NOAA Research Laboratories

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METEOROLOGICAL EFFECTS OF COOLING TOWER PLUMES

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Contribution No. 48 from the
Atmospheric Turbulence & Diffusion Laboratory

U. S. DEPARTMENT OF COMMERCE
NATIONAL OCEANIC AND ATMOSPHERIC ADMINISTRATION

Meteorological Effects of Cooling Tower Plumes

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(For presentation at the Cooling Tower Institute Winter Meeting, Houston, Texas, January 25, 1971)

Abstract

The amounts of energy and moisture emitted to the atmosphere by cooling towers are compared to the production of energy and moisture by meteorological processes. It is shown that the energy input from a large (1000 MW) power plant is a significant fraction of the energy production of mesoscale atmospheric phenomena such as thunderstorms. Methods for calculating the rise of cooling tower plumes and the possibility of cloud formation due to these plumes are described. The height of rise of a moist plume can be approximately estimated using simple formulas, but the depth and amount of cloud cannot be easily estimated.

1. Introduction

Large amounts of heat and moisture may be released to the atmosphere from cooling towers. The total heat flux from a large natural draft wet cooling tower serving a 1000 megawatt power station is about 1500 megawatts, an output that is an order of magnitude greater than the heat flux from the stack of a 1000 MW fossil fuel plant using run-of-the river cooling. Most of the heat transport from a cooling tower is in the form of latent heat, which is released to the atmosphere only when the vapor in the plume condenses. Present research with respect to these large heat and moisture additions to the atmosphere is very limited.

It is important that we are able to estimate the consequences of our artificial heat and moisture production. The purpose of this paper is to compare the energy inputs from cooling towers with the energies of natural processes, and to describe a numerical model of moist plume rise which accounts for cloud physics interactions.

2. Energetics

In order to assess the effects of cooling towers on the atmosphere, it is important to know the natural energy production of various atmospheric processes. Some typical rates of natural energy production and their area scales are listed in Table 1.

Table 1

Energy Production of Some Atmospheric Processes

Area	Natural Process	Energy Production	Production per Unit Area
$10^4 m^2$	Tornado kinetic energy (K.E.) production.	10^8 watts	10^4 watts/m^2
$10^8 m^2$	Thunderstorm K.E. production.	10^{10} watts	100 watts/m^2
	Latent heat release (1 cm rain in 30 min.).	5×10^{11} watts	5000 watts/m^2
$10^{10} m^2$	Great lakes snowsquall latent heat release (4 cm snow in hr.).	10^{13} watts	1000 watts/m^2
$10^{12} m^2$	Cyclone latent heat release (1 cm rain per day).	2×10^{14} watts	200 watts/m^2
$5 \times 10^{14} m^2$	Solar energy flux	1.75×10^{17} watts	350 watts/m^2

For comparison, the wet cooling towers at a 1000 MW power generating station release about 10^9 watts of latent heat energy to the atmosphere. Because the plume of heat and moisture diffuses as distance from the cooling tower increases, the cross-sectional area of the plume always increases. As a result, the energy flux per unit area decreases with distance. For example, if the initial energy flux were about 10^6 watts/m^2 (initial area $10^3 m^2$), at a distance of 5 km from the source the cross-sectional area of the plume would increase to about 10^5 to $10^6 m^2$ (assuming well-mixed conditions), and the energy flux per unit cross-sectional area would decrease to about 10^3 to 10^4 watts/m^2. Once the plume has the same cross-sectional area as a thunderstorm ($10^8 m^2$), the energy flux per unit area per unit time is about ten percent of the kinetic energy production per unit area of a thunderstorm.

In assessing the impact of artificial heat on the environment, it is clearly necessary to compare not only the total energy production, but also the area scales of the processes. While the total energy production of a large cooling tower is many orders of magnitude less than the rate at which solar energy is absorbed by the entire atmosphere, it is several percent of the rate of energy production in a thunderstorm. On a global scale the energy from this cooling tower has a negligible effect. However, on the scale of a thunderstorm (area $10^8 m^2$) the artificial energy input may significantly affect the natural process.

Of course, for any method of cooling, the energy ultimately reaches the atmosphere. When run-of-the-river cooling or cooling ponds are used, energy is released from the hot water mainly in the form of long wave radiation and latent heat. However the energy flux per unit area into the atmosphere due to these cooling processes is about three orders of magnitude less than the energy flux from the top of a cooling tower. Also, the energy from a cooling tower quickly rises to a height of several hundred meters, while the energy from cooling ponds or rivers diffuses vertically much more slowly. Differences in the effects of these various energy inputs on the atmosphere have not yet been fully investigated.

