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SOME RECENT ANALYSES OF PLUME RISE OBSERVATIONS

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ABSTRACT

This paper is a summary of some rather extensive comparisons of plume rise observations with plume rise formulas.[1,2] The conclusions drawn from these comparisons should be useful to meteorologists and engineers who need to estimate the dispersion of gaseous effluents that, owing to vertical momentum or buoyancy, rise above their height of emission.

It is shown that the rise of non-buoyant jets is proportional to the one-third power of the distance downwind of the source, as is predicted by a simple adaptation of the Morton, Taylor and Turner model to include the effect of wind. The "entrainment constant" appears to depend on the ratio of efflux velocity to mean wind speed. This dependence disappears when buoyancy forces begin to dominate the rise, as is shown by data on buoyant plumes which are initially momentum-dominated. A simple, semi-empirical expression is proposed to approximate this transition to buoyancy-dominated rise.

Various formulas for the rise of buoyant plumes in stable, windy conditions are compared with observations; quite clearly, the best agreement is obtained with a simple formula derived

from the modified Morton, Taylor and Turner model. This model is extended to neutral conditions by means of an inertial range atmospheric turbulence entrainment (IRATE) assumption, and is shown to give better agreement with observations than older formulas.

Introduction

Good estimates of plume rise are required to predict the dispersion of continuous gaseous emissions having large buoyancy or a high efflux velocity. The rise of such emissions above their source height often accounts for a considerable reduction of the concentrations experienced at the ground.

According to a recent critical survey[1] on the subject, several dozen programs of plume rise observations have been carried out and the results published. This alone does not solve the problem, however. The quality of these observations varies considerably, and in some cases important parameters were not measured. The picture is also blurred by the presence of turbulence in the atmosphere, which causes the plume rise to fluctuate rapidly in many situations. The great number of empirical plume rise formulas in the literature reflect these uncertainties. Each formula is based on an analysis of one or more sets of observations, but each time a different style of analysis or a different collection of observations is used, a different empirical formula results. When applied to new situations, the predictions of these formulas sometimes differ by a factor of ten. Obviously, great care in the analysis of available observations is required.

The present paper is a summary of some of the analyses of observations made in my recent critical review on plume rise[1] and in my doctoral dissertation[2]. Both of these works include extensive comparisons of observations with formulas; care was taken to categorize the data according to the type of source and the meteorological conditions, and to weight the data according to the quality and quantity of observations they represent.

Momentum Conservation and the "1/3 Law" For Jets

One of the major findings of researchers in the field of plume rise is that the radius of a plume bent over in a wind is approximately proportional to the rise of the plume centerline above its source height.[1,2,3,4] This is true for a considerable distance downwind of the source, at least several stack heights. Mathematically, we can express this by

$$r = \gamma z \qquad (1)$$

where r is a characteristic plume radius, γ is a constant (dimensionless), and z is the rise of the plume centerline. Surprisingly, this simple relationship accounts very well for the great bulk of observed plume rises, when it is used with appropriate conservation assumptions.

For instance, no outside forces act on a non-buoyant plume (jet) rising through unstratified surroundings, so we might expect that the total flux of vertical momentum in the plume is conserved. If the plume is only slightly inclined above the horizontal, is nearly the same density as the ambient air, ρ, and has a horizontal component of motion nearly equal to the mean wind speed at that height, u, then the flux of mass is approximately $\rho\pi r^2 u$. The flux of vertical momentum is then $w(\rho\pi r^2 u)$, where $w = u dz/dx$, the vertical velocity of the centerline of a plume segment moving downwind at a speed u; x is the distance downwind of the source. We then have

$$w\ u\ r^2 = u^2 \gamma^2 z^2 dz/dx = F_m = \text{constant}, \qquad (2)$$

where F_m is the initial vertical momentum flux divided by $\pi\rho$. For a jet having the same density as the ambient air, which must be true if it is non-buoyant, F_m is given by

$$F_m = w_o^2 r_o^2 = w_o^2\ D^2/4 \quad , \qquad (3)$$

where w_o is the mean efflux velocity, r_o is the radius of the stack, and D is its diameter, assuming a circular, vertical source. Integration of Equation 2 yields the prediction that

$$z^3 = (3F_m/\gamma^2 u^2)x$$

$$\Delta h = z = (3\ w_o^2\ D^2/4\gamma^2 u^2)^{1/3}\ x^{1/3} \qquad (4)$$

$$\Delta h/D = (3/4\gamma^2)^{1/3}\ R^{2/3}\ (x/D)^{1/3} \quad ,$$

where $R = w_o/u$, the ratio of the efflux velocity to the wind speed. In this paper, the "plume rise," Δh, is identified with the height of the plume centerline above the source ($\Delta h = z$).

The above prediction that the rise of a jet is proportional to the one-third power of distance downwind, the "1/3 law," is very well confirmed by the available observations on jets. They are plotted in Figure 1 for data in which $R \doteq 2$, 4, 8, 16 and 40. (the code identifying the six experiments is given in Reference 2). Surprisingly, the dotted lines representing the "1/3 law" give fair agreement with observations even in the upper-left part of the figure, where the plumes are more nearly vertical than horizontal (the derivation of Equation 4 utilizes the assumption that the plumes are only slightly inclined). However, the data indicate a stronger dependence on R than the two-thirds power. Specifically, the dotted lines represent Equation 4 with

$$\gamma = 1/3 + R^{-1} \quad . \qquad (5)$$

Thus, it appears that the "entrainment constant," γ, varies with the ratio of efflux velocity to wind speed for a jet. This turns out not to be true for a buoyant plume. Substituting this expression for γ into Equation 4, we have finally

$$\Delta h/D = 1.89 \left(\frac{R}{1+3R^{-1}}\right)^{2/3} (x/D)^{1/3} \quad . \qquad (6)$$

<u>Buoyancy Conservation and the "2/3 Law" For Buoyant Plumes</u>

If a buoyant plume is rising through unstratified surroundings and it neither gains nor loses buoyancy through radiation, ordinarily the total flux of buoyancy is conserved (for an exception, see Reference 5). Applying Newton's Second Law to a segment of a bent-over plume moving downwind with the mean speed of the wind, we find that the rate of vertical momentum flux increase equals the buoyancy flux:

$$d(w\ u\ r^2)/dt = u d(w\ u\ r^2)/dx = b\ u\ r^2 \quad , \qquad (7)$$

where b is a characteristic buoyant acceleration of the plume and bur^2 is the buoyancy flux divided by $\pi\rho$. The initial value of bur^2 is given by

$$F = g(1-\rho_o/\rho)\ w_o r_o^2 \quad , \qquad (8)$$

where g is gravitational acceleration and ρ_o is the density of the effluent at the stack. A better determination of F, that accounts for alteration of buoyancy due to dilution with ambient air, is given by

$$F = g(1-m_o/m)\ (T/T_o) w_o r_o^2 + gQ_H/(\pi c_p\ \rho T) \quad , \qquad (9)$$

where m is the mean ambient molecular weight (28.9), T is the ambient absolute temperature, c_p is the specific heat capacity of air (0.24 cal/gm - °K), and Q_H is the heat emission; subscript "o" denotes values for the efflux gas, instead of the ambient air. The quantity $g/(\pi c_p \rho T)$ is just a constant, $3.7 \cdot 10^{-5}$ (m^4/cal-sec^2), times the ratio of standard sea level pressure to the ambient pressure. If the effluent has considerable latent heat due to water vapor and condensation of the plume is likely to occur near the source, as would be expected in cold or wet weather, this latent heat may be included in the determination of Q_H; otherwise, only dry heat should be considered. If the process producing the effluent is uniform, F is proportional to the rate of production. For instance, for modern fossil fuel power plants, F is about 1.5 m^4/sec^3 times the megawatts per stack generated.

