

1 **Supporting Information for “Lagrangian Timescales**  
 2 **of Southern Ocean Upwelling in a Hierarchy of**  
 3 **Model Resolutions”**

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7 **Additional Supporting Information (Files uploaded separately)**

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9 **Introduction**

10 The supplementary information provided here serves two purposes: firstly, we provide  
 11 additional useful figures and a movie; secondly, we provide supporting text regarding:  
 12 spectral analysis in support of our hypothesis that temporally degrading velocity fields  
 13 corresponds to removing eddy energy in a comparable way to degrading horizontal reso-  
 14 lution (Text S1); sensitivity of our results to the definition of the upper ocean (Text S2);

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- 15 potential sources of error in the calculation of Lagrangian Upwelling Transport (Text S3);  
16 the inverse Gaussian fits to the transit time distributions (Text S4).

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17 **Text S1.** In the main text, we hypothesized that the temporal degradation of CM2.6  
18 velocity output from 5-day averages to monthly averages removes eddy kinetic energy at  
19 all scales, but the energetic mesoscale in particular. We begin by calculating the eddy  
20 velocities (and subsequently eddy kinetic energy) by removing the 1-year mean from the  
21 velocity output. Using code from *Rocha et al.* [2016] (python package *pyspec* available at  
22 <https://github.com/pyspec/pyspec>), we compute the wavenumber spectra of horizontal  
23 and vertical eddy kinetic energy (removing the spatial mean eddy kinetic energy and  
24 detrending in both the zonal and meridional directions) over a region at 1000 meters  
25 depth from 155°W to 135°W, 60°S to 52°S (Supplementary Fig. S2a,b). *Tamsitt et al.*  
26 [2017] found enhanced Lagrangian upwelling in this region of high eddy kinetic energy  
27 where the ACC intersects the Pacific-Antarctic Ridge. Isotropic spectra are computed  
28 from zonal-meridional spectra by interpolating to polar coordinates and selecting only  
29 the radial component (Supplementary Fig. S2c,d). We compute the spectra of each  
30 snapshot (5-day or monthly averages, depending on the experiment) in the first year of  
31 output and calculate the mean (solid lines) and spread (shaded regions, to give a sense of  
32 the variability). The similarity between the zonal, meridional, and isotropic wavenumber  
33 spectra for the vertical eddy kinetic energy suggests that the turbulence is relatively  
34 isotropic at these length scales. Focusing now on the isotropic wavenumber spectra for  
35 vertical eddy kinetic energy (panel (c)), we note that CM2.5-5day has over an order of  
36 magnitude less vertical eddy kinetic energy than CM2.6-5day at all resolved scales (and  
37 zero energy at smaller unresolved scales). The red line shows that temporal degradation of  
38 CM2.6-5day removes vertical eddy kinetic energy at all spatial scales and causes its spectra

39 to resemble that of CM2.5-5day (although it still contains some energy at scales unresolved  
40 in CM2.5-5day). Panel (d) shows the isotropic wavenumber spectrum of horizontal eddy  
41 kinetic energy, which has a steeper slope at small scales than the vertical eddy kinetic  
42 energy. We note that for the horizontal eddy kinetic energy spectrum, the difference  
43 between CM2.5-5day and CM2.6-monthly is less significant, but increases towards smaller  
44 scales. This analysis was repeated for several other regions of the Antarctic Circumpolar  
45 Current and the qualitative conclusions above are unchanged therein (not shown).

46 **Text S2.** In the paper, we define the upper ocean as the part of the ocean shallower  
47 than 300 m. Here, we show that the results of the paper are robust by exploring other  
48 definitions for the upper ocean. In Supplementary Figure S3a-e, we plot versions of Figure  
49 2a that show the Transit-Time Distributions (TTD) for particle-transport to travel from  
50 depth at 30°S to the upper ocean, where we vary the depth of the base of the upper ocean.  
51 The trend of shorter timescale when resolution is refined is shown for all definitions of the  
52 upper ocean, except perhaps the 900 m definition, which is quite close to the upper limit  
53 of the release depths of 1000-4000 m.