3. Moist Plume Rise

The theory of dry plume rise is discussed by Briggs(1969). In general, his equations predict that dry plume rise is a function of the initial buoyancy flux. With moist plumes, the effects of water vapor and liquid water must be included. In the case of moist plumes that do not condense and rise in a well-mixed environment, Briggs' equations can still be used,but the additional buoyancy flux due to the difference in molecular weight between air and water vapor must be included. The initial mixing ratios (mass of water vapor per unit mass of dry air) of the plume and the environment are denoted by q_{po} and q_{eo}, respectively. In this case, the initial buoyancy flux is given by the expression

$$\left.\begin{array}{c}\text{moist plume,}\\ \text{no}\\ \text{condensation}\end{array}\right] F = w_o \, g \, R_o^{\,2} \left\{ \frac{T_{po} - T_{eo}}{T_{po}} + .61(q_{po} - q_{eo}) \right\} \quad . \qquad (1)$$

The final plume rise, h, can then be estimated from the formula:

$$h = 4.6 \, F^{1/3} \, u^{-1} \, h_s^{\,2/3} \quad . \qquad (2)$$

where u is the wind speed and h_s is the stack height. This equation is valid for an uncondensed plume in a well-mixed environment with vertical gradients of potential temperature and mixing ratio equal to zero. The symbols in these equations and the following equations are defined below:

Subscripts:	o	:	initial value of variable
	p	:	plume variable
	e	:	environmental variable
	s	:	saturated value
	R(cm)	:	radius of plume
	w(cm/sec)	:	vertical speed of plume
	$g(cm/sec)^2$	:	acceleration due to gravity
	T(°K)	:	absolute Kelvin emperature
	U(cm/sec)	:	wind speed
	h(cm)	:	plume rise
	x(cm)	:	downwind distance
	Q_h(gm/gm)	:	large drop water content
	Q_c(gm/gm)	:	cloud water content
	q(gm/gm)	:	mixing ratio (mass of water vapor per unit mass of dry air)
	L(ergs/gm)	:	latent heat
	c_p(ergs/gm°K)	:	specific heat of air at constant pressure
	R_V'(ergs/gm°K)	:	gas constant for water vapor

The buoyancy flux from cooling towers due to the differences in water vapor content is often of the same order as the flux due to the differences in temperature. In Morton's (1957) treatment of moist plume rise, these water vapor differences were neglected. Table 2 lists the additional relative buoyancy resulting from the water vapor differences and the condensation of the excess water vapor.

Table 2

Contributions to the total buoyancy flux of a moist plume due to initial temperature differences, water vapor content differences, and latent heat.

			$.61(q_{po}-q_{eo})$			$\frac{L}{c_p T_{po}}(q_{po}-q_{eo})$		
T_{po}	T_{eo}	$\frac{T_{po}-T_{eo}}{T_{po}}$	100%RH	80%RH	60%RH	100%	80%	60%
305°K	275°K	.0984	.0150	.0155	.0160	.228	.238	.244
315	285	.0953	.0252	.0262	.0273	.372	.387	.404
305	285	.0656	.0124	.0135	.0145	.189	.206	.221
315	295	.0636	.0199	.0225	.0245	.294	.332	.361
305	295	.0328	.0078	.0098	.0117	.118	.149	.178

The terms in the last columns of Table 2 represent the additional relative temperature increase $(T_{po} - T_{eo})/T_{po}$ if all the excess humidity in the plume were to condense. These are a factor of three or four larger than the buoyancy term due to the initial temperature difference, and strongly influence plume rise if a cloud forms in the plume. Briggs (1969) shows that plume rise increases as $F^{1/4}$ during calm, stable conditions. Then if F triples, then plume rise will increase by about 30%. The following approximation can be used for the buoyancy flux in a moist plume where all excess vapor condenses:

$$\left.\begin{array}{l}\text{Moist plume.}\\ \text{All excess}\\ \text{vapor condenses}\end{array}\right] \quad F = w_o g \, R_o^{\,2} \left\{ \frac{T_{po}-T_{eo}}{T_{po}} + (q_{po}-q_{eo}) \; (.61 + \frac{L}{c_p T_{po}}) \right\} \tag{3}$$

