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Transient Motion of Floating Bodies: Application to the Computation of the Hydrodynamic Load Exerted on Ships in Collisions; A Review

by
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1- INTRODUCTION

Accidental damage to offshore installations can have potentially devastating consequences in terms of human life as well as environmental impact. Collision between vessels and platforms is now recognized as the largest single cause of offshore accidents. The problem is intensifying as offshore development proceeds at an accelerated pace throughout the world.

Existing design codes (Reports 1 and 2; Furnes and Amdahl 1980; Fjeld 1979) address the treatment of accidental loads, such as those resulting from collision, rather briefly. Furthermore, no guidance is provided for evaluating the structure's resistance against damage and collapse.

One very important factor which must be accounted for in the design of offshore platforms is the magnitude of the hydrodynamic force exerted on the striking ship during collision (Ellinas and Valsgard 1985). This force, usually referred to as the added-mass effect, is due to the motion of the water around the keel of the ship and varies from the initial moment of impact to later times (Motora et al. 1971, Petersen 1982). When the ship is in contact with the platform, this time-varying force is indirectly exerted on the platform. Yet, the estimate of this force is very crude, usually taken as a time invariant quantity equal to a fraction of the displacement of the ship, which in some circumstances may greatly fail to accurately predict the design load (Report 3; Petersen 1982; Ellinas and Valsgard 1985).

A better understanding of the hydrodynamic phenomena involved in the collision process can yield some useful information on the time-varying design loads to be used in the analysis of the resistance of an offshore platform.

In this review, we will outline the traditional methods used in the study of the motion of bodies piercing the free surface under simplifying assumptions of linearization. The equivalence, in this context, between the problems of small oscillations and transient motion of floating bodies will be shown. An application of linear theory to the transient motion of ships will be presented, with a special emphasis on the problem of collision involving ships.

Whereas linear system theory is a well developed body of knowledge, the application of which is relatively straightforward, the severe limitations of linear models are now recognized in many situations involving ocean structures and freely floating bodies. Several important aspects of the collision problem, not accounted for by linear theory, will be examined. Some mathematical techniques available to predict nonlinear effects due to the presence of a free surface and their applications for the study of collision will be reviewed.

The general features of numerical methods available to solve the linear as well as the complete nonlinear problem and the difficulties associated with their implementation will be presented.

2-THE LINEARIZED MOTION OF FLOATING BODIES

2-1 INTRODUCTION

Floating bodies without any means of propelling themselves and non-maneuvering ships are considered. In the case of a collision problem, this will correspond to a drifting, freely floating ship which doesn't disturb the surrounding water until after impact. By a change of referential, this will correspond to a still ship in still water, before impact.

2-2 THE EQUATIONS OF MOTION

2-2-1 Linearization

For a complete derivation of the equations of motion, we refer to John (1950), Wehausen (1971), Yeung (1982), Newman (1977,1983), or Huang (1987). Part of Wehausen (1971), as well as the derivation of the equations of motion are reproduced in appendix A1.

The floating body is modeled with six degrees of freedom, representing the three displacements of the center of gravity and the three rotations about fixed axes.

We will define the unknowns as $(x_1,x_2,x_3,x_4,x_5,x_6) = (x_i)_{i=1,6}$

Usual assumptions are that the surrounding fluid is inviscid and incompressible, surface tension is negligible, and the flow is initially irrotational[1]. The problem is then formulated in terms of two unknown functions, the velocity potential $\phi(x,y,z,t)$ and the equation of the free surface. At every instant t, the position of the free surface is given by

$$F(x,y,z,t) = 0. \qquad (2\text{-}2\text{-}1)$$

The resulting boundary value problem can be written in the form:

$$\Delta\phi = 0 \text{ in the fluid domain} \qquad (2\text{-}2\text{-}2)$$

$$\phi_t + gy + \frac{1}{2}\nabla\phi.\nabla\phi = 0 \qquad (2\text{-}2\text{-}3a)$$

$$F_t + \nabla\phi.\nabla F = 0 \qquad (2\text{-}2\text{-}3b)$$

(2-2-3 a&b) are taken on the free surface $F(x,y,z,t) = 0$;

Suitable boundary condition on the bottom or at infinity (for infinite depth) are:

$\phi_y(x,-h,z,t)=0$ for a finite depth h

or

$\lim_{y=-\infty}\phi_y=0$ for infinite depth (2-2-4)

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[1]The flow may not be irrotational in regions behind the body where a thin vortex sheet may exist because of the generation of lift. In this case, an appropriate cut in the fluid domain must be chosen to render the velocity potential single-valued. The extent of this problem is beyond the scope of this review.

Also, the normal velocity on the body is equal to the normal derivative of the potential since, on the surface of the body, fluid particles cannot cross the solid boundary[1] :

$$\phi_n \mid S_{body} = V.n \qquad (2\text{-}2\text{-}5a)$$

with $$V.n = \sum_{i=1}^{6} \dot{x}_i n_i \qquad (2\text{-}2\text{-}5b)$$

Although this is a well posed problem, it is unlikely to be solved in an analytical fashion when written in this manner. Some major difficulties arise from the nonlinear terms in equation (2-2-3). Also, the free surface is an unknown, as well as the position of the surface of the body on which (2-2-5) is expressed.

The linearized equations of motion (see for example Lamb 1932) can be obtained by writing the exact equations of motion and assuming the unknowns in the form:

$$\phi(x,y,z,t)=\varepsilon\phi_1(x,y,z,t)+\varepsilon^2\phi_2(x,y,z,t)+ \ldots$$
$$F(x,y,z,t) = -y +\zeta\eta_1(x,z,t)+\zeta^2\eta_2(x,z,t)+ \ldots \qquad (2\text{-}2\text{-}6)$$

By assuming that the gradients of the potential are small and that only higher order terms can be neglected[2], the equations are modified and yield (we drop the index 1 for the first order term which becomes our new unknown potential ϕ):

$$\phi_{tt}(x,0,z,t)+g\phi_y(x,0,z,t) = 0 \qquad (2\text{-}2\text{-}7)$$

taken on the undisturbed water surface y=0.

The equation of the free surface becomes $y=\eta(x,z,t)$ and

$$\eta(x,z,t) = -\frac{1}{g}\phi_t(x,0,z,t) \qquad (2\text{-}2\text{-}8)$$

The validity of such assumptions will be discussed later. A consequence of these assumptions is that the free surface is now given by y=0 and that the quadratic term has disappeared in (2-2-3).It is interesting to note that with this description, y is defined unequivocally. This doesn't allow for example for jets or plungers. Also, equation (2-2-5) is expressed on the undisturbed (or initial) position, S_0, of the floating body :

$$\phi_n \mid S_0 = V_n \qquad (2\text{-}2\text{-}9)$$

The hydrodynamic forces and moments about G can be written in the form:

[1] The expression of the normal vector n under linear assumptions is listed in appendix A1.

[2] The significance of ε and ζ will be outlined when considering perturbation techniques to solve the nonlinear problem. In the case of the linearized problem of a floating body, $\varepsilon(=\zeta)$ measures the "smallness" of the amplitude of oscillation.

$$F_H = \int_{S_0} p \,.\, n \, dS$$

$$K_H = \int_{S_0} p \, (r - r_G) \times n \, dS$$

(2-2-10)

The reaction of the fluid is expressed through the pressure acting on the surface of the body. The integrals are taken over the undisturbed position. This is a feature of the linea approximation and is a consequence of the assumed small motions. If large motions are considered, one must consider the complete equations of motion and both referentials (see Huang 1987).

The value of the pressure on the surface of the body can be derived by using Euler's equation:

$$p + \rho \, \phi_t + \rho g y + \frac{1}{2} \rho \, | \nabla \phi |^2 = 0$$

(2-2-11)

if $p = 0$ at $y = 0$.

The linearization assumption suppresses the quadratic term so that :

$$p = - \rho \phi_t - \rho g y$$

(2-2-12)

The contribution to the hydrodynamic forces are of two kind.The second term of the RHS of (2-2-12) is a purely hydrostatic term while the other is a dynamic term.

Using this expression for the pressure, one can write the equations of motion in the form:

$$m_{ij} \ddot{X}_j = - c_{ij} x_j - \int_{S_0} \rho \phi_t n_i \, dS + (F_{ext})_i \qquad (2\text{-}2\text{-}13)$$

The coefficients, following notations introduced by Ogilvie (1964), and specified in Wehausen (1971), can be found in appendix A1.

It is interesting to note that the terms c_{ij} x_i correspond to a hydrostatic contribution, therefore, the c_{ij} coefficients only depend on the static equilibrium of the body in calm water.

Again, it needs to be specified that the relations are valid for small angular displacements, provided the form of the hull underwater doesn't change appreciably

(Saint-Denis 1974). The hydrostatic terms are restoring terms. Saint-Denis gives an example of how in extreme conditions, when the position of the hull is not close to being constant, the values of these coefficients are off. As a consequence, since most hulls are designed using these coefficients, the operations of the platform should be limited, to be safe, to operations in moderate seas.

2-2-2 Separation of the Potential and Radiation Condition

It is useful at this point to separate the potential into three potentials:

$$\phi = \phi_I + \phi_D + \phi_F \tag{2-2-14}$$

ϕ_I defines a wave train in the absence of a hull. ϕ_D describes how the presence of the hull disturbs the incident wave train. The other effect of these potentials is that they will set the body into motion thus generating additional waves, coming out in all directions and corresponding to the potential ϕ_F or forcing potential. John (1950), points out that the decomposition of the velocity potential into three potentials is unique and comes naturally when applying Green's theorem to ϕ and to a suitable surface. The boundary conditions on the body become with this notation:

$$(\phi_D + \phi_I)_n \big|_{S_0} = 0 \quad \text{and} \quad \phi_{Fn} \big|_{S_0} = V_n \tag{2-2-15}$$

In addition, ϕ_D and ϕ_F must satisfy a radiation condition guaranteeing that the waves are outgoing and have proper amplitude and behavior at infinity. The mathematical equivalent of this physical condition is to find adequate behavior at infinity for the problem to have a unique solution. The form of this condition depends both on the formulation of the problem and on the nature of the body. This result was first pointed out by Sommerfeld.

1) For an initial value problem, it is sufficient to say that ϕ,ϕ_t,and $\nabla\phi$ be bounded.

2) For motions generated by an incident monochromatic wave[1] in which transients have died out, the appropriate radiation is not known for all cases. Two important cases are as follow:

[1]The importance of periodic excitation and response will become apparent later when transient motion and periodic motion are related by using the impulse response function. Again it is useful to keep in mind that this feature comes from the principle of superposition which is a consequence of the linear equations for the boundary value problem.

ϕ_I is a monochromatic wave with $\omega^2 = gk \tan kh$
and

ϕ_I, ϕ_D, ϕ_F are of the form $\phi = \text{Re}\ \varphi(x,y,z)\, e^{-i\omega t}$

(2-2-16)

a) If the fluid has a constant depth h and the body is bounded then ϕ_D and ϕ_F must satisfy

$$\lim_{R \to \infty} R^{\frac{1}{2}} [\varphi_R - ik\varphi] = 0$$

where $R = x^2 + z^2$

(2-2-17)

b) a consequence of the previous relation is the expression for problems in two dimensions:

$$\lim_{x \to \pm\infty} (\varphi_x \mp ik\varphi) = 0$$

(2-2-18)

Radiation conditions are not known for every geometric configuration. If a problem is solved in which the radiation condition is not known, by formulating it as an initial value problem and then finding the asymptotic form of the solution as $t \to \infty$, the correct solution will be found. However this method will not yield the correct radiation condition for the steady-state problem. This approach is used when numerical methods depend on what the behavior at infinity is. The free surface condition can be used to advance the solution in time. It should be noted that in the case of nonlinear problems, this approach may yield the wrong results since the solution to these problems are dependent on the initial conditions.

2-2-3 Uniqueness and Existence of a Solution to the Linear Problem

For initial value problems, uniqueness proofs have been given by Volterra (1934), John (1949), Finkelstein (1957), Wehausen (1967), Garipov (1967), and Beale (1977) .

For the steady state time harmonic problem of a freely floating body, John (1950) proved that the solution is unique provided the frequency is sufficiently large and a vertical line intersects the submerged surface of the body only once. If the body moves with the frequency of the incident wave, the restriction of large frequency is not necessary. Other uniqueness theorems are given by Kreisel (1949) for the diffraction due to floating bodies, Ursell (1947) and more recently Lenoir and Martin (1981).

Beale (1977), gave a proof of existence of a solution to the initial value problem for finite depth using a semigroup theory. Garipov (1967) for finite depth and Jami (1981) for infinite depth used another approach to derive their proofs.

For the steady-state time harmonic problem, John (1950) proves the existence of a solution if the body intersects the free surface perpendicularly. Kreisel (1949) established existence theorems for two dimensional bodies. Ursell (1949a, 1953) in his derivation of a half-submerged cylinder showed existence for this particular case.A more general existence proof was given by Lenoir and Martin (1981).

2-2-4 Hydrostatic Forces

When neglecting the time dependent term ϕ_t in the linearized equations of motion, only the hydrostatic forces exerted on a body are computed and a set of differential equations for a floating body in calm water are obtained.

2-2-5 Froude-Krylov Forces

Taking $\phi = \phi_I$ and neglecting ϕ_D and ϕ_F in the equation of motion yields what is coined the Froude-Krylov theory by naval architects. This approach is used when dimensions of the structure are small compared to the incident wave length and when inertia terms are predominant. It is assumed that the structure does not affect the wave field. The force exerted on the structure is obtained by integrating the pressure in the field, computed at points occupied by the structure. Of course, this is an approximation since ϕ_D and ϕ_F do not exactly equal zero.In some cases, a correction to the force computed in this manner can be made in the form of a force coefficient and this method provides a simple way of computing the hydrodynamic forces. For further reading one can refer to Chakrabarti (1987).

2-3 STEADY STATE, TIME-HARMONIC MOTION.

2-3-1 Derivation and Definition of the Added-Mass and Damping Coefficients

The added mass and damping coefficient come as natural definitions when one considers the response of a floating body to a monochromatic wave excitation ϕ_I. When transient motion dies out, it is legitimate to assume that the motion of the body can be written in the form:

$$x_k = \mathrm{Re}\, a_k\, e^{-i\omega t} \tag{2-3-1}$$

where the a_k are unknown in the case of a monochromatic wave or prescribed in the case of forced oscillation.

A solution for the potential ϕ_F can then be sought in the form:

$$\phi_F = \varphi_1 \cos \omega t + \varphi_2 \sin \omega t \tag{2-3-2}$$

This decomposition is of course not unique. Wehausen (1971) looks for an equivalent solution with a term in phase with the velocity and a term in phase with the displacement :

$$\phi_F = \sum_{k=1}^{6} \dot{x}_k(t)\, \varphi_{1k}(x,y,z) + \omega x_k(t) \varphi_{2k}(x,y,z) \tag{2-3-3}$$

or

$$\phi_F = \sum_{k=1}^{6} \mathrm{Re}\, -i\omega a_k \varphi_k e^{-i\omega t} \qquad \text{and } \varphi_k = \varphi_{1k} + i\varphi_{2k} \tag{2-3-4}$$

Then the ϕ_k must satisfy the following equations:

$$\Delta\varphi_k = 0 \tag{2-3-5}$$

$$\varphi_k(x,0,z) - \frac{g}{\omega^2}\varphi_{ky}(x,0,z) = 0 \tag{2-3-6}$$

$$\varphi_{1k,n}\,|_{S_0} = n_k \qquad \varphi_{2k,n}\,|_{S_0} = 0 \tag{2-3-7}$$

If the φ_k exist, then ϕ_F satisfies all the conditions imposed upon it.

