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Noname manuscript No. (will be inserted by the editor)
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1 **Near-Surface Vertical Flux Divergence in the Stable**
2 **Boundary Layer**

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6 Received: DD Month YEAR / Accepted: DD Month YEAR

7 **Abstract** Flow in the stable boundary layer is examined at four contrasting
8 sites with greater upwind surface roughness. The surface heterogeneity is dis-
9 organized and in some cases weak as commonly occurs. With low wind speeds,
10 the vertical divergence (or convergence) of the momentum and heat fluxes can
11 be large near the surface in what is normally assumed to be the surface layer
12 where such divergence is neglected. For the two most heterogeneous sites, a
13 shallow “new” boundary layer is captured by the tower observations, analo-
14 gous to an internal boundary layer but more complex. Above the new boundary

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15 layer, the magnitudes of the downward fluxes of heat and momentum increase
16 with height in a transition layer, reach a maximum, and then decrease with
17 height in an overlying regional boundary layer. Similar structure is observed
18 at the site with rolling terrain where the shallow new boundary layer at the
19 surface is identified as cold air drainage generated by the local slope above
20 which the flow undergoes transition to an overlying regional flow. Significant
21 flux divergence near the surface is generated even over an ice floe for low wind
22 speeds in a shallow Ekman layer that forms during the polar night. For higher
23 wind speeds, the magnitude of the downward fluxes decreases gradually with
24 height at all levels as in a traditional boundary layer.

25 **Keywords** Internal boundary layer · Nocturnal boundary layer · Roughness
26 change · Stable boundary layer · Vertical flux divergence

27

28 1 Introduction

29 The vertical flux divergence near the surface of the nocturnal boundary layer
30 can be sufficiently large that fluxes measured at standard observational levels
31 do not adequately estimate the surface flux. The surface layer may be confined
32 to a layer below the lowest observational level. Common shallow drainage flows
33 are an example where the fluxes may vary significantly with height even in the
34 lowest few metres. The stress can approximately vanish at the wind-speed
35 maximum of the drainage flow where the speed shear vanishes (Grisogono and
36 Oerlemans, 2001; Nadeau et al., 2013); see also Mahrt et al. (2014).

37 Fluxes can also vary rapidly with height in flow over an abrupt transi-
38 tion from rougher to smoother surfaces. An internal boundary layer develops
39 that is attached to the downwind smoother surface and is characterized by
40 smaller turbulence intensity compared to upwind locations (Garratt, 1990,

41 1994). Based on large-eddy simulations (LES), Glendening and Lin (2002)
42 found that the magnitude of the downward momentum flux increases with
43 height immediately downwind of a rough-to-smooth transition in stable con-
44 ditions. In offshore flow from a rough warm land surface over a smooth cooler
45 sea surface, Skyllingstad et al. (2005) showed a significant increase of the fric-
46 tion velocity (u_*) with height up to 100 m. Pöette et al. (2017) found that
47 downwind of a rough-to-smooth transition in a wind tunnel, the turbulent
48 energy and magnitude of the downward momentum flux both increase with
49 height up to a maximum and then decrease with further increase in height
50 (e.g., their Fig. 7). Their observations include a sequence of transitions where
51 this vertical structure is modified compared to a single transition, but the ba-
52 sic spatial structure remains intact. Details depend on the relative spacing of
53 the transitions.

54 For flow from smoother to rougher surfaces, the magnitudes of the down-
55 ward momentum flux over locally rougher surfaces may decrease with height
56 more rapidly than over homogeneous surfaces, as can be inferred from Morse
57 et al. (2002) and Dellwik et al. (2013). The fetch required for development
58 of an equilibrium layer extending to a certain height is greater in stable con-
59 ditions (Dellwik and Jensen, 2000) and depends on the particular turbulence
60 variable analyzed (Irvine et al., 1997).

61 Based on Bou-Zeid et al. (2007), a regional boundary layer above the inter-
62 nal boundary layer can be partitioned into a lower region that is horizontally
63 heterogeneous due to memory of the upwind surface variability, and an upper
64 region above the “blending height” where the flow is horizontally homoge-
65 neous; see also Schmid and Bünzli (1995). Bou-Zeid et al. (2007) emphasize
66 the difficulty of defining a unique blending height even within a LES set-
67 ting. In flow from a rough surface to a smoother surface, the magnitude of
68 the downward fluxes of heat and momentum increase with height from the

69 low turbulence intensity in the new boundary layer to the higher turbulence
70 intensity in the regional boundary layer where turbulence is advected from
71 the upwind rougher surface. Actual surfaces are commonly characterized by
72 complex surface heterogeneity rather than a single discontinuity of surface con-
73 ditions. These surfaces have been examined much less and are an important
74 part of the present study.

75 The downward momentum and heat fluxes near the surface may also in-
76 crease with height due to elevated shear generation of turbulence (Mahrt and
77 Vickers, 2002; Balsley et al., 2006; Sun et al., 2013; Acevedo et al., 2015).
78 The increase of turbulence intensity and magnitude of the downward fluxes
79 with increasing height has been attributed to the generation of turbulence at
80 higher levels where the stratification is less and shear is often maintained by
81 a low-level jet (Conangla and Cuxart, 2006; Banta et al., 2006; Van de Wiel
82 et al., 2010; Kallistratova and Kouznetsov, 2012). Elevated turbulence may
83 be transported downward toward the surface in the form of bursts (Nappo,
84 1991). Based partly on radon measurements at different heights, Williams
85 et al. (2013) constructed a classification of very stable boundary layers, which
86 includes the top-down case where the turbulence is generated primarily at
87 higher levels. Mortarini et al. (2017) show examples of complex profiles of the
88 stress associated with larger-scale surface heterogeneity, submeso motions, and
89 a low-level jet.

90 Even over homogeneous flat surfaces, the vertical structure of the fluxes
91 in very stable conditions is sometimes challenging to measure because the
92 boundary-layer depth can be less than 10 m, as found by Smedman (1988),
93 Grachev et al. (2005) and others. With the usual guidelines, surface-layer sim-
94 ilarity theory is valid only in the lower 10% of the boundary layer so that
95 measurements of the surface fluxes would be required below 1 m. Because
96 similarity theory is potentially valid only above the roughness sublayer of the

97 vegetation, the surface layer can be potentially “squeezed” out of existence.
98 These boundary layers can still be “traditional” in that the main source of tur-
99 bulence is due to the surface roughness and the turbulence decreases monoton-
100 ically with height, as in Fig. 1 of Banta et al. (2006). In addition to difficulties
101 due to the thinness of the boundary layer, Poulos and Burns (2003) found
102 maximum flux divergence at the surface. For observational levels above the
103 shallow surface layer, local similarity theory may still describe the local flux-
104 gradient relationship (Grachev et al., 2013). However, over complex surface
105 heterogeneity, local similarity theory cannot be used to predict the surface
106 fluxes based on the traditional local flux-gradient relationships and general-
107 izations are required (Grachev et al., 2018).

108 Our study investigates the momentum and heat fluxes near the surface
109 at three contrasting sites with different degrees of complex heterogeneity and
110 a fourth site with a relatively homogeneous surface. The sites are described
111 in Sect. 2.1, and the dependence of the vertical structure on wind speed and
112 stratification is discussed in Sect. 3. The profile characteristics of momentum
113 and heat fluxes are analyzed for different subclasses of the flow in Sects. 4 – 5.

114 **2 Measurements and Analysis**

115 **2.1 Field programs**

116 Sonic anemometer measurements were collected in the North Park Basin, Col-
117 orado, USA, during winter 2002–2003 in the Fluxes Over a Snow Surface II
118 field program (FLOSSII, [http : //www.eol.ucar.edu/isf/projects/FLOSSII/](http://www.eol.ucar.edu/isf/projects/FLOSSII/)).
119 See Table 1 for instrument information. On average, the basin floor is about
120 30 km wide with basin sidewalls of 1,000 m over a horizontal width of about
121 7 km. The basin is approximately 50 km from south to north. The surface at
122 the tower base consists of matted grass, sometimes with a shallow snow cover;

123 the roughness length for this site is small, less than 1 mm with snow cover.
124 Scattered short brush begins 100–200 m upwind of the tower with respect to
125 the prevailing southerly flow. Some scatter brush occur upwind for all wind
126 directions. The distribution of the brush is somewhat disorganized with a typ-
127 ical height on the order of 0.1 m. A map of the complex spatial distribution
128 of the brush is provided in Mahrt and Vickers (2005). As a measure of the
129 stratification, $\delta\theta$ is computed as the vertical difference in potential tempera-
130 ture between 1.0 and 5.0 m. The FLOSSII dataset is the longest dataset of
131 the three mid-latitude sites (4 months) and has the tallest tower (30 m) and
132 is used as the default dataset.

