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WHAT DOES THE "PROBABILITY OF PRECIPITATION" MEAN?

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## Introduction

Just what is meant when the Probability of Precipitation (PoP) is mentioned in the forecast?  In a 1968 paper, a mathematician at the University of Miami noted that:

> "There are quite a number of reasonable conjectures which can be made as to what a single-number precipitation probability might mean.  For example, it might mean:  (a) the probability of the event that some rain will fall somewhere in the forecast area sometime during the time period covered by the forecast; (b) the probability of the event that general rain will cover all of the area; (c)  the fraction of the forecast area which will receive rain in the forecast period; (d) the probability that a specific point in the forecast area will receive more than a trace of rain sometime during the forecast period.
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> Preliminary inquiries directed by the author to colleagues knowledgeable in probability and statistical theory yielded a diversity of opinions as to what the U.S.  Weather Bureau precipitation probabilities mean, but "nothing" was all too frequently the answer.  Surprisingly, in a small sample of atmospheric scientists and meteorologists, there was no consensus.  For example, a distinguished weather research scientist told the author he thought that it was the probability of the event (a)."

Actually, the correct answer is none of the above.  According to WSOM Chapter C-11, paragraph 8.3.1.b., the PoP is:

> "The likelihood of occurrence (expressed as a percent) of a precipitation event at any given point in the forecast area.  The time period to which the PoP applies must be

clearly stated (or unambiguously inferred from the forecast wording) since, without this, a numerical PoP value is meaningless."

This manual definition has some very definite implications as to the use and interpretation of the PoP by both the forecaster and the user.

## The Mathematics

To fully explore the concept of PoP, consider a network of N rain gauges that completely sample an area. At each gauge, we can define a variable, $R_i$, which has a value of one if measurable precipitation occurs during a given period ($R_i=1$ if precipitation occurs) and a value of zero if it does not occur ($R_i=0$ if no precipitation). For each gauge, the expected value of $R_i$, $E[R_i]$, is the probability that $R_i$ equals one, $Pr\{R_i=1\}$. It is also the point probability of precipitation, $p_i$, at gauge i. That is

$$p_i = E[R_i] = Pr\{R_i=1\}. \tag{1}$$

From the forecasters point of view $E[R_i]$ can be thought of as being the average value which would be observed after a large number of storm systems with exactly the same characteristics affected the official observation point. That is, with a 50% point probability of precipitation one is saying that in their judgement if the same weather system were to occur 100 times, it would produce measurable precipitation at that point 50 times.

Most of the time, however, point precipitation probabilities are summarized into one value (an average point probability) which represents the entire forecast area (not just one point). For example, forecasters usually don't try to assign different probabilities to different sections of a zone or metropolitan area, but use one value to represent the entire region. With this in mind, a single value to represent the area can be determined fairly simply. The average point probability for any given area can be expressed as:

$$\overline{P} = (1/N) \sum_{i=1}^{N} p_i = (1/N) \sum_{i=1}^{N} E[R_i], \tag{2}$$

where N is the number of gauges in the area. Obviously, equations (1) and (2) are not the same.

The interpretation of the average point probability isn't as straightforward as might be expected. The average point probability or PoP can be determined in two different ways, each of which is valid. The PoP can be derived by (1) forecasting a different probability for each point in the area, summing the values, dividing by N or (2) assigning the same probability value to all points in the forecast area. For example, three points in the forecast area with probability values of 10%, 30%, and 50%, give an average probability of (10 + 30 + 50)/3 = 30%. This PoP would also be obtained if the point probability was 30% at all the sites in the area.

The WSOM definitively states that the probability of precipitation, PoP, given in a forecast represents the entire area. Thus, it is defined according to equation (2):

$$PoP = \overline{P}. \tag{3}$$

Because of this, some additional comments can be made. The fraction of the area covered by measurable precipitation, the areal coverage (a), is simply the sum of the $R_i$'s over the area divided by the number of gauges

$$a = (1/N) \sum_{i=1}^{N} R_i. \tag{4}$$

The expected value of the areal coverage, A, is given by

$$A = E[a] = E[(1/N) \sum_{i=1}^{N} R_i] = (1/N) \sum_{i=1}^{N} E[R_i]. \tag{5}$$

However, according to equation (2) <u>this is the PoP!</u> Thus

$$A = PoP. \tag{6}$$

Therefore, because of the WSOM definition, <u>the PoP is equal to the expected areal coverage of the precipitation.</u>

## Implications

The theory shows that the PoP is the average point probability for any given area and equals the <u>expected</u> areal coverage of measurable precipitation. Just what does this mean? A typical response to this question is, "surely, it is not saying that 20% of the area will receive rain on a 20% PoP. If this were true, would not our whole concept of convective forecasting would be wrong?"

However, this is exactly what it is saying! The key word in the definition is <u>expected</u>. It is not saying that the areal coverage equals the PoP, but that the <u>expected areal coverage</u> equals the PoP. Operationally, this means that for a large enough set of cases with a particular PoP, the average areal coverage will equal the PoP. When all of the 20% forecasts made in one season are considered, perfect PoP forecasting would require that an average of 20% of the area got wet.

This expected areal coverage is offered as an alternative to the often (but incorrectly) stated explanation that the PoP represents the frequency that the existing weather pattern will produce measurable precipitation at a specific point (equation 1). It is felt that discussing the PoP in terms of expected areal coverage is more meaningful (useful) to the serious forecast user than an explanation based upon the repeatability of the weather pattern.

Also, the forecaster should realize that the PoP is not the same as the probability that precipitation will occur somewhere in the forecast area during

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the forecast period. This quantity is called the area probability, $p_a$, and can be expressed as the probability that at least one of the N gauges will receive rain. That is

$$p_a = \Pr \left\{ \sum_{i=1}^{N} R_i > 0 \right\}. \tag{7}$$

Comparing equation (7) to equations (1) and (2), we see that

$$p_a \geq p_i \tag{8}$$

and

$$p_a \geq PoP \tag{9}$$

both hold. The area probability is greater than or equal to both the point probability and the forecast PoP.

The concept of area probability can be used to give another consistent definition of PoP

$$PoP = p_a * a_c, \tag{10}$$

where $a_c$ is the conditional areal coverage or the percent of the area which will receive measurable precipitation if precipitation does occur.

This definition emphasis that the probability of precipitation is the product of the probability that the synoptic situation will cause precipitation somewhere within the area ($p_a$) and the anticipated areal coverage of the precipitation if it occurs ($a_c$). For conditions of equal synoptic uncertainty, an event which will cause widespread precipitation demands a higher PoP than one which will generate a few widely scattered showers. Thus a summertime 20% climatic potential of thundershowers would yield a higher PoP if a mesoscale convective complex (MCC) is expected than if only one-or-two isolated thundershowers are anticipated.

## Summary

The PoP can be defined in several different ways, each of which has definite implications for both the forecaster and user. The PoP, in the truest sense, is a point probability and reflects the chances of receiving precipitation at a specific point given that a large number of storm systems with exactly the same characteristics would affect that location. However, the PoP as defined and used by the NWS is an average probability whereby the same PoP value is assigned to each location in the forecast area.

Furthermore, given this WSOM definition and usage, the NWS PoP value also represents the expected areal coverage of the precipitation. Thus, the 30% chance of rain in the forecast also means the forecaster expects (considering a

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large number of forecasts) 30% of the forecast area will receive rain. The forecaster needs to be aware of these implications when making the PoP forecast.

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## References

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