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ABUNDANCE ESTIMATES BASED
ON ENCOUNTER RATES

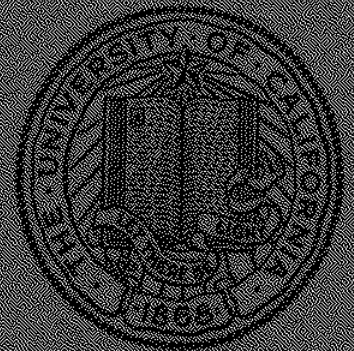
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Abundance Estimates Based on Encounter Rates

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Abstract

In this paper, a number of extensions of a recent theory for the estimation of stock abundance by encounter rates are presented. These extensions include non-Bayesian confidence intervals, multiple and unknown detection rates, learning by the fisherman, and clumping of the stock. In some of the cases, the extended theory is illustrated by application to a data set.

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INTRODUCTION

Abundance estimation - either absolute or relative abundance - is often an important part of natural resource management. Many techniques are described by Seber (1982). This paper is concerned with techniques which do not require explicit surveys (ruling out, for example transects, mark-recapture, or telemetry). The canonical example is the management of a fishery in which one tries to manage the fishery based on catch data. The traditional models based on aggregate catch and effort often do not capture the operational reality of fishing and failure of catch/effort management schemes is well known.

Recently (Mangel and Beder, 1985), a method for estimating stock abundance and predicting catches based on encounter rates was described. The method was successfully applied to a study of Pacific Ocean perch in Rennell Sound, British Columbia. The purpose of this paper is to extend the analysis presented in Mangel and Beder (1985) in a number of directions. These extensions will make the theory applicable to a much wider variety of operational situations in fisheries and wildlife. For example, although the theory presented here was developed for fisheries, it can be used just as effectively in wildlife studies (and will be used for analysis of data on porcupines in the Negev desert).

A brief review of the theory of Mangel and Beder (1985) is the following. It is based on two fundamental assumptions. These are:

1. Schools of fish are cohesive, identifiable units.
2. Given that n schools have already been fished and removed, the probability of encountering another school in the next Δt search time units is given by $(\lambda - n\epsilon)\Delta t$ where λ is unknown and ϵ is a parameter given in terms of operational variables.

The parameter ϵ , which will be called the search parameter, is characterized by $\epsilon = Wv/A$ where W is the sweep width (Koopman 1980) of the search device, v is the speed of the vessel and A is the area being searched. In some cases these parameters will be known at a reasonable level of confidence; in other cases there may be multiple ϵ 's or they may not be known at all. It is these

latter cases which motivate this work. The two assumptions lead to a binomial distribution for the catch:

$$\Pr\{\text{catch } k \text{ schools in } t \text{ units of search time}\} \quad (1)$$

$$= \binom{\lambda/\epsilon}{k} (1 - e^{-\epsilon t})^k (e^{-\epsilon t})^{\frac{\lambda}{\epsilon} - k}.$$

The value $\lambda/\epsilon \equiv N$ can be interpreted as the number of schools initially present. Given a set of data consisting of the time T_i needed to find the i^{th} school and the size of the i^{th} school (a proxy being the catch in the i^{th} school), one can compute maximum likelihood estimates for the number of schools (\hat{N}) and the biomass per aggregate (\hat{B}_A). Mangel and Beder (1985) also describe a means for calculating Bayes confidence intervals for biomass and the number of schools. They also show how one can use the theory to predict future catches given total effort (T_E).

The purpose of this paper is to extend the basic theory in a number of directions. In the next section, the case of varying search capabilities (multiple values for ϵ) and stock aggregation are treated. The third section treats the case of simultaneous estimation of the search parameter and N . The work presented in this section extends that of Allen and Punsly (1984), or Reed (1984) and has much in common with Carroll and Lombard (1983), Schnute (1983) and Littlewood and Sofer (1985). The fourth section treats the case in which the search parameter changes due to learning. Finally, the fifth section is concerned with non-Bayesian biomass estimates.

VARYING SEARCH CAPABILITIES AND STOCK AGGREGATION

In this section, two extensions of the basic model (1) are presented. The first extension involves cases in which there may be more than one value of the search parameter. Recall that

$$\epsilon = \frac{Wv}{A} \tag{2}$$

where W is the (detection) sweep width (Koopman 1980, Mangel 1985), v is the speed of the vessel while searching, and A is size of the region being searched. The sweep width usually depends upon environmental considerations. There are numerous situations in which considerably different sweep widths could arise, but one specific example concerns the sighting surveys for whales in the Southern hemisphere. In that case, the mechanism of detection is visual and the sweep width W depends strongly on the sea state and visibility conditions. The U.S. Coast Guard publishes tables that give W as a function of environmental conditions. In general, one can envision a number of situations in which encounters occur under considerably different environmental conditions (e.g. visibilities) and one does not want to lump all of the data together.

The second extension deals with aggregation or clumping of the stock - how should the analysis be modified if the stock is known to be highly aggregated?

Multiple Search Parameters

For the purposes of analysis, the data now consist of pairs (n_i, t_i) where n_i is the number of aggregations encountered in search time t_i when the search parameter is ϵ_i . Assume that there are j classes for the search parameter.

In the sequel, it helps to set

$$p_i = 1 - e^{-\epsilon_i t_i}$$

$$q_i = e^{-\epsilon_i t_i} \tag{3}$$

where

$$c_i = \frac{W_i v_i}{A} \quad (4)$$

and v_i is the vessel speed when the sweep width is W_i .