133 The Shallow Cold Pool (SCP) Experiment was conducted over semi-arid
134 grasslands in north-east Colorado, USA, from 1 October to 1 December 2012
135 ([https : //www.eol.ucar.edu/field_projects/scp](https://www.eol.ucar.edu/field_projects/scp)). The main valley is rela-
136 tively small, roughly 12 m deep and 270 m across with valley side slopes
137 generally $< 6^\circ$. Widely scattered short brush grows upwind 100–200 m from
138 the tower with respect to the prevailing wind direction of west-north-west to
139 north-west. We analyze sonic anemometer measurements taken from the main
140 tower (Table 1) with $\delta\theta$ computed between the heights of 1.0 m and 5.0 m; see
141 Mahrt and Thomas (2016) for more information on this site. The estimation
142 of the roughness length is uncertain because of frequent distortion of the wind
143 profile by the local topography.

144 A 12-m tall eddy-flux tower was deployed at the Botany and Plant Pathol-
145 ogy (BPP) Farm of Oregon State University, USA (Thomas et al., 2012). We
146 analyze observations from late August until mid-October 2011 (Table 1). Re-
147 gardless of wind direction, the upwind roughness length is greater than that
148 at the grass-covered tower location due to upwind orchards, row crops and
149 hedges and a nearby building (see vegetation map in Zeeman et al. (2015)).
150 The effective roughness length is wind-direction dependent but always greater

151 than about 25 mm and presumably influenced by the upwind vegetation. The
152 airflow generally passes over a sequence of roughness changes before arriving
153 at the tower site. The quantity $\delta\theta$ is computed by interpolating the 12 levels
154 of Omega TMTSS-020G thermocouple data to the heights 1.0 and 5 m. The
155 thermocouple levels are at 0.05, 0.1, 0.2, 0.4, 0.8, 1.5, 3, 4, 6, 8, 10 and 12 m.

156 The Surface Heat Budget of the Arctic Ocean Experiment (SHEBA) tower
157 flux measurements (Persson et al., 2002; Grachev et al., 2005) were made
158 from October 1997 to October 1998 on a multi-year ice floe drifting in the
159 Beaufort and Chukchi Seas. The measurements analyzed in our study are
160 listed in Table 1. The SHEBA site was located a few hundred kilometers
161 from land on Arctic pack ice, which had no large-scale slopes. The surface
162 is generally smooth snow-covered ice, though patches of ice rubble up to 3
163 m in height exist in most directions at distances of several hundred metres
164 to a few kilometres. The roughness lengths increase by a factor of two with
165 the height of the observations over the depth of the tower, indicating that
166 a footprint higher on the tower includes some of the rougher ice at greater
167 distances (Persson et al., 2002). Based on fluxes closest to the surface, the
168 roughness lengths are small (typically 5×10^{-4} m), indicating a relatively
169 smooth surface. Data contaminated by huts and the ship in the southerly
170 and south-westerly direction comprise about 10% of the data and have been
171 eliminated; the remaining data are predominantly from the unobstructed and
172 dominant north-easterly and north-westerly wind directions.

173 During the polar night at the SHEBA site (early November through early
174 February), stable conditions are long lasting and can reach quasi-stationary
175 states compared to those in the midlatitudes. The boundary-layer stability is
176 primarily modulated by synoptic and mesoscale disturbances and the clouds
177 they produce (Persson et al., 2017) with time scales of 12 h to 5 days. The
178 summer period has been removed because the very few cases of stably stratified

179 flow in this season are associated with warm-air advection from ice-free areas
180 and compromise the assumption of relatively homogeneous conditions. Because
181 the SHEBA data is much more stationary than the other datasets, we use the
182 fluxes from the high-frequency portion of the 13.65-min covariance spectra
183 averaged for 54 min, as described by Grachev et al. (2005). This high-frequency
184 portion represents turbulent motions with time scales of at most 2–3 min. $\delta\theta$
185 is computed between the 2.2-m and 8.9-m levels. To compare with the other
186 three sites where $\delta\theta$ is estimated over a 4-m layer, $\delta\theta$ at the SHEBA site is
187 multiplied by 4 m/6.7 m. This does not correct for the typical decrease of the
188 temperature gradient with height.

189 For the three mid-latitude datasets, we analyze only measurements between
190 2100 and 0600 LST (local standard time) to exclude daytime and transition
191 periods. No conditions were imposed on time of day for the SHEBA measure-
192 ments, but instead measurements are included only when $\delta\theta > 0.1$ K, which ex-
193 cludes neutral conditions and conditions where the stable stratification might
194 be too small to accurately measure. This condition on the stratification elimi-
195 nates some of the cloudy periods during the polar night as longwave radiation
196 from clouds sometimes produces neutral stratification (Persson et al., 2017).
197 Also, only winter and spring data are used. For the retained SHEBA data,
198 83% occurred during the polar night with little insolation while 17% represent
199 nocturnal stable boundary layers during March and April when a significant
200 diurnal cycle develops (Persson et al., 2002).

201 2.2 Challenges Computing Momentum Fluxes for Very Stable Conditions

202 Estimation of the momentum flux divergence is more sensitive to errors than
203 the momentum flux itself. The problem is particularly difficult for low wind
204 speeds and stable stratification. For wind speeds < 2 m s⁻¹, about 20% of
205 the wind directions correspond to flow through the tower, depending on the

206 site and the definition of “through the tower.” Data with flow through the
207 tower were not eliminated at the FLOSSII, SCP and BPP sites because such
208 elimination would have led to highly segmented time series for low wind speeds.

209 Loss of momentum flux due to path-length averaging is a concern with very
210 stable conditions where some of the turbulent transport might be on scales that
211 are not adequately resolved because of sonic path-length averaging. The heat-
212 flux cospectra in FLOSSII indicates that the heat flux at 2 m is fully resolved
213 (Mahrt and Vickers, 2006) such that the variation of the flux between 2 and
214 30 m is probably not affected by path-length averaging. The method of Horst
215 and Oncley (2006) indicated that the heat flux was fully resolved for 1 m and
216 above at the SCP site. The method of Moore (1986) had been applied to the
217 BPP measurements, and the impact of path-length averaging appears to be
218 small compared to the rapid increases of flux magnitude with height at the
219 BPP site.

220 Unfortunately, the above methods rely on cospectral similarity theory and
221 our cospectra for very stable conditions include cases where an inertial sub-
222 range is not evident as shown by Grachev et al. (2013) for the SHEBA measure-
223 ments. The path-length averaging flux loss should be greatest at the surface
224 where the transporting eddies are smallest and thus acts to increase the flux
225 magnitude with height. Yet the flux magnitude for low wind speeds at the
226 FLOSSII and SCP sites tends to decrease with increasing height closest to
227 the surface and increase with height only at higher levels (Sect. 5). This cir-
228 cumstantial information implies that path-length averaging is not a serious
229 problem.

230 The choice of averaging time that determines the largest scales included in
231 the flux computation becomes uncertain for very stable conditions where tur-
232 bulent and non-turbulent motions may overlap in scale. The averaging times
233 for the FLOSSII, SCP and BPP sites are chosen to be 1 min as a compromise

234 between minimizing the impact of the non-turbulent motions on the computed
235 fluctuations and minimizing the loss of turbulent flux for higher wind speeds.
236 The optimum averaging length may be a strong function of stability. Fortu-
237 nately, the results of this study based on composited (bin-averaged) quantities
238 are not particularly sensitive to the choice of averaging time for these datasets,
239 at least within the range of 30 s to 300 s.

240 A more detailed analysis of the FLOSSII measurements indicated that the
241 momentum flux depends significantly on the choice of the coordinate align-
242 ment rotation (“tilt” rotation) for very low wind speeds $< 0.5 \text{ m s}^{-1}$. With
243 larger stratification, the transporting motions are generally flatter, causing
244 smaller attack angle with respect to the anemometer, which in turn increases
245 the impact of misalignment of the sonic anemometer. Coordinate alignment
246 attempts to reduce the misalignment of the sonic anemometer with respect
247 to the surface and also reduces the impact flow distortion depending on the
248 rotation method and type of sonic anemometer. Misalignment with respect
249 to sloped terrain in the footprint of the flux measurement depends on wind
250 direction, wind speed and stability and sometimes includes the impact of mi-
251 croscale flow separation on the attack angle of the flow (Stiperski and Rotach,
252 2016).