With these notations, the hydrodynamic force X_{HF} can be written:

$$X_{HFi} = -\int_{S_0} \rho \phi_t n_i \, dS$$

$$= -\Big[\sum_{k=1}^{6} [\ \ddot{x}_k \int_{S_0} \rho \varphi_{1k} n_i \, dS + \omega \dot{x}_k \int_{S_0} \rho \varphi_{2k} n_i \, dS] +$$

$$+ \int_{S_0} \rho (\phi_{It} + \phi_{Dt}) n_i \, dS \Big] \qquad (2\text{-}3\text{-}8)$$

The following quantities can be defined:

$$\mu_{ik} = \int_{S_0} \rho \varphi_{1k} n_i dS = \int_{S_0} \rho \varphi_{1k} \varphi_{1i,n} dS \qquad (2\text{-}3\text{-}9)$$

$$\lambda_{ik} = \omega \int_{S_0} \rho \varphi_{2k} n_i dS = \omega \int_{S_0} \rho \varphi_{2k} \varphi_{1i,n} dS \qquad (2\text{-}3\text{-}10)$$

These constants depend on the geometry of the body and on the frequency ω. The first term on the right hand side of the expression giving the hydrodynamic reaction will yield a force a 180 degrees out of phase with the body acceleration and the second term will yield a force a 180 degrees out of phase with the body velocity. It can be noted at this point that there exist many "literal" definitions of the added mass and damping term. This comes from the many possibilities to take a sum of the two independent functions $\cos\omega t$ and $\sin\omega t$ in (2-3-3)[1]. Different expressions will yield different results and definitions for the added mass and damping. Whatever the choice, the hydrodynamic force comprises two terms, one which is related to the acceleration multiplied by a coefficient with dimensions of a mass, and another which is related to the velocity multiplied by a coefficient with dimensions of a damping term. It should be pointed out that different authors get different expressions for the added mass and damping and subsequently derive expressions which are consistent with their definitions but different in form (Kotik and Mangulis 1962; Greenhow 1986; Chakrabarti 1987). The added mass and damping forcing terms are usually non dimensionalized by the amplitude of the acceleration of the body multiplied by the volume of displaced water.

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[1]Saint-Denis (1974) takes the expression $\phi = \mathrm{Re}\,(\psi_k e^{i\omega t})$ which leads to a damping term in quadrature with the velocity, and a different expression of the added mass and damping coefficient. The φ_k and ψ_k can of course be easily related.

Because the flux of energy outward through any control volume must be positive, the matrix formed with the coefficients λ is definite positive. The same cannot be said for the matrix formed with the μ coefficients. Ogilvie (1963) gives an example of negative coefficients. If ω is sufficiently small, then the matrix of μ coefficients is definite positive.[1]

It is interesting that the λ_{ik} are sensitive to the asymptotic behavior of the φ_k while the μ_{ik} reflect the local behavior of the φ_k and the details of the body geometry. Kotik and Mangulis (1962) and then Kotik and Lurye (1964) derived a relation between the coefficients for heaving cylinders. This was generalized by Ogilvie (1964). An equivalent relation existed in the field of acoustics and these relations are known as the Kramer-Kronig relations[2]:

$$\mu_{ij}(\omega) - \mu_{ij}(\infty) = \frac{2}{\pi} \, \mathrm{P.V.} \int_0^\infty \lambda_{ij}(\alpha) \, \frac{d\alpha}{\alpha^2 - \omega^2} \tag{2-3-11}$$

$$\lambda_{ij}(\omega) = -\frac{2}{\pi} \, \omega^2 \; \mathrm{P.V.} \int_0^\infty [\, \mu_{ij}(\alpha) - \mu_{ij}(\infty) \,] \, \frac{d\alpha}{\alpha^2 - \omega^2} \tag{2-3-12}$$

The P.V. denotes principle value integrals (see for example Dettman pp.109-112, or a step by step derivation in Frank 1967).
Kotik and Lurye derived a relationship for the μ :

$$\int_0^\infty [\, \mu_{ij}(\omega) - \mu_{ij}(\infty) \,] \, d\omega = 0 \tag{2-3-13}$$

Further use of the Kramers-Kronig relations was made by Athanassoulis and Kaklis (1987) and some relations are given in Athanassoulis, Kaklis and Politis (1988).

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[1]The coefficients defined in the previous equations are defined with respect to G. They can easily be defined with respect to O. If they are defined with respect to O, the only differences are:

$$\mu_{4k} = \mu'_{4k} - y_G \mu'_{3k} \quad \text{and} \quad \mu_{6k} = \mu'_{6k} + y_G \mu'_{1k}$$ where the prime

refer to the quantities when they are defined with respect to O. Similar relations hold for the λ 's.

[2]These relations take different expressions, depending on the choice of the definition for the added mass and damping in equations 2-3-3 and consequently 2-3-9, 2-3-10. See (Kotik and Mangulis 1962, Greenhow 1986)

2-3-2 The Exciting Forces and Moments

These terms are given by the integral depending on the time derivative of ϕ_D and ϕ_I in expression (2-3-8). Note that to determine the μ and λ, one can simply consider forced periodic motion of the body, in which case the hydrodynamic exciting force and moments are null. Haskind (1957) pointed out that if only the body motion is of interest and ϕ_F has been determined, it is not useful to solve for ϕ_D. One can consider the region limited by the body surface So, a large vertical cylinder Σ_R of radius R, and the bottom surface B and free surface F enclosed within Σ_R. Within this region, ϕ_D and φ_k are harmonic. Therefore applying Green's equality we get:

$$\int_{S_0+B+F+\Sigma_R} (\phi_D\varphi_{kn} - \phi_{Dn}\varphi_k)\, dS = 0 \qquad (2\text{-}3\text{-}14)$$

Due to the boundary conditions on B, F and the radiation condition on Σ_R, the only non zero integral is that on the surface So:

$$\int_{S_0} \phi_D\, n_k\, dS = \int_{S_0} \phi_{Dn}\, \varphi_k\, dS \qquad (2\text{-}3\text{-}15)$$

$$= - \int_{S_0} \phi_{In}\, \varphi_k\, dS \qquad (2\text{-}3\text{-}16)$$

from the boundary condition. (2-2-15) on S_0

The components of the hydrodynamic exciting force and moment (due to ϕ_I and ϕ_D) X_{IDk} can then be evaluated:

$$X_{IDk} = - \int_{S_0} \rho\, (\phi_{It}+\phi_{Dt})\, n_k\, dS \qquad (2\text{-}3\text{-}17)$$

$$X_{IDk} = - \mathrm{Re}\, -i\omega e^{-i\omega t} \int_{S_0} \rho\, (\phi_I+\phi_D)\, n_k\, dS \qquad (2\text{-}3\text{-}18)$$

$$X_{IDk} = \mathrm{Re}\, i\omega e^{-i\omega t} \int_{S_0} \rho\, (\phi_I\varphi_{kn} - \phi_{In}\varphi_k)\, dS \qquad (2\text{-}3\text{-}19)$$

$$X_{IDk} = \mathrm{Re}\, -i\omega e^{-i\omega t} \int_{\Sigma_R} \rho\, (\phi_I\varphi_{kn} - \phi_{In}\varphi_k)\, dS \qquad (2\text{-}3\text{-}20)$$

The form (2-3-20) also follows from Green's theorem. Its advantage is that an asymptotic expression of φ_k can be used. This relation was used by Haskind (1957), Newman (1962), Kim (1969a) and Vugts (1968a) and has been implemented in numeral codes to avoid computing the force due to the diffraction potential. Asymptotic relations can be derived which can simplify the computation of μ and λ.

Other relations make use of the asymptotic representations of functions like ϕ_D or φ_k. These have been investigated in two and three dimensions by Kochin and Haskind, and in three dimensions by John (1950; see also Newman 1962; Kochin et al. 1964; Wehausen 1971;).

2-3-3 Computation of the Coefficients

There exist several ways to compute the coefficients.

1) The most efficient way is to find an analytical solution to the boundary value problem. The results are limited to bodies of simple shapes, most often in an infinite fluid (see references in Saint-Denis 1974; Sarpkaya and Isaacson 1981). Explicit solutions when a free surface is present are known only for the case of a swaying or rolling vertical plate (Kotik 1963). Most often, only a few of the 36 coefficients are given.

2) A second method is to represent each of the functions φ_k as the sum of a source function and multipoles, placed at the origin. This idea was originated by Havelock (1928) in an infinite fluid. The potentials for the singularities are first adjusted by adding to the fundamental singularity an infinite series of wave-free potentials, so that the resulting function satisfies the free surface condition, the bottom condition and the radiation condition. The strength of the singularities are then adjusted to satisfy the boundary conditions on the body. For two dimensional motion, Ursell (1950) proved that this expansion is possible. For 3 dimensions, the weak point of this method is that the condition on the surface of the body will hardly be satisfied since the singularities can only represent but a limited number of hull shapes. Approximate hull shapes are substituted to real ship shapes when this method is used. Determination of the coefficients in the multiple series requires solving an infinite set of linear equations which must be approximated. After Wehausen and Laitone (1960, p. 479), some typical functions in two dimensional problems are $\log(r)\cos(\omega t)$, $r^{-n}\cos(n\theta)\cos(\omega t)$, $r^{-n}\sin(n\theta)\cos(\omega t)$ (for $n=1,2,\ldots$ and $x = r\cos\theta$, $y=r\sin\theta$; see also Thorne 1953). The use of a series of multipoles to represent the potential for forced motion with a free surface was initiated by Ursell (1949 a and b) who considered the heaving motion of a cylinder, half-submersed in infinitely deep water. The method was further developed by he and extended to other shapes, and to water of finite depth by others. The shapes are primarily those that can be obtained from conformal mapping of a circle, using Joukovski-type mappings of the form:

$$z = \xi + \frac{b_1}{\xi} + \frac{b_3}{\xi^3} + \ldots \text{ the } b_i \text{ being real numbers} \qquad (2\text{-}3\text{-}21)$$

They are called Lewis shapes. The most extensive use of this procedure for calculating μ, λ and the relative wave amplitude at infinity has been made by Tasai (1961) and Vugts(1968a) for infinite depth. In this manner Landweber and Macagno (1957, 1959) for the heaving and swaying motion and, Landweber (1979) and Hsu and Landweber (1979) for the rolling motion derived the added mass coefficients. Porter (1960,1966) computed these quantities for heave in finite and infinite water. For finite depth, Yu and Ursell (1961) computed the heaving motion of a circular cylinder. C.H. Kim (1969) has extended this to all three modes of motion in finite depth and to the sections of ship like shapes.

Another assumption can be added to the initial small motion assumption; if the hull is assumed to be slender, the problem can be simplified by using the results derived in two dimensions. The so-called strip theory is used, assuming that the flow is everywhere in planes normal to the axis of slenderness. The hydrodynamic parameters for the three dimensional case are obtained by simple integration along the axis of slenderness of the results derived from the two dimensional flow, around a continuous sequence of colinear cylinders of arbitrary but slowly varying shape. The hydrodynamic inertia and wave damping coefficients at each section are obtained by the suitable conformal mapping of the flow around a simple body. Some results have been discussed by Newman (1970) and Saint-Denis (1974) This method will not reflect the characteristics of a three dimensional flow.

For hull sections with sharp corners, a more convenient transformation is the Schwartz-Christoffel transformation. A list of the hydrodynamic coefficients derived with this method can be found in Saint-Denis (1974).

Grim (1960) related both methods to circumvent the limitations of the singularity distribution methods and the strip theory methods. The singularities are first designed to reproduce faithfully the two dimensional flow at the hull's section. They are then modified to reflect the three dimensional character of the flow. This step is taken to satisfy to a high degree, the relationship that must exist between the shape of the hull, the frequency of oscillation and the distribution of singularities.

Havelock (1955), calculated the wave motion resulting from heaving oscillations of a half submerged sphere in infinite water extended the method of multipoles to three dimensions.Wang (1966), studied this motion in finite depth. Hulme (1982) simplified and generalized Havelock's method. Evans and Mc Iver (1984) used this approach to study the heaving of a sphere with an opening. Again it is reminded that this method is limited to shapes, such as spheres, obtained from these distributions.

3) Another procedure derives from the integral equation satisfied by the φ_k. This equation can be obtained by using a suitable Green function. If $G(x,y,z,\xi,\eta,\zeta)$ is a function which is harmonic in the lower half-plane (half-space) and such that G satisfies the free surface, bottom, and radiation conditions, then applying Green's theorem on a surface defined previously with points P and Q on the surface S_0 of the body, it can be shown that the φ_k satisfy equation:

$$2\pi\, \varphi_k(P) + \int_{S_0} \varphi_k(Q)\, G_V(P,Q)\, dS_Q = \int_{S_0} n_k(Q)\, G(P,Q)\, dS_Q \qquad (2\text{-}3\text{-}22)$$

in 3 dimension (in 2-D 2π becomes π).[1]

The values of the Green functions can be taken to suit the particular problem considered. Examples are given in Wehausen and Laitone (1960, pp.477-483). This technique serves as the basis of the numerical methods known as boundary integral methods. The integral equation is then discretized according to one of several procedures. Approximate values of the φ_k at a finite set of points on S_0 are found. Approximate values of the λ and μ are then computed. Frank (1967) for example makes use of the integral equation method to successfully compute the λ's and the μ's for a number of ship like sections. The hull contours are approximated by a variable number of segments. Wang and Wahab (1971) use this method to analyze the heaving oscillations of twin cylinders.

4) Other methods exist: Some rely on variational techniques and will lead to the finite element techniques; Results can also be derived when damping coefficients are known, by using equation (2-3-19) (see Wehausen 1971 for further reading). These methods along with the practical use of the Green Function will be described in the chapter on numerical methods.

2-3-4 Solving the Problem

Once these coefficients have been determined, the equations of motion can be written in the form:

$$(m_{ik} + \mu_{ik})\, \ddot{x}_k + \lambda_{ik}\dot{x}_k + c_{ik}\, x_k = X_{IDk} \qquad (2\text{-}3\text{-}23)$$

Assuming that $x_k = a_k\, e^{-i\omega t}$

then we get:

$$[-\omega^2\, (m_{ik} + \mu_{ik}) - i\omega\lambda_{ik} + c_{ik}]\, a_k = X_{0k} \qquad (2\text{-}3\text{-}24)$$

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[1] $G_v = [\, n_1(Q)\frac{\partial}{\partial\xi} + n_2(Q)\frac{\partial}{\partial\eta} + n_3(Q)\frac{\partial}{\partial\zeta}\,]\, G$

This can be solved for x_k $k = 1,6$ and therefore, the behavior of a floating body in a monochromatic wave is known.

