POLLUTION PRICES IN A GENERAL EQUILIBRIUM MODEL

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Environmental pollution is the classic example of an external diseconomy, and discussions of smoking chimneys and dirty laundry have been taking place in the economics literature for generations. Yet, now that pollution is regarded as one of our major public problems, the policy maker finds little in economic theory which can guide him in establishing programs of pollution control. With few exceptions, the excellent work of Allen Kneese being the most prominent, he finds that economists have not gone far beyond the observation that the correct set of taxes and subsidies can solve any problem; and the problems which have been analyzed more completely--apples and bees, chimneys and laundry, railroads and farmers--must seem rather trivial to the policy maker concerned with Los Angeles smog or the death of Lake Erie. It is not surprising that the authorities turn to the engineers for help, relying on their gadgets and rules to solve the problems.

This paper is an effort to show that many of our most pressing pollution problems fit very nicely into traditional economic analysis, and therefore economists should have some very useful advice to give on the problem of control. The emphasis throughout is on pollution of a "public bad" variety, with Los Angeles smog its prototype. There is a real difference between this sort of externality and the more common two-sided interaction; but the latter is discussed at length in existing literature, while the former seems more characteristic of current problems. And while direct interactions of the chimney-laundry type have been and will continue to be solved by civil law, the more general pollution problems will require changes in our political and economic institutions. It is advice on the required adjustments which is needed and lacking.

The analysis of the pollution problem begins with a brief discussion of existing methods of economic decision making and control, upon which most existing and proposed pollution programs are based. It is argued that benefit-cost analysis and direct regulation are totally inadequate tools for the more complex pollution problems. Ultimately, a political-economic process will have to make decisions about pollution policy and great care must be taken in designing this process. It is suggested that an explicit distinction be made between those decisions which must be made politically and those which are best made by economic calculation, and that institutions be structured accordingly. On the economic side a quasi-market in pollutants is shown to be capable of achieving specified pollution levels "efficiently" in a competitive general equilibrium model, where efficiency is defined in a sense related to, but distinct from, Pareto optimality; and the market "prices" are shown to be estimates of the marginal cost of pollution reduction. The political process can then decide the matters of welfare distribution and desired pollution levels. Finally, the practicality of the price system for pollution control is considered, and it is argued that such a system is probably the best solution for the worst problems. In fact, the more difficult the problem of control, the greater advantage the price system has over its alternatives.

The Standard Approach to Pollution Problems

While pollution has only recently been a major issue, the problem itself has been around for centuries, and steps have been taken to deal with it. The basic approach must always be to forbid or restrict the polluting activity, if "the" activity can be identified, and if it is felt that the benefits from reducing pollution exceed the costs imposed in restricting the activity. The only issues are how costs and benefits are measured, and how the offending activities are restricted.

Today, the process of measuring benefits and costs of a proposed project has become routinized, in the form of benefit-cost analysis. The shortcomings of the process, as applied to public works projects such as dams and transportation systems, are well known; because it is necessary to predict the future of prices, to estimate consumers' surplus, and to evaluate distributional effects, with all the conceptual and practical difficulties inherent in such endeavors, the results of any analysis must be treated with suspicion, especially if the project is "large" (Hines, 1962). Nonetheless, such measures of economic desirability are often the only guide to rational decisions and, if used with care, can be better than nothing (Ciriacy-Wantrup, 1964).

There is no reason, in principle, that benefit-cost analysis could not be used to help analyze pollution problems. One could simply measure, for various degrees of pollution reduction, the costs of achieving the reduction, the resulting benefits, and then choose that level which maximizes benefit minus cost, just as one chooses the optimal scale for a dam; and, if only implicitly, such calculations are the basis of many decisions about pollution programs. Since there are very good reasons to expect benefit-cost analysis to produce a bias <u>against</u> pollution reduction, it is not surprising that "economic calculation" is under bitter attack today. Further, since the nature of the pollution problem allows different decision methods which are not applicable to public works projects, it is not really necessary to rely on benefit-cost analysis; there is a preferable alternative.

The bias against pollution control suggested above, is a two-pronged weapon; benefit-cost analysis of a pollution reduction "project" will tend to underestimate the benefits and overestimate the costs. Dealing with the benefits side first, the reason for underestimation is obvious: many of the benefits are "subjective" or "noneconomic" in nature, and hence are not easy to evaluate. It is true, of course, that all values are based on subjective preferences, and it is always possible to state such subjective values in terms of marginal rates of substitution or relative prices; but it is also true that there is no way yet known to economic science to estimate these subjective magnitudes if there are no markets which compel individuals to make up their own minds on the matter and then reveal their preferences by their actions.

When confronted with this problem, the honest analyst will admit his inability to evaluate all the relevant effects and will describe them to the decisionmaker in physical terms, letting him decide how much such effects are worth.[1] Given the practical bent of the politicians and engineers who usually make such decisions, the most likely outcome is that the effects with concrete, dollar values will be given primary attention, with the nonvalued effects treated as "secondary considerations," to be taken into account if the decision is a marginal one.

The importance of this tendency to underweight nonvaluable effects depends on the size of these effects relative to those which can be assigned dollar values, and on whether there is any reason to expect more of these effects on one side or the other of the benefit-cost accounts. For public projects to which benefit-cost analysis is usually applied, the subjective effects are usually relatively small, and are as likely to be on one side as

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[1] Although he may try to put bounds on the values, or simply invent "reasonable" prices which will provide some guide. See Hirshleifter (1969) for an interesting discussion of the value of cooler water in the municipal water system of New York.

the other of the accounts.[2] But when the project concerns pollution control, it is likely that most of the subjective effects are on the benefit side; and they may be a very large share of total benefits. If we try to determine the benefits of air pollution reduction by estimating the savings in tires, paint, laundry bills, and medical expenses, treating eye irritation, painful breathing, and aesthetic values as "secondary considerations", we will certainly underestimate the benefits. It may be that even the easily valued effects will justify action in the present situation; but basing decisions on these benefits will delay action, and stop it short of optimal levels.