54 In Supplementary Figure S3f, we define the upper ocean as the surface mixed layer, as  
55 defined by the 0.03 kg/m<sup>2</sup> threshold definition used in the ocean models. For each model  
56 grid column, 5-day averaged mixed layer depths of the nearest grid-point are linearly in-  
57 terpolated in time and compared to particle depth sampled every 5 days. Qualitatively  
58 similar results are found with the constant 300 m depth definition. We note that we  
59 prefer the 300 m depth definition because it is constant across all experiments, whereas  
60 differences in the spatio-temporal distributions of mixed-layer depths across the exper-

61 iments make it difficult to know whether any differences in the timescale are due to  
62 enhanced vertical motion or lateral advection across a sloped mixed layer base. Indeed,  
63 in CM2-1deg-5day and CM2.6-monthly, the mixed-layer definition of the upper ocean  
64 causes enough particle-transport (10% total transport) to be laterally advected into deep  
65 mid-latitude mixed layers (500-800 m deep) to skew the TTD towards shorter timescales  
66 (Supplementary Fig. S6, S7). We note that in all experiments, less than 1% of particles  
67 enter mixed layers deeper than 1000 m through convective events.

### 68 **Text S3.**

69 We begin by discussing the errors in the Lagrangian overturning streamfunction of *Döös*  
70 *et al.* [2008], which is formulated to be directly comparable to the residual overturning  
71 streamfunction (which they call the “Eulerian streamfunction”). In their Figures 4a and  
72 4b, they find an upper cell of roughly 10 Sv for both the Lagrangian and Eulerian calcu-  
73 lations. In their Figure 4c, they plot the difference (error) between the Lagrangian and  
74 Eulerian streamfunctions, which appears to be less than 2 Sv for most of the upper cell  
75 but between 2 Sv and 10 Sv in some localized places. The 4 Sv contouring of the plots  
76 makes it difficult to judge the skill of the Lagrangian calculation as there could potentially  
77 be up to 2 Sv (or about 20%) error everywhere in the upper cell. They attribute this error  
78 to the looping discontinuity, time sampling, and trajectory accuracy, acknowledging that  
79 there may be some additional unknown errors.

80 We now discuss these sources of error and others for our present study. While we did  
81 not explicitly test the impact of looping in our study, we note that *Thomas et al.* [2015]  
82 showed in a study of Northern Atlantic overturning that their Lagrangian calculations

83 were insensitive to the looping discontinuity. We acknowledge that even our experiments  
84 with 5-day averaged velocities are not using the online velocities and therefore likely have  
85 some error due to the time sampling (this is the same process by which we purposefully  
86 temporally degrade our velocity fields from 5-day averages to monthly averaged to remove  
87 eddy kinetic energy).

88 In addition to the sources of error discussed in *Döös et al.* [2008], there are a few  
89 potential sources of error specific to our Lagrangian method.

90 1) Although the Lagrangian experiments are run long enough to resolve the tails of the  
91 transit-time distributions, there may still be entire modes of the transit-time distribution  
92 that are unresolved (see discussion of multi-modal distributions in Supplemental Text S4);  
93 this hypothesis is particularly relevant for the pathways that exist in the eddying models  
94 but do not exist in CM2-1deg-5day, possibly because the timescales for the particles in  
95 CM2-1deg-5day are longer than the integration time of 500 years. We do not have any  
96 reason expect this to be the case but can not rule it out from our results alone.

