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A DIFFUSION MODEL FOR GREEN BAY, LAKE MICHIGAN

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Marine Studies Center
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A DIFFUSION MODEL FOR GREEN BAY, LAKE MICHIGAN

William Frederick Ahrnsbrak

Under the supervision of Professor Robert A. Ragotzkie

A one dimensional diffusion model based on the principle of conservation of mass is applied to Green Bay. The seiche is shown to be the mechanism primarily responsible for the dispersal of Fox River water during the summer. Electrical conductivity and light transmissivity are used to observe the distribution of Fox River water in the Bay. Observed diffusivities are compared with those predicted due to seiche activity. Diffusivities in the vicinity of Long Tail Point are on the order of 0.25×10^6 $cm^2 \cdot sec^{-1}$ with an abrupt jump to $1. \times 10^6$ $cm^2 \cdot sec^{-1}$ a few km beyond and gradually increasing to 2. to $3. \times 10^6$ $cm^2 \cdot sec^{-1}$ in the central part of the Bay. A numerical model using a finite difference technique shows that approximately 400 days are required for the Bay to respond to changes in pollutant levels in the River. Fox River water is shown to be being transported through Green Bay into Lake Michigan. During the summer highest concentration gradients of Fox River water (40% km^{-1}) were found in the vicinity of Long Tail Point and along the eastern shore of the southern end of the Bay. No appreciable transverse gradients were found in the central and northern portions of the Bay. Ice during the winter inhibits the effectiveness of seiche induced mixing and advective effects are more important.

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I. INTRODUCTION

Serving as natural harbors, bays and estuaries are often the sites of fairly large, heavily industrialized cities. Those cities subject adjacent bodies of water to a relatively heavy effluent load. Since water movement in these semi-enclosed basins is constrained by physical boundaries, the threat of pollutant buildup is greater in a bay or estuary than at a point along the coast of a larger, unconfined body of water. In order to minimize the deterioration of these smaller bodies of water a means of predicting the responses to changes in the effluent source is necessary.

The ability to make such predictions requires an understanding of the degree of interaction and/or exchange of water and its properties between various parts of a bay and between a bay and its parent basin. That understanding requires a knowledge of the roles of various processes and/or mechanisms effecting the interaction and exchange.

The purpose of this research is to examine and evaluate the effects and relative roles of various circulation features as dispersive mechanisms in bays of the Laurentian Great Lakes. More specifically, the objective of this research is to establish the roles of various processes and/or mechanisms in governing the answers to the following questions:

1. What is the spatial variation of the effect of an effluent discharged into a bay (What is the concentration field of that effluent)?

2. What are the responses of the bay (in terms of question 1) to any changes in the source?

Information in the literature which yields any insight into this problem is sparse, however, some contributions to the understanding of the physical processes in tidal estuaries are available. These contributions provide a background to the present study.

The analogy between a fresh water bay and an estuary has been drawn previously (Ragotzkie et. al, 1969) and some authors have applied estuarine techniques to bays of the Great Lakes (Beeton et. al., 1967). These bays, however, are in some ways different from estuaries.

First, although there may be occasionally significant seiche activity in the bay, and the bay will no doubt respond in some way to circulation features of its parent basin, the vigorous and continuous water movement induced in most estuaries by tidal motions is absent. Second, the absence of salt eliminates density gradients due to salinity variations. Therefore, the relative roles of the various physical processes effecting the dispersal of pollutants in a bay of the Great Lakes may be significantly different than in a tidal estuary.

A secondary objective of this study is to examine the validity of the analogy between fresh water bays and tidal estuaries.

As the primary study area for this research, Green Bay, Lake Michigan, has been chosen. This elongated bay (Fig. 1), located near the northwest corner of Lake Michigan is approximately 190 km long, with an average width of 22 km, and has its principal axis oriented in a NNE-SSW direction. Green Bay, including Big and Little Bay de Noc has an area of 4212 km^2, a volume of 70 km^3 and a mean depth of 17 m (Fee, 1969). No sill separates the Bay from the Lake.

Numerous streams and rivers, tributary to Green Bay, are polluted to varying degrees. The largest of these rivers is the Fox, entering the Bay at the city of Green Bay at the extreme south end of the Bay. The Fox River drainage basin has an area of 6520 square miles (16,691 km^2) (Wis. Dept. Nat. Res., 1968a) and according to the U.S. Geological Survey (1967) the River has a 71 year average annual discharge rate of 126 $m^3 \cdot sec^{-1}$. This rate, measured 18 mi upstream from the mouth of the River, represents the discharge from 94% of the drainage basin. Lee (pers. comm.) has stated that the Fox River is a major source of domestic and industrial wastes for lower Green Bay.

Two characteristics of Green Bay have led to its selection as the study area for this research. First, the long narrow shape of the basin makes the system particularly amenable to a one-dimensional analysis. Secondly, not only does the Bay suffer from the pollution problem described in earlier paragraphs of this introduction, but also these pollutants mark the river water entering at the extreme head of the Bay and thereby provide a tracer throughout the Bay and greatly aid in answering question 1 above.

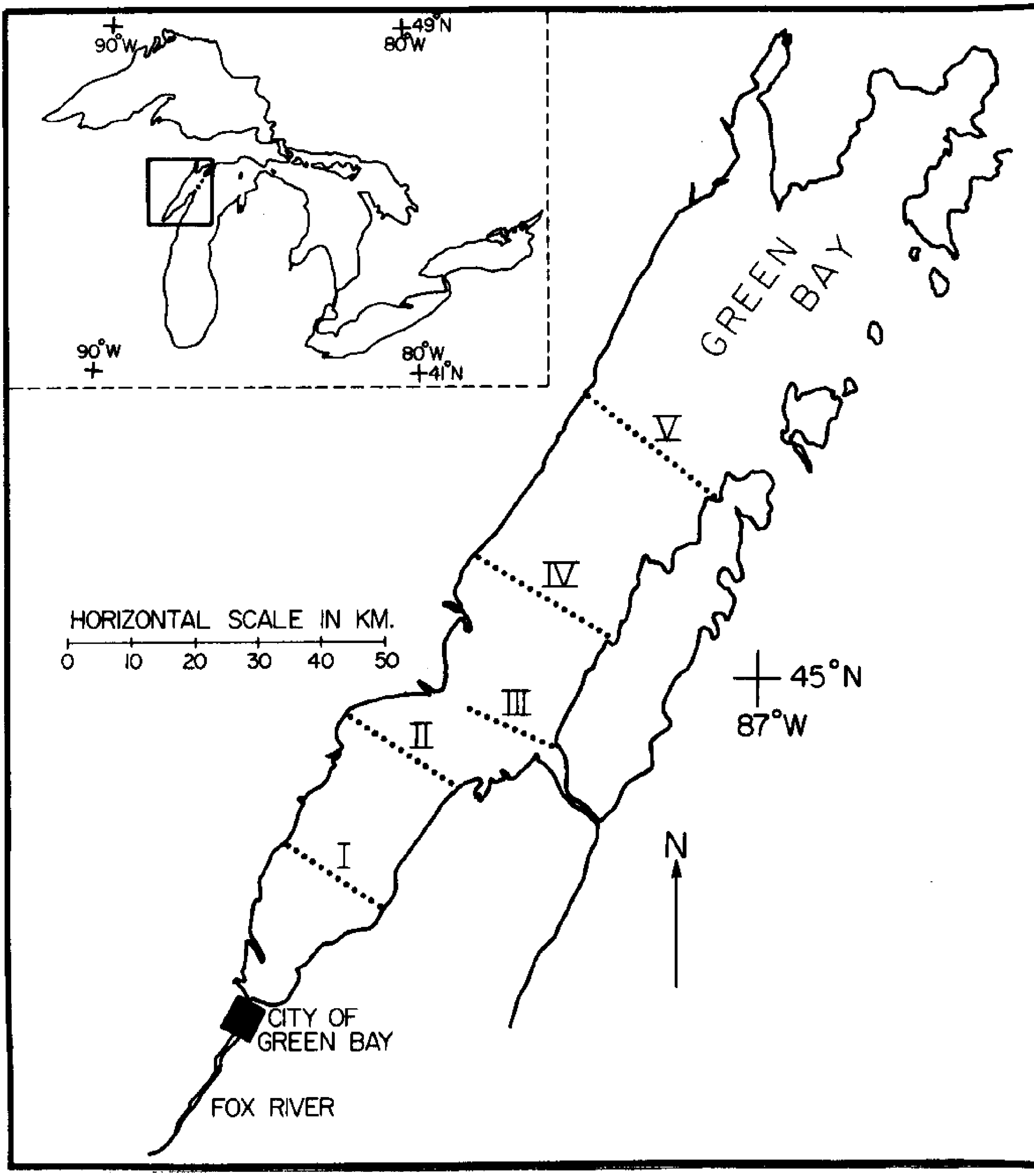

FIG. 1. Location and configuration of Green Bay, Lake Michigan. Dotted lines are locations of cross sections along which measurements for this study were taken.

A secondary reason stems from the fact that the University of Wisconsin is presently conducting an extensive interdisciplinary study of the Green Bay system which provides valuable inputs into this research, while the latter contributes to the overall Green Bay research program.

This study is intentionally not directed at any particular pollutant. The future of any basin such as Green Bay requires many decisions and each decision will involve particular pollutants, such as nutrients, waste heat, biochemical oxygen demand, and many others, each with its own characteristics, tolerable levels, and interactions with other constituents. Many of these interactions are not fully understood, while some are. It is the intention of this study to provide an understanding of the processes effecting the dispersion of pollutants of any kind.

II. LITERATURE REVIEW

The recent literature contains a large number of contributions to the understanding of turbulence phenomena and turbulent diffusion, but few of these are directly applicable to this study since most deal with diffusion in an infinite field of stationary, homogeneous turbulence.

Pertinent to this study are those which relate differences in effective diffusivities to variations in flow characteristics. Bowden (1965) has applied the ideas of Taylor (1954) to show that velocity shear in the vertical associated with a stable density gradient results in a considerable increase in effective horizontal diffusivity. Csanady (1964) has shown that the effect of a steadily changing or swinging current is to increase the effective horizontal diffusivity by a factor of two or three near the source and even more so at larger distances. He compares observed vertical and horizontal diffusivities for Lakes Huron and Erie and finds those in Lake Erie to be approximately twice as great as those in Lake Huron, presumably due to the greater turbulence intensity in the shallower Lake Erie. However, values were given for an individual plume, and only the effects of small scale turbulence are significant.

Csanady (1970) has also presented a comprehensive description of the physical factors involved in the dispersal of pollutants in the Great Lakes, and has illustrated quantitative models of small and large scale effluent plumes. Consideration is given, however, to effluent plumes which have free access to an open lake and the boundary constraints imposed in the case of a bay are not considered. Only a constraining effect of entrapment resulting from coastal "jets" near shores is considered. The thermal structure observed during this research suggests that analogous phenomena exist in Green Bay but method used in this study does not permit any evaluation of the effectiveness of that phenomenon as a barrier to horizontal mixing.

Some of the works on estuarine diffusion do provide background to the problem of dispersive processes in bays since oscillating currents (resulting from tides and seiches) are significant factors. Bowden (1967) has presented a comprehensive discussion of circulation and diffusion in estuaries, and he considers the primary parameters to be physical dimensions, river flow, and tidal conditions.

Since salinity variations and the associated density gradients in a fresh water bay are usually insignificant compared to those in an ocean connected estuary (Lee gives a value of Cl^- content of approximately 25 mg/l for the mouth of the Fox River and approximately 7 mg/l in the central part of Green Bay) and in this case are of the opposite sign, there are no salt wedges in fresh water bays.

One of the earliest models of tidal flushing was that of the tidal prism (e.g., Phelps and Velz, 1933). Since mixing is incomplete the theory is inadequate, and Ketchum (1951) has suggested an improvement in which the estuary is divided into segments in which complete mixing is assumed.

Beeton (1967) has applied the method of Ketchum to calculate flushing time for Saginaw Bay of Lake Huron, and Modlin and Beeton (1970) have used the method to compute residence times for Fox River water in Lower Green Bay, however no attempt is made to describe the physical mechanisms responsible for the observed values. In both studies residence times are based on the assumption that the advective movement of river water is solely responsible for the transport of effluents.

In addition to Beeton, other work on flushing rates of bays of the Great Lakes includes the analysis by Bryson and Stearns (1959) of seiche flushing of South Bay, Manitoulin Island, Lake Huron, which suggests a flushing rate of 10% per day. Ragotzkie, et. al. (1969) discuss flushing by seiche and by wind induced fluctuations of the thermocline in Chequamegon Bay, Lake Superior, suggesting 5% per day for the seiche and 10% per day for the fluctuating thermocline. No attempt was made to evaluate interactions between the two mechanisms, and in both works the weakness of the prism model is present in that complete mixing within and outside of the bay is assumed.

According to Bowden (1967), Aarons and Stommel (1951) attempted to relate the horizontal diffusivity, K_x, to the vigor of the water movement in a tidal estuary. The tides are regarded as turbulence superimposed on river flow and K_x is assumed to be proportional to the product of the amplitudes of the horizontal tidal displacement and the tidal current. Their method is similar to that used in this study to relate K_x to water movements in a bay.

The method of using the distribution of river water to evaluate the effective diffusivities was first proposed by Stommel (1953) and applied to freshwater in the Severn Estuary. His diffusivities are shown in figure 2a. Stommel also combined his observed diffusivities with a relaxation technique to compute steady state concentrations resulting from an arbitrary distribution of pollutant introduced into the estuary. No consideration, however, is given to temporal variation of the resulting concentration field of the pollutant in the estuary.

Kent (1958) employed essentially the same method to evaluate effective diffusivities from the salinity distribution in a scale model of the Delaware Estuary. His diffusivities are shown in figure 2b. Kent also used his diffusivities to predict the concentration field and temporal changes resulting from the introduction of a batch of pollutant at a point in the estuary. No consideration is given to a continuous source with temporal variations.

Snelleck and Pearson (1960), using a tracer substance introduced as a batch near the head of South San Francisco Bay derived a mean diffusivity value of 4.5×10^5 $cm^2 \cdot sec^{-1}$ with two maxima of approximately 1.3×10^6 $cm^2 \cdot sec^{-1}$.

In all of these studies velocity variations due to tidal action is the forcing function for the diffusion. The tracer in the works by Stommel and by Kent was fresh water, which, in a body of salt water, must be expected to be influenced by dynamic effects due to the density differences. Such effects are negligible in a fresh water bay. To the best of my knowledge no study of this nature is known to have been done in a basin where salinity variations are negligible and tidal oscillations are absent.

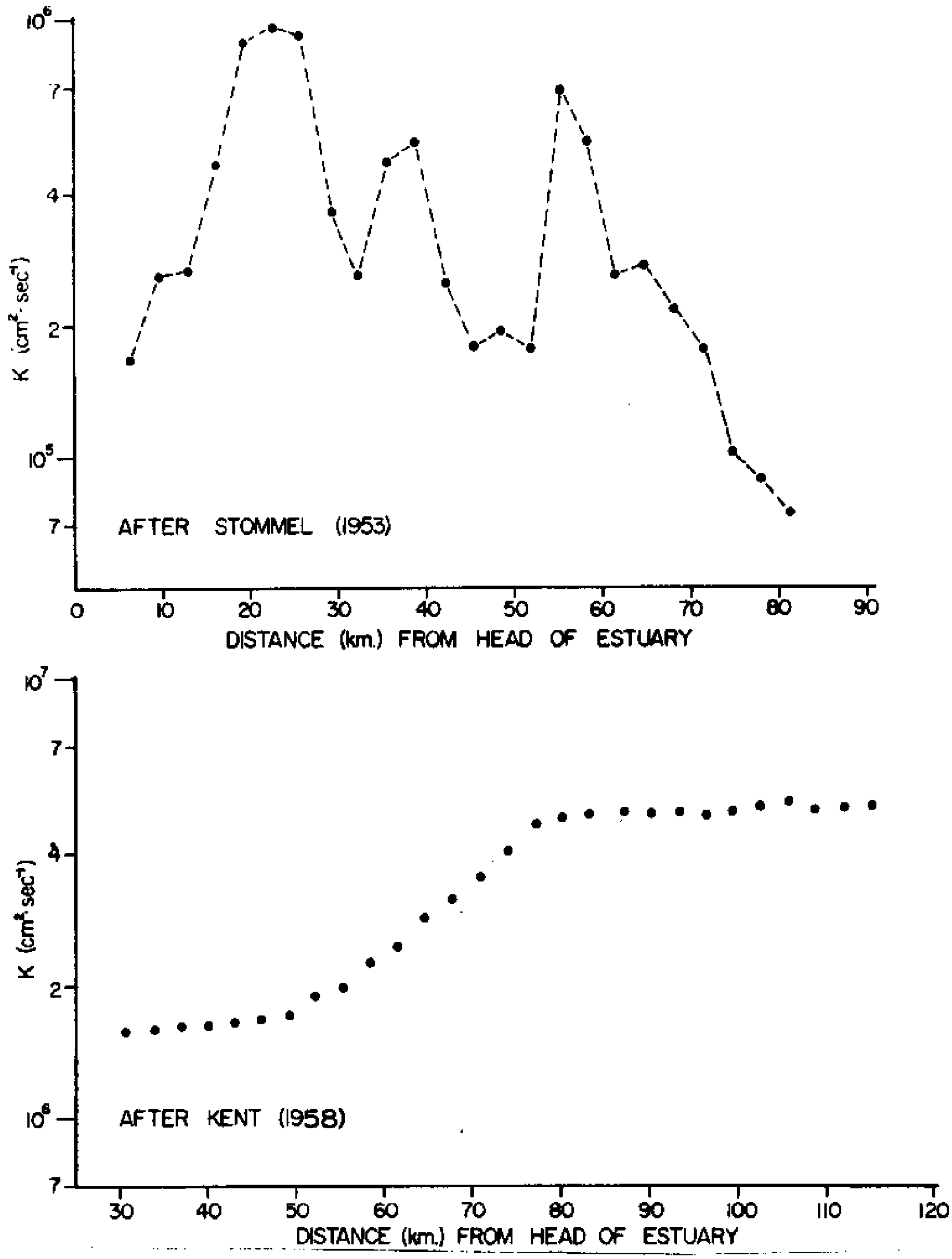

FIG. 2. Longitudinal distribution of horizontal diffusivity, K_X, in the Severn Estuary (after Stommel, 1953) and in a model of the Delaware Estuary (after Kent, 1958).

III. THEORY AND METHOD

The investigation of the Green Bay system is based on the application of a one dimensional diffusion model to the Bay. Effective eddy diffusivities, based on the gradient theory of diffusion are calculated from the observed concentration field of Fox River water in Green Bay. Resulting values are analyzed to determine the water movement processes in the Bay which could account for these observed diffusivities. Inherent in the use of the one dimensional model is the assumption that lateral and vertical mixing take place sufficiently fast that, in these dimensions, the Bay is essentially "filled," and further diffusion constrained by the lateral and lower physical boundaries. In a later section this assumption will be shown to have been valid throughout much of the Bay.

It is assumed in this study that the Fox River is the exclusive source of pollutants for Green Bay. The justification for this assumption can be derived from table 1. The four largest rivers entering the southern two thirds of the Bay are listed with their average discharge rates from the United States Geological Survey (1967) records, the average (for the years 1961 - 1968) concentrations of chlorides and suspended solids (Wis. D.N.R.; 1965, 1968b), and the net transport of those pollutants, calculated as the average concentration times the average discharge rate. Based on these net flows the Fox River, as a pollutant source, is nearly an order of magnitude larger than the sum of the other three rivers.

It is also assumed that Fox River water itself can be considered a pollutant of the Bay, and the units of concentration are the dimensionless (volumetric) ratio of River water to Bay water. The initial concentration in the River is therefore unity.

Furthermore, Fox River water is considered to be a conservative variable. To validate this assumption, evaporation and precipitation for the area of the Bay south of Section V were calculated for July, August, and September. These calculations based on evaporation rates for Lake Mendota by Dutton and Bryson (1962) and for Lakes Michigan and Huron by Folse (1929) as discussed by Hutchinson (1957). Average precipitation was calculated from USWB records from several stations around the periphery of the Bay. Precipitation minus evaporation was then compared with the monthly discharge rates for the total of the four significant rivers indicated in table 1.

TABLE 1. Average discharge rates of water, suspended solids, and chlorides for 4 rivers entering the southern lobe of Green Bay.

River Location	Discharge Rate ($m^3 \cdot day^{-1}$)	Suspended Solids		Chlorides	
		Concentration ($mg \cdot l^{-1}$)	Net Transport ($kg \cdot day^{-1}$)	Concentration ($mg \cdot l^{-1}$)	Net Transport ($kg \cdot day^{-1}$)
OCONTO at Oconto	2.35×10^6	9.8	2.3×10^4	6.7	1.57×10^4
MENOMINEE at Marinette	8.96×10^6	5.3	4.75×10^4	1.4	1.25×10^4
PESHTIGO at Peshtigo	2.25×10^6	5.6	1.26×10^4	0.6	1.35×10^4
FOX at Green Bay	11.3×10^6	17.6	19.3×10^4	12.3	13.9×10^4

TABLE 2. Evaporation, precipitation and drainage for the southern lobe of Green Bay. Evaporation estimates based on values presented by Dutton and Bryson (1962) are indicated by the abbreviation (D & B). Estimates based on values from Folse (1929) as discussed in Hutchinson (1957) are indicated by (F in H). The area of the Bay under consideration is 2200 km^2.

	July	August	September	
Evaporation from Lake Mendota (from Dutton & Bryson)	12.7	12.7	11.9	cm.
Evaporation from Lakes Michigan and Huron (from Folse in Hutchinson)	6.7	4.8	3.5	cm.
Total Evap for Bay (D & B)	2.79	2.79	2.62	$\times 10^8 m^3$
Total Evap for Bay (F in H)	1.47	1.06	.77	$\times 10^8 m^3$
Precipitation (from U.S.W.B.)	8.6	8.3	7.6	cm.
Total Precip for Bay	1.89	1.83	1.67	$\times 10^8 m^3$
River Drainage (4 rivers)	2.2	1.6	1.7	$\times 10^{10} m^3$
Total Precip--Total Evap (D & B)	-.9	-1.0	-.9	$\times 10^8 m^3$
Total Precip--Total Evap (F in H)	.42	.77	.90	$\times 10^8 m^3$
$\frac{P - E}{Drainage}$ (D & B)	-4.09	-6.25	-5.29	$\times 10^{-3}$
$\frac{P - E}{Drainage}$ (F in H)	1.91	4.81	5.29	$\times 10^{-3}$

Results of the calculations are shown in table 2. Since the largest value of the ratio of (P-E) to river runoff is about 0.5 per cent, changes due to evaporation and precipitation are negligible.

The total transport of a conservative pollutant through any section of a bay is the sum of two effects. The first is organized advection $Q \cdot C$, where Q is the total river discharge into the bay upstream from the section under consideration, and C the concentration of the pollutant at the section. The second effect is turbulent flux generated by the various water movement processes existing in the bay and is conventionally expressed as $-KA\frac{\partial C}{\partial x}$, where K is the turbulent eddy diffusivity, A is the cross sectional area, and x is the longitudinal space co-ordinate. The net lakeward flux, F, of a conservative pollutant through any section is given by:

$$F = Q \cdot C - KA\frac{\partial C}{\partial x} \qquad (1)$$

The second term on the right hand side of this equation, the mixing term, is the primary subject of this research.

The development of the theory of turbulent diffusion has followed two separate avenues. The gradient hypothesis is based on the principle of conservation of mass and treats turbulent diffusion analogously to molecular diffusion as postulated by Fick (1855). The second approach is the statistical theory and owes its origin to Taylor (1921). This theory considers the dispersion of a fluid element about its origin as being determined by the statistical properties (particularly the variance) of the displacements of a single particle. Neither theory is complete and a fully satisfactory theory of turbulent diffusion has yet to be developed.

