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Transforming Modernized NSRS Coordinates and Uncertainties from One Frame to Another

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1 Introduction

The National Geodetic Survey (NGS) is modernizing the National Spatial Reference System (NSRS). As part of that modernization, NGS will define four terrestrial reference frames which will be mathematically defined relative to the ITRF2020 frame (NGS 2021). Those four frames are NATRF2022, PATRF2022, CATRF2022 and MATRF2022 (named for the North American, Pacific, Caribbean and Mariana plates, respectively.)

A technical memorandum (Smith 2020) provided equations for transforming coordinates and variances from ITRF2020 to any of the four NSRS frames (N/P/C/MATRF2022). That paper showed how ITRF2020 positioning variance as well as variance in the rotation model (EPP2022) can both impact the variance in the NSRS coordinates. However, Smith (2020) did not address how to transform coordinates and variances *between* the NSRS frames, nor did it address transforming coordinates *from* an NSRS frame *to* ITRF2020. This paper addresses both issues.

2 From one NSRS frame to another

2.1 Coordinates

Begin with the relationship of coordinates at a common epoch, t , between some frame, F , (where F can be NATRF2022, PATRF2022, CATRF2022 or MATRF2022) and the ITRF2020 frame, which we designate I , as mentioned in NGS (2021):

$$\begin{bmatrix} X(t) \\ Y(t) \\ Z(t) \end{bmatrix}_F = R_{F,I}(t, t_0) \begin{bmatrix} X(t) \\ Y(t) \\ Z(t) \end{bmatrix}_I \quad (1)$$

Where t_0 is a reference epoch, and the rotation matrix $R_{F,I}(t, t_0)$ is defined as:

$$R_{F,I}(t, t_0) = \begin{bmatrix} 1 & \Delta t(\dot{\omega}_Z)_{F,I} & -\Delta t(\dot{\omega}_Y)_{F,I} \\ -\Delta t(\dot{\omega}_Z)_{F,I} & 1 & \Delta t(\dot{\omega}_X)_{F,I} \\ \Delta t(\dot{\omega}_Y)_{F,I} & -\Delta t(\dot{\omega}_X)_{F,I} & 1 \end{bmatrix} = \begin{bmatrix} 1 & (\omega_Z)_{F,I} & -(\omega_Y)_{F,I} \\ -(\omega_Z)_{F,I} & 1 & (\omega_X)_{F,I} \\ (\omega_Y)_{F,I} & -(\omega_X)_{F,I} & 1 \end{bmatrix} \quad (2)$$

Let us now introduce frame G , which is also one of the four NSRS frames, but different from F . Thus:

$$\begin{bmatrix} X(t) \\ Y(t) \\ Z(t) \end{bmatrix}_G = R_{G,I}(t, t_0) \begin{bmatrix} X(t) \\ Y(t) \\ Z(t) \end{bmatrix}_I \quad (3)$$

where:

$$R_{G,I}(t, t_0) = \begin{bmatrix} 1 & \Delta t(\dot{\omega}_Z)_{G,I} & -\Delta t(\dot{\omega}_Y)_{G,I} \\ -\Delta t(\dot{\omega}_Z)_{G,I} & 1 & \Delta t(\dot{\omega}_X)_{G,I} \\ \Delta t(\dot{\omega}_Y)_{G,I} & -\Delta t(\dot{\omega}_X)_{G,I} & 1 \end{bmatrix} = \begin{bmatrix} 1 & (\omega_Z)_{G,I} & -(\omega_Y)_{G,I} \\ -(\omega_Z)_{G,I} & 1 & (\omega_X)_{G,I} \\ (\omega_Y)_{G,I} & -(\omega_X)_{G,I} & 1 \end{bmatrix} \quad (4)$$

This yields the following relationship between the coordinates in F and G :

$$\begin{bmatrix} X(t) \\ Y(t) \\ Z(t) \end{bmatrix}_G = R_{G,I}(t, t_0) [R_{F,I}(t, t_0)]^{-1} \begin{bmatrix} X(t) \\ Y(t) \\ Z(t) \end{bmatrix}_F \quad (5)$$

Computationally, one may apply (5) as is to arrive at the Cartesian coordinates in frame G from those in frame F . However, for the purposes of simplification, as well as for use in the next section, we will present alternate forms of (5) which rely upon negligible approximations.