When $bur^2 = F =$ constant, Equation (7) integrates to

$$wur^2 = F_m + F\,x/u \qquad (10)$$

This relation implies that buoyancy becomes more important than the initial momentum flux when $x > uF_m/F$. For a hot effluent with about the same heat capacity and mean molecular weight as air, this occurs at $x = uw_o/(g(T_o/T - 1))$, a distance of the order of 5 seconds times the wind speed for most hot plumes. Then the effect of F_m quickly becomes negligible, and for the region in which Equation (1) applies we have

$$z^3 = (3F/2\gamma^2u^3)x^2$$

$$\Delta h = z = (3F/2\gamma^2)^{1/3}\,u^{-1}\,x^{2/3} \qquad (11)$$

$$\Delta h/L = (3/2\gamma^2)^{1/3}\,(x/L)^{2/3}$$

where $L = F/u^3$, a characteristic length for the rise of buoyant plumes.

Most of the observations available on buoyant plume rise approximate this "2/3" of rise with distance downwind. This is illustrated in Figure 2, which is a superposition of curves hand drawn through scatter diagrams of $\Delta h/L$ versus x/L for 16 individual sources[1] (the sources are identified in Reference 1). Only data for stable atmospheric conditions are omitted. There is considerably greater scatter about the "2/3 law" in this figure than there is about the "1/3 law" for jets in Figure 1. The difference is probably due to the fact that all the experiments represented in Figure 1 were made under controlled conditions in wind tunnels or modeling channels, while all the observations shown in Figure 2 were made on plumes from real stacks in the atmosphere. This introduces the possibility of aerodynamic effects caused by buildings near the stack and uneven terrain, and also assures greater fluctuations about the mean plume rise due to large, turbulent eddies in the atmosphere. Atmospheric turbulence should also lead to more rapid mixing of plumes with ambient air, and therefore a downward departure from the "2/3 law" should occur at some point downwind; however, Figure 2 offers no particular support for this expectation. This means either that L does not correlate well with the distance of downward departure ("leveling off"), or that "leveling off" occurs at greater values of x/L than those measured up until now.

There is no evidence that γ is dependent on R for buoyant plumes,[6] at least when $R > 1.2$. Below this value, downwash of the plume into the low pressure region in the wake of the stack is likely to occur. The reciprocal wind speed relationship predicted by Equation 11 with γ constant is well established[1] for buoyant plumes in neutral conditions. An analysis[1] of photographed plume diameters and concurrent plume rises of TVA plumes from single stacks showed that $\gamma \doteq 0.5$. Bringfelt[3] obtained similar results, finding an average value of γ of 0.53 for eleven plumes in slightly stable or windy conditions and 0.46 for ten plumes in strongly stable or weak wind conditions. The behavior of buoyant plumes in stable conditions is well predicted by $\gamma = 0.5$, as will be shown, but the optimum fit[1] to the "2/3 law" at large distances downwind in neutral and unstable conditions corresponds closer to $\gamma = 0.6$, or

$$\Delta h = 1.6\ F^{1/3}\ u^{-1}\ x^{2/3} \quad . \qquad (12)$$

<u>Transition to Buoyancy-Dominated Rise</u>

Equation 10 implies that a transition from the "1/3 law" for momentum-dominated rise to the "2/3 law" for buoyancy-dominated rise occurs as Fx/uF_m grows from small to large values. This prediction is confirmed by Figure 3, which plots observations of plume rise modeled in a channel by Fan.[7] The rises are divided by $L_m^{2/3}\ x^{1/3} = F_m^{1/3} u^{-2/3}\ x^{1/3}$, so there should be no variation with $Fx/uF_m = Lx/L_m^2$ in the momentum-dominated region. This is seen to be approximately true in the left-hand side of the figure, where $Lx/L_m^2 < 0.3$. However, there is a separation of the points for different values of R in this region; this is easily accounted for by letting $\gamma = 1/3 + R^{-1}$, as was done for jets. There is a clear upswing and some convergence of the points in the right-hand side of the figure, and they appear to asymptotically approach the line representing the "2/3 law," Equation 11, with $\gamma = 0.5$.

The simplest way to describe this transition mathematically is to integrate Equation 10, after substituting $r = \gamma z$ and $w = udz/dx$, and then substitute the empirical value of γ for jets in the momentum term ($\gamma = 1/3 + R^{-1}$) and the empirical value for γ for buoyant plumes in the buoyancy term ($\gamma = 0.5$). The result is

$$\Delta h = z = 3^{1/3} \left(\frac{F_m x}{(1/3+R^{-1})^2 u^2} + \frac{Fx^2}{2(0.5)^2 u^3} \right)^{1/3} . \qquad (13)$$

This equation is represented as solid lines in Figure 3, for R = 4, 8 and 16. It is seen to describe the transition region fairly well.

Stability-Limited Rise

When a plume rises in a stable environment, it entrains air and carries it upwards into regions of relatively warm ambient air. Eventually, the plume's buoyancy becomes negative and its rise is terminated. If heat is conserved, that is, the motion is adiabatic, the rate at which each plume element loses temperature relative to the ambient temperature is just its rate of rise times the ambient potential temperature gradient (potential temperature, θ, is the temperature that air would acquire if it were compressed adiabatically to a standard pressure, usually the mean pressure at sea level; the potential temperature gradient is just the real temperature gradient plus the adiabatic lapse rate, i.e., $\partial\theta/\partial z = \partial T/\partial z + 1\ ^{\circ}C/100m$ in our lower atmosphere). The resulting decay of the buoyancy flux is expressed by

$$d(bur^2)/dt = u d(bur^2)/dx = - w\ sur^2 \qquad (14)$$

where $s = (g/T)\ \partial\theta/\partial z$.

If we differentiate Equation (7) with respect to t and substitute Equation (14), we find that

$$d^2(wur^2)/dt^2 = u^2 d^2(wur^2)/dx^2 = - s(wur^2) \quad . \qquad (15)$$

This equation establishes the fact that $s^{-1/2}$ is a characteristic time for the decay of the momentum flux. If s is positive and approximately constant with height, the momentum flux is a harmonic function of $s^{1/2}t$:

$$wur^2 = F_m \cos(s^{1/2}t) + s^{-1/2} F \sin (s^{1/2}t) \quad . \qquad (16)$$

Since r always increases, the plume centerline behaves like a damped harmonic oscillator. If the wind speed is constant with height, a jet ($F \doteq 0$) reaches its maximum rise at $x = ut = (\pi/2)\, us^{-1/2}$, and a buoyant plume ($F_m \doteq 0$) reaches its maximum rise at $x = \pi us^{-1/2}$.

