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Abstract

Concentration from tall-stack buoyant plumes is ordinarily calculated using a Gaussian plume distribution that includes an effective stack height. The buoyant plume rise is, however, calculated on a different model, having a uniform distribution through the plume. In an attempt to overcome this ambiguity, a fluctuating, uniform or "top-hat" model of buoyant plumes is introduced. Mean and peak-to-mean concentration formulas are derived and compared with data. Over the range from one to about 10 stack heights downwind, the variation of the peak-to-mean ratio predicted by the model is in general agreement with observed values. P/M varies from 2 to 9 being smaller for shorter distances downwind, more stable conditions and axial, compared with surface level values.

When calculating time-averaged ground-level concentration, X_o, in the vicinity of tall stacks with buoyant plumes it is usually assumed [see Pasquill (1962), Slade (1968) and the many references they give] that the phenomenon is described by a Gaussian distribution function containing an effective stack height. This effective stack height, h, equals the actual height, h_s, plus a correction, Δh, to allow for plume buoyancy. The familiar formula is usually written

$$X_o = \frac{Q}{\pi \sigma_y \sigma_z u} \left[\exp - (h_s + \Delta h)^2 / 2\sigma_z^{\,2}\right]; \qquad (1)$$

where Q is source strength, u is the average wind speed at stack level, and $\sigma_y(x)$ and $\sigma_z(x)$ are plume material standard deviations in the y- and z-directions.

When it comes to calculating the buoyant plume rise, curiously, we assume a substantially different plume model in which the plume centerline continues to rise in a neutral atmosphere and the distribution of material across the plume differs markedly from the Gaussian. This distribution is usually approximated by a uniform, so-called "top-hat" plume distribution. The Gaussian model is based on the assumption that the governing physical phenomenon is random, turbulent diffusion of passive scalar quantities, the stack gases and particles. The "top-hat" model involves a much different idea. In it the plume is a substantial entity, rising bodily through the air because of its bulk buoyancy at a rate controlled by entrainment into the plume of outside air. This entrainment arises because of the small-scale turbulence at the plume's edge created by relative vertical motion between the plume and the ambient atmosphere.

Ultimately the initial plume buoyancy will be consumed. Unless enough buoyancy is resupplied from some source internal to the plume, such as latent heat, chemical reaction, or radioactivity, passive diffusion will then dominate plume behavior. That is, the top-hat stage ultimately must give way to the Gaussian stage. It is just a question of the point at which this happens. From Briggs' (1968, 1969, 1970) analyses it can be concluded that plumes from tall power-plant stacks are buoyancy-dominated to distances from the stack equal to at least $3h_s$ in neutral conditions, and that buoyancy is a factor to a distance of about $20h_s$. In fact all data on plume rise examined by Briggs indicate that the final stage of buoyant plume rise, in which the buoyancy effect is negligible, has not yet been observed.

We become concerned about concentrations of material from tall-stack buoyant plumes, as a rule, when the plume material is carried down to the ground and particularly when this results in a high concentration level. An appreciable part of all air pollution damage is a result of this process.* Concentration maxima from tall-stack plumes are observed to occur from time to time at all distances beyond a few stack heights. Neither the top-hat nor the Gaussian plume model can explain such occurrences, unless they result from more-or-less gradual downward plume growth. Although through this mechanism the plume can eventually reach the ground in some cases, this mostly tends to occur at fairly great distances. The idea of a rapid mixing of the plume downward by turbulence,

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*Air pollution damage in the U.S. in 1968 amounted to over 16 billion dollars, of which at least 4 billion is attributable to tall stack emissions.

called "fumigation," was proposed many years ago. While such mixing does occur for various reasons, the idea cannot be said to have lead to a very precise physical formulation, and "fumigation" remains an essentially qualitative concept.

A Gaussian plume model in which the disk elements are permitted to fluctuate, i.e. to wander irregularly from their mean position, was introduced by Gifford (1959),(1960). Plume fluctuations are often observed, and could certainly explain the occurrence of ground concentration maxima. The Gaussian plume model is however inappropriate to buoyant, tall-stack plumes for reasons just discussed. The purpose of this paper is to describe some properties of a fluctuating, top-hat plume model and compare them with available observations.

