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F. A. Gifford
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F. A. Gifford
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S. R. Hanna
Air Resources
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edited by

Kenneth E. Noll
Professor, Environmental Engineering
Illinois Institute of Technology, Chicago

Wayne T. Davis
Assistant Professor, Environmental Engineering
University of Tennessee, Knoxville

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F. A. Gifford and S. R. Hanna
Atmospheric Turbulence and Diffusion Laboratory,
National Oceanic and Atmospheric Administration,
Oak Ridge, Tennessee

INTRODUCTION

This chapter can be used in evaluating surface concentrations of sulfur dioxide emitted generally over an area. The most common example of an area source is provided by urban domestic and industrial heating, for which it is necessary to consider the sum of a large number of individual sources over an area of some convenient size. Given an area-source inventory, this chapter provides a simple, systematic procedure for calculating the surface concentration at any point, averaged over an hour, a day, or a year. The basic physical model of atmospheric diffusion employed is the same as for Chapter 5 on tall-stack emissions.

Area-source concentration calculations are a recent development compared with calculations of concentrations from isolated sources. There are discussions of some of the methods now available, but these all require the use of a high-speed electronic computer.[1] The method to be described, although very simple, has been shown[2] to provide results of a precision comparable with that of the more elaborate computer calculations and can be used with confidence for area-source calculations in urban air pollution and regional air quality studies. An even simpler area-source calculation[3] suitable for many air pollution control purposes is also described. The model is described here in such a way that all the calculations can be done by hand. For the purpose of making many applications of the model, it has also been programmed in Fortran.[4]

THE AREA-SOURCE GRID

The area source strength data are assumed to be given in the form of a regular "checkerboard" grid pattern. This area-source grid pattern is the usual one in which source information is provided. If the available source strength data are given in some other pattern, for instance by counties or districts, they should be modified to approximate a regular checkerboard pattern by choosing the largest grid-square size that will give an adequate representation of the given data. Squares of 5 x 5 km are fairly standard.

CALCULATION OF AREA-SOURCE SURFACE CONCENTRATION: ONE-HOUR PERIOD

It is assumed that the concentration is required at the center of a grid square, which is designated square "O." The procedure when the receptor is not centered on a grid square is covered under Correction Factors, below. The calculation requires evaluation of the following simple formula for the area-source concentration, χ_A:

$$u\chi_A = c_oQ_o + c_1Q_1 + c_2Q_2 + c_3Q_3 + c_4Q_4 + c_5Q_5 \qquad (1)$$

The Q's are the given area source strength emissions per unit area per second in a line of source-grid squares centered on the square containing the receptor, *i.e.*, square "O" whose strength equals Q_o, and extending *upwind*. The c's are multipliers that depend on the meteorological conditions and the grid size. For a 5 x 5 km grid the multipliers are given in Table 7.1.

Table 7.1. Grid Multipliers for Meteorological Dispersion Conditions

	Neutral	*Stable*	*Unstable*
c_o:	153	331	137
c_1:	48	124	23
c_2:	28	73	12
c_3:	20	54	8.3
c_4:	16	44	6.7
c_5:	14	38	5.3

The area-source calculation, Equation (1), requires combination of these multipliers with upwind source strengths. Since this may have to be done for a number of receptor points and wind directions, it is convenient to introduce the multipliers by means of a multiplier grid, Figures 7.1–7.3, prepared from Table 7.1.

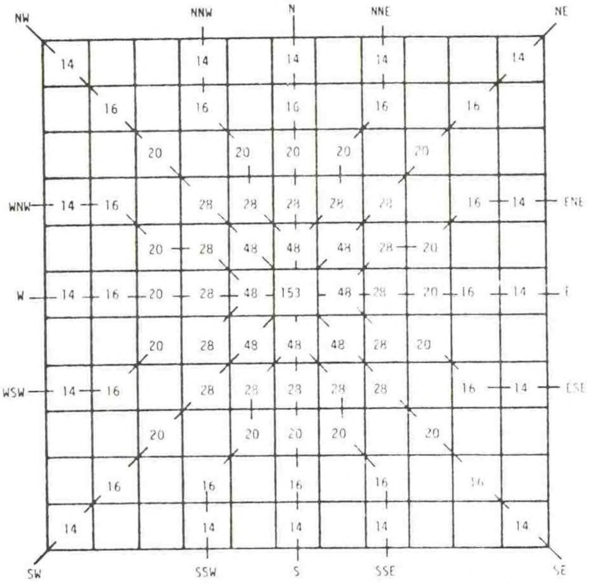

Figure 7.1. Multiplier grid for neutral meteorological conditions.