The above conclusions are based on conservation assumptions alone, and do not depend on the behavior of r. Equation 1, $r = \gamma z$, is still a good approximation for r up to the point of maximum rise; obviously, it cannot apply beyond this point, as it would imply a shrinking plume. With u constant, $w = udz/dx$, and $r = \gamma z$, Equation (16) can easily be integrated. For a jet we find that

$$\Delta h = z = \left(\frac{3Fm}{\gamma^2 us^{1/2}}\right)^{1/3} (\sin\ (x/x'))^{1/3}$$

$$= 3\ (1+3R^{-1})^{-2/3} \left(\frac{Fm}{u\ s^{1/2}}\right)^{1/3} \quad \text{(maximum rise)}\ , \qquad (17)$$

where $x' = us^{-1/2}$ and $\gamma = 1/3 + R^{-1}$. For a buoyant plume we find that

$$\Delta h = z = \left(\frac{3F}{\gamma^2 us}\right)^{1/3} (1 - \cos (x/x'))^{1/3} \qquad (18)$$

$$= 2.9 \left(\frac{F}{us}\right)^{1/3} = 2.9\ L^{1/3}\ x'^{2/3} \text{ (maximum rise).}$$

There are sufficiently detailed data to verify Equation (18), which is shown as dotted lines in Figures 4 and 5. Both these figures show plume rise, divided by $L^{1/3}\ x'^{2/3}$, versus x/x'. To determine s, the measured potential temperature gradients were averaged throughout the layer of plume rise (from the top of the stack to the top of the plume). The first figure shows centerlines of TVA[8] plumes from single stacks that were observed to level off in stable air. It also shows the observed rises of the plume tops. The second figure shows the longest plume centerlines observed in very stable air by Bringfelt.[3] Both of these figures give excellent support to the prediction that the maximum rise is obtained at $x = \pi x'$. There is only a little evidence of oscillation beyond this point; evidently, most plume centerlines experience considerable damping through continued mixing beyond this point. The leveled-off plume rises, which range from 140 to 290m for the TVA data and from 60 to 160m for the Bringfelt data, seem to be well approximated by Equation (18) on the average. For the TVA data, the scatter about the predicted rise seems to be greatest in the rising stage, which approximates the "2/3 law" when $x < 2x'$. There is less scatter in this stage in the Bringfelt data, which utilized more representative wind speed measurements and were taken during much more stable conditions.

Reference 2 also compares these observations with other formulas for buoyant plume rise in stable conditions, namely: the Holland formula,[9] minus 20% as suggested for stable conditions; Priestley's equation,[10] reduced to the case of a buoyant, point source; Bosanquet's formula,[11] similarly reduced; and Schmidt's formula,[12] with his parameter "m" set equal to 0 and 1/2. The centerline plume rises at a standard distance $x = 5x'$ were interpolated and averaged for periods in which there were at least five photographs of the plume at this distance. This yielded five periods each from the TVA and the Bringfelt observations. The ratios of calculated to observed plume rises were then calculated for each formula and each period. The median ratio and the average percentage deviation from the median ratio for each formula are shown in Table 1.

Table 1

Ratios of Calculated to Observed Plume Rises in Stable Conditions

Formula	Bringfelt	TVA	Bringfelt & TVA
Holland	0.33 ± 73%	0.81 ± 7%	0.72 ± 39%
Priestley	0.74 ± 22%	0.44 ± 5%	0.47 ± 35%
Bosanquet	1.09 ± 24%	1.22 ± 12%	1.20 ± 18%
Schmidt, m=0	0.29 ± 7%	0.28 ± 24%	0.28 ± 16%
Schmidt, m=1/2	0.94 ± 27%	0.85 ± 25%	0.90 ± 27%
Equation 18	0.89 ± 7%	0.96 ± 8%	0.93 ± 8%

The TVA heat emissions were substantially higher than those observed by Bringfelt, so the high percentage deviation exhibited by the Holland and Priestley formulas may be because of too much and too little predicted dependence of rise on heat emission, respectively. Clearly, Equation 18 for maximum rise gives the most consistently good predictions for buoyant, stability-limited plume rise.

Turbulence - Limited Rise

The simple plume rise model outlined in the preceding section, based on $r = \gamma z$ and conservation assumptions, succeeds in predicting the approximate rise behavior of all available observations of plumes bent over in a wind. It is very similar to the successful model for nearly vertical plumes suggested by G. I. Taylor in 1945 and later developed by Morton, Taylor and Turner. However, as it stands, it predicts unlimited rise in neutral ($s = 0$) and unstable environments ($s < 0$). This is contrary to the expectations of many plume rise observers, some of whom have assumed that the plume rise is the rise of the plume at the point that it becomes hard to follow. Yet, no observations made so far show any leveling-off tendencies, except in stable conditions.[1]

Nonetheless, it is quite reasonable to expect more rapid growth of the plume radius in neutral and unstable conditions, due to the presence of considerable environmental turbulence. This, in turn, leads to a reduced rise velocity and perhaps to a limited plume rise, at least in neutral conditions. The question is how to account for the enhanced growth of the plume radius. One way is to assume that only eddies of the same order of size as the plume radius are effective at mixing ambient air into the plume, and that these eddies are predominantly in the inertial subrange of the atmospheric turbulence spectrum. This part of the spectrum is adequately characterized by the eddy energy dissipation rate, ϵ, and eddies of the order of r in size have velocities of the order of $\epsilon^{1/3} r^{1/3}$. This suggests the relationship

$$dr/dt = \beta \epsilon^{1/3} r^{1/3} , \qquad (19)$$

where β is a constant (dimensionless). This should not apply at small distances, where r is small and w is large; Equation 1 gives a faster growth rate at first ($r = \gamma z$ implies that $dr/dt = \gamma w$). In References 1 and 2, I developed a model identical to the one outlined so far, except for the assumption that Equation 19 applies instead of Equation 1 beyond the distance at which $\beta\epsilon^{1/3}r^{1/3}$ becomes equal to γw. Since this model is based on an inertial range atmospheric turbulence entrainment assumption, I called this the "IRATE" plume rise model.[2]

For rise in neutral conditions, in which $bur^2 = F =$ constant, the "IRATE" model predicts a very gradual leveling of the plume centerline beyond the distance that $\beta\epsilon^{1/3}r^{1/3}$ becomes equal to γw, designated by x^*. For a buoyant plume, the "2/3 law," Equation 11, applies to the first stage of rise. The distance of transition to the second stage is then given by

$$x^* = (2/3)^{7/5}(\gamma F)^{2/5}\, u^{3/5}\, (\beta\varepsilon^{1/3})^{-9/5} \qquad (20)$$