A basis for the relationship between the eddy diffusivities, calculated according to the gradient theory of diffusion, and characteristics of the turbulence is provided by Prandtl's (1934) mixing length theory. That theory considers the distance which a turbulence element moves as being an analog of the mean free path of a molecule. It is supposed that as a result of the turbulent nature of the flow, an eddy or small volume of fluid moves across the main flow from a point Y to a point $Y + \ell$, carrying its properties to the new position, and mixes with the flow there. For continuity, another eddy must have moved from $Y + \ell$ to Y. In a statistical average the flux of a property, C, caused by the turbulence may then be expressed as:

$$F \propto -\ell\sqrt{\overline{u'^2}}\ \frac{\partial C}{\partial x} \qquad (2)$$

where the u' are the turbulent fluctuations in the mean flow $\overline{u}$ expressing the instantaneous value as

$$u = \overline{u} + u'$$

The coefficient of diffusion, K, is then proportional to

$$K \propto \ell\sqrt{\overline{u'^2}} \qquad (3)$$

Alternatively, from equation (1), given a source of known strength (the Fox River in the case of Green Bay) and a concentration field as a function of x, one can calculate effective eddy diffusivities for a conservative substance by making the following assumptions:

1. Advective effects are exclusively due to the lakeward movement of river water, and

2. The concentration field is stationary ($\partial C/\partial t = 0$).

Certainly short-term circulation patterns exist which transport Fox River water lakeward. These, however, are included in the calculated diffusivity values since measurements of actual gradients are the basis for these calculations.

The justification of the assumption of stationarity is dependent on the time and length scales of the phenomena under consideration. Modlin and Beeton (1970) have computed a residence time of 40 days for Fox River water in lower Green Bay, with larger values farther north. For purposes of this study, a survey taken over a period of three to five days is considered to be synoptic. Then the assumption of stationarity is legitimate, but not for a study of, say, the trajectory of a plume created by the Fox River as it first enters the Bay.

From equation (1), the flux through the cross section of area A at a distance i from the head of the bay can be expressed as

$$F_i = Q \cdot C_i - K_i A_i \left.\frac{\partial C}{\partial x}\right|_i . \qquad (4)$$

Similarly, the flux through the mouth of the river can be expressed as

$$F_r = Q \cdot C_r - K_r A_r \left.\frac{\partial C}{\partial x}\right|_r . \qquad (5)$$

Assuming now that the flux through the mouth of the river is by advection alone, and recalling that $C_r = 1$, we see that

$$F_r = Q \cdot 1 + 0 = Q . \qquad (6)$$

However, since river water is assumed to be conservative

$$F_i = F_r = Q . \qquad (7)$$

Therefore, by combining equations (4) and (7), we get

$$Q = Q \cdot C_i - K_i A_i \left.\frac{\partial C}{\partial x}\right|_i \qquad (8)$$

which can be rewritten for calculating the observed effective eddy diffusivities

$$K_i = - \frac{Q\ (1 - C_i)}{A_i \left.\frac{\partial C}{\partial x}\right|_i} . \qquad (9)$$

The next step is to predict the diffusivities associated with a particular water movement, for example a seiche. While the relationship between the diffusivity and the intensity of the turbulence is shown in expression (3), from dimensional considerations it can be seen that

$$[\ell] = [u] \cdot [t] \qquad (10)$$

or, that (3) can be rewritten as

$$K \propto \overline{u'^2} \cdot t \qquad (11)$$

where t is a characteristic time scale of the motion.

The proportionality constant to be inserted into (11) to make it into an equation, is initially assumed to have a value of unity. Paterns and values of diffusivities calculated from equation (11) are then compared with the diffusivities derived from the existing concentration field, to establish which water movements are primarily responsible for the dispersal of effluents in Green Bay.

IV. EXPERIMENTAL PROCEDURE

In order to observe the concentration of Fox River water in Green Bay and provide the basis for calculating effective diffusivities, two indicators were used during the summer of 1969, namely electrical conductivity and light transmissivity.

In the case of dilute solutions, electrical conductivity of water is a nearly linear function of the concentration of electrolytes dissolved in the water. For a specific ion the relationship between concentration and conductivity is well known. However, in Fox River water, a large variety of ion species is present and an empirical determination of the relationship between electrical conductivity and concentration was used (Fig. 3).

Also shown in Fig. 3 are the results of a determination of the relationship between transmissivity and concentration of Fox River water. The resulting exponential relationship is expected, since the absorption of light in a medium is governed by the equation

$$L_{(x)} = L_{(0)} \cdot e^{-k \cdot x} \qquad (12)$$

where $L_{(0)}$ is the amount of light incident on an absorber of thickness x, $L_{(x)}$ is the amount of light transmitted through the absorber, and k is the extinction coefficient. If the length of the path is held constant and the extinction coefficient is assumed to be a linear function of the concentration of suspended particulate matter the above expression yields the relationship illustrated in Fig. 3.

All field measurements were made <u>in situ</u> using a Whitney underwater temperature - conductivity meter. The observed conductivity values were converted to conductivity at 18°C using the temperature coefficients of Smith (1962). Transmissivity measurements were made with a Hydro Products model 410 BR transmissometer using a model 411 10 cm underwater sensor. This instrument provides a direct read out of transmissivity in percent. The empirically derived relationships shown in Fig. 3 were used to convert conductivity and transmissivity values to concentrations of Fox River water.

The field program consisted of three parts. Soundings of electrical conductivity and light transmissivity at one meter depth intervals were made at a number of stations along the cross sections shown in Fig. 1.

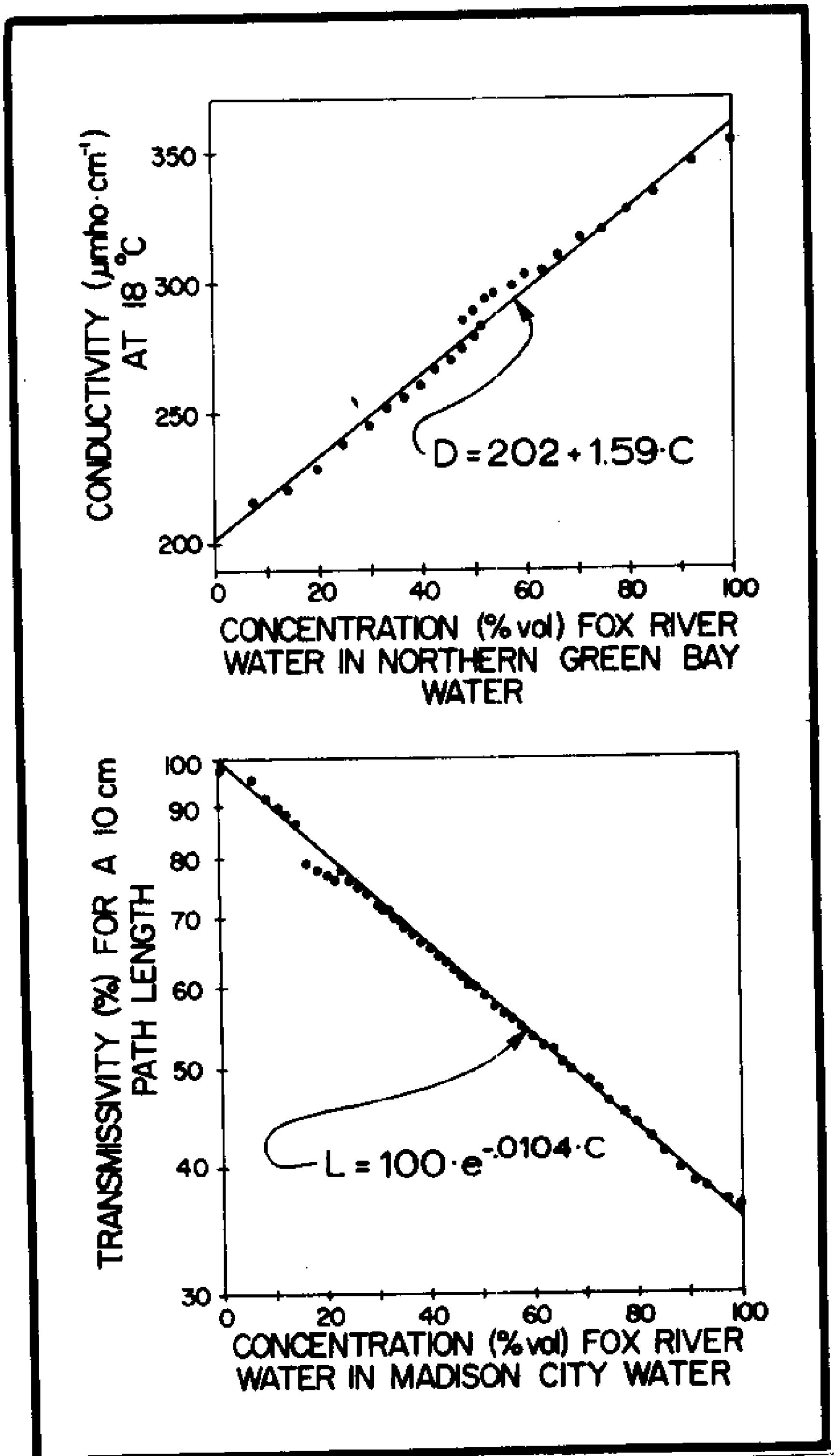

FIG. 3. Empirical relationships between concentration (percent volume) of Fox River water and electrical conductivity and between concentration and light transmissivity.

In the southern 15 km of the Bay, where gradients are strongest and the concentration field most variable, several intense surveys consisting of soundings at 75 to 100 stations were made.

The third portion of the field program consisted of two vertical sections measured along the longitudinal axis of the Bay, beginning at the mouth of the Fox River and proceeding northward along the shipping lane as far down the Bay as sea state conditions would permit. On 16 July this profile extended approximately three quarters of the length of that part of the Bay under consideration, and on 30 July the profile was completed over the entire 115 km length of the Bay, terminating off the tip of the Door County Penninsula.

A complete survey, consisting of the five cross sections plus the intense survey in the head of the Bay required three days. The cross section in the northern portion of the Bay (IV, V) took two to three hours with the shorter section in the southern portion of the Bay (I, II, III) requiring less time. The surveys in the head of the Bay required the greater portion of a day to complete. The useful data acquired are listed in table 3.

For comparison, the winter time concentration field in the southern quarter of the Bay was surveyed on 18 and 19 February 1970 using a snowmobile for transportation and drilling holes through the ice to take soundings of temperature and conductivity. Due to the cold, the meter movement in the transmissometer would not function properly and only conductivity data were taken.

While the treating of Fox River water as conservative has been justified in a previous section, the question of the conservativeness of the two chosen indicators of River water arises. The assumption that conductivity is conservative is valid. Due to the sedimentation known to be taking place in Green Bay, the conservative assumption for transmissivity does not hold.

If, however, a correction factor can be devised which takes sedimentation into account and converts apparent concentration of River water as indicated by the transmissivity measurements to real concentration, the use of transmissivity is legitimate. An attempt was made to develop such a correction factor from the budget of suspended solids for lower Green Bay.

TABLE 3. Areas of completed survey segments during summer 1969

Date	Area
15 July	Section V
16 July	Longitudinal section
17 July	Section IV
29 July	Sections I & II
30 July	Longitudinal section
14 August	Head
15 August	Head
18 August	Section III
20 August	Sections IV & V
21 August	Sections I & II
22 August	Head
10 September	Head
11 September	Head
17 September	Sections I & II
18 September	Section IV
19 September	Head and Section I

By planimetering the areas delineated and assuming a mean depth of 0.9 m for the area of 0.6 to 1.2 m depth decrease, and a depth of 1.5 m for the area of greater than 1.2 m decrease, and 0.3 m depth for those areas enclosed by areas of depth decrease less than 0.6 m, the volume of sediment accumulated over the 19 year period was calculated to be 2.19×10^8 m^3.

According to Hall (pers. comm.) the sediments in areas of greatest accumulation are mainly muds which are approximately 80 percent water by volume. Assuming a value of 2.5 $gm \cdot cm^{-3}$ for the density of the solids in the accumulated sediments, the mass of accumulated sediments is 1.10×10^{14} gm.

Several methods were used to evaluate the sediment loads of rivers entering the southern portion of the Bay. The input of solids during the 19 year period by all four rivers in table 1 calculated as mean discharge rate times the annual mean concentration of suspended solids yields a value of 1.42×10^{12} gms. The ratio of observed accumulated sediments to suspended solids input is 129:1.

Inherent in the above method of calculating the source strength of the rivers is the assumption that the concentration of suspended solids is adequately represented by the mean. Since the carrying power for suspended solids of a river is strongly dependent on discharge rate, the second method used to evaluate the inputs of solids was to calculate the source strength as being the sum over the four rivers of the sum over twelve months of the product of the monthly mean discharge rate times the monthly mean concentration. The value for the total suspended solids entering the Bay during the 19 year period, as calculated by this method, is 2.61×10^{12} gms, for which the ratio of accumulated sediments to suspended solids input is 54.5:1.

This discrepancy can be attributed to three possible factors. First it is possible that the estimates of the water content and the density of the solids in the observed sediments are in error. Table 4 shows values of the ratio of the observed sediment to calculated suspended solids input for both methods of calculating input for varying values of water content and density of the solids in the sediments. For the extreme case, 90 percent water and a density for the solids of 1.5, the ratio of 13.8:1 shows that even in this case, the mass of the observed sediments is an order of magnitude larger than the mass of the suspended solids entering the Bay.

TABLE 4. <u>Sediment accumulation in the southern lobe of Green Bay</u>

Volume of unconsolidated sediments: 2.19×10^{14} cm^3 (from Moore and Meyer, 1969)									
% water	80			85			90		
ρ solids	2.5	2.0	1.5	2.5	2.0	1.5	2.5	2.0	1.5
B: Mass of observed accumulated sediments (x 10^{13} gm)	10.9	8.8	6.6	8.2	6.6	4.9	5.5	4.4	3.3
C: Mass of sediment input calculated as:	$\sum_{j=1}^{4} \overline{Q}_j \cdot \overline{C}_j = 1.42 \times 10^{12}$ gm								
B/C:	76.8	61.7	46.3	57.8	46.3	34.6	38.5	30.8	23.1
D: Mass of sediment input calculated as:	$\sum_{j=1}^{4} \sum_{i=1}^{12} \overline{Q}_j^i \cdot \overline{C}_j^i = 2.61 \times 10^{12}$ gm								
B/D:	41.8	33.6	25.1	31.5	25.1	18.9	20.9	16.8	12.6

The other two sources of error are in the calculation of the suspended solids input to the Bay. The first is that suspended load is only a portion of the sediment transport of a river, and in this research no data are available to enable one to estimate the bed load of the Fox River as it enters Green Bay.

The other source of error in the calculation stems from the fact that the monthly suspended solids data are too sparse to base a good estimate of the sediment load on them. Spath (pers. comm.) suggests that short large flood events can carry as much as 80 percent of the suspended load of a river, and that it is important that these storm loads be adequately determined.

Lenz (pers. comm.) has suggested that for a given river, the carrying power P is proportional to the fifth power of the discharge rate Q. To evaluate whether this effect could account for the observed discrepancy, the sediment load was calculated for several years assuming that it can be expressed by

$$\text{sediment load} = \overline{Q}^a \cdot \overline{C}^a \sum_{d=1}^{365} \left(\frac{Q_d}{\overline{Q}^a} \right)^6 \tag{13}$$

The values of the dimensionless factor $\sum_{d=1}^{365} \left(\frac{Q_d}{\overline{Q}^a} \right)^6$ varied between 20 and 80, which is the same magnitude as the observed discrepancy. This suggests that it is variations of this degree in the sediment load of the Fox River which control the amount of sediments entering the Bay. This method, however, does not produce sufficient accuracy on which to base the sedimentation correction factor sought, and therefore the attempt at developing such a factor independent of the measured transmissivity values was abandoned. Therefore, in this research, results based on the conductivity based concentration field are the more reliable.

While the conductivity based concentration values provide the more sound basis for calculating effective diffusivities, the lack of a rigorously derived, independently based sedimentation correction factor does not render the transmissivity data meaningless. Qualitative descriptions of the concentration field of Fox River water in Green Bay are still valid, and by examining the effects of various correction factors on diffusivities calculated from the measured transmissivity values, limits can be placed on the sedimentation rate in Green Bay.

V. RESULTS

During the summer season two nearly complete surveys of the Bay were obtained, during the periods 18-22 August 1969, and 17-19 September 1969. Results of the surveys are shown in figures 4 and 5 for August and figures 6 and 7 for September.

Figures 5 and 7 illustrate examples of the cross sections from the surveys (see Fig. 1 for locations). Values shown are temperature (°C) and concentration (percent volume) of Fox River water, based on both transmissivity and conductivity. The transmissivity based values are uncorrected for sedimentation (as are all transmissivity based concentration values referred to hereafter unless otherwise stated).

Figures 4, 6, and 8 show surface concentrations of Fox River water in the head of the Bay for 22 August, 19 September, and 15 August, respectively. Analyses of surface concentrations in the middle and northern portion of the Bay are not illustrated since data in those regions are too sparse to provide a sound basis for any such analysis.

Cross sectional average concentrations, based on both conductivity and transmissivity are plotted as a function of distance from the mouth of the River in figure 9.

The conductivity-based concentration field and temperature structure of the longitudinal section, as observed on 16 and 30 July are shown in figures 10 and 11, respectively. (The transmissometer was not functioning properly on those occasions).

Vertical variations in the concentration field in the head of the Bay are illustrated in figures 12-15. Figures 12 and 13 show conductivity and transmissivity based concentration fields of a vertical section along the shipping lane, as observed on 14 and 22 August. Figures 14 and 15 show the concentration fields of several vertical sections located in the eastern half of the head of the Bay and oriented approximately perpendicular to the shipping lane. The figures are based on observations made on 14 August and 10 September. Locations of these sections are shown in figure 16.

The concentration field in the head of the Bay showed extreme variability, both in space and in time, approximating at times a nearly random distribution. Some degree of smoothing was therefore necessary in order to calculate diffusivities.

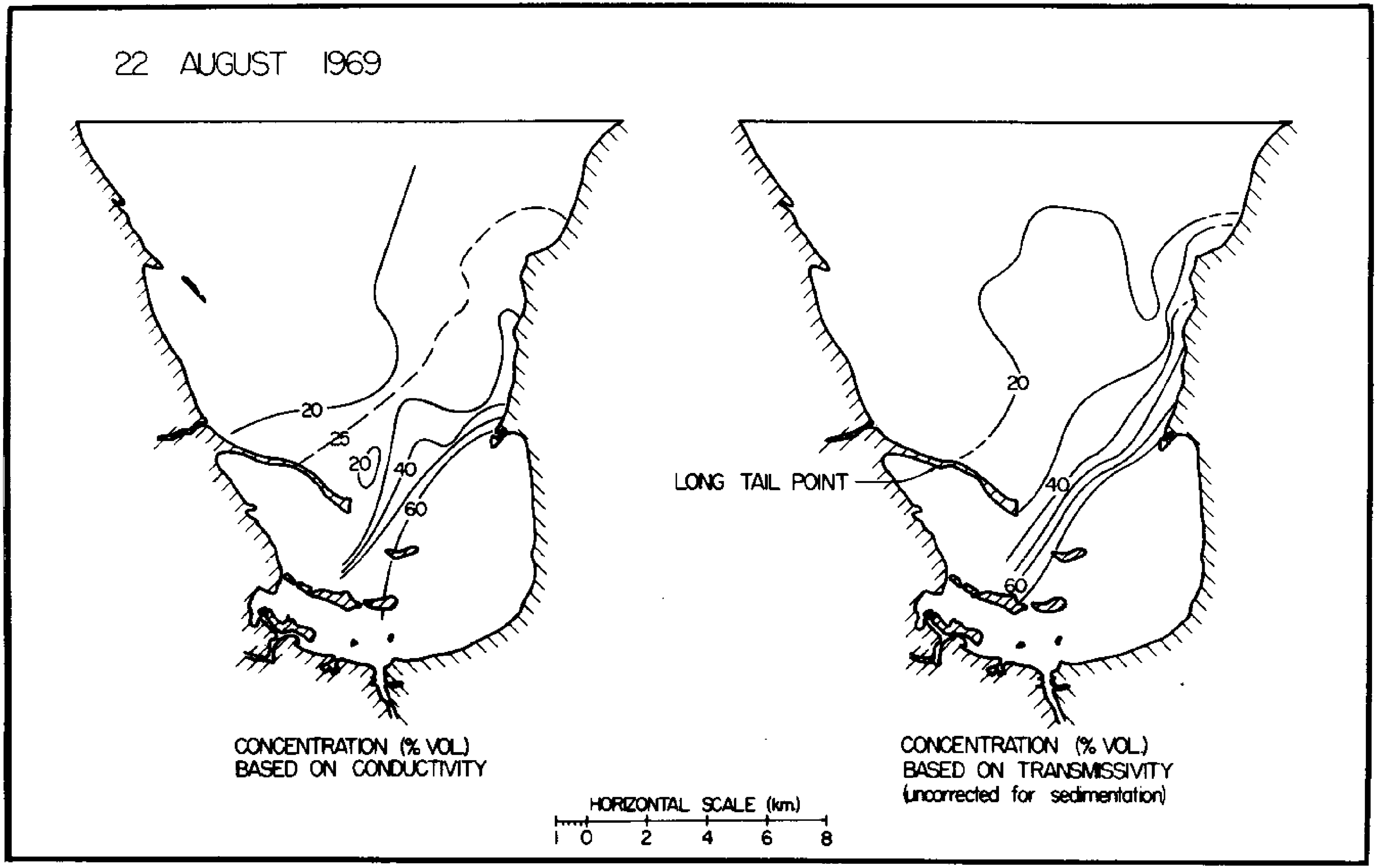

FIG. 4. Surface concentrations (percent volume) of Fox River water in the head of Green Bay on 22 August 1969. Values shown are based on electrical conductivity and light transmissivity.

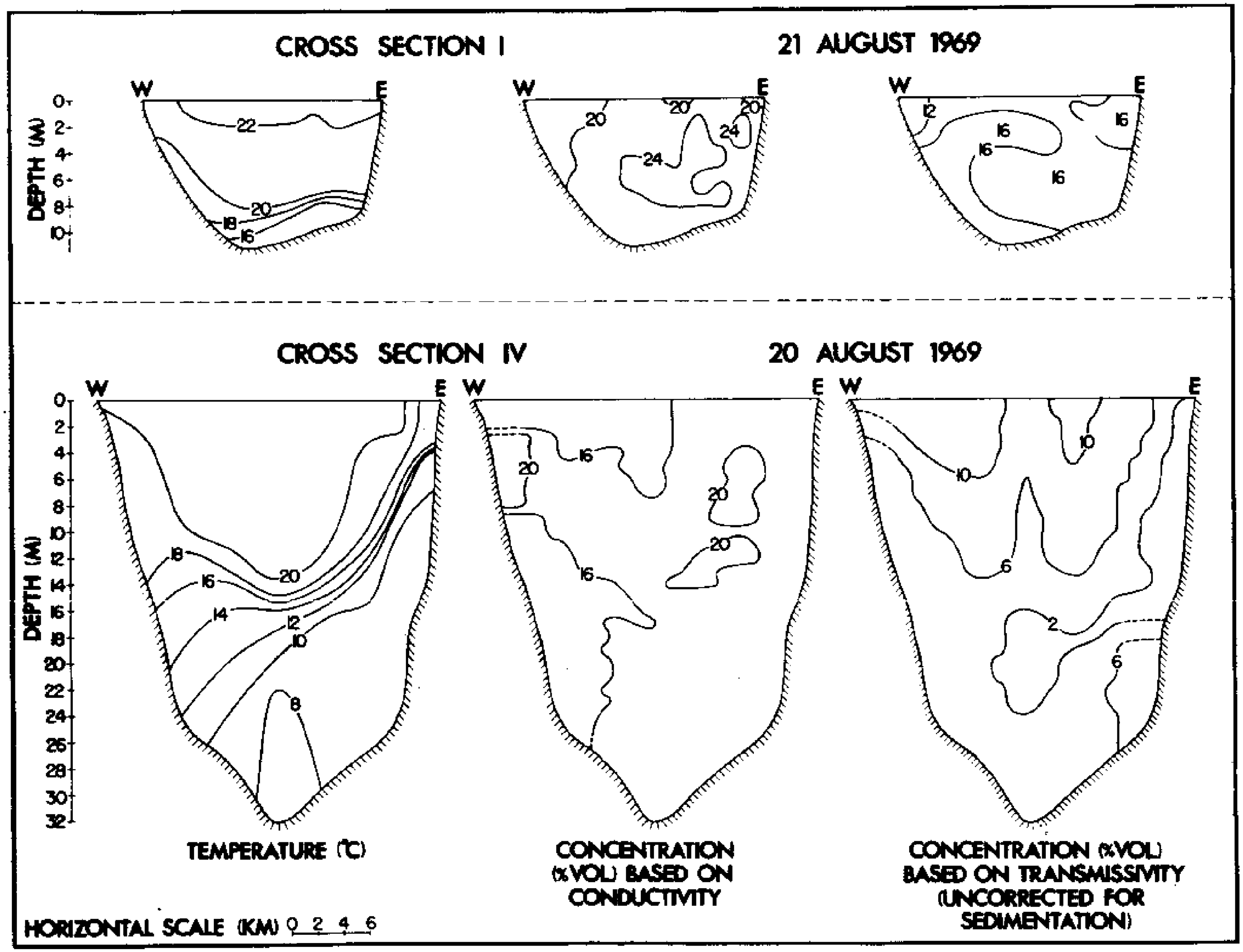

FIG. 5. Temperature (°C) and concentration (percent volume) of Fox River water based on electrical conductivity and light transmissivity for Cross Section I on 21 August and Cross Section IV on 20 August 1969. For locations of cross sections see Fig. 1.

