in: (i) an arid climate; (ii) a humid climate; and give examples of such circulation.

EXAMPLE 2.6.3 Maintenance of the Oceanic Thermocline

The oceanic main thermocline is considered to be generally maintained by the balance between turbulent downward diffusion of heat and upward advection of cold water.

(a) Use a simple relation (see Example 2.5.1) to compute the vertical velocity component from temperature observations. Assume A_z to be independent of depth.

(b) Using the temperature distribution given in TABLE 2.10 and $A_z = 1\ cm^2 sec^{-1}$, compute the vertical velocity which is necessary to keep the vertical temperature distribution stationary.

(c) Discuss briefly the shortcomings of the above-mentioned concept with regard to the occurrence of the main thermocline in the ocean.

TABLE 2.10

Mean Temperature Distribution

z (m)	50	75	100	125	150	175	200	225	250
T (°C)	25.5	25.0	22.5	19.0	16.5	14.5	13.0	12.5	12.0

EXAMPLE 2.6.4 Thermohaline Circulation in the Norwegian and Greenland Sea

The currents in the Greenland-Norwegian Sea area can be described in the following way: (i) The main current systems tend to follow the margin of the Greenland-Norwegian basin with a cyclonic sense of rotation; (ii) around islands an anticyclonic circulation is observed.

(a) Attempt to explain these observations by using a thermohaline concept based on the fact that the climate of the Norwegian-Greenland Sea area is humid.

(b) Are other explanations of the Norwegian-Greenland Sea circulation possible?

3.1 Hydrostatics

EXAMPLE 3.1.1 Gravity

(a) The effective force of gravity at the earth's surface is not directed toward the center of gravity of the earth. Explain by means of a sketch. Give equations for the accelerations involved.

(b) Assume that the earth was, at one time, spherical. Sketch the forces at latitude $\varphi = 0^\circ$, 30°, 60°, and 90° which have led to the present shape of the earth.

EXAMPLE 3.1.2 Distribution of Gravity

The acceleration of gravity, g, varies with latitude and depth (in cm) according to the following approximations:

$$g_0 = 978.049\ [1 + 0.0052884 \sin^2 \varphi - 0.0000059 \sin^2 2\varphi]$$

$$g = g_0 + 2.303 \cdot z \cdot 10^{-6}$$

(a) If the z-axis is directed vertically downwards from $z = 0$ at the sea surface, calculate in dynamic meters the depth of the surface $z = 1{,}000$ m for latitudes $\varphi = 0^\circ$, 60°, and 90°. How much energy is necessary to raise a unit mass from 1,000 m to the surface?

(b) For latitudes 0° and 60° determine the relationship between dynamic and geometric depth at 1,000 and 10,000 m. Is the latitude effect or depth effect more important?

(c) A column of water at latitude 60°N has a mean temperature of 4°C and a mean salinity of 35.00 ‰. (For the computation of the mean density, use the arithmetic mean between surface and the given depth.) Calculate the pressure in dbar at 1,000 and 10,000 m.

EXAMPLE 3.1.3 Barotropic and Baroclinic Modes

Two parallel equiscalar surfaces with scalar fields L_1 and L_2 defined in the region R are described by $F(L_1, L_2) = 0$. If, e.g., the field L_1 represents pressure p or temperature T and $F(p, L_2) = 0$ or $F(T, L_2) = 0$, then the region R is called barotropic or thermotropic, respectively. L_2 represents any other scalar field (density, salinity, etc.). If equiscalar surfaces intersect, e.g., $F(p, L_2) \neq 0$ or $F(T, L_2) \neq 0$, then the region is called baroclinic or thermoclinic.

(a) Explain the use of the barotropic coefficient $\Gamma_\rho^p = \left[\frac{\Delta\rho}{\Delta p}\right]_{\text{geom.}}$

and the piezotropic coefficient $\gamma_\rho = \left[\frac{d\rho}{dp}\right]_{\text{physical}}$

(b) What media are described by $\Gamma_\rho^p = 0,\ \gamma_\rho = 0$?

(c) Discuss the possible vertical distributions of velocity in a barotropic and a baroclinic ocean.

EXAMPLE 3.1.4 Stability Oscillations (*4.3.2)

If a particle in a linearly stratified, incompressible, inviscid ocean is displaced vertically and then released, it will oscillate with simple harmonic motion.

(a) By using a Taylor series expansion for the density ρ, evaluate the equation of motion for a water particle oscillating as follows:

$$\zeta(t) = \zeta_0 \cos(Nt)$$

(b) If you change the sign of the density gradient which you used for (a), what kind of solution is obtained for the equation of motion and what would happen to the water particle if it is displaced vertically?

(c) For a compressible ocean $N = \sqrt{g\left(E - \frac{g}{c^2}\right)}$ with

$E \cong \frac{1}{\bar{\rho}} \frac{\partial \rho}{\partial z}$ (Stability, usually reported as $E \cdot 10^8$).

Compute the Brunt-Väisälä-frequency N from stability values given in TABLE 3.1. How important is the incompressibility term?

TABLE 3.1

Stability as a Function of Depth

Depth (m)	$E \cdot 10^8$ (cm^{-1})
0	
50	0.0
100	0.0
150	1460.0
200	895.0
400	238.0
600	97.4
800	97.4
1000	97.4
1200	68.2
1500	45.4
2000	31.2
2500	11.7

(d) Describe qualitatively to what extent a neutrally buoyant Swallow float follows vertical internal wave motion, if the compressibility of the float is larger, equal, or less than the compressibility of seawater (neglect inertial effects).

3.2 Kinematics

EXAMPLE 3.2.1 Dead Reckoning

(a) Show that surface currents can be computed from the following:

The noon position of a ship was determined by Loran as 52°25'N, 42°16'W. The course for the next 24 hours was 215° true and the log showed that 225 sea miles had been traversed by the time that the noon position for the following day was determined to be 49°44'N, 46°22'W.

(b) What uncertainties are involved in this type of calculation?

EXAMPLE 3.2.2 Relative Current Measurements

Current observations were made from a drifting ship at the equator, and the following results were obtained:

Depth of meter	= 100 m
Current recorded by meter	= 188 cm/sec, 101° True
Ship's drift	= 50 cm/sec, 315° True

(a) What was the actual current at 100 m?

(b) Give an example where this may have been observed.

EXAMPLE 3.2.3 Periodic and Nonperiodic Surface Currents (*4.5.3)

Observations of the speed and direction of surface currents were made routinely from a lightship in the North Sea. TABLE 3.2 gives one day's results averaged over two-hour periods. A moderately strong shouth wind was blowing at the time the observations were made.

(a) Using square millimeter graph paper and a suitable scale, draw a progressive vector diagram of the currents.

(b) Deduce from your diagram (i) whether the periodic current is tidal or inertial; (ii) the direction and strength of the mean flow; (iii) the probable cause of the mean flow.

(c) Assuming that the mean flow was constant during the period of observation, subtract it from the observed flow and plot the remaining current vectors from a common origin. Join the endpoints of the rose.

TABLE 3.2

Surface Currents Observed from a Lightship in the North Sea

Time (Hours)	Current Direction (Degrees)	Current Speed (Knots)
0 - 2	20	1.0
2 - 4	60	0.9
4 - 6	90	0.7
6 - 8	130	0.4
8 - 10	180	0.2
10 - 12	230	0.2
12 - 14	0	0.5
14 - 16	10	0.9
16 - 18	20	1.0
18 - 20	80	0.6
20 - 22	170	0.4
22 - 24	220	0.3

EXAMPLE 3.2.4 Equation of Continuity

Derive the continuity equation for a rectangular prism of fluid. How does the resulting equation simplify for an incompressible fluid?

EXAMPLE 3.2.5 Estimation of Vertical Velocity

The mean horizontal current components (cm/sec) in the upper 50 m of an ocean have been averaged over five-degree squares and are shown in Figure 3.1..

(a) Estimate the vertical velocity at 50 m in each of the five-degree squares.

(b) What assumptions have you made, and why may your answer be erroneous?

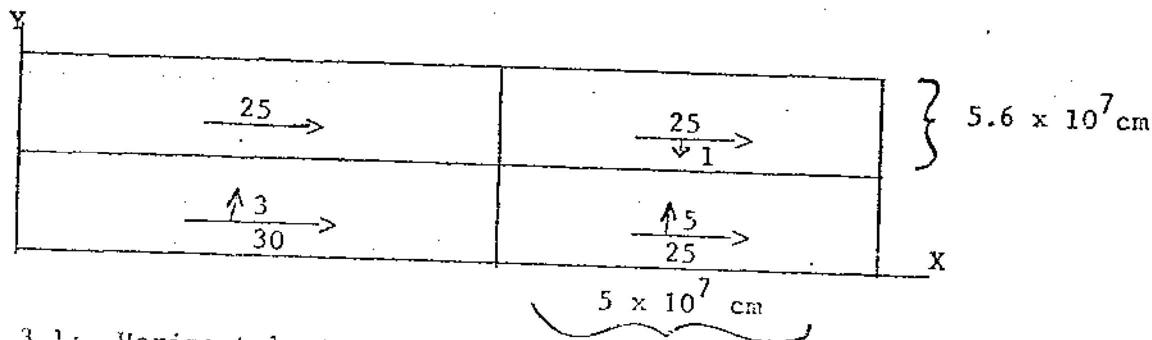


Fig. 3.1: Horizontal current components, averaged for four 5-degree squares.

EXAMPLE 3.2.6 Flow Through a Constriction

Water of constant density flows in a canal which has vertical sides and a flat bottom, but varies in width as shown in Figure 3.2. Using the continuity equation and Bernoulli's equation, calculate the depth and the velocity of water in the constriction. (You should obtain a cubic equation.)

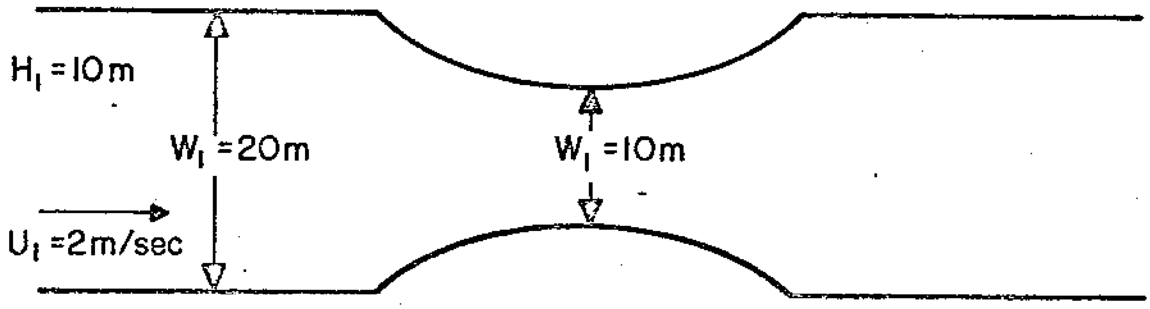


Fig. 3.2: Flow through a constriction (H = water depth).

EXAMPLE 3.2.7 Inflow into the Red Sea

Assume the Red Sea to be 2,000 km long, the Suez Canal closed, and the Straits of Bab el Mandeb to be one-tenth the average width of the sea. If surface evaporation exceeds precipitation by 1 cm/day and incoming flow is confined to the upper 50 meters, calculate the speed of the flow in cm/sec.

3.3 Dynamics

EXAMPLE 3.3.1 Acting Forces

(a) Separate the forces which generate and which influence the motion of a particle on the rotating earth into distinct categories and give examples of their effect on motion in the sea (excluding tides).

(b) Which of these forces can generate currents in a (i) homogeneous, frictionless ocean, (ii) homogeneous ocean with friction, (iii) inhomogeneous, frictionless ocean, (iv) inhomogeneous ocean with friction, (v) two-layered ocean with friction?

EXAMPLE 3.3.2 Surfaces of Equal Depth and Equal Potential (*3.1.2)

The dynamic depth of a level surface is given by

$$D = \frac{gz}{10} \quad \text{(dyn. m.)}$$

where $g_o = 9.78049\ (1 + 0.005288 \sin^2\varphi - 0.000006 \sin^2 2\varphi)[\text{m/sec}^2]$

$g = g_o + 2.303 \cdot 10^{-6} \cdot z$. What happens to a sphere which is located near the equator (a) on a surface of equal depth, and (b) on a surface of equal potential? Discuss both cases for a nonrotating and for a rotating earth. (Assume the surfaces to be plane, rigid, and frictionless.)

EXAMPLE 3.3.3 Coriolis Acceleration (*4.4.4)

(a) A plumb of mass m hangs on a 2 meter cord inside an airplane traveling eastward along the 60°N parallel of latitude at an altitude of 1,000 meters and a speed of 1,000 km/hr. At what angle to true vertical does the plumb line stand and in which direction? What would happen at 60°N if the airplane travels along the 0° meridian in a southernly direction with the same speed and altitude as before?

(b) What would happen, under momentarily the same conditions, to the plumb line in a free flying projectile having a speed of 3,000 km/hr?

(c) A particle of mass M travels due east at the equator with speed V. It is momentarily displaced to a position 1,000 meters north of the equator. Describe its subsequent motion (frictionless).

EXAMPLE 3.3.4 Deflection of a Moving Body

A billiard table is 250 cm long. Two billiard balls, 4 cm in diameter, are placed at opposite ends of the table. Neglecting friction, how fast does one ball have to be propelled initially directly toward the other in order to just barely miss the second ball due to the Coriolis force? The table is located at 43°N ($f = 10^{-4}\ \text{sec}^{-1}$). (Acc. to Hess (1959), Introduction to Theoretical Meteorology.)

EXAMPLE 3.3.5 Inertial Motions

The inertial period due to the earth's rotation is defined by

$$\tau = \frac{\pi}{|\vec{\Omega}| \sin\varphi} \quad \text{or} \quad \frac{12^h}{\sin\varphi}$$

(a) At what latitudes are the inertial periods in resonance with the semi-diurnal lunar tide, M_2, and the diurnal lunar tide, O_1?

(b) What is the shortest inertial period that an ocean current can have?

(c) Give graphically the distribution of inertial periods for latitudes between the equator and the pole.

3.4 Equation of Motion

EXAMPLE 3.4.1 Equation of Motion

The equation of motion is given by

$$\frac{\partial \vec{V}}{\partial t} + \vec{V} \cdot \nabla \vec{V} - 2\vec{\Omega} \times \vec{V} = -\frac{1}{\rho} \nabla p + g\vec{k} + \vec{K} + \vec{R}$$

(a) Give the component equations of motion.

(b) Simplify the equation of motion for (i) geostrophic, (ii) ageostrophic, (iii) cyclostrophic motions and sketch a vector diagram for each case.

(c) Explain, by means of a sketch, why the velocities in current fields with equal cyclonic and anticyclonic curvature are different, although the pressure gradients are identical.

EXAMPLE 3.4.2 Geostrophic Motion

Assuming that the Coriolis parameter can be expressed as a function of the y-coordinate, find the equation of the streamline of a parcel of water leaving the equator towards the northern hemisphere with a constant pressure gradient per unit mass (∇p) directed from south to north. There is no friction.

EXAMPLE 3.4.3 Vorticity

Vorticity, ζ, is defined $\zeta = (\text{curl}\ \vec{V})_z = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}$

The sign of ζ is determined as follows:

for rotation contra solem $\zeta > 0$

for rotation cum sole $\zeta < 0$.

By means of a sketch, show how it is possible that $\frac{\partial v}{\partial x} \gtreqless 0$ for $u = 0$.

EXAMPLE 3.4.4 Vorticity Equation, Potential Vorticity

(a) Derive the "vorticity equation"

$$\frac{d}{dt}(f + \zeta) + (f + \zeta)\, \nabla_h \cdot \vec{V} = 0$$

where ζ = relative vorticity, $(\zeta + f)$ = absolute vorticity from

$$\frac{du}{dt} - fv = -\frac{1}{\rho}\frac{\partial p}{\partial x} \qquad \frac{dv}{dt} + fu = -\frac{1}{\rho}\frac{\partial p}{\partial y} \quad .$$

(b) Using the continuity equation, derive from the vorticity equation

$$\frac{d}{dt}\left(\frac{f+\zeta}{D}\right) = 0 \quad \text{or} \quad \frac{f+\zeta}{D} = 0 \quad \text{resp. (potential vorticity)}$$

for a homogeneous layer of depth D.

(c) Consider the possible changes of a cyclonic eddy formed off Valparaiso and carried away by the Peru Current. What would happen to an anticyclonic eddy under the same circumstances?

(d) A frictionless turntable is mounted on a railway car which runs on a meridional, frictionless track from the north pole to the south pole: (i) give a qualitative description of the motion of the turntable; (ii) what would happen to a motionless car at 60°N if the turntable were rotated cum sole? (Acc. to v. Arx, 1962.)

3.5 Applications of the Equation of Motion

EXAMPLE 3.5.1 Geostrophic Current Calculation

(a) Derive the following two expressions for geostrophic current calculations:

$$c_g = \frac{g}{f}\left(\frac{z_A - z_B}{L}\right)_{p\,=\,\text{const.}}$$

$$c_g = \frac{10}{fL}\left[(\Delta D)_A - (\Delta D)_B\right]$$

where c_g = geostrophic current velocity component normal to a section between two hydrographic stations A and B.

L = distance between hydrographic stations A and B.

ΔD = dynamic depth anomaly of the reference level.

(b) Compute c_g relative to the 500 dbar-level, where no motion is assumed. Use L = 50 kilometers

$(\Delta D)_A$ = 0.837 dyn m $\qquad \varphi = 30°N$

$(\Delta D)_B$ = 0.941 dyn m

Compute the corresponding inclination of the sea surface between A and B in cm/km.