The equation for the instantaneous concentration at any point x, y, z, in a fluctuating plume having a uniform (top-hat) distribution of concentration, X(x,y,z), is

$$\frac{X}{Q} = (2\pi R^2 u)^{-1} \quad , \quad r = [(y-D_y)^2 + (z-D_z)^2]^{1/2} < R \qquad (2)$$

$$= \quad 0 \quad , \quad \text{otherwise} \quad .$$

The quantities appearing in equation (2) are identified in Figure 1. For simplicity the plume is assumed to have a circular cross-section.

Some theoretical properties of fluctuation length parameters similar to D_y and D_z have been discussed by Gifford (1959), (1960) and by F. B. Smith (see

Pasquill 1962)/ For the present purpose it is only necessary to assume that they are independently normally distributed quantities. Then the mean value M of X/Q is the convolution of equation (2) with the probability distribution function of D_y and D_z:

$$M(X/Q) = \iint X/Q \, p(D_y, D_z) \, dD_y \, dD_z \qquad (3)$$

where p, the joint probability distribution of D_y and D_z, is given by

$$p(D_y, D_z) = (2\pi)^{-1} (\overline{D_y^2} \, \overline{D_z^2})^{-1/2} \exp\left[- (D_y-y)^2 / 2\overline{D_y^2} - (D_z-z)^2/2\overline{D_z^2} \right] \qquad (4)$$

and the integration is performed over the region

$$r = \left[(y-D_y)^2 + (z-D_z)^2 \right]^{1/2} < R.$$

It is convenient to transform the origin of co-ordinates to the center of the circle R. Then the mean, M, becomes

$$M\left[X(x,y)/Q\right] = (\pi R^2 u)^{-1} \iint (2\pi)^{-1} (\overline{D_y^2} / \overline{D_z^2})^{-1/2} \cdot$$

$$\cdot \exp\left[-(D_y - y)^2 / 2\overline{D_y^2} - (D_z - z)^2/2\overline{D_z^2} \right] dD_y dD_z \qquad (5)$$

where the integration is over $r = (\overline{D_y^2} + \overline{D_z^2})^{1/2} < R$.

Equation (5) can be integrated exactly only in certain special cases. For instance in the simplest possible case; that of a receptor located on the mean plume axis, $D_y = D_z = 0$, and for which $\overline{D_y^2} = \overline{D_z^2} = \overline{D^2}$, it is readily found by transforming to polar coordinates that

$$M(X/Q) = (\pi R^2 u)^{-1} \left[1 - \exp(-R^2/\, 2\overline{D^2}) \right] . \qquad (6)$$

An integral can also be found when $(y^2 + z^2)^{1/2} = R/2$. In the general case, however, equation (5) has to be integrated numerically. Fortunately the integral occurs in many applications , and it has been extensively tabulated. Eckler (1969) gives a large number of references in connection with applications to the target covering problem, and some geophysical applications have been discussed by Crutcher (1967). For the present purpose the detailed tables provided by Groenwoud, et al. (1967) are most useful.

In the top-hat fluctuating plume model the probability of occurrence (i.e. the relative frequency) of a particular concentration X is obviously equal to the ratio of the mean concentration, M, to the peak concentration, P, where P is simply the (instantaneous) value of X given by equation (2). Thus

$$Pr\ (X/Q) = M/P = \pi R^2 u\ M(X/Q) \qquad (7)$$

and the peak to mean ratio, P/M, is just the reciprocal of this, Pr^{-1}. Figures 2 and 3 display the quantity Pr(X/Q) for values of the ratio $(\overline{D_z^2} / \overline{D_y^2})^{1/2}$ equal to 1.0 and 0.5, over a range of values of $R/(\overline{D_y^2})^{1/2}$

sufficient to cover cases of interest, and for $z/(\overline{D_z^2})^{1/2}$ equal to 0 and 1.0. These curves are based on values of Pr given in Groenwoud's extensive tables. The mean and the peak to mean ratio can easily be found by virtue of their definitions, i.e. equation (7).