The bias against action on the benefit side is strengthened by a tendency to overestimate the costs of pollution control, caused by a failure or inability to recognize the general-equilibrium nature of the economy, and hence to regard as immutable constraints many things which are not. By a well-known principle of economics, the more constraints the higher the cost of altering the situation.[3]

There are several reasons why the cost estimators regard too many things as constraints. The most obvious is that no analyst, no matter how clever, can hope to know all the subtle adjustments which could be made and therefore each concentrates on a few obvious but expensive actions. Further, even if he can imagine some of the less obvious adjustments, it is impossible to enforce most of them with the traditional methods of control; hence they must be considered unfeasible. Finally, even if enforceable, there is no way to estimate the costs involved in most of these subtle steps.

The problem of cost overestimation is illustrated clearly by some examples. In reducing the agricultural use of DDT, the California Department of Agriculture takes the existing crop mix as given, completely ignoring the possibility that a different crop mix, producible with less

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[2] Except that the analysts may work harder to value effects on one side or the other if they are trying to prove a point. Also, it is never easy to put accurate values on any of the effects; but for some it is relatively easy to find numbers which appear to have some relevance to the matter, e.g., industry sales.

[3] See Samuelson (1965) for a formal statement of the "Le Chatelier Principle."

DDT, might satisfy the same needs.[4] The future costs of reducing pollution from power plants is invariably estimated by projecting future "need" for power, and calculating the cost of cleaning up the required plants, without considering the possibility that consumers could cook with gas and insulate their homes. The estimates of the cost of reducing auto emissions often take as given the size and power of the cars and the commuting patterns of the area, even though adjustments in these variables are almost certainly part of any "best" solution to the problem. Whether the cost estimators ignore these other adjustments because they do not think of them, cannot hope to enforce them, or simply do not know how to estimate the costs involved, the effect is the same: costs are overestimated.[5]

With benefits tending to be underestimated and costs overestimated by the traditional decision processes, it is not surprising that so little action has been taken on pollution problems, until a situation has developed in which much of the population knows something is wrong, no matter what the economic calculations say. But this is only part of the problem. Even when it has been decided to do something, the traditional methods of control have been hopelessly inadequate, primarily because they have been technocratic in nature; i.e., a board of experts has studied the problem, determined the best solution, and then tried to impose it.

The shortcomings of this approach to problems should be obvious by now; they are discussed in Ruff (1970), and in considerable detail in Kneese (1964) and Kneese and Bower (1968). The basic difficulty is the one common to all centralized allocation processes: the informational and bureaucratic requirements are impossibly demanding.

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[4] This has interesting results. Since there is no "good" substitute, DDT is allowed on asparagus, but not on Brussels sprouts, where there is a "good" substitute, i.e., one only "reasonably" more expensive than DDT. The effect on relative costs is to encourage production and use of the DDT-intensive crop. Also, growers of asparagus know that their costs will be increased if they make the mistake of finding a "good" substitute for DDT.

[5] We are not discussing here the "index number problem," which will be important for large projects. It may be that costs are underestimated if measured in current prices, e.g., the price of natural gas would not stay low if all automobiles converted to it.

This makes it impossible to ever know the best solution to any problem, to enforce it, to adjust to the continual changes, to find new approaches, and so on. The control boards, always understaffed, are either far behind in their duties, or adopt an arbitrary set of "standards" which has no relation to economic efficiency, or both.

Again, some examples are instructive. Many standards are set in terms of dilution of the pollutant, e.g., X parts per million in the effluent stream. These standards are relatively easy to enforce, requiring only monitoring, and no knowledge of the production process itself. But they are pointless if, as is usually the case, it is total output of the pollutant which counts; one can reduce his parts per million either by reducing the "parts" or increasing the "millions," and the latter may be the cheaper. There are even smoke standards stated in terms of the Ringlemann Opacity Index, e.g., the smoke must not filter out more than X percent of the sunlight; literally any amount of smoke can meet any such index if it is blown fast enough from a big enough chimney. Automobile emission standards are stated in "grams per mile," as though "miles per year" were unimportant or constant. It is no surprise such regulations have disappointing results. Yet, to go beyond such uniform standards, to try to set limits on amounts rather than density of pollution, involves impossible information requirements, and/or arbitrariness of another kind; requiring uniform percentage reductions is a common practice.

An Alternative Approach to Pollution Problems

The analysis to this point suggests that the traditional approach to pollution problems is quite inadequate. The method of deciding on desired pollution levels is at best inaccurate, and at worst produces a bias against action. The typical methods of enforcement, of restricting the polluting activities, are at best inefficient, and at worst a bad joke. Unfortunately, most current attempts to deal with pollution are based on strengthening the very processes which have not worked; they will lower the Ringlemann standard from "2" to "1", and require another twenty percent reduction from all sources--subject to special permits and appeal hearings, of course. It is a fair bet such measures will not be very effective. What is needed is a fundamentally different approach.

The first step toward a more rational pollution control program is to recognize that economic calculation cannot be expected to make all decisions. Political decisions will have to be made because of the conceptual and practical difficulties already discussed; but to recognize the need for political decision is not the same as abandoning all decisions to the politicians. Rather, the suggested course of action is to design institutions which will allow explicit political decisions where required, allow economic calculation where possible, and provide sufficient feedback between the economic and political mechanisms so that the system is responsive, stable, and efficient. In the absence of such institutionalized division of authority, we will continue the existing pattern of bad economic calculation followed by misdirected political zeal.

The obvious division of authority would be to let economic calculation determine the best way to accomplish various levels of pollution reduction and the costs involved, and then let a political process choose the best level. This dodges the problem of evaluating subjective benefits in monetary terms, but raises a new difficulty; can the political process be expected to make a reasonable decision about whether to spend X dollars to reduce smog to "1940 levels"? Also, the problem of cost overestimation remains; for finite moves, a partial equilibrium view will take too many things as given and will overestimate costs.