2-3-5 Comparison with Experiments

Comparisons of these theoretical predictions with experiments can be made in three different manners. The experiments measure pressure and force on the bodies as well as wave amplitudes. They have been carried out by several researchers, some of whom are mentioned here (see Wehausen 1971 for further references):

1) Forced waves in calm water: Porter (1960) for a heaving circular cylinder and later Paulling and Porter (1962) for cylinders of ship-like sections; Yu and Ursell (1961) for the amplitude waves generated by a heaving circular cylinder; Vugts (1968a) for circular, triangular and rectangular cylinders computed the four, two dimensional coefficients. Van Oortmesen (1974) considered a swaying ship-shape section.

2) Fixed bodies in incident waves: Vugts (1968a); Dean and Ursell (1959) considered a plane wave on a half submerged cylinder.

3) Freely floating body in incident wave: Vugts (1968b) considered a rolling and swaying cylinder of rectangular section.

These results compare well with linear theory in the limitations of the assumptions made.

2-4 INITIAL VALUE PROBLEMS

2-4-1 Introduction

Assuming that the motion of the body is known at an instant which can be set as t=0, one wishes to find what the subsequent motion of the floating body is. The equations of motion in their linearized form are still valid but initial conditions need to be specified, i.e. $\phi_F(x,y,z,0)$, $\phi_{Ft}(x,y,z,0)$, $\phi_D(x,y,z,0)$, $\phi_{Dt}(x,y,z,0)$, $x_k(0)$, $\dot{x}_k(0)$ are known. ϕ_I is still a known function. There are two equivalent ways to approach this problem:

The first is to work in the physical time-space frame, making use of a time dependent Green function.

The other is to take a Fourier transform with respect to time t, of all the equations and boundary conditions. The variable becomes ω. This results in having to solve the steady state time-harmonic problem which was addressed in the previous paragraph. The interconnectivity of these approaches in ship hydrodynamics is emphasized by Cummins (1962) and later by Bishop, Burcher and Price (1973). Both methods are equivalent, since when solving with Green's functions, one will usually resort to a Laplace or a Fourier transform to solve the final set of integro-differential equations.

The approach using Green's function has been used by Volterra (1934), Finkelstein (1957), Cummins (1962), Wehausen (1967), Chung (1982), Yeung (1982b), Beck and Liapis (1987). The other approach has been used by Ursell (1964) for vertical motion of a cylinder, extended with numerical work by Maskell and Ursell (1970). Kotik and Lurye (1964) and Motora et al. (1971) considered the swaying motion of cylindrical sections. Kotik and Lurye (1968) analyzed the heaving motion of a sphere in the frequency domain.

For the first method, the first step is to derive a time dependent Green function and an integral equation for the potential ϕ. This also serves as a basis for numerical work[1]. Examples of such functions suitable for specific problems are given in Laitone and Wehausen (1960, pp.490-95), Finkelstein (1957), Chung (1962), Daoud (1975) and Huang (1987). Wehausen (1967) and Yeung (1982b) derive the equations of motion in this manner. [2]

2-4-2 Derivation of the Equations for Transient Motion

Cummins derived these equations in a somewhat different manner (1962) which is an elegant way of using physical considerations and the principle of superposition. The potential Θ corresponding to a unit impulse displacement at time t is decomposed into two potentials:

[1] See section on boundary integral in numerical methods
[2] This work is reproduced in appendix 2

$$\Theta_k(P,t) = \psi_k(P)\delta(t) + \chi_k(P,t) \tag{2-4-1}$$

The physical interpretation of these two potentials is as follows:

$\psi_k(P)$ is the initial reaction to the impulse at t=0 of the fluid. This potential, corresponding to an impulse satisfies the boundary condition $\psi_k(P) = 0$ on the surface y=0, and goes to zero at infinity. It also satisfies the body boundary condition :

$$\frac{\partial \psi_k}{\partial n} = n_k \tag{2-4-2}$$

signifying that the body is moving with a unit velocity in the k direction (see Batchelor p. 471 or Sedov p. 165 for the formalism and definition of an impulsive flow).

This impulsive flow causes an elevation of the free surface.

$\chi_k(P,t)$ represents the motion of the fluid subsequent to the impulsive phase. It is useful to note that $\chi_k(P,0) = 0$. Waves have had time to form and will dissipate but the body is no longer moving. The boundary conditions are therefore:

$$\frac{\partial \chi_k}{\partial n} = 0 \text{ on the body and } \chi_{k,tt} + g\,\chi_{k,y} = 0 \text{ on the free surface } y = 0 \tag{2-4-3}$$

Superimposing impulsive flows for successive times, of magnitude such that the velocity at time t is the actual velocity, the solution to the boundary value problem is:

$$\phi_F(P;t) = \int_{-\infty}^{t} \dot{x}_k(\tau)\,\Theta_k(P;t-\tau)\,d\tau = \int_{0}^{t} \dot{x}_k(\tau)\,\Theta_k(P;t-\tau)\,d\tau \text{ since } \dot{x}_k = 0 \text{ for } t \le 0^1 \tag{2-4-4}$$

Cummins obtains the exerted force, which is the expression which is traditionally used.

$$X_{Fi} = \int_{S_0} -\rho\phi_{Ft}(P;t)n_i dS = \int_{S_0} n_i dS \, -\rho \frac{d}{dt}\left[\int_0^t \dot{x}_k(\tau)\,\Theta_i(P;t-\tau)\,d\tau \right] \tag{2-4-5}$$

$$= -\mu_{ik}(\infty)\,\ddot{x}_k(t) - \int_0^t \dot{x}_k(\tau)R_{ik}(t-\tau)d\tau \qquad \text{since } \chi_k(P,0) = 0 \tag{2-4-6}$$

$$\text{and } \mu_{ik}(\infty) = \int_{S_0} \rho\psi_k(P)n_i dS \text{ and } R_{ik}(t) = \int_{S_0} \rho\chi_{kt}(P;t)n_i dS \tag{2-4-7}$$

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[1] Petersen (1982), gives a slightly different expression, with $\dot{x}_k = \dot{x}_0$ for $t \le 0$

In the most general case, the equation of motion can be written:

$$(m_{ik} + \mu_{ik}(\infty))\, \ddot{x}_k + \int_0^t \dot{x}_k(\tau) R_{ik}(t-\tau)\, d\tau + c_{ik}\, x_k = X_{IDk} + X_{ext} \qquad (2\text{-}4\text{-}8)$$

where X_{IDk}, and X_{ext}, correspond to the forces and moments respectively associated with, $\phi_I + \phi_D$, and the exterior exciting force.

We notice that $R_{ik}(t)$ does not depend on the motion velocity or acceleration. If the motion of the structure is sinusoidal, the above equation should turn into a frequency domain equation.

Assuming that

$$x_k = \mathrm{Re}\, a_k\, e^{-i\omega t} \qquad (2\text{-}4\text{-}9)$$

and that the motion is started for $t = -\infty$, so that the initial condition effect has disappeared, one gets :

$$X_{Fi} = -\mu_{ik}(\infty)\, \ddot{x}_k(t) - \int_{-\infty}^{t} \dot{x}_k(\tau) R_{ik}(t-\tau)\, d\tau = -\mu_{ik}(\infty)\, \ddot{x}_k(t) - \int_0^{\infty} \dot{x}_k(t-\tau) R_{ik}(\tau)\, d\tau \qquad (2\text{-}4\text{-}10)$$

$$X_{Fi} = \left[-\mu_{ik}(\infty) - \frac{1}{\omega}\int_0^{\infty} \sin(\omega\tau) R_{ik}(\tau)\, d\tau \right] \ddot{x}_k(t) - \left[\int_0^{\infty} \cos(\omega\tau) R_{ik}(\tau)\, d\tau \right] \dot{x}_k(t) \qquad (2\text{-}4\text{-}11)$$

So we get the simple relationships by comparing with equation 2-3-23:

$$\mu_{ik}(\omega) = \mu_{ik}(\infty) - \frac{1}{\omega}\int_0^{\infty} \sin(\omega\tau) R_{ik}(\tau)\, d\tau \qquad (2\text{-}4\text{-}12)$$

$$\lambda_{ik}(\omega) = \omega \int_0^{\infty} \cos(\omega\tau) R_{ik}(\tau)\, d\tau \qquad (2\text{-}4\text{-}13)$$

Therefore, $\omega^{-1}(\mu_{ik}(\omega) - \mu_{ik}(\infty))$ and $\lambda_{ik}(\omega)$ are the sine and cosine transforms of $R_{ik}(t)$. Inversely, knowing the values of $\omega^{-1}(\mu_{ik}(\omega) - \mu_{ik}(\infty))$ and $\lambda_{ik}(\omega)$, it is possible to determine $R_{ik}(t)$.This is the method used in van Oortmessen (1974) or Petersen (1982). From this relationship, the Kramers-Kronig relations can be derived as was pointed out by Ogilvie (1964). Haskind-type relations can also be derived to avoid computing ϕ_D if the φ_k are known (Wehausen 1967). The inverse Fourier tranforms are given by:

$$R_{ik}(\tau) = \frac{2}{\pi}\int_0^\infty \cos(\omega\tau)\lambda_{ik}(\omega)d\omega \tag{2-4-14}$$

$$R_{ik}(\tau) = \frac{2}{\pi}\int_0^\infty [\mu_{ik}(\infty) - \mu_{ik}(\omega)]\ \omega\sin(\omega\tau)d\omega \tag{2-4-15}$$

These relationships are widely used as we will see in the following chapters. It should be noted that different authors use different approaches which are somewhat better suited for their problems. Beck and Liapis (1987) start out with a different expression for the potential in (2-4-1):

$$\phi_F(P;t) = \int_0^t \dot{x}_k(t-\tau)\ \Theta_k(P;\tau)\ d\tau \tag{2-4-16}$$

This expression and that of Cummins are equivalent in their formulation provided that $\dot{x}_k(0) = 0$ as a simple change of variable shows. Beck and Liapis apply Green's theorem to ϕ and not ϕ_t. This is not as general as the case presented by Cummins or Wehausen. With this assumption they get:

$$X_{Fi} = - \mu_{ik}(\infty)\ \ddot{x}_k(t) - \int_0^t \ddot{x}_k(t-\tau)T_{ik}(\tau)d\tau \tag{2-4-17}$$

$$\mu_{ik}(\infty) = \int_{S_0} \rho\psi_k(P)n_i dS \ \text{ and } T_{ik}(t) = \int_{S_0} \rho\chi_k(P;t)n_i dS \tag{2-4-18}$$

The equivalence between the Cummins and the Beck and Liapis derivation (i.e. a relationship between $T_{ik}(t)$ and $R_{ik}(t)$) is found by integrating by parts but again using the fact that $\dot{x}_k(0) = 0$.

Petersen (1982) uses the same approach as Cummins and derives the equations of motion in the case where the ship has a steady non zero velocity for $t < 0$.

It is interesting to note that $\mu_{ik}(\infty)$ is a constant which depends only on the shape of the body. $R_{ik}(t)$ (or $T_{ik}(t)$, or $L_{ik}(t)$[1]) contains the memory of the fluid response. It is a function of time and of the system geometry but it is independent of the past history of motion.

This extensive paragraph was developed to show that the impulse potential can be derived in different manners. The resulting equations, giving the frequency dependent added mass or damping coefficients, and the impulse response functions will therefore be

[1] $L_{ik}(t)$ is the impulse response function derived in Appendix 2.

linked to the choice of the expression of the hydrodynamic force exerted on the floating body. This starting hypothesis is not always clearly specified in the literature and some care should be used when integrating the linearized equations of motion of a floating body.

2-4-3 Comparison with Experiments

Experiments are scarce for the study of transient motion. Ito (1977) computed the transient heaving motion of different cylindrical shapes. Bayley, Griffiths and Maskel (1976), and Beck and Liapis (1987) considered the heaving of a sphere. Most often, frequency-dependent coefficients are computed as presented in the previous section. The impulse response function is computed using this experimental data and then compared to experiments. This will be described in the following chapter. Also, time-stepping numerical schemes are usually used to model transient motion and compared to experiments. This topic will be addressed in the section on numerical methods. Some experiments have been carried out to validate second order approaches and are presented in the section on nonlinear effects.

2-5 THE ZERO AND INFINITE FREQUENCY COEFFICIENTS

When ω is null or infinite, the form of the linearized boundary value problem to solve for ϕ_F is modified. The free surface boundary condition becomes:

$$\phi_{Fy}=0 \qquad \text{for } \omega=0 \qquad (2\text{-}5\text{-}1a)$$

$$\phi_F=0 \qquad \text{for } \omega=\infty \qquad (2\text{-}5\text{-}1b)$$

The first case corresponds to a body moving in an infinite fluid or a steady state motion and the second case to a body impulsively started from rest (Batchelor p.273). Looking for a solution in the form:

$$\phi_F=\sum_{k=1}^{6} U_k\varphi'_{Fk} \quad \text{and} \quad \varphi'_{Fk}=\varphi'_{1k}+i\varphi'_{2k} \qquad (2\text{-}5\text{-}2)$$

where the U_k are the given velocities; the potential for the limiting cases $\omega=0$ and $\omega=\infty$ are solutions of 2-3-5, 2-5-1a (resp. 2-5-1b), 2-2-4[1] and $\nabla\varphi'_{Fk}$ bounded at infinity. This last condition is somewhat different from the radiation problem since no energy is radiated outwards in this case. From this radiation condition and (2-5-1) one gets:

$$\varphi'_{2k}=0 \qquad (2\text{-}5\text{-}3)$$

A consequence is that the damping coefficients are null for the limiting cases. An added mass coefficient can be defined, which is related to the kinetic energy in the fluid (see

[1]The prime must be added where it applies to account for a sign difference in the definition of ϕ_F in 2-3-3 and 2-5-2.

Batchelor pp.404-407 for a steady motion and p.472 for an impulsive motion) and with some assumptions to the force exerted on the body. Define the limiting added mass which only depends on the shape of the body:

$$\mu_{ik}(q)|_{q=0,\infty} = -\int_{S_0} \rho \varphi'_{1k}(P) n_i dS \qquad (2\text{-}5\text{-}4)^1$$

As we have seen, these coefficients play an important role for transient motion and have been researched extensively. The zero frequency added mass coefficient is related to infinite fluid flows (see Saint Denis 1974, Sarpkaya and Isaacson 1981). Flagg and Newman (1971) and Chung (1982) considered a double limiting process to obtain the zero frequency of cylinders in very shallow water.For two dimensional infinite frequency added masses see Saint-Denis, Sarpkaya and Isaacson and Sedov (1965) who derived the uncoupled coefficients for a semicircle. Landweber and Macagno (1957) and Landweber (1979) also computed the infinite frequency of swaying and rolling motion of Lewis Forms. Simon (1985) obtained asymptotic values for the heave and sway of a cylinder. Bai (1977) considered bodies in a canal. The most complete theoretical and numerical work in two dimensions is by Athanassoulis, Kaklis, Politis (1988). They present a general work which enables the computation of both coupled and uncoupled coefficients. Off diagonal terms as well as added moment of inertia fo nonsymmetric bodies are given for the first time. In three dimensions, others have investigated the infinite added mass for swaying spheroids (Landweber and Macagno 1960) or a heaving and surging sphere (Hulme 1982, Simon 1985).