The principal question of interest is the variation of the peak to mean concentration ration, P/R, which by equation (5) and (7) depends on the growth of the plume radius, R, and of the fluctuation parameters, $\overline{D_y^2}$ and $\overline{D_z^2}$. The behavior of R is fairly well known, having been discussed in detail by Briggs (1969). Except for the references given earlier, the $\overline{D^2}$'s have not been studied much. In order to establish the general pattern of the peak to mean ratio, the quantities R and $\overline{D_y^2}$ can be approximated as follows. (units are meters, grams, seconds.) The diameter of a buoyant plume from a tall stack is known empirically to equal the stack rise, Δh, very closely at least to ~ 5 h_s. Then the "2/3-law," a formula shown by Briggs (1969) to be valid to $x \cong 10h_s$ at least, gives

$$R(x) \simeq \Delta h/2 = 0.8F^{1/3}\ u^{-1}\ x^{2/3}, \tag{8}$$

where x is downwind distance. The quantity $(\overline{D_y^2})^{1/2}$ must be of the order of σ_y, the standard deviation of a diffusing plume of neutral buoyancy, over this distance range. From many experiments this is known to vary with downwind distance approximately as

$$\sigma_y \simeq .32\ x^{4/5} \tag{9}$$

in average meteorological conditions. The buoyancy flux, F, is equal to $3.7 \times 10^{-5} Q_H$ where Q_H is the stack heat emission in cal sec^{-1}, and a typical value of Q_H is 10^7 cal sec^{-1} for tall-stack electrical generating plants.

Using these approximations it follows that

$$R^2 \;/\; \overline{D_y^{\,2}} \simeq 6.25F^{2/3}\, u^{-2}\, x^{-4/15}. \tag{10}$$

From equation (10) and Figures 2 and 3 it is possible to form estimates of P/M. For an average large power plant $Q_H \simeq 10^7$ cal sec^{-1}. With an average stack-level wind speed of U = 10 m sec^{-1}, and for the distance range 300m $\leqslant x \leqslant$ 3000 m corresponding to one to ten times h_s, equation (10) given values of $R/(\overline{D_y^{\,2}})^{1/2}$ ranging from .84 to .62. The larger value corresponds to the smaller distance downwind. With these values a brief table of P/M has been prepared, based on the values of Pr that can be taken from Figures 2 and 3 or from Groenwoud's tabulation. Values from the case $z/\overline{D_z^{\,2}}^{1/2} = 0$ agree as well with equation (6) and (7).

Table I

Values of peak to mean concentration ratio, P/M, for selected conbinations of the parameters in Equations (5), (7), and (10); u = 10m sec^{-1}, $Q_H = 10^7$ cal sec^{-1}, y = 0.

$(\overline{D_z^{\,2}} \;/\; \overline{D_y^{\,2}})^{1/2}$ =	1.0			0.5		
$z/\overline{D_z^{\,2}}^{1/2}$ =	0	0.5	1.0	0	0.5	1.0
$x = h_s$	3.4	3.7	5.1	2.1	2.3	3.2
$x = 10h_s$	5.7	6.4	9.0	3.3	3.6	5.1

Table I shows, in the first place, that the range of P/M over the distance range from one to 10 stack-heights downwind is fairly small. Over this range P/M increases slightly, by a factor of about 1.7. Next, we can reasonably interpret the quantity $(\overline{D_z^2} / \overline{D_y^2})^{1/2}$ as an atmospheric stability factor. It should equal unity for neutral conditions, and the value 0.5 should correspond to stable conditions. The corresponding variation in P/M is a decrease, with increasing stability, by about the same factor of 1.7. It is further estimated that $z = \overline{D_z^2}^{1/2}$ corresponds to surface level values of P/M, then Table I shows that P/M increases between the value at the mean plume axis (z = o) and the surface $z/\overline{D_z^2}^{1/2} = 1$, a gain by a factor of 1.7 or so. Thus for the assumed parameter value P/M ranges from about 2 to 9, and increases with increasing distance downwind, decreasing stability, and increasing distance from the mean plume axis, by a factor of a little less than 2 in each case.