EXAMPLE 3.5.2 Geostrophic Current Distribution

From TABLE 3.4 calculate the geostrophic velocities at the given depths between successive pairs of stations. Draw a vertical section (profile) of velocities between Australia and Antarctica. Assume a level of no motion at 3000 meters.

TABLE 3.4

R. S. DISCOVERY Stations along 135°E Between Australia and Antarctica

z (m)	Dynamic height in centimeters at Station Numbers............ 895	894	893	892	891	890	889
0	000.0	000.0	000.0	000.0	000.0	000.0	000.0
250	034.7	033.4	033.1	028.5	024.1	018.8	017.3
500	065.3	063.5	061.3	051.0	041.7	031.1	029.0
750	095.0	093.6	085.4	069.9	055.8	041.7	039.0
1,000	120.7	121.2	106.1	085.8	067.9	051.3	048.3
1,500	158.3	164.0	139.2	111.8	088.7	068.8	065.0
2,000	185.7	194.2	164.6	134.0	107.6	085.7	081.2
2,500	208.5	219.2	186.3	153.7	125.1	101.0	096.6
3,000	229.7	242.4	207.4	172.9	141.6	115.0	110.5
Latitude	43°16'S	46°32'S	49°37'S	52°49'S	56°03'S	59°05'S	61°45'S

EXAMPLE 3.5.3 Estimation of Surface Currents

From a rapid BT-survey along 30°N latitude, you are asked to make an estimate of the surface layer current velocities. How would you proceed, using the following results and assumptions given in Figure 3.3? Calculate the mean velocities between A and B and B and C.

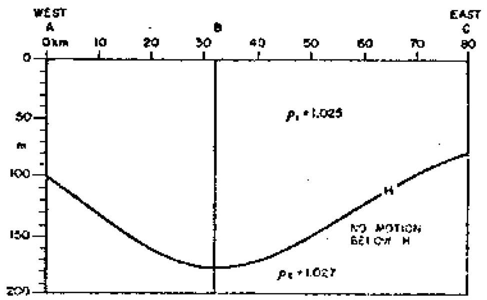


Fig. 3.3: Distribution of pycnocline depth along 30°N.

EXAMPLE 3.5.4 Ekman Drift Currents (*2.6.1)

According to Ekman, the velocity distribution of a pure drift current is given by

$$u = V_o \cdot e^{-\frac{\pi z}{D}} \cdot \cos(45^\circ - \frac{\pi z}{D})$$

$$v = V_o \cdot e^{-\frac{\pi z}{D}} \cdot \sin(45^\circ - \frac{\pi z}{D})$$

where $D = \sqrt{\frac{2A_z}{\rho f}}$ (Depth of frictional influence)

$$V_o = \frac{\tau}{\sqrt{2\rho f A_z}}$$

where τ = windstress in the y-direction

(a) If the coefficient of eddy viscosity has relative values of $A_z = 4$ and $A_z = 16$ respectively, how is the value of D affected?

(b) How are direction and speed affected at $z = D$?

(c) Find the expressions for the transport components in a wind-driven current between the surface and the depth of frictional influence by integration of the velocity components.

(d) Using the results from (c) on the direction of the mass transport in a wind-driven current, derive a schematic diagram of the meridional distribution of the height of the sea surface between 45°N and 45°S from the corresponding windstress distribution. Relate this diagram to the observed surface current systems.

EXAMPLE 3.5.5 Sverdrup-Relation

Equations (1) - (3) are the basis for the so-called Sverdrup-Relation.

$$-fM_y = -P_x + T_x|_{z=0} \qquad (1)$$

$$fM_x = -P_y + T_y|_{z=0} \qquad (2)$$

$$M_{xx} + M_{yy} = 0 \qquad (3)$$

Familiarize yourself with the given system of equations.

(a) Derive the Sverdrup-Relation.

(b) Which are the underlying assumptions made when compared to the complete equations of motion?

(c) Discuss the balance of terms in the Sverdrup-Relation and indicate where this relation may be valid in the real ocean.

(d) Split M_x and M_y into an Ekman-transport component and a geostrophic transport component and discuss interplay between both components.

EXAMPLE 3.5.6 Wind-driven Ocean Circulation

It has been shown that the wind-driven circulation in a homogeneously stratified rectangular ocean basin of constant depth H can be approximated by

$$\Psi(x,y) \sim X(x) \cdot \frac{\partial T_x}{\partial y}$$

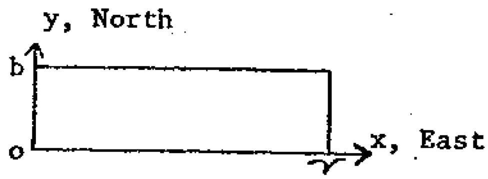
where Ψ = transport function
$X(x)$ = asymmetric distribution in East-West direction
T_x = zonal windstress component, x = eastward, y = northward

The theory shows that the number of gyres depends on the meridional distribution of the zonal component of the windstress. Assume a pertinent driving windstress to be of the form

$$T_x = -T_o \cdot \cos \frac{n\pi}{b} y$$

where b given in Fig. 3.4.

Fig. 3.4:



Sketch the distributions of the curl of the stress and the corresponding streamlines for the cases n = 0, 1, 2.

EXAMPLE 3.5.7 Western Boundary Currents

The function

$$X(x) = -\frac{2}{\sqrt{3}} e^{-\frac{kx}{2}} \cdot \cos\left(\frac{\sqrt{3}}{2} kx - \frac{\pi}{6}\right) + 1$$

represents the theoretical distribution of meridional transport through the western part of a model ocean where $k = \sqrt[3]{\frac{\beta}{A_x}}$ $\beta = \frac{\partial f}{\partial y}$

For $A_x = 5 \times 10^7$ cm^2 sec^{-1}, this distribution is given in Figure 3.5.

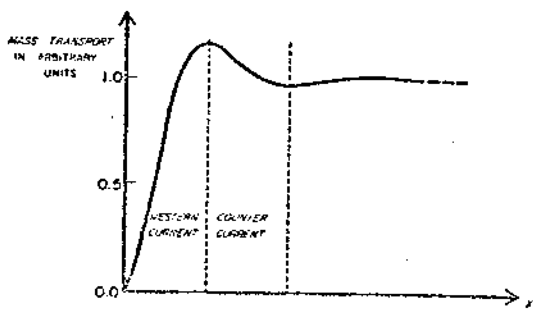


Fig. 3.5: Plot of function X(x) giving west-east variation in transport. After Stommel (1965).

where part I may be thought to represent the Gulf Stream and part II a countercurrent.

Discuss how amplitude and width would vary with values:
$A_x = 10^6\ cm^2/sec$; $A_x = 10^9\ cm^2/sec$

EXAMPLE 3.5.8 Topographic Influences on Zonal Currents

From the vorticity equation (see Example 3.4.4) one obtains the following expression

$$\frac{d\zeta}{dt} = - \beta v - (\zeta + f)\ \nabla_h \cdot \vec{V} \qquad (1)$$

with $\beta = \frac{\partial f}{\partial y}$

(a) What assumptions were made to derive equation (1)?

(b) Discuss the horizontal field of streamlines in a zonal flow, crossing a (N-S infinite) barrier as shown in Figure 3.6.

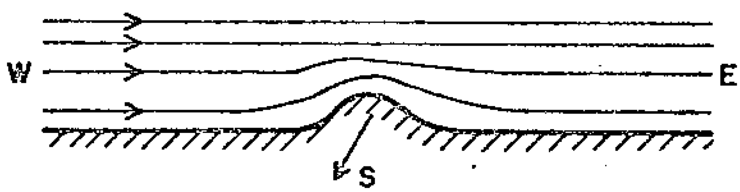


Fig. 3.6: Zonal flow over a barrier.

(c) Give examples of the occurrence of this type of current deflection (consider also atmospheric flows).

CHAPTER 4. WAVES AND TIDES

4.1 General Properties and Classification of Waves

EXAMPLE 4.1.1 Basic Properties of Harmonic Waves

Two progressive harmonic waves are described by

$$\xi_1(x,t) = a_1 \sin(\varkappa_1 x - \omega_1 t)$$

$$\xi_2(x,t) = a_2 \sin(\varkappa_2 x - \omega_2 t)$$

(a) Superimpose both waves and specify $\varkappa_1$, $\varkappa_2$, ω_1, ω_2 in order to obtain a progressive wave and a standing wave (take $a_1 = a_2$).

(b) Define the terms phase velocity and group velocity and derive their analytical form using the results from (a).

(c) Show that the total energy per unit area of a progressive wave is twice that of a standing wave.

EXAMPLE 4.1.2 Classification of Surface Waves

From the theory of waves at the air-sea interface, the following expression for the phase velocity c can be derived

$$c^2 = \left[\frac{g\lambda}{2\pi}\frac{\rho - \rho'}{\rho} + \frac{\alpha}{\rho + \rho'}\frac{2\pi}{\lambda}\right] \tanh \frac{2\pi}{\lambda} H$$

(ρ = density of water, ρ' = density of air, α = surface tension)

(a) Find the phase velocities for capillary waves, short waves, and long waves and state their dispersion behavior.

(b) Find analytical descriptions for the vertical distribution of the orbital paths of water particles for free harmonic short and long progressive waves if

$$u = -\varkappa a \cosh \varkappa(z_0 - H) \sin(\varkappa x_0 - \omega t)$$

$$w = \varkappa a \sinh \varkappa(z_0 - H) \cos(\varkappa x_0 - \omega t)$$

(x_0 and z_0 may be used instead of $x(t)$ and $z(t)$ due to the assumption of small amplitude waves)

(c) Find analytically the vertical and horizontal distribution of the orbital paths of water particles for short and long standing waves if

$$u = -\varkappa a \cosh \varkappa(z_0 - H) \sin \varkappa x_0 \cos \omega t$$

$$w = \varkappa a \sinh \varkappa(z_0 - H) \cos \varkappa x_0 \cos \omega t$$

(d) Illustrate the results obtained from (b) and (c) by simple graphs and give examples of the different type of waves.

EXAMPLE 4.1.3 Classification of Internal Waves

Internal wave motion is frequently described by a second-order partial differential equation for the vertical velocity component w. For sinusoidal internal waves of frequency ω in an incompressible, stably stratified ocean, without mean current the character of the differential equation is determined by the factor

$$q(z) = \frac{N^2(z) - \omega^2}{\omega^2 - f^2}$$

For $q(z) > 0$ (hyperbolic case), the solutions of the eigenvalue problem for w represent "ordinary" internal waves, i.e., the vertical amplitude distribution of w reaches its extremum in the interior of the interval $0 \le z \le H$. If $q(z) < 0$ in any depth intervals $0 \le z \le h_1$, $h_2 \le z \le H$, (elliptic case), the solution yields an exponentially decaying amplitude within these intervals and the resulting internal wave motions are frequently called "improper."

(a) Find the limiting periods for ordinary internal waves in the cases $\omega \lessgtr f$.

(b) Explain the following observations: Bj. Helland-Hansen observed in the latitude zone between 30°N and 74°N in the Atlantic Ocean semidiurnal internal waves at all depths, whereas he observed diurnal internal waves only in the deepest layers.

(c) Illustrate the results from (b) by a z vs. w plot. Use arbitrary amplitudes for w.

(d) Long and short internal waves are usually characterized by $N^2 \gg \omega^2$ and $f^2 \ll \omega^2$. What assumption makes this classification meaningful?

EXAMPLE 4.1.4 Wave Recording by Pressure Sensors

Assume a pressure gauge is mounted on the sea floor.

(a) List the phenomena which will be recorded. For each of the phenomena listed, name a location where extreme values will be observed. (No restrictions to length of record and instrumental accuracy.)

(b) Use the Bernoulli equation

$$\frac{\partial \bar{\phi}}{\partial t} + \frac{p}{\rho_0} - gz = 0 \qquad (\rho_0 = \text{constant})$$

and the velocity potential Φ for free progressive, harmonic surface waves

$$\Phi = a \cosh \kappa(z - H)\ e^{i(\kappa x - \omega t)}$$

to derive an expression for the pressure p at the pressure gauge. Is there any frequency dependence of the recorded amplitudes of long and short waves?

4.2 Surface Waves (Nontidal)

EXAMPLE 4.2.1 Short and Long Surface Waves

Consider a train of waves with period $\tau = 10$ sec. The speed with which a wave travels in deep water is given by

$$c_s = \sqrt{\frac{g\lambda}{2\pi}}$$

(a) Find the speed of the wave in deep water.

(b) Find the wavelength in deep water.

(c) Using the approximation that the speed of a wave in shallow water is given by $c_l = \sqrt{gH}$ show that the wavelength in shallow water is less than the wavelength in deep water for waves of the same frequency.

(d) Using

$$c = \sqrt{\frac{g}{\kappa} \tanh\ (\kappa H)}$$

compute c/c_s and c/c_l for ratios $H/\lambda = 0.05, 0.075, 0.10, 0.25, 0.50$. Plot c/c_s and c/c_l against H/λ on graph paper and find from the curves to what ratios H/λ, c_s, and c_l can be used instead of c, permitting a 5% error.

(e) Show why surf reaches a sloping beach parallel to the shore.

EXAMPLE 4.2.2 Energy Dissipation

(a) The empirical formula $E_D = 2\ A_z \pi^2 g \delta^2$ gives approximately the loss of the total energy for sea waves of the steepness $\delta = 2\ a/\lambda$ due to the turbulent viscosity.

(i) Give an example which supports the above representation.

(ii) Describe qualitatively the decay of a wind generated wave field after the wind suddenly stops blowing.

(b) A wave train approaches a gently sloping beach. Assuming wave period and forward flow of energy to be conserved, discuss qualitatively the process of nearshore energy dissipation of ocean waves. Make use of the phase-velocity relation for shallow water waves

$$c = \sqrt{g(\xi + H)}$$

which is valid for $\lambda \gg H$, but not necessarily $2a \ll H$.

EXAMPLE 4.2.3 Storm Surges

An observer notices the arrival of a dispersive train of waves from a storm at some distance. Ten hours after waves of 15-second period are observed, waves of 14-second period are observed. Assuming a point source in space and time (a) how long ago did the storm occur? (b) at what distance from the observer?

EXAMPLE 4.2.4 Seiches

Figure 4.1 shows time series of sea level variations for different locations in the Baltic Sea (location 1: northeasternmost gauge; location 8: southwesternmost gauge, see convenient map).

(a) From $c = \frac{\lambda}{\tau}$ (where τ = period of the basic eigenoscillation of an enclosed rectangular basin of length ℓ and mean depth H), find τ as a function of ℓ and H.

(b) Interpret Fig. 4.1 and use Merian's formula for an estimation of the mean depth of the Baltic basin (use ℓ = 1,450 km; estimate τ from Fig. 4.1).

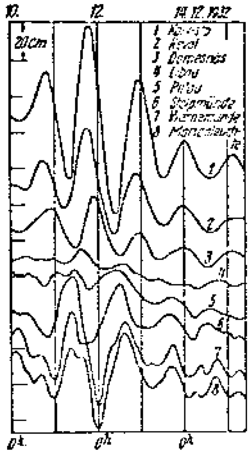


Fig. 4.1: Sea level fluctuations at eight locations in the Baltic Sea. From Dietrich (1963).

4.3 Internal Waves (Nontidal)

EXAMPLE 4.3.1 Internal Boundary Layer Waves

Assume two superposed liquids of density ρ_1 and ρ_2 ($\rho_2 > \rho_1$) which are otherwise unlimited. If the oscillations of the boundary are described by $$\xi = a \cos \kappa x \, e^{i\omega t}$$

the velocity potentials Φ_i of the upper and the lower layer are given by

$$\Phi_1 = a_1 \, e^{-\kappa z} \cos \kappa x \, e^{i\omega t}$$

$$\Phi_2 = a_2 \, e^{\kappa z} \cos \kappa x \, e^{i\omega t}$$

where the origin of z is at the mean level of the boundary and $-\kappa a_2 = \kappa a_1 = i\omega a$

(a) Using the definition of the velocity potential, $\vec{V} = \nabla\Phi$, sketch the distribution of streamlines for an instant $t = t_o$ in the x, z-plane. (Using $u/w = \Delta x/\Delta z$, draw the streamlines for several locations in the x, z-plane.)

(b) From (a) explain the phenomenon of "dead water" (disregard the infinite thickness of the upper layer).

EXAMPLE 4.3.2 Scaling Considerations (*3.1.4)

The simplest linearized equation describing internal waves is given by

$$\frac{d^2W}{dz^2} + \left\{\frac{N^2(z) - \omega^2}{\omega^2}\right\} \kappa^2 W = 0$$

with $w = W(z)\, e^{i(\kappa x - \omega t)}$

(a) Defining a local vertical length scale of the motion

$$L_z = \left[\frac{W}{W''}\right]^{\frac{1}{2}}$$

and a local horizontal length scale

$$L_H = \frac{1}{\kappa}$$

find an expression for L_H/L_z from the given differential equation.

(b) Discuss the spatial distribution of low frequency ($\omega \ll N(z)$) and high frequency ($\omega \approx N(z)$) disturbances.

EXAMPLE 4.3.3 Method of Observation

The vertical velocity component w is frequently used for the analysis of internal wave motions. However, the direct observation of w involves numerous complications. Therefore, w is indirectly determined using a conservative parameter $\Psi(x,y,z,t)$, e.g., temperature, salinity, density, etc.