The inverse in (5), computed by inverting the matrix in (2), is complicated. We therefore introduce the following small-angle approximation: *for any two angles, ω , found in (2) or (4), their product is zero*. Using this approximation, the inverse of the matrix in (2) simplifies to:

$$[R_{F,I}(t, t_0)]^{-1} = \begin{bmatrix} 1 & -\Delta t(\dot{\omega}_Z)_{F,I} & \Delta t(\dot{\omega}_Y)_{F,I} \\ \Delta t(\dot{\omega}_Z)_{F,I} & 1 & -\Delta t(\dot{\omega}_X)_{F,I} \\ -\Delta t(\dot{\omega}_Y)_{F,I} & \Delta t(\dot{\omega}_X)_{F,I} & 1 \end{bmatrix} = \begin{bmatrix} 1 & -(\omega_Z)_{F,I} & (\omega_Y)_{F,I} \\ (\omega_Z)_{F,I} & 1 & -(\omega_X)_{F,I} \\ -(\omega_Y)_{F,I} & (\omega_X)_{F,I} & 1 \end{bmatrix} = [R_{F,I}(t, t_0)]^T \quad (6)$$

We see therefore that, under the small angle approximation, a rotation matrix is an orthogonal matrix. Equation 5 therefore expands to:

$$\begin{bmatrix} X(t) \\ Y(t) \\ Z(t) \end{bmatrix}_G = \begin{bmatrix} 1 & (\omega_Z)_{G,I} & -(\omega_Y)_{G,I} \\ -(\omega_Z)_{G,I} & 1 & (\omega_X)_{G,I} \\ (\omega_Y)_{G,I} & -(\omega_X)_{G,I} & 1 \end{bmatrix} \begin{bmatrix} 1 & -(\omega_Z)_{F,I} & (\omega_Y)_{F,I} \\ (\omega_Z)_{F,I} & 1 & -(\omega_X)_{F,I} \\ -(\omega_Y)_{F,I} & (\omega_X)_{F,I} & 1 \end{bmatrix} \begin{bmatrix} X(t) \\ Y(t) \\ Z(t) \end{bmatrix}_F \quad (7)$$

If (7) is expanded out, it also grows complicated quickly. However, another application of the small angle approximation yields this simpler form:

$$\begin{bmatrix} X(t) \\ Y(t) \\ Z(t) \end{bmatrix}_G = \begin{bmatrix} X_F(t) - Y_F(t)(\omega_Z)_{F,I} + Y_F(t)(\omega_Z)_{G,I} + Z_F(t)(\omega_Y)_{F,I} - Z_F(t)(\omega_Y)_{G,I} \\ Y_F(t) + X_F(t)(\omega_Z)_{F,I} - X_F(t)(\omega_Z)_{G,I} - Z_F(t)(\omega_X)_{F,I} + Z_F(t)(\omega_X)_{G,I} \\ Z_F(t) - X_F(t)(\omega_Y)_{F,I} + X_F(t)(\omega_Y)_{G,I} + Y_F(t)(\omega_X)_{F,I} - Y_F(t)(\omega_X)_{G,I} \end{bmatrix} \quad (8)$$

or

$$\begin{bmatrix} X(t) \\ Y(t) \\ Z(t) \end{bmatrix}_G = \begin{bmatrix} X_F(t) - Y_F(t)\Delta t(\dot{\omega}_Z)_{F,I} + Y_F(t)\Delta t(\dot{\omega}_Z)_{G,I} + Z_F(t)\Delta t(\dot{\omega}_Y)_{F,I} - Z_F(t)\Delta t(\dot{\omega}_Y)_{G,I} \\ Y_F(t) + X_F(t)\Delta t(\dot{\omega}_Z)_{F,I} - X_F(t)\Delta t(\dot{\omega}_Z)_{G,I} - Z_F(t)\Delta t(\dot{\omega}_X)_{F,I} + Z_F(t)\Delta t(\dot{\omega}_X)_{G,I} \\ Z_F(t) - X_F(t)\Delta t(\dot{\omega}_Y)_{F,I} + X_F(t)\Delta t(\dot{\omega}_Y)_{G,I} + Y_F(t)\Delta t(\dot{\omega}_X)_{F,I} - Y_F(t)\Delta t(\dot{\omega}_X)_{G,I} \end{bmatrix} \quad (9)$$

We see in (9) that the coordinates in new frame G are dependent upon the coordinates in frame F as well as the rotation models for both frames F and G . Equation 9 also gives us an alternate version of (5) which may more easily be used in error propagation.