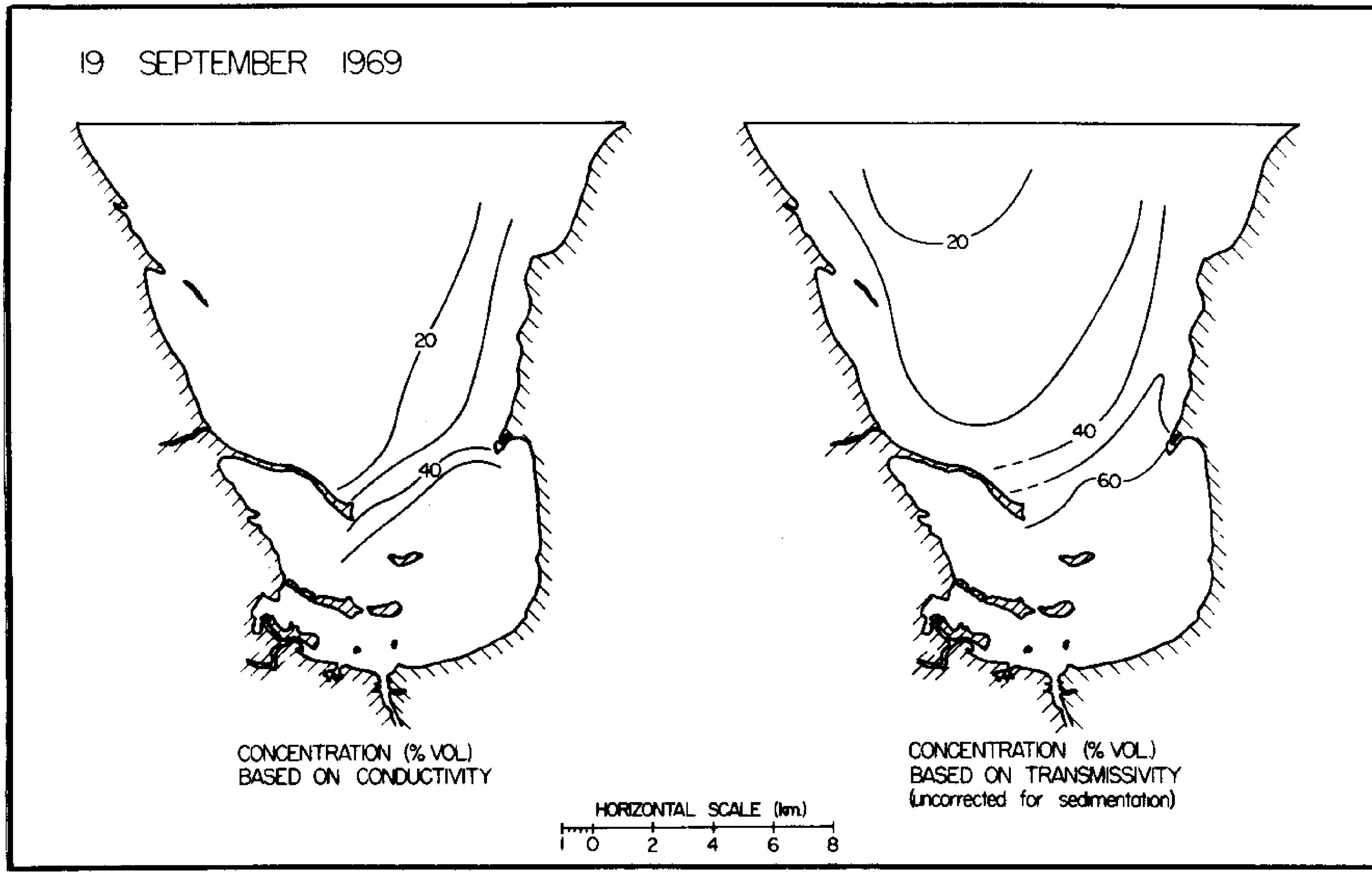

FIG. 6. Surface concentrations (percent volume) of Fox River water in the head of Green Bay on 19 September 1969. Values shown are based on electrical conductivity and light transmissivity.

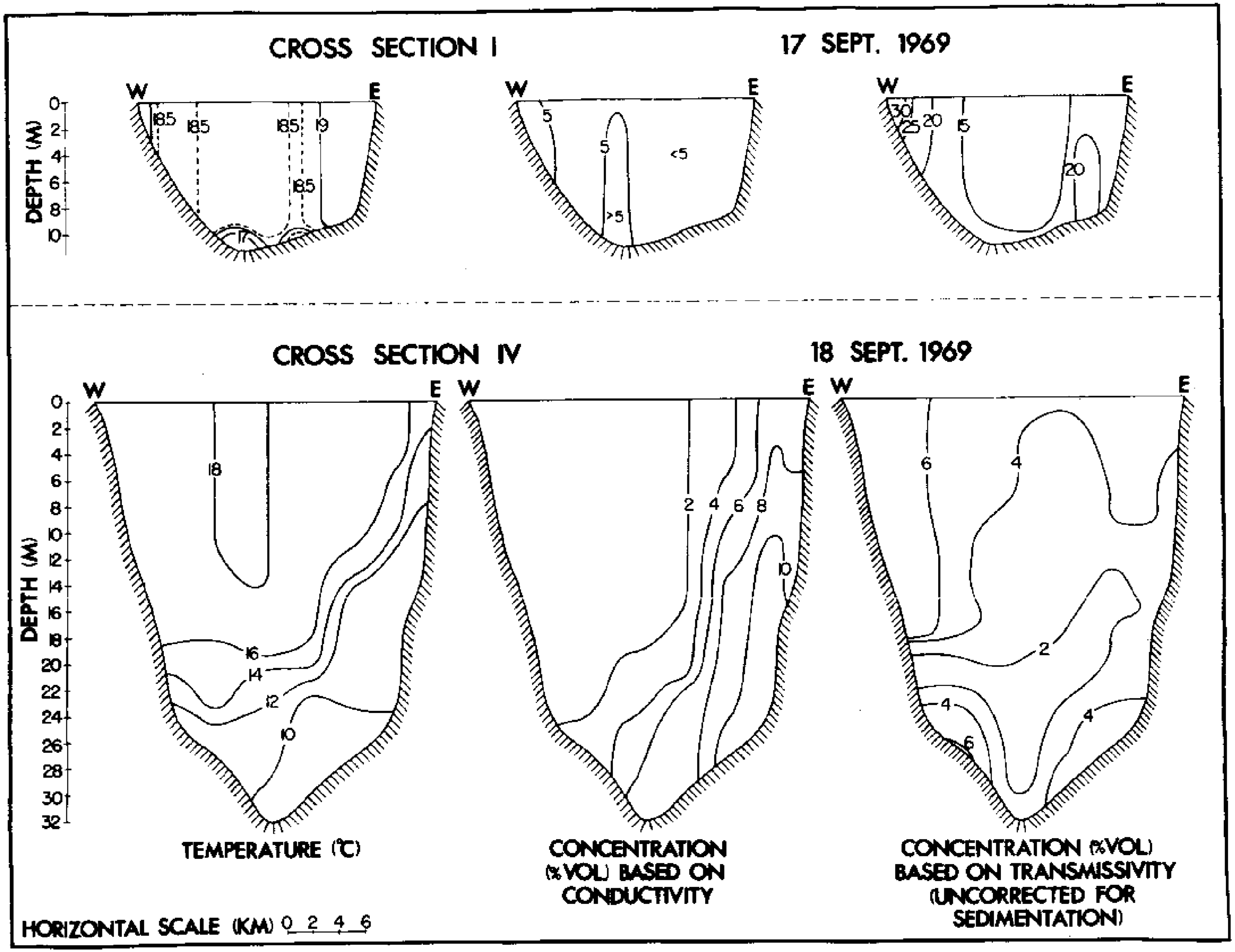

FIG. 7. Temperature (°C) and concentration (percent volume) of Fox River water based on electrical conductivity and light transmissivity for Cross Section I on 17 September and Cross Section IV on 18 September 1969. For locations of cross sections see Fig. 1.

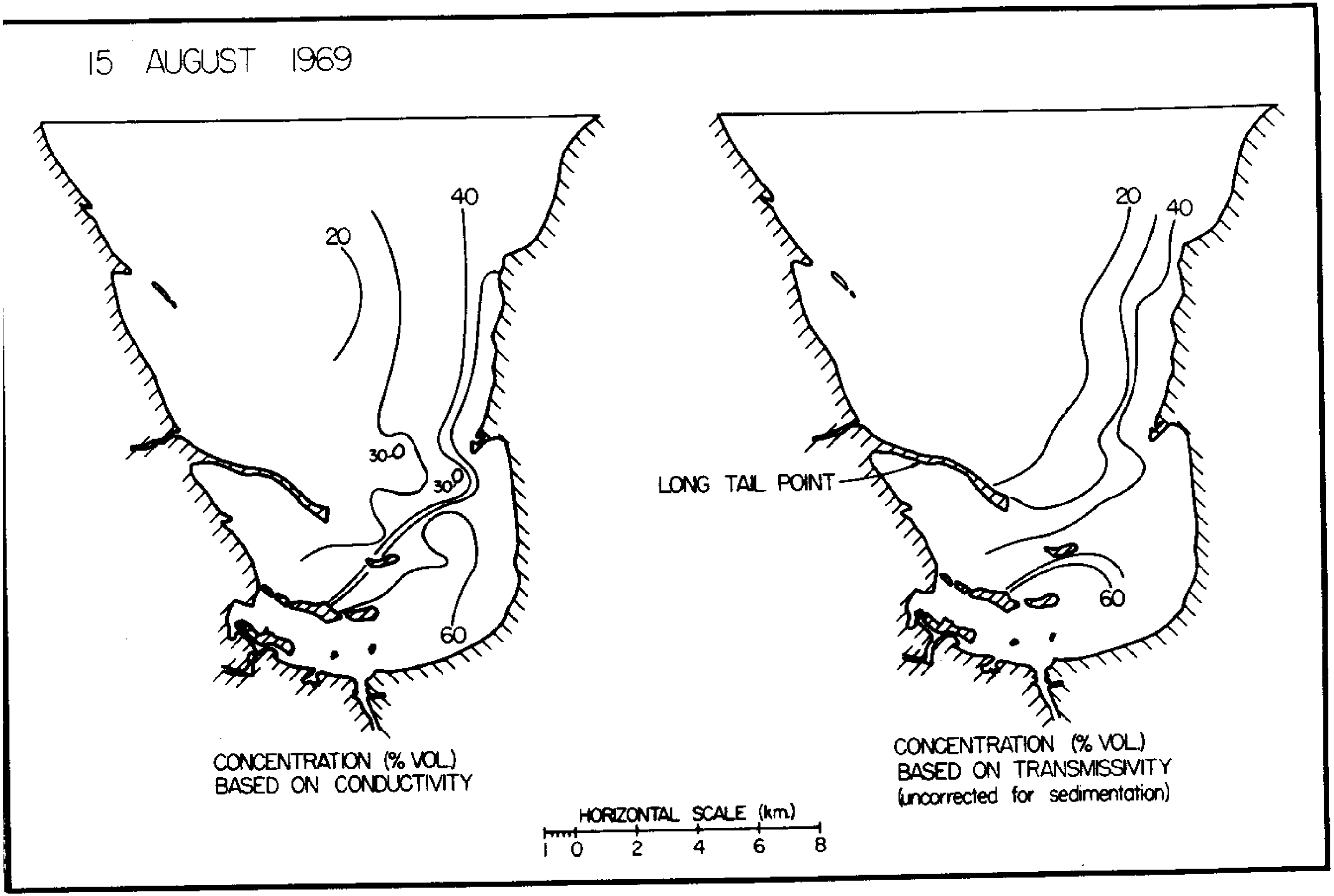

FIG. 8. Surface concentrations (percent volume) of Fox River water in the head of Green Bay on 15 August 1969. Values shown are based on electrical conductivity and light transmissivity.

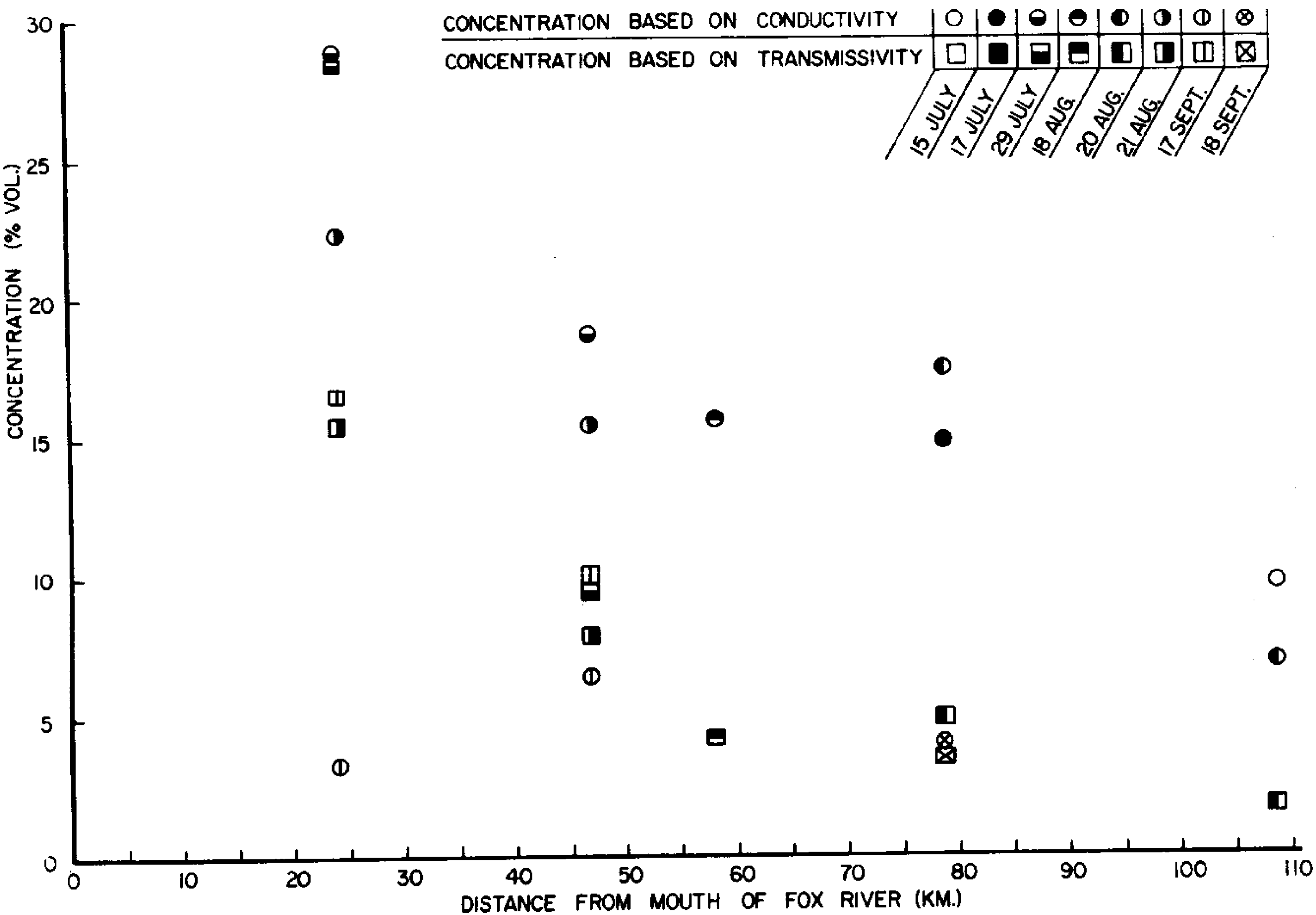

FIG. 9. Concentration (percent volume) of Fox River water in Green Bay based on electrical conductivity and light transmissivity, as a function of distance from the mouth of the River for all surveys during summer 1969.

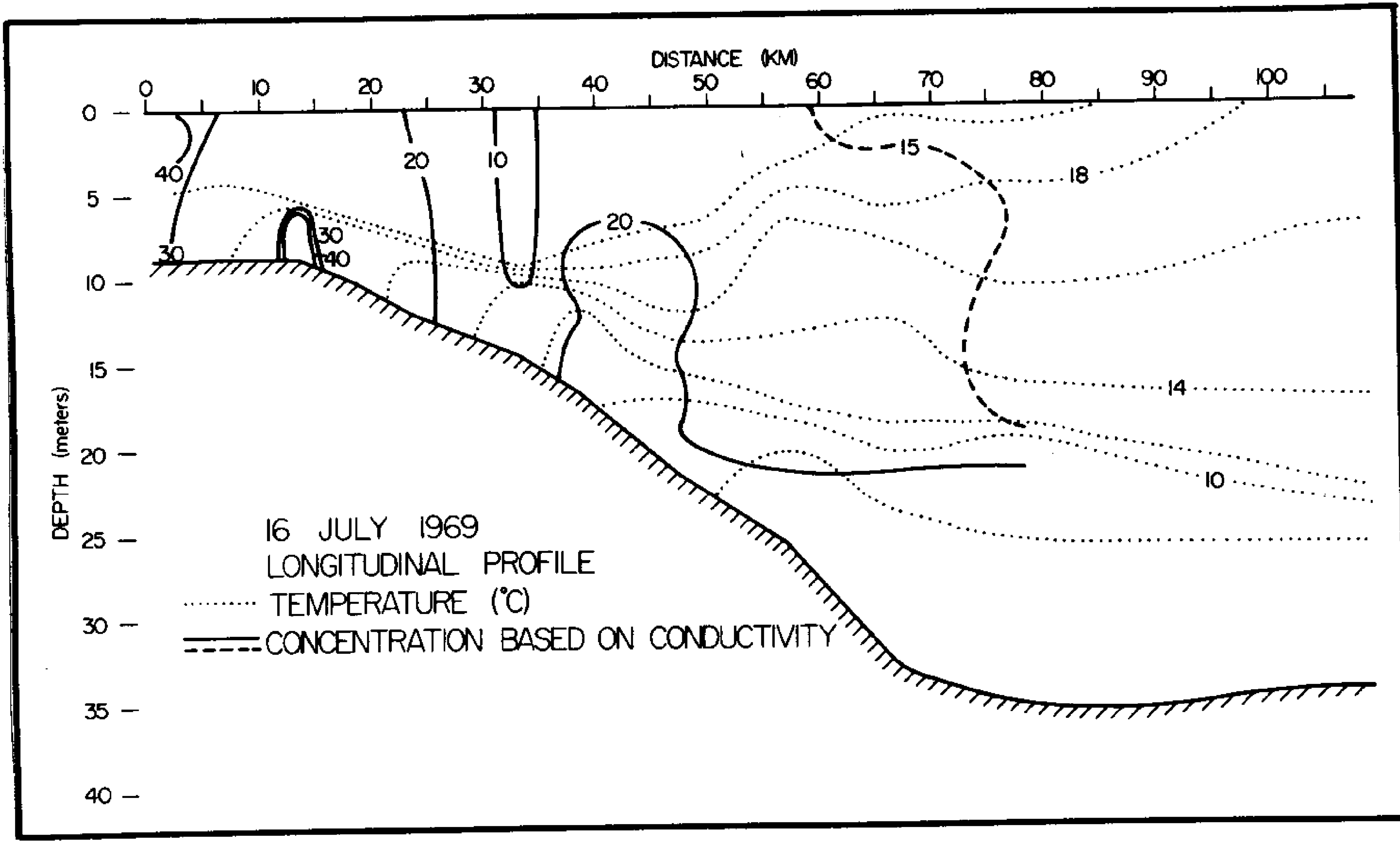

FIG. 10. Vertical variation of temperature (°C) and concentration (percent volume) of Fox River water, based on electrical conductivity, along a longitudinal section of Green Bay on 16 July 1969.

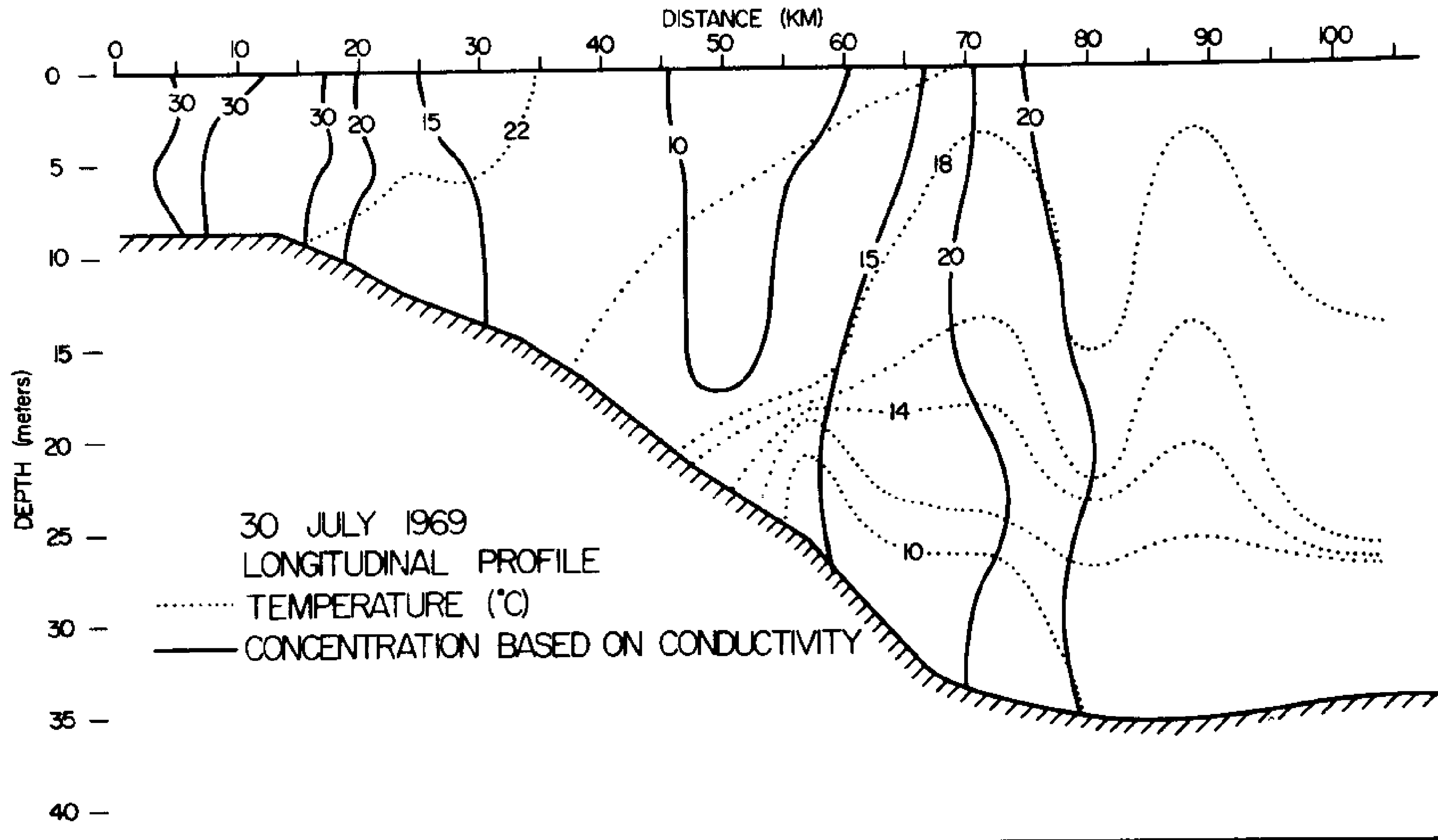

FIG. 11. Vertical variation of temperature (°C) and concentration (percent volume) of Fox River water, based on electrical conductivity, along a longitudinal section of Green Bay on 30 July 1969.

