INFORMATION AND OPTIMAL INCENTIVE CONTRACTS

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Moral hazard in risk sharing agreements often occurs when an agent's actions can't be observed directly. Quite often, however, "statistical" information on the agent's action is available through the monitoring of production or some other performance index. Contracts with "action contingent" agreements, providing for differential payment to the agent depending on whether monitoring reveals the agent's behavior to be acceptable or unacceptable, can induce efficient behavior either if monitoring information is very reliable or if incentive incompatibilities are minor. Under less ideal conditions, only second best solutions exist. Nevertheless, there are gains to monitoring production to obtain data for use in overseeing action-contingent contracts among agents with diverse preferences and incentives.

I. INTRODUCTION

It is well known that moral hazard or incentive problems can arise in risk sharing agreements between individuals under conditions of uncertainty. The problem occurs because incentive incompatibilities are sometimes created in agreements that disperse the risks of an uncertain event among several economic agents, as in an insurance contract, for example. The divergence between individuals' incentives will depend not only on the form of the contract but on the information each agent has about the actions of other agents and about the state of nature. When an agent's information is incomplete, or when information is distributed unevenly among parties, there may be gains to acquiring additional information for use in structuring the risk sharing contract. For example, the monitoring of one agent's performance can provide information on that agent's behavior. A particular case of interest occurs when performance is measured by an observed outcome that is jointly determined by some agent's action and by a state of nature, neither of which can be observed separately. In this instance, the observed outcome provides statistical information about the observed agent's behavior, assuming the distribution for different states of nature is known. Examples of monitoring occur in the typical agency relationships between employer and employee to prevent shirking, between the government and contractor to prevent cost overuns and, between insurers and insureds to prevent moral hazard.[1] In this paper, a theory of monitoring in agency relationships is developed, where the monitor is output or some other index of performance and monitoring information is incorporated into the agency contracts in terms of payments and fines for exceptionally good and bad performances. The theory is consistent with bonus and penalty clauses frequently observed in agency agreements.

The analysis is based on a standard model of the agency relationship in which there are two representative individuals, the principal and the agent. The agent takes some action resulting in an uncertain payoff to be divided between the principal and the agent. The payoff shares are arranged to disperse risks between the two parties. Incentive incompatibilities arise because the agent has a disutility for the action while the principal does not. At this point previous analyses of the theory of agency suggest one or two ways to alleviate the incentives problem. First, the agent could be made responsible for his own action by making his payment shares dependent on the payoff.[2] This would be done, however, at the expense of increasing the risk borne by the agent under the contract--(see Spence and Zeckhauser, 1971, pg. 383). The other method, suggested by Harris and Raviv (1976) and Shavell (1977), would be to acquire additional information about the agent's behavior by monitoring other variables in addition to the payoff. Payments would then make contingent on the results of monitoring and the inferences that can be drawn about the agent's behavior. This approach, although promising, is limited in its application by the cost of independent observations.

This discussion provides the point of departure for our analysis, in which we consider contracts incorporating variants of both the contingent payment and monitoring methods for alleviating moral hazard. When, according to the agreement, the agent takes some action resulting in an uncertain payoff, he receives a net payment which may vary with the observed payoff, which is dependent on the results of monitoring. However, only the payoff itself is observed, and payments are made contingent on whether the observation reveals the agent's action to be acceptable or unacceptable. For example, in the context of employer-employee relationships, the payoff resulting from an employee's action would be output. If output is unusually low (high) it is quite likely that the laborer is shirking (working diligently) and he is therefore penalized (paid a bonus). Formally, we define a penalty (bonus) as a discrete downward (upward) jump in the payment schedule. The advantage of this monitoring scheme is that it is costless, because it utilizes information on payoffs which is already available.

This theory of contracts is consistent with penalty and bonus payments frequently incorporated in agency agreements. For example, salesmen sometimes receive bonuses for sales in excess of some amount. Insurance companies offer vastly reduced (higher) rates to customers with exceptionally good (bad) accident records. The explanation we provide for the existence of bonuses and penalties is that they can be structured to provide obvious incentives for agents to act efficiently, and to allow for the results of payoff monitoring to be incorporated in the payments schedule.

The paper proceeds as follows: The formal model of the principal-agent relationship is presented in Section II. First best agreements are constructed providing for efficient agent behavior and optimal risk spreading between the principal and agent. Conditions necessary to enforce these agreements are discussed. In Section III action-contingent contracts are introduced, in which payments are made contingent on the results of monitoring as a method to eliminate incentive problems. We demonstrate that when monitoring information is very reliable or incentive incompatibilities are minor, a first best agreement can be approximated to any desired degree by an action-contingent contract. Similar results have been reported by Harris and Raviv (1976) and Mirrlees (1974). The relationship between one principal and many independent agents is considered in Section IV, where the principal pays a bonus to the agent obtaining the highest output. Agents compete among themselves for the bonus, and the agency relationship is modeled as an n-person noncooperative game. It is demonstrated, under a certain set of conditions, that agreements between principal and agent tend toward first best contracts as the number of players, n, becomes large. In Section V the conditions on monitoring and incentive incompatibilities needed to approximate first best contracts are assumed not to hold, and we consider second best solutions. We find, nevertheless, there are gains to monitoring for use in action contingent contracts when only second best solutions are available.

II. A MODEL OF THE PRINCIPAL AND AGENT

A. The Model

We consider two representative individuals, the agent and the principal referred to as individuals A and B, respectively. The agent engages in some activity resulting in an uncertain payoff to be divided between the principal and agent. Payments to the parties are arranged according to a contract which is designed to spread risks and resolve incentive conflicts.

The agent chooses some level of activity X, where X is a scalar, and is contained in the closed interval $[0,\overline{X}]$ with $0<\overline{X}<\infty$. Depending on the context, X might represent a level of effort devoted to work or a level of effort devoted to accident prevention. The activity results in certain costs and benefits to the agent, which are represented by a net payoff function, W, with

$$W = W(X,\theta) \tag{A1}$$

where W is a twice continuous differentiable function of X and θ, and θ is a random variable which represents the state of nature.[3]

The principal and agent both possess the same information on θ, θ can not be observed before the action X is taken,[4] values for θ range in the closed interval $[\underline{\theta},\overline{\theta}]$, $0 \leq \underline{\theta} < \overline{\theta} < \infty$, and θ is distributed according to the known density function $f(\theta)$ which has the properties that[5]

$$f(\theta) > 0,\ \theta \in (\underline{\theta},\overline{\theta});\ f(\underline{\theta}) \geq 0,\ f(\overline{\theta}) \geq 0 \quad \text{(A2)}$$

$$f(\theta) \text{ is continuous, } \theta \in (\underline{\theta},\overline{\theta}) \quad \text{(A3)}$$

$$\lim_{\theta \to \underline{\theta}^{+}} f(\theta) > 0 \text{ and } \lim_{\theta \to \overline{\theta}^{-}} f(\theta) > 0 \quad \text{(A4)}$$

Assumptions (A2) and (A3) are fairly innocuous, but (A3) combined with (A4) guarantees that some probability mass exists near the extreme values of θ which, as we shall explain later, is necessary in order to use observed payoffs to monitor the agent's behavior.

In many agency relationships, the principal can observe only a portion of the actual payoff because certain costs and benefits accruing to the agent are hidden. For example, the insurer can not observe the cost incurred by the insured to prevent accidents or to stay healthy. In the context of government contracting, the government can observe the costs but none of the benefits from the project that accrue to the contractor. The partial observability of payoffs is captured by assuming that W can be written in terms of two functions, $D(X,\theta)$, which can be observed at zero cost by the principal,[6] and $C(X,\theta)$, which cannot be observed, with

$$W(X,\theta) = D(X,\theta) + C(X,\theta) \quad \text{(A5)}$$

where W, D and C are twice differentiable with respect to both arguments and concave in X.

The agency contract divides the payoff between the principal and agent according to the schedule P, which specifies a payment from the agent to the principal. Clearly, P can only be made to depend on those variables, $D(X,\theta)$, and possibly X and θ, which can observed by both the principal and the agent. Thus $P = P(D(X,\theta),X,\theta)$ and the agent receives $W - P$ and the principal receives P for a given payoff W.

Associated with the agent is a twice continuously differentiable utility function u^A, which depends on final wealth given by $w_0 + W(X,\theta) - P(D(X,\theta),X,\theta)$ where w_0 is initial wealth. We assume $u^{A'} > 0$, $u^{A''} \leq 0$ and that there is a constraint on final wealth

$$w_0 + W - P > \overline{W} \tag{A6}$$

where $\overline{W}$ is either institutionally determined or it represents a subsistence level of wealth necessary for the agent ot survive.

Define $U^A(P,X)$ by

$$U^A(P,X) \equiv \int_{\underline{\theta}}^{\overline{\theta}} u^A(Z(X,\theta))f(\theta)d\theta = E_\theta\, u^A(Z(X,\theta))$$

where $Z(X,\theta) \equiv w_0 + W(X,\theta) - P(D(X,\theta),X,\theta)$ is final wealth and E_θ is the expectations operator. Given a payments schedule, P, the agent chooses an action $\hat{X}$ to solve the following problem:

$$\max_{X\varepsilon[0,\overline{X}]} \quad U^A(P,X) \tag{1}$$

assuming that $U^A(P, \hat{X}) \geq E_\theta\, u^A(W(X,\theta)) \geq u^A(w_0)$, i.e., the agent does at least as well with the contract as he would under autarky.[7] Assuming a unique interior solution to (1), $\hat{X}$ is identified by the first order condition

$$E_\theta\, U^{A'}[Z_x(X,\theta)] = 0 \tag{2}$$

We also require that $E_\theta[u^{A''}[Z_x(X,\theta)]^2 + u^{A'}[Z_{xx}(X,\theta)]] < 0$.[8]

Associated with the principal is a twice differentiable utility function u^B which depends on final wealth, $P(D(X,\theta))$, with $u^{B'} > 0$, $u^{B''} \leq 0$. Given a schedule P and a decision X, the corresponding expected utility to the principal, $U^B(P,X)$, is defined by

$$U^B(P,X) \equiv \int_{\underline{\theta}}^{\overline{\theta}} u^B(P(D(X,\theta),X,\theta))f(\theta)d\theta = E_\theta\, u^B.$$

We assume that principals compete for contracts from agents by offering attractive payment schedules. In equilibrium, competition will force the principal to offer an agency contract such that

$$U^B(P,X) = \overline{U}^B \tag{A7}$$

where $\bar{U}^B$ is the expected utility derived from the next best alternative employment for B, and X is determined by the agent according to (1).[9]

At this stage it is worthwhile to mention a few instances for which our model is not relevant. First, in our analysis θ affects only the size of the payoff, though when viewed in some contexts like health insurance, we would expect utility to be state dependent.[10] Second, we assume that all costs and benefits from an action accruing to the principal and agent can be represented in terms of monetary equivalents. Thus, instances where there is some psychic pleasure or displeasure derived from the agent's act are not dealt with in our model. Finally, our model assumes that the parties in the agency relationship, which may include both individuals and organizations, act as individual decision making units. The collective choice problems associated with establishing a utility function for a multiperson organization are ignored, and utility functions for the principal and agent are merely assumed to exist.

B. First Best Contracts

By first best contract we mean the contract that maximizes $U^A(P,X)$ while satisfying the constraints (A6) and (A7) where X can be chosen independently of (1). This contract is to be distinguished from the best contract under moral hazard, which maximizes $U^A(P,X)$ subject to X being determined by (1). A characterization of first best optimal contracts will be presented in this section, followed by a discussion of the requirements for observing X, and θ which are necessary for best moral hazard contracts to coincide with first best contracts.

Assuming it exists, the first best contract is determined as the solution to:

$$\max_{P,X} \; U^A(P,X) \tag{3}$$

subject to (A6) and (A7) where, for now, we will express P as a function only of $D(X,\theta)$. Employing standard variational techniques and assuming an interior solution for X we obtain the necessary first order conditions[11]

$$u^{A'}(W(\theta)) - \lambda\, u^{B'}\, (P(\theta)) = 0 \;\; \forall \;\theta \tag{4}$$

$$E_\theta\, u^{A'}[W_X - P'\, D_X] - \lambda\, E_\theta\, u^{B'}[P'\, D_X] = 0 \tag{5}$$

or combining (4) and (5) yields

$$E_\theta u^{A'}[W_x] = \lambda E_\theta u^{B'}[W_x] = 0 \qquad (6)$$

Equation (4) is a familiar condition for the optimal spreading of risk. Differentiating (4) with respect to θ we can obtain

$$P' = \frac{\dfrac{u^{A''}}{u^{A'}}}{\dfrac{u^{A''}}{u^{A'}} + \dfrac{u^{B''}}{u^{B'}}} \left(\frac{W_\theta}{D_\theta}\right) \qquad (7)$$

For W = D payments are expressed as a function of the entire payoff, and we see that P' varies between 0 and 1. P' can be regarded as a measure of risk sharing between principal and agent. If the principal is risk neutral and the agent is risk averse then P' = 1, the principal bears all the risk and the agent is guaranteed a certain income. This corresponds to full coverage in the context of insurance or a wage agreement in the context of employer-employee relations. A similar straightforward interpretation can be given to the case where P' = 0. To avoid any confusion, we note that final wealth accruing to the principal and agent is the same regardless of whether P is based on the total or some portion of payoffs. In particular if P and $\tilde{P}$ are the payment schedules as a function of D and W respectively, then we require $P(D) = \tilde{P}(W)$ for all θ.

We now know a little about first best optimal contracts, but in most interesting applications of the theory of agency X is chosen by the agent according to (1). A moral hazard or incentives problem arises because the agent does not take into account the effect of his action on the welfare of the principal. Under some conditions, however, the principal can obtain sufficient information to construct a payments schedule that induces the agent to act efficiently. These conditions, which are formally analyzed in Harris and Raviv (1976), Leland (1975) and Spence and Zeckhauser (1971) relate to the principal's ability to observe X and θ ex post, once the action is chosen and the state of nature is realized.[12] One case in which the agent can be induced to act efficiently is when D(X,θ) and X can be observed, for then the principal can force the agent to choose the efficient $X = X^{**}$ by making $\underline{P}(D(X,\theta),X)$ sufficiently large for $X \neq X^{**}$. Obviously, the principal can also induce X** whenever he can monitor $D(X,\theta)$ and θ, since X can be inferred from these two observations. It is rather surprising, however, that the agent can also be made to choose X** when only θ can be observed. To demonstrate this formally, note that according to (6) the first order condition for a first best optimum is $Eu^{A'}[W_x] = 0$ which can be shown to be identical to the first order conditions for the maximization in (2) when the appropriate payment schedule $P = P(\theta)$ is chosen. Finally, Harris and Raviv (1976) demonstrate that the agent will automatically act efficiently if he is risk neutral. Henceforth, we shall assume the agent is risk averse and $u^{A''} < 0$.