The usefulness of the Kramers-Kronig relations was pointed out previously. Kaklis and Athanassoulis (1987) derived other asymptotic expressions. Greenhow (1985) uses the Kramers-Kronig relations to extend Hulme's work and later generalizes this study (1986).

Relation (2-4-12) was also experimentally verified by van Oortmessen (1974).

2-6 CONCLUSION

The equations of motion to be solved when considering periodic excitation or transient motion of a floating body under linear assumptions have been derived. Solutions to these problems are related as shown when using a Fourier transform.The key to finding these solutions is to derive the so called added mass and damping terms or impulse response functions. Source and multipole expansions, conformal mapping, appropriate Green functions to derive a boundary value problem, and numerical methods can be used when no analytical methods exist, to yield results for these unknowns.

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[1] for $q=\infty$ then by comparison with 2-4-18, it is clear that $\varphi'_{1k} = -\psi_k$.

Green functions to derive a boundary value problem, and numerical methods can be used when no analytical methods exist, to yield results for these unknowns.

The linearization technique may fail to predict some important phenomena. We will now review some of the techniques which can be used to predict these phenomena and the current work on nonlinear effects when considering the motion of freely floating bodies.

3 NONLINEAR PROBLEMS

3-1 INTRODUCTION

The following assumptions have led to a simplified analysis for the motion of a floating body:

The free surface conditions are expressed on the initial position of the surface $y=0$.

The body boundary conditions are expressed on the initial position of the body.

The velocity squared terms $(\nabla\phi)^2$ are neglected in the expression of the boundary conditions on the free surface and in the expression of the pressure on the body.

Due to these assumptions, some observable effects are not apparent in the results of the linear (or first order) problem.

A number of techniques are available to predict some of these nonlinear effects and will be presented here.

3-2 NONLINEAR EFFECTS PREDICTED BY FIRST ORDER DERIVATION

Observations show that bodies freely floating in waves drift slowly in the direction of propagation. Elongated bodies turn broadside to the waves. This cannot be predicted if a solution is sought in the form (2-3-1). These time-dependent force and moment can nevertheless be derived from general momentum theorems and from first order results. This was pointed out by Haskind who computed the force and moment acting on a vertical plane barrier and later extended his work to floating bodies (see Wehausen 1971 and Newman 1967). Independently, Maruo (1960) derived the drift force in two and three dimensions and Newman (1967) extended this analysis to compute the moment. These effects are of importance in the analysis of the dynamics of moored vessels. Ogilvie (1963) analyzed these effects for a submerged cylinder. Soding (1976) used a Green approach to derive second order force on an oscillating cylinder. Kim (1967) iterated from a zero-frequency solution to study the forced heaving triangular cylinders. Grim (1977) derived approximate solutions based on low frequency assumptions for large amplitude roll.

3-3 OTHER NONLINEAR EFFECTS

The linearized equations of the boundary value problem were obtained by writing the potential and the free surface in the form (2-2-6) and by solving the first order problem. This is the first step in applying a technique which will be described here

3-3-1 The Perturbation Technique

The general feature of perturbation methods is to substitute an infinite number of linear systems to a a nonlinear problem. An expansion of the unknown quantities is sought in terms of a "small" parameter associated with the magnitude of the nonlinearities. The difficulties related to the nonlinear features of the system temporarily disappear. By matching terms of like order, simplified linear problems are substituted. The difficulty in solving for each term usually increases as the order increases. (see Van Dyke 1964; Nayfeh 1973; Kevorkian and Cole 1980 for further reading).

The convergence of the series as well as the number of terms needed to depict accurately the nonlinear effect may vary. The nonlinearities must remain weak in order to insure both convergence and an accurate prediction of the solution with a limited number of terms.

Dimensional analysis conducted before carrying out the computations is a good way to predict what the limitations of the perturbation techniques for a given problem are and what results are to be expected (Cointe and Armand 1987; Cointe, Molin and Nays 1988). Situations where perturbation techniques yield the linearized problem to first order (See appendix 1) are usually referred to as weakly nonlinear regimes (Cointe, Molin and Nays 1988) and correspond in reality to moderate wave heights and moderate wave amplitudes (Papanikolaou and Nowacki 1980). The nonlinear effects mentioned above can be taken into account in a perturbation sense where the free surface and the true position of the wetted body are written to second order, using respectively a perturbation expansion and a Taylor expansion about the initial positions.

Application to oscillatory problems.

Cointe et al consider the forced motion of a wavemaker in a tank (Cointe, Molin and Nays 1988). They show that the linearized first order free surface equations (2-2-7, 2-2-8) are valid if the acceleration of the body is much smaller than gravity . A consequence of this is $\epsilon=\zeta$ in (2-2-6). Their analysis is valid for oscillations of a body on the free surface. The small parameter in these cases is a ratio of these accelerations (Potash 1970). Second order investigations have been made. Lee (1966,1968) using multiple expansion and conformal mapping, and Parissis (1966) considered vertical oscillations of a cylinder, of ship like-sections and semicircle respectively. Potash (1970) analyzed sway, heave and roll for ship-like sections using a close-fit technique. Papanikolaou and Nowacki (1980) extended this work to cylinders of arbitrary crosssections. Kyozuka (1980) considered incoming waves in infinite depth and compared second order theory with extensive experimental data. Mc Camy (1961 and 1964) considered the motion of a heaving cylinder at shallow draft. Related to the study of the flow in the vicinity of free-surface piercing bodies, the problem of the wavemaker using perturbation techniques has received considerable attention since the work of Havelock (1928) who derived the first order solution. This was later approched in more detail by Lin (1984). Second order solutions exist for the weakly nonlinear regimes of an oscillating wavemaker in a tank (Cointe, Molin and Nays 1988). Some results derived in these studies may find applications when considering the motion of oscillating swaying bodies.

Application to transient problems.

The difficulty for these problems, as opposed to oscillatory ones, is that displacement and time are not simply related. This affects the choice of the smallness parameter which in turn affects the form of the problem to solve (Roberts 1987). The magnitude of the acceleration of the body with respect to that of gravity is also an important problem in order to determine which terms to retain in the free surface boundary condition and whether the regime is weakly nonlinear or not (Cointe, Molin and Jami 1987). The application of perturbation techniques has been widely used to model the interaction of bodies and free surface flows. Cointe and Armand (1987) studied the impact of a cylinder on the free surface completely including second order effects. Second order solutions exist for the weakly nonlinear regimes of the transient motion of a wavemaker (corresponding to a swaying motion) but not for impulsive problems[1] (Roberts 1987; Cointe, Molin and Nays 1988). Some of the results derived in these papers, especially concerning the problem of the intersection between the free surface and body can be generalized to study the transient motion of a swaying cylinder. These will be presented in chapter 5. The second order expansion to model wave-body interaction is reviewed by Ogilvie (1983).

3-3-2 The Method of Matched Asymptotic Expansion

Solutions derived in some manner (such as perturbation techniques, separation of variables or other) even for linear problem, may not be valid in the whole field. They may break down at some point where the flow is singular. Also, to simplify the problem, some physical feature on the body may be overlooked to get a solution which is valid away from the body, in an "outer domain". The flow is then examined locally by stretching the coordinates or in some other manner to obtain a solution in an "inner domain". Both solutions are matched in the limit when the domains overlap (see mentioned texts on perturbation techniques). These methods have been widely used. Lin (1984), Cointe and Armand (1987), Roberts (1987), and Cointe et al (1988) use this technique to analyze the behavior of the flow at the intersection of the body and the free surface. Simon (1985)

[1]The free surface condition becomes $\phi=0$ on $y=0$ and corresponds to the infinite frequency problem in this case.

derived asymptotic expressions for the amplitude of waves radiating due to the high frequency oscillation of floating bodies. Yeung (1981) approached the problem of a heaving vertical cylinder in this fashion. Newman, Sortland and Vinje (1984) analyze in this manner the damping of rectangular bodies close to the free surface. This work is extended by Marthinsen and Vinje (1985) to study the swaying of side by side ships with a special emphasis on the gap between both ships. Based on this study, Vinje (1987) hopes to expand this approach to compute some nonlinear forces on a berthing ship.

3-3-3. Self-Similar Flows

A flow is self-similar if "it is identical from one instant to another or from one part of the flow field to another with the exception of a change in scale". This argument may be useful to reduce the number of independent variable thus simplifying the equations. Using self-similarity arguments, nonlinear free-surface flow problems may be approached (Johnstone and Mackie 1973) and in some cases solved successfully (Cointe and Armand 1987) or for the classical problem of wedge entry. Greenhow (1987) analyzes entry related problems. Cointe, Molin and Jami (1987) show how using this techniques may simplify nonlinear impulsive problems.

3-4 EXPERIMENTS

Some experiments are geared to the study of second order effects: Vugts (1968 a and b) mentioned previously, Tasai and Koterayama (1976) and Yamashita (1977) for heaving cylinders. Also, Kyozuka (1980) compares analytical work to experiments for the motion of a cylinder in waves.

3-5 CONCLUSION

The drift force on floating bodies can be obtained from the solution of the linearized problem. Some techniques to solve flow problems have been outlined. By combining these techniques, nonlinear effects can be exhibited and higher order solutions found. These will be applied to analyze the transient motion of floating cylinders. The fully nonlinear problem can also be considered and solved using available numerical techniques. Numerical methods are usually the last step in most of the methods mentioned previously (one exception is Cointe and Armand, 1987). For perturbation theory for example, the boundary value problem that needs to be solved at every order can only be done in a numerical manner in most cases. The research on numerical methods suited for flow problems has therefore become a large part of the study of free surface flows. Features of available methods will be presented.

4 NUMERICAL METHODS IN FREE SURFACE FLOWS

4-1 INTRODUCTION

Though simple in their formulation, free surface boundary value problems are very difficult in most cases to solve analytically. To obtain answers for general problems, numerical methods have been developed which can be used to solve the linearized or the fully nonlinear problem. Three types of methods are usually distinguished according to what procedure is used to tackle the field equation; finite differences, finite elements and boundary-integral methods. Some hybrid methods incorporate features of the different primary methods in different subregions of the field or known analytical results, retaining the advantages and discarding the disadvantages of each method if this procedure is done in a consistent way. Some reviews cover these aspects in a great amount of detail and only the major concepts will be highlighted here (see Euvrard 1981, Yeung 1982). The initial boundary value problem is defined as (2-2-2 to 2-2-5). A Lagrangian approach can also be used (see Fritts and Bories 1977) but the generalities presented here apply in both cases.[1]

4-2 FINITE DIFFERENCES METHODS

This extensively used technique is particularly suited for problems dealing with a linearized free surface boundary condition and for straight body geometries. A mesh, usually rectangular, is placed on the field and by discretization of the equations of motion, the flow is computed at grid points. For nonlinear problems, the free surface boundary will not usually intersect the mesh system at grid points chosen in the still configuration and curved bodies will have to be approximated by segments. The influence of the boundary condition on the solution being strong, it is important to properly discretize these boundaries. Therefore, in the vicinity of the body and surface, the mesh needs to be refined in order for the computations to yield the same accuracy on the boundary as in the field. This will increase computation and is difficult to implement. Von Kerczek and Salvesen (1974) analyzed effect of a disturbance on a nonlinear surface and compared their results to second order theory. These agree well if the nature of the disturbance is small.

<u>Choice of discretization scheme</u>

Different forms exist to discretize the equations. Implicit schemes are the simplest to implement (values of the potential and free surface elevation at time step $t+\Delta t$ are given in terms of values at previous time-steps) but other possibilities exist. Stability analyses can

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[1]In the Lagrangian formulation of the flow problem, the boundaries are known but the field equations are nonlinear. See for example Lamb p 13 or Johnstone and Mackie 1973.

be performed to determine the minimum time step and grid space that need be chosen for the scheme to be stable and converge.

Mapping of the domain

If the boundaries are difficult to discretize using segments, or call for a complicated grid, the flow field can be mapped onto a domain where the boundaries become rectilinear. The disadvantage arises because the field equations become more complicated in the mapping process. Using this approach, Haussling and Coleman (1979) consider the motion of a cylinder under a nonlinear free surface.

Conditions at infinite

The problem addressed in this section is general to all methods. The domain's extent must, in the case of discretization, be limited. The question arises to what is a "good" numerical value for infinite and what is the proper radiation condition to consider there. If the domain is truncated to a control surface Σ_R , this surface should also propagate as the disturbance does. This problem has not been solved and is especially arduous for steady state flows. Some researchers chose to discretize the transient motion and let t go to infinite. Assuming that the scheme does not break down in the mean time, this method is not applicable to nonlinear steady state motions since the steady state solution depends on the initial condition. Some artifices are given in Yeung (1982) but the question is still open.

4-3 FINITE ELEMENT METHODS

The finite element technique, as the finite difference technique, directly discretizes the equations in the whole field. The field where the flow is considered is subdivided into a mesh of finite sub-regions or elements where the potential is approximated by trial functions, usually polynomial, written in terms of unknown parameters (see for example Bai 1977).

The weak formulation of the finite element method is obtained by substituting the trial functions into the field equations. The integrated residual based on a weighing function is then required to be null. Boundary conditions are incorporated by integration by parts. The choice of the trial function will lead to different types of elements and the choice of the weighing function will lead to different methods.[1] Different elements will be more particularly suited to different shapes of boundaries. Another possibility is to use a variational method where a functional is made stationary. This strong formulation can be related to the weak formulation.

[1]If the weighing function is the same as the trial function, this is the Galerkin method.

Super-elements which mask local troublesome effects or conditions at infinite can be implemented in finite element discretization schemes in a convenient manner.

4-4 BOUNDARY-INTEGRAL EQUATION METHODS

4-4-1 General Description

The treatment of problems in potential theory by integral equations is classical (see Kellog 1929). The main feature is to reduce the space dimension of the problem by one. The problem is formulated in terms of an integral equation, the integrals being taken on the boundary of the domain. The boundary conditions are therefore introduced directly in the problem and since the knowledge of the value of the potential in the flow field is of little interest, there is no waste of computed information.

The source of the difficulties in the boundary value problem come from the unknown position of the body and from the unknown description of the free surface. Chapman (1979) proposed a scheme of calculation in which the free surface condition is assumed linear while the other boundary condition is taken on the real position of the body. This inconsistent method, which retains only one source of nonlinearity is applicable for design purposes of floating structures. Since these have low frequencies of oscillation, even with large amplitudes, the velocity of the induced flow will be small, justifying the linearizing assumption on the free surface. Problems with a linearized free surface will be addressed first. The so-called Euler-Lagrange scheme, introduced in the much praised work of Longuet-Higgins and Cokelet (1976), and the method of inverse formulation will be presented. They provide a possibility to account for the nonlinearity on the surface.