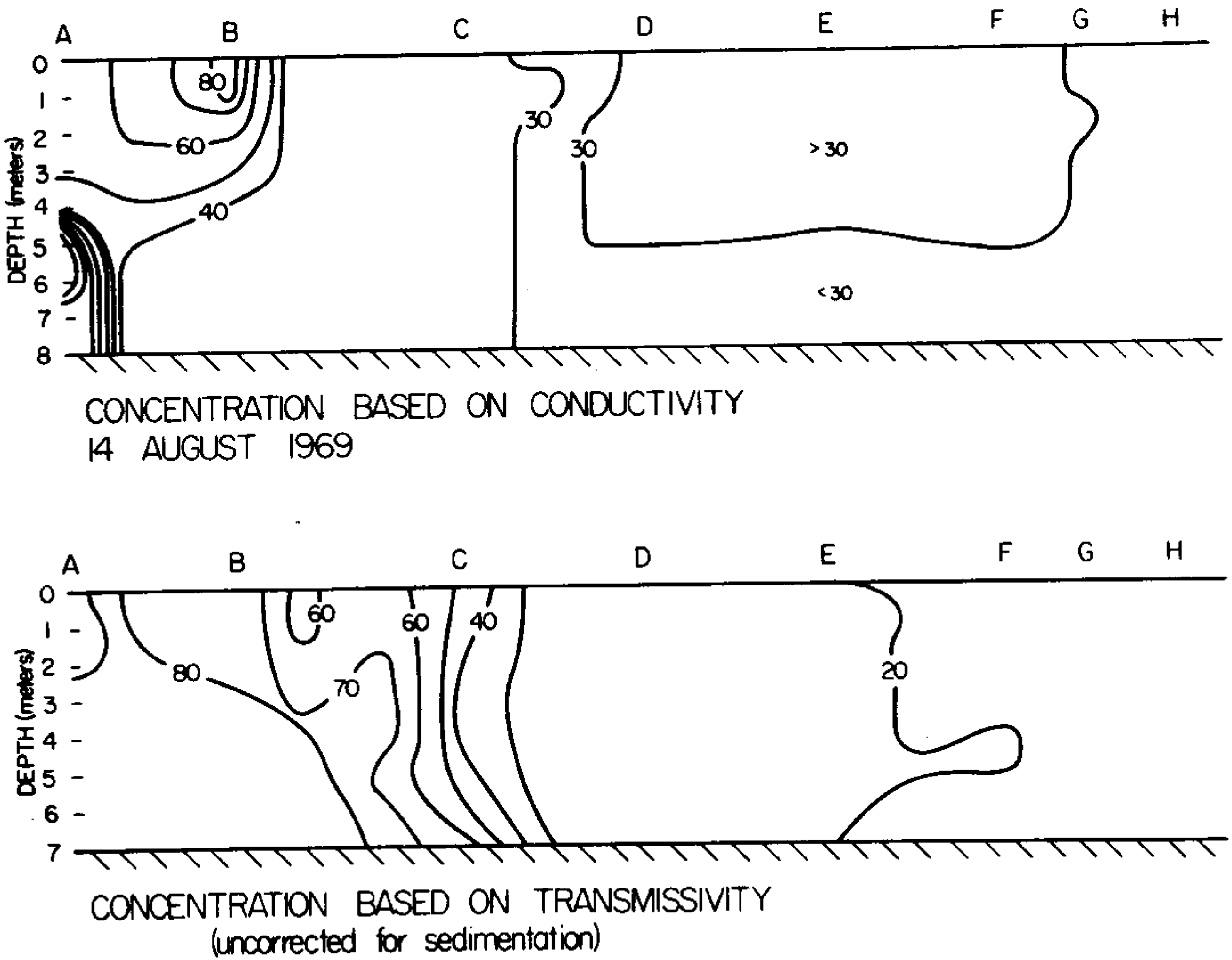

FIG. 12. Vertical variation of concentration (percent volume) of Fox River water, based on electrical conductivity and light transmissivity, along a longitudinal section in the head of Green Bay on 14 August 1969. For location of section see Fig. 16.

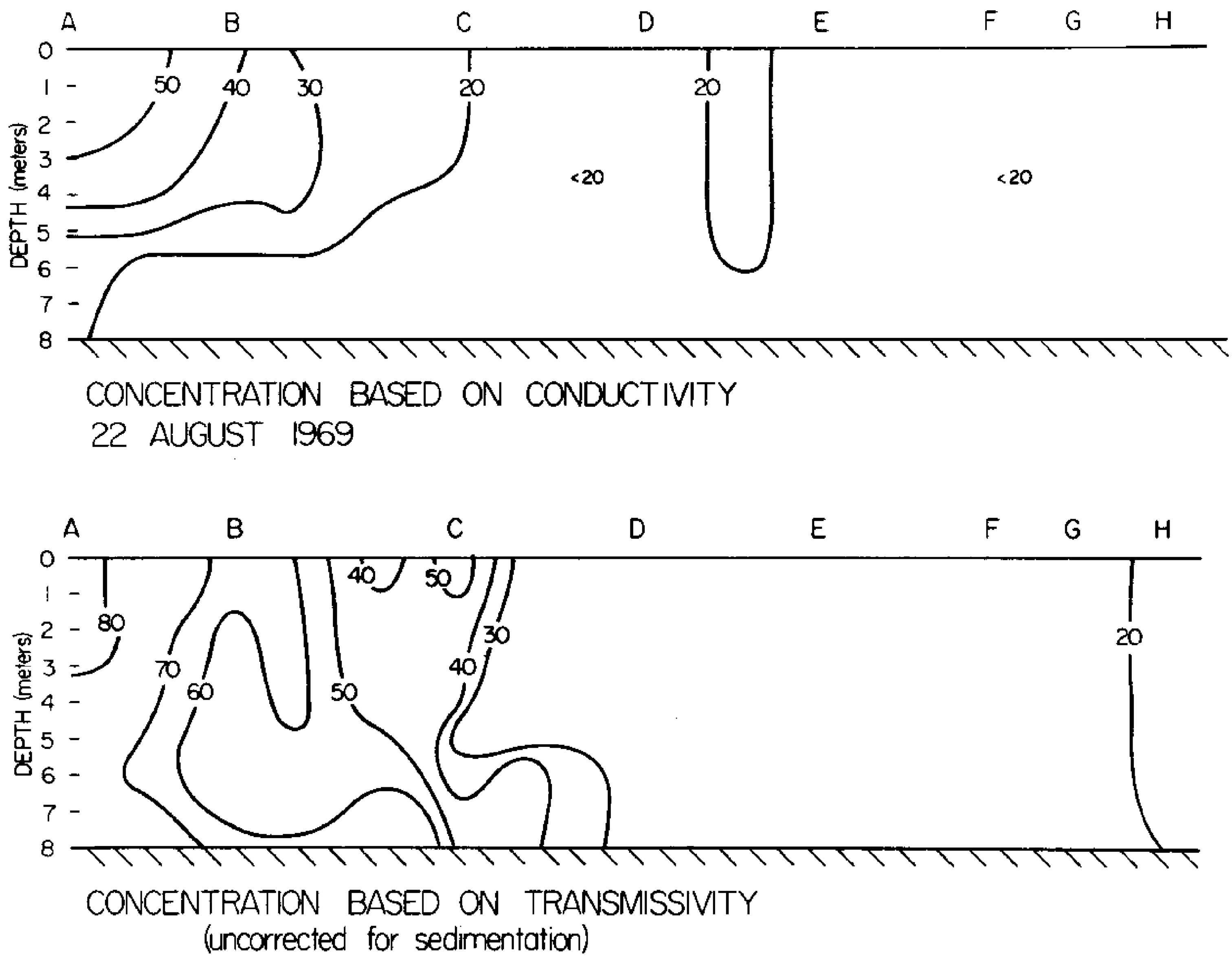

FIG. 13. Vertical variation of concentration (percent volume) of Fox River water, based on electrical conductivity and light transmissivity, along a longitudinal section in the head of Green Bay on 22 August 1969. For location of section see Fig. 16.

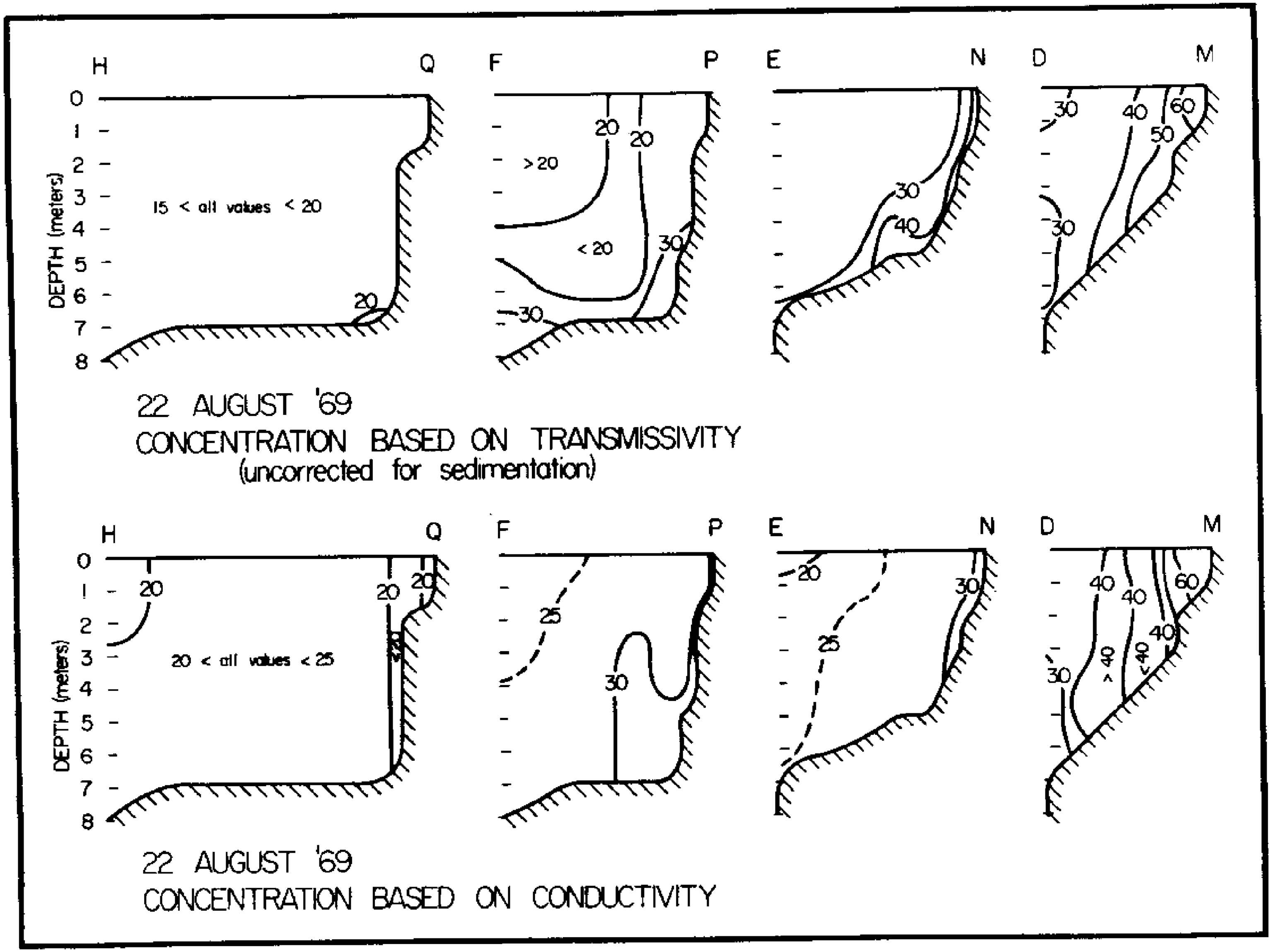

FIG. 14. Vertical variation of concentration (percent volume) of Fox River water, based on electrical conductivity and light transmissivity, along several sections perpendicular to the east shore of the head of Green Bay on 22 August 1969. For locations of sections see Fig. 16.

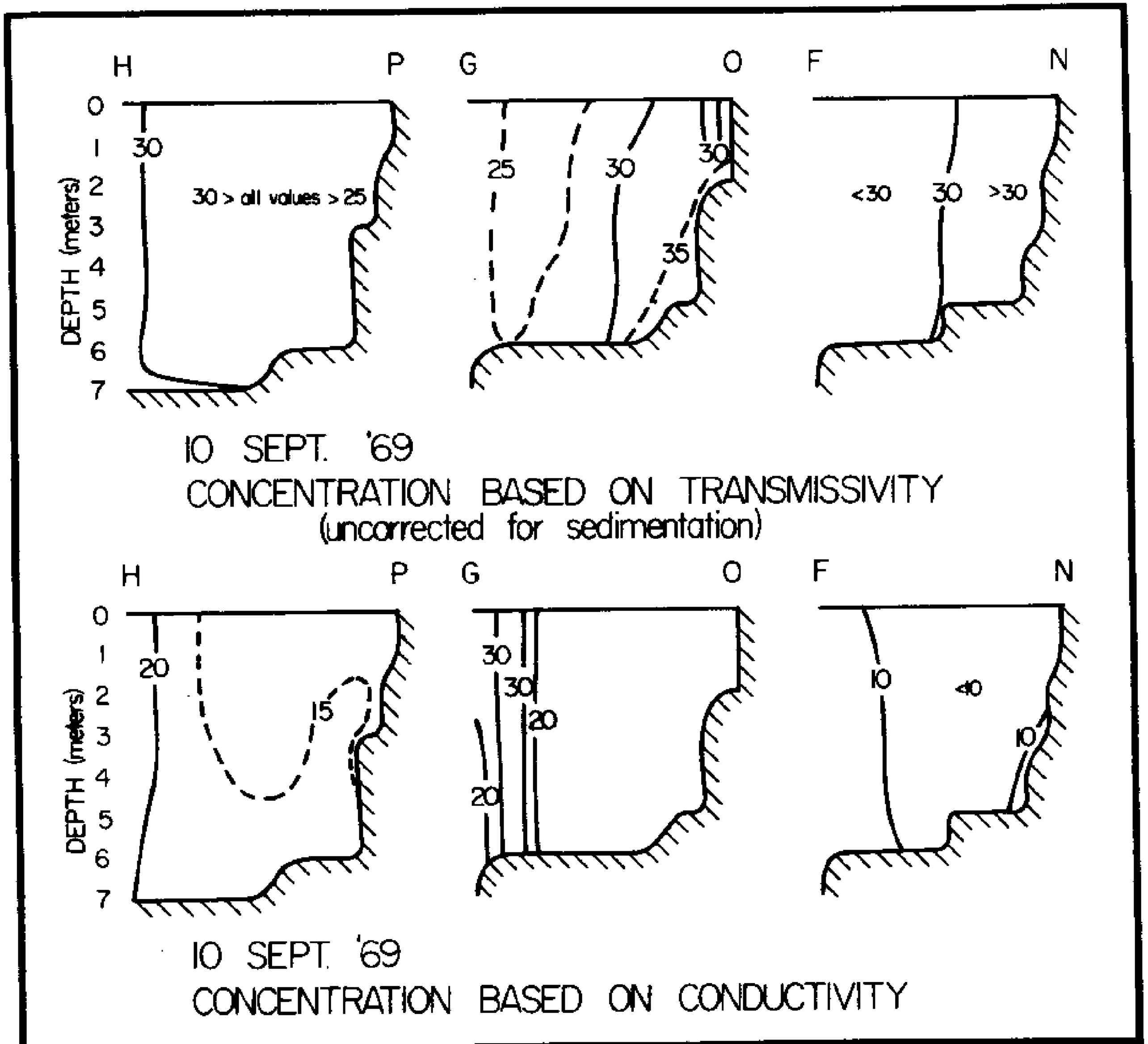

FIG. 15. Vertical variation of concentration (percent volume) of Fox River water, based on electrical conductivity and light transmissivity, along several sections perpendicular to the east shore of the head of Green Bay on 10 September 1969. Locations of sections see Fig. 16.

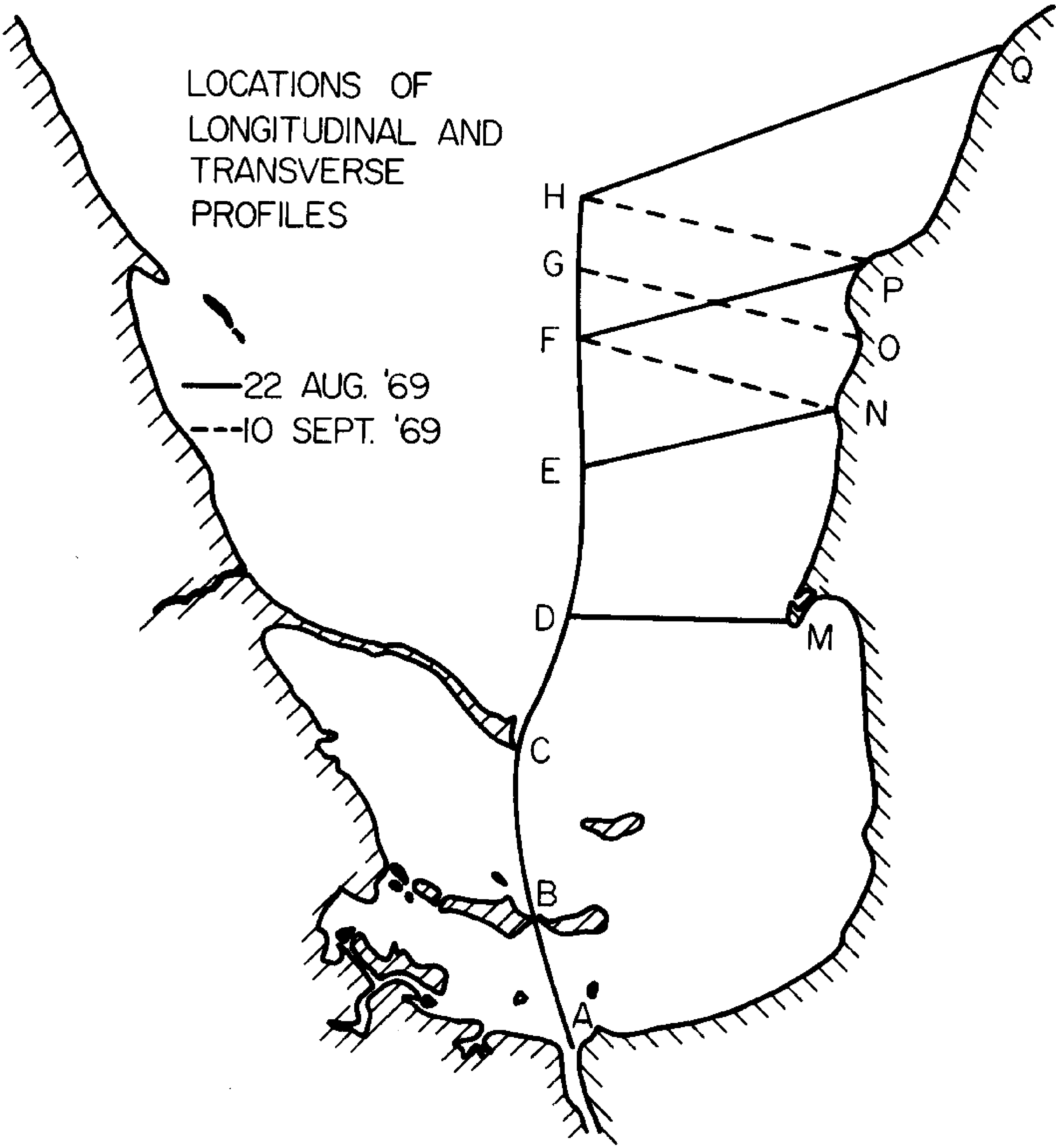

FIG. 16. Locations of vertical sections and reference stations in the head of Green Bay.

The necessary smoothing was achieved as follows: A synthetic axis of the Bay was constructed by distorting the real axis so that it is approximately perpendicular to the concentration contours. The southern 15 km of the Bay was then divided into lateral strips, 2 km wide and oriented normal to the synthetic axis. The average of all values observed between early August and mid September in a given strip for each indicator was considered to be the concentration at a point midway through the strip. Applying equation 9 to the concentration values so derived yields the diffusivity values shown in figure 17.

For the rest of the Bay, diffusivities were calculated according to equation 9 for each survey, using cross sectional mean concentrations. Values thus obtained are shown in figure 18.

To investigate the mechanisms responsible for the observed diffusivity pattern, the results of a numerical model of Green Bay circulation being developed at the Center for Great Lakes Studies at the University of Wisconsin--Milwaukee (Heaps, et al., pers. comm.) were used. This model takes into account wind stress, free oscillations in the Bay, and a forcing oscillation at the mouth of the Bay due to seiching of Lake Michigan.

Calculation of the predicted diffusivities was based on the assumption that the diffusivity is some function of the perturbation velocity u', and t, the time scale for the perturbation.

To calculate the diffusivities, the results from the numerical model of the last 10 hours of simulated time were used. The longitudinal (x) components at each grid point were averaged over the 10 hour period, approximately the period of free oscillation for the Bay. That average was assumed to be wind generated movement and residuals of the longitudinal components were assumed to be due to seiche activity. The r.m.s. values of the residuals were considered to be the perturbation velocity u' at each point. As a time scale, t, for the seiche perturbations, a value of one half of the free period of the Bay was used. Predicted diffusivities, calculated by this method are shown in figure 19. As can be seen, the proportionality constant of unity yields values comparable to the observed values.