C. Biases Under Moral Hazard

The conditions on observability and risk preferences sufficient to insure efficiency are, unfortunately, frequently not satisifed. It is more likely, for example, that the principal can observe the joint effects of X and θ as revealed by the value of $D(X,\theta)$, but that he cannot observe X or θ separately. This is the case to be considered throughout the remainder of the paper. Without direct observations on X or θ, incentive problems arise, and we can predict under a general set of conditions how this will bias the agent's choice of X.

To proceed with the analysis we will assume that W and D are monotonic with respect to θ and that W_θ and D_θ are both positive. This implies that D is a surrogate for W because both variables move in the same direction for different values of θ. We shall define $D(X,\theta)$ as being a cost (benefit) accruing to the agent from the action X whenever $D_x < 0$ (>0) for $X \epsilon [0,\overline{X}]$. Denote the first best action by X**, the optimal payment schedule by P**, and action chosen by the agent to solve (2) by $\hat{X}$. We can now state

PROPOSITION 1. The agent will choose an $\hat{X}$ greater than (less than) X** under P** (D) whenever D_x is less than (greater than) zero.

PROOF: It suffices for us to show that according to (2) which we assume is sufficient to determine $\hat{X}$ that $E_\theta u^{A'}[Z_x(X^{**},\theta)] > (<) \ 0$ as $D_x < (>) \ 0$, and that the equality in (2) holds for an $\hat{X} > (<) \ X^{**}$ as $D_x < (>) \ 0$. Define $\Phi(X) = E_\theta u^{A'}[W_x]$ and note $\Phi(X^{**}) = 0$ by (6). We will now show that the concavity of W with respect to X and the fact that for each θ and X $P^{**}(D(X,\theta)) = \tilde{P}^{**}(W(X,\theta))$, where $\tilde{P}^{**}$ is the optimal payment schedule as a function of W, imply $\Phi'(X) < 0$.

$$\begin{aligned}\Phi'(x) &= E_\theta u^{A''}[D_x + C_x - P^{**'}\,D_x][W_x] + E_\theta u^{A'}[W_{xx}] \\ &= E_\theta u^{A''}[D_x + C_x - \tilde{P}^{**'}\,W_x][W_x] + E_\theta u^{A'}[W_{xx}] \\ &= E_\theta u^{A''}[(1 - \tilde{P}^{**'})][W_x]^2 + E_\theta u^{A'}[W_{xx}] < 0\end{aligned} \tag{8}$$

From the fact that $\Phi(x^{**}) = 0$ and $P^{**'} > 0$ which follows from (7), we obtain

$$Eu^{A'}[Z_x(x^{**},\theta)] = \Phi(x^{**}) + Eu^{A'}[-P^{**'}\,D_x] \lessgtr 0 \text{ as } D_x \gtrless 0. \tag{9}$$

Thus to satisfy the first order condition in (2) for the agent's maximization, $\Phi'(x) < 0$ implies $\hat{x} \lessgtr x^{**}$ as $D_x \gtrless 0$.

A heuristic explanation of the result in proposition 1 is that because of the incomplete information available to the principal, payments must be based on only a portion of the payoff and this introduces diverse incentives between the principal and agent. When $D_x > 0$, for example, and payments are based only on benefits, then the principal shares some of the benefits but none of the costs of the agent's action. The agent has a disutility for the action while the principal does not, and consequently the agent chooses an $\hat{x}$ which is less than optimal.[13] The results of proposition 1 are illustrated with several examples of agency relationships.

(a) Employer-Employee Relationships. Without information on the level of effort put forth by the employee, the employer can only observe and make remuneration dependent on output. Consequently there is a tendency for the employee to shirk, since he has a disutility for effort while the principal does not.

(b) Government Contracting. Cost overruns are common with defense contracting since the government and contractor share only the costs of the contractor's decisions, but the contractor retains some of the benefits. These hidden benefits, which take the form of R & D advances that are useful to the firm in developing products for the private sector, induce excessive expenditures on the part of the firm in satisfying government contracts.

(c) Accident and Health Insurance. The biases introduced by prepaid insurance depend on whether actions taken by the insured which can't be observed by the insurer increase or reduce the cost of being ill or having an accident. If the action is preventive and reduces costs to the insurer, there will be insufficient attention to that action on the part of the insured. If the action, provides some intangible benefits to the insured, such as the extra attention a patient receives as a consequence of more frequent visits to the doctor, then medical facilities will be overused.

III. ACTION CONTINGENT CONTRACTS

The foregoing analysis suggests that the information required for the principal to enforce a first best optimal contract may not be available. Thus, one needs to consider best contracts under moral hazard in which the self maximizing behavior of the agent in choosing an action is explicitly acknowledged. The agent's behavior will be shaped by the payments schedule he faces, and one alternative is to choose the schedule P(D) to maximize $U^A(P,X)$ subject to (A6), (A7) and the constraint that X is chosen by the agent according to (1). Another approach is for the principal to acquire additional information about the agent's behavior by monitoring other variables in addition to the payoff, as suggested by Harris and Raviv (1976) and Shavell (1977). They demonstrate under certain conditions that contracts with payments being made contingent on the results of monitoring can dominate contracts which do not involve monitoring.

In this section we analyze a contract that incorporates the results of monitoring, where the observed payoff, $D(X,\theta)$, is utilized to make statistical inferences about the agent's behavior. An action contingent contract is introduced in which payments are made contingent on whether the observation of D reveals the agent's action to be acceptable or unacceptable. The advantage of this monitoring scheme is that it is costless, because it utilizes information on payoffs which is already available. We identify conditions under which a first best contract can be approximated to any desired degree by an action contingent contract. Finally, the application of these results is illustrated with an insurance example.

We begin by noting that observed payoffs, $D(X,\theta)$, provide statistical information about the value of X chosen by the agent. The assumption that $D_\theta > 0$ implies we can solve for θ as a function of X and D with

$$\theta = h(X,D) \tag{10}$$

Then, for an observed value of D, we can represent the conditional distribution for X, denoted by g(X/D) as

$$g(X/D) = f(h(X,D))\ |h_X| \tag{11}$$

Thus the principal has information on the relative likelihood that the agent has chosen a certain value for X from ex post observations of D. If $D_x > 0$, for example, one would expect that the conditional probability that the agent will choose an X greater than some level increases with larger observed values of D.[14]

Recall from proposition 1 that whenever $D_x > 0$ there is a tendency for the agent to choose an $\hat{X} < X^{**}$. Then for a low (high) observed value of D, the principal can infer with high probability that the agent is (is not) shirking. To reduce the incentive to shirk, the agent might be penalized (or paid a bonus) whenever extremely low or high values of D are observed. The same bonus-penalty scheme would eliminate incentives for the agent to overact when $D_x < 0$.[15]

In the action contingent contract the principal incorporates the results of monitoring into the action contingent payment schedule by penalizing the agent whenever observed D is low and by rewarding the agent whenever D is high. In this section we introduce the action contingent contract with penalties, noting that the development is similar for contracts with bonuses. To make the analysis more concrete and to simplify the exposition henceforth we restrict our attention to the case

$$D(X,\theta) \text{ concave in } X \text{ and } \theta,\ D_x > 0,\ D_{x\theta} > 0,\ C = C(X) \text{ and } C_x < 0 \qquad (A8)$$

The development of results for other cases should be apparent from our discussion. Assumptions (A8) apply to an insurance or employee-employer relationship in which the action X is beneficial in reducing insurance costs or increasing output, and associated with the action is a cost, C(X), incurred by the agent.

The action contingent contract is defined as a payment schedule denoted by $P(\hat{D})$ with the properties[16]

$$P(\hat{D}) = \begin{cases} P^{**}(D) - \delta(\hat{D},X^{**}) & \text{if } D > \hat{D} \\ \overline{P}(D,\hat{D}) & \text{if } D \leq \hat{D} \end{cases} \qquad (12)$$

Payments are given by the continuous function $\overline{P}(D,\hat{D}) > P^{**}(D)$, whenever D is less than some specified level $\hat{D}$, and the agent is penalized. Otherwise payments are given by $P^{**}(D)$ adjusted by the constant, $\delta(\hat{D},X^{**})$, to insure

(A7) is satisfied. Payment schedules vary with the principal's choice of $\hat{D}$, which is emphasized by writing P, $\overline{P}$, and δ as explicit functions of $\hat{D}$.

The constant $\delta(\hat{D},X^{**})$ is derived as follows. Given an observed D, the principal tests the hypothesis H_o: the agent is shirking, against the alternative hypothesis H_A: the agent is not shirking, and accepts H_o whenever $D \leq \hat{D}$. Assume for now that the agent can be induced to choose X**, given the schedule $P(\hat{D})$. Then for $\theta \leq \hat{\theta} = h(\hat{D},X^{**})$, $D(X^{**},\theta) \leq \hat{D}$ and the principal incorrectly accepts H_o, i.e., he commits a type II error and penalizes the agent. Otherwise, for $\theta > \hat{\theta}$, $D(X^{**},\theta) > \hat{D}$ and the agent pays $P^{**}(D) - \delta(X^{**},\hat{D})$, where δ is defined by the expression

$$\int_{\underline{\theta}}^{\hat{\theta}(\hat{D},X^{**})} u^B(\overline{P}(D,\hat{D}))f(\theta)d\theta + \int_{\hat{\theta}(\hat{D},X^{**})}^{\overline{\theta}} u^B(P^{**}(D) - \delta(X^{**},\hat{D}))f(\theta)d\theta = \overline{U}^B \tag{13}$$

The constant δ represents the variation in the payment schedule necessary to maintain the expected utility of the principal at the same level $\overline{U}^B$ for different payment-penalty schedules.

The problem in designing $P(\hat{D})$ is that type II errors are undesirable because they reduce the expected utility of the agent, nonetheless it is necessary to preserve the threat of penalizing the agent for small values of D in order to discourage shirking. In what follows we derive conditions for making the welfare loss from type II errors arbitrarily small while maintaining a penalty threat sufficient to prevent shirking. With these conditions, it will be seen that a first best optimal contract can be approximated to any desired degree by an action contingent contract.

An action level X** and payment schedule P are said to be <u>enforceable</u>, or merely X** is enforceable, if the agent can be induced to choose X**, given the schedule P. To simplify notation define

$$z^1(X,\theta) = w_o + W(X,\theta) - \overline{P}(D(X,\theta),\hat{D})$$
$$z^2(X,\theta) = w_o + W(X,\theta) - P^{**}(D(X,\theta)) + \delta(X^{**},\hat{D})$$
$E_{\theta \leq \hat{\theta}}$ = expectations operator for $\theta \in [\underline{\theta},\hat{\theta}]$

$E_{\theta > \hat{\theta}}$ = expectations operator for $\theta \in (\hat{\theta},\overline{\theta})$

Thus according to $P(\hat{D})$, $Z = Z^1$ if $D \leq \hat{D}$ and $Z = Z^2$ if $D > \hat{D}$. Then a necessary condition for enforceability of X**, derived from (2) and (12), is

$$U_x^{A'}(P(\hat{D}),X^{**}) = E_{\theta \leq \hat{\theta}}\, u^{A'}(Z^1)[Z_x^1] + E_{\theta > \hat{\theta}}\, u^{A'}(Z^2) \cdot [Z_x^2]$$

$$+ f(\hat{\theta})\, \frac{d\hat{\theta}}{dX}\, [u^A(Z^1) - u^A(Z^2)] = 0 \tag{14}$$

where $X = X^{**}$ and $\hat{\theta} = h(\hat{D},X^{**})$.

In order to reduce the probability of a type II error, we would like to construct payments schedules for which X** is enforceable as $\hat{D} \to D(X^{**},\underline{\theta})$ or $\hat{\theta} \to \underline{\theta}$. Necessary conditions for enforceability of X** in the limit as $\hat{\theta} \to \underline{\theta}$ are given in

<u>Lemma 1</u>: Necessary conditions for the enforceability of X** in the limit as $\hat{\theta} \to \underline{\theta}^+$ are that there exist a payment schedule $P(D(X^{**},\underline{\theta}))$ such that

(a) $Z^1(X^{**},\theta) > \overline{W}$

(b) $E_\theta u^{A'}(Z(X^{**},\theta))[Z_x(X^{**},\theta)] + [u^A(Z^1(X^{**},\underline{\theta}))$

$- u^A(Z(X^{**},\underline{\theta}))]\, \frac{d\theta}{dX} \lim_{\theta \to \underline{\theta}^+} f(\theta) = 0$

where $Z(X^{**},\theta) = w_o + W(X^{**},\theta) - P^{**}(D(X^{**},\theta))$.

<u>Proof</u>: The necessity of (a) follows from (A6).

To establish (b) note that enforceability requires $U_x^A(P(\hat{D}),X^{**}) = 0$. Thus enforceability in the limit as $\hat{\theta} \to \underline{\theta}^+$ requires $\lim_{\hat{\theta} \to \underline{\theta}^+} U_x^A(P(\hat{D}),X^{**}) = 0$ which is precisely the condition written out in (b). The existence of the limit is guaranteed by the continuity of the functions u^A, $\overline{P}$, f, D, and C, though the continuity assumptions are not necessary.