For many years the TVA has been making measurements of ground-level SO_2 concentrations in the vicinity of its steam plants. Much of these data are summarized in another paper at this meeting, by Dr. Montgomery. Among many interesting results, he shows that short period (3 - 5 minute), 1-hour and 24-hour averago SO_2 concentrations are in the proportion 1:2:6. This result does not seem to show a strong variation with distance from the stack. The $\overline{D^2}$ estimates used above are appropriate to an averaging period of 1/2 to 1 hour. Consequently the calculated values of P/M agrees well with what is observed. As to the longer period we do not yet have an adequate theoretical basis for a formal estimate . Slade (1968) has suggested that concentrations be reduced by a factor $(t_p/t_o)^b$ to correct for the longer averaging period; t_p is the longer period (up to 8 hours), t_o is the base period (30 mins. to one hour), and b is a constant to be determined emperically. Unfortunately there is little data on b, but what there is suggest values between 1/4 and 1/2. If the mean value is adjusted by the factor $(t_p/t_o)^b = (t_{24}/t_{1/2})^{1/2}$, then P/M is increased by the factor 7. The closeness of this number to the observed value, 6, should not be taken too seriously as the reasoning is quite crude. It is only intended to suggest the factors that are operating.

References

Briggs, G. A., 1968: CONCAWE meeting: discussion of the comparative consequences of different plume rise formulas. Atmospheric Environment, 2, 228-232.

Briggs, G. A., 1969: Plume Rise, vi and 81pp, U. S. Atomic Energy Commission, DTI.

Briggs, G. A., 1970: Some recent analysis of plume rise observations. Paper presented at 2nd Int. Clean Air Congress, Washington, D. C., December 1970; (see its transactions).

Crutcher, H. L., 1967: Applications to geophysical data, Vol. III; iv and 44pp.; Cornell Aeronautical Lab. Report No. XM-2464-G-1.

Eckler, A. R., 1969: A survey of coverage problems associated with point and area targets. Technometrics, 2, 561-589.

Gifford, F., 1959: Statistical properties of a fluctuating plume dispersion model, Adv. in Geoph. , 6, 117-138.

Gifford, F., 1960: Peak to average concentration ratios according to a fluctuating plume dispersion model. Int. J. of Air Poll., 3, 253-260.

Groenwoud, C., D. H. Hoaglin, and J. A. Vitalis, 1967: Bivariate normal offset circle probability tables, Vols. I and II. Cornell Aeronautical Lab. Report No. XM-2464-G-1, xxi and 1320pp.

Montgomery, T. L., 1971: Paper presented at American Meteor. Soc. Conf. on Air Pollution Meteorology, Apr. 1971.

Pasquill, F., 1962: Atmospheric Diffusion, xii and 297pp, D. Van Nostrand Co., London.

Slade, D. (Editor), 1968: Meteorology and Atomic Energy 1968, x and 445pp.

Legends

Figure 1. Geometry of a "top-hat" fluctuating plume model.

Figure 2. Probability, Pr, of a given relative concentration value X/Q, as a function of (nondimensional) plume radius, $R/(\overline{D_y^2})^{1/2}$, "neutral" conditions $(\overline{D_z^2} / \overline{D_y^2})^{1/2} = 1$.

Figure 3. Probability, Pr, of a given relative concentration value X/Q, as a function of (nondimensional) plume radius, $R/(\overline{D_y^2})^{1/2}$, "stable"conditions $(\overline{D_z^2} / \overline{D_y^2})^{1/2} = 0.5$.