The results show a value of 0.3×10^6 $cm^2 \cdot sec^{-1}$ in the southern most 3 km of the Bay. Beyond a sharp increase to 1.3×10^6 $cm^2 \cdot sec^{-1}$ at the next grid point, values

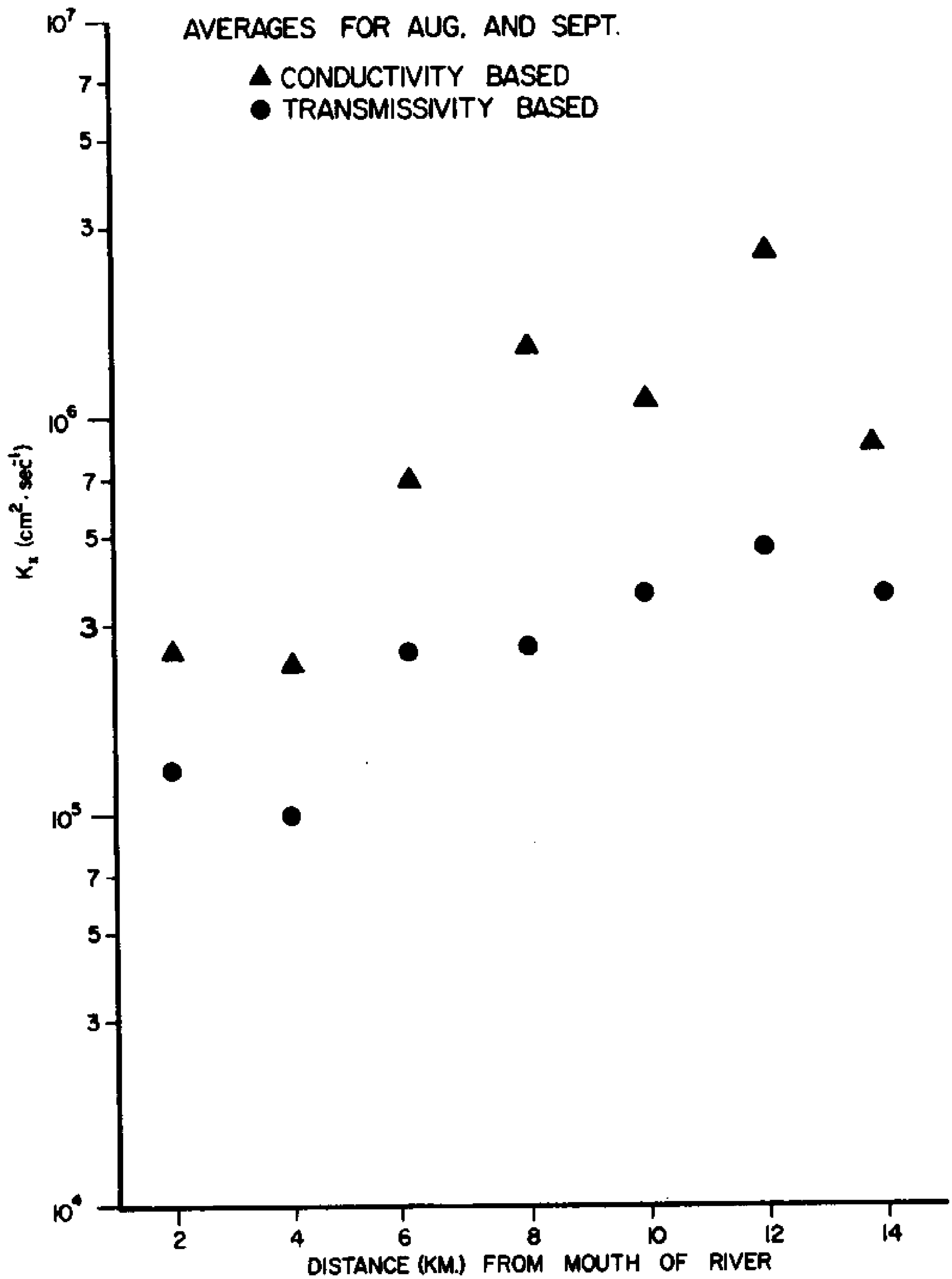

FIG. 17. Longitudinal variation of horizontal diffusivity, K_x, in the head of Green Bay. Values shown are calculated from observed concentration field of Fox River water, based on electrical conductivity and light transmissivity.

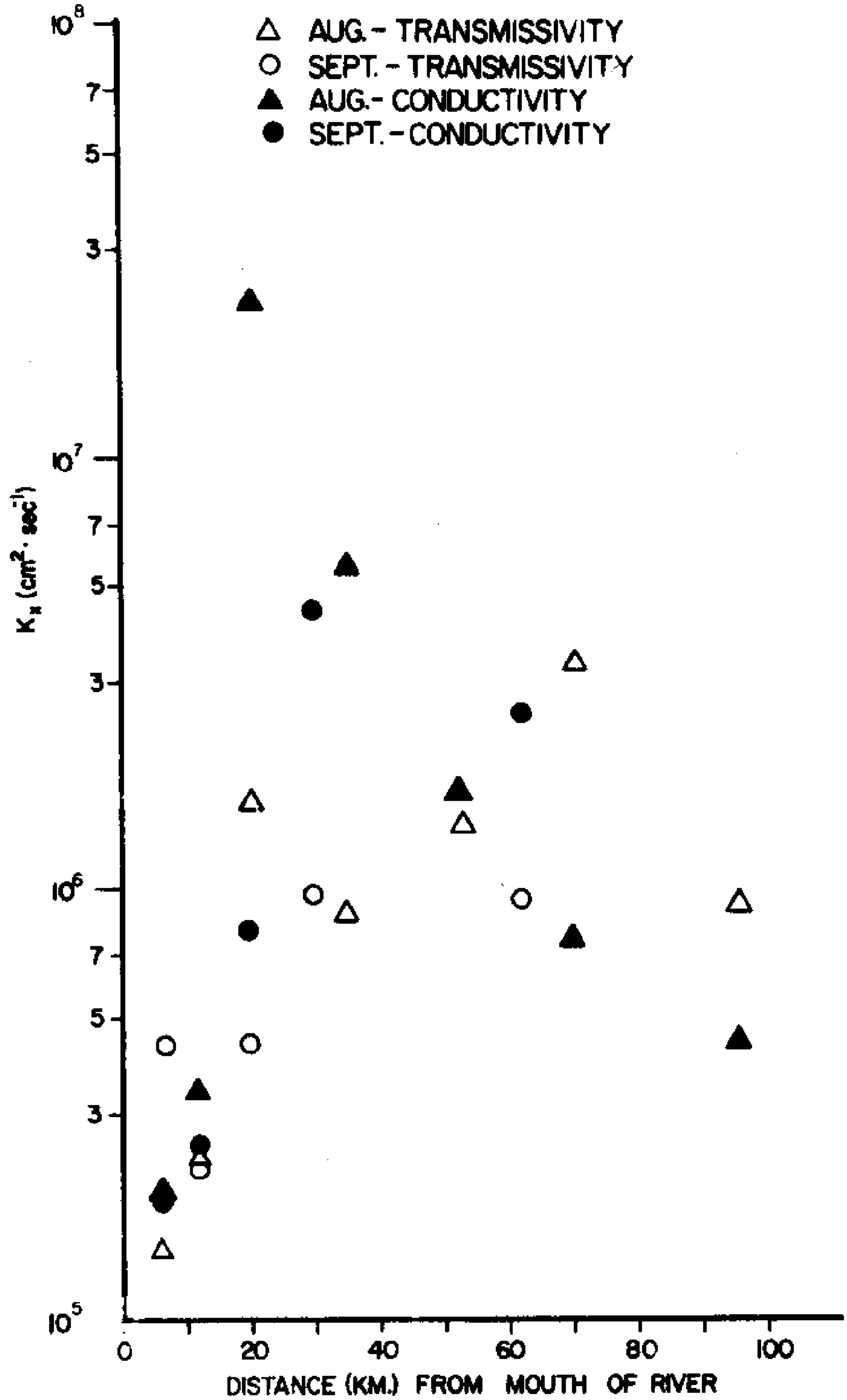

FIG. 18. Longitudinal variation of horizontal diffusivity, K_X, in Green Bay. Values shown are calculated from observed concentration field of Fox River water, based on electrical conductivity and light transmissivity.

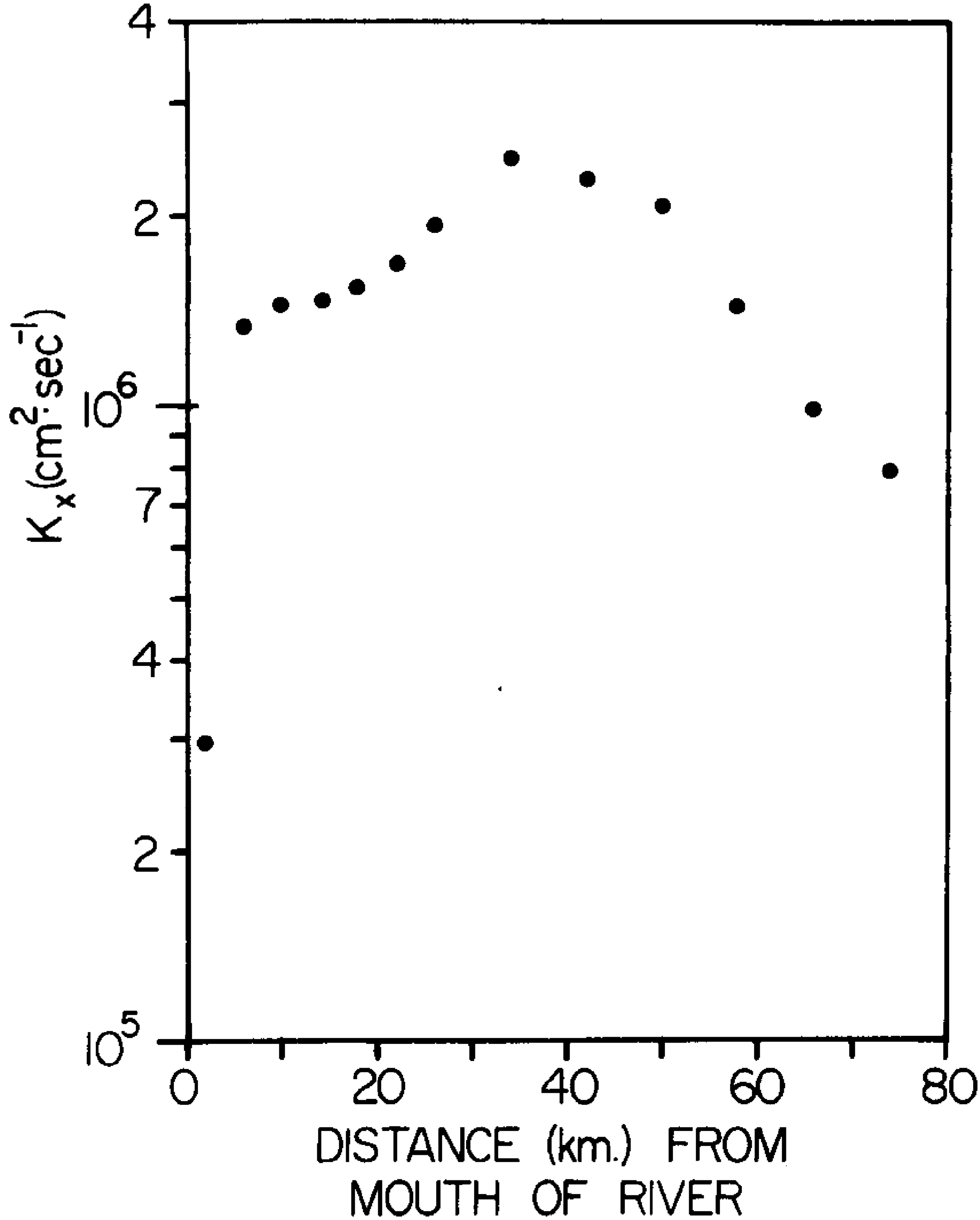

FIG. 19. Predicted longitudinal variation of horizontal diffusivity, K_x, in Green Bay due to seiche associated water movement, based on numerical model of Green Bay circulation by Heaps et. al. (1970).

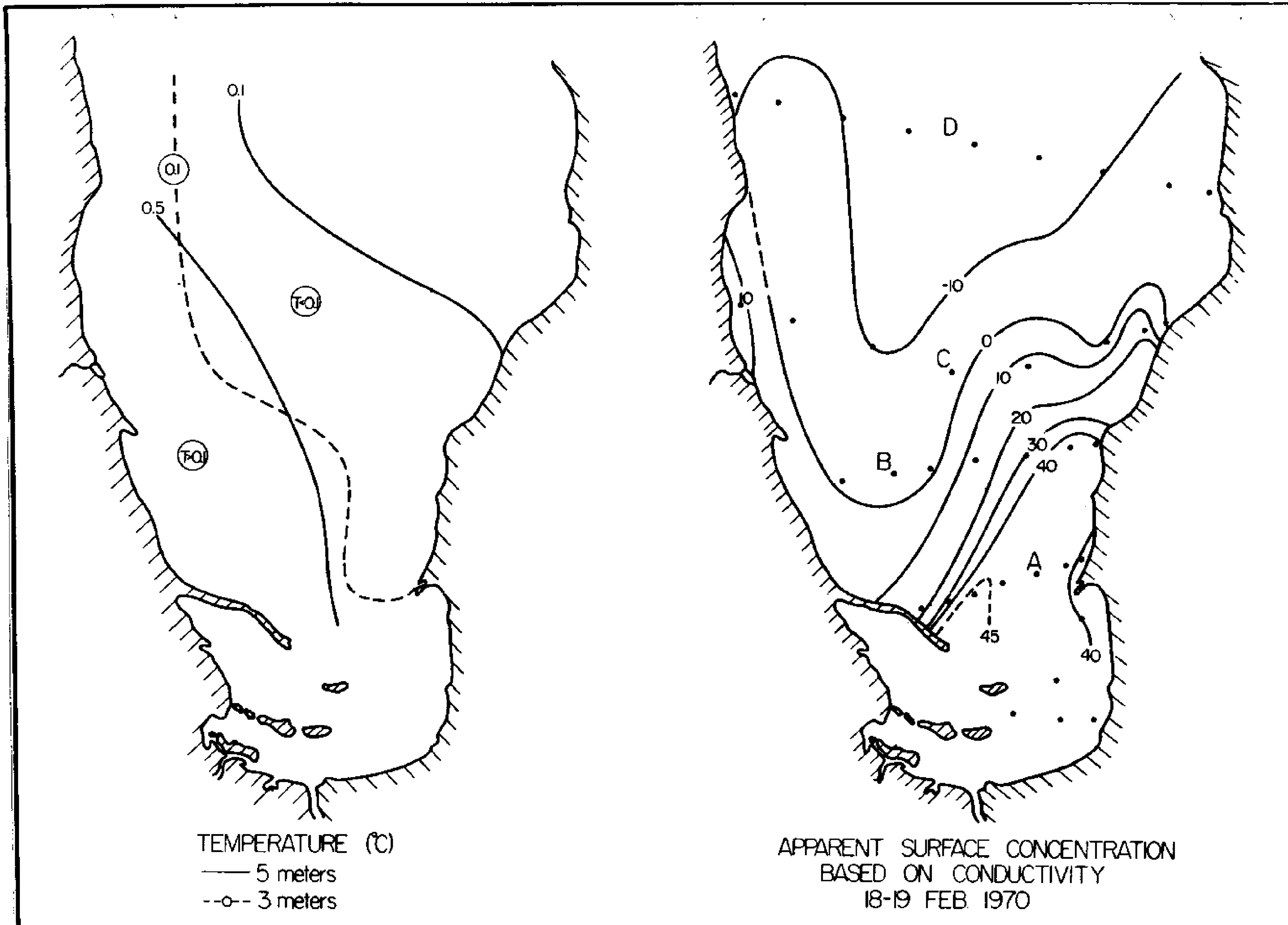

FIG. 20. Temperature (°C) and apparent concentration (percent volume) of Fox River-water, based on electrical conductivity, in Green Bay immediately beneath ice cover and at 3 meters depth on 18, 19 February 1970.

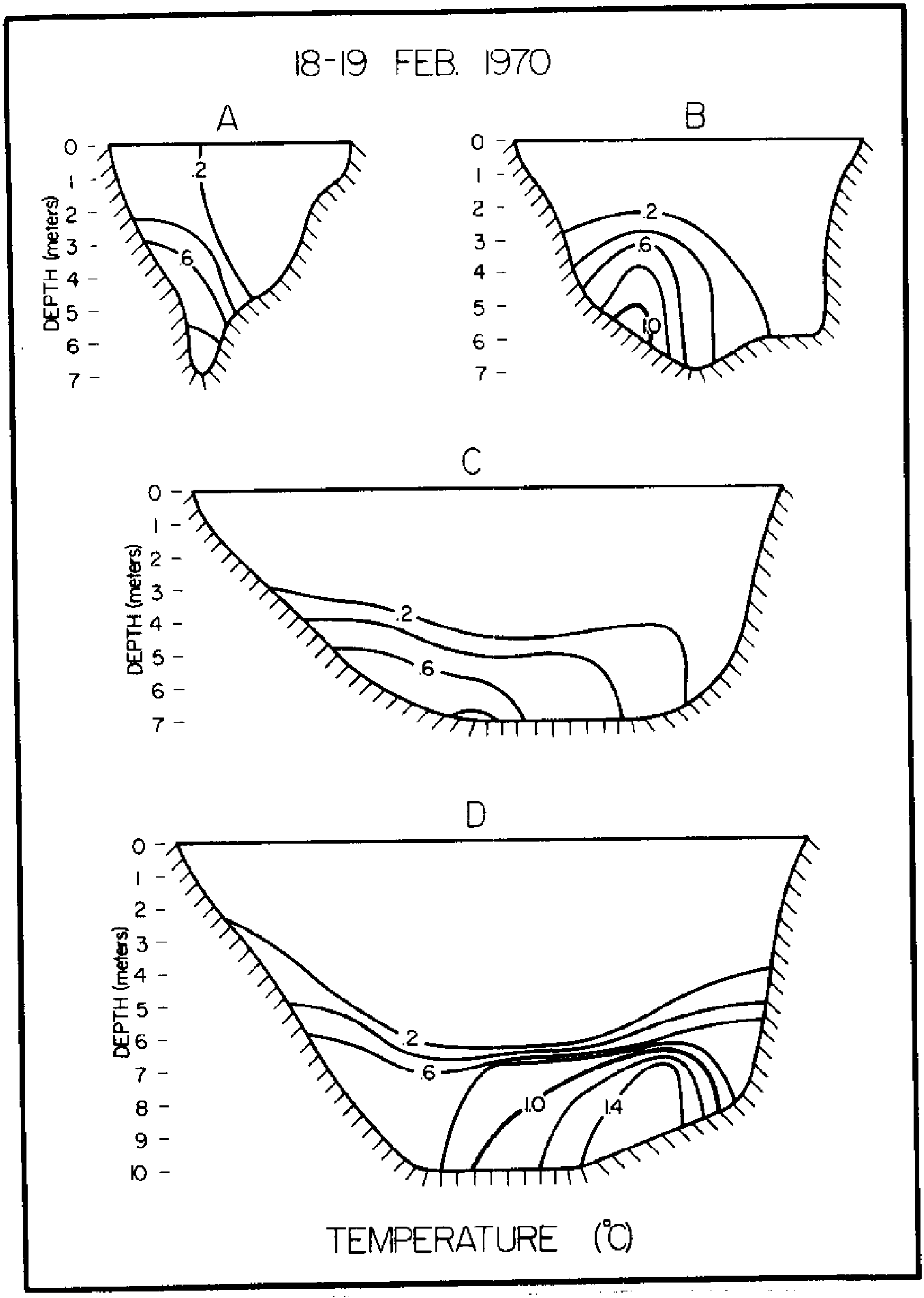

FIG. 21. Vertical variation of temperature (°C) in Green Bay on 18, 19 February 1970, along sections shown in Fig. 20.

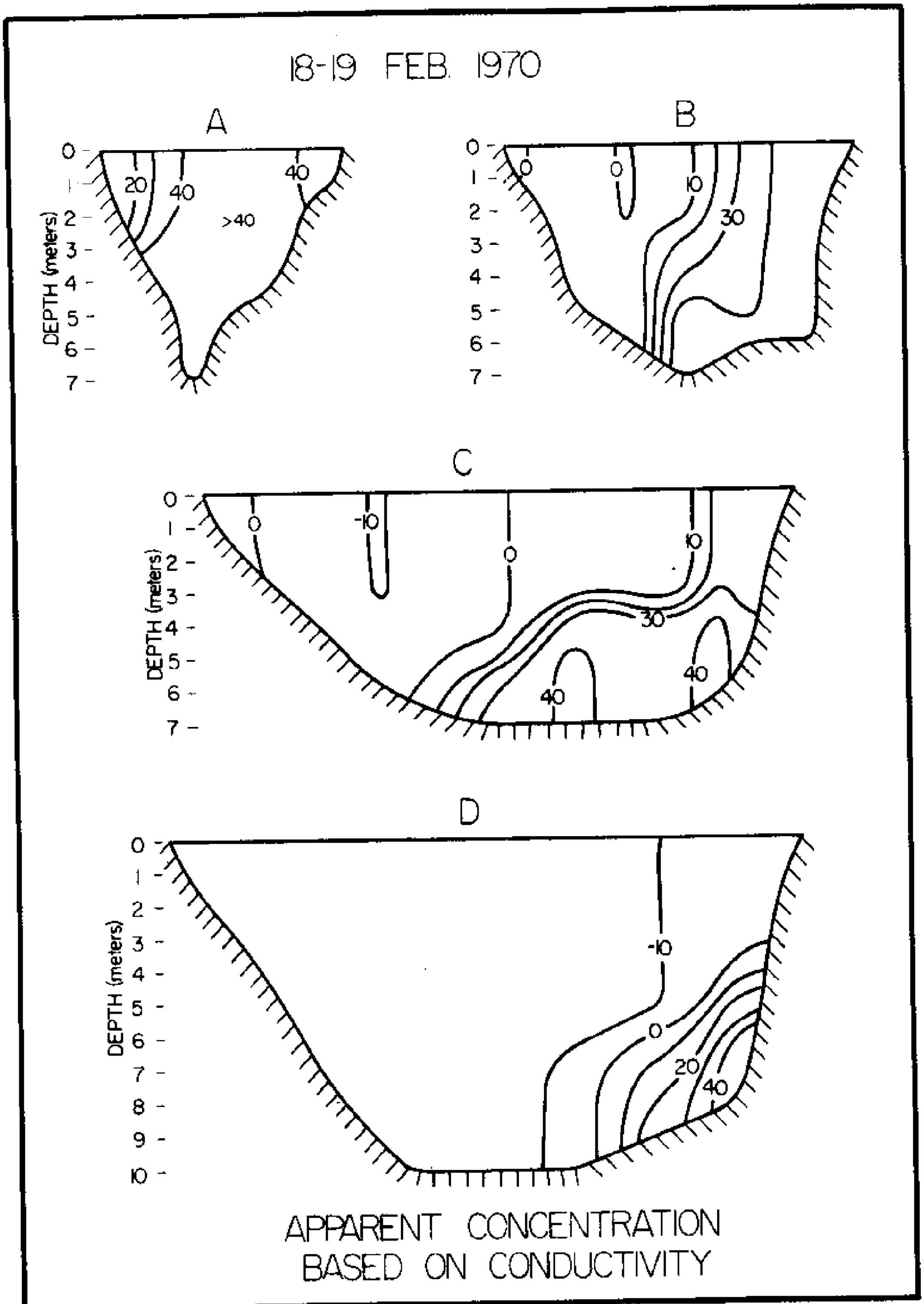

FIG. 22. Vertical variation of apparent concentration (percent volume) of Fox River water, based on electrical conductivity, in Green Bay on 18, 19 February 1970, along sections shown in Fig. 20.

increase lakeward to a value of 2.5×10^6 $cm^2 \cdot sec^{-1}$ at 35 km and then decrease to 0.8×10^6 $cm^2 \cdot sec^{-1}$ at a distance of 75 km.

Results of the wintertime survey are illustrated in figures 20, 21, and 22. Figure 20 shows apparent surface concentrations, based on conductivity, and the temperature field at 3 and 5 meters. Figures 21 and 22 respectively show the vertical structure of temperature and conductivity based apparent concentration along transverse sections, the locations of which are shown in figure 20. As in the case of the summertime data, conductivities measured in the field were reduced to 18°C according to the method of Smith (op. cit.) and then converted to concentration of Fox River water on the basis of the empirically derived relationship illustrated in Fig. 3.

VI. NUMERICAL SOLUTION OF THE DIFFUSION EQUATION

A. Description of the Model

To predict the response of the concentration field in any portion of the Bay to changes in the source strength of the Fox River effluent, a solution is needed to equation (1), expressed in the form

$$A \frac{\partial c}{\partial t} = - \frac{\partial F}{\partial x} \tag{14}$$

where

$$F = Q \cdot C - KA \frac{\partial c}{\partial x} \quad . \tag{15}$$

Kent (op. cit.) has discussed analytical solutions to equation (14) with boundary conditions appropriate to the estuarine diffusion problem. He shows that analytical solutions exist for the case of the estuary with uniform cross section area A, constant velocity U, and constant diffusivity K; for the extuary of uniform cross section area A, constant velocity U, and with the diffusivity varying linearly with x; and for the case with U, A, and K to be linear functions of x. For cases such as the irregular longitudinal variation of diffusivity values found to exist in Green Bay, Kent states that "it is unlikely that a useful general solution" can be found, and suggests that alternative methods of solution be sought.