253 The datasets varied in terms of cross-wind corrections (Liu, 2001) which
254 were found to be quite small. The datasets also varied in terms of correc-
255 tions for instrument-induced flow distortion (Horst et al., 2015) or in terms of
256 wind-tunnel calibration. For low wind speeds and strong stratification, we be-
257 lieve these errors are small compared to the difficulties of correcting for sonic
258 misalignment and potentially significant flux loss to path-length averaging. In
259 spite of the above efforts to assess flux errors, we cannot categorically assume
260 that instrumental problems do not significantly influence the estimated stress

Table 1 Site description. Total months of data, sonic-anemometer model, tower levels (m), sampling rate (SR in samples per second) and tilt-rotation method (TM) where PF = planar fit (Wilczak et al., 2001), DR = double rotation (Grachev et al., 2005) and DPF = directionally dependence planar fit (Acevedo and Mahrt, 2010).

site	months	sonic	tower levels	SR	TM
FLOSSII	4	Campbell CSAT3	1 2 5 10 15 20 30	40	DPF
SCP	2	Campbell CSAT3	0.5 1 2 3 4 5 10 15 20	20	PF
BPP	2	Metek USA-1	0.8 3	10	DPF
BPP	2	R. M. Young 81000 VRE	7.5 12	10	DPF
SHEBA	7	ATI	2.2 3.2 5.1 8.9 13.8	20	DR

261 divergence, although it is not obvious how such errors would lead to systematic
 262 variation of the flux with height.

263 2.3 Time Averaging

264 Our analysis approach is described in Mahrt and Thomas (2016) with the flow
 265 is partitioned as

$$\phi = \phi' + \bar{\phi}, \quad (1)$$

266 where the overbar designates a time average over a window of width τ , ϕ' is
 267 the deviation from such an average, ϕ is potential temperature or one of the
 268 velocity components, fluxes are computed as $\overline{w'\phi'}$ where w' is the perturbation
 269 vertical velocity, and the wind speed is determined as

$$V \equiv \sqrt{\overline{u^2} + \overline{v^2}}. \quad (2)$$

270 2.4 Averaging Over Intervals

271 Because we use small averaging windows to filter out non-turbulent motions
 272 and reduce flux bias, averages for individual windows are characterized by
 273 large uncertainty due to random-like variations. As a result, we bin-average
 274 (composite) the window averages separately for different intervals of some
 275 variable such as height above ground or the wind speed. Narrow intervals
 276 (bins) are chosen to construct a semi-continuous dependence of the turbulence

277 on the forcing variable. Broader intervals are chosen to construct classes such
 278 as the low wind class $V < 2 \text{ m s}^{-1}$ and the high wind class $V > 6 \text{ m s}^{-1}$.
 279 Either type of compositing is symbolized by square brackets such as $[\bar{\phi}]$.

280 Non-stationarity and variation of vertical structure of the flux between
 281 nights prevent interpretation of the composited variables (bin averages) in
 282 terms of an ensemble average and also preclude clear interpretation of the
 283 standard error. The standard errors are generally very small, often not even
 284 visible if plotted. These small magnitudes result from the large number of 1-
 285 min records. However, because of the lack of independence of 1-min records
 286 with typical non-stationarity, the standard error can substantially underesti-
 287 mate the actual flux uncertainty (Mahrt and Thomas, 2016).

288 2.5 Different Calculations of the Momentum Flux

289 The method of computing the momentum flux becomes important with low
 290 wind speeds. Unless otherwise noted, the x -direction refers to the along-wind
 291 coordinate at each level such that the along-wind momentum flux is $\overline{w'u'}$ and
 292 the crosswind flux is $\overline{w'v'}$. The total momentum flux in this *height-dependent*
 293 *rotated coordinate system* is written as

$$\overline{w'\mathbf{V}'} \equiv \overline{w'u'} \mathbf{i} + \overline{w'v'} \mathbf{j}, \quad (3)$$

294 where \mathbf{i} and \mathbf{j} are unit vectors in the along-wind x and crosswind y directions,
 295 respectively. The flux components are composited over all of the values within
 296 a given bin or class to obtain $[\overline{w'u'}]$ and $[\overline{w'v'}]$, and we define the shorthand
 297 notation for the magnitude of the composited momentum flux as

$$[\overline{w'\mathbf{V}'}] \equiv ([\overline{w'u'}]^2 + [\overline{w'v'}]^2)^{0.5}. \quad (4)$$

298 Here, the square brackets on the left-hand side imply that the flux components
 299 are composited first.

300 When analyzing data with wind-directional shear, a more meaningful flux
 301 divergence might be computed in a coordinate system that is fixed with height,
 302 sometimes based on the direction of the surface wind (surface-based rotation)
 303 where

$$\overline{w'\mathbf{V}'_{sfc}} \equiv \overline{w'u'_{sfc}} \mathbf{i} + \overline{w'v'_{sfc}} \mathbf{j}. \quad (5)$$

304 However, with small wind speed and significant stratification and resulting
 305 partial decoupling in the vertical, the direction of the wind vector at higher
 306 levels can become more chaotic in a surface-based coordinate system. In ad-
 307 dition, the wind vector can systematically shift with height away from the
 308 surface wind direction. As a result of compositing over flux components of
 309 varying sign, $[\overline{w'\mathbf{V}'_{sfc}}]$ decreases rapidly with height in the surface-based co-
 310 ordinate system (black, Fig. 1a) in contrast to $[\overline{w'\mathbf{V}'}]$ in the locally rotated
 311 coordinate system (red line, Fig. 1a), which increases slowly with height.

312 Comparing $[\overline{w'u'}]$ (red dashed) with $[\overline{w'\mathbf{V}'}]$ (red) indicates that the com-
 313 posited cross-wind momentum flux is small for the FLOSSII site although it
 314 can be large for individual profiles. Using 5-min averaging instead of 1-min
 315 averaging has little effect on the composited wind and stress profiles (com-
 316 pare cyan and red lines, Fig. 1a). The differences between the various profiles
 317 become significantly smaller with higher wind speeds (Fig. 1b).

318 Compositing the friction velocity

$$u_* \equiv (\overline{w'u'^2} + \overline{w'v'^2})^{0.25} \quad (6)$$

319 to obtain $[u_*]$ must be interpreted with caution. The contributions of the devi-
 320 ations of the flux components from their composited values contribute signifi-
 321 cantly to $[u_*]$ but do not contribute to $[\overline{w'\mathbf{V}'}]$. The contribution of random-like
 322 variations of $\overline{w'\mathbf{V}'}$ can be interpreted as bias in the calculation of $[u_*]$ if $[u_*]$
 323 is interpreted as a measure of the systematic momentum flux. To construct
 324 an exercise illustrating this process, we partition each flux component into the

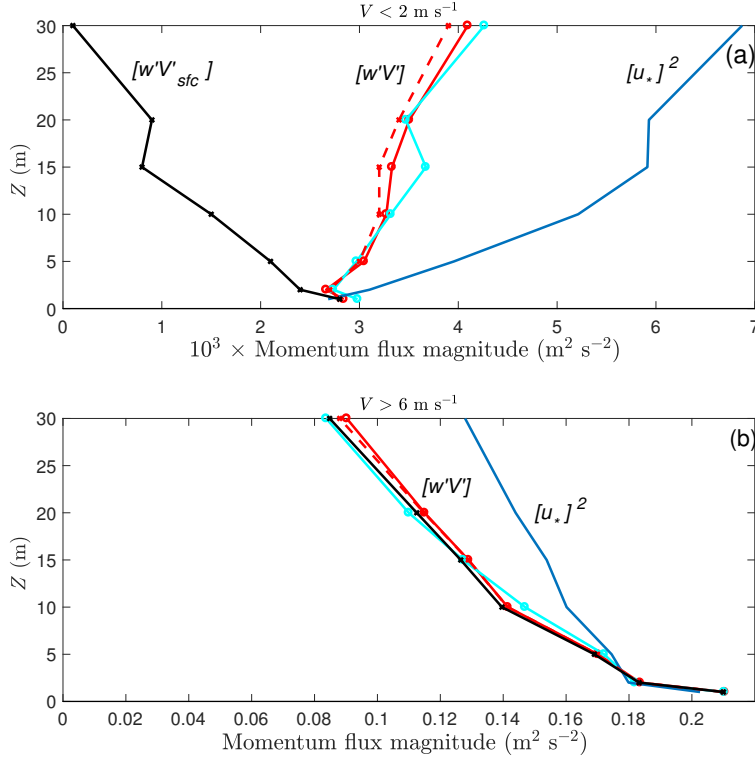


Fig. 1 Height dependence of the composited (bin-averaged) momentum flux at the FLOSSII site for a) $V < 2 \text{ m s}^{-1}$ and b) $V > 6 \text{ m s}^{-1}$. Plotted are the composited stress magnitudes based on Eq. 4 with the height-dependent rotated coordinate system (solid red, $[w'V']$), with the height-dependent rotated coordinate system but with 5-min window averages (cyan), with the along-wind flux component only (red dashed, $[w'u']$), and with the coordinate system rotated into the surface wind direction ($[w'V']_{sfc}$, black line). The blue line is the scalar composite $[u_*]^2$.