Using Briggs' (1969) notation, the upper limit on plume rise during calm, stable conditions is

$$h = 5.0F^{1/4} \; s^{-3/8} \tag{4}$$

where the atmospheric stability s is defined by the relation

$$s = \frac{g}{T_e} \left\{ \frac{\partial T_e}{\partial z} + .01°K/m \right\} \; . \tag{5}$$

In the numerical experiments described below, the temperatures and water vapor contents listed in Table 2 were used. The initial vertical speed and radius were assumed to be 5 m sec^{-1} and 30 m. For initial temperature differences (T_{po} - T_{eo}) of 10, 20, and 30°K, the initial energy flux is thus 1.4, 2.8, and 4.2 x 10^8 watts, respectively. The additional energy flux due to water vapor content differences and latent heat can be calculated using the data in Table 2. Three values of atmospheric stability s were used, s equal to 0, 3.31 x 10^{-4} sec^{-2}, and 6.62 x 10^{-4} sec^{-2}. Equation 4 was used to calculate theoretical plume rise for comparison with the numerical results. Three calculations of plume rise were made; one for dry rise in which the water vapor is not accounted for, one for uncondensed moist rise in which the differences in water vapor are account for, and one for condensed moist rise in which the latent heat is accounted for. The calculated plume rise, to the nearest ten meters, is given in Table 3.

Table 3

Theoretical Plume Rise (in meters) for $s_1 = 3.31 \times 10^{-4} sec^{-2}$, $s_2 = 6.62 \times 10^{-4} sec^{-2}$

		Dry		Moist		Moist Condensed	
T_{po}	T_{eo}	s_1	s_2	s_1, 60%RH	s_2,60%RH	s_1,100%RH	s_2,100%RH
305	275	820	630	850	660	1120	860
315	285	820	630	870	670	1250	950
305	285	740	570	780	600	1050	810
315	295	740	570	790	610	1150	890
305	295	620	480	670	520	920	710

The release of latent heat increases plume rise by several hundred meters. Of course, a cloud is present in this upper layer. Because of entrainment and variations of environmental temperature and humidity with height, these predictions cannot be expected to be highly accurate. However, they still may be useful approximations.

The complexities of moist plume rise make it difficult to come to general conclusions. In order to predict rainfall and other cloud parameters, it is necessary to consider each problem individually. Consequently a set of equations was developed to predict the variations of plume parameters with height in a calm atmosphere. Several sets of boundary conditions that were typical of cooling tower problems were considered.

The following equations apply only to cooling tower plume rise when the wind speed vanishes. It is assumed that the variables are constant across a cross-section of the plume at any height. These formulas are based on the work on cloud growth by Weinstein (1970) and Simpson and Wiggert (1970):

Equation of Motion:

$$\frac{\partial}{\partial z}\frac{w^2}{2} = g\left\{\frac{T_p - T_e}{T_p} + .61(q_p - q_e) - (Q_h + Q_c)\right\} - .2\frac{w^2}{R} \qquad (6)$$

(Acceleration) (buoyancy) (drag or entrainment)

Equation of Radius Change:

$$\frac{\partial R}{\partial z} = .2 - \frac{R}{2w^2}\frac{g}{T_p}(T_p - T_e) \qquad (7)$$

Equation for Temperature Change:

$$\left\{1 + \left[\frac{.61L^2 q_{ps}}{c_p R_v' T_p^2}\right]\right\}\frac{\partial T_p}{\partial z} = -\frac{g}{c_p}\left\{1 + \left[\frac{Lq_{ps}}{R_v T_p}\right]\right\}$$

$$- \frac{.2}{R}(T_p - T_e) - \left[\frac{.2L}{c_p R_v'}(q_s - q_e)\right] \qquad (8)$$

(entrainment) (heat used to saturate entrained air)

$$- \frac{L}{c_p}\left\{\frac{q_s - q_p}{dz}, \frac{Q_h + Q_c}{dz}\right\}\Bigg|_{min} + \left\{\frac{L}{c_p}\frac{Q}{dz} + \frac{L}{c_p}(\Delta q_s)_{w-i}\right\}$$

(resaturation of plume) (freezing processes)

The terms with square brackets in eqs. (8) (9) and (10) are included only if the plume is saturated.