4-4-2 A Linearized Free Surface

John (1950) and Stoker (1957) addressed the boundary value problem of floating bodies with a boundary-integral formulation but few problems were solved in this manner. As computers appeared, Hess and Smith (1967) in the field of aeronautics, were the first to discretize and then solve the problem for more complicated shapes in infinite fluids (see also Deruntz and Geers 1978). Adapting these methods to free surface flows took some time, due to the difficulty in the discretization with reliable accuracy of the Green functions associated with free surface conditions.

Methods based on Green's function

With the boundary of the fluid domain Ω being $\delta\Omega = S_0+F+B+\Sigma_R$ defined previously, a Green's function is defined as harmonic in the lower half plane.

Let $G(x,y,z,\xi,\eta,\zeta,t) = G(P,Q,t)$

$$= (r^{-1} + H(P,Q,t)) \text{ in 3D} \quad (5\text{-}4\text{-}1)$$

$$G(x,y,\xi,\eta,t) = (\log r + H(P,Q,t)) \text{ in 2D} \quad (5\text{-}4\text{-}2)$$

where H is harmonic for for $y \leq 0$ and $\eta \leq 0$ and satisfies equation (in the variables (ξ,η,ζ))

$$\Delta H = 0 \quad (5\text{-}4\text{-}3)$$

we will also assume in this section that G satisfies the linearized boundary condition.

a)Time harmonic problem

Suppress t in G and let:

$$-\omega^2 G(P,\xi,0,\zeta) + g\, G_y (P,\xi,0,\zeta) = 0 \quad (5\text{-}4\text{-}4)$$

Then one gets in two dimensions[1]

$$-\theta(P)\varphi_k(P) + \int_{\delta\Omega} \phi(Q)\, G_V (P,Q)\, ds_Q = \int_{\delta\Omega} \phi_V (Q)\, G(P,Q)\, ds_Q \quad (5\text{-}4\text{-}5)$$

where, $\theta(P)$ is the angle between two tangents at the boundary at P (equal to π for a smooth curve) and s is a curvilinear abscissa along $\delta\Omega$[2]. If the flow is oscillatory , and if the Green function satisfies the bottom boundary condition, radiation condition ($G=O(R^{-2})$, $G_R = O(R^{-3})$ as $R \to \infty$) then the integration is only performed on S_0 and (5-3-5) becomes (2-3-22) in the fully linearized problem.

b)Transient problem

Letting τ be a variable such that $\tau = o$ represents the moment when G appears, then if G satisfies:

$$G_{\tau\tau}(P,\xi,0,\zeta,t) + g\, G_\eta (P,\xi,0,\zeta,t) = 0 \quad (5\text{-}4\text{-}6)$$

and $G(P,\xi,0,0)=G_\tau(P,\xi,0,0)=0$ (5-4-7)

one gets an integral equation which includes a memory effect by applying Green's theorem to G and $\phi_\tau (P,\tau)$ and integrating for $\tau=0$ to $\tau= t$ (Wehausen 1971, or Yeung 1982b)

A list of references for Green's functions was given in 2-4 (Amongst these, Wehausen and Laitone 1960; and Finkelstein 1957).

There exist a number of variations to the form of the integral equations obtained. These are related to the choice of the continuation of the potential in the body. Calling ϕ' the potential inside the body, and considering a time harmonic radiation problem then :

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[1] A factor 2 will be the difference for the problem in three dimensions

[2] Then dS is an element of surface and the boundary itself is a surface.

$\phi'=0$ and $\phi_v'=0$ is called the Green's mixed distribution and leads to a Fredholm integral equation of the second kind.

$\phi'=\phi$ corresponds to a source distribution of strength $\sigma=\phi_v-\phi_v'$

$\phi_v'=\phi_v$ corresponds to a dipole distribution of strength $\sigma=\phi-\phi'$

Dirichlet conditions are imposed on the border when a source or mixed representation is used whereas, Neumann conditions are imposed when using a dipole distribution.

Among some of the works mentioned for the computation of added mass, Frank (1967) discretized an integral equation with a source distribution (Frank and Salvesen 1970). Potash (1971) used a mixed distribution for oscillating cylinders.

The problem of irregular frequencies

When the body is surface piercing, solving the integral equations by discretization is hindered by irregular frequencies which correspond to eigenvalues of the discrete system. They correspond to the resonant frequencies of the water enclosed "inside" the body (see for example, Adachi and Ohmatsu 1979 and 1980). Some methods exist to circumvent these difficulties, such as putting a lid on the free surface inside the body or by modifying G by adding concentrated singularities on $\delta\Omega$ (see Yeung 1982 for references).

Simple source formulation

This technique simply takes the wave function H=0 in (5-4-1 or 5-4-2). One interesting feature in two dimensions is the use of Cauchy theorem. Letting $w(z)=\phi+i\psi$ and $z=x+iy$ then

$$\pi i w(z)=\text{P.V.} \int_{\delta\Omega} \frac{w(z')}{z'-z}\,dz' \qquad \text{with } z \text{ and } z' \text{ on } \delta\Omega \qquad (5\text{-}4\text{-}8)$$

Yeung (1975) developed a hybrid method for linearized free surfaces which uses this relationship and is not affected by irregular frequency problems.

The simple source formulation was adopted by Chapman (1979) for large amplitude motion (retaining the linearized free surface).

4-4-3 Nonlinear Free surface

Perturbation schemes may lead to having to solve several boundary value problems. For instance, solutions to the second order radiation and diffraction problem predicting slow drift forces can be handled in a manner similar to that described in the previous section. This approach is costly and delicate. For this type of problems, Sclavounos (1987) derived a second order Green's function.

Other methods offer the possibility to handle the fully nonlinear problem.

Euler-Lagrange Scheme

This method was first presented by Longuet-Higgins and Cokelet (1976) who studied the deformation of steep surface waves. It was later adapted to study other problems (see Lin 1984; Peregrine 1987; Cointe et al 1988 for further discussion and references).

The procedure follows particles which are attached to the free surface in a Lagrangian description. The nonlinear free surface equations (2-2-3) become:

$$\frac{D\phi}{Dt} = -gy + \frac{1}{2}\nabla\phi\cdot\nabla\phi \qquad (5\text{-}4\text{-}9a)$$

$$\frac{Dx}{Dt} = \phi_x \text{ and } \qquad \frac{Dy}{Dt} = \phi_y \qquad (5\text{-}4\text{-}9b)$$

G is taken as a simple source distribution. (5-4-5) is discretized and the value of the potential and of the position of these particles is advanced in time using standard time stepping procedures. Computations using this method are very good. Analytical solutions can be incorporated to facilitate the computation around the water line but further work needs to be done (Cointe et al 1988). This method can be extended to 3 dimensions but is very costly.

Inverse formulation

The inverse formulation takes advantage of the fact that the free surface is a stream surface. Letting (x,y) become the unknowns and (ϕ,ψ) the variables, the free surface condition is nonlinear but taken on a known boundary, $\psi = 0$. Because stagnation points become singularities in this formulation, conformal mappings are generally used to transform boundaries to lines of constant ψ. Due to this requirement for the mapping to absorb the singularity, the number of geometries that can be handled in this manner is limited (see Yeung 1982 for further reading).

4-5 HYBRID METHODS

As mentioned, some methods implement different schemes in different subregions. Local analytic solutions can also be incorporated in one part of the field and matched onto a general numerical scheme in the rest of the field in a manner similar to the method of matched asymptotic expansions (See for instance Cointe , Molin and Nays 1988). A more extensive review can be found in Euvrard (1981), Jami (1981) and Yeung (1982).

4-6 CONCLUSION ON NUMERICAL METHODS

With the advent of large computers, numerical methods have proved a powerful tool in the study of linearized and nonlinear fluid problems. There is no sure way to go about doing numerical work. Depending on the problem considered, some methods will be better suited than others.

Though it was mentioned for finite difference only, the problem of the behavior at infinite is still an open one when the fluid domain is unbounded. Infinite finite elements is a promising approach.

The problem of nonlinear free surface seems to be better suited for a method using a boundary integral approach though other methods are efficient in some cases. In two dimensions, this approach is the most economical (Yeung 1982). The problem of irregular frequencies is a disadvantage of this method.

Another very difficult and unsolved problem is that of a reliable model for the behavior of the flow at the intersection of the body and the free surface. Hybrid methods using local analytical solutions will increase the accuracy of the computation (Cointe, Molin and Nays 1988).

Since there are few possibilities to check the accuracy of nonlinear codes apart from energy considerations, they should be validated by experimental results.

We will now review how these methods may be used to compute the force exerted on a ship in collision.

5-APPLICATION TO THE TRANSIENT MOTION OF COLLIDING OR BERTHING SHIPS.

5-1 INTRODUCTION

The design of piers and more recently of offshore platforms is a field of engineering which has evolved over the last century to reach its present state. A wealth of data and "hands-on" empirical rules exist which on the whole prove to be satisfactory for designs of structures under normal working conditions. But designers must also keep in mind the possibility of unusual events which can lead to potentially dangerous situations and costly repairs.

Because of such considerations, extensive programs were undertaken to estimate the impact forces on piers of berthing ships.[1] The magnitude of the impact forces being of the same order or larger than the forces due to waves and currents.

It is common practice to assume that a ship comes to a full stop when berthing, or when colliding with a platform. The kinetic energy lost in this process has to be dissipated in the form of deformation of fendering systems as well as crushing and elastic/plastic deformation of the structures involved in the collision. The energy to be absorbed is written in the form:

$$Ec = \frac{1}{2} M V^2 C_M C_E \qquad (5\text{-}1\text{-}1)$$

where, M is the mass of the ship, V the velocity prior to impact, C_M a coefficient which accounts for the energy of the entrained water around the ship, and C_E measures the eccentricity of impact with respect to the center of gravity (Blok, and Dekker 1979). In the design process, C_M has traditionally been taken to be a constant. Depending on some of the factors retained in the approach to estimate this value, it could take different values ranging from 1.3 to 2 for a small underkeel clearance (Blok, and Dekker 1979 and references herein). In the special case of ship collisions with larger keel clearance, it is taken to be 1.4 for sideways collision (Minorsky 1959). This corresponds to the added mass for vibration at infinite frequency of a ship approximated as a beam (see references in Minorsky 1959 or Ellinas and Valsgard 1985).

These values, have been contested as being too small (Blok, and Dekker 1979; Ellinas and Valsgard 1985) and have prompted experimental work in parallel with numerical calculations. Progress has been made and agreement is good when linearizing

[1]The problem of collision can easily be related to this.

assumptions are legitimate, that is for small amplitudes of oscillation (van Oortmessen 1974) and small times (Motora et al. 1971). These studies will be presented here.

5-2 APPLICATION OF LINEAR THEORY TO THE TRANSIENT MOTION OF SHIPS

5-2-1 Experimental Work and Simple Models

A vast program was undertaken at the Netherlands Ship Model Basin to estimate the influence on the coefficient C_M of different factors such as vessel size, draught, underkeel clearance, fender characteristics, berthing modes and initial berthing velocity (van Oortmessen 1974; Blok and Dekker 1979; Blok, Brozius and Dekker 1983). Very extensive experimental work was performed to realistically model the forces exerted on fenders in different configurations of berthing. The reported work encompasses mainly sideways impact, amidships with 20% keel clearance and fenders with linear or nonlinear restoring forces (1979) and eccentric collision (1983).

Motora et al (1971) point out with a different theoretical approach, that it is necessary to define **equivalent** added mass coefficients:

Equivalent **acceleration** added mass $C_{M\,acc}$, such that :

$$M \cdot C_{M\,acc}(t) = \frac{f(t)}{a(t)} \qquad (5\text{-}2\text{-}1)$$

In the case where at time t, $v(t) = \int_{-\infty}^{t} a(\tau)d\tau$ is non zero, one can define the equivalent **velocity** added mass $C_{M\,vel}$, obtained from a momentum equation such that:

$$M \cdot C_{M\,vel}(t) = \frac{\int_{-\infty}^{t} f(\tau)d\tau}{v(t)} \qquad (5\text{-}2\text{-}2)$$

Equivalent **energy** added mass $C_{M\,ener}$, obtained from a conservation of energy principle, such that:

$$M \cdot C_{M\,ener}(t) = \frac{\int_{-\infty}^{t} f(\tau)v(\tau)d\tau}{\frac{1}{2}v^2(t)} \qquad (5\text{-}2\text{-}3)$$

These equivalent added mass coefficients $C_{M\,acc}(t)$, $C_{M\,vel}(t)$, and $C_{M\,ener}(t)$ are different for a same time t. This is verified in experiments for a step or ramp excitation force f(t) (Motora et al 1971) or for linear or nonlinear restoring spring forces (Blok et al 1979, 1983). In the latter studies, t may be the time t_1 when the ships velocity is zero (end of compression phase), or t_2 when the spring displacement is zero (end of recoil stage). Expressions (3-1-2 to 4) take a slightly different form since for $t < t_0$ beginning of impact $v(t) = V$. Experimental data also shows that the energy and momentum coefficients defined in this manner are different (Blok and Dekker 1979) which can be explained when comparing the momentum and the energy in the fluid (Blok, Brozius, and Dekker 1983).

The results of these studies show that traditional values taken for C_M underestimate the measured or computed values. This is also true for large depths of water (Motora et al 1971). With a direct application to berthing, the study emphasizes the influence of different types of restoring forces. It shows that C_M is different when measured by considering momentum equations over a deceleration phase (compression of the spring) and repulsion (recoil of the spring) phase. The coefficient being larger in the second case, due to the motion "upstream" of the body. The fender design is linked to the first phase, but results are also presented on the second, or hydrodynamic phase.

The dependence of C_M on the spring rate is shown. Yet, when the berthing velocity is low, C_M is unaffected by the spring rate (Blok and Dekker 1981). For larger velocities, the energy absorbed diminishes as the spring rate increases.

An interesting feature is that for a linear spring, the compression time depends little on the initial velocity. However, this has not been studied in any more detail (Blok and Dekker 1979). Also, within the range of velocities studied, the maximum fender force seems to be linearly proportional to the velocity of approach for linear springs. This is stated in van Oortmesen (1974) and, can be deduced from fig 30. in Blok and Dekker (1979). Finally, it appears that for a linear spring, the integral from beginning of impact to end of compression is proportional to the initial velocity or that the added mass coefficient defined by the momentum equation is independent of the initial velocity. Written in equation form this becomes, taking $t_0 = 0$:

$$\left.\begin{aligned} &\int_0^{t_1} -kx(t)\,dt = MC_{M\,vel}\,\dot{x}_0 \\ &k\,x_{max} = k\,x(t_1) = \alpha\,\dot{x}_0 \\ &t_1 \text{ doesn't depend on } \dot{x}_0 \end{aligned}\right\} \qquad (5\text{-}2\text{-}4)$$

A solution to (5-2-4) proposed by Blok et al (1983) is[1] :

$$x = \dot{x}_0 \sqrt{\frac{MC_{M\,vel}}{k}} \sin\left(\sqrt{\frac{k}{MC_{M\,vel}}}\, t\right) \quad \text{and } t_1 = \frac{\pi}{2}\sqrt{\frac{MC_{M\,vel}}{k}} \qquad (5\text{-}2\text{-}5)$$

It agrees fairly well with the experimental results with some discrepancy which is also present when comparing with numerical calculations (Blok et al. 1983).