Condition (b) in lemma 1 has an intuitive interpretation. The term $E_\theta u^{A'}(Z(X^{**},\theta))[Z_x]$, which is negative, measures the incentive, in expected utility terms, for the agent to shirk. The term $[u^A(Z^1(X^{**},\underline{\theta})) - u^A(Z(X^{**},\underline{\theta}))]$ is the loss in utility from penalties. Finally, for a given $\hat{\theta}$, the probability of being penalized is $\int_{\underline{\theta}}^{\hat{\theta}(\hat{D},X^{**})} f(\theta)d\theta$. Thus, $f(\hat{\theta})\frac{d\hat{\theta}}{dX}$, is the reduction in the probability of penalization for an increase in X, X = X**. Taken together, (b) implies that if X** is enforceable in the limit, the returns from shirking are offset by the increase in the probability of being penalized weighted by the utility cost of the penalty. The prospects for enforcing X** in the limit will depend on the (1) agent's tendency to shirk, reflected in the magnitude of $E_\theta u^{A'}[Z_x]$, (2) the potential for the principal to penalize the agent, which is limited by (A6) and (3) the sensitivity of the penalty region to variations in X. The necessity of (A4) is now made apparent. The effect of penalization as a deterrent to shirking varies directly with the probability mass situated near the penalty boundary point $\hat{\theta}$.

If the necessary conditions for enforceability are satisfied in the limit, we would expect the enforceability conditions to hold for values of $\hat{\theta}$ sufficiently close to $\underline{\theta}$, and this is the message of

<u>Lemma 2</u>: If there exists a $P(D(X^{**},\underline{\theta}))$ such that conditions (a) and (b) of lemma 1 are satisfied, then there exist schedules, $P(\hat{D})$, which satisfy the necessary first and second order enforceability conditions

(a) $U_x^A\ (P(\hat{D}),X^{**}) = 0$

(b) $U_{xx}^A\ (P(\hat{D}),X^{**}) < 0$

for $\hat{\theta} \in (\underline{\theta},\theta')$ where $\hat{\theta} = \hat{\theta}(\hat{D},X^{**})$ and $\theta' > \underline{\theta}$.

<u>Proof</u>: <u>Part a</u> The limiting schedule, $P(D(X^{**},\underline{\theta}))$ can be represented by

$$\underline{P}(\overline{D}) = \begin{cases} P^{**}(D) & \text{if } D > \overline{D} \\ \overline{P}(\overline{D},\overline{D}) & \text{if } D \leq \overline{D} \end{cases}$$

where $\overline{D} = D(X^{**},\underline{\theta})$ and we note $\delta(X^{**},\hat{D}) \to 0$ as $\hat{D} \to \overline{D}$.

Consider an arbitrary continuous penalty schedule $\overline{P}_o(D,\hat{D})$ which satisfies the requirements that it be bounded below by $P^{**}(D)$, that it be bounded above by the constraint (A6), and that $\overline{P}_o(\overline{D},\hat{D}) = \overline{P}(\overline{D},\overline{D})$.

Then construct two new payments schedules

$$P_1(\hat{D}) = \begin{cases} P^{**}(D) + \delta_1(\hat{D},X^{**}) & \text{if } D > \hat{D} \\ \overline{P}_o(D,\hat{D}) - \varepsilon & \text{if } D \leq \hat{D} \end{cases}, \quad P_2(\hat{D}) = \begin{cases} P^{**}(D) + \delta_2(\hat{D},X^{**}) & \text{if } D > \hat{D} \\ \overline{P}_o(D,\hat{D}) + \varepsilon & \text{if } D \leq \hat{D} \end{cases}$$

where ε is a positive constant, small enough to maintain the boundedness properties of the penalty schedule and δ_1 and δ_2 are implicitly defined by (A7). Then for $\hat{D} = \overline{D}$ we have

$$P_1(\overline{D}) = \begin{cases} P^{**}(D) & \text{if } D > \overline{D} \\ \overline{P}(\overline{D},\overline{D}) - \varepsilon & \text{if } D \leq \overline{D} \end{cases} \qquad P_2(\overline{D}) = \begin{cases} P^{**}(D) & \text{if } D > \overline{D} \\ \overline{P}(\overline{D},\overline{D}) + \varepsilon & \text{if } D \leq \overline{D} \end{cases}$$

and it is easy to verify that $U_x^A(P_1(\overline{D}),X^{**}) < U_x^A(P(\overline{D}),X^{**}) = 0 < U_x^A(P_2(\overline{D}),X^{**})$. The schedules P_1 and P_2 are continuous functions of $\hat{D}$, so that $U_x^A(P_1,X^{**})$ and $U_x^A(P_2,X^{**})$ are continuous in $\hat{D}$ as well. Therefore there exists a θ' sufficiently close to θ such that $U_x^A(P_1(\hat{D}),X^{**}) < 0 < U_x^A(P_2(\hat{D}),X^{**})$, for $\hat{D} = D(X^{**},\hat{\theta})$, $\hat{\theta} \in (\underline{\theta},\theta')$. We wish now to show that there exists another payment schedule P_3, constructed from P_1 and P_2 such that $U_x^A(P_3,X^{**})$, which is defined relative to P_3, is zero. Consider the schedule

$$P_3(\hat{D},\mu) = \begin{cases} P^{**}(D) + \delta_3(D,X^{**},\mu) & \text{if } D > \hat{D} \\ \overline{P}_o(D,\hat{D}) + \mu & \text{if } D \leq \hat{D} \end{cases}$$

where μ ranges in the interval $[-\varepsilon, \varepsilon]$. We can easily verify that $U_x^A(P_3(\hat{D},\mu),X^{**})$ is continuous in μ since P_3 varies continuously with μ. But $U_x^A(P_3(\hat{D},-\varepsilon),X^{**}) < 0 < U_x^A(P_3(\hat{D},\varepsilon),X^{**})$ and the continuity of U_x^A with respect to μ implies there exists at least one μ_o such that $-\varepsilon < \mu_o < \varepsilon$ such that $U_x^A(P_3(\hat{D},\mu),X^{**}) = 0$.

Part b See the Appendix.

Lemmas 1 and 2 describe situations in which we can construct schedules, $\underline{P}(\hat{D})$, which satisfy the necessary enforceability conditions for $\hat{\theta} \in (\underline{\theta},\theta')$. Sufficient conditions for enforceability are given in

Lemma 3: If (a) $U^A(P^{**}(D),X)$ is concave in X

(b) Schedules $P(\hat{D})$ exist such that $U_X^A(P(\hat{D}),X^{**}) = 0$ for $\hat{\theta} \in (\underline{\theta},\theta')$, $\theta' > \underline{\theta}$

then $(P(\hat{D}),X^{**})$ is enforceable for $\hat{\theta} \in (\underline{\theta},\theta')$.

Proof: See the Appendix.

Lemmas 1-3 provide the necessary basis for establishing the primary result of this section.

Proposition 2. If there exist payment schedules, $P(\hat{D})$, such that X** is enforceable for $\hat{\theta} \in (\underline{\theta},\theta')$ then a first best contract can be approximated to any desired degree by an action contingent contract.

Proof: By the construction of $\delta(X^{**},\hat{D})$ we know $U^B(P(\hat{D}),X^{**}) = \overline{U}^B$ for all $\hat{D}$. Thus, we need to show $U^A(P(\hat{D}),X^{**}) \to U^A(P^{**},X^{**})$ as $\hat{\theta} \to \underline{\theta}^+$ to establish the proposition.

Define

(i) $$Z^{**}(X^{**},\theta) = w_o + W(X^{**},\theta) - P^{**}(D(X^{**},\theta))$$

(ii) $$\hat{Z}(X^{**},\theta) = w_o + W(X^{**},\theta) - P^{**}(D(X^{**},\theta)) + \delta(X^{**},\hat{D}) \quad \text{if } D > \hat{D}$$

$$= w_o + W(X^{*},\theta) - \overline{P}(D(X^{**},\theta),\hat{D}) \quad \text{if } D \leq \hat{D}$$

(iii) $$\underline{Z} = \inf_{\theta}(\hat{Z}(X^{**},\theta))$$

(iv) $$\Phi(\hat{D}) = U^A(P^{**},X^{**}) - U^A(P(\hat{D}),X^{**})$$

Note that $\underline{Z}$ exists since $\hat{Z}(X,\theta)$ is bounded away from $\overline{W}$ by (A6). Then we have

$$\Phi(\hat{D}) = E_{\theta \leq \hat{\theta}} [u^A(Z^{**}) - u^A(\hat{Z})] + E_{\theta > \hat{\theta}} [u^A(Z^{**}) - u^A(\hat{Z})]$$

$$\leq E_{\theta \leq \hat{\theta}} [u^A(Z^{**}) - u^A(\hat{Z})]$$

$$\leq E_{\theta \leq \hat{\theta}} [u^A(Z^{**}) - u^A(\underline{Z})] \to 0 \text{ as } \hat{\theta} \to \underline{\theta}^+.$$

Proposition 2 implies that if action contingent contracts with penalties can be used to enforce X**, then the first best optimum contract can be approximated to any desired degree, though it can never be achieved.[17] It is a mathematical curiosus that the optimal action contingent contract does not exist. Similar results occur (as demonstrated in the next section) when action contingent contracts with bonuses are used to enforce X**. Lemmas 1-3 deal with the requirements for enforceability and indicate that whenever (a) incentive problems are minor, (b) monitoring information is very reliable or (c) large penalties for shirking can be imposed, then it is possible to induce X** with an action contingent contract. These results are illustrated with the following example provided by Spence and Zeckhauser (1971) in their discussion of insurance.

In terms of our notation, the Spence-Zeckhauser example involves an insurer who is risk neutral and who faces a breakeven constraint

$$E_\theta \, P(D(X,\theta)) = 0 \tag{15}$$

and an insured who has a utility function $u^A(Z) = \log(Z)$, where $Z > 0$. The insured takes an action X resulting in a payoff $W(X,\theta) = (X\cdot\theta)^{1/2} - X$ where $D(X,\theta) = (X\cdot\theta)^{1/2}$ and $C(X) = -X$, and $\theta \in [\underline{\theta},\overline{\theta}]$ is a random variable which is uniformly distributed. Under these conditions the first best optimal contract requires that final wealth $Z = \overline{Z}$ be constant for all θ implying that

$$P^{**'} = 1 \tag{16}$$

and that X be determined by

$$E_\theta \, W_x(X,\theta) = 0 \tag{17}$$

However the insurer cannot observe X directly and, given a payment schedule P, the insured will choose $\hat{X}$ according to

$$E_\theta\, u^{A'}[Z_x(X,P)] \leq 0 \tag{18}$$

with $\hat{X} = 0$ and

$$E_\theta\, u^{A'}[-1] < 0 \tag{19}$$

for $P = P^{**}$. According to Spence and Zeckhauser, a second best solution for this problem is to choose P to maximize $U^A(P,X)$ subject to (15) and (18). The payment schedule resulting from this maximization is given by

$$\tilde{P} = D - \alpha - B(1+\gamma D)^{1/2} \tag{20}$$

where α, B, and γ are positive parameters.

However, we can construct an action contingent contract

$$P(\hat{D}) = \begin{cases} P^{**}(D) - \delta(X^{**},\hat{D}) = D + C(X^{**}) - \overline{Z} + \delta & \text{if } D > \hat{D} \\ \overline{P}(D,\hat{D}) & \text{if } D \leq \hat{D} \end{cases}$$

which dominates $\tilde{P}$ and approximates $P^{**}(D)$ to any desired degree. To see this, note that $u^A(0) = -\infty$ and thus arbitrarily large penalties can be imposed on the agent by choosing $\overline{P}(D,\hat{D})$ sufficiently large such that Z is small. In this case, the necessary limit conditions for enforceability in Lemma 1 are satisfied, assuming $\underline{\theta} > 0$, which in this example implies $\frac{d\hat{\theta}}{dX} \neq 0$. Since $U^A(P^{**},X)$ is concave in X and the necessary and sufficient conditions for enforceability in Lemma 3 are satisfied, X^{**} is enforceable for $\hat{\theta} \in (\underline{\theta},\theta')$ and Proposition 2 implies $P^{**}(D)$ can be approximated to any desired extent.

Note that the action contingent contract is unfair in that there is a small probability that the insured will be penalized severely even though he acts optimally to choose X**. Nevertheless, this injustice is outweighed by the performance of the contract and there would be a loss in efficiency if such penalties were prohibited. Alternatively, the insured might be penalized, independent of the observed value of D, only if he could not verify he had chosen X**. With random verification and large penalties for shirking it might be possible to enforce X**. However, the costs of verification borne by the insured would result in a loss of welfare as compared to the action contingent contract.[18]

One may inquire if it is desirable to construct action contingent contracts where the penalty region falls in the interior of the feasible set of D values, such that

$$P(\hat{D}_1,\hat{D}_2) = \begin{cases} P^{**}(D) + \delta(X^{**},\hat{D}_1,\hat{D}_2) & \text{if } D \notin [\hat{D}_1,\hat{D}_2] \\ \overline{P}(D,\hat{D}_1,\hat{D}_2) & \text{if } D \in (\hat{D}_1,\hat{D}_2) \end{cases}$$

where $(\hat{D}_1,\hat{D}_2)$ is a penalization interval in the interior of the feasible set of D. The problem with this schedule is that it is impossible to enforce X** because the penalization interval gets arbitrarily small as $\hat{D}_2 \to \hat{D}_1^+$. To see this, let $Z^1 = w_o + W(X^{**},\theta) - \overline{P}(D(X^{**},\theta))$, $Z^2 = w_o + W(X^{**},\theta) - P^{**}(D(X^{**},\theta) + \delta(X^{**},\hat{D}_1,\hat{D}_2)$, $\hat{\theta}_1 = h(\hat{D}_1,X^{**})$ and $\hat{\theta}_2 = h(\hat{D}_2,X^{**})$. Then

$$U_x^A(P(\hat{D}_1,\hat{D}_2),X^{**}) = E_{\theta\leq\hat{\theta}_1}\, u^{A'}(Z^2)[X_x^2] + E_{\hat{\theta}_1<\theta<\hat{\theta}_2}\, u^{A'}(Z^1)[Z_x^1] + E_{\theta\geq\theta}\, u^A(Z^2)[Z_x^2]$$
$$+ f(\hat{\theta}_1)\frac{d\hat{\theta}_1}{dX}[u^A(Z^1(\hat{\theta}_1)) - u^A(Z^2(\hat{\theta}_1))] + f(\hat{\theta}_2)\frac{d\hat{\theta}_2}{dX}$$
$$[u^A(Z^2(\hat{\theta}_2)) - u^A(Z^1(\hat{\theta}_2))] \qquad (21)$$

It follows that $\lim_{\hat{\theta}_2\to\hat{\theta}_1} U_x^A(P(\hat{D}_1,\hat{D}_2),X^{**}) = E_\theta u^{A'}(Z_x) < 0$. It is consequently impossible to enforce X** without some minimum probability of making a type II error which reduces the welfare of the agent.