Assuming independence of captures, the likelihood of a value of N , denoted by $L(N)$, given the data set $\{n_i, p_i\}$ for $i = 1, \dots, j$ is

$$L(N) = \prod_{i=1}^j \binom{N - \sum_{k=1}^{i-1} n_k}{n_i} p_i^{n_i} q_i^{N - \sum_{k=1}^{i-1} n_k} \quad (5)$$

with the understanding that $\sum_1^0 \equiv 0$. The MLE \hat{N} for N is found by setting

$$\frac{L(N+1)}{L(N)} = 1 \quad (6)$$

which gives the equation

$$\left\{ \prod_{i=1}^j \binom{N+1 - \sum_{k=1}^{i-1} n_k}{n_i} \right\} \prod_{i=1}^j q_i = \prod_{i=1}^j \binom{N+1 - \sum_{k=1}^{i-1} n_k}{n_i} \quad (7)$$

The solution of (7), subject to integer values of \hat{N} , is found after some algebra to be

$$\hat{N} = \text{INT} \left[\frac{\prod_{i=1}^j \binom{N+1 - \sum_{k=1}^{i-1} n_k}{n_i}}{1 - \prod_{i=1}^j q_i} \right] - 1 \quad (8)$$

where $\text{INT}(x)$ is the integer part of x . Equation (8) is a natural generalization of the case of a single search parameter. Note that the estimate

for \hat{N} depends only upon the total number of encounters $n = \sum_{i=1}^j n_i$ and the parameter

$$Q = \exp\left\{-\sum_{i=1}^j \epsilon_i t_i\right\} \quad (9)$$

which is a measure of total search effectiveness.

In addition to the MLE estimate \hat{N} , it is good to have an idea about the distribution of \hat{N} . Mangel and Beder (1985) used a Bayesian approach and showed that the posterior distribution for N when the prior was uniform was virtually the same as the posterior distribution using the noninformative prior distribution. If $g(N|(n,t))$ denotes the posterior distribution on N when the data are $(n,t) = (n_1, \dots, n_j, t_1, \dots, t_j)$ and one assumes a uniform prior distribution, then the posterior distribution is

$$g(N|(n,t)) = \frac{L(N)}{\sum_{N=n} L(N)} \quad \text{for } N \geq n \quad (10)$$

where $L(N)$ is given by (5). In order to compute (10) the following identities are useful

$$\left. \begin{aligned} \binom{M}{p} \binom{M-p}{r} &= \binom{p+r}{p} \binom{M}{p+r} \\ \sum_{M=p}^{\infty} \binom{M}{p} q^{M-p} &= (1-q)^{-p-1} \end{aligned} \right\} \quad (11)$$

Thus, the denominator in (10) is given by

$$\sum_{N=n} L(N) = \prod_{i=1}^j p_i^{n_i} \left\{ 1 - \prod_{i=1}^j q_i \right\}^{\sum_{i=1}^j n_i - 1} \left\{ \prod_{i=1}^{j-1} q_i^{\sum_{k=i+1}^j n_k} \right\} \quad (12)$$

$$\left\{ \prod_{I=1}^{j-1} \left(\sum_{k=I+1}^j n_k \right) \right\}$$

Combining (5) and (12) shows that the posterior distribution is

$$g(N|(n,t)) = \binom{N}{n} (1-Q)^{n+1} Q^{N-n} \quad (13)$$

where, recall $n = \sum_{i=1}^j n_i$ is the total catch and Q is given by (9).

To close this section, it is worthwhile to consider the distribution of the number of remaining schools Y . To find it, note that

$$Y = N - \sum_{i=1}^j n_i \quad \text{so that} \quad (14)$$

$$\Pr\{Y=y\} = \Pr\{N = y + \sum_{i=1}^j n_i\}$$

Thus, if one sets

$$r = \sum_{i=1}^j n_i + 1 \quad \text{and} \quad p = 1-Q \quad (15)$$

it follows from (13) that

$$\Pr\{Y=y\} = \binom{y+r-1}{y} p^r (1-p)^y \quad (16)$$

so that the number of remaining schools has a negative binomial distribution.

Aggregation or Clumping of the Stock

It is commonly observed that many stocks aggregate (the conditional probability of a school being present increases as if another school is present). For the purposes of modeling, aggregation usually means that the variance of the catch rate exceeds the mean - sometimes by a large amount. One way of modeling such a situation is to assume that the catch rate, conditioned on λ , is a Poisson process with parameter λ . If λ has a gamma density with parameters ν and α , so that

$$f(\lambda) = \frac{e^{-\alpha\lambda} \alpha^\nu \lambda^{\nu-1}}{\Gamma(\nu)} \tag{17}$$

then the unconditional distribution of the catch rate, C , is negative binomial with mean and variance given by

$$E\{C\} = \frac{\nu}{\alpha} \tag{18}$$

$$\text{Var}\{C\} = \frac{\nu}{\alpha} + \frac{1}{\nu} \left(\frac{\nu}{\alpha}\right)^2$$

(see Mangel 1985 for details).