Fortunately, however, the nature of most pollution problems provides an option which is lacking for most public works projects; a series of marginal decisions, or a "gradient search method," may lead to an optimal solution. For, unlike a dam or canal, a pollution control program can be started on a small scale, and expanded if necessary, until an optimal scale is achieved; we have *ex post* as well as *ex ante* continuity of project scale.[6]

The *ex post* continuity of the control program greatly simplifies the decision process. Given a level of control, we need only consider the benefits and costs of a *marginal* reduction in pollution in order to decide which way to move. Presumably a political process can be designed which can decide whether a small reduction is worth its cost; and economic calculation can measure the marginal cost of further reduction, since *marginal* cost is independent of

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[6] This is still "approximately" true even when control for any single polluter exhibits economies of scale, so long as there are many polluters who must be controlled separately.

the number of constraints assumed fixed.[7] Marginal cost may even, in fact, be estimated automatically if the proper control methods are used.

The division of authority suggested here can be contrasted with that of Musgrave (1959). Musgrave suggests an Allocative Branch, a Distributive Branch, and a Stabilization Branch of the Government. Assuming full employment always, so that the Stabilization Branch is not needed, Musgrave's structure would leave the entire pollution control problem in the Allocative Branch. The Allocative Branch would recognize the need for action to correct the market failure due to the externality, would decide how much pollution reduction to accomplish, using some form of benefit-cost analysis, and would move the system to a Pareto optimal state. The Distributive Branch would then act to correct any undesirable changes in income distribution.

This distribution of authority does effectively separate political and economic decisions, when allocative decisions require no political action. But when political decisions are required in allocative matters, as they are in pollution problems, this division of authority begins to break down. The political decision about the desirability of further reduction in pollution levels is not really different from the decision about the desirability of changing the income distribution, and hence similar processes must be used for both decisions.[8] The attempts to construct clever voting schemes to get people to reveal their true preferences about such matters may yet prove successful; but even if they do, it is clear that these decisions are rather far removed from the traditional domain of the allocative economist.

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[7] If everything is adjusted to the existing situation, the slope of the MC curve, not MC itself, increases as the number of immutable constraints increases. This is a standard result in the theory of the firm. A further advantage of the marginal approach is that it avoids the index number problem.

[8] At least in societies where redistribution and pollution policy is based on the preferences of the citizens, rather than being imposed. If pollution levels and some index of income equality enter into individual utility function, they must be treated symmetrically.

A distribution of authority slightly different from Musgrave's seems more useful for pollution problems. Let there be an Efficiency Branch and a Political Branch of the Government. The task of the Efficiency Branch is to achieve "efficiency" in a special sense to be defined more precisely, taking as given the distribution of income and specified levels of pollution. Once an efficient state has been achieved, the Political Branch must decide whether a redistribution of income or a change in the specified pollution levels is called for. If so, they are made; the Efficiency Branch finds a new efficient state, and so on. With luck, this process eventually produces a state which is optimal, at least relative to the existing political structure.

The form of the political decision process is not the primary concern here. It must be capable of making decisions of the simple form: is it worth $X to reduce pollution Y%? Because the issues are so simple, the process could be very direct; referendum is a possibility, although decision by an elected body seems more practical. But, if democratic principles are adhered to, the process should be responsive to the wishes of "the people" to prevent its capture by the polluters or by overly zealous anti-polluters. However the decision is made by the Political Branch, the Efficiency Branch must be able to accomplish the specified levels efficiently, and to provide accurate estimates of the costs of further reductions.

The suggested device for efficient economic calculation is, of course, an extension of the price system to cover polluting activities. In the next section we show that such a price system can do the job, in theory; and we then outline, in the last section, how it could be applied in practice.

General Equilibrium With Pollution Prices: The Simplest Case

The model we use to analyze general equilibrium with pollution prices is a simple extension of standard general equilibrium models. Each of M individuals is assumed to have an ordinal utility indicator, which is a function of his individual consumption of N ordinary economic goods, and the levels of L types of pollution. For individual j, the utility function is

$$U^j(q^j; Z) = U^j(q^j_1, \ldots, q^j_N; Z_1, \ldots, Z_L) ,$$

where q_i^j = consumption of good i by individual j,

Z_ℓ = level of pollution of type ℓ.

We assume there is always some good which the individual desires.

Individual j has a budget b^j, which depends on prices, taxes, subsidies, etc. There is some sort of redistribution scheme which keeps income distribution "fair," perhaps operating by allocating the proceeds of the pollution control activity; all we need assume is that all purchasing power is ultimately in the hands of the consumers, and each individual's budget is independent of his consumption decisions.

On the production side, there are K profit maximizing competitive firms, which use ordinary goods as inputs and outputs, and which produce pollution in the production process. For now, production constraints are not affected by aggregate pollution levels. For firm k, the production constraint is

$$g^k(x^k;\, z^k) \equiv g^k(x_1^k, \ldots, x_N^k;\, z_1^k, \ldots, z_L^k) \geq 0 ,$$

where x_i^k = net output of good i from firm k,

z_ℓ^k = net output of pollutant ℓ from firm k.

Neither x_i^k nor z_ℓ^k need be non-negative.

The aggregate pollution levels are simply the sums of individual pollution outputs of firms,

$$Z_\ell \equiv \sum_{k=1}^{K} z_\ell^k , \quad \ell = 1, 2, \ldots, L.$$

Of course, by letting L be large enough, this formulation can handle any producer-consumer interaction, including the chimney-laundry two-sided externality. However, the theorems to follow assume "competitive" conditions, in which firms take pollution prices as given, and individuals take aggregate pollution levels as given; such behavior is unreasonable unless, for each ℓ, there are many producers of pollutant ℓ.