(a) Derive

$$w = - \frac{\frac{\partial \Psi^{(1)}(x,y,z,t)}{\partial t}}{\frac{\partial \Psi^{(0)}(x,y,z)}{\partial z}} \quad \text{from} \quad \frac{d\Psi(x,y,z,t)}{dt} = 0$$

and discuss the assumptions made. Use

$$\underset{\text{observation}}{\Psi(x,y,z,t)} = \underset{\text{mean}}{\Psi^{(0)}(x,y,z)} + \underset{\text{perturbation}}{\Psi^{(1)}(x,y,z,t)}$$

where $|\Psi^{(1)}| \ll |\Psi^{(0)}|$

(b) Try to explain why the choice of $\Psi^{(0)}$ and, consequently, $\Psi^{(1)}$ depends on the spatial scale of the problem to be investigated.

(c) Which phenomena are likely to cause differences in computing w from the depth variations of an isoline Ψ = const compared to measuring the variations of Ψ at a fixed level z = const?

EXAMPLE 4.3.4 Energy of Internal Waves

The total energy per unit area of internal gravity waves of order n is given by

$$E_n = \frac{1}{8} \bar{\rho}\, g\Gamma_o\, a_n^2\, H$$

For surface waves it was found that

$$E_o = \frac{1}{4} \bar{\rho} g\, a_o^2$$

(a) Assuming the maximum internal wave amplitudes to be 1/10 of the depth interval available ($a_n = H/10n$), find the ratio of the maximum energies of internal waves of the order 1 to 5.

(b) Find the ratio E_o/E_n for $n = 1,\ldots,5$, assuming $a_o = 100$ cm, $a_n = H/20n$, $H = 10^5$cm, $\Gamma_o = \frac{\bar{\rho}_H - \bar{\rho}_O}{\bar{\rho}H}$, $\bar{\rho}_H - \bar{\rho}_O = 4{:}10^{-3}$ g cm^{-3}.

4.4 Tide Generating Forces

EXAMPLE 4.4.1 The Earth-Moon System

(a) Assume the masses of earth and moon to be concentrated at their center points. Compute the approximate location of the center of mass of the earth-moon system, earth's mass = $6 \cdot 10^{27}$g, moon's mass = $7.3 \cdot 10^{25}$g, distance = $3.84 \cdot 10^{10}$cm.

(b) Explain graphically the notation "revolution without rotation" for the movement of the earth-moon system around their common center of mass. (Use a meridional cross-section of the earth's sphere and show its location and orientation for several instants. Draw the tracks of two points fixed on the cross-section.)

EXAMPLE 4.4.2 Tidal Potential

The tidal potential is represented by

$$V = \gamma M \left(\frac{1}{r} - \frac{1}{R} - \frac{r_o \cos\theta}{R^2}\right)$$

where γ = gravity constant
M = mass of celestial body
r = distance gravity center celestial body - observation location earth
R = distance gravity center celestial body - gravity center earth
r_o = distance gravity center earth - observation location earth
θ = zenith angle of the celestial body at the observation location

(a) Show by an expansion of r into powers of $(\frac{r_o}{R})$, that the third-order term $V^{(3)}$, which contains the third powers of $(\frac{r_o}{R})$, is given by

$$V^{(3)} = K \cdot (3\cos\theta + 5\cos 3\theta) \qquad (1)$$

where

$$K = \frac{\gamma M}{8} \cdot \frac{r_o^3}{R^4}$$

(b) Assuming a spherical earth with radius r_1 and a mean distance $R_{earth-moon} = c$, derive $K = \frac{G}{6} \cdot \sin\Psi$ using the tidal constant

$$G(r_1) = \frac{3}{4}\gamma M \frac{r_1^3}{c^3} = 26.2 \cdot 10^3 \text{ cm}^2\text{sec}^{-2} \qquad (2)$$

and the parallax Ψ of the moon. (Parallax in this case is used from the definition of the "equatorial horizontal lunar parallax.")

(c) Using (1) and (2) and $\Psi = 57'$, compute the difference in height between zenith and nadir high tides of the lunar equilibrium tide.

EXAMPLE 4.4.3 Horizontal Component of Lunar Tidal Forces

(a) By means of a diagram of sufficient accuracy, sketch the distribution of the horizontal component of the lunar tide generating forces at the earth's surface. Assume the moon to be in the zenith at 25°N latitude.

(b) Using the diagram obtained in (a), plot (qualitatively) for latitudes 0°, 25°N, 60°N, and 90°N central vector diagrams of the horizontal components at 0, 3, 6, 9, and 12 hours lunar time.

EXAMPLE 4.4.4 Magnitude of Tidal Forces (*3.3.3)

(a) Compare the magnitude of the horizontal component of the tide generating forces, F_H,

(i) to the wind stress, which acts on the sea surface and is empirically described by

$$\vec{T} = 3.2 \times 10^{-6}\ W^2 \text{ dyn cm}^{-2}$$

(W - windspeed at a height of 10 m, given in cm sec^{-1})

(ii) to the Coriolis force, which acts on a moving particle and is described by

$$C = 2|\vec{\Omega}| \sin\varphi \cdot V_o \cdot m \quad \text{g cm sec}^{-2}$$

where m = mass

V_o = current speed, for this example take the speed of the wind-driven current, which is given by

$$V_o = \frac{\lambda \cdot W}{\sqrt{\sin\varphi}}, \quad \lambda = 1.26 \cdot 10^{-2}$$

(iii) to the force due to a horizontal pressure gradient in the ocean, which causes a geostrophic current of the speed V_o. Use W = 10 m/sec, $\varphi = 30^\circ$N and report all forces in dynes.

(b) Why is the vertical component of the tide generating forces negligible for oceanic tides? Is this also true for the tides of the atmosphere and the solid earth?

4.5 Analysis of Tidal Observations

EXAMPLE 4.5.1 Characteristic Features of Tidal Records

Figure 4.2 shows parts of tide gauge records from four different locations.

(a) State the type of the tides and name one characteristic location for each of the examples given.

(b) Mark the inequalities you can recognize in Figure 4.2 and give their relation to moon phases as well as their delay.

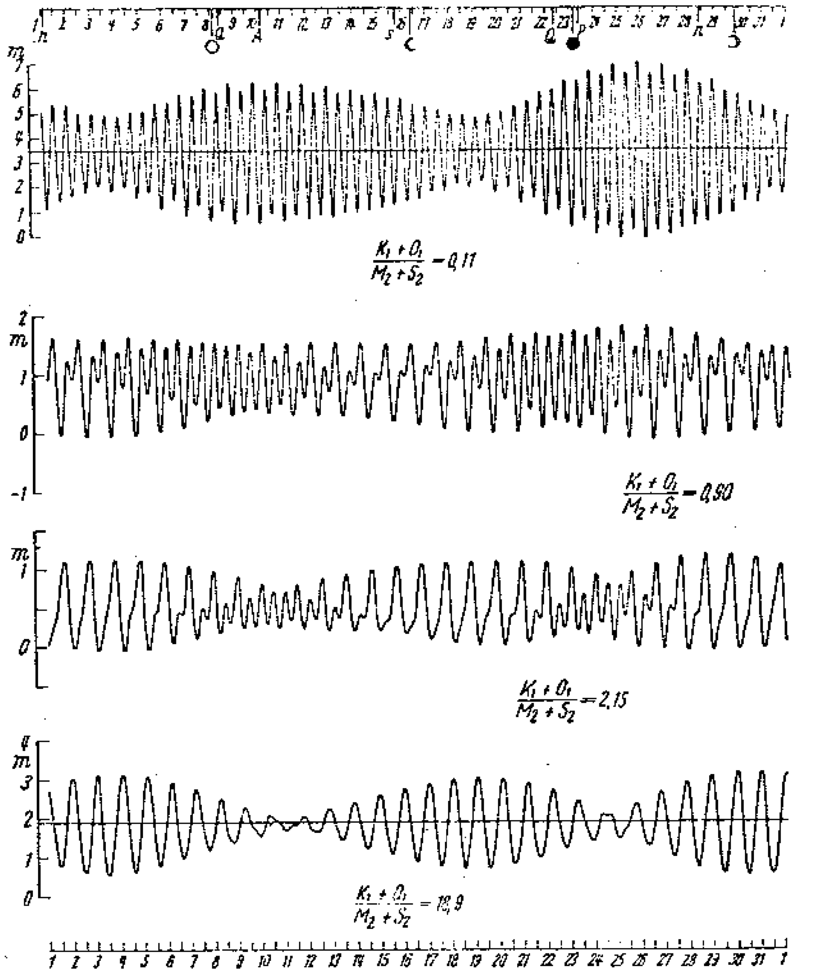


Fig. 4.2: Tide gauge from four different geographic locations. From Dietrich and Kalle (1963).

Abbreviations:
- A = Apogee point of moon's orbit
- P = Perigee point of moon's orbit
- N = Maximum northern declination of the moon
- S = Maximum southern declination of the moon
- Q = Time when moon crosses the equator

EXAMPLE 4.5.2 Harmonic Analysis of Tidal Currents

TABLE 4.1 gives the results of hourly current observations from a North Sea lightvessel.

(a) Plot the observed values as a sequence of hourly current vectors (progressive vector diagram), state the main period observed and find the residual current components graphically. Assume the residual currents to be constant during both tidal cycles.

(b) Plot the observed values in separate u/t and v/t-diagrams.

(c) Compute amplitude and phase of both u- and v-components for 24.8 and 12.4 hour periods by harmonic analysis. Divide the interval of

24.8 hours = 2π into $2n = 24$ equal parts of $\frac{2\pi}{2n}$ each. Read the velocity values u_i and v_i at $\Psi_i = \Psi_{i-1} + \frac{2\pi}{2n}$ for $i = 1, \ldots 2n$. Compute separately for u and v

$$\begin{Bmatrix} au_\nu \\ av_\nu \end{Bmatrix} = \frac{1}{n} \sum_{i=1}^{2n} \begin{Bmatrix} u_i \\ v_i \end{Bmatrix} \cos \nu \Psi_i \quad , \nu = 0,\ 1,\ 2$$

$$\begin{Bmatrix} bu_\nu \\ bv_\nu \end{Bmatrix} = \frac{1}{n} \sum_{i=1}^{2n} \begin{Bmatrix} u_i \\ v_i \end{Bmatrix} \sin \nu \Psi_i$$

and from these expressions amplitude $A_\nu = a_\nu^2 + b_\nu^2$, and

phase $\varphi_\nu = \tan^{-1} \frac{b_\nu}{a_\nu}$. (The analyzed period is $\tau_\nu = \frac{1}{\nu} \cdot 24.8$ [h].)

(d) Compare the results of the harmonic analysis with the results obtained in (a) and (b).

TABLE 4.1

Hourly Current Observations from a lightvessel in the North Sea

Local time (h)	N-comp ($cm\ sec^{-1}$)	E-comp ($cm\ sec^{-1}$)	Local time (h)	N-comp ($cm\ sec^{-1}$)	E-comp ($cm\ sec^{-1}$)
0	- 5.3	4.1	13	7.3	10.0
1	15.5	16.0	14	18.6	16.9
2	25.5	22.9	15	22.5	18.9
3	24.9	22.8	16	16.2	11.7
4	19.2	17.2	17	9.8	3.8
5	7.8	11.9	18	- 0.4	6.3
6	- 9.1	5.5	19	-17.9	- 0.6
7	-26.2	- 6.2	20	-15.9	- 5.5
8	-39.6	-17.8	21	-37.3	-10.1
9	-35.1	-21.3	22	-30.6	-11.0
10	-30.6	- 4.9	23	-23.8	- 4.7
11	-21.6	- 3.1	24	- 8.7	- 3.5
12	- 6.4	+ 3.1	25	+ 3.2	+ 2.8

EXAMPLE 4.5.3 Current Ellipses (*3.2.3)

TABLE 4.1 gives the results of hourly current observations from a North Sea lightvessel.

(a) State the mean period observed and find the residual current components numerically. Assume the residual currents to be constant during each of the observed dominant cycles.

(b) Eliminate the residual currents from the observations given in TABLE 4.1, and plot the resulting hourly vectors originating all from the same point (central vector diagram).

(c) Assume that the results of a Fourier analysis of current observations revealed

$$u = u_o \cos(\omega t + \varphi_1)$$

$$v = v_o \cos(\omega t + \varphi_2), \quad \omega = \frac{2\pi}{\tau}, \quad \tau = \text{period}$$

Determine analytically the length and orientation of the major semi-axis and the minor semi-axis of the current ellipse (find the normalized equation of the ellipse by rotating the coordinate system).

(d) Using the results of (c) and the harmonic constants derived for the semidiurnal tidal component in Example 4.5.2, draw the computed tidal ellipse and compare it with the graph obtained in (b).

EXAMPLE 4.5.4 Harmonic Synthesis

For the port of Nereus on the African coast, the harmonic constants of the principal tidal components are given in TABLE 4.2. The tidal height ξ may be computed from

$$\xi = M_2 \cos\left(\frac{2\pi}{\tau_{M_2}} t + V_{o_{M_2}} + K_{M_2}\right)$$

$$+ S_2 \cos\left(\frac{2\pi}{\tau_{S_2}} t + V_{o_{S_2}} + K_{S_2}\right)$$

$$+ K_1 \cos\left(\frac{2\pi}{\tau_{K_1}} t + V_{o_{K_1}} + K_{K_1}\right)$$

(a) What was the height of the tide at 14.30 hours January 3, 1969?

(b) Was the tide rising or falling at 14.30 January 3, 1969?

(c) What was the time of high tide and low tide for January 3, 1969?

(d) Plot hourly tidal components for 0 to 24 hours, January 3, 1969, add them graphically and use the result for checking (a) - (c). Are the tides mainly diurnal, semidiurnal, or mixed?

TABLE 4.2

Harmonic Constants for Port of Nereus

Tidal Constituent	Phase K	fn	Amplitude cm	Period τ hrs	V_o at 0000 hrs Jan. 1, 1969
M_2	+ 60°	cos	72	12.42	273°
S_2	+ 72°	cos	50	12.00	20°
K_1	+250°	cos	48	23.93	92°

EXAMPLE 4.5.5 Barotropic and Baroclinic Tides

The results of a harmonic analysis for the M_2-period of current observations at 30°N, 28°W are given in TABLE 4.3.

(a) Do the observations allow for an interpretation in terms of a barotropic tide only? Give the reasons.

(b) The observations given in TABLE 4.3 were interpreted in terms of barotropic and baroclinic tidal waves (0-order internal mode equivalent to barotropic mode). According to the concept of internal wave motion, to be eigen oscillations of a vertically stratified ocean, the vertical distribution of the v-amplitude due to harmonic internal waves at the observation site can approximately be given by

$$v(z) = v_o + \sum_{n=1}^{m} v_n \cos \frac{n\pi(H-z)}{H}$$

the indices o and n indicating the order of the barotropic and baroclinic modes; H = 294 m.

Using the results of an approximation of the observed currents by zero and first-order M_2-internal modes given in TABLE 4.3 for the depth z = 19 m, plot the vertical distribution of the computed zero and first-order v-amplitudes and compare their sum to the observed v-amplitudes. Complete TABLE 4.3.

TABLE 4.3

Harmonic Analysis for the M_2-period

	M_2- Tidal Current Observations				Approximations for v-component					
	Ampl. (cm sec^{-1})	Phase (degr.)			0.-order mode		1.-order mode		0.+1.-order modes	
z (m)	u	φ_u	v	φ_v	v_o	φ_{v_o}	v_1	φ_{v_1}	v	φ_v
19	20.4	316	19.8	257	16.0	267	6.1	220	?	?
151	17.1	339	17.2	261	?	?	?	?	?	?
283	10.4	358	12.6	292	?	?	?	?	?	?

4.6 Tides in the Ocean

EXAMPLE 4.6.1 Equilibrium Tide

(a) State the main objection to the equilibrium tide theory.

(b) What change of one of the earth's parameters would make the objection meaningless? Take the world ocean's depth to be 4...

EXAMPLE 4.6.2 Tidal Waves in the Ocean Basins

(a) Why does the head of a tidal current vector in the ocean generally follow an ellipse? Is this also true for the currents observed during the passage of a tsunami wave?

(b) State the assumptions which lead to the generation of an amphidromic point in a quadratic basin. Several amphidromic points are known to exist in the real ocean basins, although the assumptions for their generation seem to be rather restrictive. Give an explanation.

(c) How would you determine the location of an amphidromic point if you have the facilities to measure (i) currents or (ii) surface elevation at three different locations? Propose convenient arrays and give the reasons.

EXAMPLE 4.6.3 Tides in Adjacent Seas

For the tides of adjacent seas or bays, three different types of oscillations must be considered: (i) eigen-oscillations, (ii) co-oscillating tides, (iii) independent astronomic tides.

(a) What causes the three different types of oscillations, and what factors are decisive for their periods and amplitudes?

(b) What are the differences between (ii) and (iii) at the entrance of the adjacent sea or bay?

(c) Give one example each of where (i) is of large influence and where (ii) is of small influence on the tides of the corresponding areas.

EXAMPLE 4.6.4 Topographic Influences on Tides

(a) Idealize the English Channel to be a channel of a width of 100 km and of a uniform depth of 80 m. Assume a semidiurnal tidal wave is propagating through the channel in northernly direction. What is the difference in tidal amplitude between the "English" and the "French" coast?

(b) What important factors determine additionally the actual distribution of tidal amplitudes in the English Channel area?

(c) Although the horizontal components of the tide generating forces are of the order of 10^{-7} x g, the tidal current velocities over shelf areas are of the order of 1 kn. Give an explanation.