If ϵ is approximately constant above the height at which this transition occurs, the second stage rise ($x > x^*$) is given by

$$\Delta h = (3F/2\gamma^2)^{1/3} u^{-1} x^{*2/3} \left[\frac{55}{16} - \frac{243}{32} \frac{(x/x^* + 5/8)}{(x/x^* + 5/4)^2} \right]. \qquad (21)$$

While this not a very simple formula, note that it is just the "2/3" law times a function of x/x^* . A final rise, equal to 55/16 times the rise at the transition point, is approached, but only at a very great distance; 90% of the asymptotic rise is achieved at $x = 20x^*$, and x^* can be as large as a kilometer for a very buoyant or very high source. It is likely that the maximum ground concentration occurs well before this point, especially since Equation 19 predicts an extremely large radius at $x = 20x^*$. If $\gamma = 0.5$ and the bottom of the plume is taken to be a distance $(\Delta h - r)$ above the source height, we find that the plume bottom begins to descend at about $x = 2x^*$ and spreads down to the source height $(r = \Delta h)$ at $x = 5x^*$. Since the growth of the plume radius is quite rapid at this point, the highest ground concentration should ordinarily occur in this neighborhood. It therefore seems prudent to use the rise at $x = 5x^*$, 2.3 times the rise at the transition point, as the "final" plume rise in neutral conditions. This rise is the same as that given by the "2/3 law" with $x = 3.5x^*$, and Equation 21 deviates from the "2/3 law" by only 11% at $x = 3.5x^*$. This suggests a much more practical prediction procedure for buoyant plume rise in neutral conditions:

$$\Delta h = (3F/2\,\gamma^2)^{1/3}\, u^{-1}\, x^{2/3} \qquad \text{when } x < 3.5x^*$$

$$\Delta h = (3F/2\,\gamma^2)^{1/3}\, u^{-1}\, (3.5x^*)^{2/3} \qquad \text{when } x > 3.5x^*. \qquad (22)$$

With this approach, in a simple way we recognize the observed fact that plume rise is substantially a function of distance, yet we have a usable "final" rise formula.

In order to use the formulas based on the "IRATE" model, an estimate of x^* is needed; this requires values of β and ϵ for substitution into Equation 20. In Reference 1, β was conservatively estimated to be about unity, on the basis of observations on the growth rates of puffs and particle clusters (in order to infer the value of β from Equation 19, it was necessary to estimate values of ϵ, as this quantity was not measured in the puff and particle experiments). It is well known that $\epsilon = 2.5u^{*3}/\bar{z}$ in the neutral surface layer,[13] but this layer extends only to a height of the order of $10^{-2}u^*/f$ in neutral conditions (u^* is the friction velocity, $\bar{z}$ is the height above the ground, and f is the Coriolis parameter); in mid-latitudes, this height is of the order of 10 seconds times the wind speed. However, most plumes rise to heights above this layer, where ϵ is less dependent on wind speed and height.

In a convective mixing layer, such as exists the lowest few hundred to few thousand meters on any sunny, non-windy day, the average value of ϵ is about $(1/2)\ gH/(c_p \rho T)$, where H is the heat flux transported upwards from the ground;[2] a fairly strong heat flux, 1 cal/cm^2-min, corresponds to $\epsilon = 30\text{cm}^2/\text{sec}^3$; the eddy dissipation rate is relatively constant with height, except near the ground, where the neutral surface layer expression dominates. Less is known about the variation of ϵ above the surface layer in neutral conditions. If ϵ ceased to decrease with height at the top of this layer, it would be proportional to fu^{*2}, which is approximately proportional to fu^2. In Reference 1, measurements of ϵ at heights from 15 to 1200m were shown to fit $\epsilon \propto u$ slightly better than $\epsilon \propto u^2$ or ϵ = constant (measurements made in stable or convective conditions were excluded from this analysis). This result was especially convenient for application to the IRATE plume rise model, as $\epsilon \propto u$ cancels out the wind speed in Equation 20; above the neutral surface layer, x^* is approximately independent of u. However, the great variation in ϵ at these heights, of the order of $\pm 50\%$, can be expected to account on considerable variations in the plume rise.

In the above analysis, definite decrease of ϵ values with height above the ground was also noted. Above a height somewhere between 100 and 300m the variation become much less. The best fit in the 15 to 300m range was given by

$$\epsilon = 0.068 \ (m/sec)^2 \ u/\bar{z} \ . \qquad (23)$$

Substituted in Equation 20, this gives

$$x^* = 2.16 \ m(F/ \ m^4/sec^3)^{2/5} \ (\bar{z}/m)^{3/5}. \qquad (24)$$

In References 1 and 2, it was suggested that conservative values of x^* and Δh would result by evaluating ϵ at the source height, h_s. Thus, $\bar{z} = h_s$, but no more than $\bar{z} = 300m$, was substituted in Equation 24; for the present, let us designate this estimation of x^* by x_1^*. A simpler estimate of x^* is suggested by a plot of <u>total</u> plume height, $h_s + \Delta h$, versus buoyancy flux. Using TVA,[8] CERL,[14,15] Bringfelt[16] and other observations, a quite conservative value for $\bar{z}$ is given by

$$\bar{z} = 22 \ m \ (F/ \ m^4/sec^3)^{3/8} \leq \ 100 \ m \ . \qquad (25)$$

The 100m maximum value of $\bar{z}$ may be overly conservative when the stack height itself is greater than 100m, but consider also the fact that ϵ does not diminish very much with height above this elevation. The predicted final value for Δh only depends on $\bar{z}^{0.4}$, and the scatter in the few plume rise data at large distances renders tentative any conclusion about the best evaluation of x^* for these purposes. Equations 23 and 25 give a particularly simple way to evaluate x^*, and can even be applied to ground sources without difficulty. Let us designate this estimate of x^* by x_2^*:

$$x_2^* = 14\text{m}\,(F/\,\text{m}^4/\text{sec}^3)^{5/8} \qquad \text{when } F < 55\ \text{m}^4/\text{sec}^3$$

$$x_2^* = 34\text{m}\,(F/\,\text{m}^4/\text{sec}^3)^{2/5} \qquad \text{when } F > 55\ \text{m}^4/\text{sec}^3. \tag{26}$$

A number of plume rise formulas for buoyant plumes in neutral conditions were compared with all available observations in Reference 1; TVA[8] and CERL[14, 15] data each comprised about one-third of the data analyzed. The more recent observations by Bringfelt[3,16] were added to the comparisons in Reference 2. A similar analysis is summarized here in Table 2. Since the relationship $\Delta h \propto u^{-1}$ seemed well verified[1] for neutral conditions, the average value of $u\Delta h$ for each source was calculated at the greatest distance downwind that was represented by a least three 30-120 min periods of observations with at least five Δh determinations each. The Bringfelt data had to be handled differently, because there was only one period of observation for many of the sources; accordingly, they are weighted only one-third as much as the "Reference 1" observations in the last column of Table 2. When appropriate measurements were available, it was required that $x \leq 2x'_.$, in order to exclude cases of stability-limited plume rise. Cases of probable downwash, terrain effects, etc. were eliminated in "select" set of observations in Reference 1, and agreement with almost every formula improved. Of 25 periods chosen by Bringfelt for analysis,[3] only 12 are selected here (periods 7, 8, 11b, 15, 18, 27a, 27b, 31, 29, 41, 47a, and 27b); the periods rejected greatly increase the scatter in the ratios of calculated to observed values for every formula tested, tending to obscure the comparison. Table 2 shows the median value of the ratio of calculated to observed plumes rises, and the average percentage deviation of these ratios from the median, for eight different formulas of the $\Delta h \propto u^{-1}$ type. The last column combines the two sets of select data, with appropriate weighting.