In the absence of any control on pollution levels, we assume a general competitive equilibrium is established[9], in which pollution levels are considered to be too high. Therefore, it is decided to impose control, in the form of a L-dimensional pollution fee or tax vector, $\bar{\tau}$. The system readjusts, reaching a new equilibrium, defined by

<u>Definition I</u>: An allocation $\bar{q}$, $\bar{x}$, $\bar{z}$ is said to be a competitive general equilibrium allocation, and $\bar{p}$ is said to be a competitive general equilibrium price vector, relative to the tax vector $\bar{\tau}$ (and relative to the income distribution) if:

(a)
$$\bar{p}'\bar{q}^j = b^j, \quad \bar{q}^j \geq 0, \quad \text{and for all } q^j \geq 0$$
$$\text{such that } \bar{p}'q^j \leq b^j,$$
$$U^j(q^j;\, \bar{Z}) \leq U^j(\bar{q}^j;\, \bar{Z}), \quad j = 1, 2, \ldots, M;$$

(b)
$$g^k(\bar{x}^k;\, \bar{z}^k) \geq 0, \quad \text{and for all } x^k, z^k$$
$$\text{such that } g^k(x^k;\, z^k) \geq 0,$$
$$\bar{p}'x^k - \bar{\tau}'z^k \leq \bar{p}'\bar{x}^k - \bar{\tau}'\bar{z}^k, \quad k = 1, 2, \ldots, K;$$

(c)
$$\sum_{k=1}^{K} \bar{x}_i^k \geq \sum_{j=1}^{M} \bar{q}_i^j, \quad \bar{p}_i \geq 0,$$

and

$$\bar{p}_i \left[\sum_{k=1}^{K} \bar{x}_i^k - \sum_{j=1}^{M} \bar{q}_i^j \right] = 0, \quad i = 1, 2, \ldots, N;$$

(d)
$$\bar{Z}_\ell = \sum_{k=1}^{K} \bar{z}_\ell^k, \quad \ell = 1, 2, \ldots, L.$$

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[9] We are concerned here with the question of efficiency of price equilibria, rather than with the more difficult question of <u>existence</u> of such equilibria. However, if consumers' preferred-to sets are convex for every set of pollution levels, and production sets are convex, it would not be difficult to prove that a price equilibrium exists for every set of desired pollution levels. See Arrow (1951).

This definition is the standard one for a competitive equilibrium, with the addition of pollution as a public good. Individuals are choosing their consumption vectors to maximize their utility, subject to their budget constraint, and ignoring the impact of their consumption choices on pollution levels. Firms are maximizing profits, given prices, taxes and their production constraints. Production of any good does not fall short of consumption, and exceeds it only if the price of the good is zero.

Because of the externality operating through pollution levels, there is no presumption that the competitive price equilibrium allocation is Pareto optimal. It may fail to be Pareto optimal for two distinct reasons. The first is that the levels of pollution may be such that everybody would gain from an increase or decrease in these levels, with the corresponding decrease or increase in income levels. This brings us back to the problem of choosing the optimal pollution levels, which we are avoiding by relying on a political decision. The second possible reason for the price allocation failing to achieve Pareto optimality is that it may not be efficient, in the sense of

Definition II: An allocation will be said to be efficient if it is feasible, and if there is no feasible allocation which has the same aggregate pollution levels, and yet is Pareto preferred, i.e., makes somebody better off without harming anyone.

That is, even if the "correct" pollution levels are achieved, the allocation must be efficient in this sense in order to be Pareto optimal.

It is this concept of efficiency which is important for the political-economic mechanism discussed earlier. The given pollution levels must be accomplished efficiently, and there must be an estimate of the marginal cost of further reduction, if the political decision concerning changes in the levels is to be an informed one. The following two theorems prove that the extended price system satisfies both these requirements.

Theorem I: A price equilibrium defined by Definition I is efficient in the sense of Definition II.

Proof: Let $\bar{q}$, $\bar{x}$, $\bar{z}$ be the allocation of Definition I, and let $\hat{q}$, $\hat{x}$, $\hat{z}$ be any feasible allocation with the same pollution levels, i.e.,

$$\sum_k \hat{z}_\ell^k = Z_\ell = \sum_k \bar{z}_\ell^k , \qquad \ell = 1, 2, \ldots, L.$$

From the profit maximization condition, we know

$$\bar{p}'\bar{x}^k - \bar{\tau}'\bar{z}^k \geq \bar{p}'\hat{x}^k - \bar{\tau}'\hat{z}^k, \qquad k = 1, 2, \ldots, K.$$

Or, summing over all firms and changing the order of summation,

$$\sum_i \bar{p}_i \left[\sum_k \bar{x}_i^k - \sum_k \hat{x}_i^k \right] \geq \sum_\ell \bar{\tau}_\ell \left[\sum_k \bar{z}_\ell^k - \sum_k \hat{z}_\ell^k \right].$$

Since the pollution levels are the same in the two allocations, the right-hand-side vanishes, and we have

$$\text{(a)} \qquad \sum_i \bar{p}_i \sum_k \bar{x}_i^k \geq \sum_i \bar{p}_i \sum_k \hat{x}_i^k .$$

Now, suppose the $\hat{q}$, $\hat{x}$, $\hat{z}$ allocation is Pareto preferred to the $\bar{q}$, $\bar{x}$, $\bar{z}$ allocation. By the assumption of nonsatiation in consumption, this implies the $\hat{q}^j$ bundles must cost more (at prices $\bar{p}$) for some and no less for all consumers than the $\bar{q}^j$ bundles, and hence the total cost of all bundles must be greater, i.e.,

$$\text{(b)} \qquad \sum_i \bar{p}_i \sum_j \bar{q}_i^j < \sum_i \bar{p}_i \sum_j \hat{q}_i^j .$$