The necessary conditions for enforcing an optimal action X** presented here and in the next section are admittedly quite strong and frequently will not be satisfied. In section V, however, we will demonstrate that an action contingent contract is beneficial for both the principal and agent under quite general conditions, even when it is not possible to approximate first best optima.

IV. N PLAYER BONUS SYSTEM

In agency relationships involving one principal and several agents, one frequently observes what we shall call an N player bonus system. Employer-employee relationships in which the worker with the highest output or the salesman with the greatest sales is paid a bonus are examples of the bonus system. The opportunity to earn a bonus provides incentives for all participants to try harder. In this section we demonstrate that the N player bonus system behaves similarly to the action contingent contract as a device to prevent shirking.

Formally, the bonus system is described as an N person noncooperative game. There are N independent, identical agents directed by a single principal. Each agent A_i chooses an action X_i resulting in a payoff $w(X,\theta_i) = D(X_i,\theta_i) + c(X)$, where the functions W, D and C have the properties listed in (A8). The random variables θ_i are identical and jointly distributed according to the continuous joint density function $f_N(\theta_1 \ldots \theta_N)$. Marginal density functions are denoted by $f_N(\theta_i)$ and are assumed to have the property that $f_N(\theta_i) > 0$ for $\theta_i \in [\underline{\theta},\overline{\theta}]$.[19] Generally we would expect the θ_i's to be correlated with each other to reflect the similarity in working conditions confronting individual agents.

For mathematical simplicity we shall assume the principal is risk neutral, although the results we are to present hold under risk aversion as well. If the principal could enforce a first best contract he would choose a schedule $P_N^{**}(D)$ and an X_N^{**} (N denoting the dependence on the number of agents) as the solution to

$$\max_{P_N, X} \; \mathop{E}_{\theta_1 \ldots \theta_N} \left[\sum_{i=1}^{N} u^{A_i}[w_0 + W(X_i,\theta_i) - P_N(D(X_i,\theta_i))] \right] \tag{22}$$

subject to a breakeven constraint for the principal

$$\mathop{E}_{\theta_1 \ldots \theta_N} \sum_{i=1}^{N} P_N(D(X_i,\theta_i)) - G = 0 \tag{23}$$

where G is a market determined competitive return for the principal. Equation (23) constitutes a particular form of revenue sharing where the principal expects to collect a return G while he redistributes net payoffs to the agents to maximize the expected sum of their utilities. The solution to (22) requires $P_N^{**\prime}(D) = 1$ and that each agent receive the same net payoff, independent of $D(X_i,\theta_i)$. Confronted with the schedule P_N^{**} each agent will shirk and choose an $X_i = 0$.

The bonus system may be used to prevent shirking; one particular form of the system is given by the schedule

$$\tilde{P}_N = \begin{cases} P_N^{**}(D) + \delta_N & \text{if } D < \max_i (D(X_i,\theta_i)) \\ P_N^{**}(D) - B_N & \text{if } D = \max_i (D(X_i,\theta_i)) \end{cases}$$

where $B_N < \infty$ is the positive reward given to the worker or salesman with the highest output or sales and δ_N, defined by

$$E_{\theta_1 \ldots \theta_N} [\Sigma \tilde{P}_N(D(X^{**},\theta_i))] - G = 0$$

is a compensation to the principal for the extra bonuses paid out under $\tilde{P}_N$, assuming $X_i = X^{**}$ for all i. Bonus payments such as B_N are frequently used to reward the employee with the best apparent performance based on sales or output. Bonuses given to the worker with the least number of absences or the best accident record also are examples of payment systems that reward hard work and diligence.

We assume there is no collusion and, given a payment schedule, each agent independently chooses an action to maximize his perceived expected utility. For a particular $D(X_i,\theta_i) = D_i$, the probability, denoted by $P_r(B_N|D_i,X')$, that agent A_i will receive a bonus, givern the other N - 1 agents choose an action X', is the probability that

$$\theta_j \le \hat{\theta}(D_i,X') = h(D_i,X') \text{ for } j \neq i \text{ or}$$

$$Pr(B_N|D_i,X') = F_{N-1}(\hat{\theta}\ldots\hat{\theta}|\theta_i) \tag{25}$$

where $F_{N-1}(\hat{\theta}\ldots\hat{\theta}|\theta_i)$ is the conditional probability that $\theta_j \le \hat{\theta}$ for $j \neq i$ given θ_i.

Define

$$Z(X_i,\theta_i) = \begin{cases} Z^1(X_i,\theta_i) = w_0 + W(X_i,\theta_i) - P_N^{**}(D(X_i,\theta_i) - \delta_N & \text{if } D < \max_i D_i \\ Z^2(X_i,A_i) = w_0 + W(X_i,\theta_i) - P_N^{**}(D(X_i,\theta_i)) + B_N & \text{if } D = \max_i D_i \end{cases}$$

and

$$V^{A_i}(D_i,X') = Pr(B_N|D_i,X')\ u^{A_i}(Z^2) + [1-Pr(B_N|D_i,X')]\ u^{A_i}(Z^1)$$

where V^{A_i} is understood to be a function of X_i. (The explicit dependence of some functions on their arguments will be dropped where no confusion exists). Then the expected utility of agent i given $\tilde{P}_N$, X_i, and X' is given by

$$U^{A_i}(\tilde{P}_N,X_i,X') = E_{\theta_i} V^{A_i} \tag{28}$$

and agent i chooses $X_i \in [0,\overline{X}]$ to maximize (28).[20] Assuming a unique interior solution, a necessary condition for the maximization of (28) is

$$U_x^{A_i} = \int_{\underline{\theta}}^{\overline{\theta}} \{Pr \cdot u^{A_i'}(Z^2)[Z_x^2] + (1-Pr)\ u^{A_i'}(Z^1)[Z_x^1]\}\ f_N(\theta_i)\ d\theta_i$$

$$+ \int_{\underline{\theta}}^{\overline{\theta}} [u^{A_i}(Z^2) - u^{A_i}(Z^1)]\ Pr_x\ f_N(\theta_i)\ d\theta_i = 0 \tag{29}$$

We assume the agents are symmetric so that each one uses the same strategy in choosing X_i. The above construction defines a noncooperative game with incomplete information, with agents competing against one another for the bonus. Because the game is symmetric, and equilibrium strategy is one that is optimal for each agent if each other agent uses it. In this section we identify necessary conditions underwhich X_N^{**} is an equilibrium strategy and thus a choice of X_N^{**} for each agent is enforceable. For two reasons, we are particularly interested in the properties of the N-player bonus sytem as the number of players becomes large. First, large scale bonus competitions involving many participants exist in the real world in the form of factorywide competitions among workers and national and regional competitions among salesmen of a certain product. Second, we demonstrate in Proposition 3, that whenever X_N^{**} exists as an equilibrium strategy given $\tilde{P}_N$ then the bonus contract approaches the first best contract for N sufficiently large. Necessary conditions for a unique equilibrium strategy X_N^{**} to exist for N large are contained in lemmas 4-6.

To proceed further with the analysis we shall assume

$$F_{N-1}(\hat{\theta}\ldots\hat{\theta}|\theta_i) \text{ is a differential function of } \hat{\theta} \text{ for } \hat{\theta} \in [\underline{\theta},\overline{\theta}] \tag{A9}$$

$$F_{N-1}(\ldots|\theta_i) \geq F_{N'-1}(\ldots|\theta_i), N < N' \quad \text{(A10)}$$

$$F_{N-1}(\ldots|\theta_i) \to 0 \text{ as } N \to \infty;\ \hat{\theta} < \overline{\theta} \quad \text{(A11)}$$

$f_N(\theta_i)$ is continuous in θ_i and it converges uniformly to some function (A12)

$f(\theta_i)$, where $f(\theta_i) > 0$ for $\theta_i \in [\underline{\theta}, \overline{\theta}]$

$$X^{**}_N \to X^{**} \text{ as } N \to \infty \quad \text{(A13)}$$

$P^{**}_N(D)$ converges uniformly to some function $P^{**}(D)$ as $N \to \infty$ (A14)

Assumptions (A9) and (A12) are fairly unexceptional, (A10) and (25) imply that $Pr(B_N|D_i,X')$ decreases as the number of players increase, and (A11) and (25) imply that as the number of players becomes large, the probability that an agent can win the reward by drawing a $\theta < \overline{\theta}$ approaches zero. Assumptions (A12), (A13) and (A14) are regularity conditions that insure the continuity of the first best optimal solution as $N \to \infty$.

Suppose schedules $\tilde{P}_N$ exist that induce an equilibrium at X^{**}_N so that each agent's best choice given $X' = X^{**}_N$ is X^{**}_N. Then the condition in (29) holds and evaluating $EV_x^{A_i}$ at $X_i = X^{**}$, and $X' = X^{**}$ and integrating by parts yields

$$\int_{\underline{\theta}}^{\overline{\theta}} \{F_{n-1}(\theta\ldots\theta|\theta)\, u^{A_i}(Z^2)[Z^2_x] + [1-F_{N-1}(\theta\ldots\theta|\theta)][u^{A_i}(Z^1)[Z^1_x]]\}\, f_N(\theta)\, d\theta$$

$$+ F_{N-1}(\theta\ldots\theta|\theta)\, \frac{D_x}{D_\theta} f_N(\theta_i)\left[u^{A_i}(Z^2) - u^{A_i}(Z^1)\right]\Bigg|_{\theta_i = \underline{\theta}}^{\theta_i = \overline{\theta}} \quad (30)$$

$$-\int_{\underline{\theta}}^{\overline{\theta}} F_{N-1}(\theta\ldots\theta|\theta)\, \frac{d}{d\theta}\left[\frac{D_x}{D_\theta} f_N(\theta_i)\left[u^{A_i}(Z^2) - u^{A_i}(Z^1)\right]\right] f_N(\theta)\, d\theta = 0$$

where $Pr(B_N|D_i,X') = Pr(B_N|D_i,X_i) = F_{N-1}(\theta\ldots\theta|\theta)$ with $\theta = \theta_i$. For any converging sequence, $B_N \to B$, our convergence assumptions on F_{n-1}, f_N, P^{**}_N, and X^{**}_N imply that $\lim_{N\to\infty} E^{A_i}V_x$ exists. An necessary condition for enforceability in the limit, obtained by evaluating (30) as $N \to \infty$ is

$$\int_{\underline{\theta}}^{\overline{\theta}} u^{A_i}(Z^1)[Z^1_x]\ f(\theta)d\theta + [u^{A_i}(Z^2(\overline{\theta})) - u^{A_i}(Z^1(\overline{\theta}))]\ f(\overline{\theta})\ \frac{D_x(X^{**},\overline{\theta})}{D_\theta(X^{**},\overline{\theta})} = 0 \quad (31)$$

where $Z^1 = w_0 + W(X^{**},\theta) + P^{**}(D(X^{**},\theta))$ and $Z^2 = w_0 + W(X^{**},\theta) - P^{**}(D(X^{**},\theta)) + B$

We have thus established

Lemma 4. A necessary condition for the enforceability of $\tilde{X}^{**}_N$ in the limit as $N \to \infty$ is that there exists a payment schedule $\tilde{P}_N$ such that

$$E_{\theta_i}\ u^{A_i}(Z^1)[Z^1_x] + [u^{A_i}(Z^2(\overline{\theta})) - u^{A_i}(Z^1(\overline{\theta})]\ f(\overline{\theta})\ \frac{D_x(X^{**},\overline{\theta})}{D_\theta(X^{**},\overline{\theta})} = 0$$

This condition can be interpreted similarly to condition (b) in Lemma 1. The term $E_{\theta_i}\ u^{A_i}(Z^1)[Z^1_x]$, which is negative is a guage of moral hazard given the first best contract P^{**}. It measures the incentive, in expected utility terms, for the agent to shirk. The positive effect of the bonus B on expected utility is given by the second term where

$$f(\overline{\theta})\ \frac{D_x(X^{**},\overline{\theta})}{D_\theta(X^{**},\overline{\theta})} = \lim_{N\to\infty} \frac{dPr}{dx}\Bigg|_{\substack{x_i = x^{**} \\ \theta = \overline{\theta}}}$$

Taken together, the condition implies that if X^{**} is enforceable in the limit as $N \to \infty$ then the incentive to shirk is offset by the increase in expected utility from rewards. Note that (31) is stronger than condition (b) of Lemma 1, since the latter does not require that $f(\overline{\theta}) > 0$, but only that $\lim_{\theta \to \overline{\theta}^-} f(\theta) > 0$.

As an immediate consequence of Lemma 3 we have

Lemma 5. If there exists a B such that (31) is satisfied, then there exist schedules $\tilde{P}_N$ such that $U^{A_i}_x(\bar{P}_N, X^{**}_N, X^{**}_N) = 0$ for all $N \geq N'$ where N' is sufficently large. The proof is omitted since it is formally similar to the proof of Lemma 2.