Table 2

Ratios of Calculated to Observed Plume Rises in Neutral Conditions

Formula	Reference 1 (all data)	Reference 1 (select)	Bringfelt (select)	Weighted, Select Data
Holland[9]	0.44 ± 37%	0.47 ± 26%	0.26 ± 32%	0.40 ± 35%
Stümke[17]	0.79 ± 27%	0.72 ± 24%	0.82 ± 35%	0.74 ± 29%
Moses & Carson[18]	0.54 ± 34%	0.48 ± 19%	0.53 ± 28%	0.48 ± 23%
Priestley[10,19]	1.44 ± 26%	1.41 ± 18%	1.42 ± 15%	1.41 ± 17%
Lucas, et.al.[14]	1.36 ± 21%	1.24 ± 22%	1.36 ± 13%	1.35 ± 19%
Lucas[20]	1.18 ± 20%	1.16 ± 14%	1.12 ± 17%	1.16 ± 15%
Equation 11*	1.17 ± 23%	1.17 ± 12%	1.08 ± 13%	1.17 ± 13%
Equation 22* ($x^* = x_1^*$)	1.12 ± 21%	1.13 ± 8%	0.99 ± 15%	1.11 ± 10%
Equation 22* ($x^* = x_2^*$)	1.12 ± 17%	1.13 ± 6%	1.00 ± 13%	1.11 ± 9%

*with $\gamma = 0.5$

Of the first three formulas, which are empirical, it is seen that the Moses and Carson[18] formula in which $\Delta h \propto Q_H^{1/2}$ gives the most consistent fit to the select data; the fit would be optimized by multiplying by a correction factor of 2. The next three formulas are based on the Priestley[10] model, the first being the asymptotic prediction of the first stage[19] that $\Delta h \propto Q_H^{1/4} u^{-1} x^{3/4}$. This formula and the "2/3 law," Equation 11, are very similar, and neither predict any "final" rise; yet both of these formulas give better agreement than the empirical formulas. The scatter in the Bringfelt data makes it difficult to conclude that any one of the last six formulas is superior, as about ± 15% seems to be the lowest possible scatter. The differences between the last five formulas are also slight in the Reference 1 data, except in the select set. In this set, as well as in the weighted select data, it is seen that Equation 22 gives the best fit; this equation is simply the "2/3 law," Equation 11, terminated at a distance $x = 3.5x^*$. The second estimate for x^*, x_2^*, as given by Equation 26, seems to have a slight edge over $x^* = x_1^*$; the amount of scatter and the scarcity of data at large values of x/x^* makes this comparison of $x^* = x_1^*$ and $x^* = x_2^*$ inconclusive. Which value of x^* to be preferred is mostly just a matter of convenience.

When it is not clear whether plume rise is turbulence-limited or stability limited, an analysis[2] of the IRATE model with both factors included shows that Equation 18 or Equation 22, whichever gives the <u>lowest</u> rise in a given situation, offers a good approximation of the very complicated prediction that results when both ϵ and s are greater than zero. In unstable conditions, there is no strong evidence that the average plume rise differs much from its value at the same wind speed in neutral conditions, but the rise is more variable.[1,2]

Summary and Conclusions

Direct analysis of plume rise observations and several comparisons of observations with a number of empirical and theoretical formulas have shown that very satisfactory predictions of plume rise are given by a rather spare physical-mathematical model. This model was briefly outlined here and is more rigorously developed in Reference 2; it basically consists of the assumptions that momentum, buoyancy and potential temperature are conserved, that the horizontal component of motion of plume elements is essentially equal to the mean wind speed, u, and that $r = \gamma z$ in a first stage of rise and $dr/dt = \beta\epsilon^{1/3}r^{1/3}$ in a second stage of rise (r is a characteristic plume radius, z is the rise of the plume centerline above the source height, ϵ is the eddy dissipation rate of ambient atmospheric turbulence, and γ and β are dimensionless constants). Empirical guidance is used in evaluating γ, β, and ϵ.

The assumptions that $r = \gamma z$ and that momentum is conserved in a non-buoyant plume (jet) in unstratified surroundings lead to a simple "1/3 law" of rise that fits a large variety of observed jet center lines:

$$\Delta h/D = 1.89\left(\frac{R}{+3R^{-1}}\right)^{2/3} (x/D)^{1/3}, \qquad (6)$$

where x is the distance downwind, D is the stack diameter, and R is the ratio of efflux velocity to wind speed. To derive Equation 6, one must assume that $\gamma = 1/3 + R^{-1}$. On the other hand, there is no evidence that γ is a function of R for buoyant plumes. The assumptions that buoyancy is conserved and that the initial plume momentum is neglible for a very buoyant plume in unstratified surroundings lead to the often-cited "2/3 law" of rise:

$$\Delta h = 1.6F^{1/3}\, u^{-1}\, x^{2/3}\ , \qquad (11)$$

where F is the initial buoyancy flux divided by $\pi\rho$; complete expressions for F are given by Equations 8 and 9. The constant in Equation 11 is based on the best fit to data shown in Table 2, and corresponds to $\gamma = 0.6$. Only equations that include the second stage entrainment assumption that $dr/dt = \beta\epsilon^{1/3} r^{1/3}$ give a better fit to observations of the rise of hot plumes is near-neutral conditions. For plumes in which both momentum and buoyancy are significant, Equation 13 gives a semi-empirical transition between Equations 6 and 11. Buoyancy becomes the dominant factor for most hot plumes at a distance downwind of the order of 5 seconds times the wind speed.