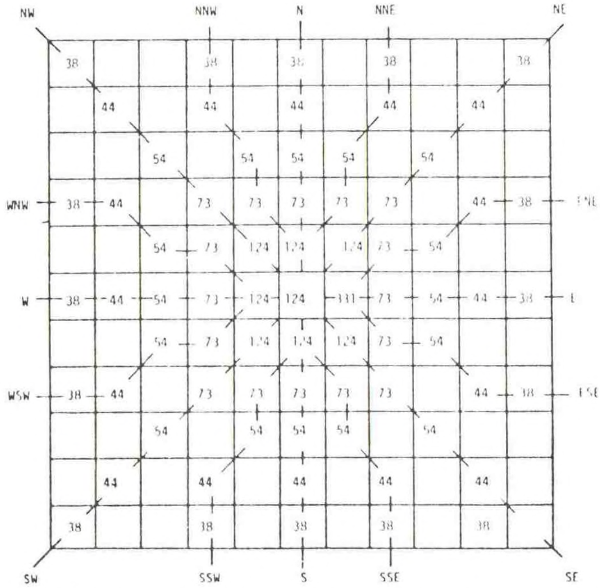

Figure 7.2. Multiplier grid for stable meteorological conditions.

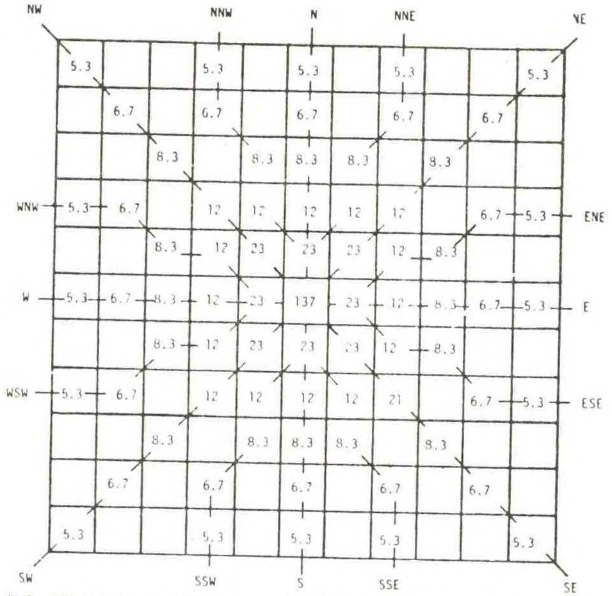

Figure 7.3. Multiplier grid for unstable meteorological conditions.

Step 1 — Plot the area-source strength data at the centers of 1/2-inch squares. This is the "source grid." It is convenient for the following steps to do this on tracing paper or, better, to make a transparency.

Step 2 — Select the multiplier grid, Figures 7.1, 7.2, or 7.3, appropriate to the given meteorological conditions. Center the source grid over the multiplier grid so that square "O" corresponds to the source grid square that contains the receptor point.

Step 3 — Six numbers on the multiplier grid, including the central number at square "O," correspond to each wind direction. Multiply these by the six numbers in the corresponding source-grid squares and sum these products.

Step 4 — Divide the result of Step 3 by the given mean wind speed, u. The result is the required hourly average area-source concentration value.

Step 5 — Relocate the multiplier grid to a new receptor square and repeat Steps 1 through 5.

EQUATIONS FOR AREA SOURCE

The equation for surface concentrations from an area source:

$$\chi_A = \left(\frac{2}{\pi}\right)^{1/2} \frac{(\Delta x/2)^{1-q}}{b(1-q)} \left[Q_0 + \sum_{i=1}^{5} Q_i \left[(2i+1)^{1-q} - (2i-1)^{1-q}\right] \right] \tag{2}$$

is based directly, on Equation (2) of Chapter 5; Δx is the size of the source-strength grid, and the rest of the terms are defined in the nomenclature. Equation (2) must be multiplied by the frequency f_d of wind from the d direction, to obtain the annual average.

Comparing Equations (1) and (2), it can be seen that the dimensionless multipliers c_i, $i = 0, 1, \ldots$ are given by:

$$c_0 = \left(\frac{2}{\pi}\right)^{1/2} \frac{(\Delta x/2)^{1-q}}{b(1-q)} \tag{3}$$

$$c_1 = \left(\frac{2}{\pi}\right)^{1/2} \frac{(\Delta x/2)^{1-q}\,(3^{1-q}-1)}{b(1-q)} \tag{4}$$

$$c_2 = \left(\frac{2}{\pi}\right)^{1/2} \frac{(\Delta x/2)^{1-q}\,(5^{1-q}-3^{1-q})}{b(1-q)} \tag{5}$$

and so on. The values appearing in Table 7.1 were calculated from these equations.