325 composited momentum flux and a deviation from this value. Then each value
 326 of u_* can be written as

$$u_* = ((\overline{[w'u']} + \widehat{\overline{w'u'}})^2 + (\overline{[w'v']} + \widehat{\overline{w'v'}})^2)^{0.25}, \quad (7)$$

327 where the hat symbol above the overbar designates deviations from the com-
 328 posited value that include random variations. To avoid the need for a complex
 329 series expansion, we raise Eq. 7 to the fourth power to obtain

$$u_*^4 = \overline{[w'u']^2} + 2\overline{[w'u']\widehat{\overline{w'u'}}} + \widehat{\overline{w'u'}}^2 + \overline{[w'v']^2} + 2\overline{[w'v']\widehat{\overline{w'v'}}} + \widehat{\overline{w'v'}}^2. \quad (8)$$

330 Compositing each side and noting that $[\overline{w'u'}\widehat{w'u'}]$ and $[\overline{w'v'}\widehat{w'v'}] = 0$,

$$[u_*^4] = \overline{w'u'}^2 + \overline{w'v'}^2 + (\widehat{w'u'}^2 + \widehat{w'v'}^2). \quad (9)$$

331 The terms $\widehat{w'u'}^2$ and $\widehat{w'v'}^2$ augment $[u_*^4]$ but not $\overline{w'V'}$ and $\overline{w'u'}$; $[u_*]$
 332 increases monotonically with increasing $[u_*^4]^{1/4}$ in a way that depends on the
 333 frequency distribution of $[u_*]$ not pursued here.

334 The augmentation of $[u_*]$ above $\overline{w'V'}$ and $\overline{w'u'}$ for the current data is
 335 quantified in Fig. 1 (compare blue line with red solid and dashed lines). If
 336 the square brackets represented a true ensemble average, the contributions of
 337 $\widehat{w'u'}^2$ and $\widehat{w'v'}^2$ could be viewed as inadvertent conversion of random error
 338 to systematic momentum flux. Although u_* represents the magnitude of the
 339 momentum flux for an individual averaging window, $[u_*]$ cannot be thought of
 340 as a pure measure of the systematic momentum flux. Instead, the composited
 341 friction velocity, $[u_*]$, provides a measure of the intensity of the turbulence
 342 that is based on the momentum flux. For the FLOSSII dataset, σ_w and u_*
 343 are linearly correlated with a ratio of about 1.4, within the range for stable
 344 conditions summarized in Garratt (1994) and also in general agreement with
 345 Fig. 4 in Pahlow et al. (2001) and Fig. 6 in Basu et al. (2006). However, in
 346 our datasets, σ_w is more likely to increase with height near the surface than
 347 u_* (not shown) noting that σ_w unavoidably includes some influence of non-
 348 turbulent motions. On the other hand, Acevedo et al. (2009) advises against
 349 using u_* as a scaling variable for low wind conditions because of its sensi-
 350 tivity to mesoscale variability and recommends use of σ_w instead. For our
 351 measurements, the vertical profile of u_* lies between the profiles of σ_w and the
 352 along-wind momentum flux $\overline{w'u'}$.

353 2.6 Vertical Structure

354 Section 3 examines the behaviour of vertical structure based on the traditional
 355 scaling variable u_* which is used as a measure of the intensity of the turbulence
 356 based on the momentum flux. To avoid fitting complex individual profiles that
 357 occur in very stable conditions, we compute the vertical difference based on
 358 simple finite differencing defined as

$$\delta_z u_* \equiv u_*(z_2) - u_*(z_1), \quad (10)$$

359 where the operator δ_z takes the difference of u_* between the two levels. We
 360 evaluate Eq. 10 as close to the surface as possible yet include sufficient layer
 361 thickness to control the impact of observational errors on the computed vertical
 362 differences. As a compromise, the heights z_1 and z_2 are chosen to be the 1-
 363 and 5-m levels for the FLOSSII and SCP sites, the 0.8- and 7.5-m levels for
 364 the BPP site and the 2- and 8-m levels for the SHEBA site. To reduce the
 365 effect of the larger vertical interval, we multiply the differences by 0.6 for the
 366 BPP site and 0.67 for the SHEBA site.

367 Section 4 examines the vertical structure of the along-wind momentum flux
 368 $\overline{[w'u']}$ to identify the behaviour of the systematic part of the momentum flux.
 369 Because the crosswind momentum flux is of variable sign, the contribution of
 370 crosswind momentum flux to the composited profiles is generally small except
 371 for the SCP data where shallow drainage flows induce significant directional
 372 shear.

373 **3 Dependence of $\delta_z u_*$ on V and $\delta\theta$**

374 3.1 Joint Distribution

375 We composite the distribution of $\delta_z u_*$ with respect to joint intervals of $\delta_z\theta$
 376 and V at the FLOSSII site (Fig. 2); the advantages of joint distributions

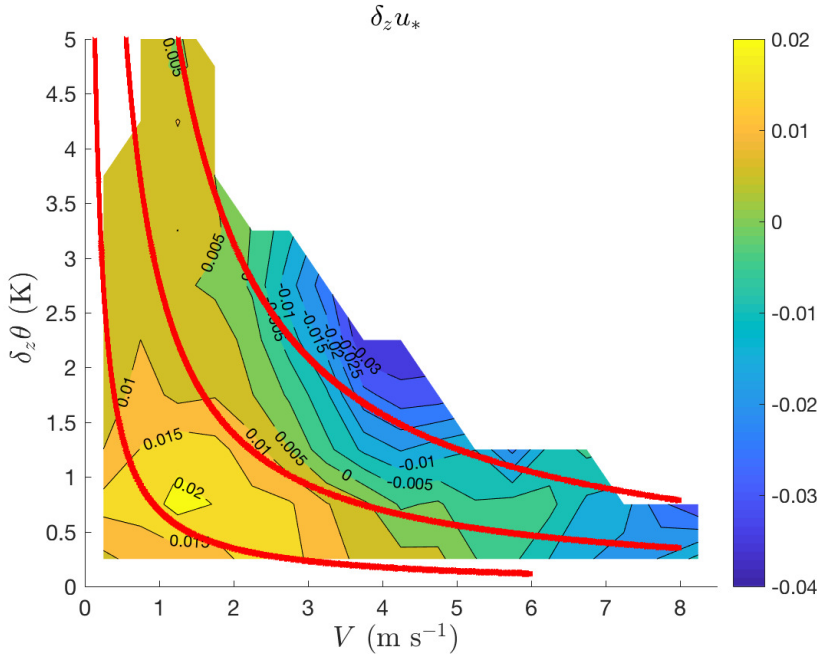


Fig. 2 Dependence of $[\delta_z u_*]$ on V and $\delta\theta$ for the FLOSSII site where $[\delta_z u_*]$ represents compositing jointly over intervals of V and $\delta\theta$. Upper left corresponds to more stable conditions, and the lower right corresponds to near-neutral conditions. Red lines correspond to $V_{\delta\theta} = 0.05, 0.1$ and 0.15 m s^{-1} (Eq. 13).

377 are discussed in Williams et al. (2013). On average, $\delta_z u_* > 0$ for small V ,
 378 particularly for small $\delta\theta$. Advection of larger turbulence intensity from the
 379 rougher brush upwind from the site (Sect. 2.1) over the less turbulent air
 380 above the grass surface probably contributes significantly to the increase of
 381 u_* with height ($[\delta_z u_*] > 0$), discussed further in terms of the along-wind
 382 momentum flux in Sect. 4.

383 $[\delta_z u_*]$ is < 0 with larger V at the FLOSSII site (Fig. 2). Decreasing u_*
 384 with decreasing height is characteristic of traditional boundary layers. For a
 385 given large magnitude of V , the magnitude of $[\delta_z u_*]$ tends to increase with
 386 increasing $\delta_z \theta$. Larger $\delta\theta$ generally leads to smaller boundary-layer depth for a
 387 given magnitude of V such as implied by the usual decrease of boundary-layer
 388 depth with increasing bulk Richardson number. This argument assumes that

389 $\delta_z u_*$ is proportional to $(u_*)_{sfc}/h$ where h is the stability-dependent boundary-
 390 layer depth.