97 2) Many particles that are released with an initially southward velocity will nevertheless  
98 travel north of 30°S at some point before they reach the upper ocean; in all cases, these  
99 particles are omitted from our analysis because our domain ends one grid cell north of  
100 30°S. Had these particles been included in our analysis, some small subset of particles may  
101 have spent time north of 30°S after their release and before traveling south to upwell into  
102 the upper layers of the Southern Ocean. Despite the fact that these particles are released  
103 with an initial southward velocity and eventually reach the upper ocean south of 30° S, we  
104 justify omitting them because their southward transport will already be counted by the

105 new particles released at the locations and times that the original particles last crossed  
106 south of 30°S. For yet a smaller subset of these particles which spend only the first 30 days  
107 of their journey north of 30°S, we argue these particles will not be counted by a future  
108 particle release and thus should be included in estimates of LUT. Because our domains end  
109 one grid cell north of 30°S, we are unable to consistently resolve trajectories in this buffer  
110 region (which depends strongly on model resolution) to correct for this effect. In CM2-  
111 1deg-5day, our model with the largest buffer region (30°S to 29°S), we estimate a lower  
112 bound of 0.2 Sv of particles that are omitted. This estimate may increase as resolution  
113 increases and the large scale motions become more turbulent. We do not expect this  
114 source of error to be important here because our buffer zone is so small but note that in  
115 other models we have worked with where the buffer zone was 10° wide and the analysis  
116 included particles that spent time in the buffer zone, the error due to this double counting  
117 was as big as the signal itself!

118 3) Regridding the velocity field from its native B-grid to an A-grid is convenient for  
119 computing trajectories in CMS but complicates the representation of the no-normal flow  
120 boundary condition at topography; as a result, we find that some particles become trapped  
121 in corners of topography where flow is into the wall of the A-grid. We expect this to bias  
122 our LUT estimate low but note that many of these particles were initially released with  
123 small velocities very near bottom topography and thus both represent little overall trans-  
124 port (less than 10% of the total) and were anyways unlikely to be classified as upwelling  
125 Circumpolar Deep Water.

126 In a worst case estimate (where we take the high end of error from *Döös et al.* [2008]  
 127 and add on the high end of our model-specific sources of error), we estimate an upper  
 128 bound of 30% error on our estimate of LUT. It is unclear how these errors would affect the  
 129 upwelling timescale, except that the particles trapped at topography and particles with  
 130 timescales longer than the integration times may add to the slow tails of the distribution.  
 131 Such long-timescale particles would likely not effect the mode of the TTD anyways, which  
 132 we highlight as our primary timescale metric.

133 **Text S4.** We fit a three-parameter inverse Gaussian distribution separately to each of the  
 134 six individual pathways in a given experiment by the least-squares method. The inverse  
 135 Gaussian distribution is an analytical solution for the probability distribution of tracer age  
 136 in a steady uniform flow with constant diffusivity and has been previously used to describe  
 137 tracer-derived TTD [*Waugh et al.*, 2003]. A constant scaling parameter is multiplied to  
 138 the inverse Gaussian function to allow the integral of the analytical TTD to be greater  
 139 than one, thus accounting for additional particle-transport that may not be captured  
 140 within the integration time limits of the experiment. The scaled inverse Gaussian,  $f(t)$ ,  
 141 is defined as follows:

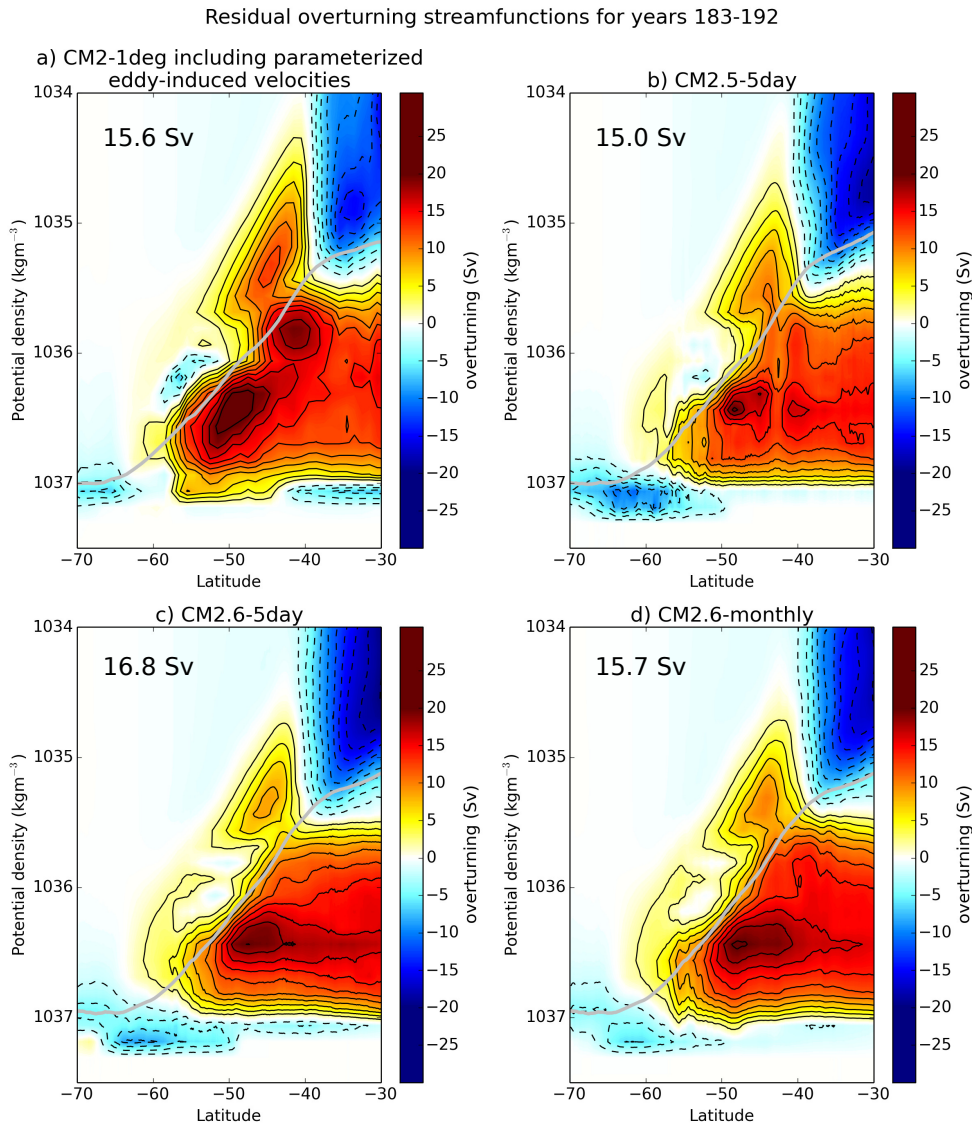
$$f(t) = C \sqrt{\frac{\Gamma^3}{4\pi\Delta^2 t^3}} \exp\left(\frac{-\Gamma(t - \Gamma)^2}{4\Delta^2 t}\right) \quad (1)$$

142 where  $t$  is the transit-time [*Waugh et al.*, 2003] and the three parameters are:  $C$ , the  
 143 multiplicative scaling parameter that accounts for particle-transport that has not yet up-  
 144 welled;  $\Gamma$ , the mean transit-time; and  $\Delta$ , the width of the analytical TTD. The analytical  
 145 TTD for each pathway is then scaled by the corresponding particle-transport such that