Kim (1983) uses another simple model based on these experimental observations, making use of the zero-frequency sway-added-mass and shows that for collision amidships:

$$F_{max} = k\, x_{max} \approx \sqrt{\frac{M + \frac{\mu_{11}(0)}{2}}{k}}\, \dot{x}_0 \qquad (5\text{-}2\text{-}6)$$

$\mu_{11}(0)$ can then be computed, by making use of the two dimensional theory of sway added mass (Kim 1975), Sedov's (1965) definition of a blockage coefficient and a slender body theory which describes the flow near the hull (Newman 1969). This method provides in the case of small velocities a relatively efficient way of predicting the maximum deflection of the fender and agrees well with experimental data (Blok, and Dekker 1979) and numerical calculations (van Oortmesen 1974).

5-2-2 Numerical Calculation for Colliding Ships Using the Impulse Response Function

Experimental results have been compared to numerical calculations performed by using the equations of motion derived in the previous chapter.

$$(m_{ik} + \mu_{ik}(\infty))\, \ddot{x}_k + \int_0^t (\dot{x}_k(t-\tau) - \dot{x}_{k0}) R_{ik}(\tau)\, d\tau + c_{ik}\, x_k = X_{ext} \qquad (5\text{-}2\text{-}7)$$

where it is assumed that for $t < 0$, $\dot{x}_k(t) = \dot{x}_{k0}$. X_{ext} being the exciting force takes different values when considering a step or ramp input (Motora, Fujino, Sugiura, and Sugita 1971), a linear spring restoring force proportional to the displacement (van Oortmessen 1974; Petersen 1982) or a nonlinear restoring or plastic reaction force (Petersen 1982; Petersen and Pedersen 1981). These computations compare well with the experiments and simple models presented in the previous paragraph. They imply the knowledge of the impulse response function or, of either the damping coefficient or the added-mass and infinite added-mass coefficients which represents the core of the numerical work if these are

[1]In this particular case of swaying the only degree of freedom is x.

unknown. The equations of motion can easily be solved on a "desktop computer" (Petersen 1982). Motora et al. solve for the displacement by transforming the problem in the Fourier domain and obtaining the solution by an inverse transform (see appendix 3). They show that the values of C_M vary, depending on the length of the collision, and that $C_M = 1.4$ (for sideways collision in deep water) is too small and only valid at the inception of collision. This is clear when letting t go to 0 in 3-3-1 since $\mu_{xx}(\infty)$ is of the order of 0.4 m_{xx} for sideways collision of a cylindrical section.

Petersen (1982) uses strip theory and these equations to analyze ship collision. In the special case of sideways impact or a constant excitation force, his computations agree well with the mentioned experiments and theoretical works. Different excitation forces are considered and the local nonlinear behavior is implemented. This is coupled with the dynamic motion of a platform in Petersen and Pedersen (1981). The localized deformation of the platform is modeled as a nonlinear spring force, used as an externally applied force on the ship.

5-2-3 A Direct Approach to the Linear Problem

The transient motion of a floating body was approached using an impulse response function technique. Generally, this method uses the frequency-dependent added mass or damping coefficients to compute the values of the response function. Some authors have tackled the problem by solving for the impulse function potential directly, using a time stepping approach. The method, based on work by Finkelstein (1957), was extended by Wehausen (1967) and others. It leads to an integral equation of the problem which must then be discretized. Most of the literature reviewed deals with heave or roll motion. A complete derivation of the transient motion of a heaving cylinder is given by Yeung (1982b) and compared with experiments performed by Ito (1977). Some of the problems encountered in the numerical work addressed in chapter 4 are described. One interesting feature of this work is that the velocity of the body is computed at each time step and is a function of the hydrodynamic force at each time step. Similar numerical and experimental work was performed by Beck and Liapis (1987) in three dimensions.

As previously discussed, Chapman(1979) devised a novel so-called inconsistent method where the boundary condition is expressed on the actual position of the body. Again, his method is compared to theoretical predictions for the vertical motion of bodies.

5-2-4 Conclusion

Within the assumptions of linear theory, the prediction of the energy to be dissipated in collision or absorbed by the fenders in berthing is a fairly straightforward task. It can be performed by using the impulse response function computed directly or indirectly, and the transient equations of motion or by solving the problem in the frequency domain. Both approaches are equivalent when the linear problem is considered. In both cases, knowledge of the frequency dependent added mass and/or damping coefficients or of the impulse potential is essential and represents the bulk of the numerical work or of scaled experiments to be performed (see chapter 2). Due to the amount of work that has been performed in the field of freely floating bodies, a great deal of information in the form of computed two or three dimensional frequency-dependent coefficients is available. The results for some known shapes can also be extrapolated for similar shapes (Petersen 1982) and the equations of motion can readily be computed. When strip-theory assumptions are made, two-dimensional coefficients can be used.

This approach is valid when the body undergoes small displacements. It is also easier for heaving and rolling bodies than for swaying bodies, which unfortunately, describes more accurately the case of a berthing or sideways colliding ship. Some of the problems encountered will be presented next.

5-3 OTHER ASPECTS OF THE TRANSIENT MOTION OF A SWAYING BODY

5-3-1 Introduction

For small motions, the transient swaying motion of a floating body can be described using the impulse response function as computed from the frequency dependent damping coefficients (see appendix 4). Two dimensional motion will be presented here with a special emphasis on the behavior of the flow at the intersection point. A distinction will be made between weakly nonlinear problems, where to first order, the linearized equations of motion yield satisfactory solutions and other problems.

5-3-2 The Behavior of the Flow at the Intersection of the Body and of the Free Surface

The considerable attention given to the study of wavemakers yields useful results for the study of swaying surface-piercing bodies. Both weakly nonlinear regimes (for which the acceleration of the wavemaker is much smaller than gravity) and impulsive accelerations are considered (Lin 1984; Roberts 1987; Cointe, Molin and Nays 1988). The former case will correspond to slowly berthing ships, whereas the latter case will correspond to the limiting model of a drifting ship stopped suddenly when hitting a structure. Results are presented only when the intersection of the body is at straight angles with the surface of the water. At this point, a singularity appears due to the two different boundary conditions which must be satisfied (the body boundary condition and the free surface boundary condition). In the case of the weakly nonlinear regime, this singularity is exhibited by Cointe et al (1988) using a technique similar to that of Kravtchenko (1954) in the periodic case. Their work agrees with the results derived by Roberts (1987) who uses a matched asymptotic expansion method to study the transient case. In the case of a fully impulsive motion, the behavior of the flow near the intersection of the wavemaker and the free surface is still an unknown (Roberts 1987; Cointe et al. 1988). A simplification, in this case, may be obtained by seeking a self-similar solution in an inner domain (Cointe, Jami and Molin 1987). Though this yields a possibility for the aft side of the body, it appears that further linearization in the inner domain will lead to a dead end for the front side of the body (Cointe et al. 1987, 1988).

The importance of the behavior at the intersection point is crucial for the well-conditioned behavior of numerical codes used to solve fully nonlinear transient flows (Cointe et al. 1988). The results derived for wavemakers are very important and applicable to the theoretical and numerical analyses of the transient motion of the swaying water piercing bodies. Generalizations need to be made for more complicated shapes and non vertical sides.

5-3-3 Other Studies

Marthinsen and Vinje (1985) modeled the swaying of side by side ships, with a special emphasis on the gap between both ships where nonlinear and viscous effects are considered. Vinje (1987) in a preliminary study presented a technique to crudely model the berthing of a ship by a flat plate approaching a wall.

A nonlinear numerical study for two dimensional motion of ships using the Euler-Lagrange scheme is presented in Vinje,Maogang and Brevig (1982) and outlines some of the shortcomings mentioned in Cointe et al. (1988). The problem at the intersection between the flow and the free surface is also addressed using a matching technique.

6-CONCLUSION

The value of the hydrodynamic load exerted on floating bodies in the context of linear theory can be estimated from the amount of existing literature and simplified, yet accurate analytical methods. Second-order effects can be predicted in some cases using simple mathematical techniques.

Need for further studies

The transient study of swaying bodies piercing the free surface needs to be investigated when large motion and large free surface amplitudes are considered.

For sideways collision of ships, the impact velocity is larger than that of a berthing ship due to the uncontrolled nature of the event. The transient analysis of the fully nonlinear problem should therefore be addressed, as well as the limiting case of the fully impulsive problem.

Experiments are much needed to validate results for the theoretical weakly nonlinear problems, as well as numerical calculations.

The behavior at the intersection between the free surface and the body is an important question which still needs to be solved in the case of a horizontal impulsive motion . Attention should also be given to configurations where the intersection between the side of the ship and the free surface is not horizontal.

Further research on these topics should improve the overall assessment of the design load to consider when a ship strikes another ship or a platform.

A1 THE EQUATIONS OF MOTION

A1-1 The Coordinate Systems

(O,x,y,z) is a referential, fixed in space, with y pointing upwards; (O,x,z) is the undisturbed plane of water. This referential is particularly suited to describe the motion of the water around the body.

Consider a referential fixed with the body, with origin at the center of gravity of the body: (G,X,Y,Z). At time t=0, $G(0,y_G,0)$ in (O,x,y,z), and for later times, $G(x_1,y_G+x_2,x_3)$.

If O and G coincide, (G,X,Y,Z) is obtained from (O,x,y,z) by:

rotating Ox by the roll angle x_4 around Ox;

rotating Oy by the yaw angle x_5 around Oy;

rotating Oz by the pitch angle $x_6$around Oz.

A point P has coordinates (x,y,z) in Oxyz and (X,Y,Z) in GXYZ.

Assuming the angles are small, we get by linearizing (taking the cosines of the angles equal to 1 and the sines of the angles equal to the angles in radians):

$$x=x_1+X-x_6Y+x_5Z$$
$$y=x_2+y_G+x_6X+Y-x_4Z \qquad \text{(A1-1-1)}$$
$$z=x_3-x_5X+x_4Y+Z$$

The unknowns are $(x_1,x_2,x_3,x_4,x_5,x_6) = (x_i)_{i=1,6}$

A1-2 The Nonlinear Boundary-Value Problem

The fluid is inviscid and incompressible and the flow is initially irrotational. The problem can be formulated in terms of the velocity potential $\phi(x,y,z,t)$ and a function parametrizing the free surface F(x,y,z,t). At every instant t, the position of the free surface is given by F(x,y,z) = 0. The resulting boundary value problem can be written in the form:

$$\Delta\phi = 0 \text{ in the fluid domain} \qquad \text{(A1-2-1)}$$
$$\phi_t + gy + \frac{1}{2}\nabla\phi.\nabla\phi = 0 \qquad \text{(A1-2-2a)}$$
$$F_t + \nabla\phi.\nabla F = 0 \qquad \text{(A1-2-2b)}$$

taken on the free surface F(x,y,z,t) = 0;

A boundary condition on the bottom or at infinity (for infinite depth)

$\phi_y(x,-h,z,t)=0$ for a finite depth h

or

$\lim_{y\to-\infty}\phi_y=0$ for infinite depth (A1-2-3)

A body condition on the boundary.

$\phi_n \mid S_{Body} = V.n$ (A1-2-4)

A1-3 The Linear Boundary-Value Problem

The linearized boundary value problem can be written in the form:

$\Delta\phi = 0$ in the fluid domain (A1-3-1)

$\phi_{tt}(x,0,z,t)+g\phi_y(x,0,z,t) = 0$ (A1-3-2)

$y=\eta(x,z,t)$ and $\eta(x,z,t) = -\frac{1}{g}\phi_t(x,0,z,t)$ (A1-3-3)

A boundary condition on the bottom or at infinity (for infinite depth)

$\phi_y(x,-h,z,t)=0$ for a finite depth h

or

$\lim_{y\to-\infty}\phi_y=0$ for infinite depth (A1-3-4)

A body condition on the boundary.

$\phi_t \mid S_0 = V_n$ (A1-3-5)

with $$V.n = \sum_{i=1}^{6} \dot{x}_i \, n_i$$ (A1-3-6)

n_1, n_2, n_3, are the components of the outward normal and

$n_4 = (y - y_G)\, n_3 - z\, n_2$

$n_5 = z\, n_1 - x\, n_3$

$n_6 = x\, n_2 - (y - y_G)\, n_1$ (A1-3-7)

A1-4 The Linear Hydrodynamic Forces and Moments

The hydrodynamic forces and moments about G can be written in the form:

$$F_H = \int_{S_0} p \, . \, n \, dS$$

$$K_H = \int_{S_0} p \, (r - r_G) \times n \, dS \qquad \text{(A1-4-1)}$$

The value of the pressure on the surface of the body can be derived by using the linearized form of Euler's equation:

$$p = - \rho \phi_t - \rho g y \qquad \text{(A1-4-2)}$$

A1-5 The Equations of Motion

Using this expression for the pressure, one can write the equations of motion in the form:

$$m_{ij} \ddot{x}_j = -c_{ij} x_j - \rho \int_{S_0} \phi_t \, n_i \, dS + (F_{ext})_i \qquad \text{(A1-5-1)}$$

Where the coefficients are given as:

$$m_{ij} = m \, \delta_{ij} \quad \text{and} \quad m_{i+3,j+3} = I_i \quad \text{for } i, j = 1, 2, 3$$

$$m_{45} = m_{54} = - I_{12}$$

$$m_{46} = m_{64} = - I_{13}$$

$$m_{56} = m_{65} = - I_{23}$$

$$m_{ij} = 0 \text{ for other values of i and j} \qquad \text{(A1-5-2)}$$

Where m is the mass of the floating body, δ_{ij} is the Kronecker δ, I_i and I_{ij} are the moments and products of inertia.

Also define

W = waterplane area in equilibrium position.

V = displaced volume of water in equilibrium position

$$x_c = \frac{1}{W}\int_W x dS \qquad z_c = \frac{1}{W}\int_W z dS \quad (y_c = 0)$$

$$d_1^{\,2} = \frac{1}{W}\int_W x^2 dS \quad d_3^{\,2} = \int_W z^2 dS \quad J_{13} = \int_W xz dS$$

$$x_B = 0 \qquad y_B = \frac{1}{V}\int_V (y - y_G)\, dV \qquad z_B = 0$$

(A1-5-3)

Define H_1 and H_3, the two metacentric heights for inclination about axes through the point (x_c, y_c, z_c) and parallel to Ox and Oz, respectively. These are defined by:

$$Wd_1^{\,2} + Vy_B = VH_1 + Wx_c^{\,2}$$

$$Wd_3^{\,2} + Vy_B = VH_3 + Wz_c^{\,2}$$

The metacentric heigths must be positive for hydrostatic stability.

The c_{ij} coefficients only depend on the static equilibrium of the body in calm water:

$$c_{22} = \rho g W$$

$$c_{24} = c_{42} = -\rho g W z_c$$

$$c_{26} = c_{62} = \rho g W x_c$$

$$c_{44} = \rho g (VH_3 + Wz_c)$$

$$c_{46} = c_{64} = -\rho g J_{13} \qquad c_{66} = \rho g (VH_1 + Wx_c) \quad \text{and } c_{ij} = 0 \text{ for other i,j}$$

(A1-5-4)

A2 DERIVATION OF THE EQUATIONS FOR TRANSIENT MOTION USING A GREEN'S FUNCTION.