2.2 Error propagation

We now build upon and expand the discussion found in section 2 of Smith (2020), and seek an expression for the dispersion of the coordinates in frame G . We begin by applying the dispersion operator to (9):

$$D \left\{ \begin{bmatrix} X(t) \\ Y(t) \\ Z(t) \end{bmatrix}_G \right\} = D \left\{ \begin{bmatrix} X_F(t) - Y_F(t)\Delta t(\dot{\omega}_Z)_{F,I} + Y_F(t)\Delta t(\dot{\omega}_Z)_{G,I} + Z_F(t)\Delta t(\dot{\omega}_Y)_{F,I} - Z_F(t)\Delta t(\dot{\omega}_Y)_{G,I} \\ Y_F(t) + X_F(t)\Delta t(\dot{\omega}_Z)_{F,I} - X_F(t)\Delta t(\dot{\omega}_Z)_{G,I} - Z_F(t)\Delta t(\dot{\omega}_X)_{F,I} + Z_F(t)\Delta t(\dot{\omega}_X)_{G,I} \\ Z_F(t) - X_F(t)\Delta t(\dot{\omega}_Y)_{F,I} + X_F(t)\Delta t(\dot{\omega}_Y)_{G,I} + Y_F(t)\Delta t(\dot{\omega}_X)_{F,I} - Y_F(t)\Delta t(\dot{\omega}_X)_{G,I} \end{bmatrix} \right\} \quad (10)$$

In (9), we see that there are nine stochastic variables of interest (three coordinates in frame F , three rotation rates for frame F and three rotation rates for frame G). To compute the dispersion on the left, we must linearize (9) and use a Jacobian, as per (ibid). To simplify things, we expand the abbreviation set from (ibid) and define the nine stochastic variables as vector \mathbf{X} (comprised of three sub-vectors \mathbf{X}_1 , \mathbf{X}_2 and \mathbf{X}_3) and our three, to-be-determined, Cartesian coordinates in frame G as vector \mathbf{Y} :

$$\mathbf{X} = \begin{bmatrix} \mathbf{X}_1 \\ \mathbf{X}_2 \\ \mathbf{X}_3 \end{bmatrix} = \begin{bmatrix} X_F(t) \\ Y_F(t) \\ Z_F(t) \\ (\dot{\omega}_X)_{F,I} \\ (\dot{\omega}_Y)_{F,I} \\ (\dot{\omega}_Z)_{F,I} \\ (\dot{\omega}_X)_{G,I} \\ (\dot{\omega}_Y)_{G,I} \\ (\dot{\omega}_Z)_{G,I} \end{bmatrix} \quad (11)$$

$$\mathbf{Y} = \begin{bmatrix} X_G(t) \\ Y_G(t) \\ Z_G(t) \end{bmatrix} \quad (12)$$

This allows us to write the non-linear (9) as:

$$\mathbf{Y}(\mathbf{X}) = \begin{bmatrix} X_G(X_F, Y_F, Z_F, (\dot{\omega}_Y)_{F,I}, (\dot{\omega}_Z)_{F,I}, (\dot{\omega}_Y)_{G,I}, (\dot{\omega}_Z)_{G,I}) \\ Y_G(X_F, Y_F, Z_F, (\dot{\omega}_X)_{F,I}, (\dot{\omega}_Z)_{F,I}, (\dot{\omega}_X)_{G,I}, (\dot{\omega}_Z)_{G,I}) \\ Z_G(X_F, Y_F, Z_F, (\dot{\omega}_X)_{F,I}, (\dot{\omega}_Y)_{F,I}, (\dot{\omega}_X)_{G,I}, (\dot{\omega}_Y)_{G,I}) \end{bmatrix} \quad (13)$$