This approach cannot be used here, since the underlying Poisson model does not include depletion. However, the general idea can be used. That is, assume now that the binomial distribution

$$\text{Pr}\{n \text{ schools caught in search time } t\} \tag{19}$$

$$= \binom{\lambda/\epsilon}{n} (1-e^{-\epsilon t})^n (e^{-\epsilon t})^{\lambda/\epsilon - n}$$

is a conditional distribution of the catch, given λ . If λ has the gamma density (54), then the unconditional probability of the catch is (Mangel and Clark 1983, Mangel 1985).

$$\begin{aligned} \Pr\{n \text{ schools caught in search-time } t\} &\equiv P_n(t, v, \alpha) \\ &= \frac{1}{n!} (1 - e^{-\epsilon t})^n e^{-\epsilon n t} \frac{\alpha^v}{\Gamma(v)} \sum_{j=1}^n \frac{A(j, n)}{\epsilon^j} \frac{\Gamma(j+v, (\alpha+t)\epsilon n)}{(\alpha+t)^{j+v}} \end{aligned} \quad (20)$$

where $\Gamma(\mu, x)$ is the incomplete gamma function (Abramowitz and Stegun 1965)

$$\Gamma(\mu, x) = \int_x^\infty e^{-s} s^{\mu-1} ds \quad (21)$$

and the constants $A(j, n)$ satisfy the following recursion formulas

$$\begin{aligned} A(n, n) &= 1 \\ A(k, n+1) &= A(k-1, n) - nA(k, n) \quad k = 2, 3, \dots, n-1, n \\ A(1, n+1) &= -nA(1, n) . \end{aligned} \quad (22)$$

Furthermore, if λ has the gamma density (17), the coefficient of variation of λ is $1/\sqrt{v}$. That is, the "level of aggregation" is determined solely by v . This suggests that one can fix v by comparison with the same species in different regions or similar species in the same region. If v is fixed this way, and ϵ is assumed to be known, the $P_n(t, v, \alpha)$ can be viewed as the likelihood for α , given n and t . The derivative of $P_n(t, v, \alpha)$ is easily computed; setting it equal to zero shows that the MLE for α must satisfy the equation

$$\begin{aligned}
 & \nu \alpha^{\nu-1} \sum_{j=1}^n \frac{A(j,n)}{\epsilon^j} \frac{\Gamma(j+\nu, (\alpha+t)n\epsilon)}{(\alpha+t)^{j+\nu}} \\
 & = \alpha^\nu \sum_{j=1}^n \frac{A(j,n)}{\epsilon^j} \left\{ \frac{j+\nu}{(\alpha+t)^{j+\nu+1}} \Gamma(j+\nu, (\alpha+t)n\epsilon) \right. \\
 & \quad \left. - e^{-(\alpha+t)n\epsilon} (n\epsilon)^{j+\nu+1} \right\}
 \end{aligned} \tag{23}$$

The solution of (23) is easily found numerically. It is actually more instructive, however, to plot the likelihood (23) over a range of values of α .

As an example, consider the Pacific Ocean perch data used in Mangel and Beder (1985) (these data are described in detail in the Appendix). They are

$$\left. \begin{aligned}
 n &= 13 \\
 t &= 181 \text{ hrs} \\
 \nu &= 1.0
 \end{aligned} \right\} \tag{24}$$

Figure 1 shows the likelihood $P_n(t, \nu, \alpha)$ over a wide range of α . The MLE is $\hat{\alpha} = 14$, giving

$$E\{\lambda\} = \frac{\nu}{\alpha} = .0714 \tag{25}$$

and since

$$E\{N\} = E\{\lambda/\epsilon\}, \tag{26}$$

the MLE for $N = 714$ aggregations - similar to the value 646 reported by Mangel and Beder (1985). Note, too that the peak in $P_n(t, \nu, \alpha)$ is rather broad. If α_+ and α_- are the values of α at which the likelihood is 90% of its maximum value, then

$$\alpha_+ = 21.5 \qquad \alpha_- = 8.5 \tag{27}$$

showing the broad range of α , and thus \hat{N} , when stock aggregation is included.

SIMULTANEOUS ESTIMATION OF N AND THE SEARCH PARAMETER

Consider now the situation in which there is only one value of ϵ , but it is unknown so that one is simultaneously estimating N and ϵ . The easiest approach would be to use moment estimators which, it turns out, correspond with the methods of Leslie (1952) and DeLury (1947, 1951) for estimating stock abundance. That is, consider a sequence of "periods" (e.g. weeks) and let H_i denote the harvest in period i . From (1), one has

$$E\{H_i | H_j \text{ for } j=1, \dots, i-1\} = (1 - e^{-\epsilon t_i}) \left(N - \sum_{j=1}^i H_j \right) \tag{28}$$

where t_i is the search time in period i . This search time is actually related to the catch if the total effort is fixed (see Mangel and Beder 1985 pg. 160, equation (5.5) and the subsequent discussion). For simplicity, define k and E_i by $1 - e^{-\epsilon t_i} \equiv kE_i$ where E_i is the "effort" in period i and k is a constant. If $CPUE_i$ is the expected harvest in period i divided by E_i , then

$$CPUE_i = k \left\{ N - \sum_{j=1}^i H_j \right\} \tag{29}$$

Equation (29) is set up for Leslie's method: one regresses the observed $CPUE_i$ against $\sum_{j=1}^i H_j$ and obtains moment estimators for N and k . DeLury's method

is obtained by scaling time on the original assumptions so that one deals essentially with a continuous harvest process.

The methods of Leslie and DeLury can be very unstable. For example, if the data used in the regression related to (29) are quite noisy -- and thus do not show a pronounced decline with increasing periods -- one can obtain negative estimates for k or N .

The solution to this problem is to try a maximum likelihood approach as well. But, in order to do this, more information is needed than is included in the binomial likelihood (1). To see this need, rewrite (1) as a likelihood

$$L(N, \epsilon) = \binom{N}{n} (1 - e^{-\epsilon t})^n (e^{-\epsilon t})^{N-n} \quad (30)$$

If $p = 1 - e^{-\epsilon t}$, (30) becomes

$$L(N, p) = \binom{N}{n} p^n (1-p)^{N-n} \quad (31)$$

Since $L(N, p) \leq 1$ and $L(n, 1) = 1$, the MLEs are $\hat{N} = n$, $\hat{p} = 1$ and these are clearly unsatisfactory.