PART B - ANSWERS AND REFERENCES

CHAPTER 1

EXAMPLE 1.1.1

Fofonoff, N. P. (1962): The Sea, Vol. 1, "Physical Properties of Sea Water," Interscience Publ., pp. 4-6.

EXAMPLE 1.1.2

(a) Fofonoff, N. P. (1962): Loc. cit. Example 1.1.1, pp. 8-10.

(b) $T = 4.70°C$, $S = 34.74$ ‰ ; $\sigma_t = 27.52$. Density is not a linear function of salinity and temperature.

(c) 76 cm

EXAMPLE 1.1.3

(a) Fofonoff, N. P. (1962): Loc. cit. Example 1.1.1, pp. 12-17.

(b)

Depth (m)	T°C	θ°C
2,000	2.25	2.10
3,000	1.64	1.41
4,000	1.60	1.27
5,000	1.72	1.26
6,000	1.86	1.25
7,000	2.01	1.25
8,000	2.15	1.23
9,000	2.31	1.19
10,000	2.48	1.17

Apparent instability if T is used.

(c) Flow into Cayman Trench over an approximate sill depth of 1,600m.

EXAMPLE 1.1.4

(a) Fofonoff, N. P. (1962): Loc. cit. Example 1.1.1, pp. 17-22.

(b) $T = -1.3°C$, $S = 24.7$ ‰

Dietrich, G. and K. Kalle (1963): General Oceanography, Interscience Publ., pp. 55-57.

EXAMPLE 1.2.1

(a) $\varkappa = (\mu + p\frac{d\mu}{dp})/(1 - \mu p)$; $4540 \cdot 10^{-9}$ dbar^{-1}; $4725 \cdot 10^{-9}$ dbar^{-1}.

(b) Dietrich, G. and K. Kalle (1963): Loc. cit. Example 1.1.4, pp. 38-73.

(c) 960 cm^3

(d) 1483 m/sec; 1454 m/sec.

EXAMPLE 1.2.2

(a), (b) Neumann, G. and W. J. Pierson, Jr. (1966): Principles of Physical Oceanography, Prentice-Hall, pp. 48-51.

EXAMPLE 1.2.3

(a) Pressure, $g_1 = 0.018\ sec^{-1}$

(b) Neumann, G. and W. J. Pierson, Jr. (1966): Loc. cit. Example 1.2.2, pp. 48-51.

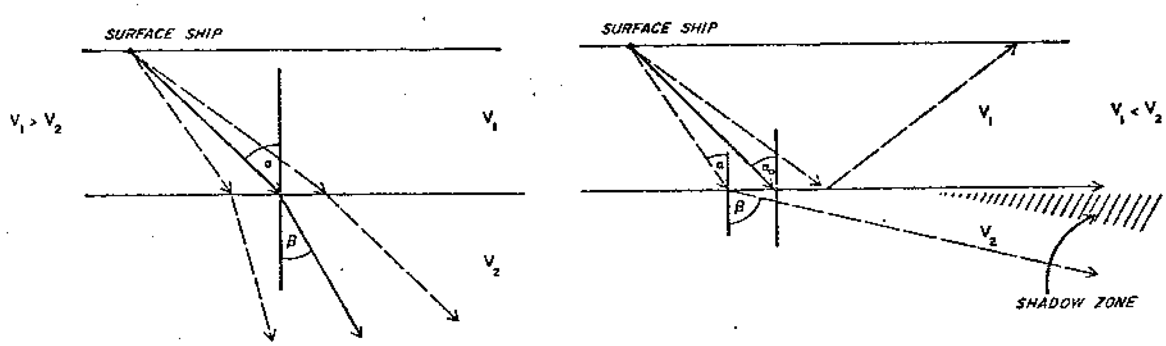
(c) 86.7 km

EXAMPLE 1.2.4

(a), (b)

Depth (m)	Time in seconds using c $t = \frac{2H}{c}$	Time in seconds using 1500m/sec	Absolute Error (m) $E = 1500\frac{\Delta t}{2}$	Relative Errors in ‰ $R = \frac{E}{H} \cdot 100$
125	0.164	0.167	+2.25	+1.8
175	0.230	0.233	+2.25	+1.3
350	0.462	0.466	+3.0	+0.9
750	0.997	1.0	+2.25	+0.3
950	1.265	1.269	+3.0	+0.3

EXAMPLE 1.2.5



(i) S = CONSTANT ; t_1 = 25°C , t_2 = 10°C (NO TOTAL REFLECTION POSSIBLE)

(ii) t = CONSTANT ; S_1 = 20 ‰ ; S_2 = 35 ‰ (TOTAL REFLECTION POSSIBLE SINCE $\beta \geq 90°$)

Fig. 1.1: Answer to Example 1.2.5.

EXAMPLE 1.3.1

(a)

Attenuation =	Absorption	+ Scattering
pure water	molecular	molecular
seawater	molecular } solution dissolved } suspended matter	molecular } solution dissolved } suspended matter

Equation describes only absorption for a specified wavelength over a small depth range.

(b) Sverdrup, H. U., M. W. Johnson, and R. H. Fleming (1942): The Oceans, Prentice-Hall, pp. 82-88.

EXAMPLE 1.3.2

(a), (b) Neumann, G. and W. J. Pierson, Jr. (1966): Loc. cit. Example 1.2.2, pp. 67-69.

(c) Predominantly foreward scattering (downward) occurs.

(d) Oceanic water would match color of coastal water shifting towards green-yellow.

EXAMPLE 1.3.3

(a) Planck's radiation law; assumption of black body radiation.

(b) 10 mμ, airborne sensing possible; space borne sensing may be influenced by O_3-absorption.

(c) von Arx, W. S. (1962): An Introduction to Physical Oceanography, Addison-Wesley Publ., pp. 144-146.

(d) Order of magnitude of effected layer is 1 mm.

EXAMPLE 1.3.4

(a) Dietrich, G. and K. Kalle (1963): Loc. cit. Example 1.1.4, pp. 38-42.

(b) Gaul, R. D., Editor, (1963): Marine Science Instrumentation Vol. 2, Instr. Soc. America, pp. 10, 19-24.

EXAMPLE 1.3.5

(a), (b) von Arx, W. S. (1962): Loc. cit. Example 1.3.3, pp. 260-279.

EXAMPLE 1.4.1

(a) Proudman, J. (1953): Dynamical Oceanography, Methuen, pp. 101-102.

(b) See Fig. 1.2.

EXAMPLE 1.4.2

(a), (b) Hinze, J. O. (1959): Turbulence, McGraw-Hill, pp. 13-22.
Defant, A. (1961): Physical Oceanography I, Pergamon Press, pp. 100-109.

EXAMPLE 1.4.3

(a), (b), (c) Proudman, J. (1953): Loc. cit. Example 1.4.1, pp. 111-119.

EXAMPLE 1.4.4

(a), (b) Defant, A. (1961): Loc. cit. Example 1.4.2, pp. 393-398.

EXAMPLE 1.5.1

(a), (b) Dietrich, G. and K. Kalle (1963): Loc. cit. Example 1.1.4, pp. 38-57.

(c) North Sea no; Baltic Sea yes.

EXAMPLE 1.5.2

(a), (b) Neuman, G. and W. J. Pierson, Jr. (1966): Loc. cit. Example 1.2.2, pp. 82-84.

EXAMPLE 1.5.3

(a), (b), (c) Neuman, G. and W. J. Pierson, Jr. (1966): Loc. cit. Example 1.2.2, pp. 85-87.

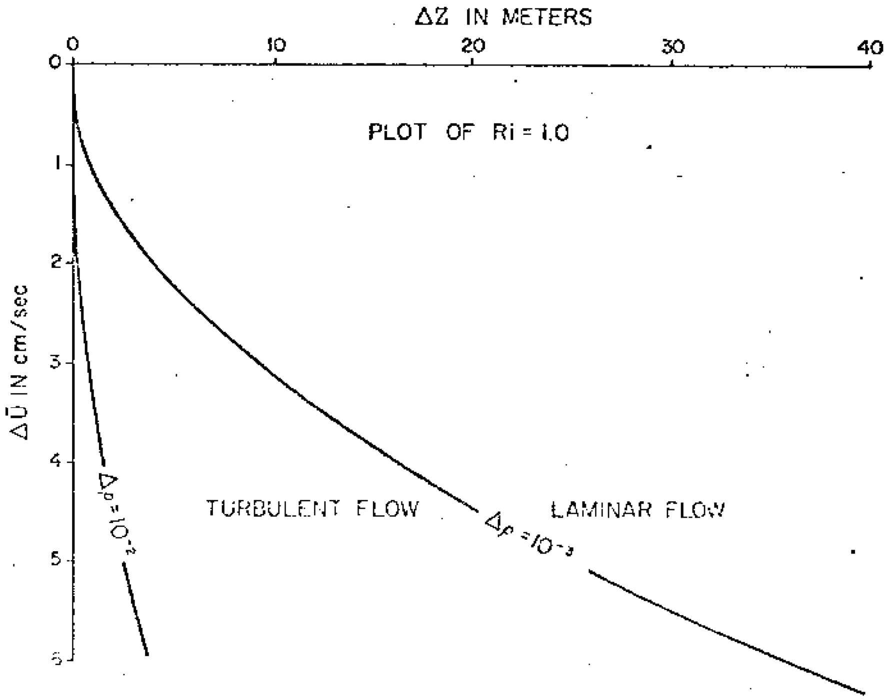


Fig. 1.2: Answer to Example 1.4.1(b).

CHAPTER 2

EXAMPLE 2.1.1

Dietrich, G. and K. Kalle: Loc. cit. Example 1.1.4, pp. 152-154.

EXAMPLE 2.1.2

von Arx, W. S.: Loc. cit. Example 1.3.3, pp. 141-142.

EXAMPLE 2.1.3

(a) Dietrich, G. and K. Kalle: Loc. cit. Example 1.1.4, pp. 151-158.

(b), (c) von Arx, W. S.: Loc. cit. Example 1.3.3, pp. 144-146.

EXAMPLE 2.1.4

(a), (b) Sverdrup, H. U., M. W. Johnson, R. H. Fleming: Loc. cit. Example 1.3.1, pp. 104-110.

EXAMPLE 2.1.5

(a) Annual heat storage in ocean = $7.2 \cdot 10^{22}$ cal
Annual heat storage in land = $8.4 \cdot 10^{21}$ cal
Daily incoming radiation = $1.5 \cdot 10^{21}$ cal/day
Different capacities essential due to difference in specific heat, occurrence of turbulent exchange processes in water.

(b) 57.2°C and 6.6°C

(c), (d) Defant, A.: Loc. cit. Example 1.4.2, pp. 110-117.
Sverdrup, H. U., M. W. Johnson, R. H. Fleming: Loc. cit. Example 1.3.1, pp. 124-128.

EXAMPLE 2.1.6

(a) - (c) Dietrich, G. and K. Kalle: Loc. cit. Example 1.1.4, pp. 159-169.

EXAMPLE 2.2.1

(a) Ocean: $1.7 \cdot 10^{18}$ cal; atmosphere: $1.2 \cdot 10^{20}$ cal

(b) Ocean: $1.1 \cdot 10^{21}$ cal/day; atmosphere: $3.6 \cdot 10^{21}$ cal/day

(c) Latent heat in atmosphere: $9 \cdot 10^{21}$ cal

(d) Advective heat transport in oceanic currents (e.g., western boundary currents).

EXAMPLE 2.2.2

The heat transport per second has been related to 0°C. The reference temperature is arbitrary, since the net heat transport is an absolute value.

Surface Water:	150×10^{12} cal/sec
Central Water:	10×10^{12} cal/sec
Deep Water:	-27×10^{12} cal/sec
Bottom Water:	2×10^{12} cal/sec
Net heat transport northward	135×10^{12} cal/sec
	$= 5.7 \times 10^{11}$ kw

EXAMPLE 2.2.3

(a) Heat flux = 600 cal cm^{-2} sec^{-1}; power = 12.6×10^{11} kwatts

(b) Kinetic energy per unit mass: 2.7×10^{-4} cal
Excess heat content per unit mass: 4 cal

EXAMPLE 2.2.4

(a) 1 day : Depth = 0.2 m for molecular thermal diffusivity
Depth = 28.7 m for eddy thermal diffusivity

1 year: Depth = 3.6 m for molecular thermal diffusivity
Depth = 544.0 m for eddy thermal diffusivity

(b) Lag of 180°: 28.7 m for 1 day; 544.0 m for 1 year
Lag of 360°: 75.4 m for 1 day; 1088.0 m for 1 year

(c) 1 day : phase velocity = 6.6×10^{-2} cm/sec
1 year: phase velocity = 3.5×10^{-3} cm/sec

EXAMPLE 2.2.5

(a) $\left.\frac{d\rho}{dz}\right|_{crit.} = \frac{\bar{\rho}\, u_o^2}{50 \cdot gH^2}\left(\frac{H - z}{H}\right)^{-8/5}$; critical depth = 25m

(b) 235 cm/sec

(c) Dietrich, G. and K. Kalle: Loc. cit. Example 1.1.4, pp. 471-474.

EXAMPLE 2.3.1

(a) Sverdrup, H. U., M. W. Johnson, R. H. Fleming: Loc. cit. Example 1.3.1, pp. 124-126.

(b) Three main reasons: (i) Atlantic is main freshwater source for rivers and lakes in adjacent continents; (ii) deficit of zonal water vapor transport in the southern west wind drift due to precip[illegible] along the Andes; (iii) presence of the arid European Mediterranean.

EXAMPLE 2.3.2

Sverdrup, H. U., M. W. Johnson, R. H. Fleming: Loc. cit. Ex[illegible] 1.3.1, pp. 124-126.

EXAMPLE 2.3.3

(a), (b) Defant, A.: Loc. cit. Example 1.4.2, pp. 154-156.

(c) Meteor: 48m; Altair: 47m

(d) Meteor: 916m; Altair: 900m. In presence of a thermocline A_z will be drastically decreased.

EXAMPLE 2.3.4

(a) 40.6 cm

(b) $2.78 \cdot 10^8$ kwh

(c) 96 years

EXAMPLE 2.4.1

(a) With E-P = 50 cm/year, S(z = 10m) = 35.055 ‰

(b) With E-P = -50 cm/year, S(z = 10m) = 34.945 ‰

EXAMPLE 2.4.2

(a) See Fig. 2.1. Also, Schott, F. (1966): Der Oberflächensalzgehalt in der Nordsee, Deut. Hydr. Zeitschr., A(8), Nr. 9.

EXAMPLE 2.5.1

(a), (b) Sverdrup, H. U., M. W. Johnson, R. H. Fleming: Loc. cit. Example 1.3.1, pp. 153-163.

EXAMPLE 2.5.2

(a) See Fig. 2.2.

(b) See Fig. 2.3.

(c) Dietrich, G. and K. Kalle: Loc. cit. Example 1.1.4, pp. 198-202.

(d) See Fig. 2.4.

vapor transport in the southern west wind drift due to precipitation along the Andes; (iii) presence of the arid European Mediterranean.

EXAMPLE 2.3.2

Sverdrup, H. U., M. W. Johnson, R. H. Fleming: Loc. cit. Example 1.3.1, pp. 124-126.

EXAMPLE 2.3.3

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(a), (b) Sverdrup, H. U., M. W. Johnson, R. H. Fleming: Loc. cit. Example 1.3.1, pp. 153-163.

EXAMPLE 2.5.2

(a) See Fig. 2.2.

(b) See Fig. 2.3.

(c) Dietrich, G. and K. Kalle: Loc. cit. Example 1.1.4, pp. 198-202.

(d) See Fig. 2.4.

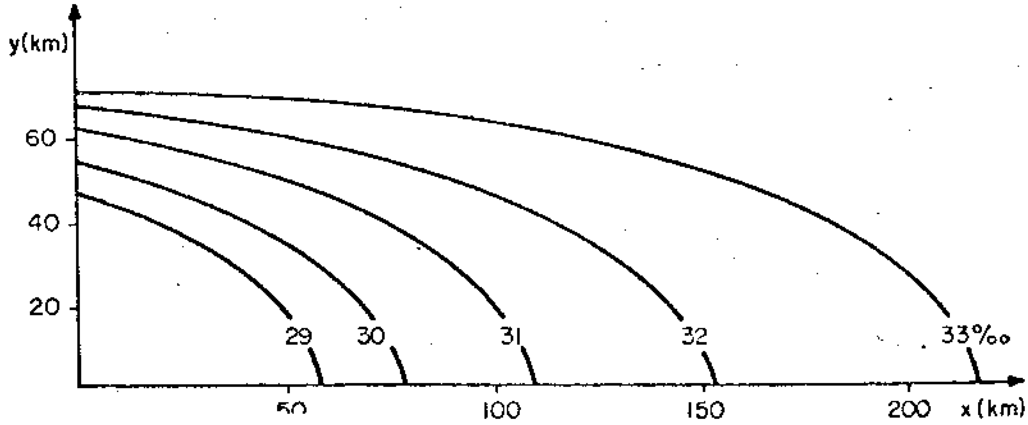


Fig. 2.1: Answer to Example 2.4.2. (After Schott, 1966.)