We now introduce the 9×9 dispersion matrix for the pre-determined \mathbf{X} vector as $\Sigma_{\mathbf{X}}$. This matrix will be block-diagonal and known. The lower right pair of 3×3 blocks will each be dispersions for the micro-rotation rates ($\Sigma_{\mathbf{X}_2}$ and $\Sigma_{\mathbf{X}_3}$) for frames F and G respectively and will be part of the definition of the plate fixed frames in the modernized NSRS. These values will be identical every time we use them, and we assume no correlation between the rotation rates for the two frames. The upper left 3×3 block will be dispersion for the frame F Cartesian coordinates ($\Sigma_{\mathbf{X}_1}$), and may come from any positioning software which uses frame F as its geodetic control. That is:

$$+ \begin{bmatrix} 0 & -Z_F(t)\Delta t & Y_F(t)\Delta t \\ Z_F(t)\Delta t & 0 & -X_F(t)\Delta t \\ -Y_F(t)\Delta t & X_F(t)\Delta t & 0 \end{bmatrix} \Sigma_{X_3} \begin{bmatrix} 0 & -Z_F(t)\Delta t & Y_F(t)\Delta t \\ Z_F(t)\Delta t & 0 & -X_F(t)\Delta t \\ -Y_F(t)\Delta t & X_F(t)\Delta t & 0 \end{bmatrix}^T$$

$$= \begin{bmatrix} 1 & -\Delta t(\Delta\dot{\omega}_Z)_{F,G} & \Delta t(\Delta\dot{\omega}_Y)_{F,G} \\ \Delta t(\Delta\dot{\omega}_Z)_{F,G} & 1 & -\Delta t(\Delta\dot{\omega}_X)_{F,G} \\ -\Delta t(\Delta\dot{\omega}_Y)_{F,G} & \Delta t(\Delta\dot{\omega}_X)_{F,G} & 1 \end{bmatrix} \begin{bmatrix} \sigma_{X_F(t)}^2 & \sigma_{X_F(t),Y_F(t)} & \sigma_{X_F(t),Z_F(t)} \\ \sigma_{Y_F(t),X_F(t)} & \sigma_{Y_F(t)}^2 & \sigma_{Y_F(t),Z_F(t)} \\ \sigma_{Z_F(t),X_F(t)} & \sigma_{Z_F(t),Y_F(t)} & \sigma_{Z_F(t)}^2 \end{bmatrix} \begin{bmatrix} 1 & -\Delta t(\Delta\dot{\omega}_Z)_{F,G} & \Delta t(\Delta\dot{\omega}_Y)_{F,G} \\ \Delta t(\Delta\dot{\omega}_Z)_{F,G} & 1 & -\Delta t(\Delta\dot{\omega}_X)_{F,G} \\ -\Delta t(\Delta\dot{\omega}_Y)_{F,G} & \Delta t(\Delta\dot{\omega}_X)_{F,G} & 1 \end{bmatrix}^T$$

$$+ \begin{bmatrix} 0 & Z_F(t)\Delta t & -Y_F(t)\Delta t \\ -Z_F(t)\Delta t & 0 & X_F(t)\Delta t \\ Y_F(t)\Delta t & -X_F(t)\Delta t & 0 \end{bmatrix} \begin{bmatrix} \sigma_{(\dot{\omega}_X)_{F,I}}^2 & \sigma_{(\dot{\omega}_X)_{F,I},(\dot{\omega}_Y)_{F,I}} & \sigma_{(\dot{\omega}_X)_{F,I},(\dot{\omega}_Z)_{F,I}} \\ \sigma_{(\dot{\omega}_Y)_{F,I},(\dot{\omega}_X)_{F,I}} & \sigma_{(\dot{\omega}_Y)_{F,I}}^2 & \sigma_{(\dot{\omega}_Y)_{F,I},(\dot{\omega}_Z)_{F,I}} \\ \sigma_{(\dot{\omega}_Z)_{F,I},(\dot{\omega}_X)_{F,I}} & \sigma_{(\dot{\omega}_Z)_{F,I},(\dot{\omega}_Y)_{F,I}} & \sigma_{(\dot{\omega}_Z)_{F,I}}^2 \end{bmatrix} \begin{bmatrix} 0 & Z_F(t)\Delta t & -Y_F(t)\Delta t \\ -Z_F(t)\Delta t & 0 & X_F(t)\Delta t \\ Y_F(t)\Delta t & -X_F(t)\Delta t & 0 \end{bmatrix}^T$$