A finite difference technique was therefore developed to obtain a numerical approximation to the solution of (14). The assumption that such a solution exists must be made *a priori*.

The model used was initialized with a given concentration field of River water in the Bay, and each time through the sequence the value of F_i^n, the Fox River input was specified. In this discussion, the superscript n (= 1, 2, 3, ...) denotes the *n*th time step, and the subscript i ($1 \leq i \leq L$) denotes the *i*th grid point.

From the known concentration field, centered differences were then used to evaluate the spatial derivatives on the right hand side of equation (14), i.e.:

$$\frac{\partial F}{\partial x}\bigg|_i^n = \frac{F_{i+1}^n - F_{i-1}^n}{2\Delta x} \tag{16}$$

where

$$F_i^n = Q^n C_i^n - (K_i A_i)\left(\frac{C_{i+1}^n - C_{i-1}^n}{2\Delta x_1}\right). \tag{17}$$

Since there is insufficient information to evaluate expression (17) at the mouth of the Bay (the L̲th grid point), the one sided technique defined by

$$F_L^n = -\frac{K_L A_L}{2\Delta x}\,(C_{L-2}^n - 4C_{L-1}^n + 3C_L) \tag{18}$$

was used. The approximation for a space derivative on the right hand side of the above equation is derived as follows:

Consider a function G having values G_1, G_2, G_3, etc., at points 1, 2, 3, ... with equal spacing Δx near the end of the domain of the function

$$\underbrace{\overset{G_1}{\underset{1}{\cdot}} \quad \overset{G_2}{\underset{2}{\cdot}}}_{\Delta x} \quad \overset{G_3}{\underset{3}{\cdot}} \quad \overset{G_4}{\underset{4}{\cdot}} \quad . \ \text{etc.}$$

Express the values of G_2 and G_3 as Taylor series expansions in terms of G_1 and $\frac{\partial G}{\partial x}\Big|_1$

$$G_2 = G_1 + \frac{\partial G}{\partial x}\bigg|_1 \Delta x + \frac{(\Delta x)^2}{2}\,\frac{\partial^2 G}{\partial x^2}\bigg|_1 + \theta(\Delta x)^3 \tag{19a}$$

$$G_3 = G_1 + \frac{\partial G}{\partial x}\bigg|_1 \Delta x + 2(\Delta x)^2\,\frac{\partial^2 G}{\partial x^2}\bigg|_1 + \theta(\Delta x)^3 \tag{19b}$$

Neglecting terms of order $(\Delta x)^3$ and higher, solving the simultaneous equations (19a) and (19b) for $\frac{\partial G}{\partial x}$ by eliminating $\frac{\partial^2 G}{\partial x^2}$ yields

$$\frac{\partial G}{\partial x} = \frac{4G_2 - G_3 - 3G_1}{2\Delta x} \quad (20)$$

the expression used in equation 18, and also in equation 22.

The advective term is omitted in equation 18 because that term is implicitly equal to zero. Some boundary condition was necessary to complete the system of equations and the assumption was made that an infinite sink exists at the mouth of the Bay, or that $C_L^n \equiv 0$.

Having evaluated the right hand side of equation (16) for the nth time step, a forward difference technique was used to evaluate the concentration values for the n+1th time step at all internal gridpoints (1<i<L). The formula for evaluating C is given by

$$C_i^{n+1} = C_i^n - \frac{\Delta t}{2A_i \Delta x}\left(F_{i+1}^n - F_{i-1}^n\right). \quad (21)$$

At the head of the system, again lacking sufficient information to evaluate C by the above method, the following formula was used:

$$C_1^{n+1} = C_1^n - \frac{\Delta t}{2A_1 \Delta x}\left(4F_2^n - F_3^n - 3F_1^n\right). \quad (22)$$

B. Verification of the Model

To check the model, it was applied to two synthetic cases to which analytical solutions can be found for the steady state problem. Those cases were: (I) Q, K, and A are constant, and (II) Q and K are constant while A varies linearly with distance.

The analytical solutions are

$$\text{(I)} \quad C(x) = 1 - e^{x-1} \qquad 0 \le x \le 1$$

and

$$\text{(II)} \quad C(x) = 1 - x^{0.5} \qquad 0 \le x \le 1$$

respectively.

The numerical solutions to these two cases are illustrated in figures 23a and 23b. The resulting concentrations (in percent) are shown as a function of distance down the Bay (along the ordinate) and time

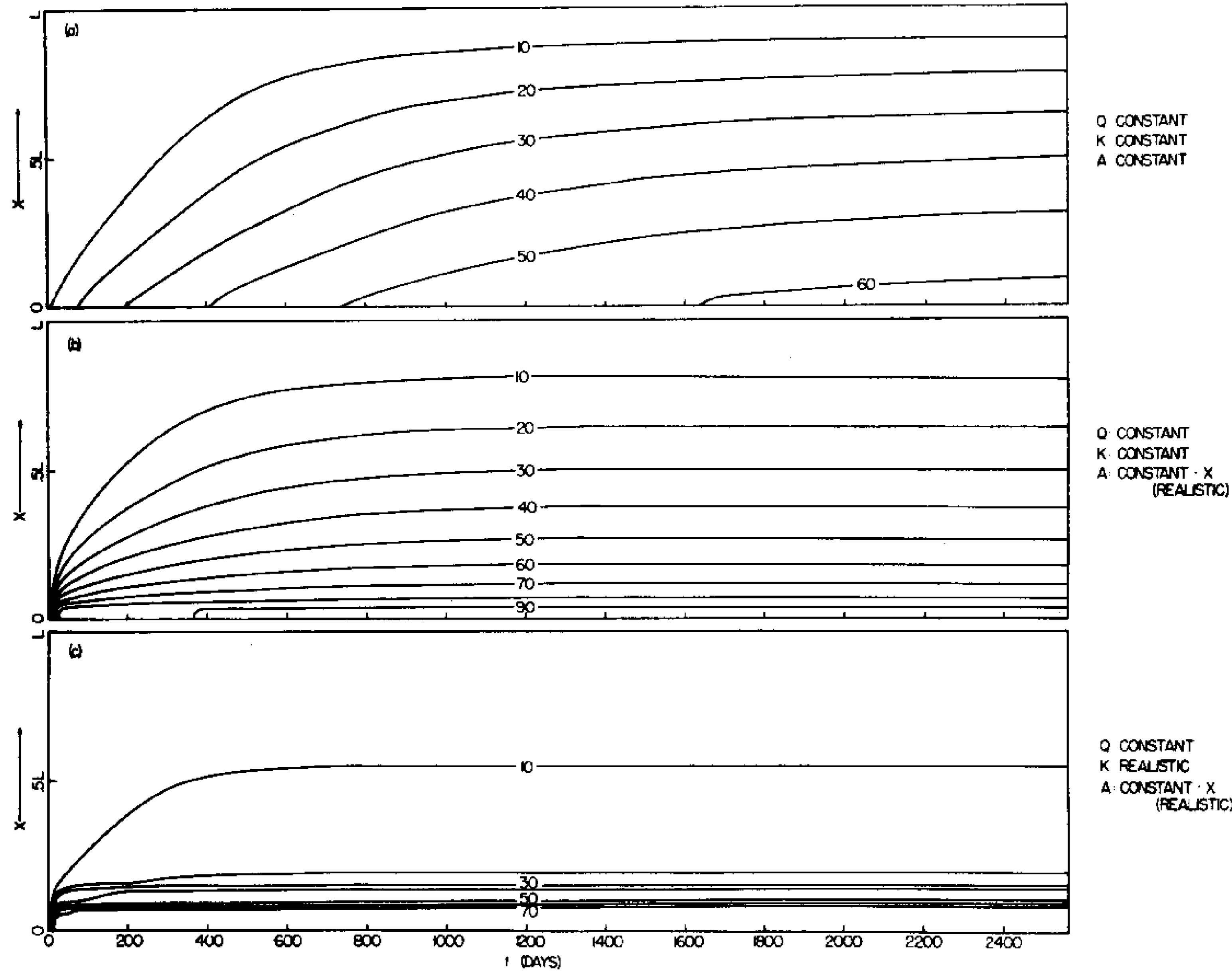

FIG. 23. Numerically predicted concentration (percent volume) of Fox River water in Green Bay, as a function of time and of distance from the source. Variations of horizontal diffusivity, K, cross sectional area, A, and river discharge rate, Q, are as shown. Fox River water is assumed to be conservative.

(along the abcissi). The steady state concentrations for all 30 grid points for both the analytical and numerical solutions are given in table 5.

In the case of Q, K, and A constant the agreement between the analytical and approximate solutions is excellent. At all grid points the two values are within 0.5 percent of each other. In the case of the linearly varying A, the agreement is not quite as good. At the grid point closest to the head of the Bay the numerical solution exceeds the analytical solution by approximately 10 percent. This disagreement decreases to about 4 percent at approximately 0.1 L from the head of the Bay. By a distance of 0.4 L the disagreement is reduced to less than one percent.

This discrepancy is most likely due to the computational effect of the formula for area of the Bay. To avoid a zero cross sectional area at the head of the Bay (which would result in an infinite value for the concentration), the expression

$$A(x) = \frac{2 \cdot x + \varepsilon}{L}$$

was used for the longitudinal variation of area. This ε introduces an error of 25 percent of the area at the second grid point, with decreasing effect down the Bay. This spurious effect is one which will have to be tolerated in the use of this model.

Figure 23c shows the results of the most realistic case tested. In that case cross sectional area varied linearly with distance, and the variation of K used was that observed in this study. The discharge rate and pollutant concentrations of the Fox River were held constant. Following turn on of the river into a clean Bay, equilibrium is reached after approximately 450 days. For purposes of this study, equilibrium is considered to have been reached at that time after which all concentration changes are less than 0.1 percent.

The resulting concentration field has maximum concentrations of slightly over 70 percent at the first two grid points (7 km), with a sharp decrease to 45 percent at the third grid point (10.5 km). Beyond this, concentrations decrease more slowly, to a value of 10 percent slightly beyond mid Bay, and, in accordance with the imposed boundary condition, to a value of 00 percent where the Bay connects with the lake.

TABLE 5. Numerical and analytical solutions to the diffusion equation for conservative River water.

	Q: constant A: constant K: constant		Q: constant A: realistic K: constant		Q: constant A: realistic K: realistic
Grid Point	Analytical Solution	Numerical Solution	Analytical Solution	Numerical Solution	Numerical Solution
1	62.0	62.4	82.0	91.8	74.0
2	60.8	61.1	74.6	80.7	76.7
3	59.5	59.8	68.9	73.4	45.1
4	58.1	58.4	64.1	67.7	42.8
5	56.8	57.0	59.8	62.9	22.1
6	55.4	55.6	56.0	58.6	19.9
7	53.9	54.1	52.5	54.7	18.2
8	52.4	52.6	49.2	51.1	16.7
9	50.8	51.0	46.1	47.8	15.5
10	49.2	49.4	43.2	44.6	14.5
11	47.5	47.7	40.4	41.7	13.6
12	45.8	46.0	37.8	38.9	12.8
13	44.0	44.2	35.2	36.2	12.2
14	42.2	42.3	32.8	33.6	11.6
15	40.3	40.4	30.4	31.2	11.0
16	38.4	38.5	28.2	28.8	10.5
17	36.3	36.4	25.9	26.5	10.0
18	34.3	34.3	23.8	24.2	9.5
19	32.1	32.2	21.7	22.1	8.9
20	29.9	29.9	19.7	20.0	8.3
21	27.6	27.6	17.7	18.0	7.8
22	25.2	25.3	15.8	16.0	7.1
23	22.7	22.8	13.9	14.0	6.5
24	20.2	20.3	12.0	12.1	5.7
25	17.6	17.6	10.2	10.3	5.0
26	14.9	14.9	8.4	8.5	3.4
27	12.1	12.1	6.7	6.7	2.6
28	9.2	9.2	5.0	5.0	2.6
29	6.2	6.2	3.3	3.3	1.7
30	3.2	3.1	1.6	1.6	.8

The agreement between this predicted concentration field and the observed concentration field (Fig. 9) confirms that this model generates a sufficiently realistic representation of the concentration field of Fox River water in Green Bay.

C. Effects of a Non-Conservative Pollutant

Throughout this study Fox River water has been treated as being conservative in its transit through Green Bay. Some of the constituents of River water, however, are non-conservative, for example, suspended solids subjected to flocculation and sedimentation. To examine effects of a non-conservative pollutant the model was applied to River water considered to have three different loss rates, of 10^{-1}, 10^{-2}, and 10^{-3} days^{-1}.

The effects of "turning on" a non-conservative river into a clean bay are shown in figure 24. The times required for equilibrium, as previously defined, to be reached are 80, 360, and 700 days respectively, and the resulting concentration fields are listed along with the equilibrium concentration field for conservative river water in table 6. As can be seen, in the case of the loss rate of 10^{-1} day^{-1}, the 10 percent concentration limit is displaced toward the head of the Bay to a position approximately 0.15 L from the mouth of the River, hence the effects of any pollutant with a loss rate greater than 10^{-1} day^{-1} are confined to the 5 km of the Bay closest to the mouth of the River. For the case of the loss rate of 10^{-3} day^{-1} the differences from the steady state concentration field for conservative River water are slight, and hence it is concluded that a pollutant with a loss rate of less than 10^{-3} day^{-1} is, in Green Bay, a conservative pollutant.

D. Effects of Decreasing Pollutant Concentrations in the Source

One of the questions which can be answered with this model is "How quickly and to what degree will the concentration field of pollutants in the Bay respond to a change of pollutant concentration in the River?"

Four levels of pollutant decrease in the River were tested. In those four cases, C_r, the concentration of pollutants in the River, was instantaneously decreased to 0.0, 0.2, 0.5, and 0.9 times the present level.

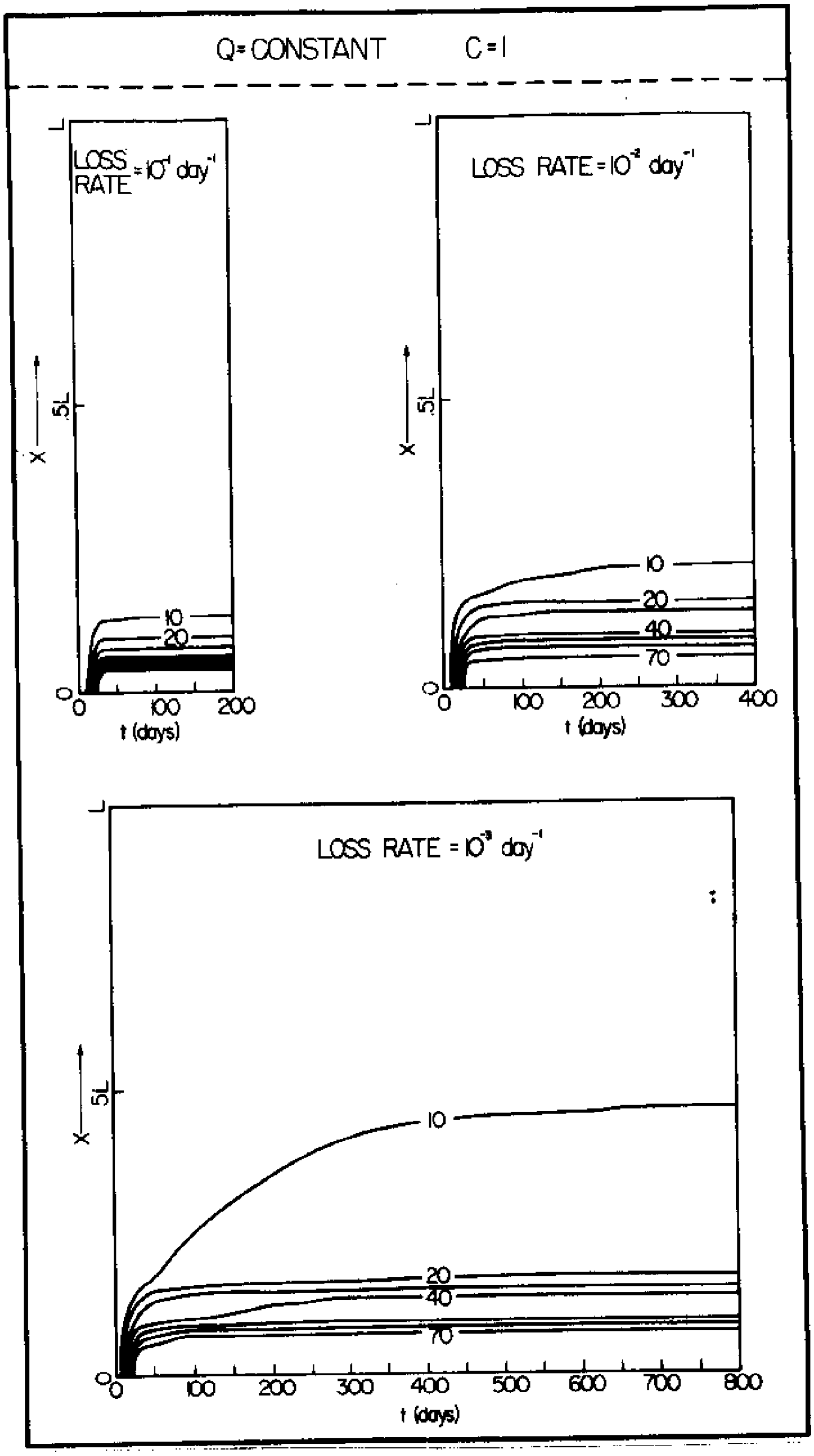

FIG. 24. Numerically predicted concentration (percent volume) of Fox River water in Green Bay, as a function of time and of distance from the source. Values of K, A, and Q are as shown in Fig. 23c. Fox River water is assumed to be non-conservative with loss rates of .1, .01, and .001 per day.

TABLE 6. Numerically predicted quilibrium concentration field of Fox River water in Green Bay assuming Fox River water to be nonconservative with loss rates of .1, .01, and .001 per day, and assuming Fox River water to be conservative.

Grid Point	CONCENTRATION (Percent Volume)			
	Loss Rate $10^{-1}.day^{-1}$	Loss Rate $10^{-2}.day^{-1}$	Loss Rate $10^{-3}.day^{-1}$	Loss Rate $00.day^{-1}$
1	85.3	77.3	74.3	74.0
2	36.1	65.7	75.1	76.7
3	19.2	37.1	43.8	45.1
4	9.9	31.2	40.9	42.8
5	3.3	13.3	20.5	22.1
6	1.7	10.5	18.1	19.8
7	1.6	9.6	16.5	18.1
8	0.9	7.9	15.0	16.7
9	0.9	7.3	13.9	15.5
10	0.5	6.2	12.8	14.5
11	0.5	5.9	12.1	13.6
12	0.2	5.0	11.3	12.8
13	0.2	4.9	10.7	12.2
14	0.1	4.2	10.1	11.6
15	0.1	4.1	9.6	11.0
16	0.1	3.6	9.1	10.5
17	0.1	3.5	8.7	10.0
18	0.0	3.1	8.1	9.4
19	0.0	3.0	7.7	8.9
20	0.0	2.6	7.1	8.3
21	0.0	2.4	6.7	7.8
22	0.0	2.1	6.0	7.1
23	0.0	1.9	5.5	6.5
24	0.0	1.6	4.9	5.7
25	0.0	1.4	4.3	5.0
26	0.0	1.1	3.6	4.2
27	0.0	0.9	2.9	3.4
28	0.0	0.6	2.2	2.6
29	0.0	0.4	1.5	1.7
30	0.0	0.2	0.7	0.8

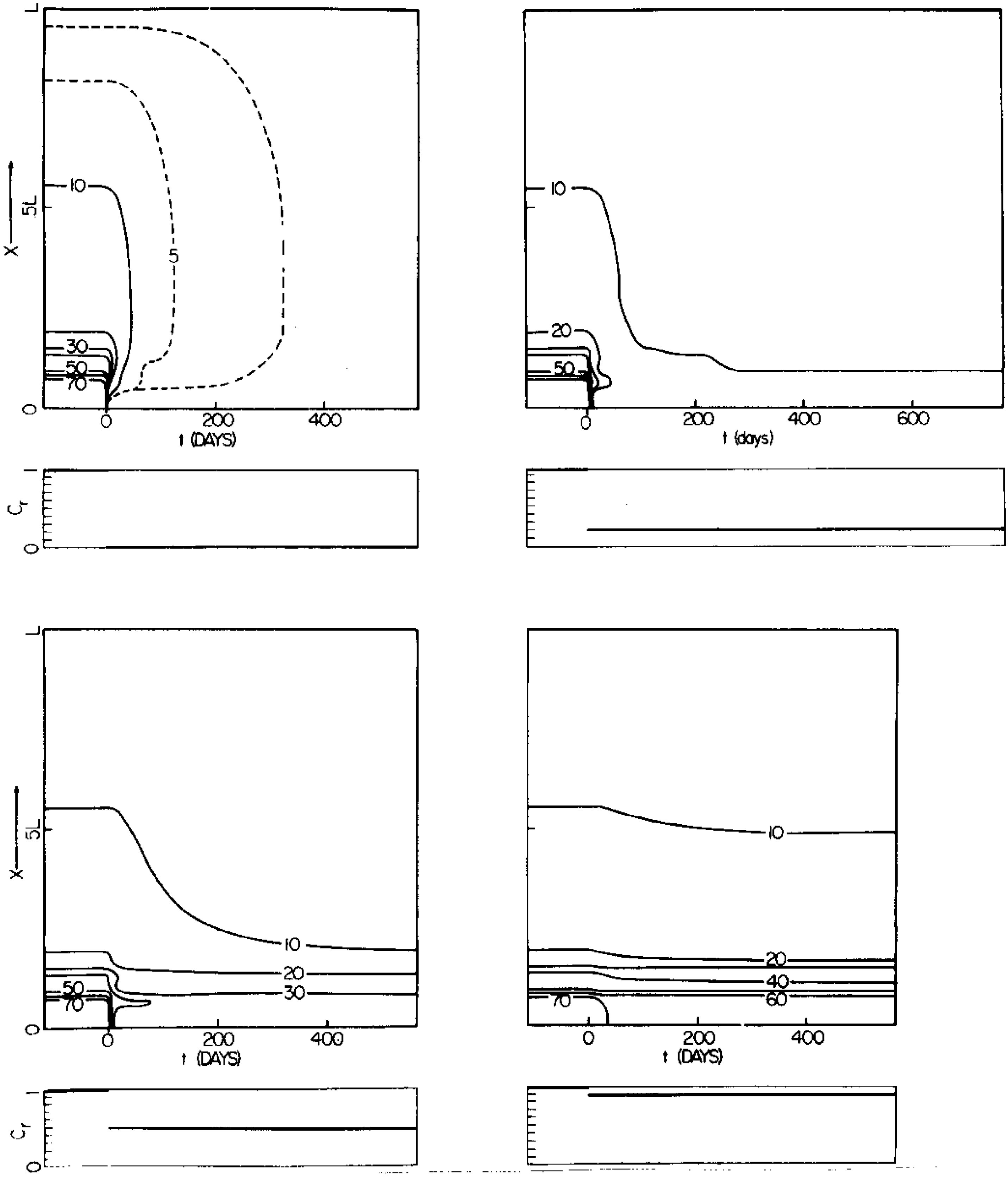

FIG. 25. Numerically predicted concentration (percent volume) of Fox River water in Green Bay as a function of time and of distance from the source. Decreases in pollutant levels in the River, C_r, are shown. Values of K, A, and Q are as shown in Fig. 23c.

The Bay was approximated by the same conditions illustrated in figure 23c. Those are: cross sectional area of the Bay varying linearly with distance from the head, the longitudinal variation of diffusivity observed in Green Bay, and a constant discharge rate of the River (thereby maintaining the advective effect of the lakeward flow of River water). Results of the tests are illustrated in figure 25.

The response of the Bay to negative changes in the source is fast. Approximately 100 days after a complete clean up of the River concentration levels throughout the Bay are less that 5% of those presently existing in Fox River water. In just over 300 days, concentrations of present day River water throughout the Bay are one percent or less. As can be seen in figures 25b, c, and d, the response to the lesser decreases in pollutant concentration in the Fox River requires a time of approximately 400 days before a new equilibrium is reached. The response time, previously defined, is approximately equal to that required for equilibrium following turn on of the River into a clean Bay, illustrated in figure 23c.