The assumption that the potential temperature of entrained air is conserved leads to the prediction that a buoyant plume attains a maximum rise at a distance $x = \pi u s^{-1/2}$ in stable air ($s = (g/T)\partial\theta/\partial z$, g is gravitational acceleration, T is the absolute ambient temperature, θ is the ambient protential temperature, and $\partial\theta/\partial z = \partial T/\partial z + 1°C/100m$). This prediction is very well confirmed by plots of plume rise versus distance in stable conditions ($\partial\theta/\partial z$ and u are averaged from the top of the stack to the top of the plume). These plots also indicate that the "2/3 law" is approximated when $x < 2us^{-1/2}$ and that the plume centerlines level off at a height

$$\Delta h = 2.9 \left(\frac{F}{us}\right)^{1/3} \qquad (18)$$

This corresponds to the maximum rise given by $r = \gamma z$ with $\gamma = 0.5$. Equation 18 gives substantially better agreement with observations than other formulas tested for buoyant, stability-limited rise. It should be noted that in very light winds the well-proven[1,2] formula of Morton, Taylor and Turner[21] best applies if it gives a <u>lower</u> plume rise than Equation 18:

$$\Delta h = 5.0\, F^{1/4}\, s^{-3/8} \qquad (27)$$

In neutral conditions, a limited rise results only after the second stage entrainment assumption is utilized. A good approximation to the complete prediction for buoyant plumes in neutral conditions is given by

$$\Delta h = 1.6\, F^{1/3}\, u^{-1}\, x^{2/3} \qquad \text{when } x < 3.5x^{*}$$

(22)

$$\Delta h = 1.6\, F^{1/3}\, u^{-1}\, (3.5x^{*})^{2/3} \qquad \text{when } x > 3.5x^{*} ,$$

where x^{*} is the distance of transition from the first stage to the second stage of rise. This equation gives a somewhat better fit to observations than any other formula tested when x^{*} is estimated by:

$$x^{*} = 14\text{m}\, (F/\, \text{m}^4/\text{sec}^3)^{5/8} \qquad \text{when } F < 55\ \text{m}^4/\text{sec}^3$$

(26)

$$x^{*} = 34\text{m}\, (F/\, \text{m}^4/\text{sec}^3)^{2/5} \qquad \text{when } F > 55\ \text{m}^4/\text{sec}^3$$

This equation for x^{*} should be considered tentative, since it is based on limited empirical determinations of β and ϵ, and there is too much scatter in the few observed plumes rises at large values of x/x^* to make any strong conclusions about x^*. Equations 22 and 26 apply satisfactorily to the mean rise in unstable conditions as well, and also in slightly stable conditions if they give a lower rise than Equation 18.

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ORNL-DWG 69-13875

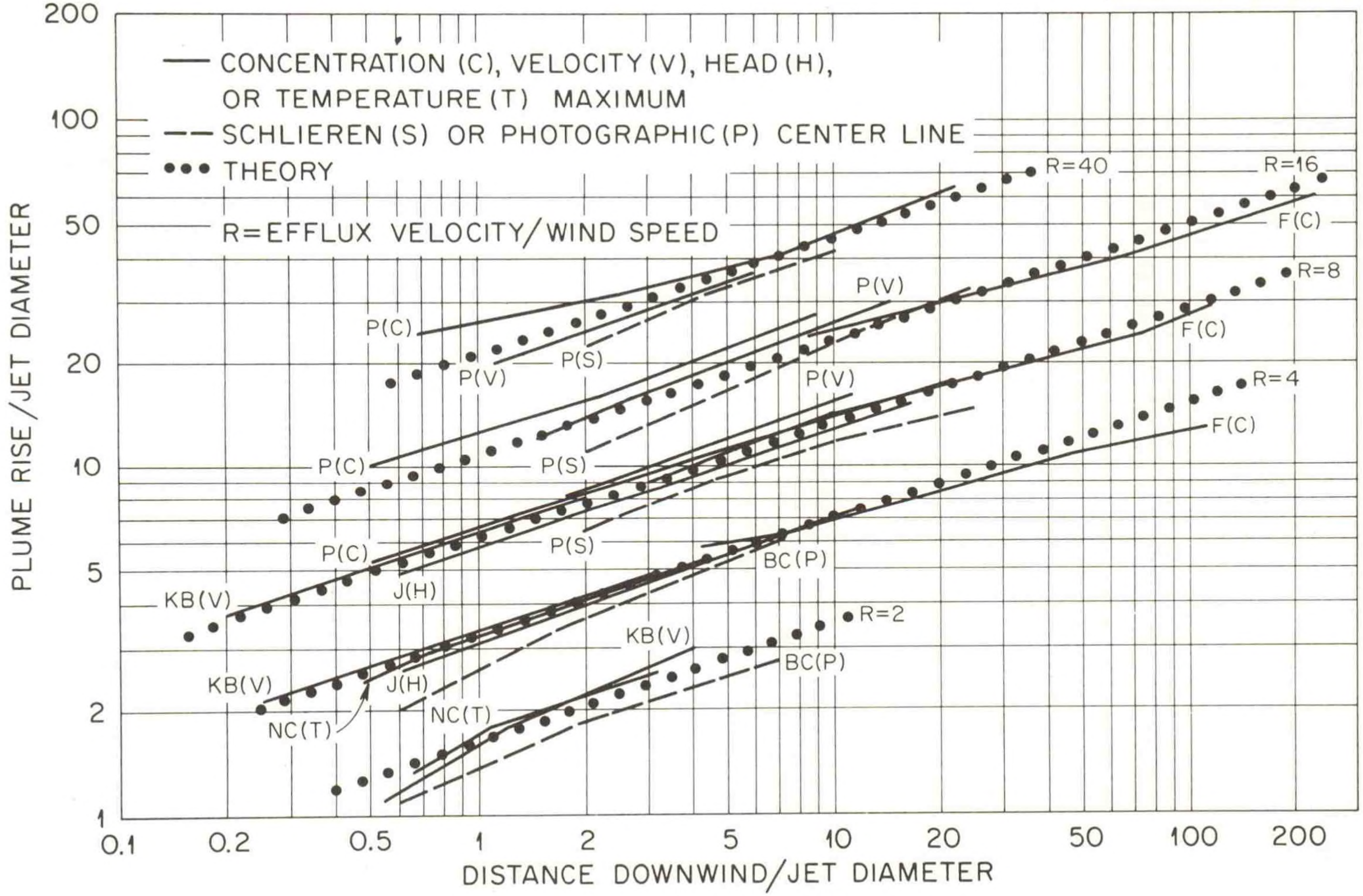

Figure 1 - Rise of jet centerlines versus distance downwind for R = 2, 4, 8, 16 and 40 (dotted lines are Equation 6) (see Reference 2 for code identifying experiments)