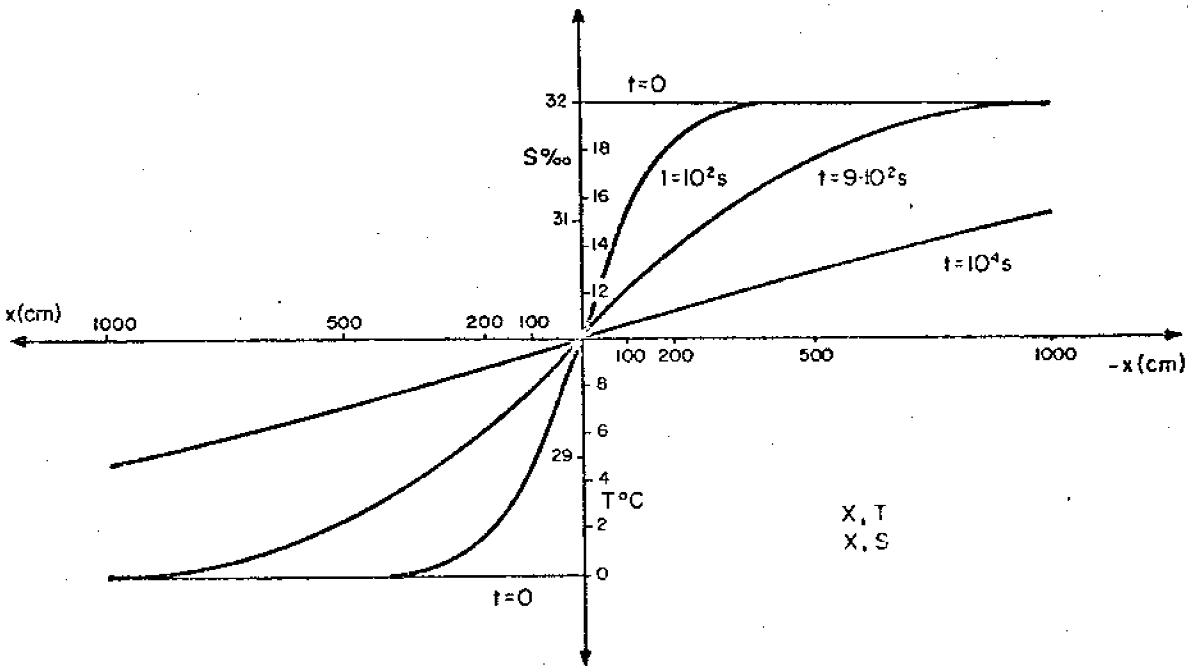


Fig. 2.2: Answer to Example 2.5.2(a).

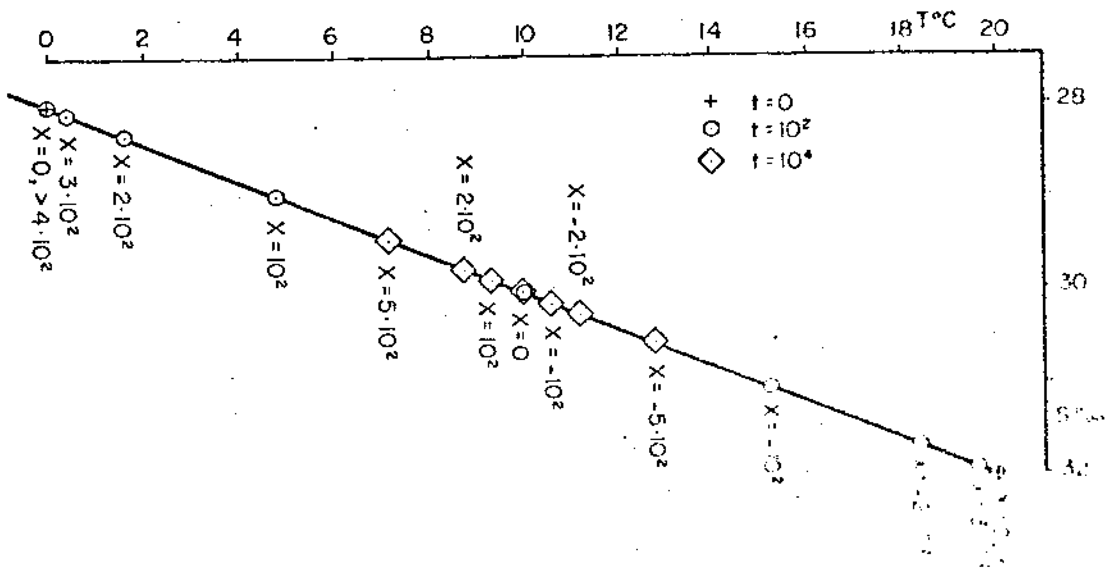


Fig. 2.3: Answer to Example 2.5.2(b).

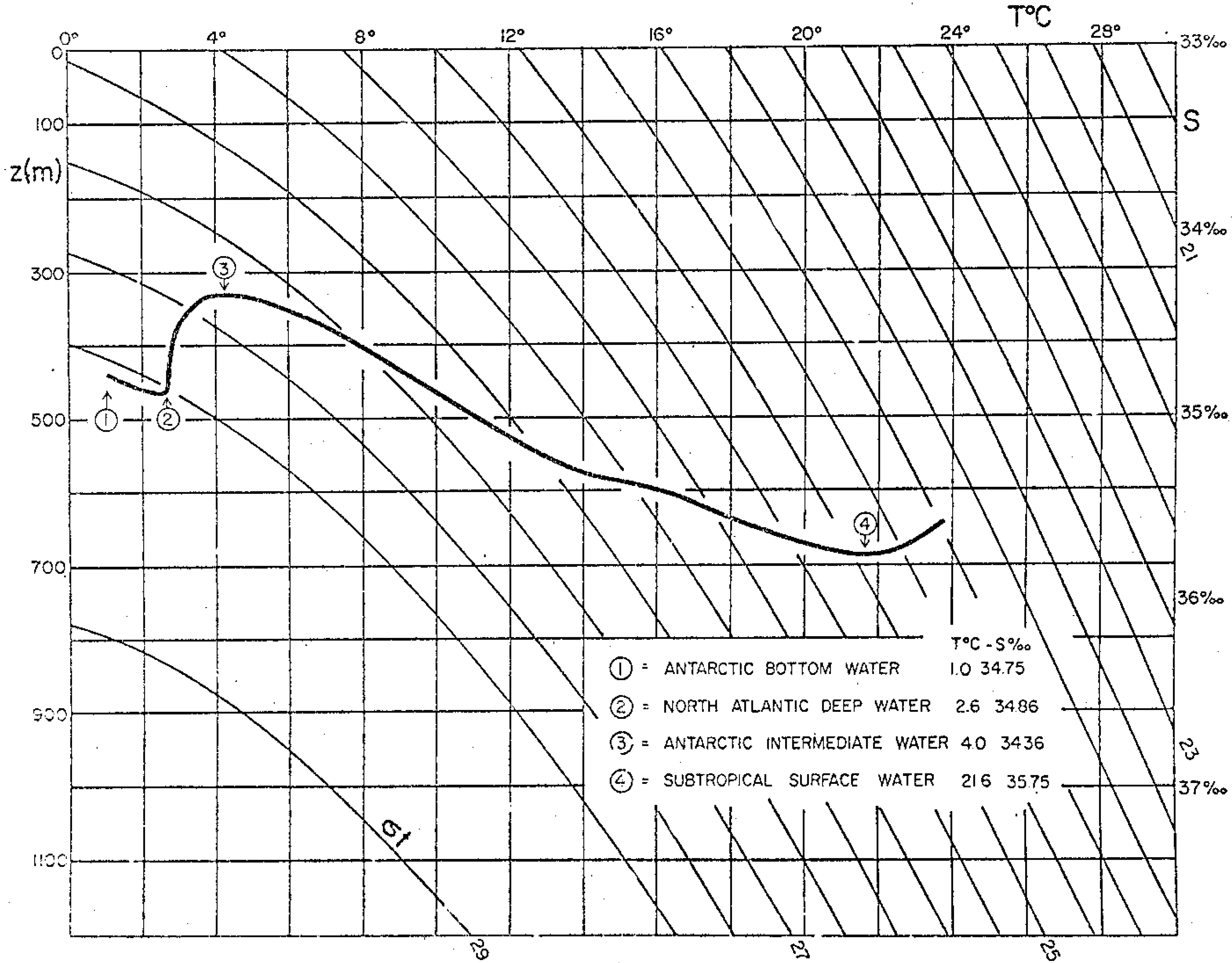


Fig. 2.4: Answer to Example 2.5.2(d).

EXAMPLE 2.5.3

(a), (b) See analogue example in Defant, A.: Loc. cit. Example 1.4.2, pp. 214-216.

EXAMPLE 2.5.4

(a), (b), (c) Broecker, W. (1963): The Sea, Vol. II, "Radioisotopes and Large-scale Oceanic Mixing," Interscience Publ., pp. 88-95. Also for (c) Neuman, G. and W. J. Pierson, Jr.: Loc. cit. Example 1.2.2, pp. 465-478.

EXAMPLE 2.5.5

(a) See Fig. 2.5 to 2.7.

(b) April; Mid-May; Winter water from 1957.

(c) Evaporation is greater in fall and winter when surface winds are stronger. During summer wind speeds are low and precipitation exceeds evaporation and during fall there is excess evaporation.

(d) During summer T } more important effect
During winter S
For vertical convection see Fig. 2.7.

(e) (i) Combined effects of winter cooling and evaporation; (ii) winter cooling; (iii) wind mixing (winter cooling lowers surface density below $4^\circ C$ because $S < 24.7$ ‰).

(f) See Fig. 2.8 a, b.

(g) Dietrich, G. and K. Kalle: Loc. cit. Example 1.1.4, pp. 172-176.

EXAMPLE 2.6.1

(a) - (c) Dietrich, G. and K. Kalle: Loc. cit. Example 1.1.4, pp. 349-352.

EXAMPLE 2.6.2

(i) and (ii) See Figs. 2.9 a, b.

EXAMPLE 2.6.3

(a) $w \frac{\partial T}{\partial z} = A_z \cdot \frac{\partial^2 T}{\partial z^2}$

(b)

z (m)	50	75	100	125	150	175	200	225	250
$w \cdot 10^5$ (cm sec^{-1})	-	-53	-13	-13	-9	-11	-40	0	-

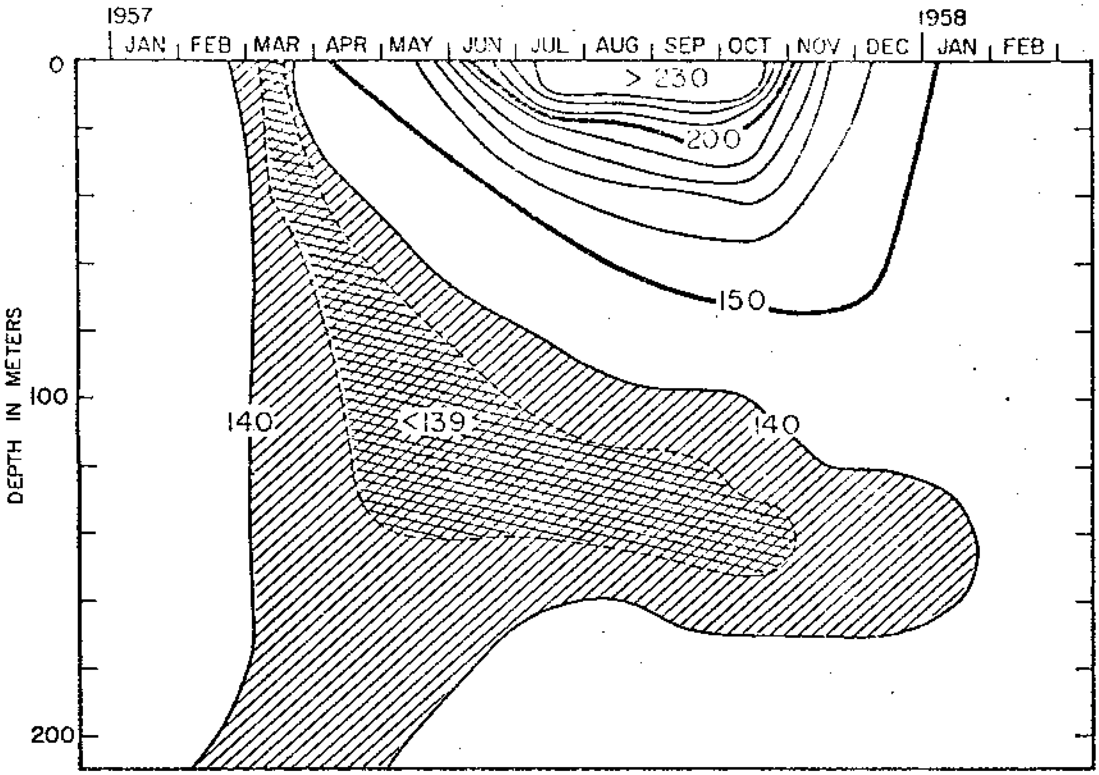


Fig. 2.5: Answer to Example 2.5.5(a).

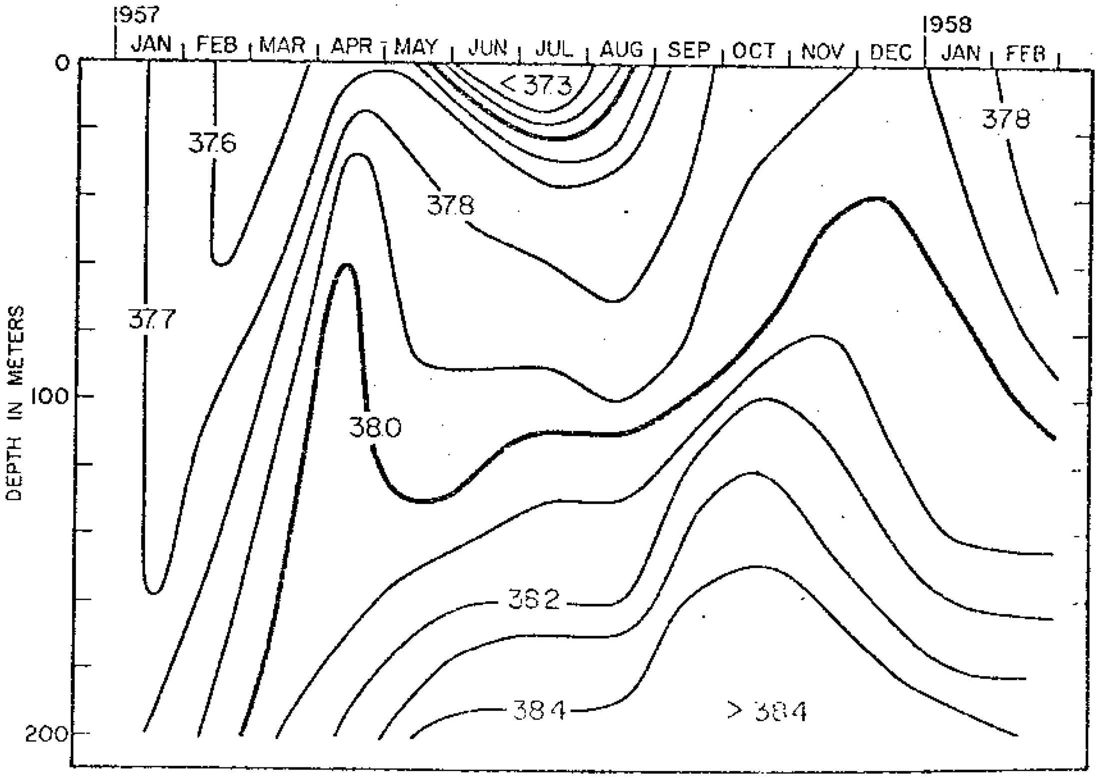


Fig. 2.6: Answer to Example 2.5.5(a)

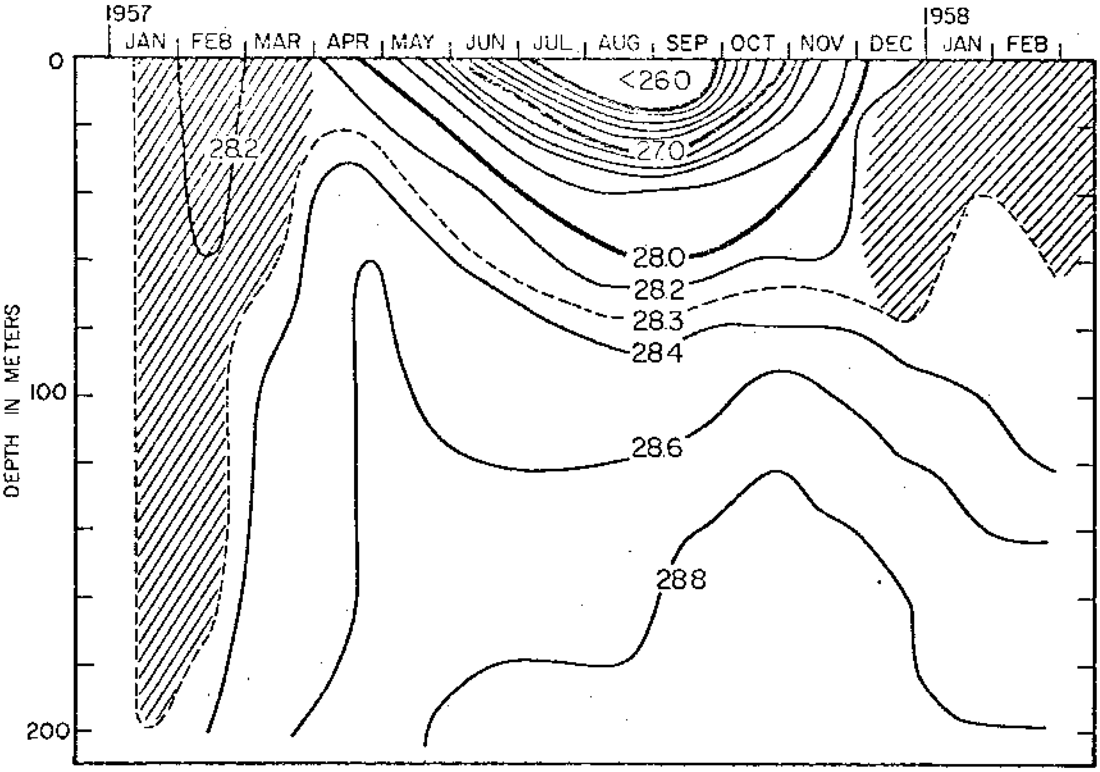


Fig. 2.7: Answer to Example 2.5.5(a)