CALCULATION OF ANNUAL AVERAGE AREA-SOURCE CONCENTRATION

The procedure is the same as the previous one. The exception is that the annual average wind direction frequency must be included. It is assumed that a standard, 16-point wind direction frequency distribution (wind rose) is available, giving the fraction, f_d, of the time the wind blows *from* each direction, d. The average annual wind speed is used and neutral meteorological conditions are assumed to apply. Because the wind rose varies from place to place, it is necessary to calculate a new multiplier grid for each application (each city). The complete procedure is as follows.

Step 1 — Form an annual average multiplier grid by multiplying each of the values in the hourly average multiplier grid for *neutral meteorological conditions,* Figure 7.1, by the corresponding numbers in the wind direction frequency grid, Figure 7.4, after first summing the f_d's as indicated in Figure 7.4; f_1 is the relative frequency (fraction) of winds from the NNE, f_2 that from NE, . . ., f_{16} that from N.

f_{14}			f_{15}		f_{16}		f_1			f_2
	f_{14}		f_{15}		f_{16}		f_1		f_2	
		f_{14}		f_{15}	f_{16}	f_1		f_2		
f_{13}	f_{13}		f_{14}	f_{15}	f_{16}	f_1	f_2		f_3	f_3
		f_{13}	f_{13}	f_{14}	$f_{15}+f_{16}+f_1$	f_2	f_3	f_3		
f_{12}	f_{12}	f_{12}	f_{12}	$f_{11}+f_{12}+f_{13}$	1	$f_3+f_4+f_5$	f_4	f_4	f_4	f_4
		f_{11}	f_{11}	f_{10}	$f_7+f_8+f_9$	f_6	f_5	f_5		
f_{11}	f_{11}		f_{10}	f_9	f_8	f_7	f_6		f_5	f_5
		f_{10}		f_9	f_8	f_7		f_6		
	f_{10}		f_9		f_9		f_7		f_6	
f_{10}			f_9		f_8		f_7			f_6

Figure 7.4. Wind direction frequency grid.

Step 2 — Center the source-grid square containing the receptor point over the multiplier grid. It is convenient for this purpose to use a transparent source grid. (If the annual average source strength differs from the short period average value, then of course the annual source strengths must be used.)

Step 3 — Multiply *all* the number pairs of these two grids, and sum. (Where a square of the multiplier overlay falls outside the source grid area, the product is zero.)

Step 4 — Divide the result of Step 3 by the annual average wind speed to get χ_a.

Step 5 — Relocate the multiplier grid to a new receptor square and repeat Steps 1 through 5.

Correction Factors

The following procedures should be applied, if required, to the values determined by the foregoing procedure.

One-Hour to 24-Hour or Other Averaging Time

To calculate average area-source concentrations for 24-hour or other (*e.g.*, monthly) periods, follow the same procedures as for the annual average except that wind direction frequencies appropriate to the period in question must be used.

Grid Size Other Than 5 x 5 km

Multiply the value obtained at Step 4 by the appropriate correction factor shown in Table 7.2.

Table 7.2. Grid Size Correction Factors-Meteorological Conditions.

Grid Size (km)	*Neutral*	*Stable*	*Unstable*
1	.94	.97	.93
2	.97	.98	.96
3	.98	.99	.98
4	.99	1.00	.99
5	1.00	1.00	1.00
10	1.03	1.01	1.03
15	1.04	1.02	1.05
20	1.05	1.02	1.07
25	1.06	1.03	1.08

Receptor Point Not at Center of a Source-Grid Square

Calculate χ_a for the center of the source-grid square containing the receptor point, and for the next nearest square beyond the receptor point. Interpolate linearly between these values.

Worked Example

Problem

The source inventory for sulfur dioxide assuming no decay for a (real) city is given in Figure 7.5. The grid spacing is 5 km. (1) Calculate the sulfur dioxide concentration for a receptor located at the center of source square with the heavy outline, for the case of a WNW wind at 10 mph (3.4 m/sec) under neutral meteorological conditions. (2) Given the following annual wind frequency distribution, calculate the annual average concentration for a receptor located at the center of the same source square.

.15	.16	.27	.18	.14	.07
.39	.45	.58	.27	.20	.07
.22	.64	1.42	.36	.24	.14
.10	.34	.50	.14	.05	.05
.08	.31	.13	.05	.05	.05
.05	.12	.26	.08	.10	.11

Figure 7.5. Source inventory grid, annual average particulate emissions ($\mu g/m^2$ sec).