But, since all allocations must be feasible, and prices $\bar{p}$ are nonnegative, we can say

$$\sum_j \hat{q}_i^j \leq \sum_k \hat{x}_i^k ,$$

hence

$$\bar{p}_i \sum_j \hat{q}_i^j \leq \bar{p}_i \sum_k \hat{x}_i^k ,$$

and

$$\text{(c)} \qquad \sum_i \bar{p}_i \sum_j \hat{q}_i^j \leq \sum_i \bar{p}_i \sum_k \hat{x}_i^k .$$

Combining (b) and (c) above with (c) of Definition I, we finally obtain

$$\text{(d)} \qquad \sum_i \bar{p}_i \sum_k \bar{x}_i^k = \sum_i \bar{p}_i \sum_j \bar{q}_i^j < \sum_i \bar{p}_i \sum_j \hat{q}_i^j \leq \sum_i \bar{p}_j \sum_k \hat{x}_i^k .$$

Since inequalities (d) and (a) are contradictory, it follows that it is impossible to find an allocation which is feasible, has the same pollution levels, and yet is Pareto-preferred to a price-equilibrium allocation. Q.E.D.

<u>Theorem II</u>: In the general equilibrium of Definition I, the tax-price of pollutant ℓ, $\bar{\tau}_\ell$, is the marginal cost, in terms of "national income" as ordinarily defined,[10] and at current prices, of reducing pollutant ℓ. That is, defining national income at current prices, $\bar{Y}$, by

$$\bar{Y} \equiv \sum_k \bar{p}'\bar{x}^k \equiv \sum_j \bar{p}'\bar{q}^j ,$$

it is true that

$$\text{(a)} \qquad \bar{\tau}_\ell = \left.\frac{\partial \bar{Y}}{\partial Z_\ell}\right|_{p=\bar{p}} \equiv \sum_j \bar{p}' \frac{\partial \bar{q}^j}{\partial Z_\ell} \equiv \sum_k \bar{p}' \frac{\partial \bar{x}^k}{\partial Z_\ell} .$$

<u>Proof</u>: For simplicity, we will assume continuous, differentiable functions for this proof. Then, the profit maximization conditions imply, for all k, i, j,

$$\text{(a)} \qquad \frac{1}{\bar{\tau}_j} \cdot \frac{\partial g^k}{\partial z_j^k} + \frac{1}{\bar{p}_i} \frac{\partial g^k}{\partial x_i^k} = 0 ,$$

$$g^k(\bar{x}^k; \bar{z}^k) = 0 .$$

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[10] The qualification "as ordinarily defined" is required to distinguish this Y from any measure of "welfare." Y, as defined here and in ordinary GNP accounts, cannot be considered a reliable index of welfare because it ignores changes in the environment. But changes in Y <u>can</u> be used as estimates of the "economic" cost of improving the environment.

Now consider a differential change in the parameters $\bar{\tau}$, to $\bar{\tau} + \Delta\tau$. Taking a total differential of $g^k = 0$ yields

$$\sum_i \Delta x_i^k \frac{\partial g^k}{\partial x_i^k} + \sum_j \Delta z_j^k \frac{\partial g^k}{\partial z_j^k} = 0 .$$

Using (a), this relation can be rewritten

$$\frac{1}{p_i} \frac{\partial g^k}{\partial x_i^k} \left[\sum_i \Delta x_i^k \cdot \bar{p}_i - \sum_j \Delta z_j^k \cdot \bar{\tau}_j \right] = 0 ,$$

from which it follows that

$$\sum_j \Delta z_j^k \bar{\tau}_j = \sum_i \Delta x_i^k \bar{p}_i .$$

Or, summing over all k,

$$\sum_j \bar{\tau}_j \cdot \Delta Z_j \equiv \sum_j \bar{\tau}_j \sum_k \Delta z_j^k = \sum_i \bar{p}_i \sum_k \Delta x_i^k .$$

But, if the tax changes $\Delta\tau_j$ are chosen so that the aggregate levels of all pollutants except ℓ are unchanged, i.e., if $\Delta Z_j = 0$, $j \neq \ell$, then

$$\bar{\tau}_\ell = \sum_i \bar{p}_i \sum_k \frac{\Delta x_i^k}{\Delta Z_\ell}$$

from which (a) follows in the limit. Q.E.D.

These two theorems demonstrate that problems of pollution can be analyzed with the standard tools of economic analysis, and they suggest that the political-economic process outlined above may be a very practical approach. In fact, these theorems are not really a significant theoretical or (certainly) mathematical extension of the more familiar theorems regarding efficiency and price equilibria. If we regard "waste disposal services" as productive factors, with the acceptable level of pollution determining the total amount of such services available, we have a standard problem of allocating the publicly owned scarce capacity; that a price system accomplishes this allocation efficiently, and that the competitive rental price reflects the marginal value of the capacity, is no surprise. Still, it is a useful way of viewing the problem and does suggest how an effective pollution control program might be structured, a matter to which we return presently.

The tax rate $\bar{\tau}_\ell$ does estimate the marginal cost of reducing pollutant ℓ, and this cost can be used in the political decision process which decides whether to increase or decrease Z_ℓ. However, it must be made clear to the decision makers that this cost is less than the increase in their taxes which would result from an increase in tax rates and unchanged behavior on their part. They can and will make adjustments, which will lower their pollution taxes. In addition, any increase in tax revenues is rebated, either through lower taxes of other kinds, or more public services. If this rebate feature is not understood, tax increases will be difficult to sell.

Some Extensions of the Simplest Case

The model of Theorems I and II has assumed there are several distinct types of pollutant, which are produced by firms and which act directly on consumers. Often, however, the primary effluents interact in the environment to produce synthesized pollutants, which then act on individuals. Smog in Los Angeles is an example, with the more obnoxious components being generated photochemically in the atmosphere, with industrial and automotive emissions providing raw materials. A similar situation arises if it is possible to specify "isodamage" curves or surfaces for some of the pollutants, on the basis of medical findings or cost estimates, for example.