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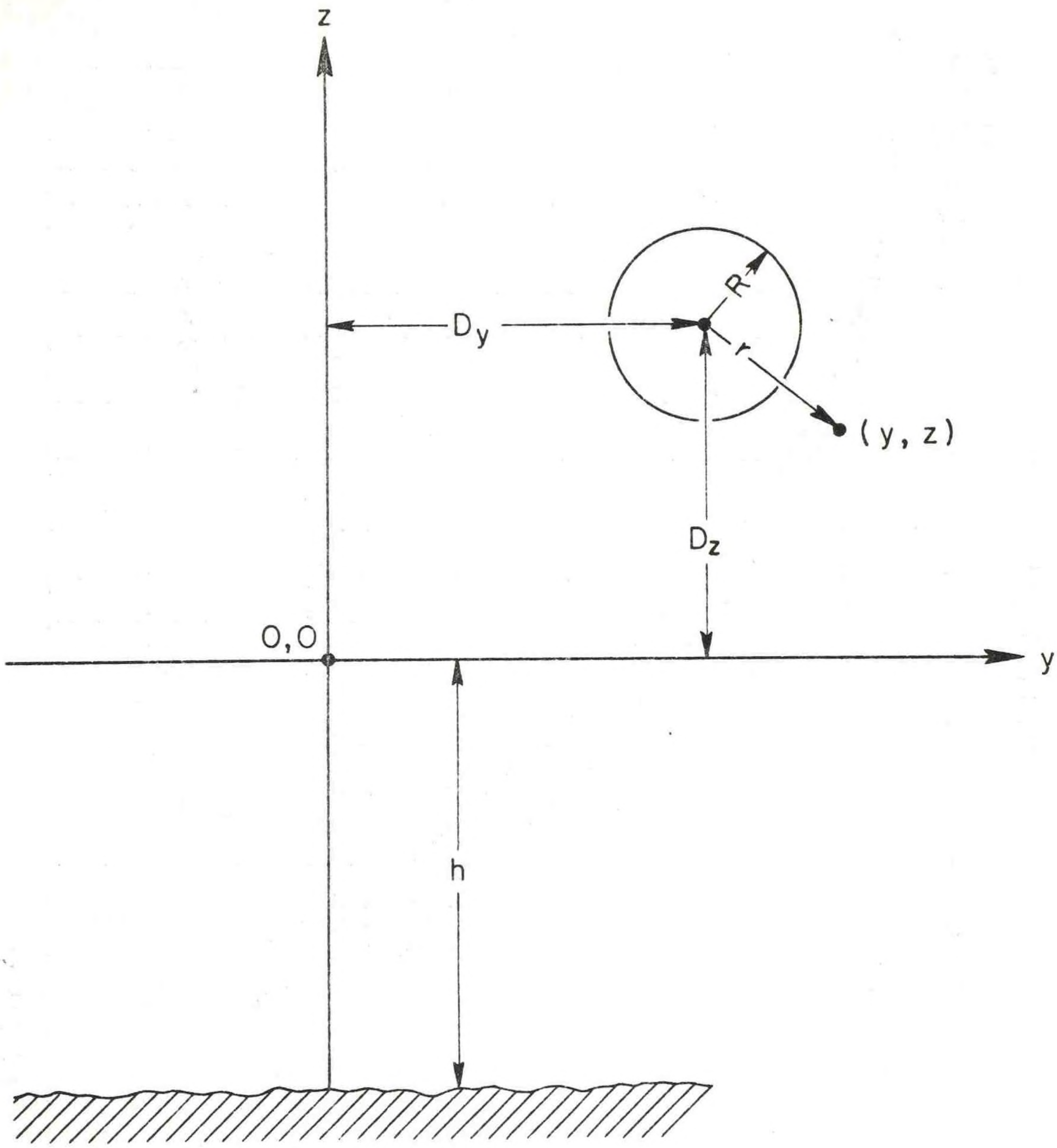

Figure 1. Geometry of a "top-hat fluctuating plume model.