A2-1Wehausen Approach

The transient equation of motion is derived in a different manner using a Green's function (see Finkelstein 1957; Wehausen 1967; Yeung 1982b). In the domain defined previously (consisting of S_0, B, F and Σ_R), one applies Green's identity to the functions ϕ_t (P,Q,τ) and G(P,Q,t-τ), where G is a "suitable" function. The integral over the surface B vanishes identically and that over Σ_R converges to 0 as $R\rightarrow\infty$. An easy manipulation with respect to the integral taken on the free surface F puts it into the form of a derivative with respect to τ. Integrating from 0 to t one gets:

$$\Im\{\phi\} = 2\pi\phi(P;0) + \int_{S_0} \phi(Q;0)\, G_v(P;Q;t)\, dS_Q + \int_0^t d\tau \int_{S_0} \phi_{vt}(Q;\tau)\, G(P;Q;t-\tau)\, dS_Q$$

$$- \int_F [\eta_t(\xi,\zeta;0) G(P;\xi,0,\zeta;t) + \eta(\xi,\zeta;0)\, G_t(P;\xi,0,\zeta;t)]\, d\xi d\zeta \qquad \text{(A2-1-1)}$$

where $\Im$ is the linear operator:

$$\Im\{\phi\} = 2\pi\phi(P;t) + \int_{S_0} \phi(Q;t)\, G_v(P;Q;0)\, dS_Q + \int_0^t d\tau \int_{S_0} \phi(Q;\tau)\, G_{vt}(P;Q;t-\tau)\, dS_Q$$

(A2-1-2)

Where one replaces 2 by 4 in 3D and where 2π becomes π when P is a point on the surface (in 2D). The integral equation is satisfied separately by ϕ_F and $\phi_W = \phi_I + \phi_D$ where one takes $\phi_{vt} = 0$ for ϕ_W We have assumed that we know the quantities when t = 0.

Defining $\phi_F{}^\circ = \phi_F - \phi_{FC}$, then ϕ_{FC} represents the motion which would take place if the motion of the body started in otherwise calm water.

Solution for body started in still water. Define Φ_k as the solution of

$$\Im\{\Phi_k\} = \int_{S_0} n_k(Q) G(P;Q;t)\, dS \qquad \text{(A2-1-3)}$$

k=1,2,...6 This will correspond to a motion in which starting from rest, the body makes a jump of unit magnitude in the kth mode of motion. This is a mathematical tool and one may show that:

$$\phi_{FC}(P;t) = \int_0^t \mathcal{X}_k(\tau)\, \Phi_k(P;t-\tau)\, d\tau$$

(A2-1-4)

This is the analog for time harmonic motions of the decomposition:

$$\phi_F = \sum_{k=1}^{6} \mathrm{Re}\, - i\omega a_k \varphi_k e^{-i\omega t} \qquad \text{and } \varphi_k = \varphi_{1k} + i\varphi_{2k} \tag{A2-1-5}$$

With that expression, we get

$$\int_{S_0} -\rho\phi_{FCt}\,(P;t) n_i dS = \int_{S_0} n_i dS\, -\rho \frac{d}{dt}\Big[\int_0^t \ddot{X}_k(\tau)\, \Phi_k(P;t-\tau)\, d\tau \Big] \tag{A2-1-6}$$

$$= -\,\mu_{ik}\,(\infty)\, \ddot{X}_k(t) - \int_0^t \ddot{X}_k(\tau) L_{ik}(t-\tau) d\tau \tag{A2-1-7}$$

$$= X_{FCi} \tag{A2-1-8}$$

$$\text{and } \mu_{ik}\,(\infty) = \int_{S_0} \rho\Phi_k\,(P;0+) n_i dS \quad \text{and } L_{ik}(t) = \int_{S_0} \rho\Phi_{kt}\,(P;t) n_i dS \tag{A2-1-9}$$

It may be shown that $\Phi_{kt}\,(P;0+) = 0$, $L_{ik}(0+) = 0$ and $\mu_{ik}\,(\infty) = \mu_{ki}\,(\infty)$

These quantities are related to the values μ_{ik} and λ_{ik} defined in the steady harmonic problem in the following manner:

$$\mu_{ik}\,(\omega) = \mu_{ik}\,(\infty) + \int_0^\infty \cos(\omega\tau) L_{ik}(\tau) d\tau \tag{A2-1-10}$$

$$\lambda_{ik}\,(\omega) = \omega \int_0^\infty \sin(\omega\tau) L_{ik}(\tau) d\tau \tag{A2-1-11}$$

In the most general case, the equation of motion can be written:

$$(m_{ik} + \mu_{ik})\ddot{X}_k + \int_0^t \ddot{X}_k(t-\tau) L_{ik}(\tau) d\tau + c_{ik}\, x_k = X_F{}^\circ + X_k + X_{ext} \tag{A2-1-12}$$

where $X_F{}^\circ$, X_k, and X_{ext},correspond to the force and moments respectively associated with, $\phi_F{}^\circ$, $\phi_I + \phi_D$, and the exterior exciting force. When there are no incoming waves and the water is initially at rest, then $X_F{}^\circ + X_k = 0$. The Laplace transform reduces this system of integro-differential equations to a set of linear algebraic equations.

A2-2 Equivalence with Cummins Approach.

Integrating $\int_0^t \ddot{x}_k(t-\tau)L_{ik}(\tau)d\tau$ by parts yields

$$\int_0^t \ddot{x}_k(t-\tau)L_{ik}(\tau)d\tau = \dot{x}_k(0)L_{ik,t}(t) - \dot{x}_k(t)L_{ik,t}(0) + \int_0^t \dot{x}_k(t-\tau)(L_{ik,t}(\tau))d\tau$$

(A2-2-1)

Assuming that $\dot{x}_k(0) = 0$, one will get a form equivalent to Cummins if $L_{ik,t}(0) = 0$ which needs to be proven.

A3 A FREQUENCY DOMAIN ANALYSIS OF A SWAYING TANKER USING STRIP THEORY

Motora, Fujino, Sugiura, and Sugita (1971) consider the motion of a ship undergoing sideways motion. The equation of motion with the conventions defined in the preceeding chapters are:

$$(m_{xx} + \mu_{xx}(\omega))\, \ddot{x}(t) + \lambda_{xx}(\omega)\, \dot{x}(t) = f(t) \qquad \text{(A3-1)}$$

We will simplify the notations by writing $(m_{xx} + \mu_{xx}(\omega)) = m$, and we will drop the indices so that $m = (m_{xx} + \mu_{xx}(\omega))$ and $\lambda = \lambda_{xx}(\omega)$. Using a Fourier transform one can apply it to equation (A3-1) and get:

$$A(\omega) = H(\omega)F(\omega) \qquad \text{(A3-2)}$$

with

$$F(\omega) = \int_{-\infty}^{\infty} f(t)\, e^{-iwt} \qquad \text{(A3-3)}$$

$$A(\omega) = \int_{-\infty}^{\infty} \ddot{x}(t)\, e^{-iwt} \qquad \text{(A3-4)}$$

$$H(\omega) = \frac{i\omega}{i\omega m + \omega\lambda} \qquad \text{(A3-5)}$$

and a(t) can be obtained by the inverse Fourier transform.

$$a(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} A(\omega)\, e^{iwt} \qquad \text{(A3-6)}$$

Motora et al. take a step function exciting force. We have

$$f(t) = \begin{cases} 0 \text{ if } t \leq 0 \\ 1 \text{ if } t \geq 0 \end{cases} \qquad \text{(A3-7)}$$

then

$$F(\omega) = \pi\, \delta(\omega) + \frac{1}{i\omega} \qquad \text{(A3-8)}$$

By substitution we get:

$$a(t) = \frac{1}{\pi} \int_{-\infty}^{\infty} \frac{\pi\, m(\omega)\omega^2\delta(\omega)}{m^2(\omega)\omega^2+\lambda^2(\omega)} \cos\omega t\, d\omega + \frac{2}{\pi} \int_{0}^{\infty} \frac{\lambda(\omega)}{m^2(\omega)\omega^2+\lambda^2(\omega)} \cos\omega t\, d\omega$$

(A3-9)

Neglecting higher order terms (Tasai 1961), this can be rewritten:

$$a(t) = \frac{1}{m(0)} + \frac{2}{\pi} \int_0^{\infty} \frac{\lambda(\omega)}{m^2(\omega)\omega^2+\lambda^2(\omega)} \cos\omega t \, d\omega \qquad \text{(A3-10)}$$

The integral in equation (A3-10) can be computed by making use of a strip theory for the striking ship. Results can be obtained for 2 dimensionnal ship shape forms for the damping λ and added-mass $m(\omega) = (m_{xx}+\mu(\omega))$ enabling the computation of a(t). The acceleration can therefore be computed theoretically and one can obtain an equivalent time-dependent added masses such that:

Equivalent **acceleration** added mass μ_{acc}, such that:

$$(M+\mu_{acc}(t)) = \frac{f(t)}{a(t)} \qquad \text{(A3-11)}$$

In the case where at time t , $v(t) = \int_{-\infty}^{t} a(\tau)d\tau$ is non zero,one can define the equivalent **velocity** added mass μ_{vel}, such that:

$$(M+\mu_{vel}(t)) = \frac{\int_{-\infty}^{t} f(\tau)d\tau}{v(t)} \qquad \text{(A3-12)}$$

Equivalent **energy** added mass μ_{ener} , such that:

$$(M+\mu_{ener}(t)) = \frac{\int_{-\infty}^{t} f(\tau)v(\tau)d\tau}{\frac{1}{2}v^2(t)} \qquad \text{(A3-13)}$$

These equivalent added masses $\mu_{acc}(t)$, $\mu_{vel}(t)$, and $\mu_{ener}(t)$ are different for a same time t. As t goes to zero, Motora shows that, for the cases where f(t) is a step or input function, the equivalent added masses all converge towards $\mu(\infty)$. In conclusion of this study, the energy to be dissipated by the crushing of the colliding structures can be realistically taken as a constant only if the impact has a very short duration. The Fourier analysis or the time domain integration are equivalent and lead to the same results as shows a comparison between these two methods (Petersen 1982).

A4 A TIME STEPPING COMPUTATION FOR THE SWAYING OF A SEMICIRCLE

A practical application for the swaying motion of a semicircle is presented here, using the existing results for the damping and added mass coefficients in two dimensions (Tasai 1961; Frank 1967; Vugts 1968a). These coefficients vary slightly from one reference to another when they are nondimensionalized and an average between the three results was taken.

The equation to solve, since only the swaying motion is considered, is:

$$(m_{xx}+\mu_{xx}(\infty))\,\ddot{x}_1+\int_0^t(\dot{x}_1(t-\tau)-\dot{x}_0)R_{xx}(\tau)d\tau = X_{ext} \qquad \text{(A4-1)}$$

Let $D=\frac{1}{2}\rho\pi R^2$ be the displacement where R is the radius of the cylinder. This assumes that at rest, the cylinder floats exactly with half of its surface under the free surface, therefore $m_{xx}=D$. We also have $\mu_{xx}(\infty)=\frac{4}{\pi^2}D$ according to Sedov (p. 171)[1].

$$\text{Calling } M=\frac{(m_{xx}+\mu_{xx}(\infty))}{D}=1+\frac{4}{\pi^2} \qquad \text{(A4-2)}$$

we have

$$M\ddot{x}_1+\int_0^t(\dot{x}_1(t-\tau)-\dot{x}_0)\frac{R_{xx}(\tau)}{D}d\tau=\frac{X_{ext}}{D} \qquad \text{(A4-3)}$$

$$\text{and} \quad h(\tau)=\frac{R_{xx}(\tau)}{D}=\int_0^\infty\lambda_{xx}(\omega)\sqrt{\frac{K}{g}}\cos(\omega\tau)d\omega \qquad \text{(A4-4)}$$

in nondimensional form. The values of the damping coefficients are given in the forementionned papers and the computation of the impulse response function can therefore be performed.

The impulse response function is obtained from the damping terms (eq. 2-4-14) rather than from the added mass (eq. 2-4-15) because, although asymptotic expansions of the high frequency coefficients are given in both cases, the damping coefficient converges

[1] It is interesting to note, as Sedov does, that the value of the potential derived in the case of the infinite frequency of a swaying cylinder yields a vertical velocity which is infinite in magnitude at both intersections between the cylinder and the free surface. The potential derived in this manner does not correspond to a physically realistic solution . Sedov finds a solution for the flow behind the cylinder by assuming that there is a cavity behind the body, this assumption leading to a realistic value of the velocity. In front of the cylinder, the problem is similar to the problem of the impulsively started wavemaker or flat plate and the results presented by Simon (1985) would need to be further investigated in view of the findings of Roberts (1987) and Cointe et al (1987,1988).

more rapidly to zero than $\mu_{xx}(\omega)$-$\mu_{xx}(\infty)$ (See Greenhow 1986 who uses asymptotic values derived by Simon 1985). According to these derivations,

$$\lambda_{xx}(\omega)\sqrt{\frac{R}{g}} \approx \frac{8}{\pi\alpha^3}\left(1+\frac{4}{\pi\alpha^2}(\ln\beta+\gamma+\ln 2-2)\right) \qquad \text{(A4-5)}$$

where γ is the Euler constant (See Abramovitz and Steigun) and $\alpha=\omega\sqrt{\frac{R}{g}}$.

taking simply $\lambda_{xx}(\omega)\sqrt{\frac{R}{g}} \approx \frac{8}{\pi\alpha^3}$, an estimate of the error is given when the integral (A4-4) is truncated for a given value of ω.

The curve yielding the expression for the damping coefficients obtained from Frank (1967) was therefore faired smoothly into the curve $\frac{8}{\pi\alpha^3}$ as α increased.

To obtain results with order of magnitude similar to those of Motora et al. (1971), similar numerical values were taken:
The displacement D is 53 kg. A consequence of this choice is that R=0.18m. Several exciting forces were considered, constant, linear spring and nonlinear spring as well as different initial velocities. We present here the case where the constant force is taken to be a falling weight of 1 kg : $\frac{X_{ext}}{D}$ =0.185 and the initial velocity is null.