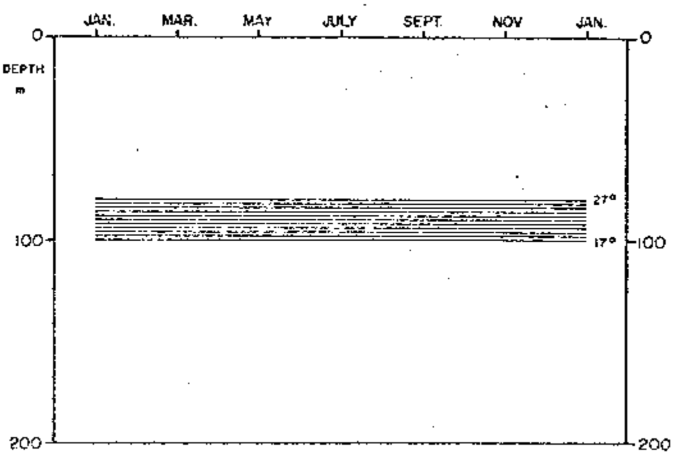


Fig. 2.8a: Answer to Example 2.5.5(f)

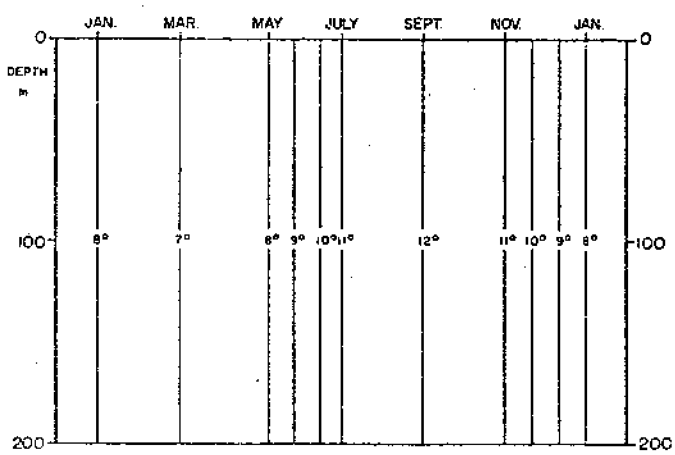


Fig. 2.8b: Answer to Example 2.5.5(f)

(c) Neuman, G. and W. J. Pierson, Jr.: Loc. cit. Example 1.2.2 pp. 445-446.

EXAMPLE 2.6.4

(a), (i) Runoff from Norway and Greenland generates lighter water in coastal areas and, hence, an offshore directed pressure gradient.

(ii) Runoff from islands (e.g., Iceland) has same effect.

(b) An additional component of the Norwegian-Greenland Sea circulation is due to the inflow of the Northeast Atlantic Current along Norway and a compensating outflow along Greenland.

Fig. 2.9a: Answer to Example 2.6.2, arid climate.

The Atlantic Ocean, Straits of Gibraltar, Mediterranean Sea.

The Indian Ocean, Strait of Bab-el-Mandeb, Red Sea.

The Indian Ocean, Strait of Hormuz, Persian Gulf.

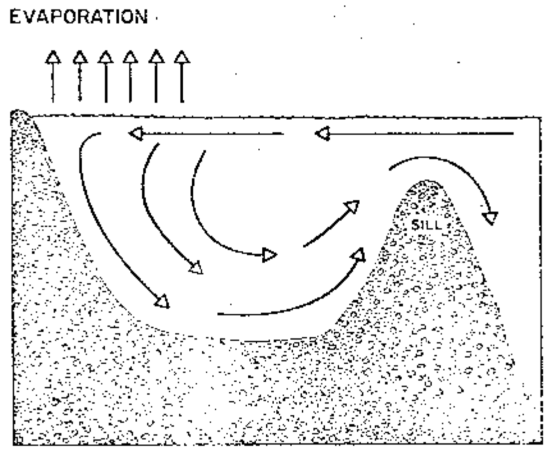
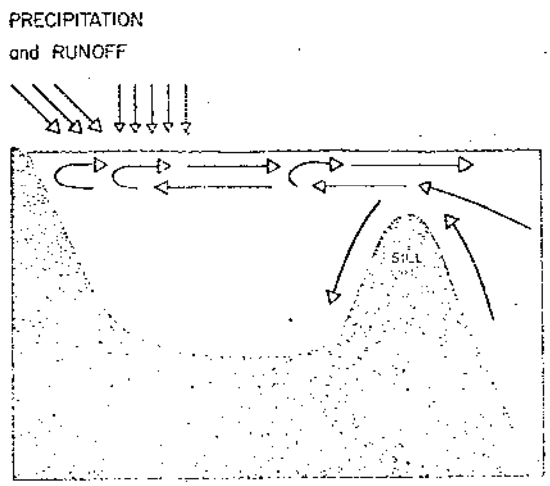


Fig. 2.9b: Answer to Example 2.6.2, humid climate.

The North Sea, Kattegat, Baltic Sea.

The Mediterranean Sea, Bosporus, Black Sea.

Atlantic Ocean, most of Norwegian fjords.



CHAPTER 3

EXAMPLE 3.1.1

(a), (b) von Arx, W. S.: Loc. cit. Example 1.3.3, pp. 36-38.

EXAMPLE 3.1.2

(a) 978.28 dyn m, 983.23 dyn m, 984.23 dyn m. Depth in dyn m expresses energy.

(b)

	$\varphi = 0°$	$\varphi = 60°$
1,000 m	D = 0.978 h	D = 0.983 h
10,000 m	D = 0.980 h	D = 0.985 h

Latitude effect is more important.

(c) 1,012 dbar, 10,500 dbar

EXAMPLE 3.1.3

(a), (b) Defant, A.: Loc. cit. Example 1.4.2, pp. 304-308.

(c) von Arx, W. S.: Loc. cit. Example 1.3.3, pp. 130-133

EXAMPLE 3.1.4

(a), (b) Krauss, W. (1966): Interne Wellen, Borntraeger-Verlag, pp. 16-17.

Phillips, O. M. (1966): The Dynamics of the Upper Ocean, Cambridge University Press, pp. 14-19 and pp. 161-165.

(c)

Depth (m)	$N_1 = \sqrt{gE}$ (sec^{-1})
0	
50	0
100	0
150	120×10^{-4}
200	94×10^{-4}
400	48×10^{-4}
600	9.8×10^{-4}
800	9.8×10^{-4}
1,000	9.8×10^{-4}
1,200	8.3×10^{-4}
1,500	6.7×10^{-4}
2,000	5.6×10^{-4}
2,500	3.4×10^{-4}

(d) Larger compressibility of float ⟶ overshooting
Smaller compressibility of float ⟶ undershooting
Equal compressibility of float ⟶ perfect following

EXAMPLE 3.2.1

(a), (b) Defant, A.: Loc. cit. Example 1.4.2, pp. 343-344.

EXAMPLE 3.2.2

(a) 150 cm/sec towards 90°

(b) Equatorial undercurrent.

EXAMPLE 3.2.3

(a) See Fig. 3.1.

(b)(i) Tidal; (ii) 0.5 kn towards 48°; (iii) Ekman drift

(c) See Fig. 3.2.

EXAMPLE 3.2.4

Neumann, G. and W. J. Pierson, Jr.: Loc. cit. Example 1.2.2, pp. 119-120.

EXAMPLE 3.2.5

(a)

Top left	2.68×10^{-4}	cm/sec down
Top right	3.57×10^{-4}	cm/sec down
Bottom right	1.43×10^{-4}	cm/sec down
Bottom left	7.68×10^{-4}	cm/sec up

(b) Errors:
(i) Ignored world outside the four areas.
(ii) Density may vary.
(iii) Ignored cyclonic motion on the left, and its dynamic effects.

EXAMPLE 3.2.6

$$u_2^3 - u_2 \; (2gH_1 + u_1^2) + \frac{2gu_1W_1H_1}{W_2} = 0$$

$u_2 = 4.3$ m/sec; $H_2 = 9.3$ m

EXAMPLE 3.2.7

$V \cong 4$ cm/sec

EXAMPLE 3.3.1

(a),(b) Dietrich, G. and K. Kalle: Loc. cit. Example 1.1.4, pp. 313-314.

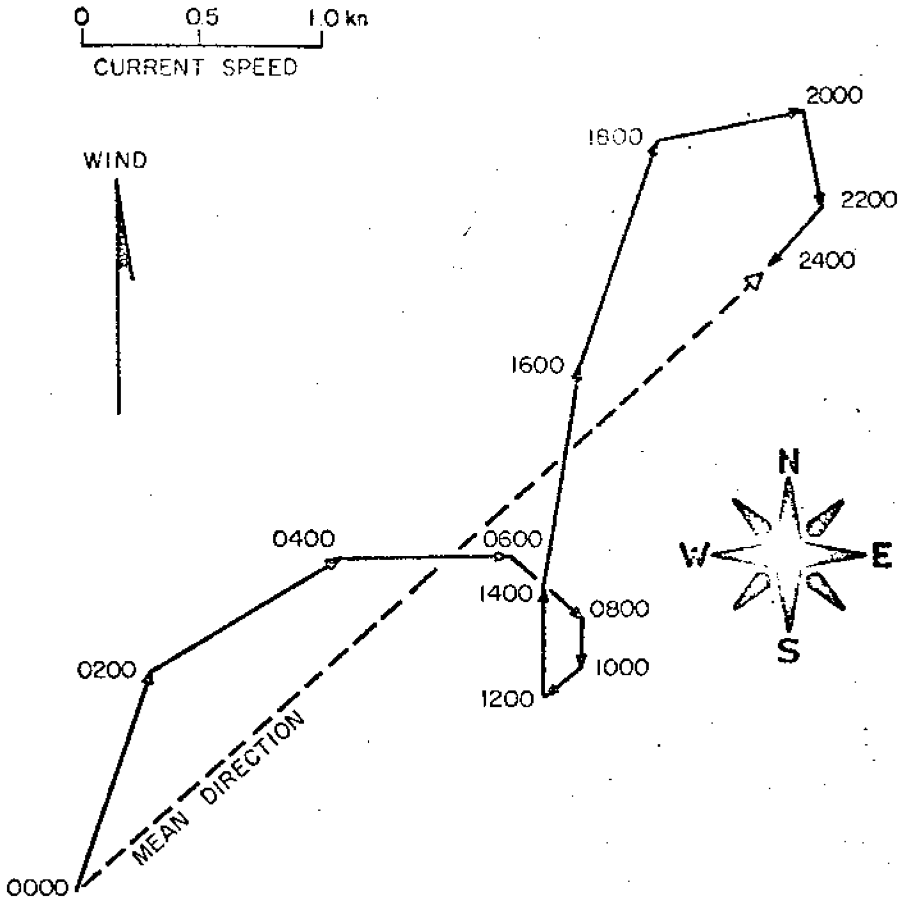


Fig. 3.1: Answer to Example 3.2.3(a).

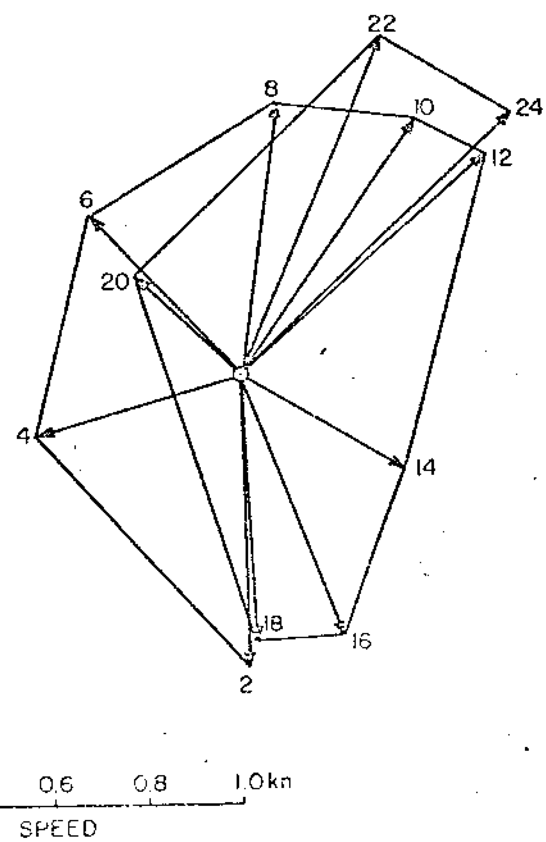


Fig. 3.2: Answer to Example 3.2.3(c).

EXAMPLE 3.3.2

(a), (b)

	z = const	D = const
$\vec{\Omega} = 0$	poleward acceleration	unaccelerated
$\vec{\Omega} \neq 0$	cycloidal path in zonal direction	unaccelerated

EXAMPLE 3.3.3

(a) 12' towards the south
Momentarily 12' deflection toward the west.

(b) No deflection from the vertical.

(c) Particle oscillates around the equator.

EXAMPLE 3.3.4

Initial speed 1.56 cm sec^{-1}

EXAMPLE 3.3.5

(a) M_2: 75° N and S; O_1: 28° N and S

(b) 12 hours at the North pole

(c) See Fig. 3.3.

EXAMPLE 3.4.1

(a) - (c) Neumann, G. and W. J. Pierson, Jr.: Loc. cit. Example 1.2.2, pp. 114-186.

EXAMPLE 3.4.2

$x = \frac{\nabla p}{f} (t - \frac{1}{f} \sin ft)$; $y = \frac{\nabla p}{f^2} (1 - \cos ft)$, representing cycloidal motion in zonal direction.

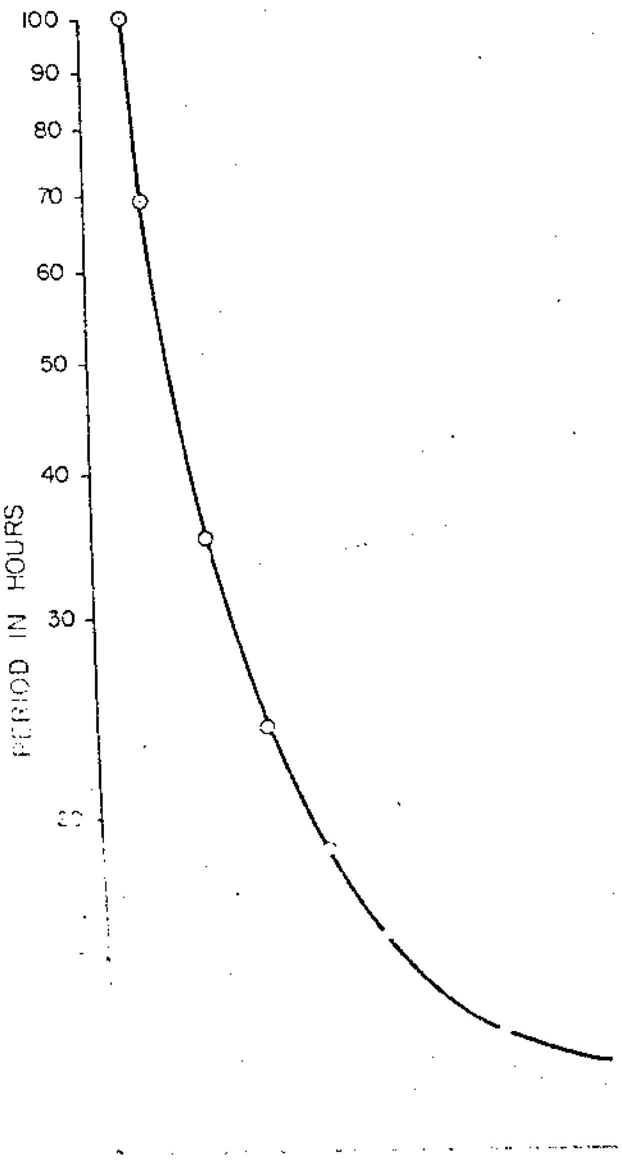
EXAMPLE 3.4.3

See Fig. 3.4.

EXAMPLE 3.4.4

(a), (b) Stommel, H. (1965): The Gulf Stream, 2d ed., University of California Press, pp. 108-111.

(c) von Arx, W. S.: Loc. cit. Example 1.3.3, pp. 107-111.