Lemmas 4 and 5 identify necessary conditions for enforceability, but we should also like to derive sufficient conditions as well. Unfortunately there seems to be no natural or easy characterization of the density functions and payment schedules that will insure that U^{A_i} is concave in X^{21} and that will guarantee the existence of an equilibrium strategy at X^{**}_N. Further, we should like to know conditions under which equilibrium strategies are unique, assuming they exist. Fortunately, we can make some progress in this direction. Suppose an equilibrium strategy exists for some schedule $\tilde{P}_N$ at $X = X^0$. Then (29) implies $U_x^{A_i}(\tilde{P}_N, X^0, X^0) = 0$. Differentiating $U_x^{A_i}$ with respect to X and substituting for Pr from (25), we obtain (note that $X_i = X' = X$ for all X)

$$\frac{d}{dx} U_x^{A_i} = \frac{d}{dx}\int_{\underline{\theta}}^{\overline{\theta}} F_{N-1}(\theta\ldots\theta|\theta)[u^{A_i}(Z^2)Z_x^2]\, f_N(\theta)d\theta$$

$$+ \frac{d}{dx}\int_{\underline{\theta}}^{\overline{\theta}} (1 - F_{N-1}(\theta\ldots\theta|\theta))(u^{A_i}(Z^1)\, Z_x^1)\, f_N(\theta)d\theta \qquad (31)$$

$$+ \frac{d}{dx}\int_{\underline{\theta}}^{\overline{\theta}} [u^{A_i}(Z^2) - u^{A_i}(Z^1)]\, f_N(\theta\ldots\theta)\, \frac{D_x}{D_\theta}\, f_N(\theta)d\theta$$

The first two terms are negative by the concavity of $u^{A_i}(Z^1)$ and $u^{A_i}(Z^2)$ with respect to X. The last term is harder to sign as it depends on the sign

$$\{Pr_x \frac{d}{dx}(u^{A_i}(Z^2) - u^{A_i}(Z^1) + (u^{A_i}(Z^2) - u^{A_i}(Z^1))\frac{d}{dx} Pr_x\} \qquad (32)$$

where $Pr_x = \frac{D_x}{D_\theta} f_N(\theta\ldots\theta)$. The first term in (32) is positive and the second term, which equals $(u^{A_i}(Z^2) - u^{A_i}(Z^1))\, f_N(\theta\ldots\theta)\, \frac{d}{dx}(\frac{D_x}{D_\theta})$, is negative so that sign in (32) depends on relative magnitudes. Rearranging terms, we obtain that the sign in (32) depends on

$$\text{sign}\left\{\frac{\frac{d}{dx}(u^{A_i}(Z^2) - u^{A_i}(Z^1))}{u^{A_i}(Z^2) - u^{A_i}(Z^1)} + \frac{\frac{d}{dx} Pr_x}{Pr_x}\right\} \qquad (33)$$

The expressions in (33) and consequently the expression in (31) will be negative if as X increases the percentage decrease in Pr_x is greater in absolute terms than the percentage increase in $u^{A_i}(Z^2) - u^{A_i}(Z^1)$, the gain in utility from the bonus. This is a stability condition which implies that the marginal expected gain in utility from the bonus is always positive but decreases with larger X. Consequently, if the stability condition is satisfied for X, $\frac{d}{dx} U_x^{A_i} < 0$, an equilibrium strategy is unique whenever it exists. We have thus established

Lemma 6. If an equilibrium strategy exists and if the stability condition $\frac{d}{dx}[u^{A_i}(Z^2) - u^{A_i}(Z^1)]\ Pr_x < 0$ is satisfied, the strategy is unique.

We now turn to the primary result of this section

Proposition 3: If there exist payment schedules, $\tilde{P}_N$, such that X_N^{**} is enforceable for all $N \geq N'$, then the N player bonus contract approaches the first best contract for N sufficiently large.

Proof: By construction of δ_N, $U^B = \bar{U}^B = G$ for all N, so we need to show $U^{A_i}(\tilde{P}_N, X_N^{**}, X_N^{**}) \to U^{A_i}(P_N^{**}, X_N^{**}, X_N^{**})$ for N sufficiently large. Define

(i) $Z_N^{**} = w_0 + W(X_N^{**}, \theta) - P_N^{**}(D(X_N^{**}, \theta))$

(ii) $\Phi(n) = U^{A_i}(P_N^{**}, X_N^{**}, X_N^{**}) - U^{A_i}(\tilde{P}_N, X_N^{**}, X_N^{**})$

For $N \geq N'$

$$0 < \Phi(n) = \int_{\underline{\theta}}^{\overline{\theta}} Pr[u^{A_i}(Z_N^{**}) - u^{A_i}(Z_N^2)]\ f_N(\theta)\, d\theta$$

$$+ \int_{\underline{\theta}}^{\overline{\theta}} [1 - Pr][u^{A_i}(Z_N^{**}) - u^{A_i}(Z_N^1)]\ f_N(\theta)\, d\theta$$

$$\leq \int_{\underline{\theta}}^{\overline{\theta}} [1 - Pr][u^{A_i}(Z_N^{**}) - u^{A_i}(Z^1)\ f_N(\theta)\, d\theta \tag{34}$$

We now show $Z^1 \to Z_N^{**}$ as $N \to \infty$. Let $\Sigma_N = E_{\theta_i \ldots \theta_N} \sum_{i=1}^{N} P_N^{**}(D(X_N^{**}, \theta_i))$ From the definition of δ_N we have[22]

$$\delta_N(N-1) + \Sigma_N = B_N + G \tag{35}$$

or

$$\delta_N = \frac{B_N}{N-1} \tag{36}$$

since $\Sigma_N = G$ by (23). Clearly $\delta_N \to 0$ as $N \to \infty$ since B_N is bounded above by assumption. Thus $Z^1 \to Z_N^{**}$ as $N \to \infty$ and

$$0 < \Phi_N \leq \int_{\underline{\theta}}^{\overline{\theta}} [1 - Pr][u^{A_i}(Z_N^{**}) - u^{A_i}(Z^1)]\ f_N(\theta)\, d\theta \to 0 \tag{37}$$

as $N \to \infty$

Proposition 3 lends some theoretical support to the notion that large scale bonus competitions can be used to prevent shirking. However, as we found in the previous section, the conditions for enforcing an optimal action X are quite strong and frequently will not be satisfied. We now turn to a more general case where the enforcement of optimal actions is not possible.

V. ACTION CONTINGENT CONTRACTS IN SECOND BEST SITUATIONS

The foregoing analysis has dealt with the use of action-contingent payments and bonus systems to approximate first best contracts in agency relationships. To approximate such agreements, the action contingent contract must provide incentives to avoid penalties, or to achieve bonuses, that are sufficiently great to offset the tendency for the agent to shirk. Frequently, though, the conditions on monitoring and constructing payment schedules necessary to induce optimal agent behavior are too stringent. In such cases, only second best contracts exist where the diversity of incentives between principal and agent restrict the set of feasible agency agreements.

In this section we construct second best solutions involving the use of action contingent contracts. The principal result we obtain is that under quite genral conditions there are gains to monitoring for use in contingent contracts even when the requirements for approximating first best agreements are not satisfied. In fact, it is seen that the requirements for first-best approximation are but a special case of the general conditions for gains to monitoring. To demonstrate this, we begin with the contract derived from the standard moral hazard problem, considered by Spence and Zeckhauser, Mirrlees, and others, which is to choose a continuous payment schedule $P(D)$ to maximize $U^A(P,X)$ subject to the constraint that X is chosen by the agent to maximize his expected utility $U^A(P,X)$. Denoting $P^*(D)$ and X^* as the solution to this problem, we show that under the assumptions of section III, (A1-A8), there exists an action contingent contract that dominates $P^*(D)$. With the contingent contract, penalties are assessed against (bonuses are paid to) the agent whenever extremely low, (high) values of D are observed. The information from monitoring and the use of contingent payments increase the expected utility of the agent even though conditions for first-best contract approximation are not necessarily satisfied.

A. Derivation of P*(D)

As a benchmark for the evaluation of action contingent contracts, we consider the contract and action level $(P^*(D),X^*)$ derived as the solution to the problem:

$$\max_{P(D),X} \quad U^A(P,X) \tag{38}$$

subject to (A6), (A7), and

$$E_\theta u^{A'}[Z_x] - \gamma = 0 \tag{39}$$

where $Z = w_0 + W(X,\theta) - P(D(X,\theta))$. This is the standard formulation of the moral hazard problem where the principal chooses a payment schedule P(D) according to (38) given that it is impossible for him to monitor X. Normally, the parameter γ is set equal to zero in (39), indicating that the agent chooses a level of activity X to maximize expected utility, given the schedule P. Our reason for retaining the general form of the constraint (39) will become clear below. The formulation in (38) is subject to two difficulties. First, the constraint that the agent choose X to maximize $U^A(P,X)$ given P, is equivalent to (39) only if, for example, U(P,X) is concave in X. But the concavity of U depends on the schedule P which is unknown. Consequently, one must verify that the constraint (39) is valid once P*(D) is determined. Second, the problem in (38) seems well suited for control theoretic techniques, but it is impossible to write (38) as an optimal control problem. The difficulty is that if we take P as the control function, the equation of motion (39) contains the derivative of the control.[23] Instead we can treat this as a problem in the calculus of variations. Assuming the optimal payment schedule is twice continuously differentiable, we obtain the necessary conditions for a unique interior solution

$$\lambda\, U^A_{xx} + \Phi\, U^B_x = 0 \tag{40}$$

$$\lambda u^{A''}[C_x] + u^{A'}[1 + \lambda \frac{d}{d\theta}(\frac{D_x}{D_\theta}) + \lambda \frac{D_x}{D_\theta}\frac{f'(\theta)}{f(\theta)} - \Phi u^{B'} = 0 \;\forall\; \theta \tag{41}$$

along with (A6), (A7) and (39), where λ and Φ are the multipliers attached to the constraints (39) and (A7) respectively. To simplify notation, denote $(P^*_\gamma, X^*_\gamma, \lambda_\gamma, \Phi_\gamma)$ as the solution to (38) to signify its dependence on the parameter γ, with $(P^*_0, X^*_0, \lambda_0, \Phi_0)$ being the normal case where $\gamma = 0$.

Our strategy for finding an enforceable action contingent contract and action level $(P(\hat{D}),X)$ that dominates the pair (P_0^*,X_0^*) in the sense that $U^A(P(\hat{D}),X) > U^A(P_0^*,X_0^*)$ is as follows: First a new contract and action, $(\tilde{P},\tilde{X})$, is constructed that dominates (P_0^*,X_0^*) and for which $U_x^A(\tilde{P},\tilde{X}) = \gamma$, where γ is small and negative. Then we construct an action contingent contract with penalities denoted by $P(\hat{D})$ (the development of contingent contracts with bonus payments is similar) such that

$$P(\hat{D}) = \begin{cases} \tilde{P}(D) - \delta(\hat{D},\tilde{X}) & \text{if } D > \hat{D} \\ \overline{P}(D,\hat{D}) & \text{if } D \leq \hat{D} \end{cases} \tag{42}$$

where $\overline{P}(\hat{D},\hat{D}) > \tilde{P}(\hat{D})$ and δ is a constant which depends on $\hat{D}$ and $\tilde{X}$ and is implicitly defined by $U^B(P(\hat{D}),\tilde{X}) = \overline{U}^B$. The contingent schedule $P(\hat{D})$ is constructed assuming $X = \tilde{X}$, and P and $\overline{P}$ are written explicitly as a function of $\hat{D}$ to emphasize their dependence on the cutoff value of D for penalization. Although the contract $(\tilde{P},\tilde{X})$ is not enforceable, i.e., $U_x^A(\tilde{P},\tilde{X}) < 0$, the threat of penalization in the action contingent contract, can be used to enforce $\tilde{X}$, and if $\tilde{X}$ is enforceable in the limit as $\hat{D} \to D(\underline{\theta},\tilde{X})$ we can establish that $U^A(P(\hat{D}),\tilde{X}) \to U^A(\tilde{P},\tilde{X}) > U^A(P_0^*,X_0^*)$.

Consider the case $\gamma = 0$, for the problem in (38). From (40)

$$\lambda_0 = \frac{-\Phi\, U_x^{\;B}}{U_{xx}^A} \tag{43}$$

The multiplier λ has the physical interpretation of being the rate of increase in the objective functional $U^A(P,X)$ for a decrease in the parameter γ.[24] In (43), $\Phi_0 > 0$, and $U_{xx}^A(P_0^*,X_0^*) < 0$ (assuming X maximizes $U^A(P_0^*,X)$) so that the sign of λ_0 is positive if and only if $U_x^{\;B} > 0$ and the principal would like X to be further increased if he could control it given the payment schedule. We would expect λ_0 to be positive, certainly given the agent's propensity to shirk. In this case we can simply choose $(\tilde{P} = P^*_{\gamma'},\ \tilde{X} = P^*_{\gamma'})$ for γ small and negative to find a new contract $(\tilde{P},\tilde{D})$ dominating (P^*,X^*) with $U_x(\tilde{P},\tilde{X}) < 0$. However we have been unable to find general assumptions that exclude the possibility $\lambda_0 < 0$, though we can establish the sign of λ_0 under certain conditions.

<u>Lemma 7</u>. Suppose the principal is risk neutral, and $Z(X,\theta)$ is concave in X, given the schedule $P_0^*(D)$, then $\lambda_0 = -\Phi_0\, U_x^B/U_{xx}^A > 0$.