The problem is discretized on a MacIntosh™ 512K using Lightspeed Pascal™ . The discretization is performed as in Petersen (1982); to compute the unknown $\Delta\ddot{x}$ with[2]:

$$\ddot{x}(t+\Delta t) = \ddot{x}(t) + \Delta\ddot{x} \qquad \text{(A4-6)}$$

$$\dot{x}(t+\Delta t) = \dot{x}(t) + \Delta t\,\ddot{x}(t) + \frac{\Delta t}{2}\Delta\ddot{x} \qquad \text{(A4-7)}$$

$$x(t+\Delta t) = x(t) + \Delta t\,\dot{x}(t) + \frac{\Delta t^2}{2}\ddot{x}(t) + \frac{\Delta t^2}{6}\Delta\ddot{x} \qquad \text{(A4-8)}$$

and

$$\int_0^{t+\Delta t} h(\tau)[\dot{x}(t+\Delta t-\tau) - \dot{x}_0]d\tau = \int_0^{t} h(\tau)[\dot{x}(t-\tau) - \dot{x}_0]d\tau + \ldots$$

$$\ldots + \Delta t \int_0^{t} h(\tau)\ddot{x}(t-\tau)d\tau + \frac{\Delta t^2}{2}[h(t)\ddot{x}(0) + \int_0^{t} h(\tau)\ddot{x}'(t-\tau)d\tau] + \ldots$$

(A4–9)

For the first time step Δt, we have :

[2]The index 1 caracterizing the displacement in the horizontal direction is dropped.

$$\ddot{x}(\Delta t) = \frac{X_{ext}}{MD} \tag{A4-10}$$

For the computation of the impulse response function, the maximum value of the nondimensional frequency $\frac{\alpha}{D}$ was taken as 7 . This corresponds to an error inferior to 0.01 for the impulse response function. the damping coefficient is evaluated at 500 frequencies using polynomial interpolation of data obtained from Tasai (1961), Frank (1967), and Vugts (1968). The impulse function is computed using Simpson's rule for 1000 discrete values of the nondimensional time $t\sqrt{\frac{g}{R}}$ with a maximum value of 10, corresponding with the numbers given above to 1.35 seconds for dimensional time. The displacements are evaluated every 0.01 unit of nondimensional time.

Plots of the damping coefficients, impulse response function and of the displacement, velocity and acceleration are given in figure 1-5. They give results qualitativily in agreement with Motora's (see Petersen 1982).

This technique is easy to implement. The impulse response function computation need only be done once. This is easily generalized to more degrees of freedom and different exciting forces.

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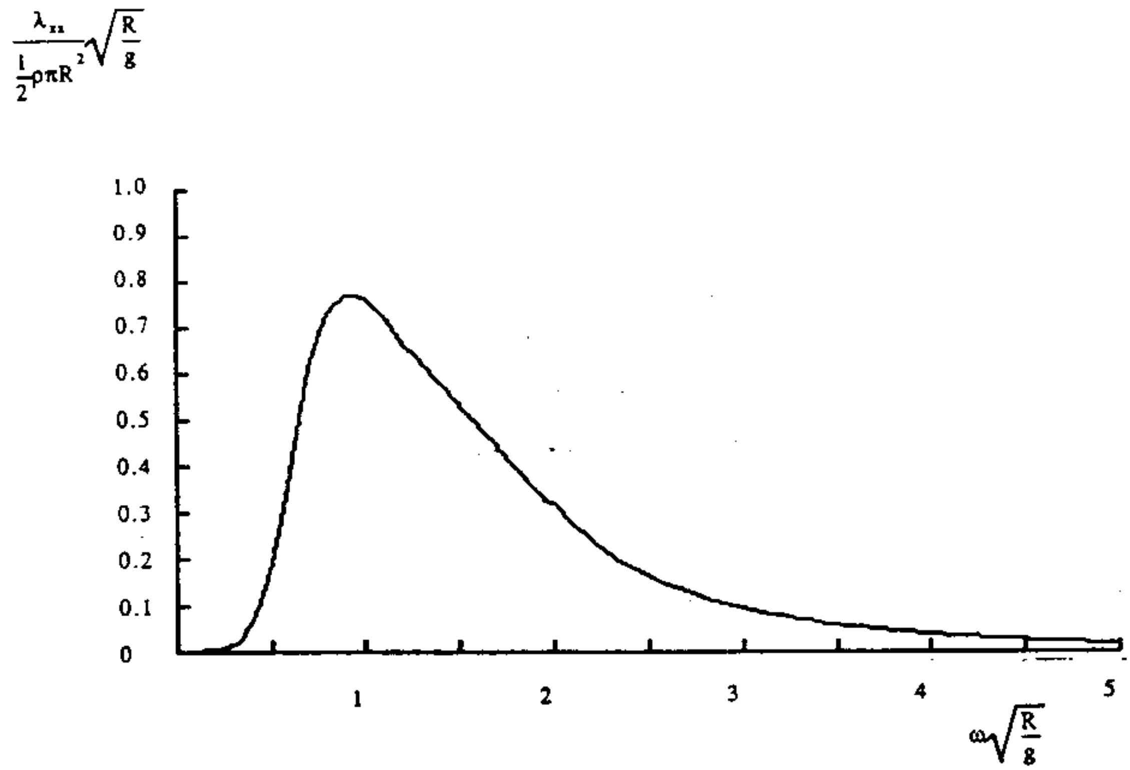
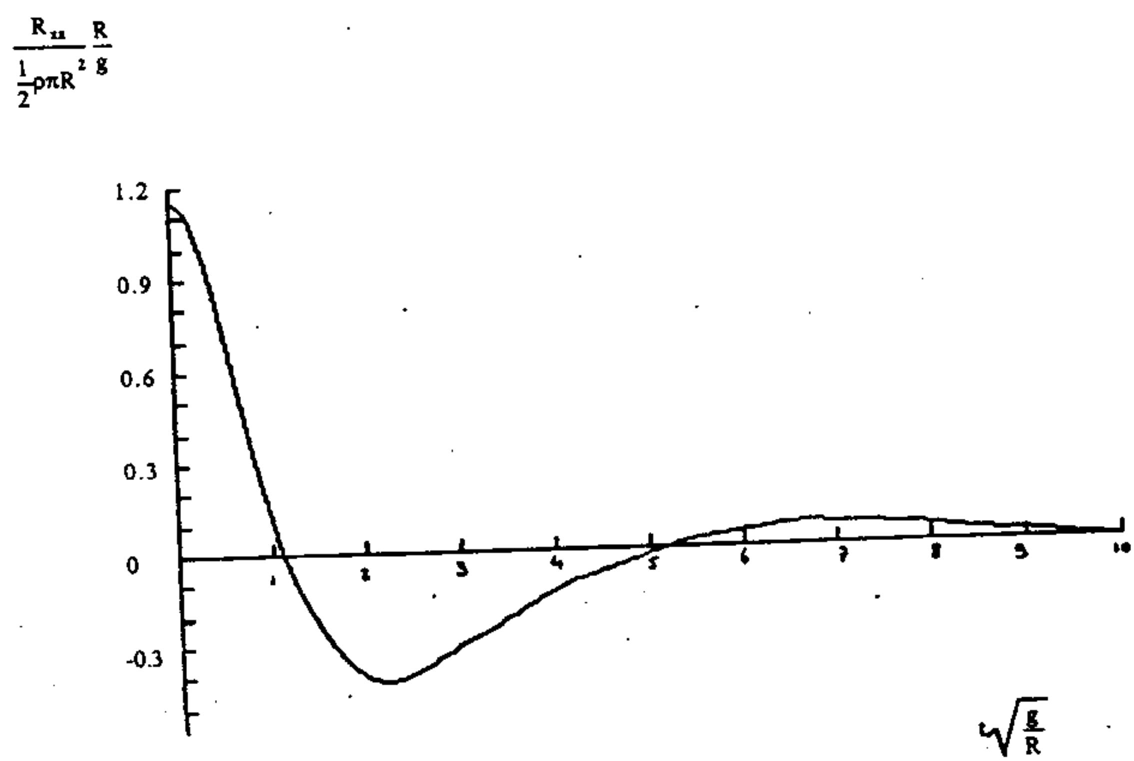


Damping coefficient (Frank 1967) for swaying semicircle

Figure 1



Impulse response function for swaying semicircle

Figure 2

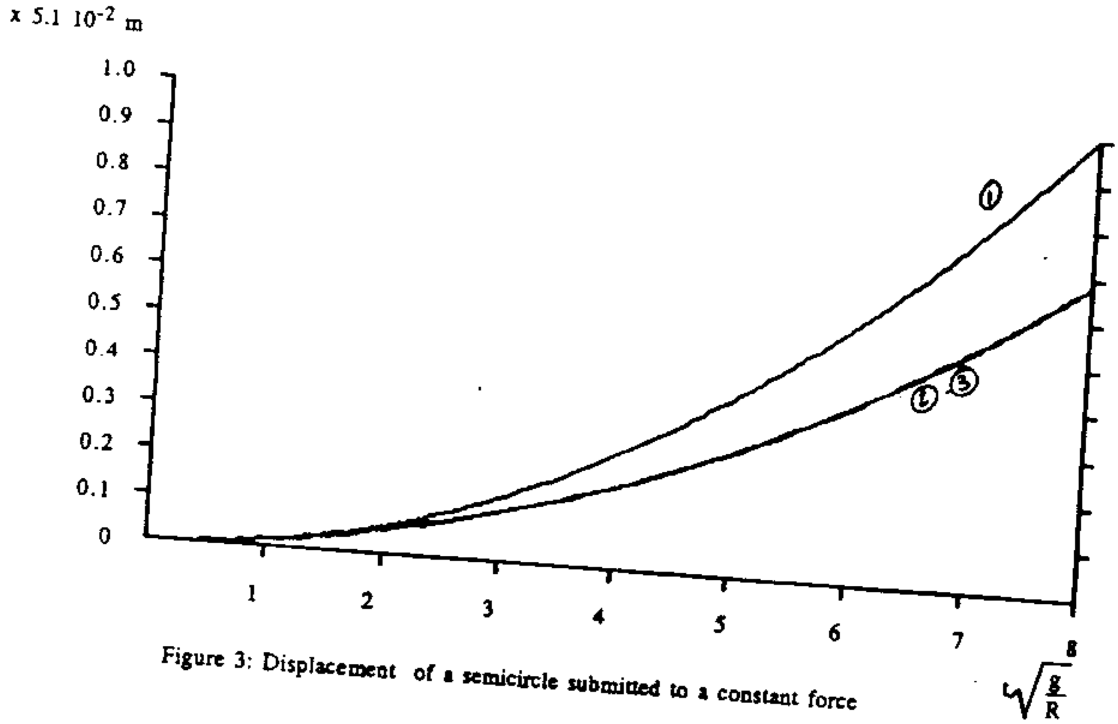


Figure 3: Displacement of a semicircle submitted to a constant force

Curve 1 No memory effect, no convolution integral
Curve 2 One term expansion of convolution integral
Curve 3 Two term expansion of convolution integral

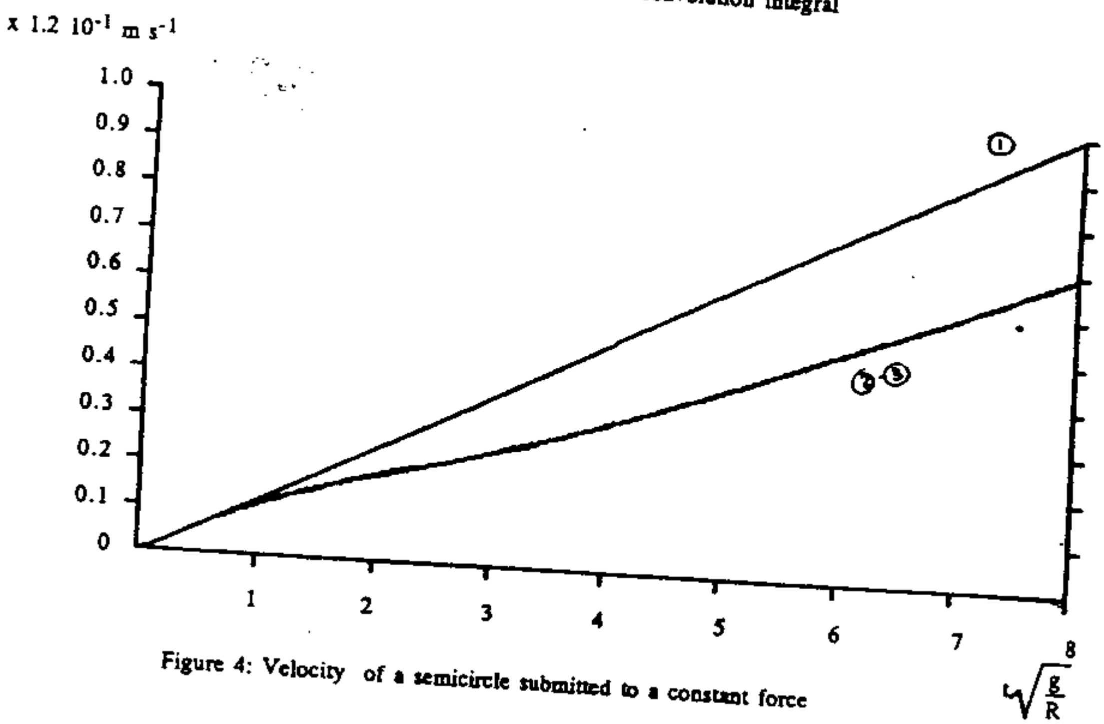


Figure 4: Velocity of a semicircle submitted to a constant force

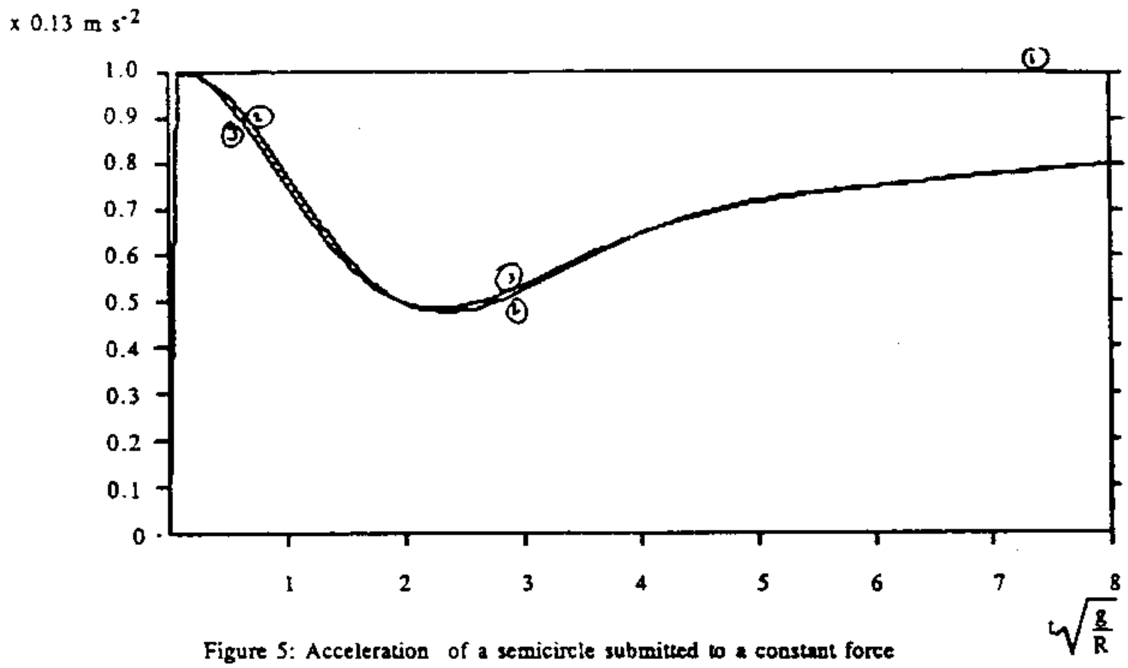


Figure 5: Acceleration of a semicircle submitted to a constant force