The needed additional information involves the individual search times. That is, by assumption if T_i is the time needed to find the i^{th} aggregation, then T_i has an exponential distribution with parameter $\lambda - \epsilon(i-1)$. Thus, the likelihood $f(\{T_i\})$ of a set of discovery times T_1, \dots, T_n is given by

$$\begin{aligned} f(\{T_i\}) &= \prod_{i=0}^{n-1} (\lambda - i\epsilon) e^{-(\lambda - i\epsilon)T_{i+1}} \\ &= \prod_{i=0}^{n-1} (N - i)\epsilon e^{-(N - i)\epsilon T_{i+1}} \end{aligned} \quad (32)$$

where $N \equiv \lambda/\epsilon$. It can be shown that the likelihood (32) does not yield satisfactory estimates of ϵ and N either (that is, (32) gives $N = n$).

Satisfactory estimates of ϵ and N can be obtained by an extension of (32) that proceeds as follows. Augment the data set T_i by the time S since the last aggregation was found. The augmented likelihood $g(\{T_i\}, S)$ is

$$g(\{T_i\}, S) = \left\{ \prod_{i=0}^{n-1} (N-i)\epsilon e^{-(N-i)\epsilon T_{i+1}} \right\} e^{-(N-n)\epsilon S} \quad (33)$$

(This likelihood, remarkably enough, arises in the theory of computer software reliability - see Joe and Reid 1985, Littlewood 1981 or Littlewood, Ghaly and Chan 1985 and the references therein.) The MLEs derived from (33) are

$$\hat{\epsilon} = \frac{n}{(\hat{N}-n)S + \sum_{i=0}^{n-1} (\hat{N}-i)T_{i+1}} \quad (34)$$

and where \hat{N} is the MLE for N and satisfies the following equation

$$0 = F(N) = \sum_{i=0}^{n-1} \frac{1}{N-i} \quad (35)$$

$$\frac{-n \left(\sum_{i=0}^{n-1} T_{i+1} + nS \right)}{(N-n)S + \sum_{i=0}^{n-1} (N-i)T_{i+1}}$$

If $F(N)$ is such that $F'(N)$ is bounded away from zero near the MLE, then (35) can be efficiently solved using Newton's method. As an example, consider the following data on Pacific ocean perch (see Appendix 1 for more details)

<u>i</u>	<u>T_i (hrs.)</u>
1	13.9
2	4.0
3	4.5
4	5.6
5	1.3
6	11.7
7	31.5
8	22.6
9	11.1
10	16.5
11	5.0
12	30.3
13	23.1

The value of S is not known, but one can parameterize the MLEs by S . Results of computations are shown in Table 1. The results are in accord with intuition: as S grows, \hat{N} decreases (one expects, in fact, that $\hat{N} \rightarrow n$ as $S \rightarrow \infty$) and $\hat{\epsilon}$ increases. Perhaps somewhat unexpected is the relative consistency of $\hat{N} \hat{\epsilon}$. Note that $\hat{N} \hat{\epsilon}$ is the estimate for the initial catch rate (and $(\hat{N} - n) \hat{\epsilon}$ for the catch rate after catching n aggregations).

In numerical experimentation with other data sets, it has turned out that the likelihood (33) is quite flat, so that the Newton iterates for the MLE did not converge well. In cases such as those, the following procedure seemed to work well. To begin, fix a maximum possible value of ϵ , ϵ_M , determined on the basis of external considerations. Now, if ϵ is fixed the MLE for N derived from (33) satisfies the equation

$$\sum_{i=0}^{n-1} \frac{1}{N-1} = \epsilon(T + S) \tag{36}$$

where $T = \sum T_i$ is the total search time.

TABLE 1
MLEs for N and ϵ using Pacific Ocean Perch Data

S (hrs)	\hat{N}	$\hat{\epsilon}$	$\hat{N} \hat{\epsilon}^*$
5	386	2.1×10^{-4}	.0814
10	199	4.1×10^{-4}	.0806
15	137	5.8×10^{-4}	.0799
20	106	7.5×10^{-4}	.0793
25	87	9.0×10^{-4}	.0786
30	75	1.0×10^{-4}	.0780
40	60	1.3×10^{-4}	.0769
50	51	1.5×10^{-3}	.0759
60	45	1.7×10^{-3}	.0748
70	41	1.8×10^{-3}	.0738
80	38	1.9×10^{-3}	.0728
90	36	2.0×10^{-3}	.0718
100	34	2.1×10^{-3}	.0708

* \hat{N} and $\hat{\epsilon}$ reported above are rounded values; $\hat{N} \hat{\epsilon}$ is computed from the pre-rounded values.

This equation is easily solved for \hat{N} by setting

$$F(N) = 0 = \sum_{i=0}^{n-1} \frac{1}{N-i} - \epsilon(T + S) \quad (37)$$

The advantage in using (37) over (35) is that the derivative $F'(N)$ of (37) is bounded away from zero more than the derivative of (35) - that is, the likelihood is not as flat. Thus, the following algorithm was used. First, fix $\epsilon \leq \epsilon_M$, find the MLE \hat{N} from (37) and evaluate the log-likelihood

$$\hat{L} = \sum_{i=0}^{n-1} \log(\epsilon) + \log(\hat{N}-i) - \epsilon(\hat{N}-i)T_{i+1} - \epsilon(\hat{N}-n)S \quad (38)$$

The log-likelihood in (38), \hat{L} , is really a function of ϵ too $\hat{L}(\epsilon)$ and the idea is to choose the $\epsilon \leq \epsilon_M$ (say by simple search) that maximizes $\hat{L}(\epsilon)$.