Proof: If the principal is risk neutral (A7) becomes a breakeven constraint, $E_\theta P(D(X,\theta)) = G$ where G represents a normal competitive return for the principal. Fixing X at X_0^*, the first best solution to the agency problem would require that $P_0^{*\prime} = 1$ and that $Z(\theta)$ be constant, with $Z = E_\theta(Z) = w_0 + E_\theta W(X^*,\theta) - G$. In the solution to (38), there clearly exists at least one $\tilde{\theta} \in (\underline{\theta},\overline{\theta})$ such that $Z(\tilde{\theta}) = E_\theta(Z)$ for otherwise since $Z(\theta)$ is continuous this would imply $Z(\theta) < E_\theta(Z)$ or $Z(\theta) > E_\theta(Z)$ for all θ which would cause a violation of the breakeven constraint. Suppose $\lambda_0 < 0$, contrary to our assertion. Then from (43) $P^{*\prime} < 0$ for some θ and it follows that $Z(\theta) \neq E(Z)$ over some range. Assume, without losing generality,[25] that $Z(\theta) \geq (\leq) E(Z)$ (with strict inequality for some θ) for $\theta < (>) \tilde{\theta}$ and consider a new schedule $Z(\theta) + \alpha h(\theta)$ where $h(\theta) = E(Z) - Z(\theta)$ and $\alpha \in [0,1]$. Then for $\alpha > 0$, $h \neq 0$ and $u^{A''} < 0$ we have

$$u^A(Z + \alpha h) - u^A(Z) = \int_0^h u^{A'}(Z + \alpha\xi)d\xi > u'(Z(\tilde{\theta}))\alpha h = u'(E(Z))\alpha h$$

for all θ

and it follows that for $\alpha > 0$

$$E_\theta u^A(Z(\theta) + \alpha h(\theta)) - Eu^A(Z(\theta)) > \int_{\underline{\theta}}^{\tilde{\theta}} \alpha h\, u^{A'}(Z(\tilde{\theta}))\, f(\theta)d\theta + \int_{\tilde{\theta}}^{\overline{\theta}} \alpha h\, u^{A'}(Z(\tilde{\theta})\, f(\theta)d\theta$$

$$= u^{A'}(Z(\tilde{\theta}))\int_{\underline{\theta}}^{\overline{\theta}} \alpha h(\theta)\, f(\theta)d\theta = 0.$$

Let $Z^\alpha(\theta) = Z(\theta) + \alpha h(\theta)$. Since $\lambda_0 < 0$, and $E_\theta u^A(Z^\alpha(\theta)) > Eu^A(Z(\theta))$ it necessarily follows from the interpretation of λ that $E_\theta u^{A'} Z_x^\alpha(\theta) > 0$ at least for α small. It is easy to verify that $E_\theta u^{A'} Z_x^\alpha(\theta)$ is a continuous function of α and that $E_\theta u^{A'} Z_x^\alpha(\theta) < 0$ for $\alpha = 1$. Thus there exists at least one $\hat{\alpha} \in (0,1)$ such that $E_\theta u^{A'} Z_x^{\hat{\alpha}}(\theta) = 0$. Furthermore,

$$\frac{d}{dx} E_\theta u^{A'}[Z_x^{\hat{\alpha}}(\theta)] = E_\theta[u^{A''}[Z_x^{\hat{\alpha}}(\theta)]^2 + u^{A'}[(1-\hat{\alpha})Z_{xx} + \hat{\alpha}E_\theta Z_{xx}]] < 0$$

since Z is concave in X, so that X^* maximizes $E_\theta u^A(Z^{\hat{\alpha}}(\theta))$. But $E_\theta u^A(Z^{\hat{\alpha}}) > E_\theta u^A(Z)$ which implies $(P^{\hat{\alpha}}(D),X^*)$ dominates $(P^*(D),X^*)$ where $P^{\hat{\alpha}} = P^* + \alpha h$ is the payment schedule implicitly defined by $Z^{\hat{\alpha}}$. Consequently we have a contradiction, which implies $\lambda_0 > 0$.

The assumptions in Lemma 7 are strong and we suspect that the sign of λ_0 can be established under weaker conditions. However even for the unlikely case $\lambda_0 < 0$ we can construct another contract $(\tilde{P},\tilde{X})$ such that $U^A(\tilde{P},\tilde{X}) > U^A(P_0^*,X_0^*)$ with $U_x^A(\tilde{P},\tilde{X}) < 0$ as follows. To find a contract that dominates (P_0^*,X_0^*) consider the schedule

$$\tilde{P}(\hat{\hat{D}}) = \begin{cases} P_0^*(D) & \text{if } D \leq \hat{\hat{D}} \\ \hat{P}(D,\hat{\hat{D}}) & \text{if } D > \hat{\hat{D}} \end{cases} \tag{44}$$

where $\hat{P}(D,\hat{\hat{D}})$ is derived as the solution to

$$\max_{\tilde{P}} \int_{\theta(\hat{\hat{D}},X_0^*)}^{\overline{\theta}} u^A(\tilde{Z})\ f(\theta)d\theta$$

subject to

$$\int_{\theta(\hat{\hat{D}},X^*)}^{\overline{\theta}} u^B(\hat{P}(D(X_0^*,\theta),\hat{\hat{D}}))\ f(\theta)d\theta = \overline{U}^B - \int_{\underline{\theta}}^{\theta(\hat{\hat{D}},X^*)} u^B(P_0^*(D))f(\theta)d\theta \tag{45}$$

where $\tilde{Z} = w_0 + W(X^*,\theta) - \tilde{P}(D(X^*,\theta))$. $\tilde{P}$ is constructed to maximize expected utility of the agent over the interval $[\theta(\hat{D},X^*),\overline{\theta}]$, holding $X = X^*$ such that $U^B(\tilde{P},X_0^*) = \overline{U}^B$. Clearly for $\theta(\hat{D},X^*) < \overline{\theta}$

$$U^A(\tilde{P}(\hat{\hat{D}}),X_0^*) - U^A(P_0^*,X_0^*) = \int_{\theta(\hat{\hat{D}},X^*)}^{\overline{\theta}} [u^A(\tilde{Z}) - u^A(Z)]\ f(\theta)d\theta > 0 \tag{46}$$

by construction.

Next, to show that we can find a $\hat{\hat{D}}$ such that $U_x^A(\tilde{P}(\hat{D}),X^*)$ is arbitrarily small and negative we note that:

(a) $U_x^A(\tilde{P}(D(\overline{\theta},X_0^*),X_0^*) = U_x^A(P_0^*,X_0^*) = 0$

(b) $U_x^A(\tilde{P}(D(\underline{\theta},X_0^*),X_0^*) = U_x^A(\hat{P}(D),X_0^*) < 0$. To assume contrarily that $U_x^A(\tilde{P}(D(\underline{\theta},X_0^*),X_0^*) \geq 0$. Clearly, $U_x^A(\tilde{P}(D(X_0^*,\underline{\theta}),X_0^*) = 0$ is impossible since (46) would imply $(\tilde{P}(D(X_0^*,\underline{\theta}),X_0^*)$ is the solution to (38). Suppose $U_{\underline{x}}^A(\tilde{P}(D(X_0^*,\underline{\theta}),X_0^*)) > 0$. The first order conditions for (45) imply $\frac{\partial \hat{P}(D,D)}{\partial D} \in (0,1)$ and therefore $U_x^B > 0$. Thus given the schedule $\tilde{P}$, both A and B can be made better off by increasing X to $\overline{X}$ or to the point where $U_x^A = 0$, but this would again imply that (P_0^*,X_0^*) is not a solution to (38).

(c) Given our continuity and differentiability assumptions about W, D, C, u^A, u^B, and f it can be shown that $U_x^A(\tilde{P}(\hat{\hat{D}}), X_0^*,), X_0^*)$ is a continuous function of $\hat{D}$.[25]

Therefore (a) - (c) imply by the intermediate value theorem that there exists at least one $\theta \in [\underline{\theta}, \bar{\theta}]$ such that $U_x^A(\tilde{P}(D(X_0^*, \theta), X_0^*) = \gamma$ for each $\gamma \in [0, U_x^A(\tilde{P}(D(X_0^*, \underline{\theta})), X_0^*)]$.

Having identified contracts and action levels (P_γ^*, X_γ^*) for the case $\lambda_0 > 0$ and the pair $(\tilde{P}(\hat{D}), X_0^*)$ for the case $\lambda_0 < 0$ that dominate $(P_0^*\ X_0^*)$ we now wish to construct action contingent schedules as described in (42) that are enforceable with $U_x^A(P(\hat{D}), X) = 0$ and that can be made to approximate schedules P_γ^* and $\tilde{P}(\hat{D})$ to any desired degree. Results on enforceability for cases $\lambda_0 > 0$ and $\lambda_0 < 0$ are summarized in Lemmas 8 and 9 respectively.

Lemma 8. Assume solutions $(P_\gamma^*, X_\gamma^*, \lambda_\gamma, \Phi_\gamma)$ to the problem (38) exist, that they are continuous with respect to γ and that $\lambda_0 > 0$. Then there exists a $\gamma < 0$ such that

i (P_γ^*, X_γ^*) dominates $(P_0^*\ X_0^*)$

ii There exists an action contingent contract

$$P(\hat{D}) = \begin{cases} P_\gamma^*(D) - \delta(\hat{D}, X_\gamma^*) \text{ if } D > \hat{D} \\ \bar{P}(D, \hat{D}) \text{ if } D \leq \hat{D} \end{cases} \tag{47}$$

which satisfies the necessary and sufficient conditions for enforceability such that

(a) $U_x^A\ (P(\hat{D}), X_\gamma^*) = 0$

(b) $U_x^A\ (P(\hat{D}), X) \gtrless 0$ as $X \lessgtr X_\gamma^*$

so that $(P(\hat{D}), X_\gamma^*)$ is enforceable

for $\hat{D} \in (D(X_\gamma^*, \underline{\theta}), D')$; $D > D(X_\gamma^*, \underline{\theta})$ or equivalently

for $\hat{\theta} \in (\underline{\theta}, \theta')$; $\hat{\theta} = \hat{\theta}(\hat{D}, X_\gamma^*)$

Proof:

Part (i) follows from the assumption that $\lambda_0 > 0$, which implies $U^A(P_\gamma^*, X_\gamma^*)$ increases monotonically as γ decreases for $\gamma \in (\gamma', 0)$, $\gamma' < 0$.

To prove part (ii a) that there exists a $\gamma \in (\gamma', 0)$ such that $U_x^A(P(\hat{D}), X_\gamma^*) = 0$ for $\hat{\theta} \in (\underline{\theta}, \theta')$, we first need to show there exists a γ

such that $U_x(P(\hat{D}), X^*_\gamma) = 0$ as $\hat{\theta} \to \underline{\theta}^+$ and then we can rely on a proof similar to that for Lemma 2 to establish our result. To simplify notation define

$$Z = w_0 + W(X^*_\gamma, \theta) - P^*_\gamma(D(X^*_\gamma, \theta))$$

$$Z_1 = w_0 + W(X^*_\gamma, \theta) - \overline{P}(D, \hat{D})$$

$Z_0 = \inf Z \equiv$ the smallest Z taken over all values of γ.

Note that (A6) implies $Z_0 > \overline{W}$. Given the contingent schedule in (47) we have:

$$\lim_{\hat{\theta}\to\underline{\theta}^+} U_x^{A'}(P(\hat{D}), X^*_\gamma) = E_\theta u^{A'}(Z)[Z_x] + [u^A(Z^1(\underline{\theta})) - u^A(Z(\underline{\theta}))] \frac{d\underline{\theta}}{dx} \lim_{\hat{\theta}\to\underline{\theta}^+} f(\hat{\theta}) \tag{48}$$

Rearranging (48), and noting that the first term on the right hand side of (48) is $U_x^A(P^*_\gamma, X^*_\gamma) = \gamma$ we obtain that $\lim_{\hat{\theta}\to\underline{\theta}^+} U_x^A(P(\hat{D}), X^*_\gamma) = 0$ if and only if

$$u^A(Z^1(\underline{\theta})) = u^A(Z(\underline{\theta})) - \frac{\gamma}{\frac{d\underline{\theta}}{dx} \lim_{\hat{\theta}\to\underline{\theta}^+} f(\hat{\theta})} \tag{49}$$

The second term on the right hand side of (47) is negative and can be made arbitrarily small in absolute value by choosing γ sufficiently close to zero. Let $Z^1(\underline{\theta}) = Z(\underline{\theta}) - \varepsilon$ where we choose $\varepsilon > 0$ such that $Z_0 - \varepsilon > \overline{W}$ to insure (A6) is satisfied. Then it follows that there exists a γ close to zero such that (47) is satisfied. Given $\lim_{\hat{\theta}\to\underline{\theta}} U_x(P(\hat{D}), X^*_\gamma) = 0$ for some γ, we can make the obvious alterations in the proof to Lemma 2 to establish $U_x(P(\hat{D}), X^*_\gamma) = 0$ for $\hat{\theta} \in (\underline{\theta}, \theta')$.

The proof of (ii b) is established in the Appendix.

We now state without proof Lemma 9 which establishes enforceability conditions for the case $\lambda_0 < 0$, that are analogous to the conditions for the case $\lambda_0 > 0$, stated in Lemma 8. The proofs of Lemmas 8 and 9 are essentially the same.

Lemma 9. Assume the solution $(P^*_0, X^*_0, \lambda_0, \Phi_0)$ to the problem in (38) exists and that $\lambda_0 < 0$. Then there exists a schedule $\tilde{P}(\hat{D})$, characterized in (44), such that i $(\tilde{P}(\hat{D}), X_0^*)$ dominates (P_0^*, X_0^*)

ii There exists an action contingent contract

$$P(D) = \begin{cases} \tilde{P}(\hat{D}) - \delta(\hat{D}, X_0^*) & \text{if } D > \hat{D} \\ \overline{P}(D, \hat{D}) & \text{if } D \le \hat{D} \end{cases}$$

such that

(a) $U_x^A(P(\hat{D}), X_0^*) = 0$

(b) $U_x^A(P(\hat{D}), X) \gtrless 0$ as $X \lessgtr X_0^*$

and thus $(P(\hat{D}), X_*^0$ is enforceable

for $\hat{D} \in (D(X_0^*, \underline{\theta}), D')$; $D' > D(X_0^*, \underline{\theta})$ or equivalently

for $\hat{\theta} \in (\underline{\theta}, \theta')$; $\hat{\theta} = \hat{\theta}(\hat{D}, X_0^*)$

Lemmas 8 and 9 suggest that there exists a contract and action level denoted here by $(\tilde{P}(D), X)$ that dominates the solution pair (P_0^*, X_0^*) to the standard moral hazard problem posed in (38). The problem is that $(\tilde{P}(D), X)$ is not enforceable as $U_x(\tilde{P}(D), X) < 0$. Nevertheless, under (A1) – (A8) we can always construct an action contingent contract $P(\hat{D})$ such that the pair $(P(\hat{D}), X)$ is enforceable and $P(\hat{D})$ can be made to approximate $\tilde{P}(D)$ to any desired degree. A special case, discussed in Section 3, occurs when $P(\hat{D})$ can be used to enforce (P^{**}, X^{**}). From Lemmas 8 and 9 it follows that $U^A(P(\hat{D}), X) > U^A(P_0^*, X_0^*)$. To demonstrate this, define:

$$\Delta = U^A(\tilde{P}(D), X) - U^A(P_0^*, X_0^*) > 0$$

$$\Delta(\hat{\theta}) = U^A(\tilde{P}(D), X) - U^A(P(\hat{D}), X) > 0$$

$$Z = w_0 + W(X, \theta) - \tilde{P}(D(X, \theta))$$

$$Z^1 = w_0 + W(X, \theta) - \overline{P}(D, \hat{D})$$

$$Z^2 = w_0 + W(X, \theta) - \tilde{P}(D(X, \theta)) + \delta(\hat{D}, X)$$

where it is understood that $\hat{\theta} = \hat{\theta}(\hat{D}, X)$. Then for $\hat{\theta} \in (\underline{\theta}, \theta')$

$$\Delta(\hat{\theta}) = \int_{\underline{\theta}}^{\hat{\theta}} [u^A(Z) - u^A(Z^1)]\, f(\theta)\,d\theta + \int_{\hat{\theta}}^{\overline{\theta}} [u^A(Z) - u^A(Z^2)]\, f(\theta)\,d\theta$$

$$< \int_{\underline{\theta}}^{\hat{\theta}} [u^A(Z) - u^A(Z^1)]\, f(\theta)\,d\theta \to 0 \text{ as } \hat{\theta} \to \underline{\theta}$$

But $U^A(P(\hat{D}),X) = U^A(P_0^*,X_0^*) + \Delta - \Delta(\hat{\theta})$ which implies that we can choose a $\hat{\theta}$ sufficiently close to $\underline{\theta}$ such that

$$U^A(P(\hat{D}),X) > U^A(P_0^*,X_0^*).$$

This result, and Lemmas 8 and 9 establish

Proposition 4. If there exist solutions $(P_\gamma^*,X_\gamma^*,\lambda_\gamma,\Phi_\gamma)$ to the standard moral hazard problem as posed in (38), then an action contingent contract $P(\hat{D})$ and an action level X exist such that $U^A(P(\hat{D}),X) > U^A(P_0^*,X_0^*)$.