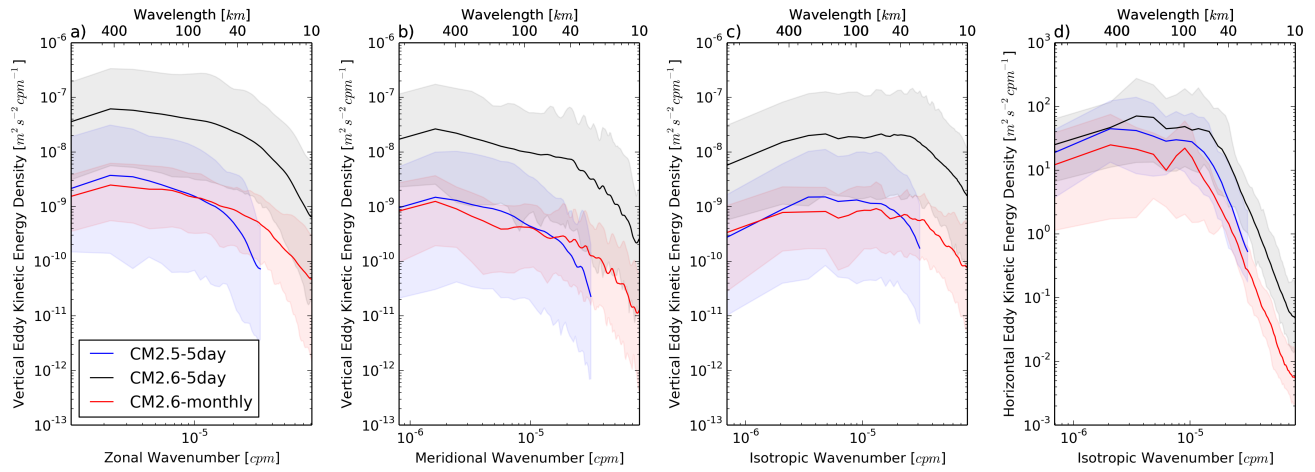


146 its integral from 0 years to infinity is an estimate of the particle-transport expected to  
147 reach the upper ocean from 30°S, independent of the Lagrangian experiment time limit  
148 (Supplementary Fig. S5a-d). This extrapolation allows us to extend the inverse Gaussian  
149 curves analytically to account for the  $< 3\%$  additional particle-transport that is expected  
150 to upwell after the integration time of the experiments. While this approach does not  
151 significantly change the results of this paper because most of the particle-transport is  
152 resolved, this approach may be useful to reconstruct a TTD when only part of the TTD is  
153 resolved from data. Studies on observational tracers and modeled Lagrangian trajectories  
154 provide examples of bimodal TTDs that fit the inverse Gaussian form poorly [*Haine et*  
155 *al., 2002; Iudicone et al., 2008*]. In order to determine an accurate analytical fit to a  
156 potentially multimodal total TTD, we sum the transport-scaled analytical TTDs of the  
157 six individual pathways (Supplementary Fig. S5e).

