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CENTRAL REGION TECHNICAL ATTACHMENT 86-3

WHY THE BRIER SCORE IS A "PROPER" SCORING SYSTEM

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The question was once asked of me, "Why square anything?", made in reference to the Brier Score (BS) used in Probability of Precipitation forecast verification. There is a fairly simple way of showing how the Brier Score works and why it has been used all these years. The following development is similar to that given in the explanation of why the BS is a "proper" scoring system according to Murphy and Epstein (1967).

The definition of the Brier Score is

$$BS = \frac{1}{N} \sum_{i=1}^{N} (f_i - o_i)^2, \tag{1}$$

where f_i is the forecast PoP (expressed as a decimal fraction), o_i is the observed event (0 for no precipitation and 1 for precipitation), and N is the total number of forecasts. Rewriting (1) over 12 classes of allowable f values (.00, .05, .10, .20, ... , 1.00) we have

$$BS = \frac{1}{N} \sum_{k=1}^{12} \sum_{j=1}^{M_k} (f_{jk} - o_{jk})^2, \tag{2}$$

where M_k is the number of forecasts in the kth class.

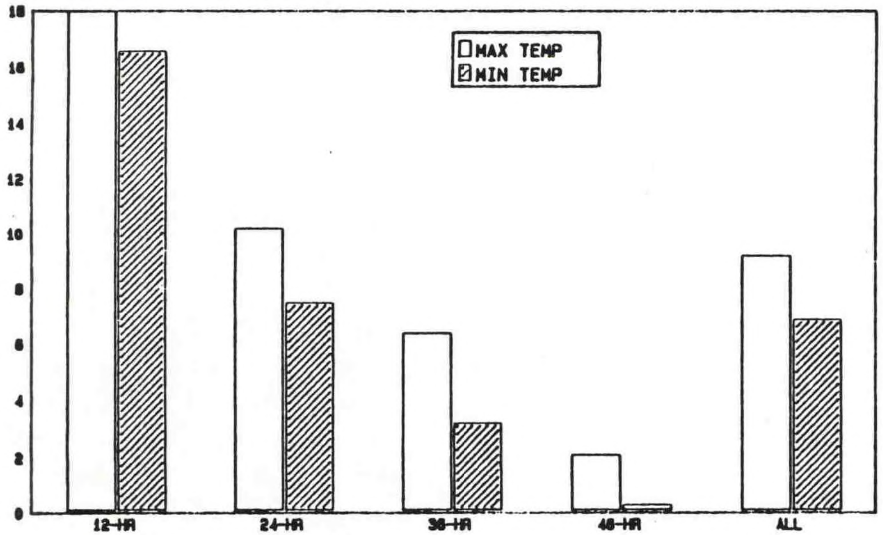
Expanding (2) we get

$$BS = \frac{1}{N} \sum_{k=1}^{12} \sum_{j=1}^{M_k} (f_{jk}^2 - 2\, o_{jk}\, f_{jk} + o_{jk}^2)\ . \tag{3}$$