E. Effects of Variations in the Discharge Rate of the Source.

Bowden (1967) states that one of the primary physical factors governing diffusion in an estuary is the discharge rate of rivers entering the estuary. To examine the effects of variations in that rate for the Green Bay case the model was applied to three different variations of the discharge rate of the Fox River into Green Bay. While maintaining the concentrations of pollutants in the River at their present level and using the same longitudinal variation of K and A as in the previous section, the discharge rates tested are as follows:

a) A sinusoidally varying discharge rate with a period of one year and an amplitude equal to the mean discharge rate of the Fox River;

b) A discharge rate equal for each day to the daily average rate for the period 1961 - 1967; and

c) The daily discharge rate for 1969.

The latter two of these discharge rates are illustrated in figure 26. Results of these applications of the model are shown in figure 27.

In all of these cases, the fact that the discharge rate exceeds unity and the method of evaluating the space derivative at the boundary point of the system result in spurious concentration values at the first two grid points, hence the analyses do not include those points. Since the model is flux conserving however, some valid conclusions may be drawn from the model results in the rest of the Bay in terms of phase lag and amplitude decrease for the various discharge rates tested.

Based on the three discharge rates, the longitudinal variation in response lag is shown in figure 28, and figure 29 shows longitudinal variation of annual mean concentration and standard deviation for all three cases of varying discharge rates tested.

It can be seen in figure 28 that lakeward the time required for the concentration field to respond to variations in the discharge rate is least for the sinusoidally varying daily discharge rate and greatest for the 1969 daily discharge rate. Approximately 25 days are required for the response at mid Bay in the case of the 1969 discharge rate, while approximately 100 days are required in the case of the sinusoidal discharge rate.

Figure 29 shows that beyond that 10 percent of the Bay closest to the source, the annual mean concentration does not differ significantly among the three cases tested. The standard deviations at each point do show some differences between the three cases. The value of the ratio of standard deviation to mean concentration is greatest in the case of the sinusoidally varying discharge rate, reaching approximately 0.27 in mid Bay. The multi-annual mean discharge rate yields the smallest value of the ratio, approximately 0.11 in mid Bay, and the 1969 discharge rate yields the intermediate value of approximately 0.15 in mid Bay.

The results of the 3 cases tested, illustrated in figure 27 also validates the assumption that a 3 day survey can be considered synoptic. While short term fluctuations in the concentration field due to seiching and the direct response to winds could only be resolved by a true synoptic picture, in terms of the response

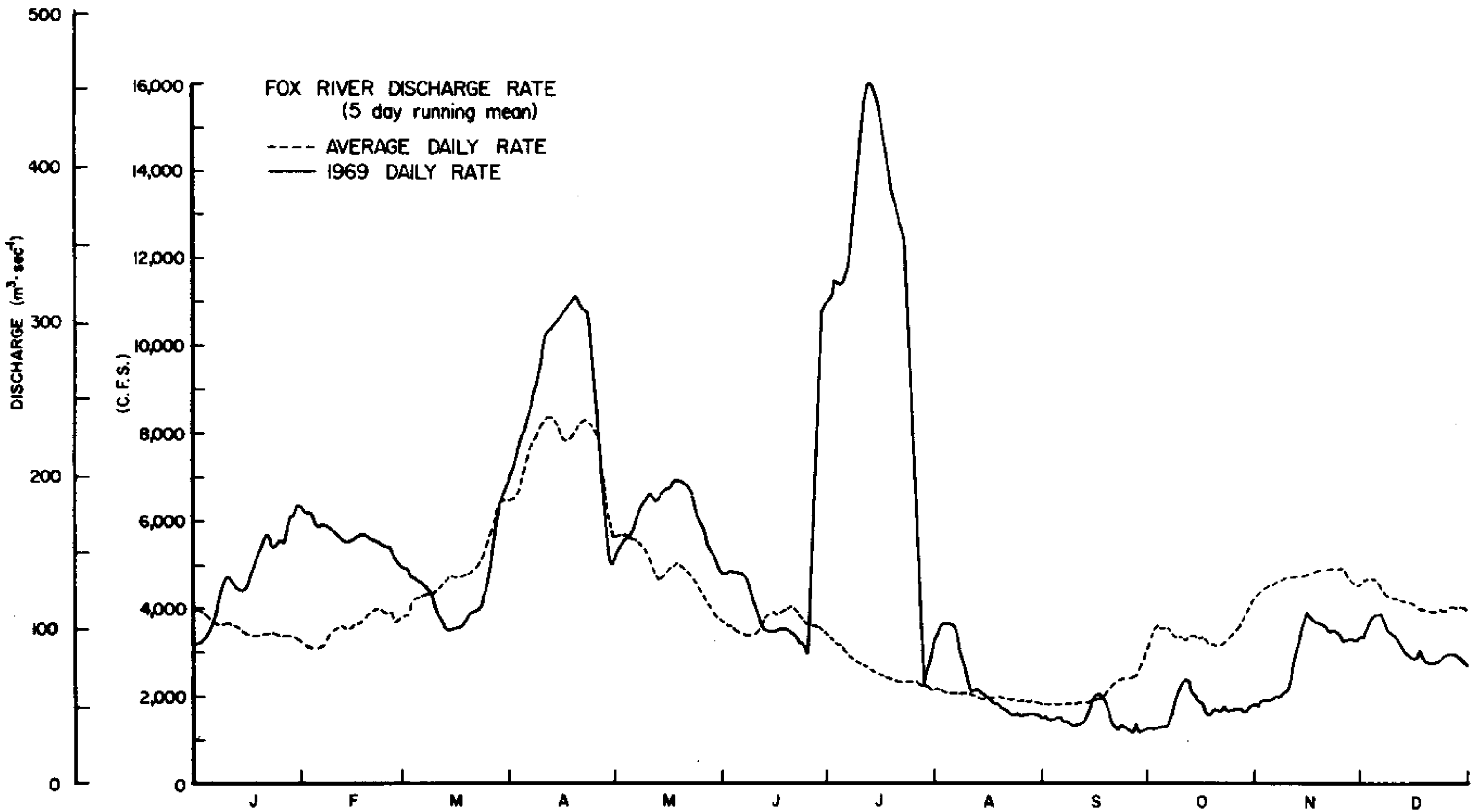

FIG. 26. 7 year mean daily and 1969 daily discharge rates of the Fox River, in c.f.s. and $m^3 \cdot sec^{-1}$.

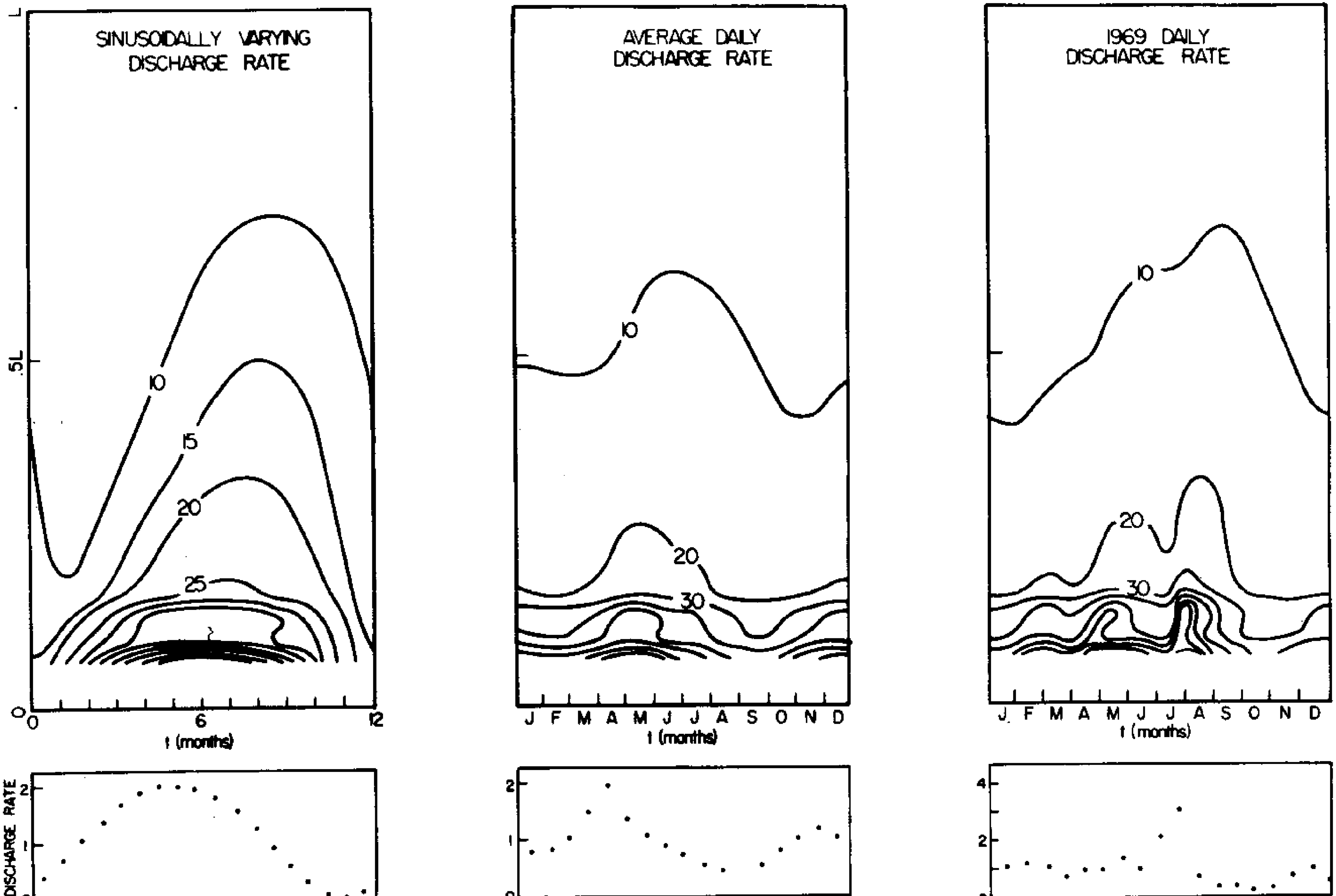

FIG. 27. Numerically predicted concentration (percent volume) of Fox River water in Green Bay as a function of time and of distance from the source. Values of K and A are as shown in Fig. 23c. Variations in A are (a) sinusoidally varying, (b) 7 year mean daily, and (c) 1969 daily discharge rates, shown in Fig. 26.

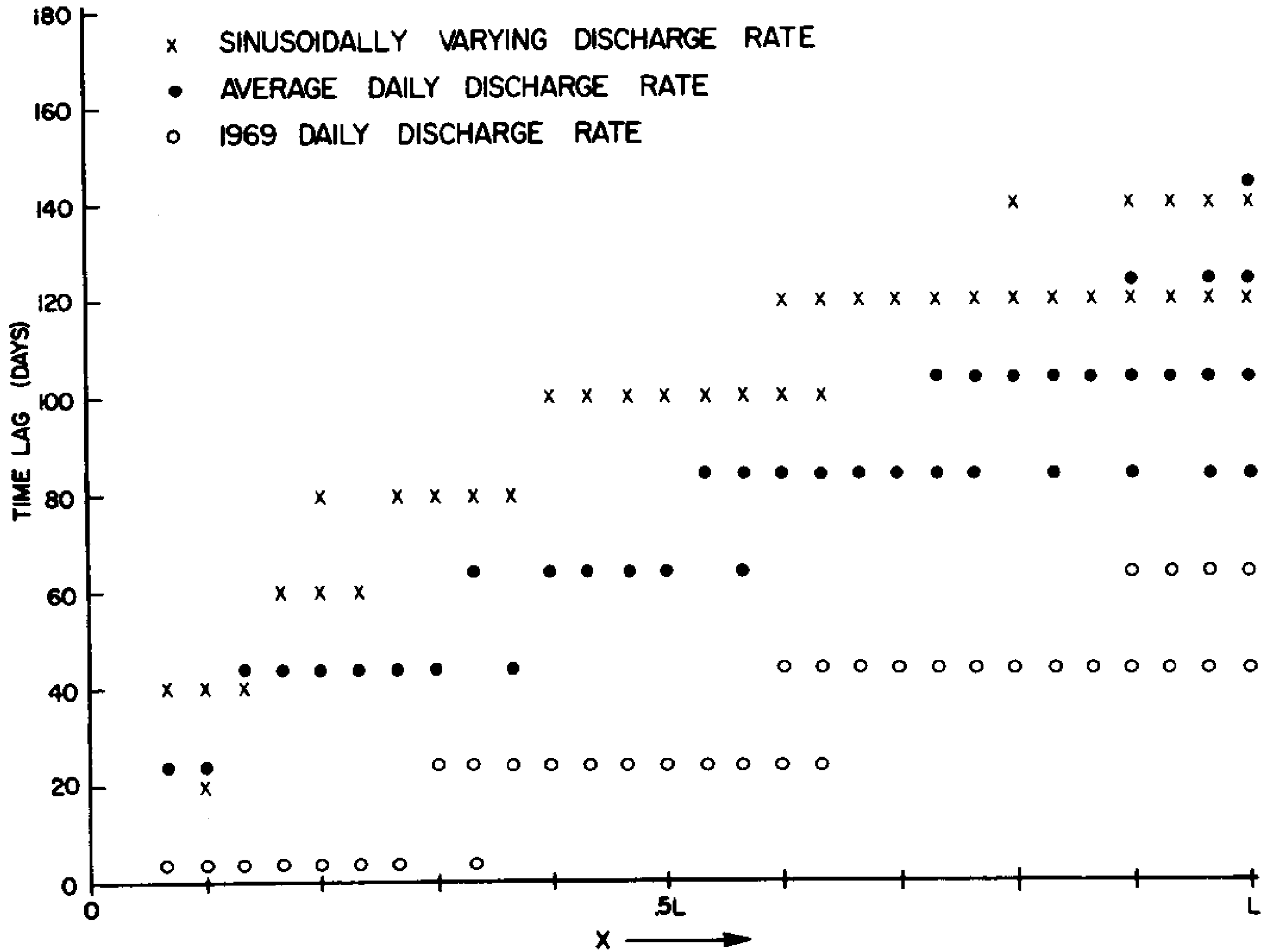

FIG. 28. Time required, as a function of distance from the source, for Green Bay to respond to each of the variations in Fox River discharge rate shown in Fig. 27.

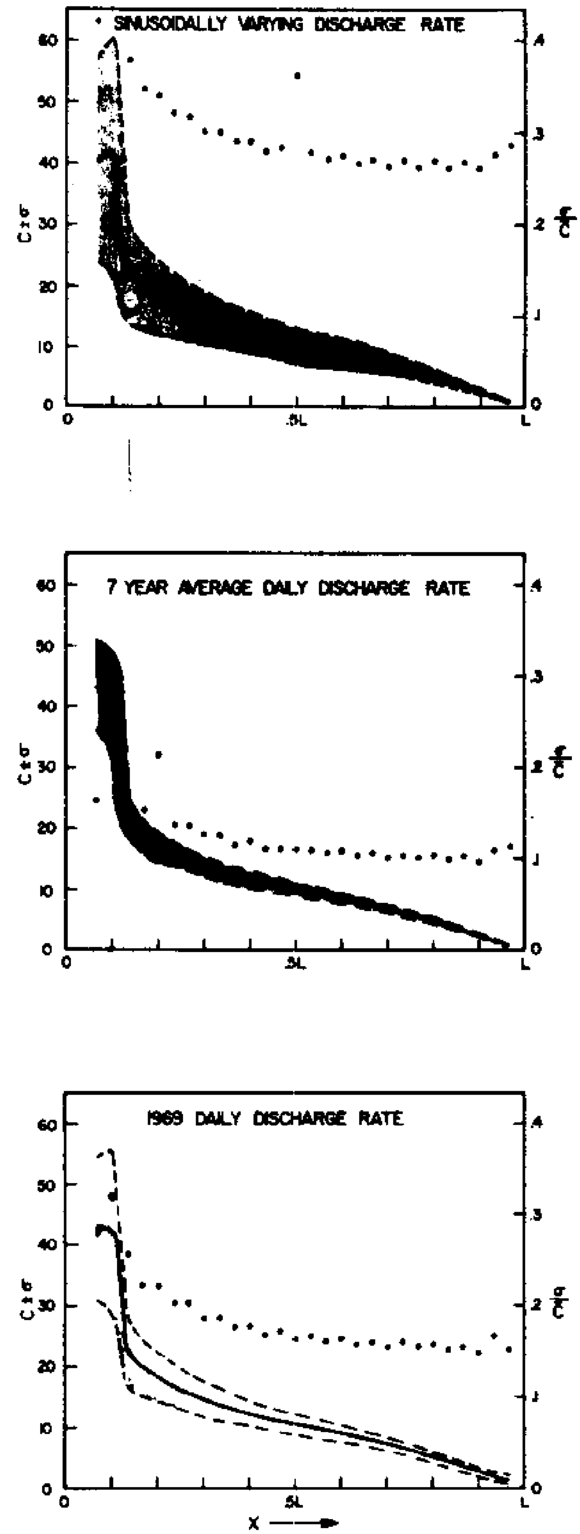

FIG. 29. Numerically predicted annual mean concentration (percent volume) of Fox River water in Green Bay, as a function of distance from the mouth of the River, for the three variations in discharge rate shown in Fig. 27. The dashed lines represent the limits of $C \pm \sigma$ and the dots represent the values of $\sigma/\overline{C}$.

of the concentration field to variations in discharge rate of the river, a three day survey can indeed be considered synoptic.

VII. *DISCUSSION*

A. Summertime Concentration Field

The summertime concentration field of Fox River water in Green Bay is characterized by extremely high concentrations south of Long Tail Point (Fig. 10), with values ranging from 50 to 80 percent Fox River water (Figs. 4, 6, 8). In the vicinity of Long Tail Point the concentration decreases very rapidly, a gradient of 30 to 40%·km^{-1} observed on occasion. Beyond this maximum gradient zone, at a distance of 15 to 20 km from the mouth of the River, concentrations are on the order of 20 to 30 percent. Lakeward the concentrations continue to decrease, but more gradually, with values of a few percent just south of the tip of the Door Peninsula, which separates the Bay from the Lake.

One of the most pronounced characteristics of the concentration field in the vicinity of Long Tail Point is its variability, both in space and in time. To illustrate the temporal variability the means and standard deviations for all concentration values from three stations in the head of the Bay are tabulated below. The three stations are B and C (the locations of which are shown in Fig. 15) and station B1, approximately midway between B and C.

Station	Mean	Standard Deviation
B	65.8	16.6
B1	63.7	22.6
C	35.1	11.0

Exemplifying the spatial variability is the fact that on at least one occasion the concentration very near the mouth of the Fox River was measured to be only 35 percent, while on another occasion an apparent concentration of 88 percent was measured north of Long Tail point.

The extreme variability of the concentration field in this region is due to at least two factors. The turbulent nature of the mixing processes in this region leads to a nearly random distribution. In addition, fluctuations in the composition of the pollutants in the Fox River add to the seeming randomness of the concentration field.

As a consequence of the variability in the head of the Bay, the analyses in Figs. 4, 6, and 8 have been smoothed in order to show average trends and patterns rather than detailed patterns of concentrations.

Likewise, the stated concentration values for south of Long Tail Point of 50 to 80 percent are not meant to be taken as limiting, but rather as being representative.

More realistic representations of the concentration field can be seen in the vertical sections along the shipping lane (Figs. 12 and 13). These figures show a general decrease in concentration away from the mouth of the river, although this tendency reverses itself at times. This is exemplified by the reappearance of the 30 percent isoline in the conductivity based values in figure 12, by the folding of the 70 percent isoline in the same figure, and the 60 percent isoline in the transmissivity based values in figure 13. These reversals in the tendency are again illustrative of the extreme variability of the concentration field in this region.

In some cases the water with lower concentration overlays water with a higher concentration, as in the region between stations B and C in the transmissivity based values in figure 12, and in other cases, as in the region between stations D and F in the conductivity based values for the same profile, the opposite is true.

As stated previously, there usually exists a zone of maximum horizontal gradient in the vicinity of Long Tail Point and approximately coincident with the bar extending from Long Tail Point to Point Sable on the eastern shore of the Bay (figure 30). In some cases this strong gradient was found to be displaced slightly towards the river, and at other times it was displaced lakeward. On several occasions disturbances in this zone, perhaps associated with eddies breaking off were observed. In the mean, however, the position of this maximum gradient zone is very close to the aforementioned bar.

Under conditions of light and/or southerly winds there is a tendency for a tongue of water with relatively high concentration, 30-40 percent River water, to develop extending northward along the east shore of the Bay. While this tendency appears in the analysis of a majority of the surveys made in this study, this is due to a bias in the data in the head of the Bay toward conditions under light and southerly winds. This was due to the limitations of the small craft used in making the measurements, since the rough sea state under northerly winds made taking measurements under those conditions impracticable. On 10 September,

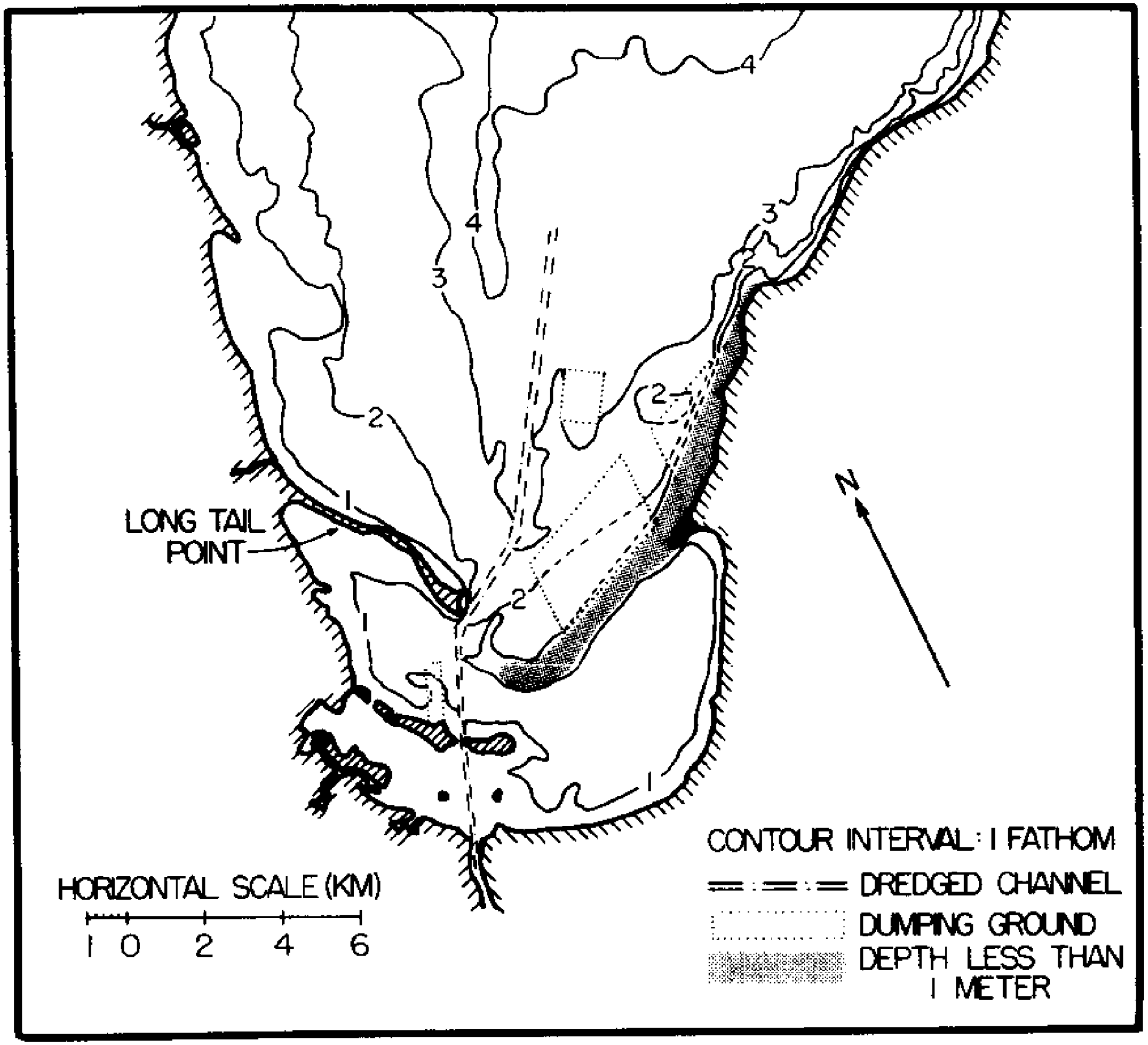

FIG. 30. Bottom topography of head of Green Bay (after U.S. Lake Survey Chart No. 703). Contour interval = 1 fathom.

however, under conditions of moderate northerly winds, this tongue was absent. The vertical structure of several cross sections along the east shore, under conditions of both the absence and presence of the tongue of concentrated river water, shown in figures 14 and 15 shows little vertical variation, as illustrated by the nearly vertical concentration contours. There is a slight tendency for the concentrations to be slightly higher just above the bottom. Since this tendency is more pronounced in the transmissivity based concentration values, it is thought to be a consequence of the sedimentation taking place rather than a case of the tongue spreading out along the bottom.