391 The joint distribution of $\delta_z u_*$ for the SCP site (not shown) is similar to
 392 that at the FLOSSII site in spite of the influence of the down-valley drainage
 393 flow at the SCP site. However, the joint distribution for the heterogeneous
 394 BPP site is less organized beyond a general increase of positive $\delta_z u_*$ with
 395 increasing V , in contrast to the FLOSSII and SCP sites, presumably due to
 396 an increase of advection of turbulence from the rougher surface immediately
 397 upwind from the BPP site (Sect. 2.1). The wind speed is generally confined to
 398 small magnitudes at the BPP site, corresponding to a small subdomain of Fig.
 399 2. The joint distribution for the SHEBA site, which includes predominantly
 400 small stratification, is less organized. The turbulence and δu_* are smaller at
 401 the SHEBA site compared to the other sites (Sect. 4) partly because of the
 402 very small roughness length of the snow-covered ice.

403 3.2 Stability-Dependent Velocity Scales

404 We now consider two stability-dependent velocity scales based on the mean
 405 flow that might better predict $\delta_z u_*$ than predicted by V alone. For the first
 406 velocity scale, we begin with the bulk Richardson number

$$Rb \equiv \frac{gz\delta\theta}{\Theta V^2}, \quad (11)$$

407 where g is the acceleration due to gravity and z is the height of the wind-speed
 408 measurement above the ground (Sect. 2.1). A stability-dependent velocity scale
 409 can be formulated from these variables as

$$V_F \equiv C_F V / \sqrt{\frac{\delta\theta}{\Theta}}, \quad (12)$$

410 so that V_F is proportional to the inverse square root of the bulk Richardson
 411 number but has units of wind speed and thus formally allows a dimensionally

412 consistent examination of the variation of $[\delta_z u_*]$. The parameter C_F is chosen
 413 to be 0.04 in order that V_F has roughly the same range as V to facilitate
 414 comparison; $[V_F]$ does not better explain the variation of $[\delta_z u_*]$ than does $[V]$
 415 (compare Fig. 3b with Fig. 3a). This result implies that the stability based on
 416 the bulk Richardson number does not systematically describe the variation of
 417 $[\delta_z u_*]$. Evidently, stability is not the greatest influence on $[\delta_z u_*]$.

418 The joint distribution (Fig. 2) motivates an alternative stability-dependent
 419 velocity of the form

$$V_{\delta\theta} \equiv C_{\delta\theta} V \sqrt{\frac{\delta\theta}{\Theta}}. \quad (13)$$

420 The coefficient $C_{\delta\theta}$ is chosen to be 300 in order that $V_{\delta\theta}$ has roughly the
 421 same range as V . $[\delta_z u_*]$ varies more systematically with $[V_{\delta\theta}]$ (Fig. 3c) than
 422 with $[V]$ or $[V_F]$ with the possible exception of the very complex BBP site.
 423 In addition, $[\delta_z u_*]$ for all four sites decreases with increasing $[V_{\delta\theta}]$ whereas
 424 $[\delta_z u_*]$ may increase or decrease with increasing $[V]$ or increasing $[V_F]$. Positive
 425 $[\delta_z u_*]$, where the turbulence increases with height, occurs for small $[V_{\delta\theta}]$ for all
 426 of the sites. For a given magnitude of V , $\delta_z u_*$ is more likely to be positive with
 427 small $\delta\theta$ (smaller $V_{\delta\theta}$). With larger $\delta\theta$, a shallow traditional boundary layer
 428 can develop near the surface (Sect. 4), and $\delta_z u_*$ more likely becomes negative.

429 However, $[\delta_z u_*]$ at the very heterogeneous BPP site is more sensitive to
 430 $[V]$ than to $[V_{\delta\theta}]$. In addition, use of $V_{\delta\theta}$ did not categorically decrease the
 431 within-bin standard deviations compared to use of V or V_F . Although $V_{\delta\theta}$
 432 generally better explains the variation of the stress divergence at a given site,
 433 compared to V , progress toward a more universal relationship requires other
 434 information such as surface roughness, heterogeneity and advection. Such a
 435 relationship is beyond the scope of this investigation.

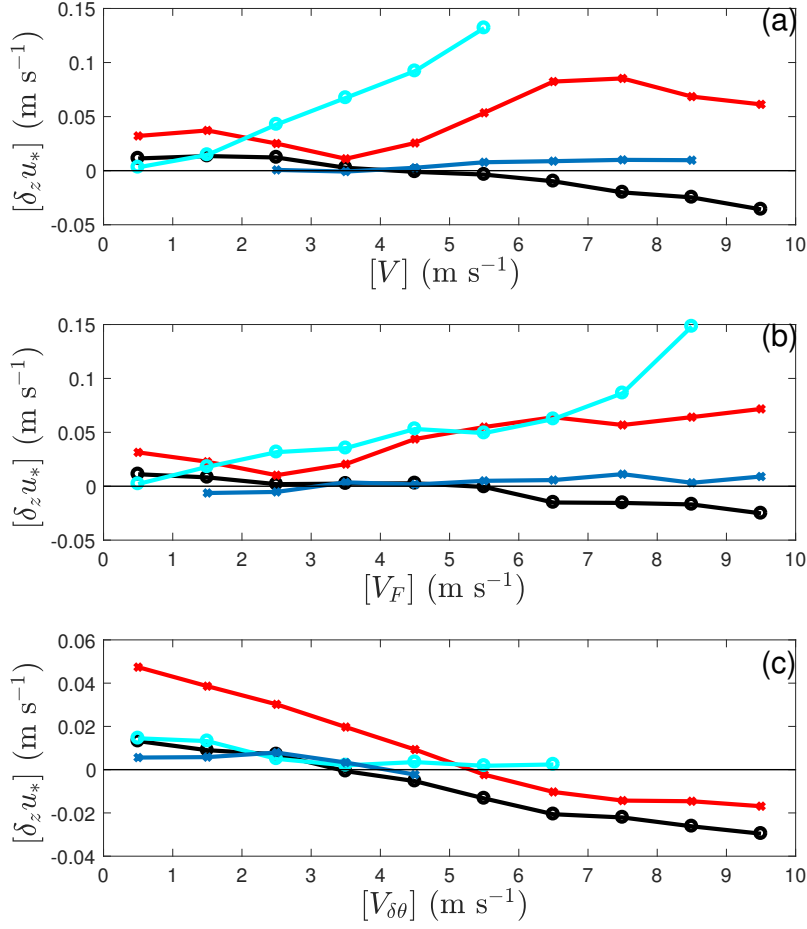


Fig. 3 Dependence of $[\delta_z u_*]$ on the velocity scales for the FLOSSII site (black), the SCP site (red), the BPP site (cyan) and the SHEBA site (blue) for a) $[V]$, b) $[V_F]$ and c) $[V_{\delta\theta}]$. $[V_{\delta\theta}]$ is small for the SHEBA site because of the generally small magnitudes of $\delta\theta$.

436 4 Vertical Structure for High Wind Speeds

437 We examine the vertical structure of the fluxes separately for the high-speed
 438 class ($V(1\text{ m}) > 6\text{ m s}^{-1}$) and the low speed class ($V(1\text{ m}) < 2\text{ m s}^{-1}$) as
 439 was done in Fig. 1. The low speed class corresponds to the low speed part of

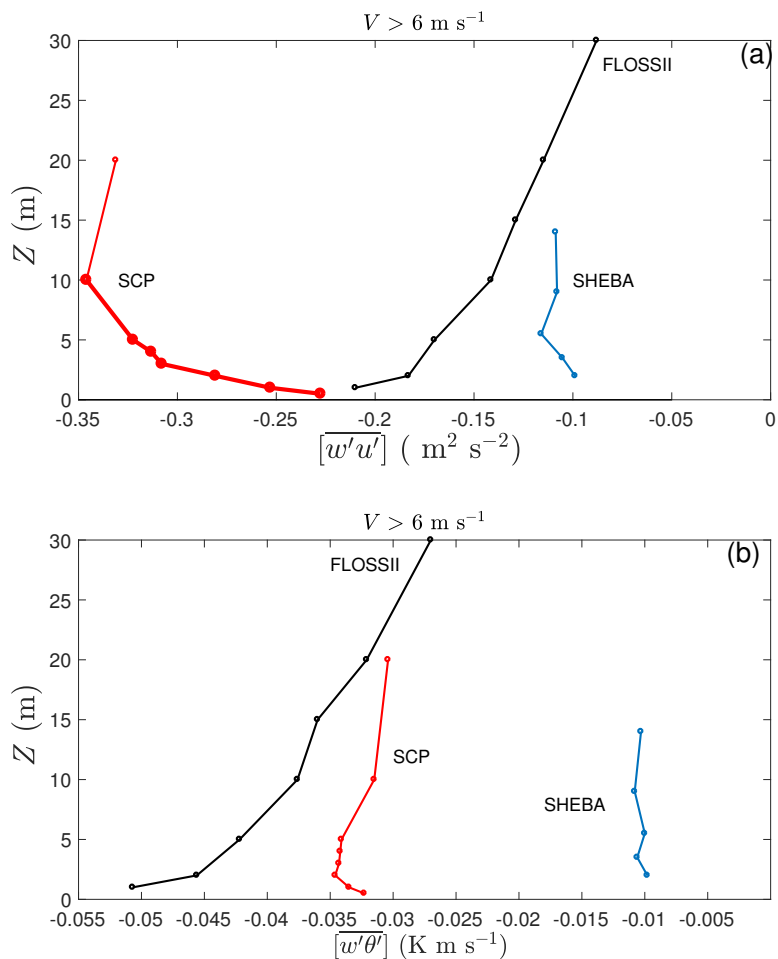


Fig. 4 a) Composed profiles of $\overline{w'u'}$ for $V(1 \text{ m}) > 6 \text{ m s}^{-1}$ and b) composed $\overline{w'\theta'}$, for the FLOSSII (black), SCP (red) and SHEBA (blue) sites. The thicker line identifies a reversed vertical flux gradient (transition layer). The BPP site did not contain sufficient data for the class $V > 6 \text{ m s}^{-1}$.