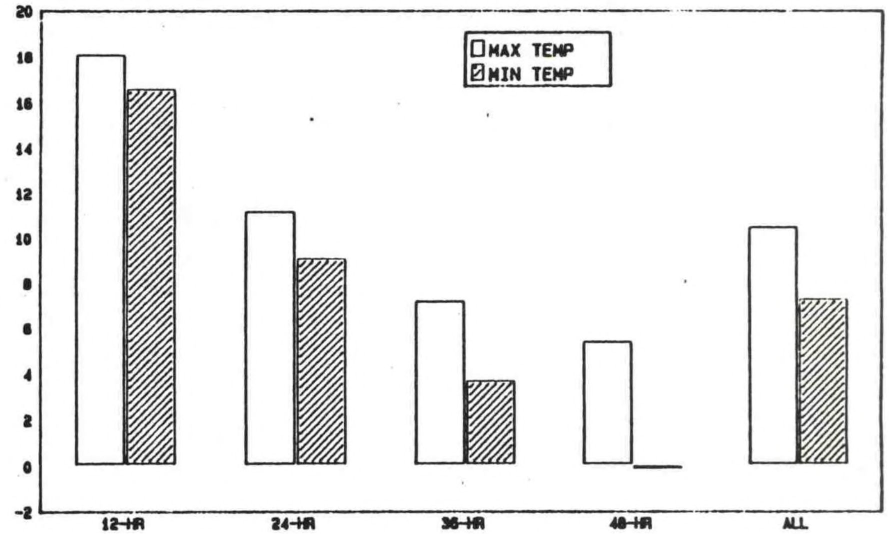
Now

$$\sum_{j=1}^{M_k} f_{jk}^2 = M_k\, f_k^2\ ,$$

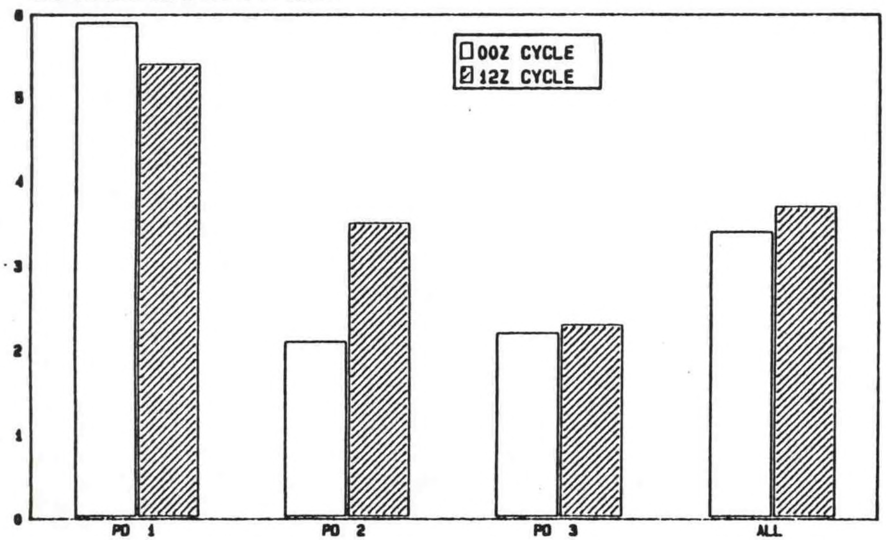
NWS TEMPERATURE VERIFICATION OCT 1-DEC 31, 1985
MAE MOS-LCL WSFO %IMP OVER MOS



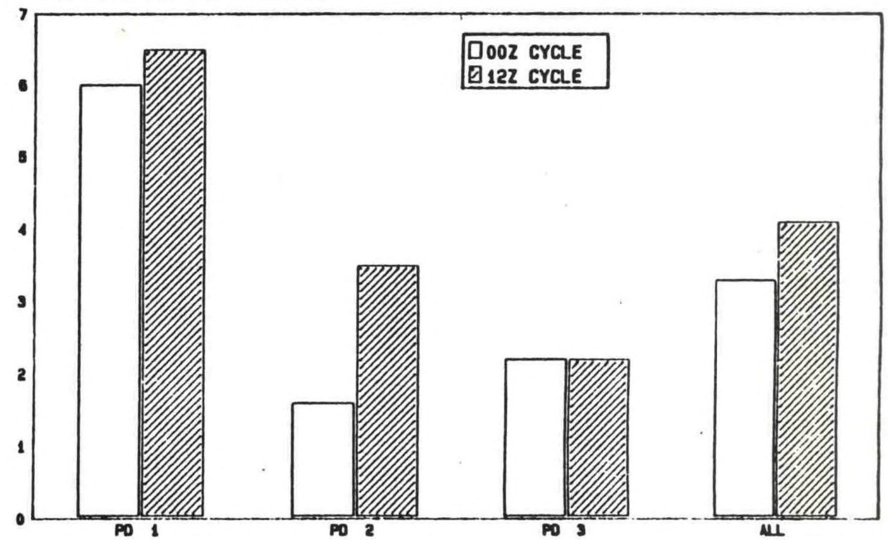
CENTRAL REGION TEMPERATURE VERIFICATION OCT 1-DEC 31, 1985
MAE MOS-LCL WSFO %IMP OVER MOS



NWS PRECIPITATION VERIFICATION OCT 1-DEC 31, 1985
WSFO-MOS BRIERSCORE %IMP OVER CLIMAT



CENTRAL REGION PRECIPITATION VERIFICATION OCT 1-DEC 31, 1985
WSFO-MOS BRIERSCORE %IMP OVER CLIMAT



since f_j is constant for any given k, and

$$\sum_{j=1}^{M_k} o_{jk} f_{jk} = R_k \cdot f_k ,$$

where R_k is the number of times precipitation occurred when the kth class was forecast (the precipitation frequency). Likewise

$$\sum_{j=1}^{M_k} o_{jk}^2 = R_k .$$

So

$$BS = \frac{1}{N} \sum_{k=1}^{12} [M_k f_k^2 + R_k (1-2f_k)]. \quad (4)$$

To get the minimum value of BS, we take now one class of f and partially differentiate BS with respect to f_k,

$$\frac{\partial(BS)}{\partial f_k} = 2 f_k M_k - 2 R_k , \quad (5)$$

and set it equal to zero giving

$$f_k = \frac{R_k}{M_k} . \quad (6)$$

This says that for a class of forecasts where the PoP is a constant, the best score will be obtained when the forecast PoP is equal to the observed relative frequency of precipitation for that class. Thus, the Brier Score encourages the forecaster to make forecasts equal to the observed relative frequency, which is the goal of probability forecasting.

It should be emphasized that this does not mean that the forecaster should try to tinker with the score by putting "sure things" in a forecast class just to make the relative frequency equal to the forecast probability in that class. The only thing the forecaster really has command of is the forecast probability, and proper use will, over the long term, give the best verification score.

Reference:

Murphy, A.H., and E.S. Epstein, 1967: A note on probability forecasting and "Hedging". <u>J. of Appl. Meteor.</u>, <u>6</u>, 1002-1004.