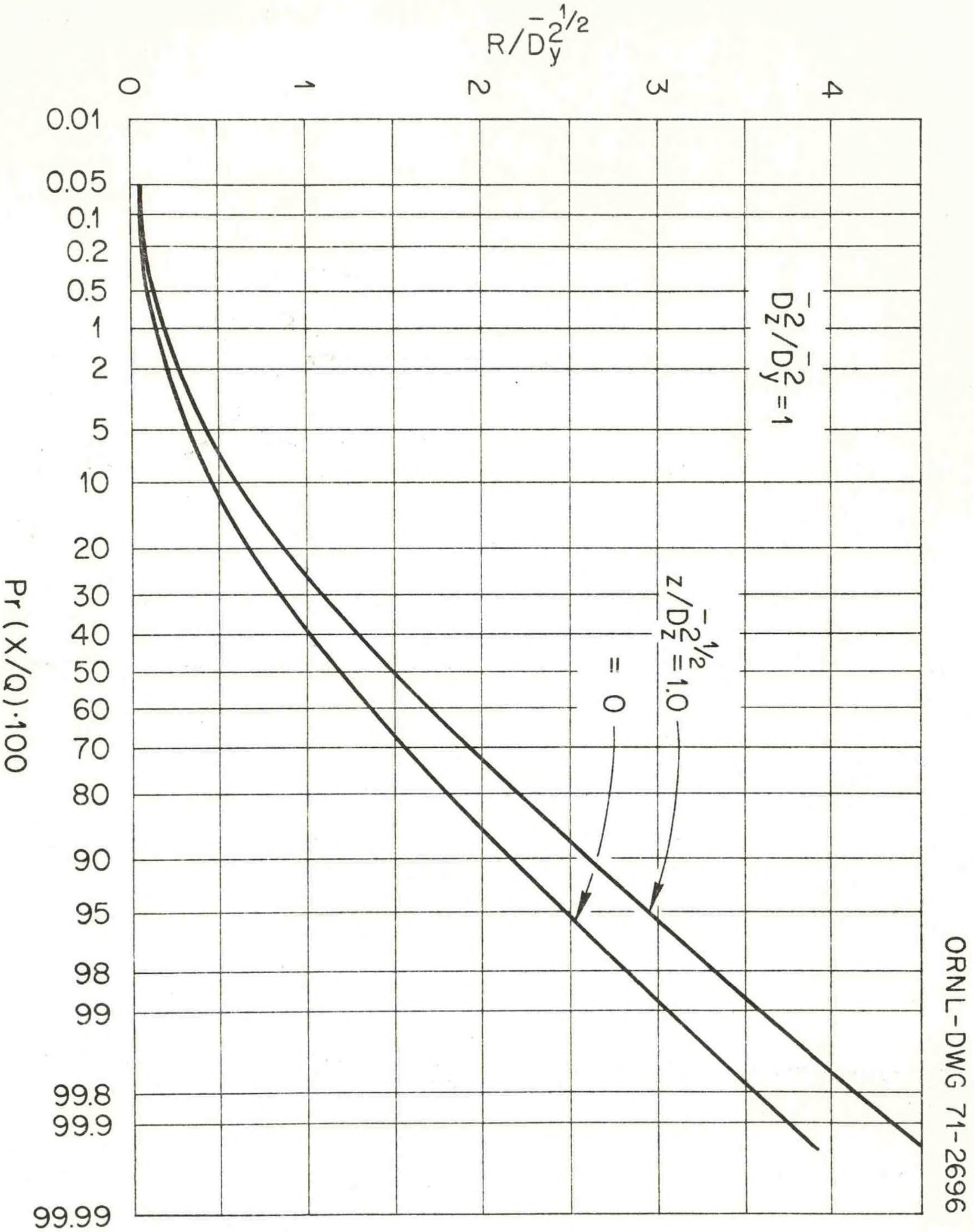

Figure 2. Probability, Pr, of a given relative concentration value X/Q, as a function of (nondimensional) plume radius, $R/(\overline{D_y^2})^{1/2}$ "neutral" conditions $(\overline{D_z^2} / \overline{D_y^2})^{1/2} = 1$.