Direction	f_d	*Direction*	f_d
NNE	.02	SSW	.05
NE	.04	SW	.05
ENE	.08	WSW	.05
E	.08	W	.05
ESE	.08	WNW	.12
SE	.05	NW	.14
SSE	.04	NNW	.07
S	.04	N	.02

Solution

Procedure for Hourly Average Concentration. Select the multiplier grid for neutral conditions, Figure 7.1. Center over this the square of the source inventory overlay (Figure 7.5) containing the receptor point. Corresponding to WNW wind, it is seen that Equation (1) gives:

$$\chi_0 = \frac{1}{u}(153 \times .05 + 48 \times .05 + 28 \times .50 + 20 \times .34 + 16 \times .22 + 14 \times 0) = 34.4/3.4 = 10.\mu g/m^3$$

Procedure for Annual Average Concentration. Form an annual average multiplier grid by multiplying the corresponding numbers in Figure 7.6 and Figure 7.1. The result is Figure 7.7. Figure 7.6 was obtained by substituting the actual wind direction frequencies in the above table into the annual wind frequency grid, Figure 7.4. The receptor point of Figure 7.5 is now centered over the multiplier grid, Figure 7.7, and all corresponding numbers are multiplied and added; the result equals 24.03. The annual average concentration is:

$$24.03/3.4 = 7.1\ \mu g/m^3$$

.14			.07		.02		.02			.04
	.14		.07		.02		.02		.04	
		.14		.07	.02	.02		.04		
.12	.12		.14	.07	.02	.02	.04		.08	.08
		.12	.12	.14	.11	.04	.08	.08		
.05	.05	.05	.05	.22	1.0	.24	.08	.08	.08	.08
		.05	.05	.05	.13	.05	.08	.08		
.05	.05		.05	.05	.04	.04	.05		.08	.08
		.05		.05	.04	.04		.05		
	.05		.05		.04		.04		.05	
.05			.05		.04		.04			.05

Figure 7.6. Wind direction frequency grid for example.

A Simple Approximation to the Area-Source Calculation. Area-source strength values do not in practice vary much from one grid square to another. Furthermore, the grid multipliers of Table 7.1 show that the contribution to χ_a from the zeroth square (strength =

Q_o) is weighted most heavily. This suggests the following simple approximation to Equation (1):

$$u\chi_a = CQ_o \tag{6}$$

where the coefficient C is obtained by summing the values given in Table 7.1, *i.e.*,

$$C = c_o + c_1 + c_2 + c_3 + c_4 + c_5 \tag{7}$$

thus C equals 279 for neutral conditions, 664 for stable conditions, and 192 for unstable conditions.

Corresponding to the conditions of the worked example, Equation (6) provides the result, $\chi_a = 4.1\ \mu g/m^3$. This is in reasonably good agreement with the results using Equation (1), especially the annual average value. Hanna[3] has made a number of comparisons and finds Equation (6) to be a valid, useful area-source formula.

2.52			.98		.28		.28			.56
	2.24		1.12		.32		.32		.64	
		2.80		1.40	.40	.40		.80		
1.68	1.92		3.92	1.96	.56	.56	1.12		1.28	1.12
		2.40	3.36	6.72	5.28	1.92	2.24	1.66		
.70	.80	1.00	1.40	10.56	153	11.52	2.24	1.60	1.28	1.12
		1.00	1.40	2.40	6.24	2.40	2.24	1.60		
.70	.80		1.40	1.40	1.12	1.12	1.40		1.28	1.12
		1.00		1.00	.80	.80		1.00		
	.80		.80		.64		.64		.80	
.70			.70		.56		.56			.70

Figure 7.7. Annual average multiplier grid for example.

NOMENCLATURE

b = vertical dispersion coefficient corresponding to given atmospheric stability (m^{1-q})
c = multiplier that depends on the meteorological conditions and the grid size
Δx = size of the source-strength grid (m)
f_d = fraction of the time the wind blows from each direction (dimensionless)
p = crosswind dispersion exponent corresponding to a given atmospheric stability (dimensionless)
q = vertical dispersion exponent corresponding to a given atmospheric stability (dimensionless)
Q_N = area source strengths of grid square N ($\mu g/m^2$ sec)
u = mean wind speed in plume region (m/sec)
χ_A = area source 1-hour average surface sulfur dioxide concentrations for the center of the source-grid square A ($\mu g/m^3$)

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