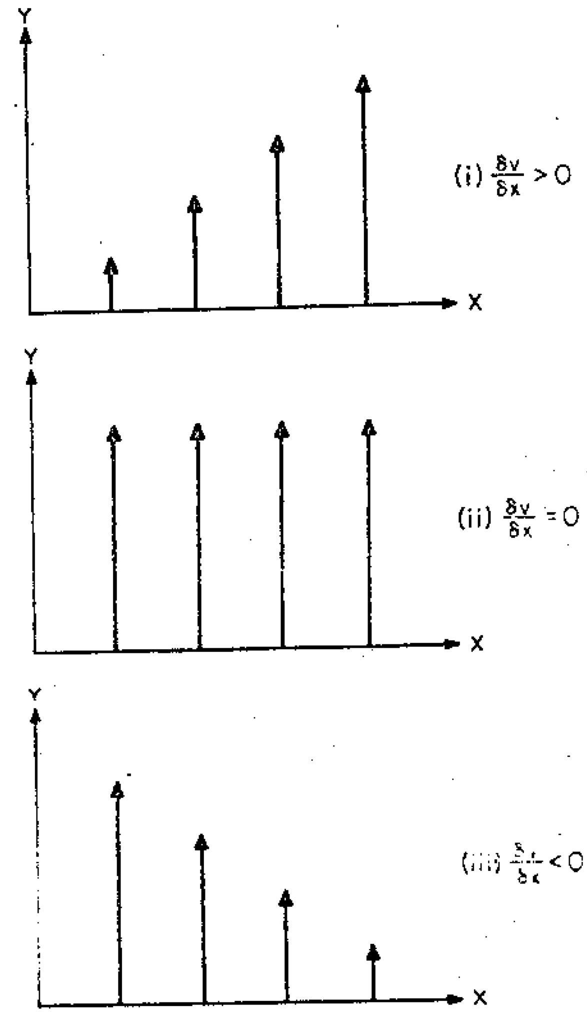


Fig. 3.4: Answer to Example 3.4.3.

(d)(i) Northward / Southward motion causes anticyclonic / cyclonic rotation of the turn table.

(ii) Car moves towards the north.

EXAMPLE 3.5.1

(a) Sverdrup, H. U., M. W. Johnson, and R. H. Fleming: Loc. cit. Example 1.3.1, pp. 460-465.

(b) 29 cm/sec; $2 \cdot 10^{-6}$ cm/km

EXAMPLE 3.5.2

See Fig. 3.5. and Dietrich, G. and K. Kalle: Loc. cit. Example 1.1.4, pp. 328-335.

EXAMPLE 3.5.3

Between A and B: 67 cm/sec towards north
Between B and C: 54 cm/sec towards south

EXAMPLE 3.5.4

(a) - (d) Neumann, G. and W. J. Pierson, Jr.: Loc. cit. Example 1.2.2, pp. 191-205 and pp. 450-453.

EXAMPLE 3.5.5

Stommel, H.: Loc. cit. Example 3.4.4, pp. 154-156.

EXAMPLE 3.5.6

Stommel, H.: Loc. cit. Example 3.4.4, pp. 81-103 and see Fig.

EXAMPLE 3.5.7

Stommel, H.: Loc. cit. Example 3.4.4, pp. 93-103.

$A_x = 10^6$ cm^2/sec: Narrow, slow boundary current
$A_x = 10^9$ cm^2/sec: Broad, fast boundary current

EXAMPLE 3.5.8

(a) - (c) Haltiner, G. J. and F. L. Martin (1957): Dynamical and Physical Meteorology, McGraw-Hill, pp. 356-357.

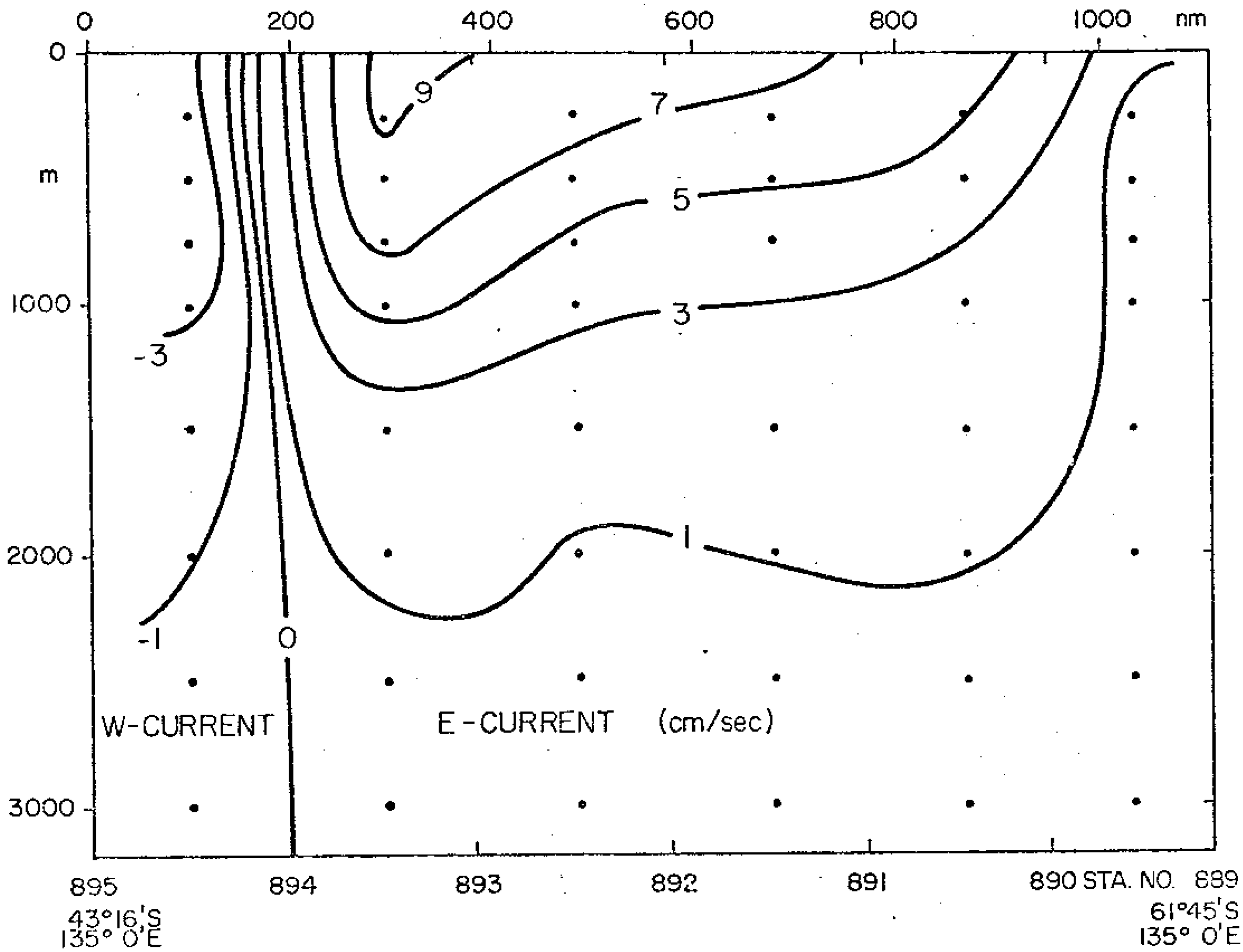


Fig. 3.5: Answer to Example 3.5.2.

CHAPTER 4

EXAMPLE 4.1.1

Sears, F. W. (1958): Mechanics, Wave Motion and Heat, Addison-Wesley, Chapters 16, 17.

EXAMPLE 4.1.2

(a) Kinsman, B. (1965): Windwaves, Prentice Hall, pp. 126-128.

(b) Equation for orbital path of progressive waves obtained by integration with respect to t:

$$\frac{(x - x_0)^2}{\left[\frac{\varkappa}{\omega}a \cdot \cosh \varkappa(z_0 - H)\right]^2} + \frac{(z - z_0)^2}{\left[\frac{\varkappa}{\omega}a \cdot \sinh \varkappa(z_0 - H)\right]^2} = 1,$$

for short waves

$$\begin{cases} \cosh \varkappa(z_0 - H) \approx \dfrac{e^{-\varkappa z} \cdot e^{\varkappa H}}{2} \\ \sinh \varkappa(z_0 - H) \approx - \dfrac{e^{-\varkappa z} \cdot e^{\varkappa H}}{2} \end{cases}$$

for long waves

$$\begin{cases} \cosh \varkappa(z_0 - H) \approx \varkappa H \\ \sinh \varkappa(z_0 - H) \approx 1 \end{cases}$$

The equation of the orbital path simplifies for

short waves to:

$$\frac{(x - x_0)^2}{(\frac{\varkappa}{\omega} a^* e^{-\varkappa z_0})^2} + \frac{(z - z_0)^2}{(\frac{\varkappa}{\omega} a^* e^{-\varkappa z_0})^2} = 1$$

where $a^* = ae^{\varkappa H}$

long waves to:

$$\frac{(x - x_0)^2}{(\frac{\varkappa}{\omega} a)^2} + \frac{(z - z_0)^2}{(\frac{\varkappa^2}{\omega} a (z_0 - H))^2} = 1$$

(c) Equation of orbital path for <u>standing waves</u> obtained by integration with respect to t:

$$\frac{z - z_0}{x - x_0} = - \tanh \varkappa(z_0 - H) \cot \varkappa x_0$$

simplification for

short waves: $\frac{z - z_0}{x - x_0} = \cot \varkappa x_0$

long waves: $\frac{z - z_0}{x - x_0} = -\varkappa(z_0 - H) \cot \varkappa x_0$

(d) Dietrich, G. and K. Kalle: Loc. cit. Example 1.1.4, pp. 333, 362, 394.

Kinsman, B.: Loc. cit. Example 4.1.2, pp. 138-140.

EXAMPLE 4.1.3

(a) - (d) Krauss, W.: Loc. cit. Example 3.1.4, pp. 15-16 and pp. 26-29. (See also English summaries p. 216.)

EXAMPLE 4.1.4

$\mathrm{Re}\{p\} = -\omega \cdot a \cdot \rho_0 \cosh \varkappa(z - H) \cdot \sin(\varkappa x - \omega t) + g\rho_0 z$, amplitudes of long and short waves are frequency-dependent.

EXAMPLE 4.2.1

(a) 156 m

(b) 15.6 m/sec

(c) $\lambda_s = \sqrt{\lambda_s / 2\pi \cdot H} \cdot \lambda_1$

(d) Kinsman, B.: Loc. cit. Example 4.1.2, pp. 128-132.

(e) Dietrich, G. and K. Kalle: Loc. cit. Example 1.1.4, pp. 382-384.

EXAMPLE 4.2.2

(a) 139 hours before first observation.

(b) 11,700 km

EXAMPLE 4.2.3

(a), (b) Dietrich, G. and K. Kalle: Loc. cit. Example 1.1.4, pp. 380-384.

EXAMPLE 4.2.4

(a) $\tau = \frac{2\ell}{\sqrt{gH}}$

(b) Dietrich, G. and K. Kalle: Loc. cit. Example 1.1.4, pp. 398-405.

EXAMPLE 4.3.1

(a), (b) Krauss, W.: Loc. cit. Example 3.1.4, pp. 43-45.

EXAMPLE 4.3.2

(a), (b) Phillips, O. M.: Loc. cit. Example 3.1.4, pp. 161-165.

EXAMPLE 4.3.3

(a) Krauss, W.: Loc. cit. Example 3.1.4, pp. 145-146.

(b) Since internal wave processes are controlled by a dispersion relation the choice of $\Psi^{(o)}$ requires a corresponding choice of the spatial scale.

(c) $\frac{\partial \Psi}{\partial z} = f(z)$, especially step-like fine structure of the stratification.

EXAMPLE 4.3.4

(a), (b) Krauss, W.: Loc. cit. Example 3.1.4, pp. 46-49.

EXAMPLE 4.4.1

(a) 4,600 km from the earth's center.

(b) Defant, A. (1960): Physical Oceanography II, Pergamon Press, pp. 254-258.

EXAMPLE 4.4.2

(a)-(c) Neumann, G., W. J. Pierson, Jr.: Loc. cit. Example 1.2.2, pp. 302-304. Dietrich, G., and K. Kalle: Loc. cit. Example 1.1.4, pp. 423-425. Bartels, J. (1957): Handbuch der Physik, Vol. 48, "Gezeitenkräfte," Springer-Verlag, pp. 734-746.

EXAMPLE 4.4.3

(a), (b) Defant, A. (1958): Ebb and Flow, University of Michigan Press, pp. 26-33.

EXAMPLE 4.4.4

(a) $F_H = 0$ (10^{-4} dynes); $\vec{T} = 3.2$ dynes/cm^2; $C = 1.3 \cdot 10^{-3}$ dynes; pressure gradient: $1.3 \cdot 10^{-3}$ dynes.

(b) Ratio of gravity, g, to tidal induced changes, Δg, approximately $9 \cdot 10^6:1$. For tides of the atmosphere and the solid earth, see Defant, A., loc. cit. Example 4.4.3, pp. 106-117.

EXAMPLE 4.5.1

(a), (b) Defant, A.: Loc. cit., Vol. II, Example 4.4.1, pp. 305-308.

EXAMPLE 4.5.2

(a) - (d) See Figures 4.1 and 4.2.

EXAMPLE 4.5.3

(a) 0-12h: u = 3.9, v = -6.2; 13-25h: u = 2.6, v = -5.0.

(b) See Fig. 4.3.

(c) Major and minor axes, direction δ

$$(1/\sqrt{2})\left\{u_1^2 + u_2^2 + v_1^2 + v_2^2 \pm [(u_1^2 + u_2^2 + v_1^2 + v_2^2)^2 - 4(u_1 v_2 - u_2 v_1)^2]^{\frac{1}{2}}\right\}^{\frac{1}{2}}$$

$$\tan 2\delta = 2(u_1 v_1 + u_2 v_2)/(u_1^2 + u_2^2 - v_1^2 - v_2^2)$$

$$u_1 = u_o \cos \Psi_1;\ v_1 = v_o \cos \Psi_1;\ u_2 = -u_o \sin \Psi_2;\ v_2 = -v_o \sin \Psi_2$$

(d) See Fig. 4.3.

EXAMPLE 4.5.4

(a) -23.8 cm

(b) rising

(c) HT at 0200 hours; LT at 1000 hours

(d) See Fig. 4.4.
Form factor given by $\frac{K_1}{M_2 + S_2} \approx 0.39$; hence mixed, but predominantly semidiurnal tides.

EXAMPLE 4.5.5

(a) No, because of variations in the vertical distribution of amplitudes and phases.

(b) Approximations for v-component

0.-order mode		1.-order mode		0.+1.-order modes	
v_o	φ_{v_o}	v_1	φ_{v_1}	v	φ_v
16.0	267	6.1	220	20.6	255
16.0	267	0.6	040	15.6	269
16.0	267	5.5	040	12.9	285

EXAMPLE 4.6.1

(a) Dietrich, G. and K. Kalle: Loc. cit. Example 1.1.4, pp. 443-444.

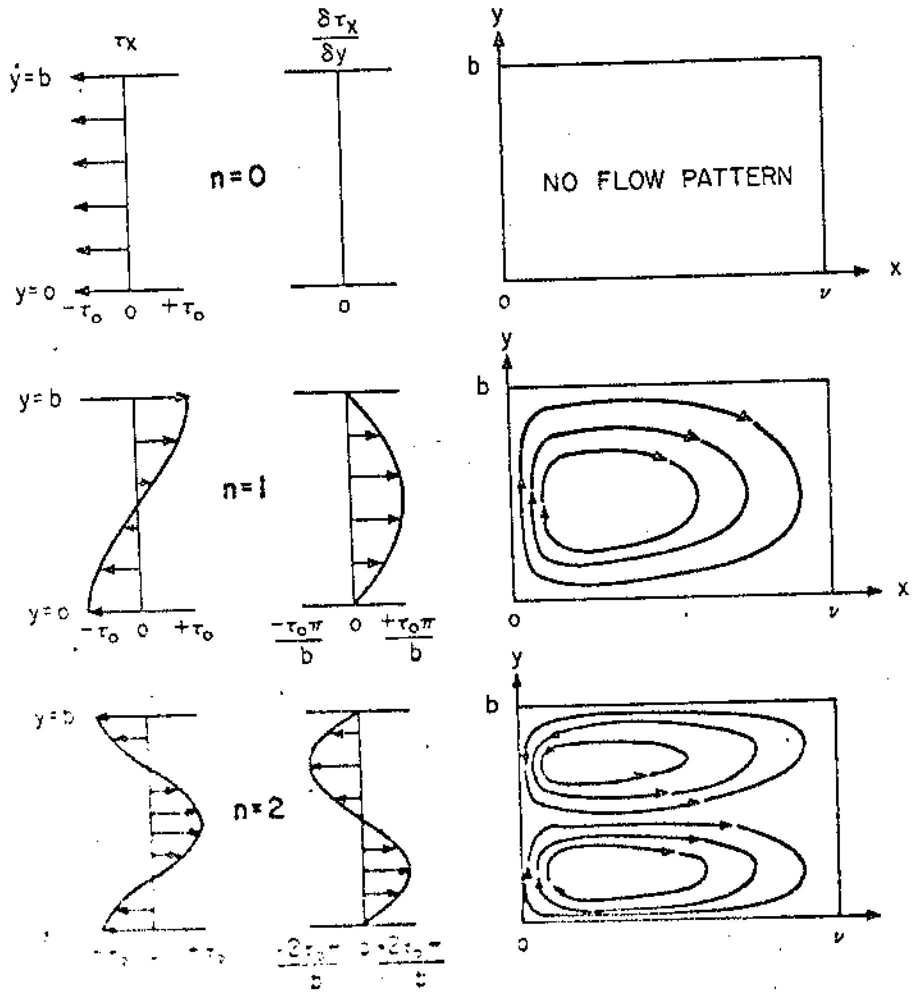


Fig. 3.6: Answer to Example 3.5.6.