440 the “hockey stick” dependence of u_* on V (Sun et al., 2012), and the high
 441 speed class includes only wind speeds substantially higher than the speed at
 442 the hockey stick transition.

443 The low wind (high wind) cases account for 39% (9%) of the FLOSSII
 444 records, 61% (2%) of the SCP records, 97% (0%) of the BPP records and 18%

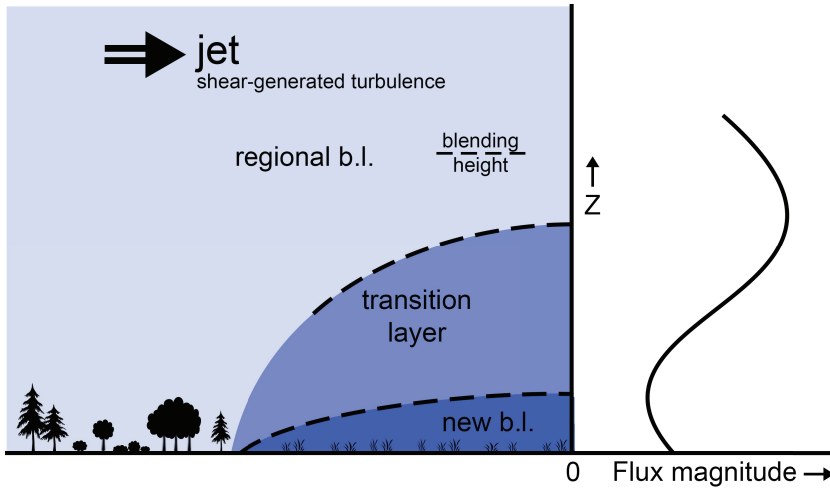


Fig. 5 Plausible example of the new boundary layer at the surface, an overlying transition layer and regional boundary layer, based partly on evidence in the literature (Introduction) and the results of our study. This terminology is more vague compared to that for internal boundary layers (Garratt, 1990) in order to accommodate less-understood flow over complex surface heterogeneity. The blending height partitions the regional boundary layer into a lower layer of horizontal heterogeneity and an upper layer of horizontal homogeneity. In the limit of a single sharp change from rough to smooth surfaces, the new boundary layer and transition layer combine to form the internal boundary layer. In addition, elevated turbulence can be generated by shear instability at higher levels as might occur on the underside of a low-level wind maximum. The diagram on the right illustrates an idealized profile of the magnitude of the downward momentum flux, which reaches a maximum value at the top of the transition layer.

445 (15%) of the SHEBA records. We now composite the along-wind momentum
 446 flux $\overline{w'u'}$ and the heat flux for the high wind class (Fig. 4). The surface flow
 447 at the SCP site is influenced by the local slopes and discussed separately in
 448 Sect. 5.4.

449 For $V > 6 \text{ m s}^{-1}$ at the FLOSSII site, the magnitudes of the downward
 450 momentum and heat fluxes decrease systematically with height (black, Fig.
 451 4) corresponding to traditional boundary-layer structure. Extrapolation of the
 452 momentum flux and heat flux to zero both correspond to a boundary-layer
 453 depth of roughly 60 m. For $V > 6 \text{ m s}^{-1}$ at the SCP tower within a shallow
 454 valley (red, Fig. 4a), the magnitude of $[\overline{w'u'}]$ increases significantly with height
 455 up to 10 m where it reaches a maximum within the vertical resolution of the

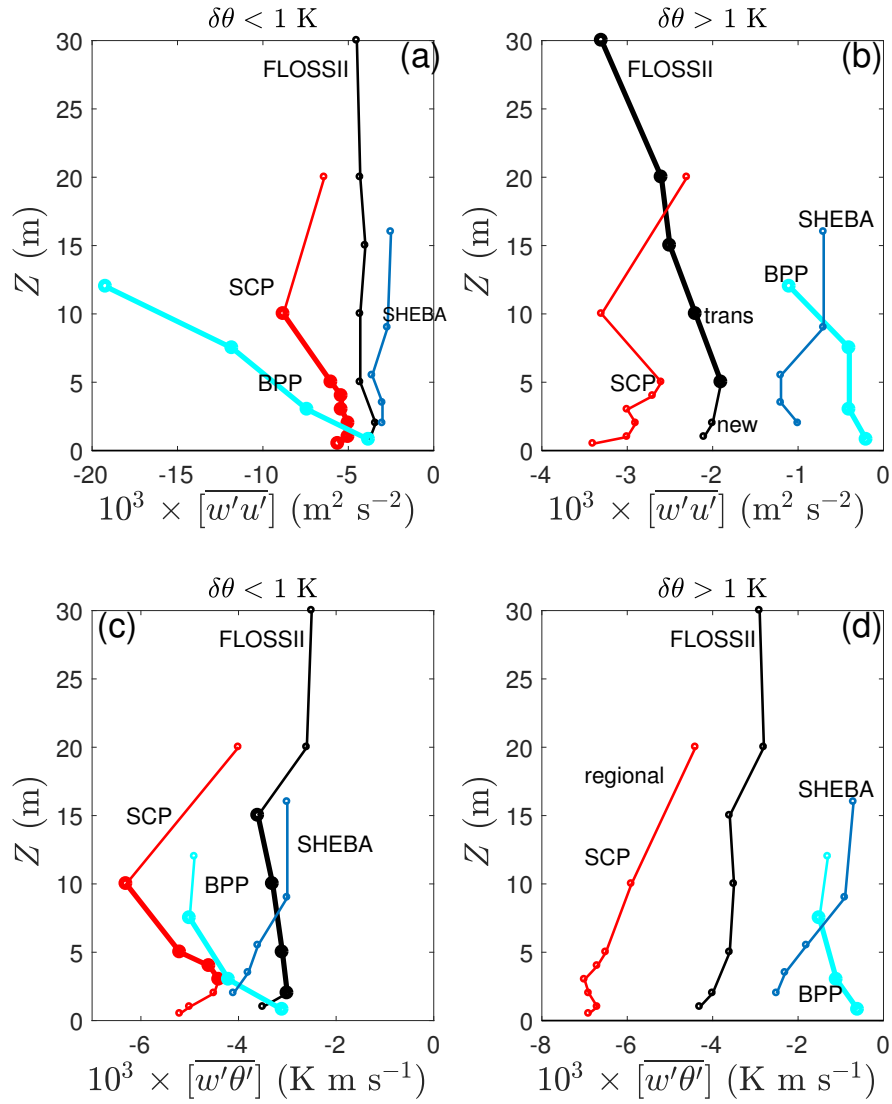


Fig. 6 Profiles of the composited (bin-averaged) stress $[\overline{w'u'}]$ for $V < 2 \text{ m s}^{-1}$ for a) $\delta\theta < 1 \text{ K}$ and b) $\delta\theta > 1 \text{ K}$ and composited $[\overline{w'\theta'}]$ for c) $\delta\theta < 1 \text{ K}$ and d) $\delta\theta > 1 \text{ K}$. Profiles are shown for the FLOSSII (black), SCP (red), BPP (cyan) and SHEBA (blue) sites. Thicker lines identify reversed vertical flux gradients (transition layer). The labels “new” and “trans” in b) identify examples of the new boundary layer and the transition layer. The label “regional” in d) identifies an example of a regional boundary layer, which is above the tower layer for most of the other profiles.