$$+ \begin{bmatrix} 0 & -Z_F(t)\Delta t & Y_F(t)\Delta t \\ Z_F(t)\Delta t & 0 & -X_F(t)\Delta t \\ -Y_F(t)\Delta t & X_F(t)\Delta t & 0 \end{bmatrix} \begin{bmatrix} \sigma_{(\dot{\omega}_X)_{G,I}}^2 & \sigma_{(\dot{\omega}_X)_{G,I},(\dot{\omega}_Y)_{G,I}} & \sigma_{(\dot{\omega}_X)_{G,I},(\dot{\omega}_Z)_{G,I}} \\ \sigma_{(\dot{\omega}_Y)_{G,I},(\dot{\omega}_X)_{G,I}} & \sigma_{(\dot{\omega}_Y)_{G,I}}^2 & \sigma_{(\dot{\omega}_Y)_{G,I},(\dot{\omega}_Z)_{G,I}} \\ \sigma_{(\dot{\omega}_Z)_{G,I},(\dot{\omega}_X)_{G,I}} & \sigma_{(\dot{\omega}_Z)_{G,I},(\dot{\omega}_Y)_{G,I}} & \sigma_{(\dot{\omega}_Z)_{G,I}}^2 \end{bmatrix} \begin{bmatrix} 0 & -Z_F(t)\Delta t & Y_F(t)\Delta t \\ Z_F(t)\Delta t & 0 & -X_F(t)\Delta t \\ -Y_F(t)\Delta t & X_F(t)\Delta t & 0 \end{bmatrix}^T$$

$$= \Sigma_{Y_1} + \Sigma_{Y_2} + \Sigma_{Y_3}$$

Where Σ_{Y_1} is that portion of NSRS XYZ coordinate uncertainty due to XYZ positioning uncertainty in frame F (Σ_{X_1}), while Σ_{Y_2} is that portion of NSRS XYZ uncertainty due to EPP2022 uncertainty of frame F (Σ_{X_2}) and Σ_{Y_3} is that portion of NSRS XYZ uncertainty due to EPP2022 uncertainty of frame G (Σ_{X_3}).

Equation 17 lets NGS (or any NSRS user) separately compute the impact of positioning uncertainty (Σ_{Y_1}) from the impact of rotation model uncertainty (Σ_{Y_2} and Σ_{Y_3}) and determine if/whether to apply the latter. As per the decision made in section 2.1 of Smith (2020), NGS products and services will not propagate plate rotation model uncertainty into coordinate uncertainty, but such values will be computed and available for users, if they so desire them. That is, when a user asks for Σ_Y NGS will use $\Sigma_Y = \Sigma_{Y_1}$, but will also provide Σ_{Y_2} and Σ_{Y_3} .

Once the dispersion has been computed in XYZ for frame G , it can be rotated into an ENU frame using the same method as section 2.1 of Smith (2020).

3 From an NSRS plate fixed frame to ITRF2020

The simpler case is transforming coordinates between the NSRS plate-fixed frame and ITRF2020. Much of this information can be determined by examining Smith (2020) and re-arranging equations, and by following the logic of the previous section. Therefore substantial redundant details are omitted from the derivations below.

3.1 Coordinates

If we are provided with Cartesian coordinates in some plate-fixed NSRS frame, called F (where F can be NATRF2022, PATRF2022, CATRF2022 or MATRF2022), and are interested in coordinates in ITRF2020 we can use (1) from earlier in this report, and invert it:

$$\begin{bmatrix} X(t) \\ Y(t) \\ Z(t) \end{bmatrix}_I = [R_{F,I}(t, t_0)]^{-1} \begin{bmatrix} X(t) \\ Y(t) \\ Z(t) \end{bmatrix}_F \quad (18)$$

As above, this equation may be coded as is, but for simplicity and for the purposes of error propagation we use the small angle approximation, repeatedly, to arrive at:

$$\begin{aligned} \begin{bmatrix} X(t) \\ Y(t) \\ Z(t) \end{bmatrix}_I &= [R_{F,I}(t, t_0)]^T \begin{bmatrix} X(t) \\ Y(t) \\ Z(t) \end{bmatrix}_F = \begin{bmatrix} 1 & -(\omega_Z)_{F,I} & (\omega_Y)_{F,I} \\ (\omega_Z)_{F,I} & 1 & -(\omega_X)_{F,I} \\ -(\omega_Y)_{F,I} & (\omega_X)_{F,I} & 1 \end{bmatrix} \begin{bmatrix} X(t) \\ Y(t) \\ Z(t) \end{bmatrix}_F \\ &= \begin{bmatrix} X_F(t) - Y_F(t)\Delta t(\dot{\omega}_Z)_{F,I} + Z_F(t)\Delta t(\dot{\omega}_Y)_{F,I} \\ Y_F(t) + X_F(t)\Delta t(\dot{\omega}_Z)_{F,I} - Z_F(t)\Delta t(\dot{\omega}_X)_{F,I} \\ Z_F(t) - X_F(t)\Delta t(\dot{\omega}_Y)_{F,I} + Y_F(t)\Delta t(\dot{\omega}_X)_{F,I} \end{bmatrix} \end{aligned} \quad (19)$$