Wherever they come from, suppose we can specify a set of functions of the form

$$S_r = \phi_r(Z), \quad r = 1, 2, \ldots, R, \quad R < L,$$

where it is the vector S which enters into individual utility functions, rather than Z; that is, $U^j(q^j; Z)$ is replaced by $U^j(q^j;S)$, but otherwise the model is unchanged. Now, when a set of pollution prices $\bar{\tau}$ is specified and a general equilibrium is achieved with effluent outputs of $\bar{Z}$, the levels of synthesized pollutants (using this interpretation, for concreteness) are $\bar{S} = \phi(\bar{Z})$. Theoren II tells us there is no Pareto-preferred way to achieve emission levels $\bar{Z}$, but says nothing about better ways to accomplish $\bar{S}$. There are many Z's, each with its own tax vector τ, which will produce $\bar{S}$.[11] How can we know when we have found an efficient one? At least a partial answer is provided by

<u>Theorem III</u>: A price equilibrium defined by Definition I, with levels of synthesized pollution $\bar{S} = \phi(\bar{Z})$, is efficient, in the sense that there is no Pareto-preferred way to accomplish $\bar{S}$, <u>if</u>

$$\sum_{\ell} \bar{\tau}_{\ell} \bar{Z}_{\ell} \geq \sum_{\ell} \bar{\tau}_{\ell} Z_{\ell} \quad \text{for all } Z_{\ell} \text{ such that } \phi(Z) = \phi(\bar{Z}).$$

That is, if there is no way to rearrange production so as to increase tax revenue at current rates, without also changing the levels of synthesized pollution, the allocation is efficient.

<u>Proof</u>: Let $\bar{q}$, $\bar{x}$, $\bar{z}$ be the allocation and $\bar{p}$ the price vector for the general competitive equilibrium corresponding to tax vector $\bar{\tau}$, and assume the hypothesis of the theorem. Then, if $\hat{q}$, $\hat{x}$, $\hat{z}$ is any feasible allocation with the same levels of synthesized pollution, i.e., $\phi(\sum_k \bar{z}^k) = \phi(\sum_k \hat{z}^k)$,

we know.

$$\text{(a)} \qquad \sum_{\ell} \bar{\tau}_{\ell} \bar{z}_{\ell} \geq \sum_{\ell} \bar{\tau}_{\ell} \hat{z}_{\ell} .$$

Just as in Theorem I, profit maximization and feasibility imply

$$\sum_{i} \bar{p}_i \left[\sum_k \bar{x}_i^k - \sum_k \hat{x}_i^k \right] \geq \sum_{\ell} \bar{\tau}_{\ell} \left[\sum_k \bar{z}_{\ell} - \sum_k \hat{z}_{\ell}^k \right] .$$

Since (a) implies the right-hand-side above is non-negative, we have

$$\sum_i \bar{p}_i \sum_k \bar{x}_i^k \geq \sum_i \bar{p}_i \sum_k \hat{x}_i^k ,$$

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[11] As long as $R < L$, as assumed. If $R \geq L$, then it becomes impossible, in general, to find a set of L tax-rates which will produce specified levels of R pollutants; at best, there is no room for choice of τ, and hence no question of efficiency. We assume away the problem of more goals than instruments. In any case, if $R \geq L$, we may as well stay with the primary pollutants.

which is just (a) of Theorem I, from which the rest of the proof follows exactly as in Theorem I. Q.E.D.

Theorem III seems to suggest that the pollution control authorities can achieve any level of synthesized pollution efficiently, by choosing a set of pollution taxes which maximizes tax revenue, subject to the constraint that smog be at the specified levels. This interpretation is not quite correct, however, primarily because the taxing authority cannot be allowed to exploit its inevitable monopoly position. For example, suppose that in a two-pollutant, single-synthesized-product world, the pollution authorities select taxes τ_1^0 and τ_2^0, and in the resulting equilibrium primary pollutant levels are Z_1^0, Z_2^0, with "smog" level of S°, as in Figure I. The curve A-Z°-Z^1-B is an iso-smog curve, and will have the suggested shape if the primary pollutants exhibit "increasing marginal damage", in a generalized sense. The curve C-Z°-D is an iso-profit curve for firms; that is, at pollution levels along this curve, firms can produce goods with the same profit for themselves, and can get higher profits only by producing pollution levels above the curve.

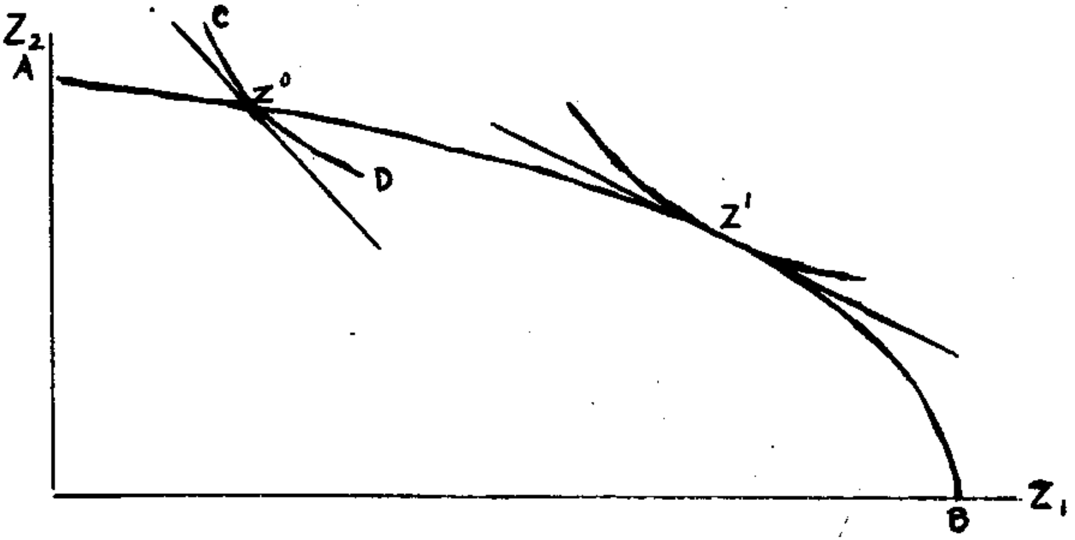

Figure I.