Equations for Water Vapor Change:

$$\frac{\partial q_p}{\partial z} = \left[\frac{\partial q_{ps}}{\partial z}\right] + \left\{\underbrace{\frac{q_s-q_p}{dz}, \quad \frac{Q}{dz}}_{\text{(resaturation of plume)}}\right\}\Bigg|_{min} - \underbrace{\frac{.2}{R}(q_p-q_e)}_{\text{(entrainment)}} \tag{9}$$

Equation for Cloudwater Content Change:

$$\frac{\partial Q_c}{\partial z} = \left[-\frac{\partial q_s}{\partial z}\right] - \underbrace{10^{-3}(Q_c-.0005)/w}_{\text{(conversion to } Q_h)} - \underbrace{.0052\, Q_c(1000\, Q_c)^{\frac{7}{8}}/w}_{\text{(collection by } Q_h)}$$

$$- \underbrace{\left\{\frac{q_{ps}-q_p}{dz}, \quad \frac{Q_c}{dz}\right\}\Bigg|_{min}}_{\text{(resaturation)}} - \underbrace{\frac{.2}{R}(q_p-q_e+Q_c)}_{\text{(entrainment)}} \tag{10}$$

Equation for Large Water Drop Change:

$$\frac{\partial Q_h}{\partial z} = \underbrace{10^{-3}(Q_c-.0005)/w}_{\text{(conversion)}} + \underbrace{.0052\, Q_c(1000\, Q_c)^{\frac{7}{8}}/w}_{\text{(collection)}}$$

$$- \underbrace{4.5\, Q_h(1000\, Q_h)^{\frac{1}{8}}/wR}_{\text{(rainout)}} - \underbrace{\frac{.2Q_h}{R}}_{\text{(entrainment)}} \tag{11}$$

The last term in equation (9) applies only if the plume is unsaturated.

Equation for Saturation Specific Humidity:

a) $273.16°K < T < 373.°K$

$$\ln q_s = 1.335 + 2.303 \left\{ 10.79574 \,(1 - T_1/T) + 1.50475 \times 10^{-4} \,(1 - 10^{-8.2969(T/T_1 - 1)}) + .42873 \times 10^{-3} (10^{4.76955(1 - T_1/T)} - 1) \right\} - 5.028 \ln T/T_1 - \ln p$$

(12)

b) $T < 273.16° K$

$$\ln q_s = 1.335 + 2.303 \left\{ - 9.09685(T_1/T - 1) + .87682 \,(1 - T/T_1) \right\} - 3.56654 \ln T_1/T - \ln p$$

where $T_1 = 273.16°K$ and p is the pressure in millibars, which is calculated by means of the hydrostatic equation. These empirical equations are the "Goff-Gratch" formulas, as given by the World Meteorological Organization (1966). This set of equations accounts for the important physical processes that govern moist plume rise. The cloud water and large drops are separated here so that rain falling from the cloud in the plume can be accounted for. Given an initial set of plume parameters at the mouth of the cooling tower and the manner in which the atmospheric temperature and specific humidity vary with height, these equations are integrated upward using a high-speed, digital computer. The calculations are stopped when the vertical speed w drops below zero. The height at which this occurs represents the height of maximum plume rise.

A vertical height increment of one meter was used to insure computational stability. Initially, the vertical speed of the plume is 5 m sec^{-1} and the radius is 30 m for all the runs. The initial temperatures, water vapor contents, and stabilities are the same as in the examples in Tables 2 and 3. The plume is saturated, and Q_h and Q_c each initially equal .001. Solutions are obtained for all combinations of these conditions.

For an adiabatic or well-mixed environment ($\partial T_e/\partial z = -\ .98°K/100m$), the plume always continues to rise. Depending on the initial conditions, a cloud forms at a height of one or two kilometers and persists up to a height of ten kilometers, where the computation is halted arbitrarily. The initial cloud water existing at the opening of the cooling tower evaporates at heights of ten or twenty meters. Above this, no liquid water is present until the cloud forms. The results are summarized in Table 4, where it is assumed that the cloud base occurs at the height where the cloud water content Q_c reaches .0001.