As an example, consider the following data on the crested porcupine, Hystrix indica (see the appendix for details of the data collection). Porcupines were trapped, marked, and released. Because they were marked, it is as if the porcupines were removed from the population. Table 2 shows data on the time required to trap the i^{th} porcupine. Using the previously described algorithm, a maximum value of $\epsilon_M = .001$ was picked. The values of the log-likelihood for various values of ϵ and the MLE $\hat{N}(\epsilon)$ are shown in Table 3. The extreme flatness of the log-likelihood function is seen in this table. The MLE $\hat{N}(\epsilon)$ with the largest log-likelihood is 60 porcupines, with a corresponding $\epsilon = .00048$. Other estimates for the population level gave about 50 porcupines (P. Alkon, personal communication). These two estimates are thus in relatively good agreement.

TABLE 2
Porcupine Trapping Data

<u>Capture Number, i</u>	<u>T_i</u> (trap nights)*
1	3
2	1
3	17
4	5
5	4
6	6
7	2
8	4
9	8
10	3
11	11
12	2
13	2
14	6
15	2
16	2
17	10
18	2
19	4
20	2

* One trap night (the unit of effort) is nominally 8 hours.

TABLE 3

$\hat{N}(\epsilon)$ and Log-Likelihood for the Porcupine Data

<u>ϵ</u>	<u>$\hat{N}(\epsilon)$</u>	<u>Log-Likelihood</u>
.001	37	-92.92
.0009	39	-92.82
.0008	42	-92.73
.0007	46	-92.65
.0006	52	-92.55
.0005	55	-92.56
.00048	60	-92.53
.00042	66	-92.54
.00036	74	-92.55
.00032	84	-92.63
.00024	99	-92.75

In many situations, unfortunately, one will not have individual search times, but will have aggregated data. For example, assume that the data consist of pairs (n_i, t_i) where n_i is the number of schools encountered in period i with total search time t_i . For a situation like this one, the following new model (which is a Poisson approximation) is a good one. Assume that in the i^{th} period, the catch follows a Poisson distribution with parameters

$$\lambda_i = \epsilon t_i \left(N - \sum_{k=1}^{i-1} n_k \right). \quad (39)$$

Thus, the likelihood of a set $\{n_i\}$ is

$$L = \prod \frac{e^{-\lambda_i} \lambda_i^{n_i}}{n_i!}. \quad (40)$$

Setting $C_i = \sum_{k=1}^{i-1} n_k$ to be the catch up to period i and using (39), the log-likelihood \hat{L} is

$$\begin{aligned} \hat{L} = \sum - \epsilon t_i (N - C_i) + \log(n_i!) \\ + n_i [\log(\epsilon) + \log(t_i) + \log(N - C_i)] \end{aligned} \quad (41)$$

Setting the derivatives of (41) equal to zero gives

$$0 = \sum - t_i (N - c_i) + \frac{n_i}{\epsilon} \quad (42)$$

$$0 = \sum - \epsilon t_i + \frac{n_i}{N - c_i} \quad (43)$$

Solving (43) for ϵ gives

$$\epsilon = \frac{1}{t} \sum \frac{n_i}{N-c_i} \quad (44)$$

where $t = \sum t_i$ is the total search time. Using (44) in (42) gives a single equation for N :

$$Nt - \sum t_i c_i = \frac{nt}{\sum \frac{n_i}{N-c_i}} \quad (45)$$

where $n = \sum n_i$ is the total catch. This equation is easily solved for N by an iterative scheme such as Newton's method.

As an example, consider the following data, pertaining to *Sardinella maderensis*, *Sardinella aurita*, *Scomber japonicus* and *Brachideuterus auritus* off the Ivory Coast (Cury and Roy, 1985a,b). Search times and catch data are given in two weak blocks, and geographically divided into 8 zones. Each fortnight can be identified as one period. Based on empirical observation (P. Cury, personal communication), it was decided to assume that school required three sets. Data for the first five fortnights in 1966 in zone 4 are shown below

<u>Period</u>	<u>T_i (hrs)</u>	<u>Catch (tons)</u>	<u>n_i</u>
1	216	119	11
2	2280	594	85
3	3312	1133	102
4	2064	370	43
5	1008	322	57

These data give an average of 8.5 tons/school. Using these data in (44) and (45) gives estimates for N and ϵ

$$\left. \begin{aligned} \hat{N} &= 5871 \\ \hat{\epsilon} &= 5.8 \times 10^{-6} \end{aligned} \right\} \quad (46)$$

note that predictions based on the Poisson approximation are actually quite simple. For example, if N_c is the best current estimate of N , the expected catch given a future search time s is

$$E\{\text{catch} \mid s\} = \hat{\epsilon} N_c s \quad (47)$$

In addition to the mean, one is interested in the range of possible catches. This can be found by summing the Poisson distribution to encompass as much of the total probability as desired. (This idea roughly corresponds to a confidence interval).