Given the tax vector τ°, producers are in equilibrium at Z°, which is clearly an inefficient position; points such as Z^1, for example, have higher profits and hence consumption of greater utility, with the same smog levels. They also have higher tax revenues at the present rates τ°. Therefore, the authorities should encourage movement toward Z^1, and the obvious way to do this is to lower τ_1, relative to τ_2, and adjust the absolute levels of both until some point such as Z^1 is reached. At Z^1, there is no Z with the same $\phi(Z)$ and higher tax revenues <u>at the prices</u> τ^1. However, whether total tax revenue $\Sigma\tau_\ell^1 Z_\ell^1$ is greater or less than it was originally, $\Sigma\tau_\ell^\circ Z_\ell^\circ$, is impossible to say; this depends on the elasticities of demand for waste disposal services. Hence, Theorem III does <u>not</u> imply the control authorities should maximize tax revenues; but it does suggest a tax-adjustment rule they might be able to use to find an efficient solution, and which requires them to know only the $\phi(Z)$. And there is still a simple method of estimating the marginal cost of reducing the level of S_r, given the "marginal product" of primary pollutant Z_ℓ in the production of S_r, $\partial\phi_r/\partial Z$:

$$\left.\frac{\partial Y}{\partial S_r}\right|_{p=\bar{p}} = \frac{\bar{\tau}_\ell}{\dfrac{\partial\phi_r}{\partial Z_\ell}} \quad .$$

Of course, Theorem III (as well as I and II) is a theorem of the form: <u>If</u> there is a price equilibrium, it is efficient. There is no <u>a priori</u> guarantee that there is an equilibrium, tax-revenue-maximizing solution at all, in which case Theorem III is of little help to the authorities. For example, the tax vector τ° might lead to a situation such as in Figure II, with the smog level S° produced efficiently at point Z°, where tax-revenue-subject-to-constraint-and-given-rates, is minimized.

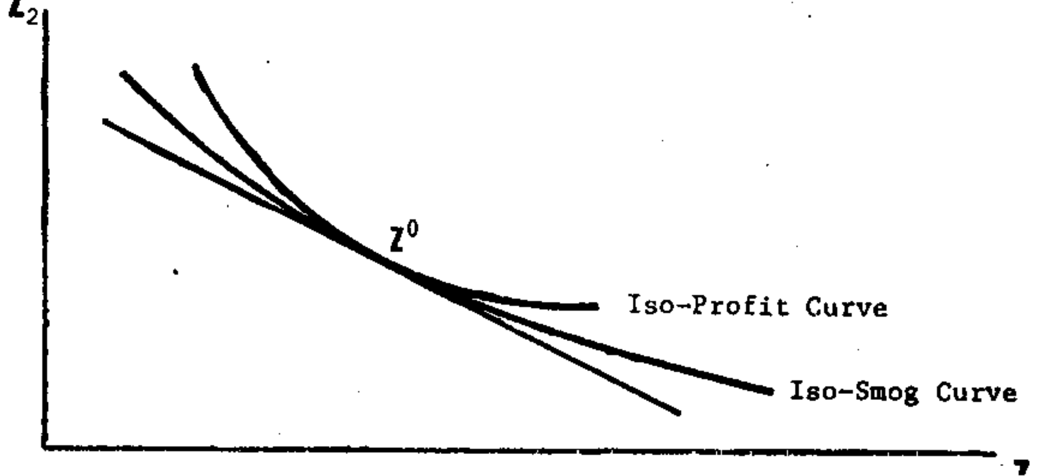

Figure II

The control authorities can distinguish this efficient situation from the similar inefficient one, where the iso-smog curve lies on or *above* the iso-profit curve, only by knowing the production relations of the firms. Even more difficult problems arise if the iso-profit curves are concave downward; the efficient point might not be a price equilibrium at all. We do not pursue these standard difficulties here, although they might be very important in any application of the taxing scheme.

Another form of difficulty arises if we admit that pollution affects production functions. This external effect is no doubt a diseconomy in the vast majority of cases--smog damages agricultural crops, rots tires, peels paint; but in some instances pollution can be a benefit--oxygen-depleted waters cannot support marine life which attacks piers and boats; some marine life flourishes near sewage or heat outfalls. It is not really known how important such effects are, and it is not unreasonable to assume that they are small relative to the direct effects on consumers; such things as health problems, destroyed aesthetic values, lowered property values, are captured by putting pollution into individual utility functions rather than into production functions, and it is these effects which are the most widely discussed and condemned. But, positive or negative, significant or negligible, such production-related effects do complicate the analysis.

When we write production constraints in a form which allows these effects, say in the form $g^k(x^k; z^k; ZZ) \geq 0$, or $g^k(x^k; z^k; S) \geq 0$ for the synthesis cause, Theorems I and III still hold;[12] there is no Pareto-preferred way to accomplish the *same* pollution levels. But now there is the possibility that we can decrease pollution levels *and* produce more goods, since decreases in pollution levels may make all processes more productive. Of course, if pollution were this bad, any political decision would almost certainly demand its reduction. And, long before we got to a reasonable level of pollution, further reduction would cease to be free.

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[12] Assuming that, in choosing their profit-maximizing input-output combinations, firms ignore their individual impact on pollution levels.