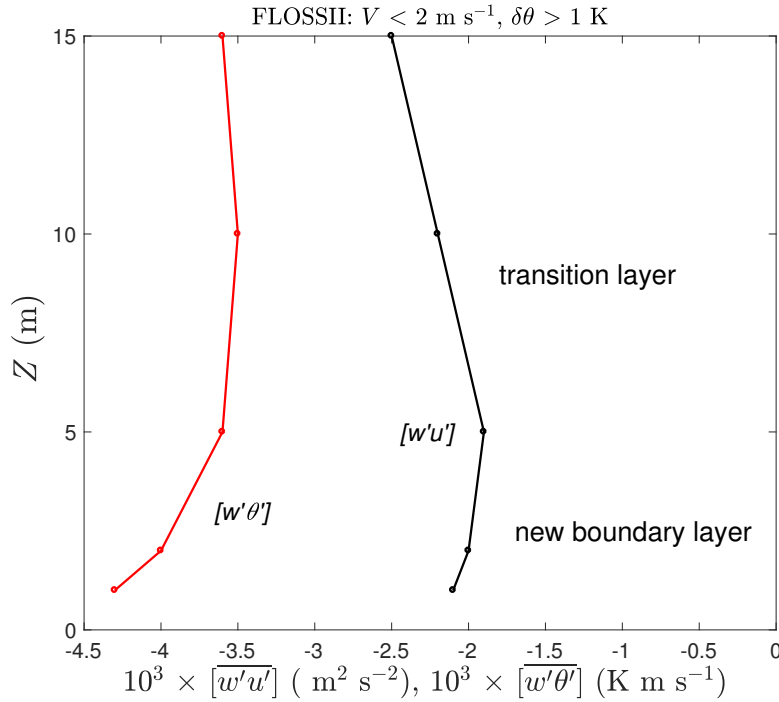


Fig. 7 Composed profiles at the FLOSSII site in the lowest 15 m for $\overline{[w'u']}$ (black) and $\overline{[w'\theta']}$ (red) for $V < 2 \text{ m s}^{-1}$ and $\delta\theta > 1 \text{ K}$. A shallow new boundary layer of roughly 5-m depth, where the flux magnitudes decrease with height, evidently represents at least partial adjustment to the local smaller roughness length.

456 data. Lee-generated turbulence contributes to this maximum in spite of the
 457 small amplitude of the topography, about 12 m depth (Mahrt, 2017). At the
 458 homogeneous SHEBA site (blue, Fig. 4), the magnitudes of the downward
 459 momentum and heat flux are relatively independent of height with respect to
 460 the estimated errors in u_* of 0.015 m s^{-1} (Persson et al., 2002). Generation
 461 of turbulence on the underside of low-level jets may maintain the turbulence
 462 fluxes at higher levels and prevent significant decrease of the magnitudes of
 463 the downward fluxes with height within the tower layer (Persson and Vihma,
 464 2017).

5 Low Wind Speeds

We partition the vertical structure of $\overline{w'u'}$ and $\overline{w'\theta'}$ for small V into sub-classes of smaller and larger $\delta\theta$. We first consider the two sites with complex heterogeneity of surface vegetation. The flow response to such common less organized changes of surface roughness is not understood, but the concept of an internal boundary layer offers guidance. We organize our discussion of low wind speeds over complex heterogeneity in terms of an idealized vertical structure (Fig. 5). We use the purposely vague terms “new boundary layer” and “regional boundary layer” and the transition between these two layers. The flow in the new boundary layer is strongly influenced by the local surface conditions but may not be accommodated by internal boundary-layer theory that was developed for flow past a single discontinuity at the surface. Because we are unable to assess the degree of adjustment of the new boundary layer to the smoother surface, we do not use the term “equilibrium layer” as in internal boundary-layer theory. We sequentially discuss the new boundary layer at the surface, the transition layer and the overlying regional boundary layer at the heterogeneous FLOSSII and BPP sites using the terminology defined in Fig. 5. Because the upwind heterogeneity is complex, the interpretation of the results must be considered somewhat speculative and incomplete.

5.1 New Boundary Layer

For $V < 2 \text{ m s}^{-1}$ and larger stratification ($\delta\theta > 1 \text{ K}$) at the FLOSSII site, the magnitudes of both the downward momentum and heat flux decrease with height in the lowest 5 m (black, Fig. 6b, d and Fig. 7), suggestive of a shallow new boundary layer that has adjusted, to some degree, to the small local roughness (Sect. 2.1). For small stratification ($\delta\theta < 1 \text{ K}$), a very shallow new boundary layer might cause the observed decrease of the magnitudes of both

491 the heat and momentum flux between 1 and 2 m (black, Fig. 6a, c) as in a
492 new boundary layer, but this inference is based on only the two lowest levels
493 and must be considered as speculation. The new boundary layer is apparently
494 too shallow to be resolved by the measurements at the BPP site for both
495 stratification categories (cyan, Fig. 6), probably because the upwind greater
496 roughness is close to the tower at the BPP site, only a few tens of metres
497 upwind for the prevailing wind directions.

498 5.2 Transition Layer

499 The flux magnitudes increase with height in the transition layer between the
500 new boundary layer and the overlying regional boundary layer (Fig. 6). For
501 $\delta\theta < 1$ K at the FLOSSII site, the magnitude of the downward momentum
502 flux $\overline{w'u'}$ increases only very slowly with height up to at least the top of the
503 tower layer (black, Fig. 6a) while the magnitude of the downward heat flux
504 increases more significantly with height but only up to 15 m where it reaches
505 a local maximum (black, Fig. 6c).

506 For $\delta\theta > 1$ K at the FLOSSII site (black, Fig. 6b and Fig. 7), the magnitude
507 of the downward momentum flux increases more rapidly with height above 5 m
508 compared to the subclass of small stratification. This transition layer extends
509 upward to the top of the tower and presumably above. Because the magnitude
510 of the fluxes cannot increase indefinitely with height, a maximum of the flux
511 magnitude and an overlying regional boundary layer is inferred above the tower
512 layer.

513 A well-defined transition layer occupies the entire 12-m tower layer for the
514 BPP site for both $\delta\theta$ subclasses (Fig. 6, cyan). The magnitudes of both the
515 downward momentum and heat fluxes increase more rapidly with height at
516 the BPP site (cyan, Fig. 6a,b) compared to that at the FLOSSII site (black)
517 probably because the increase of the height of the upwind vegetation is greater

518 and closer to the tower compared to the scattered upwind brush at the FLOS-
519 SII site. The magnitude of the downward heat flux at the BPP site for the low
520 wind class reaches a weak maximum at 8 m for both $\delta\theta$ subclasses (Fig. 6c-d),
521 indicating the bottom of an overlying regional boundary layer. The vertical
522 profile of the heat flux is again characterized by a smaller depth scale than
523 that for the momentum flux.

524 The different vertical structures for the heat and momentum flux under-
525 score the need for caution in making analogies with internal boundary layers.
526 The heat flux profile is directly influenced by the constraint of the surface
527 energy budget and also by the rapid decrease of the stratification with height
528 for very stable conditions. The momentum flux can be maintained at higher
529 levels due to shear on the underside of low-level jets. All four sites experience
530 low-level jets (not shown). Gravity waves appear to be better organized away
531 from the surface and may also effectively transport momentum.

532 5.3 Regional Boundary Layer

533 Maximum magnitudes of the fluxes at the top of the transition layer are ex-
534 plicitly observed for the heat flux at the FLOSSII site for small stratification
535 and to a lesser degree at the BPP site. A maximum in the magnitude of the
536 momentum flux is inferred at some unknown higher level because the flux mag-
537 nitude cannot increase indefinitely with height. At the FLOSSII site, elevated
538 turbulence can be generated by hills that rise up to 200 m above the tower
539 base, beginning 2–4 km upwind from the tower site with respect to the pre-
540 vailing wind direction. Higher mountains occur farther upwind. The elevated
541 turbulence is presumably advected over the tower site.

542 Elevated turbulence was observed from the Wyoming King Air aircraft dur-
543 ing FLOSSII (Cardon, 2007). The Wyoming King Air flew nine early morning
544 flights over the FLOSSII tower in February and March of 2003 at 30 m and

545 60 m above the ground surface. On average, the 30-m aircraft-measured fluxes
546 agreed well with those measured at the top of the tower although the aircraft
547 fluxes were more variable (not shown) probably due to smaller sample size.
548 The magnitude of the momentum flux generally decreased between the 30-m
549 and 60-m levels even with low wind speeds when the flux magnitude tended
550 to increase with height across the tower layer. These observations are consis-
551 tent with a maximum of the downward momentum flux between 30 and 60
552 m although the aircraft measurements represent a small amount of data. The
553 magnitude of the aircraft-measured heat fluxes at 60 m were often near zero
554 within suspected observational error. Thus the depth scale for the vertical vari-
555 ation of the heat flux again appears to be smaller than that for the momentum
556 flux. The aircraft soundings showed transient elevated wind maxima but not
557 persistent low-level jets.