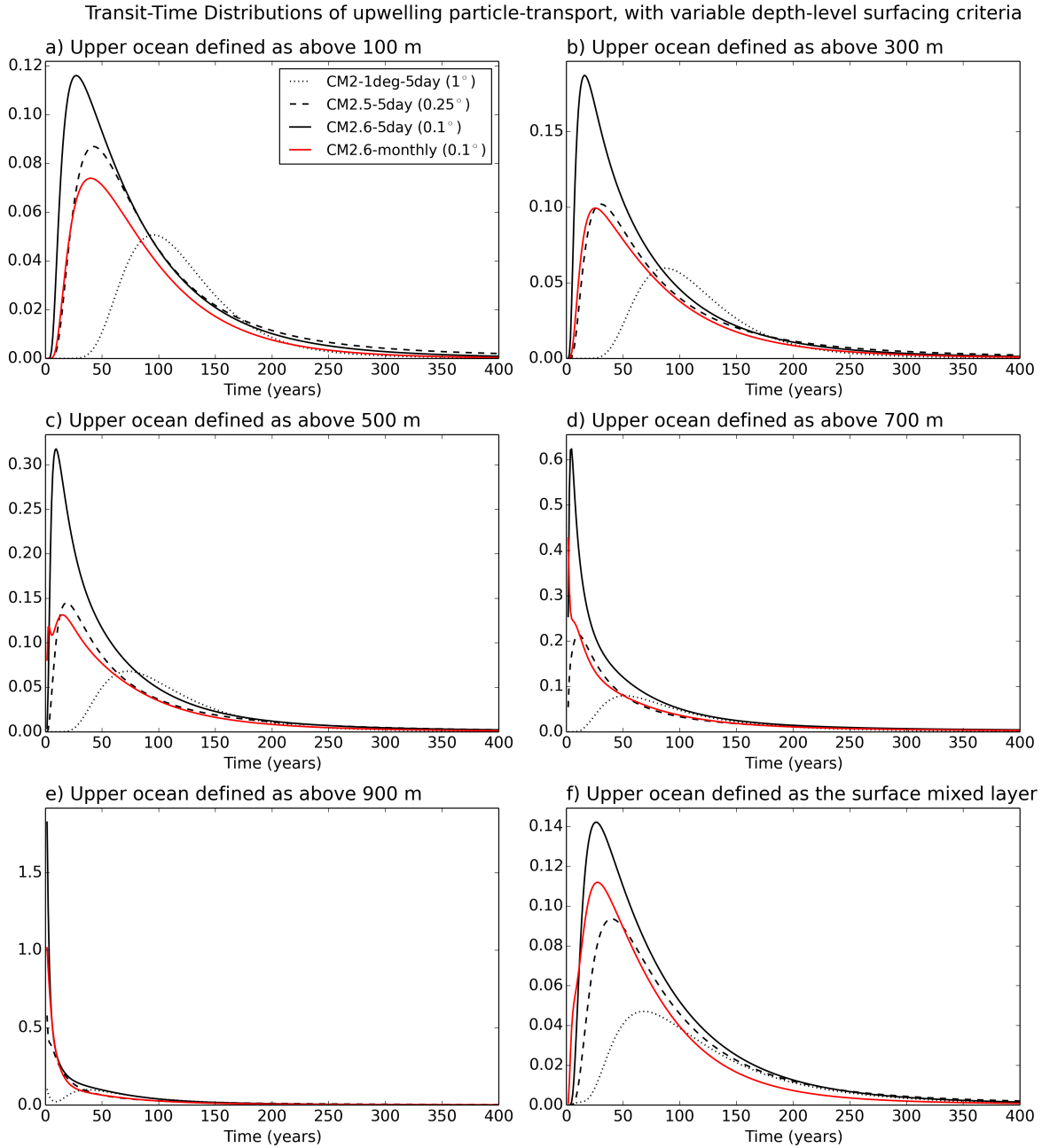
158 **Movie S1.** The 30 year time-evolution of Lagrangian trajectories of particle-transport  
159 between a release depth of 2000 m at 30°S and the upper layers of the Southern Ocean. The  
160 trajectories selected for the movie correspond to the subset of particle-transport released  
161 on January 15th of the model year 183 and on the model depth level closest to 2000 m.  
162 Vertically exaggerated model bathymetry is shown in blue and model land elevation (1/3  
163 vertical scale of ocean) is shown in green-brown-white.