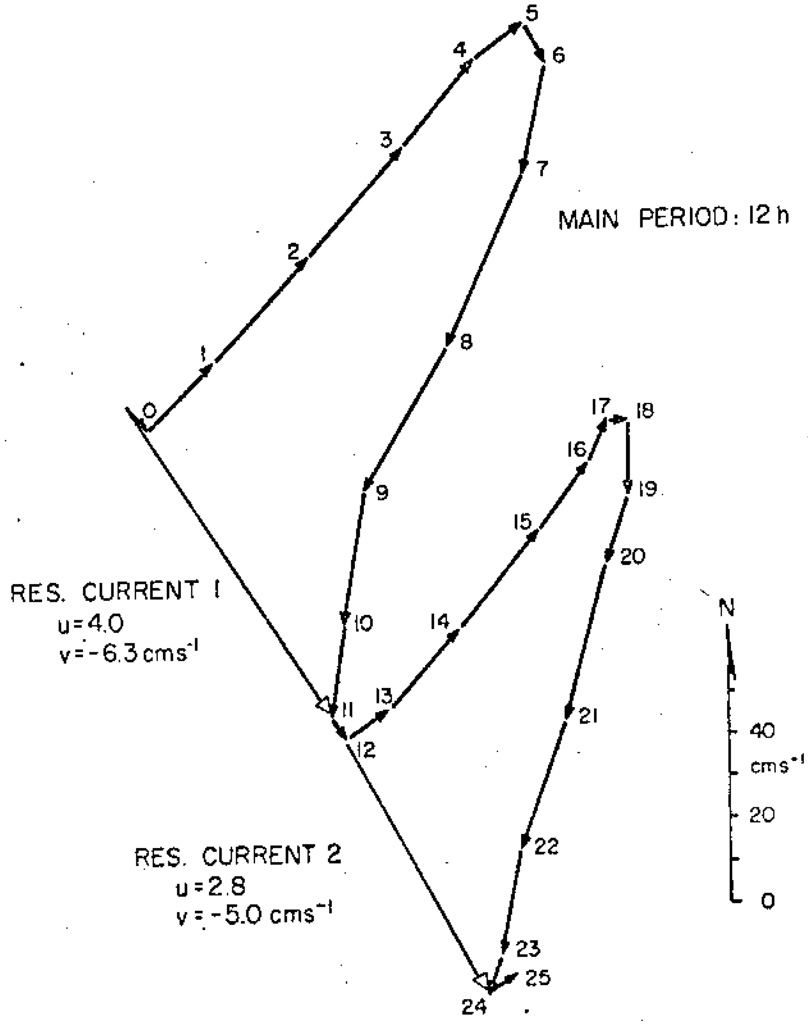
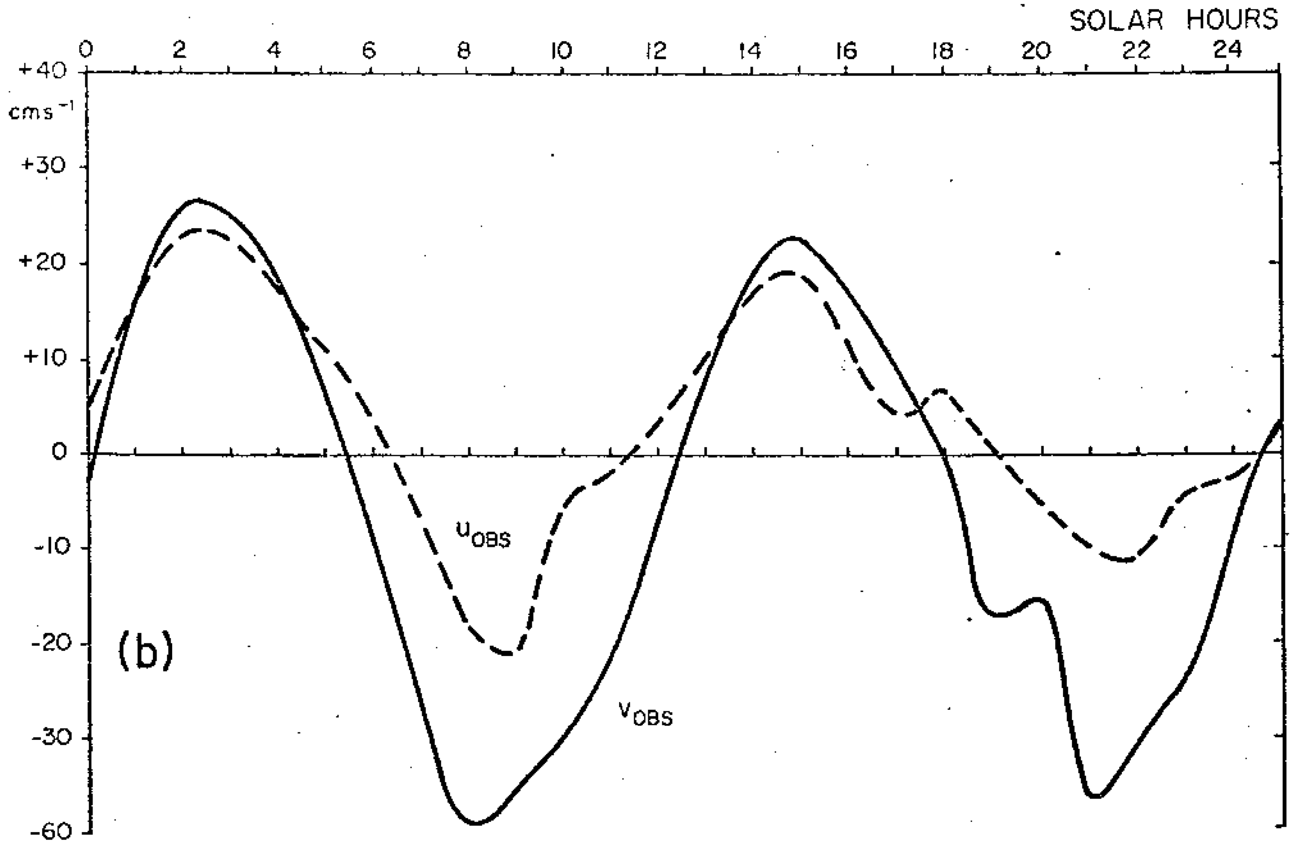


Fig. 4.1: Answer to Example 4.5.2(a).



(c)

NORTH COMP v

ν	Tν	Aν	φν
0	∞	-3.0	–
1	24.8	2.6	15
2	12.4	29.9	81

EAST COMP u

ν	Tν	Aν	φν
0	∞	2.4	–
1	24.8	1.7	5
2	12.4	16.2	79

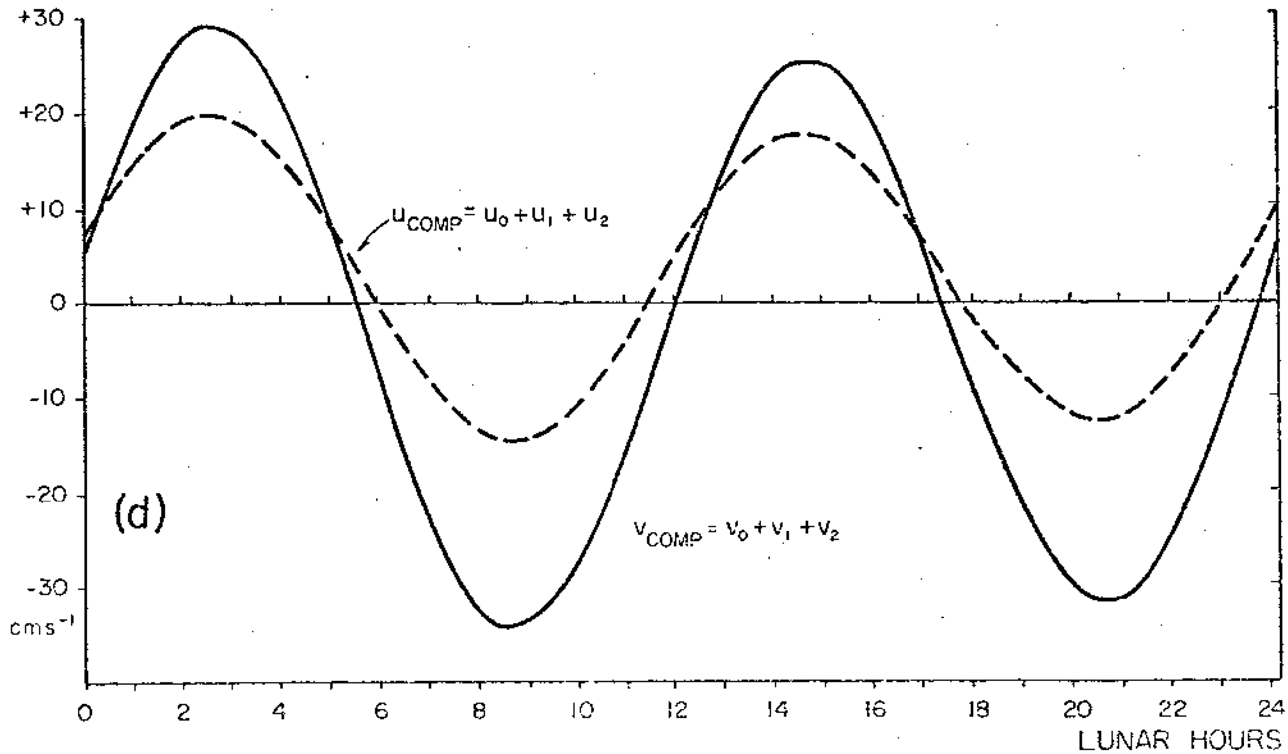


Fig. 4.2: Answer to Example 4.5.2(b), (c), (d).

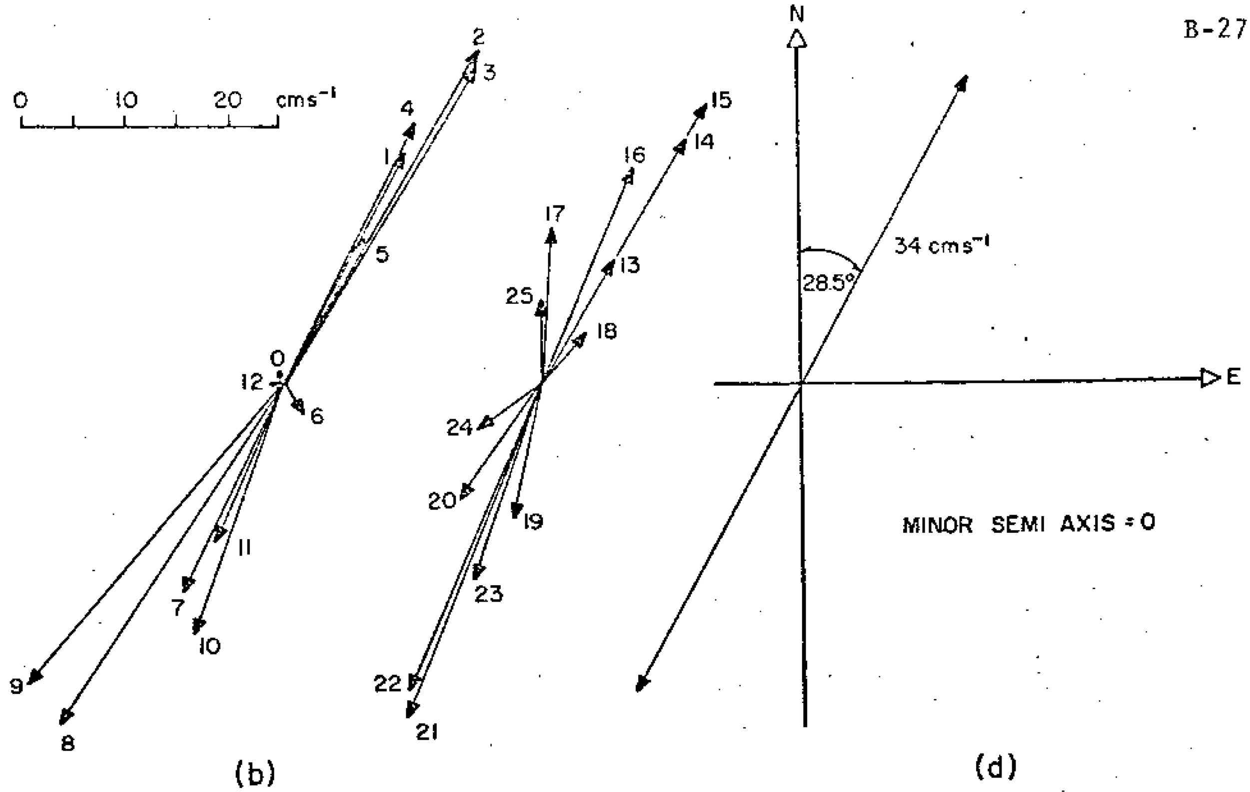


Fig. 4.3: Answer to Example 4.5.3(b), (d).

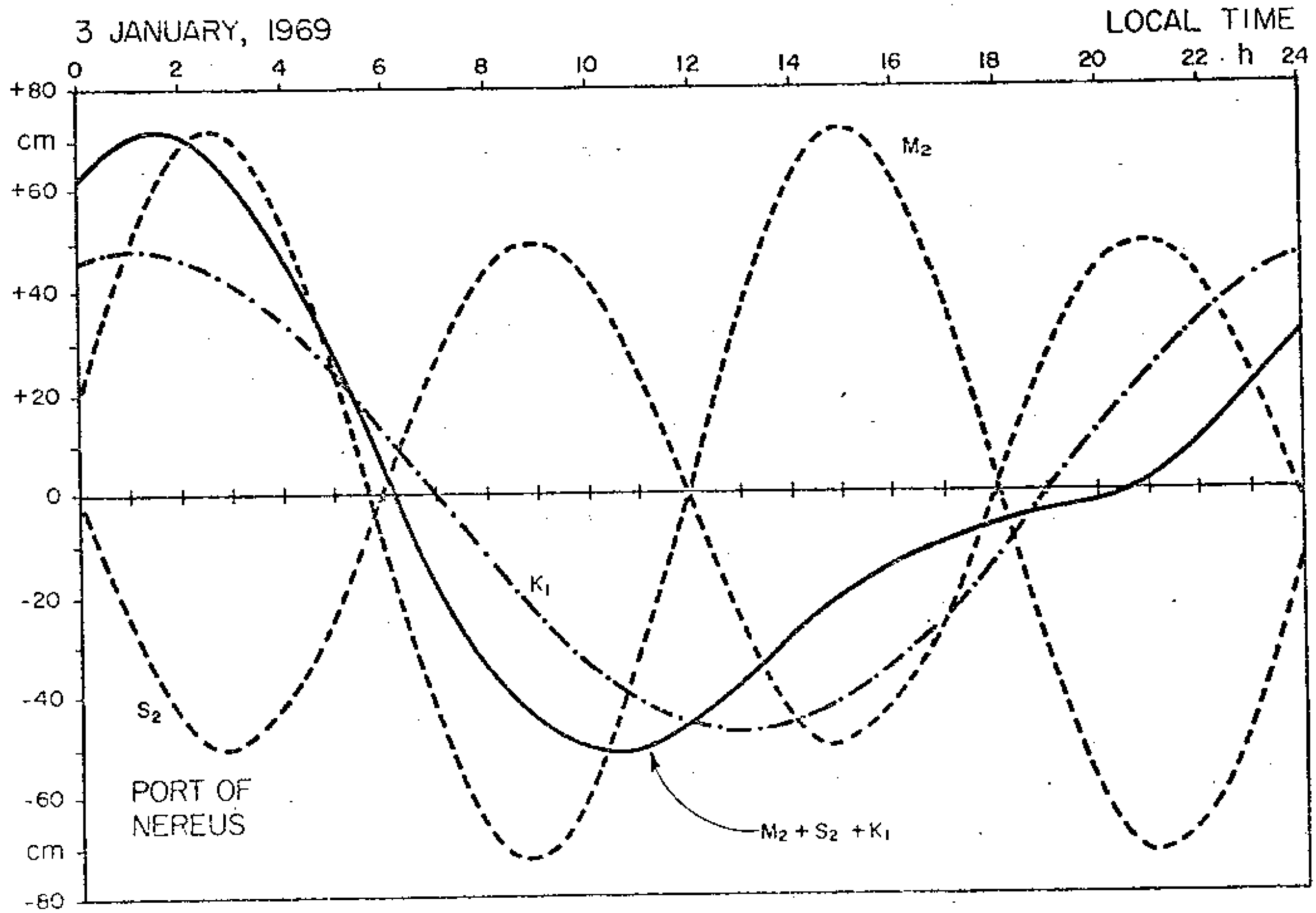


Fig. 4.4: Answer to Example 4.5.4(d).

(b) Substantial increase of oceanic water depth so that $c = \sqrt{gH}$ would equal the velocity of the lunar zenith.

EXAMPLE 4.6.2

(a) Because of the deflection of Coriolis force. Tsunamis are not affected because of much lower periods (ellipse degenerates essentially to a straight line).

(b) Defant, A.: Loc. cit. Example 4.4.3, pp. 70-75.

(c) Current maximum and elevation minimum characterize amphidromic point.

EXAMPLE 4.6.3

(a) - (c) Defant, A: Loc. cit. Example 4.4.3, pp. 65-88.

EXAMPLE 4.6.4

(a) Assuming an idealized Kelvin wave, the tidal amplitude along the "English" coast will be 67% of the amplitude along the "French" coast.

(b) Real coastline deviates strongly from idealized channel.

(c) Dietrich, G. and K. Kalle: Loc. cit. Example 1.1.4, p. 465.