When the tongue does occur, it loses its marked identity by a distance of 15 to 20 km north of the mouth of the River. A weak cross Bay gradient was sometimes observed, with a tendency for the concentrations of River water to be slightly higher on the east side of the Bay than on the west at cross section I. Since on other occasions the distribution of Fox River water in cross section I is nearly random, however, it cannot be said that the tendency for Fox River water to be more concentrated along the east shore than the west is any persistent summertime feature in central Green Bay.

North of cross section I no persistent cross Bay gradients were observed. On some occasions the water on the east side of the Bay had distinctly lower concentrations than on the west side, as in the case of the transmissivity based values from 20 August (Fig. 5). On other occasions, the opposite was found to be the case, for example in the conductivity based values from 18 September (Fig. 7). In still other cases, no apparent patterns are discernible.

The apparent randomness of the concentration field on a horizontal scale of a few km in the southern portion of the Bay and on the scale of 10 km in the central and northern portion of the Bay serves to validate the treatment of the problem as one of turbulent diffusion, since on a scale approximately equal to the scale of the turbulence elements effecting the diffusion, the distribution expected is a random one. Had more persistent, systematic patterns been observed, it could have been argued that advective processes are dominant over turbulent diffusion processes in the dispersal of Fox River water.

B. Wintertime Concentration Field

From the concentration values shown in Figs. 20-22, it is apparent that of the two conversions which must be applied to the field observed conductivity measurements, namely the reduction to conductivity at 18°C and the application of the summertime conductivity-concentration relationship for the winter case, at least one, if not both, is in error.

This is evidenced by two facts. First, the presence of the low concentration values in the vicinity of the mouth of the Fox River (the maximum apparent concentration observed was 48%) and secondly, the existence of apparent negative concentrations at a distance of less than 25 km from the mouth of the River. Although the absolute values of the concentrations are in error, some conclusions as to the nature of the wintertime concentration field can be drawn from the patterns and gradients shown.

First, that, as during the summer, there exists in the southernmost portion of the Bay, an area occupied by extremely concentrated Fox River water (Fig. 20). That region seems to act as a stilling well, absorbing and smoothing many of the variations in the discharge of the Fox River.

Second, the maximum horizontal gradient observed is slightly less than maximum gradients observed in summer. Nonetheless, a zone of maximum horizontal gradient exists in the vicinity of Long Tail Point, separating the portion of the head of the Bay south of the Point from the rest of the lower Bay. This zone of maximum gradient is displaced farther away from the mouth of the River in the winter than in the summer.

A tongue of Fox River water more clearly defined during the winter than the summer can be observed extending northward down the Bay at the surface (i.e. immediately below the ice) on the right (east) side for a distance of approximately 25 km. This case of river water traveling down the right hand shore is similar to that stated by Cameron and Pritchard (1963) as being the case in a tideless, frictionless estuary.

From the concentration cross sections (Fig. 22) it can be seen that in the southernmost two cross sections vertical variations in the concentration field are slight. Beyond that point at which the River plume can no longer be observed at the surface, it is clearly identifiable below the surface. That is to say the Fox River water retains its identity, but sinks below the surface.

The fact that a distinct plume of Fox River water is identifiable much farther north in the winter than in the summer has at least two implications: First that since lateral and vertical mixing take place much slower in winter than in summer, the intensity of the turbulence effecting the dispersal of Fox River water is significantly lower in winter due to the shielding effect of the ice cover, decreasing both direct wind generation of currents as well as seiche generation. Second that the pronounced systematic lateral and vertical variations in the concentration field in winter render the one dimensional analysis used for the summertime case invalid for the wintertime case.

These two considerations imply that the summer and winter concentration fields and the physical processes affecting them come under two separate and distinct regimes.

C. Diffusivities--Observed vs. Predicted

The agreement between the observed effective diffusivities and those independently predicted as being attributable to the seiche in Green Bay is remarkable. In diffusivities calculated from both the smoothed (Fig. 17) and unsmoothed (Fig. 18) concentration fields, diffusivities closest to the mouth of the River have a value of approximately 0.2×10^6 $cm^2 \cdot sec^{-1}$. The values of the diffusivities based on the smoothed concentration field (Fig. 17) remain approximately the same to a distance of approximately 5 km from the mouth of the River. Continuing out into the Bay, the values increase, quite steadily, to a value of approximately 1.0×10^6 $cm^2 \cdot sec^{-1}$ at a distance of 12 to 14 km from the mouth of the River.

The diffusivities based on the unsmoothed concentration field (Fig. 18) show a more gradual increase, to a value of approximately 0.25×10^6 at a distance of 20 km from the mouth of the River. By a distance of 20 km from the mouth of the River the values show a rapid increase to a value of approximately 0.8×10^6 $cm^2 \cdot sec^{-1}$. In the case of the predicted, seiche associated diffusivities (Fig. 19) this abrupt increase is more marked, and takes place closer to the head of the Bay.

Both the observed and predicted diffusivities continue to increase lakeward, and both show a maximum of 2.0 to 3.0 $\times 10^6$ $cm^2 \cdot sec^{-1}$ at a distance of approximately 40 km from the River. Beyond this point, both the observed and predicted diffusivities decrease, to a value of approximately 0.8×10^6 $cm^2 \cdot sec^{-1}$ at a distance of 72 km for the predicted values, and approximately 0.7×10^6 at a distance of 90 km for the observed values.

The most significant discrepancy between the observed and predicted diffusivity values is the distance from the mouth of the River at which the abrupt increase in diffusivity occurs.

The most probable reason for this discrepancy is that the grid on which the mathematical model is based is too coarse to resolve the effects of Long Tail Point and the bar which seems to be its extension (Fig. 30). With the exception of a small area near the dredged shipping channel, depths of the water over this bar are less than one meter. It is postulated that the Point and bar are effectively the end of the Bay for these large scale motions (seiches and larger). Therefore, considering the mouth of the River as the outfall site for the effluent is an error, and the effective discharge of Fox River water into Green Bay takes place in the vicinity of Long Tail Point and the bar which is its extension.

Subtracting 6 km (the distance from the mouth of the River to Long Tail Point) from the distance in figure 6, or displacing the origin by that amount brings the two sets of data into good agreement.

D. <u>Sedimentation Rate</u>

A budget approach is inadequate for evaluating the sedimentation rate in the Bay. In an attempt to establish limits for the rate at which sedimentation is taking place, rates between 10^{-1} km^{-1} and 10^{-4} km^{-1} were applied to the observed apparent concentration field based on transmissivity and equation (9) was applied to the resulting concentration values. Values of the ratio (diffusivity calculated with an assumed sedimentation rate) : (diffusivity calculated assuming no sedimentation) are plotted as a function of the assumed sedimentation rate in figure 31. With the exception of one data point, for values of the sedimentation rate of less than 10^{-2} km^{-1}, the diffusivities calculated by including assumed sedimentation effects are in agreement with diffusivities calculated without sedimentation within a factor of two. For larger values of the sedimentation rate the discrepancy is greater, by as much as a factor greater than 10^3 (this point is not shown in Fig. 31, nor are several points at which the given ratio has a negative value for large values of the sedimentation rate).

Since diffusivities calculated by assuming no sedimentation do not differ significantly from diffusivities calculated from the conductivity based concentration field, in

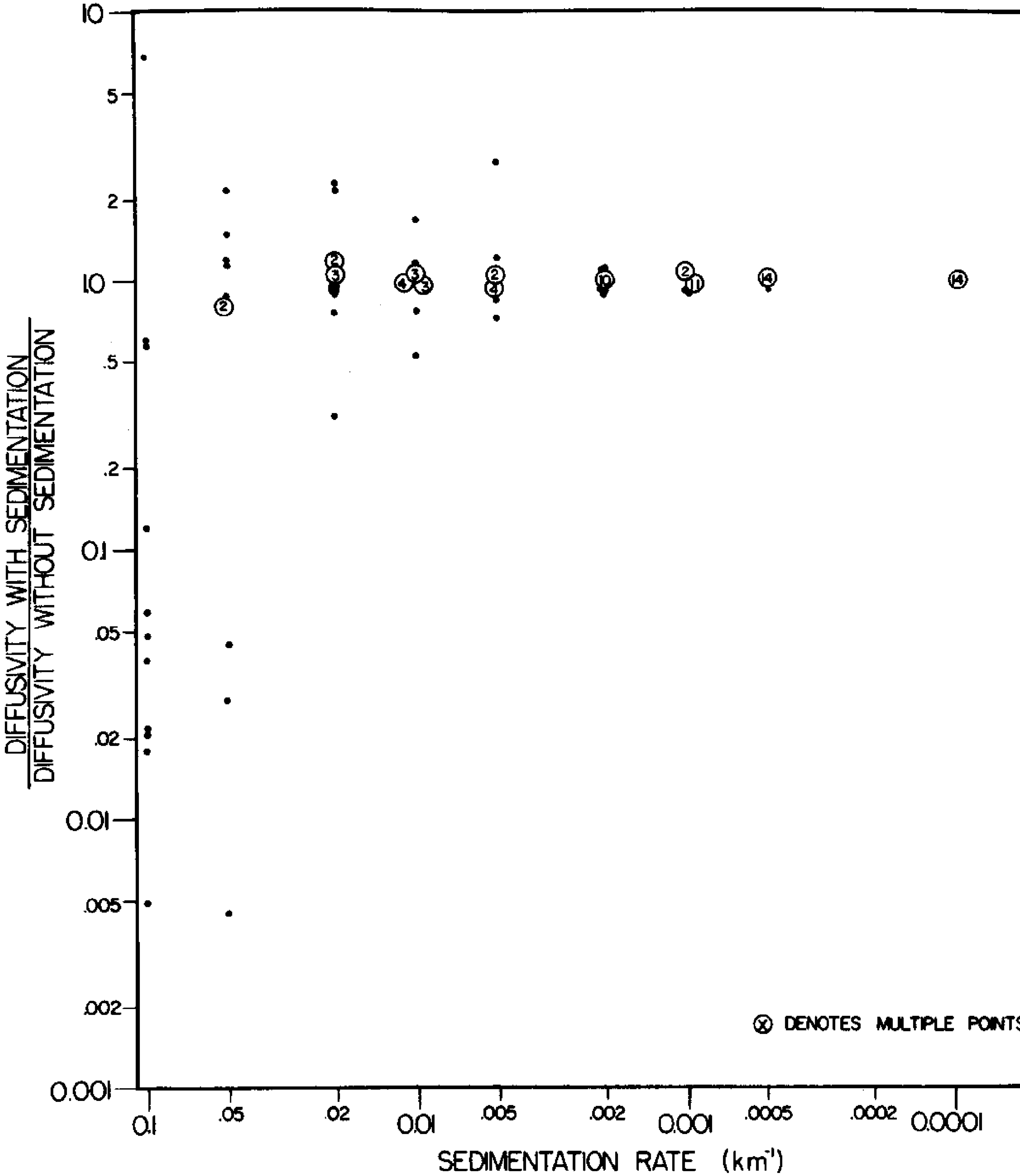

FIG. 31. Variation of the ratio (diffusivity with sedimentation correction): (diffusivity assuming no sedimentation) as a function of assumed sedimentation rate.

which a reasonable degree of confidence can be placed, it is concluded that the sedimentation rate in Green Bay is no greater than 10^{-2} km^{-1}.

E. Exchange Between Bay and Lake

This study has shown that the primary mechanism for the transport of Fox River water from the mouth of the River to where the Bay connects to Lake Michigan is the seiche of the Bay, with advective processes being the second most significant process, far less important than seiche effects. The question remains as to the nature of the processes governing the exchange of water between Lake and Bay. Unfortunately, no data are available which would make possible a direct assessment of the roles of those processes. Nonetheless, some statement can be made about them.

The numerical prediction model developed in this study has shown that by assuming that all of the Fox River water is transported through the Bay, a realistic representation of the existing concentration field of Fox River water in Green Bay can be derived. Therefore, the assumption of through the Bay transport is realistic. If there were not considerable exchange of water between Bay and Lake, the build up of River water in the large area of the Bay immediately adjacent to the Lake would be significantly greater than it is. Therefore, we conclude that Fox River water is being transported into Lake Michigan.

Since the advective effect of the lakeward movement of River water is proportional to the concentration of River water at any point, this process is responsible for the transport of a negligible percentage of the total River water passing through the Bay.

Since it can be concluded that River water is being transported from Green Bay into Lake Michigan, and it has been shown that advective effects are negligibly small, it is suggested that the primary processes responsible for the exchange of water between the Bay and the Lake are two: first, seiches, particularly that seiche with a node located in Lake Michigan just outside of the mouth of Green Bay. This position of the node has been located by Mortimer (pers. comm.), and the effectiveness of this process has been demonstrated by Bryson and Stearns (1959). Second, diffusivities were calculated for this region by the same method as those in the central portion of the Bay, except in this region the y component of the velocities as predicted by the numerical model were used. The eight values obtained are, from south to north: 7.47, 1.19, 0.95, 0.64, 0.37, 0.45, and 0.42 x 10^6 $cm^2 \cdot sec^{-1}$, generally slightly lower than the diffusivities in the central portion of the Bay, but nonetheless, of a magnitude sufficiently large that the contribution of this process to the total exchange of water is significant.

The second process believed to play a significant role in water exchange is also of an oscillating nature, however the period of the oscillation is much longer. The reversals of surface water flow in direct response to changes in wind direction associated with the passage of atmospheric weather systems, and the associated upward and downward movement of the thermocline has been found by Ragotzkie, et al. (1969) to have a flushing effectiveness of the same order of magnitude as the seiche with a nodal line across the mouth of Chequamegon Bay, Lake Superior. They have suggested that that effect is typical of other semi-enclosed bays of the Great Lakes. It is extremely likely that effects of this phenomenon are also significant in the case of Green Bay, since it is known that currents through the openings at the mouth of the Bay are swift and variable, as is attested to by the name of one of those openings which has claimed the lives of many mariners--Porte Des Mortes Passage.

F. Mixing Processes in Bays

The primary flushing mechanisms in Bays of the Great Lakes are periodic motions. In Green Bay the process by which most of the transporting of Fox River water is accomplished is the surface seiche acting within the Bay, yielding maximum diffusivity values in mid Bay, at and near the nodal line of the uninodal sieche. In the case of Green Bay, due to the resonance between the period of the lunar semidiurnal tide and the free period of the Bay, tidal effects do play a role. The vigorous horizontal water movements due to seiching within Chequamegon Bay have also been shown to play a significant role in the mixing of water within that Bay.

The exchange of water between a bay and its parent basin is also largely due to seiches. In the case of bays of three different sizes for which quantitative data are available, namely Chequamegon Bay of Lake Superior, South Bay of Lake Huron, and Green Bay of Lake Michigan, a nodal line is known to exist near the mouth of the Bay, and that seiche associated water movement plays a significant role in water exchange.

Another periodic oscillation, the periodic rise and fall of the thermocline due to wind changes or internal seiches and the associated horizontal water movements into and out of the Bay is known to contribute significant water exchange in the case of Chequamegon Bay and is postulated to play a similar role in Green Bay.

In the nearshore regions of a bay, the effectiveness of seiche induced mixing is greatly reduced as horizontal water movements tend to zero due to boundary constraints. This is evidenced by the sharp decrease in values of the effective diffusivities based on both the observed concentration field and in the predicted diffusivities, however, the horizontal range of this "end effect" is extended lakeward due to the presence of islands, bars, and peninsulas which are not taken into consideration by the numerical model. Although the presence of those obstructions does expand the spatial range of the "end effect", in the case of an effluent introduced at or near the head of a bay, i.e. at the antinode, advective effects are the dominant process in pollutant dispersal due to the higher concentrations near the outfall.

Advective effects, however, have been shown in this study to be of relatively little import during the summer in the transport of effluents through the central portions of a system such as Green Bay, since persistent paterns were not found to exist in the central and northern portion of the Bay. Due to the low concentrations, it can be seen that in mid Bay, advective transport contributes only a few percent of the total transport.

In the winter, as evidenced by the presence of a well defined plume extending through at least one third of the length of the Bay, advective processes, rather than turbulent phenomena, could be the primary transport mechanism.

Since advective transport is nearly negligible during the ice free seasons, residence times for basins based solely on those effects are in error. Results of the predictive model developed for this study, which includes the effects of diffusion, show that the time for the Bay to reach equilibrium following a change in the source strength of the Fox River is on the order of 400 days.

It is believed that expressions might be derived which would give an _a priori_ method of determining the relative roles in mixing within a bay and between bay and lake of advective effects, seiching, and wind induced current variations. It is suggested that factors to be included in such an expression might be: the period of the various nodal seiches within the bay, the ratio of the cross sectional area of the mouth of the bay to either the volume or the surface area of the bay, and the advective residence time of the bay.

G. Bays Versus Estuaries

In the case of an ocean connected or tidal estuary, the primary flushing mechanism is a periodic oscillation, the tide. In the case of fresh water bays, for example bays of the Great Lakes, the primary flushing mechanisms are also periodic motions. While there are therefore some similarities, there are significant differences between the two basins.

The primary difference stems from the differing wave lengths of the oscillations in the two systems. In the case of tidal estuaries the wavelength of the oscillation is on the order of half the diameter of the ocean and the node of the oscillation is well out to sea. In the case of fresh water bays, since the oscillation is induced by wind stress on the basin itself, the wavelength is on the order of the length of the basin, and at least one node will exist within the bay.

Since the maximum horizontal water movement associated with both of these oscillations is at and near the node, in the case of a tidal estuary the most efficient mixing takes place beyond the mouth, with the effects decreasing toward the head. In the case of the fresh water bay the most efficient mixing takes place near mid bay, with other mechanisms contributing significantly to the water exchange between bay and lake.

VIII. CONCLUSIONS

1. Green Bay, south of Long Tail Point, is severely polluted by Fox River water, sometimes in the form of a tongue of polluted water extending down the east shore of the Bay for approximately 20 km.

2. North of Long Tail Point the concentration of Fox River water decreases rapidly, a value of 25 percent seldom being exceeded 25 km from the mouth of the River. Lakeward, concentrations are very low and the effect of the Fox River is small.

3. Diffusivities in the southernmost 15 km of the Bay are approximately $0.25 \times 10^6\ cm^2 \cdot sec^{-1}$. Beyond this distance an abrupt increase to approximately $1.00 \times 10^6\ cm^2 \cdot sec^{-1}$ occurs, followed by a gradual increase to approximately $3.00 \times 10^6\ cm^2 \cdot sec^{-1}$ at 30 km. Lakeward the diffusivities decrease to approximately $0.7 \times 10^6\ cm^2 \cdot sec^{-1}$ at 95 km.

4. Diffusivity values of 0.2 to $0.3 \times 10^6\ cm^2 \cdot sec^{-1}$ in the vicinity of Long Tail Point suggest a barrier to horizontal mixing in this area, while northward the larger diffusivities imply strong mixing and rapid transit of Fox River water.

5. Diffusivities calculated on the basis of a mathematical model of seiche-induced and wind driven circulation in the Bay agree with those calculated from the concentration gradients of Fox River water. This pattern suggests that high diffusivity values in the central portion of the Bay are primarily due to seiche activity.

6. Maximum horizontal gradients in the vicinity of Long Tail Point, and the down Bay displacement of the transition to strong mixing implies that the effective discharge for the effluent of Fox River water into Green Bay takes place in the vicinity of Long Tail Point and its connected bar rather than at the mouth of the River.

7. The flux conserving numerical method of solving the equation $\frac{\partial C}{\partial t} = \frac{\partial}{\partial x} (Q \cdot C - K \cdot A . \frac{\partial C}{\partial x})$ generates a realistic representation of the one dimensional concentration field of Fox River water in Green Bay, implying that a significant portion of the pollutants entering Green Bay through the Fox River are being transported into Lake Michigan.

8. The time required for the central portion of the Bay to respond to changes in Fox River discharge rate is approximately 50 days, assuming a sinusoidal variation with a period of one year.

9. Assuming that the dispersive processes governing the summertime concentration field are in effect all year long, the time required for the central portion of the Bay to respond to changes in pollutant concentration, without changing the discharge of the Fox River, is approximately 400 days.

10. During the winter ice shields the water from wind effects and greatly decreases the effectiveness of turbulence as a dispersive agent, particularly in the lateral and vertical dimensions. Consequently the concentration fields of Fox River water in Green Bay have different characteristics in wintertime than during the summer, insomuch as a well defined plume of Fox River water can be identified through at least one third of the length of the Bay under the ice.

APPENDIX

The following is a listing of the FORTRAN program used to generate the solutions to the diffusion equation discussed in chapter VI. All variables are in non-dimensional form, i.e. units of distance from the mouth of the River are expressed as X/L where L is the length of the Bay. The definitions of the variables used are as follows:

DT = Time Increment

DX = Space Increment

E = Epsilon (To avoid zero area at head)

N = Number of grid points

X = Print out frequency

BE = Diffusivity

A = Area

C = Concentration

Q = Source Strength

F = Flux of River Water

```
      DIMENSION C(50),F(50),B(50),A(50),D(50),IC(50),BK(50)
      READ 90,DT,DX,E,X
      READ 91,N
      NM = N-1
      DO 10 I = 1,N
      TEST(I) = 0
      TN = I-1
      RN = N-1
      A(I) = ((2.*TN)+E)/RN
      READ 92,BE(I)
      B(I) = BE(I)*A(I)
10    C(I) = 0
      KOUNT = 0
20    KOUNT = KOUNT+1
      Q = 1
      S = Q
      F(1) = S
      DO 30 J = 2,NM
30    F(J) = -(((B(J)/(2.*DX))*(C(J+1)-C(J-1)))-Q*C(J))
      F(N) = -((B(N)/(2.*DX))*(3.*C(N)-4.*C(N-1)+C(N-2)))
      C(1) = C(1)-(DT/A(I)*2.*DX))*(4.*F(2)-F(3)-3.*F(1))
      DO 50 I = 2,NM
50    C(I) = C(I)-(DT/(A(I)*2.*DX))*(F(I+1)-F(I-1))
      OUNT = KOUNT
      UNT = OUNT/X
      JUNT = UNT
      ONT = JUNT
      IF (ONT-UNT)20,55,20
55    DO 60 J = 1,N
      D(J) = 1000.*C(J)
60    IC(J) = D(J)
      MM = N-2
      IDAYS = KOUNT/10
      PRINT 103,IDAYS,Q,S
      PRINT 101,(IC(J),J=1,MM)
65    ITEST = 0
```

```
      DO 74 J = 1,N
      IF (.05-ABS(TEST(J)-D(J)))72,72,74
72    ITEST = 1
74    CONTINUE
      IF (ITEST)76,89,76
76    DO 78 J = 1,N
78    TEST(J) = D(J)
      GO TO 20
89    CONTINUE
90    FORMAT (4F7.4)
91    FORMAT (I4)
92    FORMAT (F7.4)
101   FORMAT (29(1X,I3))
103   FORMAT (1X,I6,2(3X,F8.4))
      END
```

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