The next five search times in the data of Cury and Roy are $T_6 = 1608$ hrs, $T_7 = 600$ hrs, $T_8 = 504$ hrs, $T_9 = 360$ hrs, and $T_{10} = 1008$ hrs. Using these data, one can compare the actual number of schools encountered with the expected number and with the range corresponding to 90% of the summed probability. These comparisons are shown below

<u>Period</u>	<u>Number of Schools Encountered</u>		
	<u>Actual</u>	<u>Expected</u>	<u>Range for 90% of the Probability</u>
6	53	55	(43,67)
7	23	20	(13,27)
8	22	17	(10,24)
9	13	12	(6,18)
10	44	34	(24,44)

The approximation seems to work reasonably well. Other tests, using both the Ivory Coast data and additional data on tuna in the South Pacific, are currently underway. The general situation which is arising from the results reported here

and the additional work is the following one. The simultaneous estimation of ϵ and N from a relatively limited data set is difficult because of the limited information in the data (for example, low ϵ and high N versus high ϵ and low N may be virtually indistinguishable). The problem of trying to extract too much information from the data is, essentially, the source of the instability of the Leslie and DeLury methods. One should try to determine ϵ by other means, such as the operational definition given early in the paper.

LEARNING BY THE FISHERMAN

There are many operational situations in which learning by the operator is important. In this paper, learning will be characterized as follows. Recall the definition that $\epsilon = Wv/A$. In order to model learning, let A be a decreasing function of search time t . That is, learning is summarized by a reduction in the area searched. Three possible models are

$$\left. \begin{aligned}
 A(t) &= A_0 / (1 + \theta t) \\
 A(t) &= A_0 e^{-\theta t} \\
 A(t) &= \begin{cases} A_0(1 - \theta t) & t < \frac{1}{\theta} \\ 0 & t \geq \frac{1}{\theta} \end{cases}
 \end{aligned} \right\} \quad (48)$$

(Another possibility is that the area searched depends not only on the elapsed search time, but also on the current catch; $A = A(t, n)$). Now, fundamental assumption (2) is equivalent to

$$\Pr \left\{ \begin{array}{l} \text{another discovery in next } dt \text{ units of search} \\ \text{time} \mid n \text{ discovered in the first } t \text{ units} \\ \text{of search time} \end{array} \right\} = (N-n)\epsilon dt \quad (49)$$

When learning is included, $(N-n)\epsilon$ in (49) is replaced by $(N-n)\epsilon(t)$. It is easily verified that the probability density for the time until the i^{th} detection is

$$q_i(t_i) dt \equiv \Pr\{i^{\text{th}} \text{ detection occurs in the interval } (t_i, t_i + dt)\} \quad (50)$$

$$= \epsilon(t_i) (N-(i-1)) \exp \left\{ -(N-(i-1)) \int_{\tilde{T}_{i-1}}^{\tilde{T}_{i-1} + t_i} \epsilon(s) ds \right\} dt$$

where \tilde{T}_{i-1} is the total elapsed search time for the first $i-1$ detections.

For example, consider the first of the three models in (48). Then

$$q_i(s) = \frac{Wv}{A_0} (1+\theta s) (N-(i-1)) \exp \left\{ -(N-(i-1)) \frac{Wv}{A_0} \left[s + \frac{\theta}{2} (s^2 + 2\tilde{T}_{i-1}s) \right] \right\} \quad (51)$$

For a given set of discovery times $\{T_1, \dots, T_n\}$, the likelihood is now

$$\mathfrak{L}(\{T_i\}) = \prod_{i=0}^{n-1} \frac{Wv}{A_0} (1+\theta T_{i+1}) (N-i) \exp \left\{ -(N-i) \frac{Wv}{A_0} \left[T_{i+1} + \frac{\theta}{2} (T_{i+1}^2 + 2T_{i+1} \sum_{k=0}^{i-1} T_k) \right] \right\} \quad (52)$$

and the log-likelihood is given by

$$\log \mathfrak{L} = \sum_{i=0}^{n-1} \log(1+\theta T_{i+1}) + \log(N-i) - (N-i) \frac{Wv}{A_0} \left(T_{i+1} + \frac{\theta}{2} [T_{i+1}^2 + 2T_{i+1} \sum_{k=0}^{i-1} T_k] \right) \quad (53)$$

+ terms independent of (N, θ) .

The MLEs for N and θ thus satisfy

$$0 = \sum_{i=0}^{n-1} \frac{1}{N-1} - \frac{Wv}{A_0} \left(T_{i+1} + \frac{\theta}{2} [T_{i+1}^2 + 2T_{i+1} \sum_{k=0}^{i-1} T_k] \right) \quad (54)$$

$$0 = \sum_{i=0}^{n-1} \frac{T_{i+1}}{1+\theta T_{i+1}} - \frac{(N-1)Wv}{2A_0} \left(T_{i+1}^2 + 2T_{i+1} \sum_{k=0}^{i-1} T_k \right) \quad (55)$$

These are easily computed by solving (54) for $\theta = \theta(N)$ and substituting into equation (55) to obtain a single equation for N . This was done using the Pacific Ocean perch data and it yielded results which indicated that learning was not a factor in that data set.

An alternate approach for estimating the learning parameter θ is a Bayesian one. Note first that the distribution of catches is a time varying binomial

$$\Pr\{n \text{ catches in search time } t\} = \binom{N}{n} (1 - \exp[-\epsilon_0(t + \frac{\theta}{2}t^2)])^n \exp[-\epsilon_0(t + \frac{\theta}{2}t^2)] \quad (56)$$

where $\epsilon_0 = Wv/A_0$. Equation (56) can be rewritten as

$$\begin{aligned} &\Pr\{n \text{ catches in search time } t\} \\ &= \binom{N}{n} \exp[-(N-n)\epsilon_0 t] (1 - p e^{-w\theta})^n e^{-w(N-n)\theta} \end{aligned} \quad (57)$$

where

$$p = e^{-\epsilon_0 t} \quad w = \frac{t^2 \epsilon_0}{2} \quad (58)$$

Now, if a prior distribution is placed on θ , either (56) or (57) can be viewed as conditional quantities. A versatile choice of prior densities for the learning parameter is the gamma family

$$f(\theta) = \frac{e^{-\alpha\theta} \theta^{\nu-1}}{\Gamma(\nu)} \alpha^\nu \quad (59)$$

with parameters ν and α .