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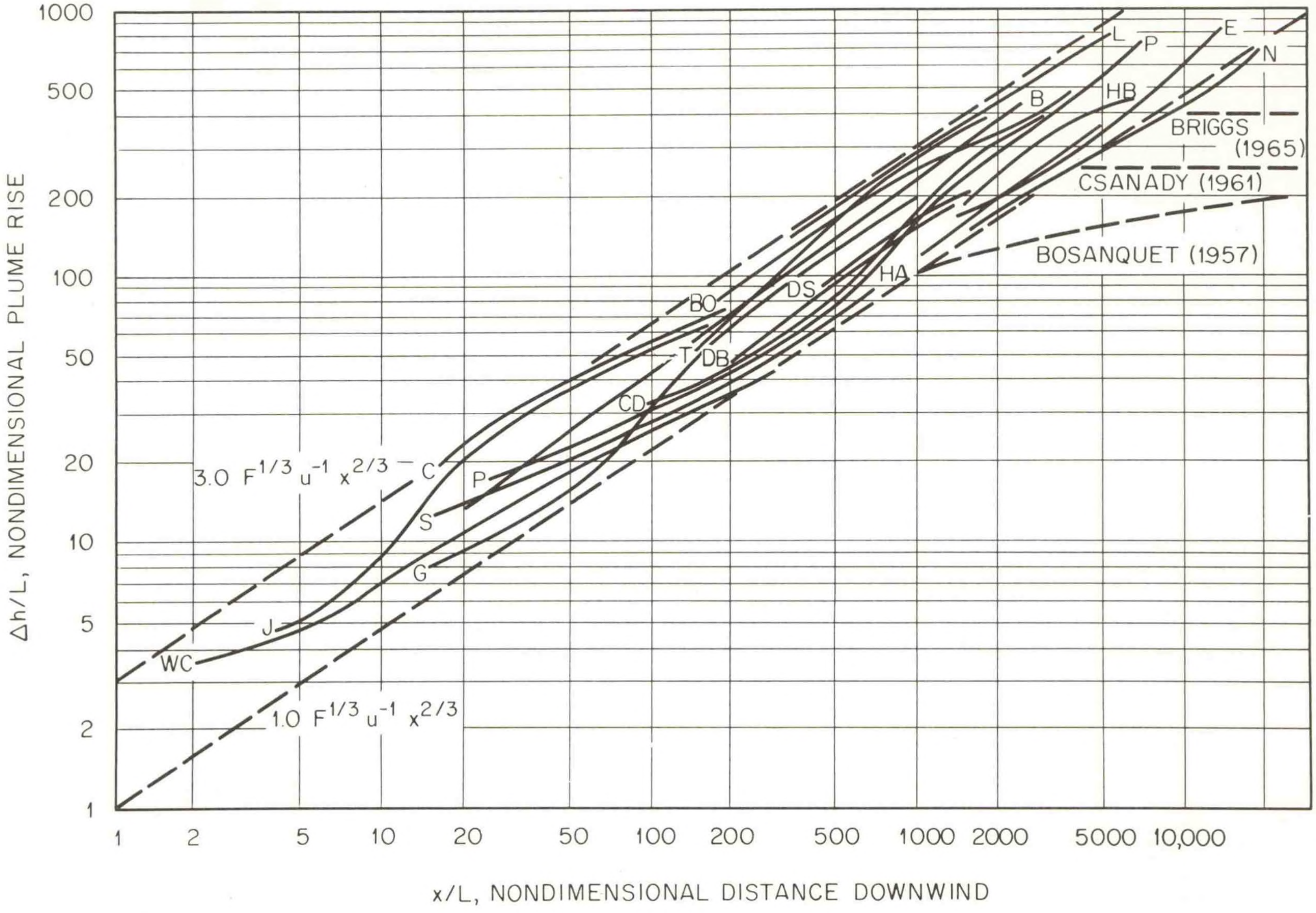

Figure 2 - Buoyant plume rise versus distance downwind, both nondimensionalized by $L = F/u^3$ (see Reference 1 for code identifying source)

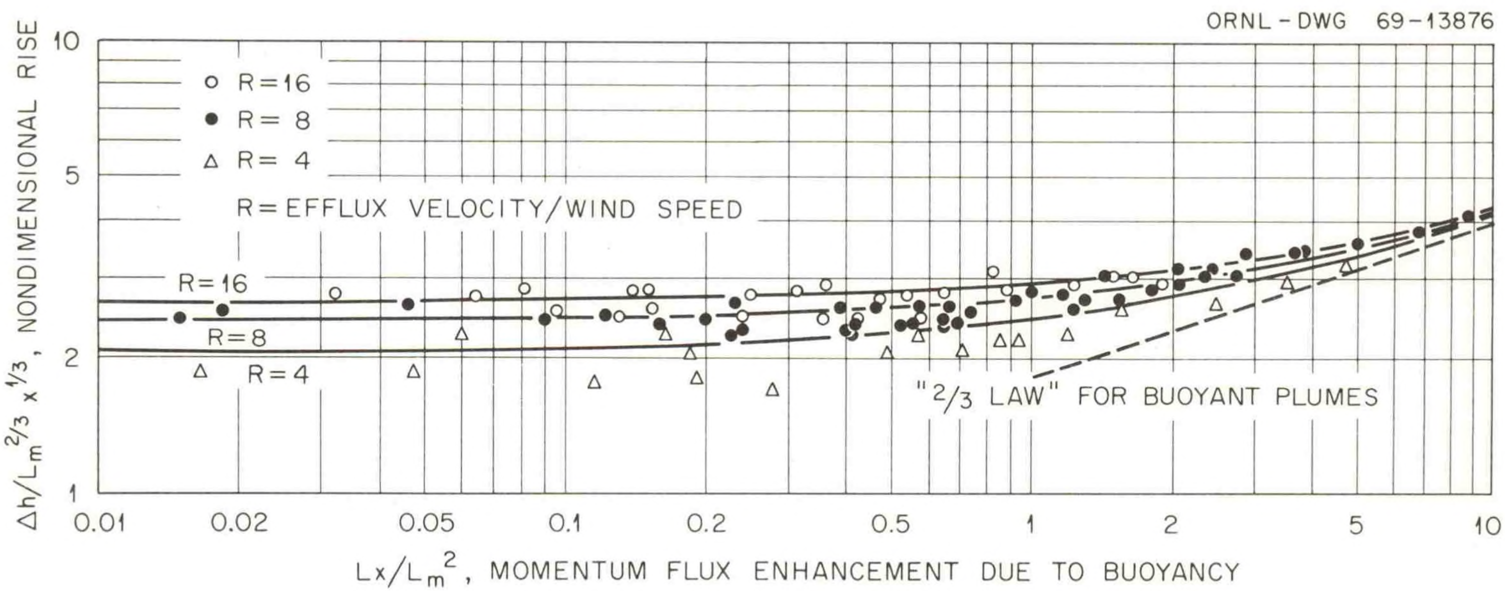

Figure 3 - Nondimensionalized rise of model plume centerlines versus ratio of buoyancy-induced momentum to initial momentum (solid lines are Equation 13).