We can illustrate the application of Lemmas 7-9 and proposition 4 with two examples

(a) Spence-Zeckhauser Insurance Problem. For this case, it trivially follows that we can choose an enforceable action and action contingent contract $(P(\hat{D}),X)$ to dominate (P_0^*,X_0^*), since we recall from our in discussion of this example in Section III that it is possible to construct contingent schedules that approximate the first best contract to any desired degree. This result is driven by the fact that the penalties for shirking can be made infinitely large by allowing net payments, Z, to approach zero in the penalty region. If, however, we reduce the power to penalize by requiring $Z > \overline{W} > 0$ we may be confined to second best solutions. We can nevertheless apply the results of Lemmas 7 and 8 and proposition 4 to construct contingent contracts that dominate (P_0^*,X_0^*). For this example, Lemma 7 implies $\lambda_0 > 0$ since the principal is risk neutral and $U^A(P_0^*,X)$ is concave in X (see 20). Assuming solutions $(P_\gamma^*,X_\gamma^*,\lambda_\gamma,\Phi_\gamma)$ exist to the problem (38) with the added restriction that $Z > \overline{W} > 0$,[27] then Lemma 8 and proposition 4 together imply that there exists a pair $(P(\hat{D}),X_\gamma^*)$ such that $U^A(P(\hat{D}),X_\gamma^*) > U^A(P_0^*,X_0^*)$.

(b) Fixed Royalty Leases. A fixed royalty contract is commonly used in government leasing of mineral and natural gas and oil reserves to private developers. Expressed in terms of our model the contract requires the developer to pay the government a fixed sum $\overline{P}$ plus some constant percentage, p, (royalty) of the total revenues from the lease, denoted by $D(X,\theta)$ where X is the effort devoted to recovering the resource and θ, which is unknown, is the amount of the resource available. The cost of providing effort is $C(X)$ and θ, the resource abundance, is distributed according to $f(\theta)$. Taking a standard approach, the lease is determined as the solution to

$$\max_{\overline{P},p,X} U^A(\overline{P},p,X) = \max_{\overline{P},p,X} E_\theta u^A(w_0 - \overline{P} + (1-p)D(X,\theta) + C(X)) \tag{50}$$

subject to

$$U^A_x(\overline{P},p,X) = \gamma^{28} \tag{51}$$

and the breakeven constraint for the government

$$E_\theta u^B(\overline{P} + p\ D(X,\theta)) = \overline{U}^B \tag{52}$$

which might reflect a government policy to recover some fair share of the rents from the resource reserves. Assuming solutions $(\overline{P}^*_\gamma, p^*_\gamma, X^*_\gamma, \lambda_\gamma, \Phi_\gamma)$ to (50) exist, then the maximization reveals that

$$\lambda_0 = -\frac{E_\theta u^{B'}[p^*_0\ D_x]}{\Phi_0\ U^A_{xx}(\overline{P}^*_0, p^*_0, X^*_0)} \qquad \text{where}$$

λ and Φ are the multipliers attached to the constraints (51) and (52), respectively, and λ_0 is interpreted as the increase in U^A for a small decrease in γ away from zero. Using an argument similar to the proof of Lemma 7 we can show that $\lambda_0 > 0$. Therefore, by Lemma 8, we can find a contract and action $(\overline{P}^*_\gamma, p^*_\gamma, X^*_\gamma)$, with $\gamma < 0$, that dominates $(\overline{P}^*_\gamma, p^*_\gamma, X^*_\gamma)$ such that X^*_γ can be enforced by an action contingent contract which can be made to approximate the contract characterized by $(\overline{P}^*_\gamma, p^*_\gamma)$ to any desired degree. Proposition 4 then implies that

$$U^A(P(\hat{D}), X^*) > U^A(\overline{P}^*_0, p^*_0, X^*_0).$$

SUMMARY

This paper describes a method of contracting to alleviate incentive problems occurring in risk sharing agency agreements. Problems arise because the principal possesses only partial information on the agent's action. In cases of incomplete information, the agent's activity level tends to be too low (high) if contract payments are based only on observed benefits (costs). We consider action contingent contracts to alleviate this problem, where payment between the principal and the agent are allowed to shift in accord with the observed level of output.

These variations in the payment schedule take the form of penalties or bonuses which are assessed according to whether the observed level of output reveals the agent's action to be unacceptable or acceptable based on some statistical criteria. Similar payment schemes have been explored by Harris and Raviv (1976) and Shavell (1977), but they require observations on other variables besides output, which may be costly to obtain.

When the potential for penalizing or rewarding the agent is great, or incentive incompatibilities are relatively minor, we can approximate a pareto optimal contract to any desired degree with a suitably constructed action contingent contract. Under less ideal conditions, only second best solutions exist, but monitoring output nevertheless results in gains to both principal and agent in action contingent contracts.

The primary result of this paper is that information contained in observation of output can be more fully utilized in action contingent contracts that provide for bonus and penalty payments. The arguments for bonus and penalty contracts presented here can be interpreted as providing some theoretical support for the pervasive use of bonus and penalties in agency agreements like insurance, wage contracts, lease agreements, and government defense contracting.

FOOTNOTES

1. The theory of hierarchy and supervision in employer-employee relationships is discussed in Mirrlees (1976) and Stiglitz (1975). The problem of cost overruns in defense contracts is discussed by Cummins (1977). A small sample of the literature on moral hazard in insurance includes Arrow (1971), Townsend (1976), Pauly (1968), Shavell (1977), Spence and Zeckhauser (1971), and Zeckhauser (1970). The relationship between the lessor and lessee is covered by Cheung (1968), Leland (1975), Newberry (1976), and Stiglitz (1974).
2. Agreements providing for partial insurance, and for profit and cost sharing among employees and employers are examples of payments that are contingent on payoffs.
3. The analysis could be generalized by making W a function of a vector of activity variables and a vector of state variables. The generalization allowing for more state variables would seem to be straightforward, but the inclusion of additional action variables would appear to complicate the analysis significantly.
4. Therefore our analysis is not pertinent for those instances where the agent has more information than the principal about the state of nature.
5. The notation, $\lim_{\theta \to \underline{\theta}^+}$ and $\lim_{\theta \to \overline{\theta}^-}$ refers to the limit as θ approaches $\underline{\theta}$ from above, and the limit as θ approaches $\overline{\theta}$ from below, respectively.
6. Townsend (1976) and Shavell (1977) consider cases where observation costs are positive.
7. If P is continuous, a solution to (1) exists since $U^A(P,X)$ is continuous and X is chosen from a compact set. In addition we shall assume that the $\hat{X}$ solving (1) is unique.
8. Throughout the paper, the derivative of a function is denoted by a "prime," i.e., $f'(\theta) = \frac{df(\theta)}{d\theta}$, and partial derivatives are denoted by subscripts, i.e., $D_\theta = \frac{\partial D(X,\theta)}{\partial \theta}$. Note, $U^A_{XX}(P,X) = E_\theta u^{A''}[D_X + C_X - P'D_X]^2 + E_\theta u^{A'}[D_{XX} + C_{XX} - P'D_{XX} - P''D_X]$. This $U^A(P,X)$ is concave in X so long as the payments schedule is not too concave.

9. In many agency relationships, there is either competition on the agent's side of the market, or negotiations between principal and agent are bilateral so that the principal captures some of the rent from the contract. The assumption in our model, which has competition among principals driving their share of the rents to zero, is made only for convenience and is not crucial to the analysis. However, a more serious objection to our model comes from the fact that we are ignoring some of the strategic aspects that exist in contract bidding. Cox (1977) and Wilson (1975) present interesting analysis of contract bidding under different institutions.

10. See Zeckhauser (1970).

11. The concavity of u^A, u^B and W guarantee the sufficiency of conditions (4) and (5).

12. We provide only an intuitive justification for these conditions as they are formally derived in Harris and Raviv, Leland, and Spence and Zeckhauser.

13. It is interesting to note that when D = W, the agent generally will still not choose the optimal action, in this case $\hat{X} \gtrless X^{**}$ according to whether the optimal payment schedule is a strictly concave, linear, or strictly convex function of W. See Leland (1975).

14. Define $G(\tilde{X}/D) = \int_0^{\tilde{X}} g(X/D)\, f(X)dX$. Then the conditional probability that $X \geq \tilde{X}$ is $1 - G(\tilde{X}/D)$. It is surprising that we cannot rule out the possibility that $\frac{dG(\tilde{X}/D)}{dD} > 0$ so the probability that $X \geq \tilde{X}$ may actually decrease over a certain range of increasing values for D. It is possible to show, however, that $\frac{dG(\tilde{X}/D)}{dD} < 0$ for X sufficiently small.

15. According to proposition 1, $\hat{X} > X^{**}$ whenever $D_X < 0$. Thus, for a low (high) value of D, the principal can infer with high probability that the agent is (is not) overacting.

16. P is similar to the dichotomous contract introduced by Harris and Raviv (1976).

17. Some threat of penalization must be preserved in order to enforce X**.
18. See Townsend for a discussion of stochastic verification.
19. The property that $f_N(\theta_i) > 0$ for $\theta \in [\underline{\theta},\overline{\theta}]$ is slightly stronger than (A4) as it requires that there be some density at the end points of the support of θ_i.
20. A solution to this maximization problem exists since U^{A_i} is continuous in X and X_i is chosen from a compact set.
21. This appears to be a common problem with formulations such as ours. For example Mirrlees (1975) and Shavell (1977) encounter the same concavity problems but in a different context.
22. In writing (35) we can effectively ignore the unlikely event which occurs with probability zero that two or more agents achieve $\max_i D(X,\theta_i)$.
23. See Holmstrom (1977) and Girsanov (1972) for an interesting discussion of this problem.
24. See Dreyfus (1964), pp. 125-7.
25. A similar, but merely more complicated mathematical argument is needed to establish Lemma 7 when this assumption is not fulfilled.
26. To formally establisy (c) we can rely on the Euler equations for a maximum of (44) and our continuity and differentiability assumptions on W, D, C, u^A, u^B and f to establish that given X*, $\overline{P} = \overline{P}(\hat{D},\theta)$, where $\overline{P}$ is continuously differentiable in $\hat{D}$ and θ. It follows from the construction of $\tilde{P}(\hat{D})$ that $U^A_x(\tilde{P}(\hat{D}),X^*)$ is a continuous function of $\hat{D}$. See Pontryagin (1960), pp. 192-200.
27. Of course a solution may not exist because the feasible set of Z is open. We could enhance the possibilities for obtaining a solution by allowing the set to contain its lower boundary, and requiring $Z = w^o \geq \overline{W}$. However, with this convention the ability of the principal to penalize the agent is eliminated if $Z = w_o + W(X^*_Y,\hat{\theta}) - P^*_Y(D(X^*_Y,\hat{\theta})$ should happen to equal w^o at the cutoff point $\hat{\theta} = \hat{\theta}(\hat{D},X^*_Y)$. This does not appear to be a significant problem, however, since we can always construct contingent contracts that utilize rewards instead of penalties.
28. It is easy to verify that $U^A(\overline{P},p,X)$ is concave in X so that (49) is sufficient to describe the agent's maximizing choice.

APPENDIX

We wish to verify (a) part b of lemma 2, (b) lemma 3, and (c) part (iib) of lemmas 8 and 9. The proofs of all these parts are similar and are thus presented together in a general proof given below. To establish a consistent notation for this discussion we denote X^o as an action level which is to be enforced by an action contingent contract, denoted by $P(\hat{D})$, where $P(\hat{D})$ is constructed with reference to a particular contract $\tilde{P}(D)$ such that

$$P(\hat{D}) = \begin{cases} \tilde{P}(D) - \delta(\hat{D},X^o) & \text{if } D > \hat{D} \\ \bar{P}(D,\hat{D}) & \text{if } D \leq \hat{D} \end{cases}$$

where: (P1)

$\bar{P}(\hat{D},\hat{D}) > \tilde{P}(\hat{D})$,

$\delta(\hat{D},X^o)$ is implicitly defined by $U^B(P(\hat{D}),X^o) = \bar{U}^B$, and $\tilde{P}(D)$ is constructed so that $U_x^A(\tilde{P}(D),X^o) < 0$

the constraint (A6) holds.

Enforceability of $(P(\hat{D}),X^o)$ requires that X^o be the solution to

$$\max_X U^A(P(\hat{D}),X) \tag{P2}$$

and first and second order necessary conditions for enforceability are

$$U_x^A(P(\hat{D}),X^o) = 0 \qquad \text{(f.o. necessary condt.)} \tag{P3}$$

$$U_{xx}^A(P(\hat{D}),X^o) < 0 \qquad \text{(s.o. necessary condt.)} \tag{P4}$$

In terms of the notation presented here, part b of lemma 2 is equivalent to statement A.