To do the integral of (57) against the gamma density, set

$$I(\theta) = \int [1 - \rho e^{-w\theta}]^n e^{-w(N-n)\theta} f(\theta) d\theta \quad (60)$$

Expanding the term $[1 - \rho e^{-w\theta}]^n$ and integrating termwise gives

$$I(\theta) = \sum_{k=0}^n \binom{n}{k} (-\rho)^k \left[\frac{\alpha}{\alpha + w(k + N - n)} \right] \quad (61)$$

BIOMASS ESTIMATES

The procedures given in this paper yield an estimate \hat{N} for the number of schools or aggregations present and an estimate \hat{B}_A for the biomass per school or aggregate. The latter is computed by

$$\hat{B}_A = \frac{1}{n} \sum_{i=1}^n B_i \quad (62)$$

where B_i is an estimate for the biomass of the i^{th} school.

The total biomass B is estimated by

$$\hat{B} = \hat{N} \hat{B}_A \quad (63)$$

and the question is then: what can one say about the distribution of \hat{B} ? Mangel and Beder (1985) used a posterior Bayesian distribution on B , computed as follows:

$$\Pr\{m \leq B \leq m+dm\} = \sum_{k=1} \Pr\{k \text{ aggregates present}\} \Pr\{B_A = m/k\} \quad (64)$$

The Bayesian confidence intervals for the data on Pacific Ocean perch reported by Mangel and Beder are quite large (the 85% confidence level is [3000,36000] tons and the 90% confidence level is [1000,3800] tons).

An alternate procedure uses Goodman's (1959) formula for the exact variance of products. According to this formula (modified for this special case).

$$\text{Var}\{\hat{B}\} = \text{Var}\{\hat{N} \hat{B}_A\} = \hat{N}^2 \frac{S_B^2}{n} + \hat{B}_A^2 \frac{S_N^2}{n} - \frac{S_B^2 S_N^2}{n^2} \quad (65)$$

where S_B^2 and S_N^2 are the usual unbiased estimates for the variances of B_A and N respectively. For the Pacific ocean perch data used by Mangel and Beder (1985)

$$\begin{aligned} \hat{B}_A &= 32 \text{ tons} & \hat{N} &= 646 \\ S_B^2 &= 524 & S_N^2 &= 31654 \end{aligned} \quad (66)$$

Using the data (66) in equation (65) gives

$$\sqrt{\text{Var}\{\hat{B}\}} = 6925 \text{ tons} \quad (67)$$

The mean $\hat{B} = \hat{N} \hat{B}_A = 20672$, so that

$$\text{mean} \pm \sqrt{\text{Var}(\hat{B})} = [13747, 27597]$$

(68)

$$\text{mean} \pm 2\sqrt{\text{Var}(\hat{B})} = [6322, 34522]$$

The form of (68) is chosen to mimic the central limit theorem (although its not clear that the theorem applies here). The limits given by the second equation in (68) are remarkably close to the Bayesian ones.

CONCLUSIONS AND DISCUSSION

This paper represents another step towards a general theory of abundance estimation based on encounter rates. Further work needs to be done in two general areas: 1) adding various operational effects and 2) including population dynamics and/or treatment of open populations.

Two operational effects which may be important are the effect of school structure on detection of schools and non-random search. That is, in visual or acoustic search the larger schools may be detected first and this kind of phenomenon could be modeled, although it introduces yet another parameter into the estimation process.

Preliminary analysis of data on tuna in the South Pacific seems to indicate the following operational situation: fishermen locate patches containing schools of tuna on a relatively random basis, but the search within the patches is non-random. This operational situation can be treated by the methods of this paper, with minor modification and work along those lines is currently in progress.

All the work in this paper and in Mangel and Beder (1985) assumed a closed population. The population could be open for a number of reasons, for example emmigration during the season or birth and death of fish during the season. There are two ways to treat such open populations. To begin, one can write that the population at time t , $N(t)$, is given by

$$N(t) = N(0) + \sum_{s=0}^{t-1} b(s) - d(s) + i(s) - e(s) \quad (69)$$

where $b(s)$ is the number of births, $d(s)$ the number of deaths, $i(s)$ the number of immigrations and $e(s)$ the number of emigrations at time s . One can then try to estimate each of these parameters separately. Another approach is to simply treat $N(t)$ as a random variable. For example, the following approach is currently under investigation and may turn out to be quite appropriate. First, let $N(i)$ be value of N at the i^{th} detection. Second, write

$$N(i) = n + \tilde{N} \quad (70)$$

where n is the total number of detections in the entire observation period and \tilde{N} is a random variable. For example, if \tilde{N} has a gamma distribution with parameters v and α , then the density for the i^{th} detection is given by

$$\begin{aligned} & \epsilon(n-i)e^{-\epsilon(n-i)t} \left(\frac{\alpha}{\alpha+\epsilon t}\right)^v \\ & + e^{-\epsilon(n-i)t} \frac{\epsilon\alpha^v}{\Gamma(v)} \frac{\Gamma(v+1)}{(\alpha+\epsilon t)^{v+1}} \end{aligned} \quad (71)$$

and one can try to estimate the parameter set (ϵ, v, α) . An even simpler approach to the open population is simply to write $N(i) = \tilde{N}$ and ignore depletion, so that conditioned on $N(i)$ the time to the i^{th} detection has conditional density

$$\epsilon N(i) e^{-\epsilon N(i)t} \quad (72)$$

so that the unconditional density is

$$\epsilon \frac{a^v}{\Gamma(v)} \frac{\Gamma(v+1)}{(a+\epsilon t)^{v+1}} \quad (73)$$

The utility of (71) - (72) for the study of open populations is currently under investigation.