Even when it is impossible to get less pollution *and* more production, the tax rates τ_ℓ are no longer good estimates of the cost of pollution reduction when pollution levels affect production; Theorem III does not hold. In this case it becomes necessary to study each process, in order to determine the savings in production costs which would result from a decrease in pollution levels, and to deduct this from the tax-rate estimate of the costs of reducing pollution; the "net cost" figure must be used in the political decision process. While such estimates are difficult to make, they do not involve the same level of difficulty as estimates of the *consumption* benefits of reduction.

The Practicality of Pollution Prices

Obviously, the simple theorems proved here ignore many difficulties which might arise in trying to use a system of pollution prices in a control program. Some of the possible theoretical complications have been mentioned briefly: non-convexities may make impossible a marginal approach to finding the optimal levels; calculations may be needed to estimate marginal costs of further reduction, or to verify that an equilibrium is a maximum and not a minimum. It must be remembered, of course, that if these problems are present, it is due to the perversity of the world, and *any* control program will have to deal with the world as it is. A price system may help solve, and can never worsen, these problems. And, as long as the production side does not exhibit significant non-convexities, prices can still be used to accomplish desired levels of pollution, however the "desired levels" are chosen.

Beyond these theoretical difficulties in a practical objection, which at first glance strikes most people as insurmountable, there is just no way effectively to monitor pollution outputs from the thousands of plants, automobiles, homes, ships, etc., which produce pollutants. Certainly, such monitoring presents formidable problems, and it must be conceded that the system of prices which would be perfect in a world of zero monitoring and enforcement costs, is out of the question. But there are ways to establish price systems, even for such difficult-to-monitor sources as automobiles, which will approximate the ideal. In any case, any system of control must solve this measurement problem, and using prices as control instruments cannot make the situation worse.

There is a simple "dominance" argument for the price system. Set up any system of pollution control you choose, with given standards, methods of monitoring, penalties for violation, and so on; obviously, there is a set of prices which exactly duplicates this system, and which could be established by the addition of only a fee collection agency. Once this implicit set of prices is recognized, it is almost always obvious that it is irrational: it contains discontinuous and contradictory prices, makes only very crude distinctions, and allows very little individual variation. It is always easy to imagine a better price structure, even within the specified monitoring and enforcement system.

The problem of automobile emissions offers an excellent example of the advantages of a price system. Current control programs require certain gadgets on cars, and require new cars to meet certain standards, usually stated in terms of grams of emission per mile. This discontinuous price structure makes no distinction between cars driven a lot and those driven a little, or among geographic areas. It provides little incentive to drive less by shortening commutes, taking the bus, or forming car pools, all of which must be part of any "optimal" solution to the pollution problem. It applies only to new cars, and is very difficult to apply to old cars; should the standards be different for every make, model and year? It provides no incentive for manufacturers to develop new control methods; in fact, there is real incentive to delay, to retard development, to mislead the authorities about future possibilities.

A very practical system of automobile registration fees and fuel surtaxes, increasing with the "dirtiness" of the car and the fuel, could eliminate most of the shortcomings of the present system, with its discontinuous price. With the costs of owning and driving a car increasing with pollution produced, there is continuous pressure on everyone to take the steps which can reduce pollution efficiently. Those who drive more would have much greater incentive to drive smaller, cleaner cars. Higher costs of driving would encourage other means of transport; higher costs of transportation services would discourage commuting and suburban sprawl;

higher costs in cities would discourage urbanization.[13] In short, all possible adjustments, short-term and long-term, would be made, in the correct proportions, and without central control or direction. There is no need for legislators to negotiate technological details with industry experts, no impossible standards to fight over, no way to use the emission rules to exclude foreign competitors or to force consumers to buy expensive gadgets, little to be gained from collusion or pay-off. It is doubtful that all the rigid standards, threats of lawsuits, boycotts, or legislative bluffs, will ever accomplish what even a very crude price system could accomplish.

For stationary polluters, the problem of monitoring is much easier; and once it is solved, control becomes a matter of issuing bills and collecting fees. A system relatively easy to monitor and police is one with upper limits on pollution (stated in pounds per day, for example), which the polluters would set for themselves, paying a fee per unit; a clever variation due to J. H. Dales (1968) would set the total number of pounds per day beforehand, and then auction off "pollution rights."[14] Existing methods of enforcing standards can be used to enforce these rights. In any case, <u>any</u> system of pollution control must know how much of <u>what</u> is being produced by whom; only a price system need know nothing else.

Conclusions

This discussion of the theory of pollution prices obviously has not solved many of the problems involved in the theory or

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[13] Higher costs of living would discourage population growth? Perhaps. But one should not push his faith in downward-sloping demand curves too far. Speaking of elastic demand curves, it is important to remember that a tax on cars is equivalent to a subsidy on trains <u>only</u> in a partial equilibrium world, where demand for "transportation services" is inelastic. If the goal is to <u>dis</u>courage pollution, rather than to <u>en</u>courage commuting, tax gasoline rather than subsidize subways.

[14] This method avoids the problem of uncertainty and fluctuations which might arise with a fixed price. Of course, in a static, perfect-knowledge world, it makes no difference whether price or quantity is chosen; in an uncertain world, it may make a great deal of difference.

its application. We have ignored the question of how the political process should be structured, and how the voters can be convinced they "should" want to pay to clean up the environment. We have ignored the problem of distribution of costs, imperfect competition,[15] and non-convexities in production or preference sets. But we have illustrated a way of bringing pollution problems into the mainstream of allocative economics, and suggested a possible approach to solving these very important problems. A price system may be able to reduce pollution efficiently with a minimum of information and interference, and provide estimates of costs of further reduction, allowing political decisions about desired levels to be made simply and rationally. In the long run, some such system will have to be instituted, if we are to avoid both undesirable degradation of the environment and unnecessary economic disruption.

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[15] One effect of imperfect competition is worthy of note, however. A firm which is a monopsonist in the purchase of pollution rights or waste disposal services, facing an upward-sloping politically determined supply curve, will pollute <u>less</u> than will a competitive industry.

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