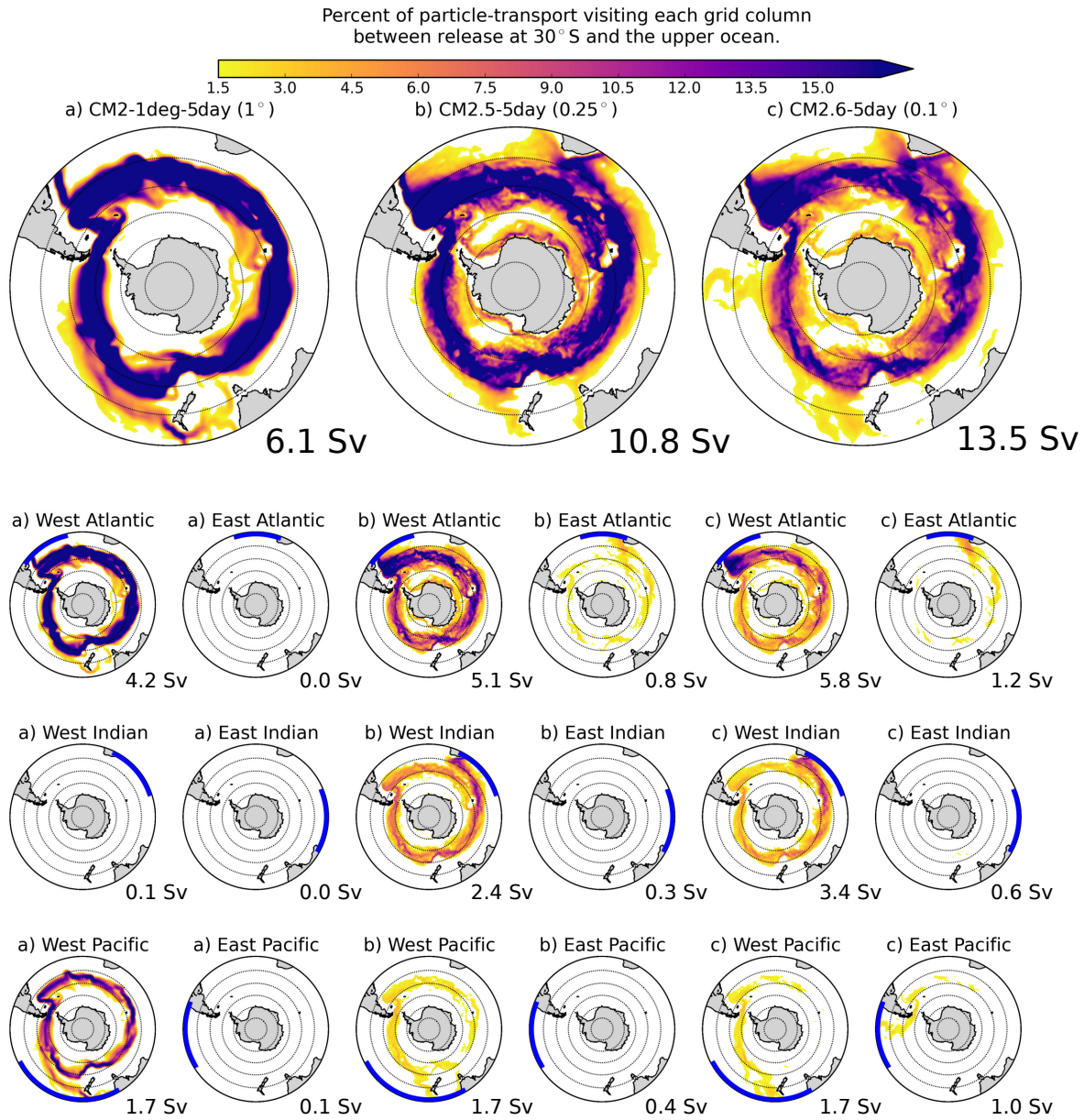
**Figure S1.** The residual overturning streamfunctions poleward of 30°S averaged over release years 183-192 in a) CM2-1deg-5day (1°, including parameterized eddy-induced transports), b) CM2.5-5day (0.25°), c) CM2.6-5day (0.1°), d) CM2.6-monthly (0.1°, temporally degraded velocities). Red colors denote clockwise circulation (2 Sv solid black contour lines) and blue colors denote counter-clockwise circulation (2 Sv dashed black contour lines). Overturning transports are binned online into potential density layers referenced to 2000 m. The thick grey line shows the zonally-averaged density of the 300m depth surface as a function of latitude. In the top left is the maximum value of the residual overturning streamfunction at 30S.



**Figure S2.** Mean (solid lines) and spread (shading) vertical eddy kinetic energy wavenumber spectra in CM2.5-5day (blue), CM2.6-5day (black), CM2.6-monthly (red) calculated from all snapshots in the first year of velocity output as a function of (a) zonal, (b) meridional, and (c) isotropic wavenumber in cycles per meter. (d) Same but for isotropic wavenumber spectrum of horizontal eddy kinetic energy. Ticks designate tenths of the next largest integer power of 10 wavenumber. Wavelengths in km are given along the top axis.