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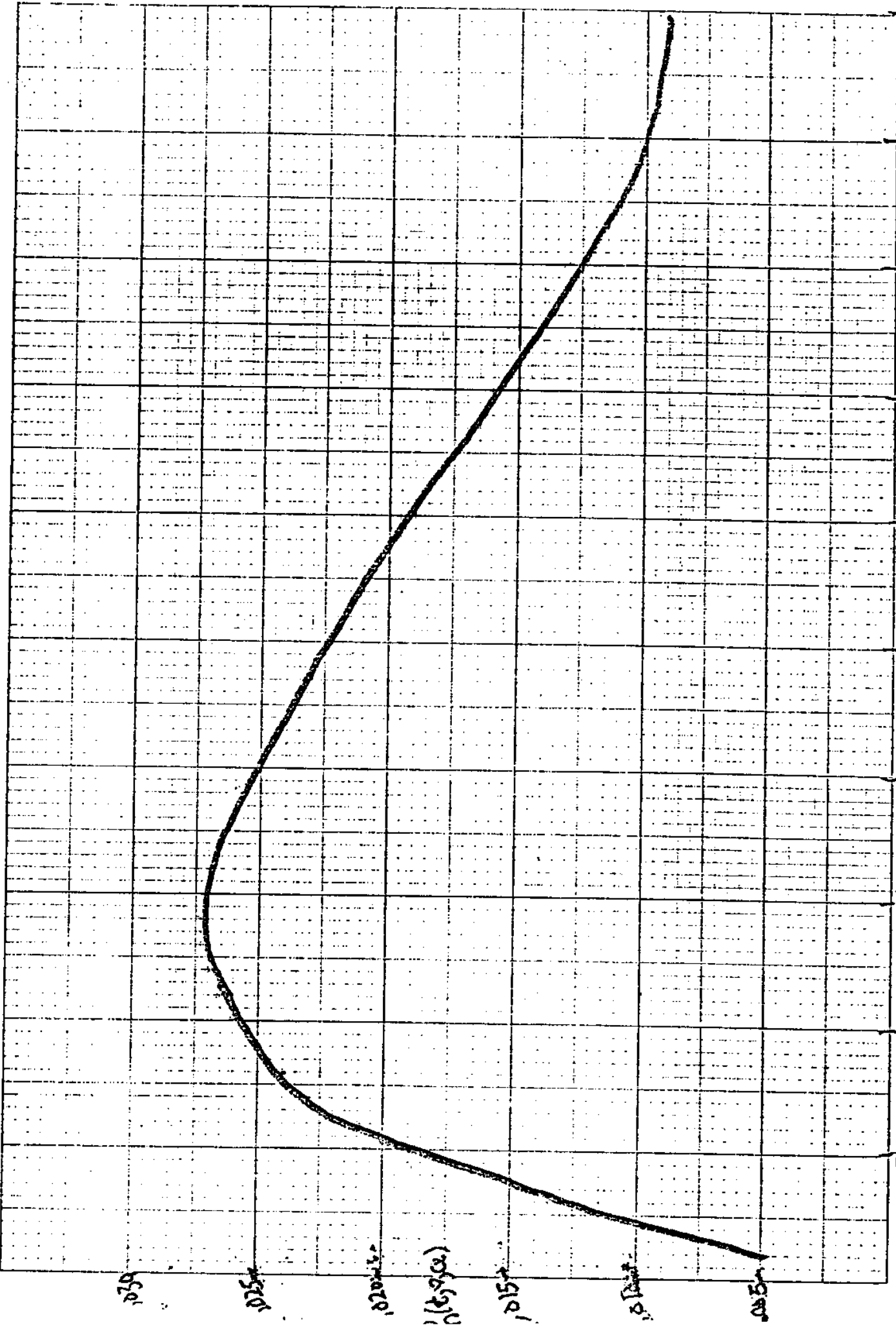
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Caption

Figure 1 The likelihood $P_n(t, v, \alpha)$ as a function of α for the POP data and $v = 1$. —



Appendix: Data Sources

The data for Pacific Ocean perch (Sebastes alutus) were given in Mangel and Beder (1985). They were obtained from log-books provided by Bruce Leaman and Rick Stanley, Pacific Biological Station, Nanaimo, BC, Canada.

The data for the crested porcupine (Hystrix indica) were provided by Phil Alkon, Blaustein Center for Desert Research, Bersheva, Israel. These data were obtained directly from trap log-books; the trapping was done for the purpose of telemetric behavioral studies. Although porcupines were not removed during the study, because of the radio tagging, recaptured porcupines could be ignored when trap counts were made.

The data for the Ivory Coast Fisheries were graciously provided by Phillip Cury who was visiting the Pacific Environmental Group, Monterey, California. The data were analyzed, from log-books, as described in Cury and Roy (1985a,b). That is, raw data were not used here, as in the previous two cases.