Table 4

Height of Cloud Base and Maximum Vertical Speed of a Moist Plume in an Adiabatic Atmosphere

T_{po}	T_{eo}	Relative Humidity	Cloud base	Max W at	Height
305°K	275°K	60%	1300m	$12.1\frac{m}{sec}$	5100m
		80%	1180	13.0	4670
		100%	780	14.6	4040
315	285	60%	1980	16.1	6290
		80%	1530	18.8	5730
		100%	1130	22.2	5400
305	285	60%	1900	14.8	6420
		80%	1460	17.5	5800
		100%	970	20.4	5370
315	295	60%	2290	23.0	7350
		80%	1970	26.2	7100
		100%	1490	29.9	7070
305	295	60%	2120	22.0	7340
		80%	1880	25.1	7040
		100%	1290	28.4	6980

There are several general observations that can be made. For constant initial conditions, the cloud base decreases and maximum vertical speed increases as environmental relative humidity increases. It is to be expected that a deeper cloud will form and more latent heat will be released in a more humid environment. Furthermore, the maximum vertical speed is correlated more with the initial environmental temperature T_{eo} than with the difference $(T_{po} - T_{eo})$. The basic result is that the cloud base is at a height of one to two kilometers and the maximum vertical speed is about 15 to 30 m/sec, which is slightly greater than the updrafts observed in thunderstorms.

For the two stable temperature gradients that are considered ($\partial T_e/\partial z$ = 0, + .98°K/100m), maximum plume rise is less than two kilometers. In all cases with 100% humidity, a cloud forms, while in no cases with 60% humidity does a cloud form. The values of plume rise observed in the numerical model are listed in Table 5, which can be compared with the simple predictions in Table 2.

Table 5

Plume Rise (in meters) Observed in the Numerical Model

T_{po}	T_{eo}	Moist (no cloud) s_1,60%RH	s_2,60%RH	Moist condensed (cloud) s_1,100%RH	s_2.100%RH
305°K	275°K	1040m	740m	1450m	900m
315	285	1070	760	1570	910
305	285	840	590	1240	720
315	295	890	620	1340	730
305	295	660	450	950	510

The moist (no cloud) plume rise in the numerical model is from 85% to 131% of that predicted in Table 2 using the simplified theory. In general the greater the initial temperature difference ($T_{po} - T_{eo}$), the greater the ratio of observed to predicted plume rise. The observed moist condensed (cloud) plume rise is from 72% to 130% of that predicted in Table 2 assuming condensation of all excess vapor. Low values of observed rise are caused by failure of all the excess vapor to condense. High values are caused by the latent heat added by moist air entrained into the plume. Nevertheless, the predicted plume rise is within 30% of the observed rise for all cases. Thus the simple predictions are adequate for many purposes.

The depth of the cloud and its liquid water content cannot be predicted as easily as plume rise. With 80% relative humidity a cloud may or may not form, and when it does form it is usually thin. With 100% relative humidity the cloud thickness decreases as stability increases and initial temperature difference decreases. The cloud thicknesses observed in the numerical model and maximum cloud water concentrations are listed in Table 6.

Table 6

Cloud Thicknesses and Maximum Cloud Water Content

T_p	T_e	Relative Humidity	Thickness s_1	Thickness s_2	Q_c Max s_1	Q_c Max s_2
305°K	275°K	80%	220m	10m	.00029	.00012
		100%	730	300	.00091	.00072
315	285	80%	190	20	.00033	.00014
		100%	720	280	.00132	.00102
305	285	80%	50	0	.00017	0
		100%	570	230	.00108	.00081
315	295	80%	50	0	.00020	0
		100%	580	220	.00150	.00114
305	295	100%	439	160	.00114	.00083

The rate of rainfall from these numerical clouds is very light in all cases. By themselves, the clouds that may form in stable air above large cooling towers may not precipitate. However, in these studies the environment is always cloud free. In future studies the enhancement of rainfall by the moist plume should be studied.

4. Conclusions

Large cooling towers emit heat and moisture at rates that are a few per cent of the rates of energy production by mesoscale atmospheric processes such as thunderstorms. Numerical experiments show that it is possible for clouds to form above cooling towers. Plume rise can be estimated approximately using simple theory, but cloud depth and maximum water content cannot be estimated easily.

It is possible, using the numerical model described above, to calculate the growth of the moist plume for a variety of different plumes and environmental temperature and humidity distributions. These methods can be applied to estimate the effects of cooling towers on the atmosphere in any location.

Similar formulas can be used to calculate plume and cloud growth when the wind speed is greater than zero. The bases for all these formulas are the theory of plume rise from isolated stacks and the theory of cumulus cloud growth.

Acknowledgement

This research was performed in an agreement between the U. S. Atomic Energy Commission and National Oceanic and Atmospheric Administration.

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