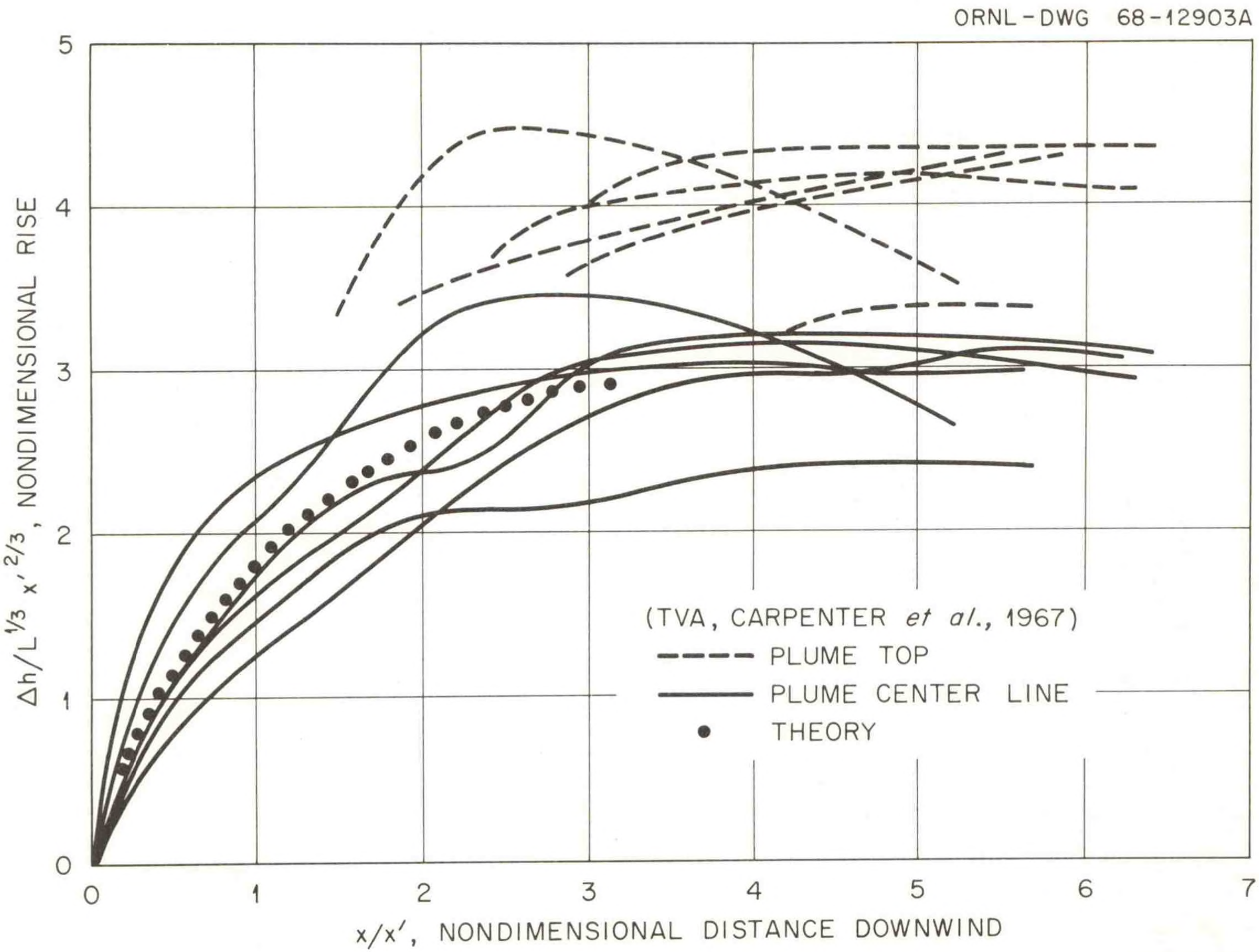

Figure 4 - Nondimensionalized rise versus nondimensionalized distance downwind for single-stack TVA plumes in stable conditions (dotted line is Equation 18)

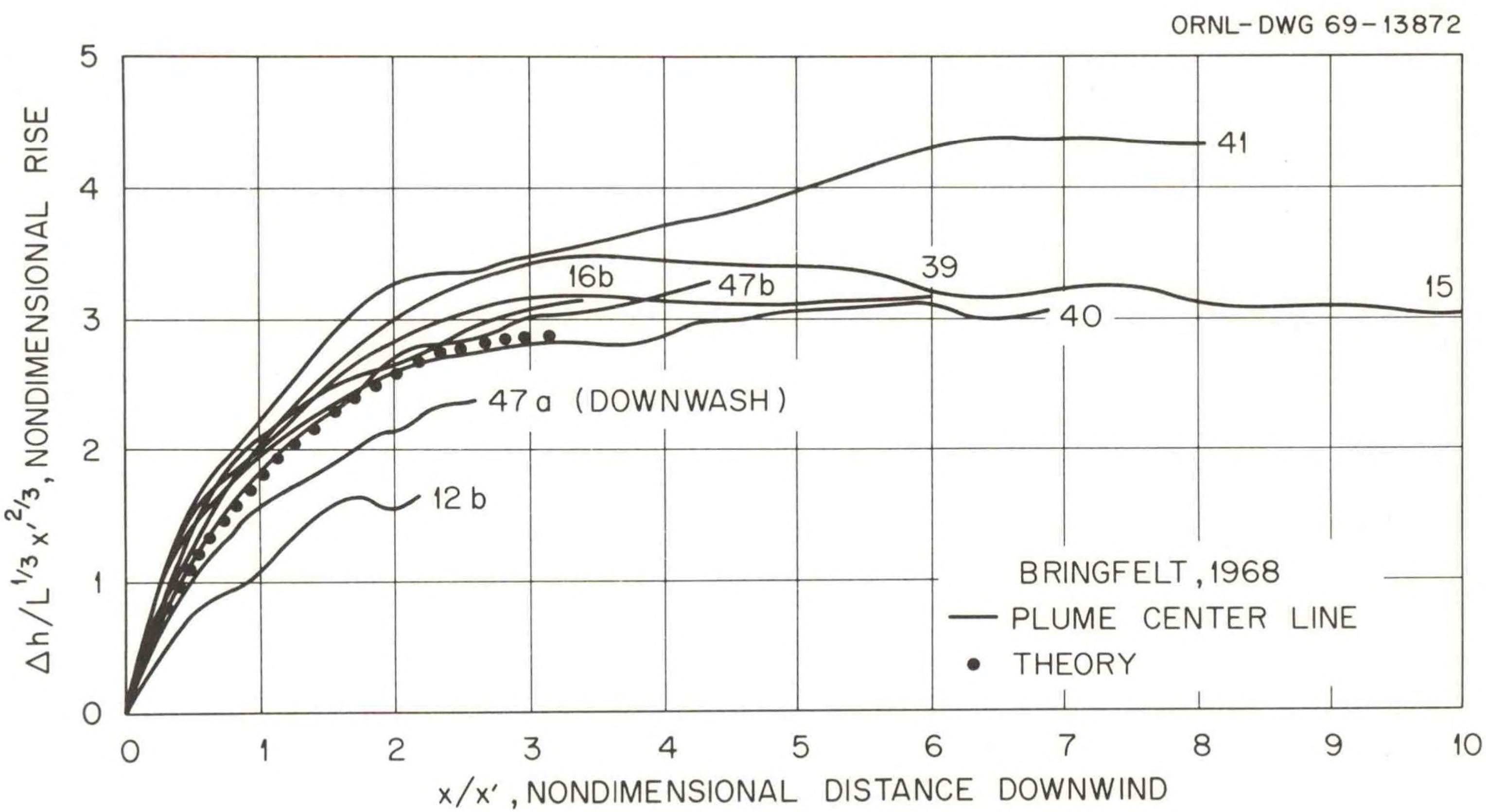

Figure 5 - Nondimensionalized rise versus nondimensionalized distance downwind for plumes measured by Bringfelt in stable conditions (dotted line is Equation 18)