558 5.4 Three-Layer Structure over Gentle Topography

559 The flow at the SCP tower site for low wind speeds also includes a shallow
560 boundary layer associated with the local surface, a transition layer and a re-
561 gional boundary layer. However the generation of the flow is different compared
562 to the FLOSSII site. The flow near the surface for low wind speeds at the SCP
563 site is often down the slope of the valley floor. The magnitude of the downward
564 momentum flux at the SCP site decreases with height up to about 5 m for
565 both classes of $\delta\theta$ (red, Fig. 6a-b), defining a shallow new boundary layer at
566 the surface that is responding to the local slope that extends a few hundred
567 metres upwind for the tower. The increase of the magnitude of the downward
568 momentum flux with height between 5 and 10 m is due to generation of turbu-
569 lence by shear in the vertical transition between the underlying drainage flow
570 and an overlying regional flow (Mahrt et al., 2014), which sometimes leads to
571 significant directional shear.

572 The magnitudes of the downward heat and momentum fluxes reach a maxi-
573 mum at 10 m for small stratification (red, Fig. 6a, b). For strong stratification,
574 the magnitude of the downward momentum flux reaches a relative maximum
575 at 10 m while the heat flux magnitude reaches a relative maximum at 3 m (red,
576 Fig. 6c, d). These relative maxima define the bottom of the regional bound-
577 ary layer. Even gentle small-scale topography can lead to complex vertical
578 structure of the flux profile.

579 5.5 Homogeneous Case

580 For low wind speeds at the quasi-homogeneous SHEBA site, the magnitudes of
581 the fluxes still decrease significantly with height in the lowest 10 m, more so for
582 the heat flux than for the momentum flux (blue, Fig. 6). Although shallow, this
583 layer can still be referred to as a regional boundary layer although such termi-
584 nology serves only to make an analogy with the heterogeneous FLOSSII and
585 BPP sites. The shallow boundary layer at the SHEBA site is apparently asso-
586 ciated with the very small roughness length and establishment of an Ekman
587 boundary layer where the boundary layer reaches approximate equilibrium
588 during the polar night and the boundary-layer depth becomes constrained by
589 the Earth's rotation (Grachev et al., 2005). The magnitude of the downward
590 momentum flux does not decrease further above 10 m (blue, Fig. 6) possibly
591 because of the influence of background turbulence, turbulence generated by
592 overlying jets, and the impact of cases of deeper boundary layers that are
593 also included in the composite. Grachev et al. (2005) identified several dif-
594 ferent types of boundary layers that contribute to the composited profiles of
595 our present study. The percentage decrease of the magnitude of the downward
596 heat flux with height is greater compared to that for the momentum flux for
597 both subclasses of $\delta\theta$. As a result, the relatively shallow boundary layer is
598 more clearly revealed by the heat flux profile than the momentum flux profile.

6 Errors in the Surface Flux due to Flux Divergence

600 For low wind speeds, the lowest flux measurement level may not be sufficiently
601 close to the surface to estimate the surface fluxes. To assess potential flux
602 errors, we assume that the 1-m values of the momentum flux are a reasonable
603 approximation to the surface fluxes and examine the errors associated with
604 use of 5-m fluxes. For the different methods of compositing the momentum
605 flux and the different $\delta\theta$ classes in Figs. 6–7, the surface flux error using the
606 composited 5-m fluxes is typically 5%–20%. The estimated errors for the heat
607 flux are generally a little larger. Error estimates based on extrapolation of
608 linear regression of the fluxes to the surface are somewhat larger than those
609 based on the 1-m fluxes. The errors would be roughly twice the above values
610 with use of 10-m fluxes.

611 The problem becomes more complex with microscale heterogeneity because
612 the flux varies with height due to the increasing footprint with height. This
613 issue is not an instrumental problem but rather a problem of representative-
614 ness and intent of the flux calculation. Defining the surface flux to be at the
615 ground surface for use in the surface energy balance would correspond to a
616 vanishing footprint. Ideally the site is homogeneous on the microscale where
617 the increasing footprint with height close to the surface does not affect the
618 flux. The microscale heterogeneity appears to be small at our sites with the
619 possible exception of the SCP site. However, nearby flux stations in the net-
620 work on the valley floor showed, on average, minimal spatial variations of the
621 1-m momentum flux. At higher levels on the FLOSSII, SCP and BPP towers,
622 the flux footprint increases with height to the extent that it is capturing some
623 of the rougher surface upwind, and this defines the transition layer studied in
624 Sect. 5.

625 The material above did not consider deviations between the stress, wind
626 and shear directions. Although such deviations can be large for individual
627 events, vector averaging the stress and wind directions over the datasets re-
628 vealed little systematic difference between the wind and stress directions ex-
629 cept at the SCP site. At this site for low wind speeds, the systematic differences
630 between the stress and wind directions based on composited wind and stress
631 components reach 60° in the transition layer between the shallow down-valley
632 flow at the surface and the overlying regional flow (not shown).

633 **7 Conclusions**

634 We examined turbulent flux measurements over four different surfaces where
635 the surface roughness is greater upwind of the observational sites but disor-
636 ganized. With low wind speeds, the vertical divergence of the momentum and
637 heat flux is often large such that measurements at standard observational lev-
638 els (2–10 m) are inadequate for estimation of the surface fluxes. For low wind
639 speeds at the four sites, the mean surface-flux error using 5-m measurements
640 was estimated to be 5%–20%. Such errors lead to miscalibration of similarity
641 theory and contribute to errors in the surface energy budget.

642 With flow from complex rougher surfaces to smoother surfaces, the magni-
643 tudes of the downward momentum and heat fluxes close to the surface some-
644 times decreases rapidly with height in a shallow new boundary layer adjust-
645 ing to the new surface. The new boundary layer is analogous to an internal
646 boundary layer that develops with flow past a single sharp decrease of surface
647 roughness. Above the new boundary layer, the flux magnitudes increase with
648 height in a transition layer, reaching a maximum at the top of the transition
649 layer as idealized in Fig. 5. The flux magnitudes then decrease with height
650 in an overlying regional boundary layer. Such significant flux divergence (con-
651 vergence) can occur even with disorganized weak surface heterogeneity. Over

652 the two sites with more significant surface heterogeneity, the depth scale of
653 the heat flux profile is smaller than that for momentum, suggesting that any
654 analogy with internal boundary layers is incomplete, although the cause of
655 this difference of depth scales was not isolated.

656 For the measurements of our study with low wind speeds, the magnitude of
657 the downward momentum and heat fluxes close to the surface decrease rapidly
658 with height in common shallow drainage flows and also in shallow polar Ekman
659 layers over flat ice/snow surfaces (5-m depth, Grachev et al. (2005)) where the
660 flow is relatively stationary and the surface roughness length is small. Elevated
661 generation of turbulence and fluxes, often associated with a low-level jet, can
662 also contribute to the increase of the flux magnitudes with height. With higher
663 wind speeds, the vertical divergence of the flux close to the surface is generally
664 less important in our study, except with flow over the gentle topography at the
665 SCP site where lee-generated turbulence sometimes leads to a rapid increase
666 of the flux magnitudes with height.

667 Because the behaviour of the flux divergence is site dependent, analysis of
668 more datasets is required. In addition, long datasets are required to partition
669 the analysis into different wind directions and into different regimes based
670 on stability, non-stationarity, low-level jets and so forth. The flux divergence
671 is more vulnerable to flux errors than the flux itself, discussed in Sect. 2.2.
672 Although it is not obvious how the flux errors could explain the systematic
673 height dependence of the flux, more investigation is required. To estimate
674 surface fluxes, measurements are required close to the surface, preferably at
675 1 m or lower, provided that the flux loss due to path-length averaging is not
676 important and the measurements are above the roughness sublayer. Smaller
677 path lengths or supplemental instrumentation such as hot film anemometers
678 would be potentially useful.

679 **Acknowledgements** We gratefully acknowledge the extensive comments of the reviewers
680 that led to major improvements of the manuscript. Discussions with Ivana Stiperski signifi-
681 cantly improved our perspective on the impact of sloped terrain on flux measurements. This
682 project received support from Grant AGS-1614345 from the National Science Foundation.
683 The Earth Observing Laboratory of the National Center for Atmospheric Research pro-
684 vided the measurements from the FLOSSII and SCP campaigns. We acknowledge the hard
685 work by scientists and staff involved in collection of the SHEBA turbulence data, especially
686 Christopher Fairall, Peter Guest, and the late Ed Andreas. The SHEBA data collection and
687 analysis was supported by Grants OPP-97-01766 and OPP-00-84323 from the U. S. Na-
688 tional Science Foundation. OP and AG were supported by funds from the National Oceanic
689 and Atmospheric Administration/Earth System Research Laboratory/Physical Sciences Di-
690 vision during the preparation of this manuscript. Emily Andreas Moynihan prepared Fig.
691 5.

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