3.2 Error propagation

We follow, but simplify much of the discussion in section 2.2. Beginning with:

$$\begin{aligned} D \left\{ \begin{bmatrix} X(t) \\ Y(t) \\ Z(t) \end{bmatrix}_I \right\} &= D \left\{ \begin{bmatrix} X_F(t) - Y_F(t)\Delta t(\dot{\omega}_Z)_{F,I} + Z_F(t)\Delta t(\dot{\omega}_Y)_{F,I} \\ Y_F(t) + X_F(t)\Delta t(\dot{\omega}_Z)_{F,I} - Z_F(t)\Delta t(\dot{\omega}_X)_{F,I} \\ Z_F(t) - X_F(t)\Delta t(\dot{\omega}_Y)_{F,I} + Y_F(t)\Delta t(\dot{\omega}_X)_{F,I} \end{bmatrix} \right\} \\ &= \Sigma_X = \begin{bmatrix} \Sigma_{X_1} & 0 \\ 0 & \Sigma_{X_2} \end{bmatrix} = \begin{bmatrix} \sigma_{X_F(t)}^2 & \sigma_{X_F(t),Y_F(t)} & \sigma_{X_F(t),Z_F(t)} & & & \\ \sigma_{Y_F(t),X_F(t)} & \sigma_{Y_F(t)}^2 & \sigma_{Y_F(t),Z_F(t)} & & & 0 \\ \sigma_{Z_F(t),X_F(t)} & \sigma_{Z_F(t),Y_F(t)} & \sigma_{Z_F(t)}^2 & & & \\ & & & 0 & & \\ & & & & \sigma_{(\dot{\omega}_X)_{F,I}}^2 & \sigma_{(\dot{\omega}_X)_{F,I},(\dot{\omega}_Y)_{F,I}} & \sigma_{(\dot{\omega}_X)_{F,I},(\dot{\omega}_Z)_{F,I}} \\ & & & & \sigma_{(\dot{\omega}_Y)_{F,I},(\dot{\omega}_X)_{F,I}} & \sigma_{(\dot{\omega}_Y)_{F,I}}^2 & \sigma_{(\dot{\omega}_Y)_{F,I},(\dot{\omega}_Z)_{F,I}} \\ & & & & \sigma_{(\dot{\omega}_Z)_{F,I},(\dot{\omega}_X)_{F,I}} & \sigma_{(\dot{\omega}_Z)_{F,I},(\dot{\omega}_Y)_{F,I}} & \sigma_{(\dot{\omega}_Z)_{F,I}}^2 \end{bmatrix} \end{aligned} \quad (20a)$$

Using the same approach (Jacobian) as above, we arrive at:

$$\begin{aligned} &\Sigma_Y = \\ &= \begin{bmatrix} 1 & -\Delta t(\dot{\omega}_Z)_{F,I} & \Delta t(\dot{\omega}_Y)_{F,I} \\ \Delta t(\dot{\omega}_Z)_{F,I} & 1 & -\Delta t(\dot{\omega}_X)_{F,I} \\ -\Delta t(\dot{\omega}_Y)_{F,I} & \Delta t(\dot{\omega}_X)_{F,I} & 1 \end{bmatrix} \Sigma_{X_1} \begin{bmatrix} 1 & -\Delta t(\dot{\omega}_Z)_{F,I} & \Delta t(\dot{\omega}_Y)_{F,I} \\ \Delta t(\dot{\omega}_Z)_{F,I} & 1 & -\Delta t(\dot{\omega}_X)_{F,I} \\ -\Delta t(\dot{\omega}_Y)_{F,I} & \Delta t(\dot{\omega}_X)_{F,I} & 1 \end{bmatrix}^T \\ &\quad + \begin{bmatrix} 0 & Z_F(t)\Delta t & -Y_F(t)\Delta t \\ -Z_F(t)\Delta t & 0 & X_F(t)\Delta t \\ Y_F(t)\Delta t & -X_F(t)\Delta t & 0 \end{bmatrix} \Sigma_{X_2} \begin{bmatrix} 0 & Z_F(t)\Delta t & -Y_F(t)\Delta t \\ -Z_F(t)\Delta t & 0 & X_F(t)\Delta t \\ Y_F(t)\Delta t & -X_F(t)\Delta t & 0 \end{bmatrix}^T \end{aligned} \quad (21)$$