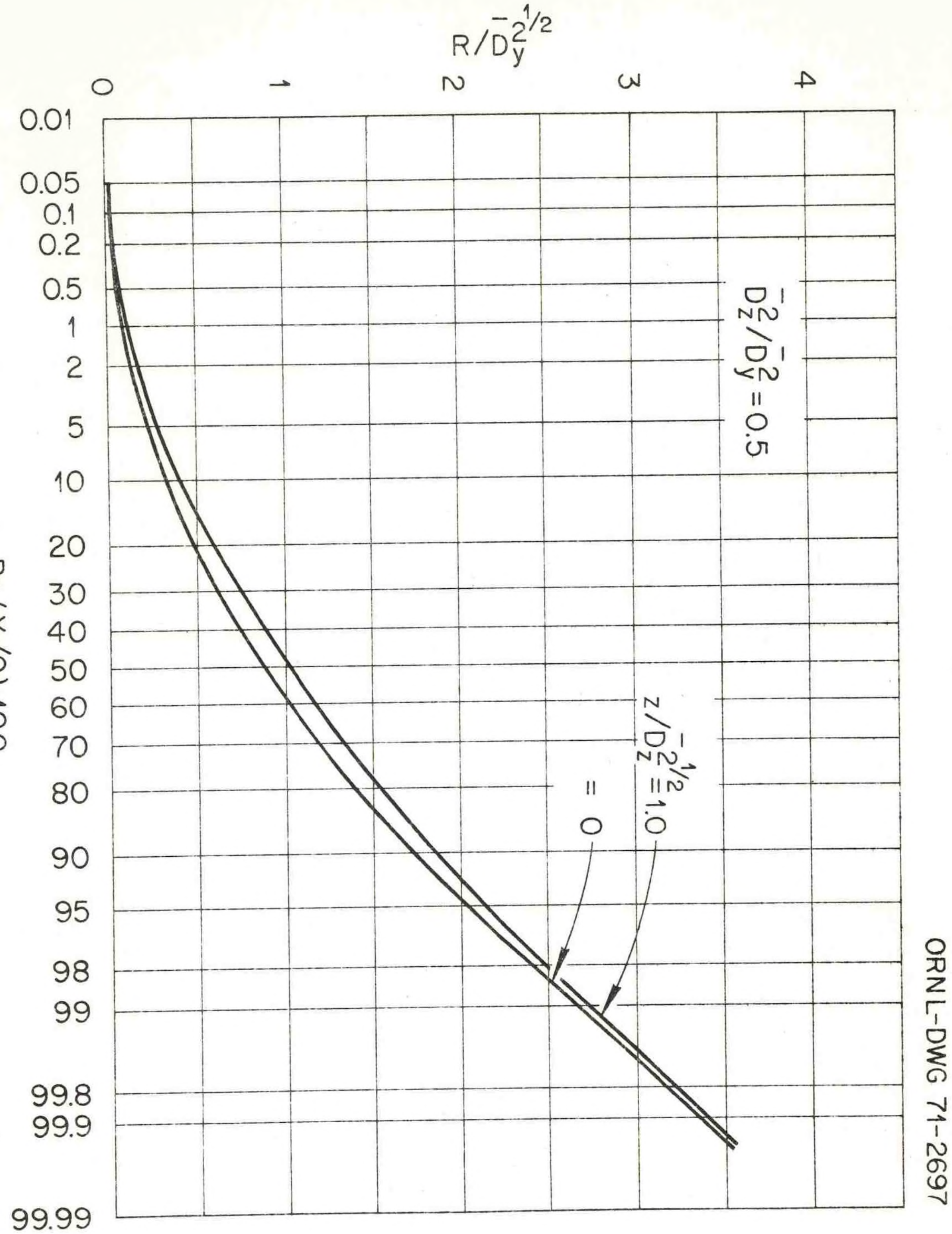

Figure 3. Probability, Pr, of a given relative concentration value X/Q, as a function of (nondimensional) plume radius, $R/(\overline{D_y^2})^{1/2}$, "stable" conditions $(\overline{D_z^2} / \overline{D_y^2})^{1/2} = 0.5$.