(A) $U_{xx}^A(P(\hat{D}),X^o) < 0$ for $\hat{D} \in (\underline{D},D')$

Lemma 3 and part (iib) of lemmas 8 and 9 are represented by

(B) $U_x^A(P(\hat{D}),X) \gtrless 0$ as $X \lessgtr X^o$ for $\hat{D} \in (\underline{D},D')$

The conditions $U_x^A(P(\hat{D}),X) = 0$ and (B) and the fact that $U^A(P(\hat{D}),X)$ is continuous in X are sufficient to establish $U^A(P(\hat{D}),X^o) > U^A(P(\hat{D}),X)$ for $X \neq X^o$ and that therefore $(P(\hat{D}),X^o)$ is enforceable.

In statements (A) and (B) we understand $\hat{D}$, $\underline{D}$, and $\hat{\theta}$ to be given by $\hat{D} = D(X^o,\hat{\theta})$, $\underline{D} = D(X^o,\underline{\theta})$ and $\hat{\theta} = \hat{\theta}(\hat{D},X^o)$, respectively.

The proof of (A) and (B) is constructed as follows:

(i) Consider contingent contracts in (P1) with penalty schedules of the form

$$\bar{P}(D,\hat{D}) = \begin{cases} \bar{P} - \alpha D & \text{for } D \in [\underline{D},\hat{D}] \\ \bar{P} - \alpha\underline{D} + (D-\underline{D}) & \text{for } D < \underline{D} \end{cases} \qquad \text{(P5)}$$

where α and $\bar{P}$ are constants which both depend on $\hat{D}$. Define the following variables

$$Z^1 = W_o + W(X,\theta) - \tilde{P}(D) + \delta(\hat{D},X^o)$$

$$Z^2 = W_o + W(X,\theta) - (\bar{P} - \alpha D)$$

$$Z^3 = W_o + W(X,\theta) - (\bar{P} - \alpha\underline{D} + (D-\underline{D})$$

(ii) Suppose $\lim_{\hat{\theta}\to\underline{\theta}^+} U_x^A(P(\hat{D}),X^o) = 0$ where

$$\bar{P}(\underline{D},\underline{D}) = P_o = \bar{P} - \ \underline{D} \qquad \text{(P6)}$$

for some $\bar{P}$ and α. Then as in the proof of lemma 2, part a, we construct two new contingent schedules $P_1(\hat{D})$ and $P_2(\hat{D})$ with penalty schedules given by

$$\bar{P}_1(D,\hat{D}) = \begin{cases} \bar{P} + \quad - \alpha D & \text{for } D \in [\underline{D},\hat{D}] \\ \bar{P} + \varepsilon - \alpha\underline{D} + (D-\underline{D}) & \text{for } D < \underline{D} \end{cases}$$

$$\bar{P}_2(D,\hat{D}) = \begin{cases} \bar{P} - \varepsilon - \alpha D & \text{for } D \in [\underline{D},\hat{D}] \\ \bar{P} - \varepsilon - \alpha\underline{D} + (D-\underline{D}) & \text{for } D < \underline{D} \end{cases}$$

It follows from lemma 2 that there exists a contingent schedule $P_3(\hat{D})$ with a corresponding penalty schedule

$$\bar{P}_3(D,\hat{D}) = \mu\bar{P}_1(D,\hat{D}) + (1-\mu)\bar{P}_2(D,\hat{D}), \quad \mu \in (0, 1)$$

such that (P3) is satisfied for $\hat{D} \in (\underline{D},D')$. Furthermore, this is true for each $\bar{P}$ and α satisfying (P6) so that for a given α, one can find a $\bar{P}$ such that (P3) is satisfied for $\hat{D} \in (\underline{D},D')$ with contingent contracts given by (P1) and (P5).

(iii) Differentiating $U^A(\hat{P}(D),X^o)$ twice with respect to X we obtain

$$U^A_{xx}(\hat{P}(D),X^o) = E_{\theta>\hat{\theta}}\{U^{A''}(Z^1)\cdot[Z^1_x]^2 + U^{A'}(Z^1)\;[Z^1_{xx}]\}$$
$$+ E_{\theta\leq\hat{\theta}}\{U^{A''}(Z^2)\cdot[Z^2_x]^2 + U^{A'}(Z^2)\;[Z^2_{xx}]\}$$
$$+ [u^A(Z^2(\hat{\theta})) - u^A(Z^1(\hat{\theta}))]\;\frac{d}{dx}\left[f(\hat{\theta})\;\frac{d\hat{\theta}}{dx}\right] \qquad (P7)$$
$$+ 2[u^{A'}(Z^2(\hat{\theta}))\cdot[Z^2_x] - u^{A'}(Z^1(\hat{\theta}))\cdot[Z^1_x]]\quad\left[f(\hat{\theta})\;\frac{d\hat{\theta}}{dx}\right]$$

From (P7) we have $U^A_{xx}(\hat{P}(D),X^o) < 0$ if and only if

$$Z^2_x(\hat{\theta}) > \Gamma(\hat{\theta}) = \frac{-A - B - C + 2u^{A'}(Z^1(\hat{\theta}))[Z^1(\hat{\theta})]\;f(\hat{\theta})\;\frac{d\hat{\theta}}{dx}}{2u^{A'}(Z^2(\hat{\theta}))\;f(\hat{\theta})\;\frac{d\hat{\theta}}{dx}} \qquad (P8)$$

where the first three terms on the right hand side of (P7) are represented by A, B, and C, respectively.

From part (ii) we know that for a fixed α there exists a sequence of $\bar{P}$ which depends on $\hat{D}$ such that (P3) is satisfied for each $\hat{D}$ sufficiently close to $\underline{D}$. Suppose (P3) is satisfied at some $\hat{D}$ given a choice of α, then substituting for C in terms of (P3), (P8) becomes

$$Z^2_x(\hat{\theta}) > \Gamma(\hat{\theta}) = \frac{[-A-B]\left[1 + f(\hat{\theta})\;\frac{d\hat{\theta}}{dx}\left[\frac{d}{dx}\;f(\hat{\theta})\;\frac{d\hat{\theta}}{dx}\right]\right] + 2u^{A'}(Z^1(\hat{\theta}))[Z^1_x(\hat{\theta})]\;f(\hat{\theta})\;\frac{d\hat{\theta}}{dx}}{2u^{A'}(Z^2(\hat{\theta}))\;f(\hat{\theta})\;\frac{d\hat{\theta}}{dx}} \qquad (P9)$$

It can be shown that $\Gamma(\hat{\theta})$ is continuous in $\hat{\theta}$ given a fixed α, and that the right hand side of (P9) approaches

$$\Gamma(\underline{\theta}) = \frac{[-A]\;1 + \bar{f}(\underline{\theta})\;\frac{d\hat{\theta}}{dx}\left[\lim_{\hat{\theta}\to\theta^+}\frac{d}{dx}\;f(\hat{\theta})\;\frac{d\hat{\theta}}{dx}\right] + 2u^{A'}(Z^1(\underline{\theta}))[Z^1_x(\underline{\theta})]\quad\bar{f}(\underline{\theta})\frac{d\hat{\theta}}{dx}}{2u^{A'}(Z^2(\underline{\theta}))\;\tilde{f}(\underline{\theta})\frac{d\theta}{dx}} \qquad (P10)$$

as $\hat{\theta} \to \underline{\theta}^+$, where $\tilde{f}(\underline{\theta}) = \lim_{\theta\to\underline{\theta}^+} f(\theta)$. Note $\Gamma(\underline{\theta})$ is independent of α (see (P6)) so that we clearly choose an $\hat{\alpha}$ sufficiently large such that

$$Z^2_x(\underline{\theta}) = (1 + \hat{\alpha})\;D_x\;(X^o,\underline{\theta}) + C_x\;>\Gamma(\underline{\theta}) \qquad (P11)$$

But both $Z^2_x(\hat{\theta})$ and $\Gamma(\hat{\theta})$ are continuous so that

$Z^2_x(\hat{\theta}) > \Gamma(\hat{\theta})$ for $\hat{\theta}$ sufficiently close to $\underline{\theta}$

which implies $U^A_{xx}(P(\hat{D}),X^o) < 0$ is satisfied for $\hat{\theta} \varepsilon (\underline{\theta},\theta')$, $\theta' > \underline{\theta}$. Thus we have proved statement (A).

(iv) The difficulty one encounters in establishing (B) is that according to (P1) and (P5) $U^A_x(P(\hat{D}),X)$ is discontinuous at two points X_1 and X_2 defined by

$$D(\overline{\theta},X^1) = \hat{D} \tag{P11}$$

$$D(\underline{\theta},X^2) = \hat{D} \tag{P12}$$

and thus it is not sufficient to establish $U^A_{xx}(P(\hat{D}),X) < 0$. Consider each of three cases.

(a) $\underline{X < X_1}$ Suppose X is sufficiently small such that $D(X,\overline{\theta}) \leq \underline{D}$. Then it is easy to verify

$$U^A_x(P(\hat{D}),X) = E_\theta u^{A'}(Z^3)[Z^3_x] < 0.$$

Instead, suppose that X is sufficiently large such that $D(X,\overline{\theta}) > \underline{D}$. Then defining $\theta_1 = \theta(\underline{D},X)$ and $\theta_2 = \theta(\hat{D},X)$,

$$U^A_x(P(\hat{D}),X) = E_{\theta \leq \theta_1} u^{A'}(Z^3)[Z^3_x] + E_{\theta\varepsilon(\theta_1,\theta_2)} u^{A'}(Z^2)[Z^2_x]$$

Now let $\hat{D}$ approach $\underline{D}$, then for those X which are still less than X_1 (note X_1 increases with $\hat{D}$) we have

$$U^A_x \rightarrow E_\theta u^{A'}(Z^3)[Z^3_x] < 0 \quad \text{as} \quad \hat{D} \rightarrow \underline{D}$$

so that there exists a $\hat{D}$ sufficiently close to $\underline{D}$ such that $U^A_x(P(\hat{D}),X) < 0$.

(b) $\underline{X_1 \leq X \leq X_2}$ Note that $X^o \varepsilon [X_1X_2]$. A proof similar to the development in part (iii) suffices to show $U^A_{xx}(P(\hat{D}),X) < 0$ for $X \varepsilon [X_1,X_2]$, $\hat{D}$ sufficiently close to $\underline{D}$. This together with the condition $U^A_x(P(\hat{D}),X^o) = 0$ implies $U^A_x(P(\hat{D}),X) \gtrless 0$ as $X \lessgtr X^o$ for $X \varepsilon [X_1,X_2]$.

(c) $\underline{X > X_o}$ For $X = X^o$, values of D range in the interval $[D(X,\underline{\theta}), D(X,\overline{\theta})]$, and thus the schedule $\tilde{P}(D)$ is not explicitly defined for $D > D(X^o,\overline{\theta})$. Clearly we can select any schedule for $D > D(X^o,\overline{\theta}) \equiv \overline{D}$ without affecting the crucial necessary condition for enforceable that $U^A_x(P(\hat{D}),X^o) = 0$, so that we want to choose a schedule such that $U^A_x(P(\hat{D}),X) < 0$ for $X > X^o$ to insure enforceability. In particular let

$$\tilde{P}(D) = \tilde{P}(D(X^o,\overline{\theta})) + \alpha(D-\overline{D}) \text{ for } D > \overline{D} \tag{P13}$$

where α is large, and define $Z^{o} = W_{o} + W(X,\theta) - \tilde{P}(D(X^{o},\bar{\theta}) - \alpha(D(X,\theta) - \bar{D} + \delta(\hat{D},X^{o})$. Then for $X > X^{o}$

$$U_{x}^{A}(P(\hat{D}),X) = E_{\theta \leq \theta(\bar{D},X)} u^{A'}(Z^{1})Z_{x}^{1} + E_{\theta > \theta(\bar{D},X)} u^{A'}(Z^{o})Z_{x}^{o} \qquad \text{(P14)}$$

Recall from (P1) that for $D \in [\underline{D},\bar{D}]$, $\tilde{P}(D)$ is constructed so that $U_{x}^{A}(\tilde{P}(D),X^{o}) < 0$. Thus as $X \to X^{o+}$ and $\hat{D} \to \underline{D}^{+}$ we have $U_{x}^{A}(P(\hat{D}),X) \to U_{x}^{A}(\tilde{P}(D),X^{o}) < 0$. Consequently for $\hat{D}$ sufficiently close to $\underline{D}$, and X sufficiently close to X^{o}, $U_{x}^{A}(P(\hat{D}),X) < 0$. On the other hand for larger X, the term $E_{\theta > \theta(\bar{D},X)} u^{A'}(Z^{o})Z_{x}^{o}$ can be made sufficiently large and negative by specifying a large α in (P13), to insure $U_{x}^{A}(P(\hat{D}),X) < 0$.

The three cases (a)-(c) combine to yield the result in statement B which together with the fact that $U^{A}(P(\hat{D}),X)$ is continuous in X implies that $(P(\hat{D}),X^{o})$ is enforceable if $U_{x}^{A}(P(\hat{D}),X^{o}) < 0$ (see Figure 1).

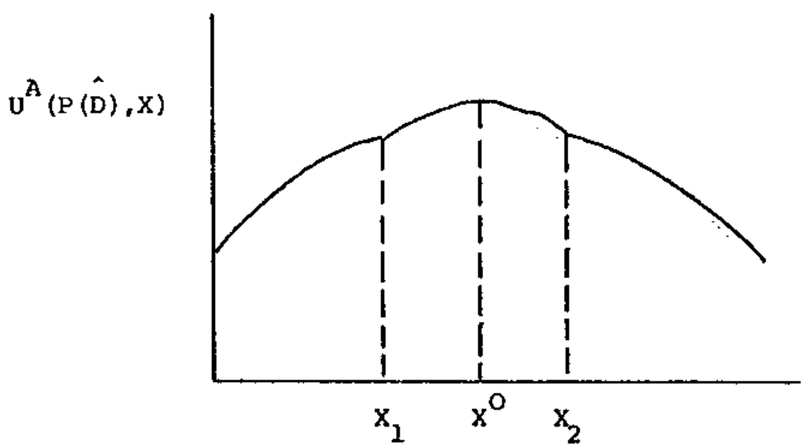

Figure 1

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