$$= \begin{bmatrix} 1 & -\Delta t(\dot{\omega}_Z)_{F,I} & \Delta t(\dot{\omega}_Y)_{F,I} \\ \Delta t(\dot{\omega}_Z)_{F,I} & 1 & -\Delta t(\dot{\omega}_X)_{F,I} \\ -\Delta t(\dot{\omega}_Y)_{F,I} & \Delta t(\dot{\omega}_X)_{F,I} & 1 \end{bmatrix} \begin{bmatrix} \sigma_{X_F(t)}^2 & \sigma_{X_F(t),Y_F(t)} & \sigma_{X_F(t),Z_F(t)} \\ \sigma_{Y_F(t),X_F(t)} & \sigma_{Y_F(t)}^2 & \sigma_{Y_F(t),Z_F(t)} \\ \sigma_{Z_F(t),X_F(t)} & \sigma_{Z_F(t),Y_F(t)} & \sigma_{Z_F(t)}^2 \end{bmatrix} \begin{bmatrix} 1 & -\Delta t(\dot{\omega}_Z)_{F,I} & \Delta t(\dot{\omega}_Y)_{F,I} \\ \Delta t(\dot{\omega}_Z)_{F,I} & 1 & -\Delta t(\dot{\omega}_X)_{F,I} \\ -\Delta t(\dot{\omega}_Y)_{F,I} & \Delta t(\dot{\omega}_X)_{F,I} & 1 \end{bmatrix}^T$$

$$\begin{aligned}
& + \begin{bmatrix} 0 & Z_F(t)\Delta t & -Y_F(t)\Delta t \\ -Z_F(t)\Delta t & 0 & X_F(t)\Delta t \\ Y_F(t)\Delta t & -X_F(t)\Delta t & 0 \end{bmatrix} \begin{bmatrix} \sigma_{(\dot{\omega}_X)_{F,I}}^2 & \sigma_{(\dot{\omega}_X)_{F,I},(\dot{\omega}_Y)_{F,I}} & \sigma_{(\dot{\omega}_X)_{F,I},(\dot{\omega}_Z)_{F,I}} \\ \sigma_{(\dot{\omega}_Y)_{F,I},(\dot{\omega}_X)_{F,I}} & \sigma_{(\dot{\omega}_Y)_{F,I}}^2 & \sigma_{(\dot{\omega}_Y)_{F,I},(\dot{\omega}_Z)_{F,I}} \\ \sigma_{(\dot{\omega}_Z)_{F,I},(\dot{\omega}_X)_{F,I}} & \sigma_{(\dot{\omega}_Z)_{F,I},(\dot{\omega}_Y)_{F,I}} & \sigma_{(\dot{\omega}_Z)_{F,I}}^2 \end{bmatrix} \begin{bmatrix} 0 & Z_F(t)\Delta t & -Y_F(t)\Delta t \\ -Z_F(t)\Delta t & 0 & X_F(t)\Delta t \\ Y_F(t)\Delta t & -X_F(t)\Delta t & 0 \end{bmatrix}^T \\
& = \Sigma_{Y_1} + \Sigma_{Y_2}
\end{aligned}$$

Where Σ_{Y_1} is that portion of ITRF2020 XYZ coordinate uncertainty due to XYZ positioning uncertainty in frame F (Σ_{X_1}), while Σ_{Y_2} is that portion of ITRF2020 XYZ uncertainty due to EPP2022 uncertainty of frame F (Σ_{X_2}).

Equation 21 lets NGS (or any NSRS user) separately compute the impact of positioning uncertainty (Σ_{Y_1}) from the impact of rotation model uncertainty (Σ_{Y_2}) and determine if/whether to apply the latter. As per the decision made in section 2.1 of Smith (2020), NGS products and services will not propagate plate rotation model uncertainty into coordinate uncertainty, but such values will be computed and available for users, if they so desire them. That is, when a user asks for Σ_Y NGS will use $\Sigma_Y = \Sigma_{Y_1}$, but will also provide Σ_{Y_2} .

4 Bibliography

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