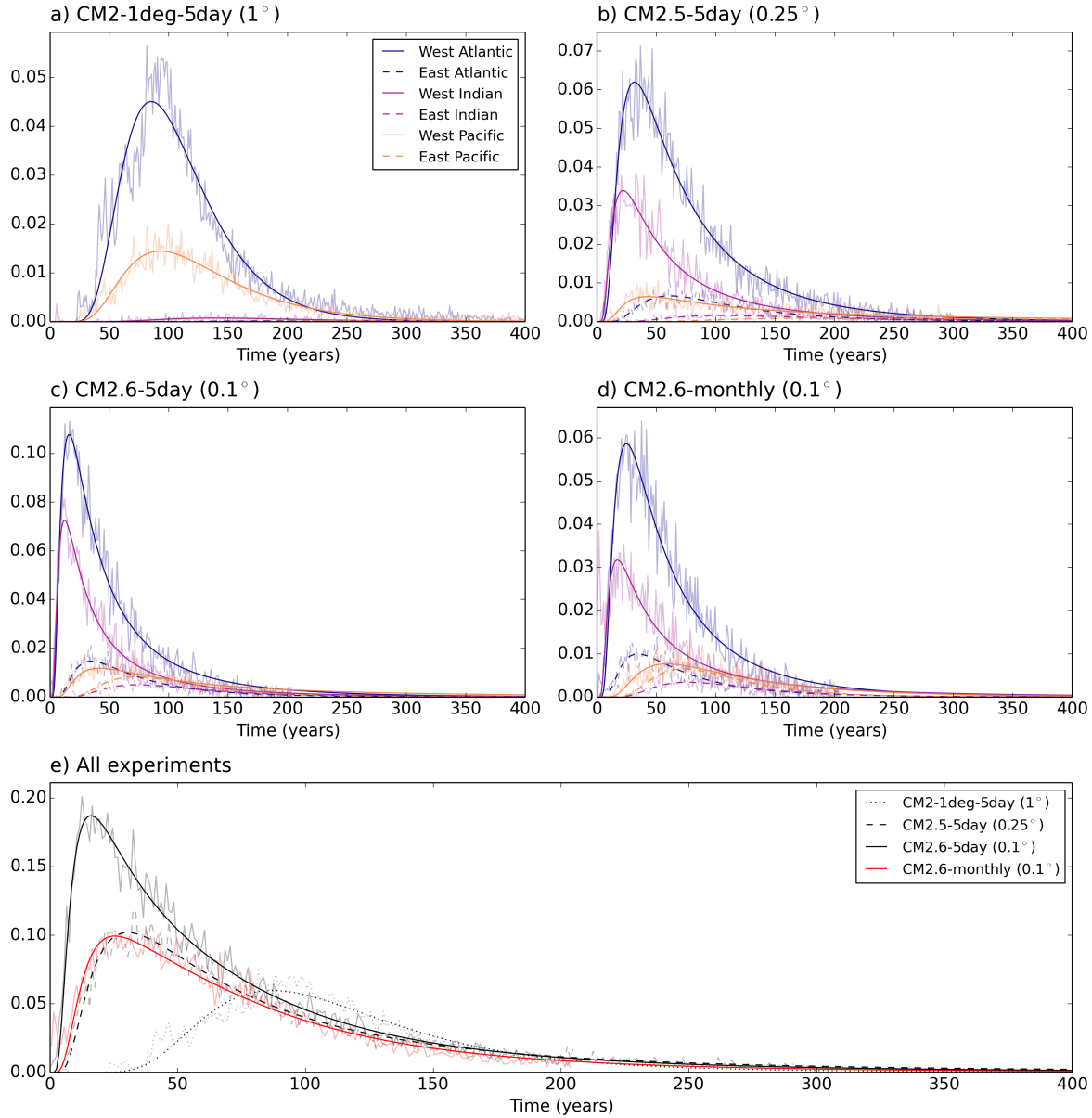


**Figure S3.** As in Figure 2a (upwelling TTD with upper ocean defined as the ocean above 300 m depth), but using the following different definitions of the upper ocean: a) shallower than 100m depth, b) shallower than 300 m depth, c) shallower than 500 m depth, d) shallower than 700 m depth, e) shallower than 900m depth, f) within the surface mixed layer.

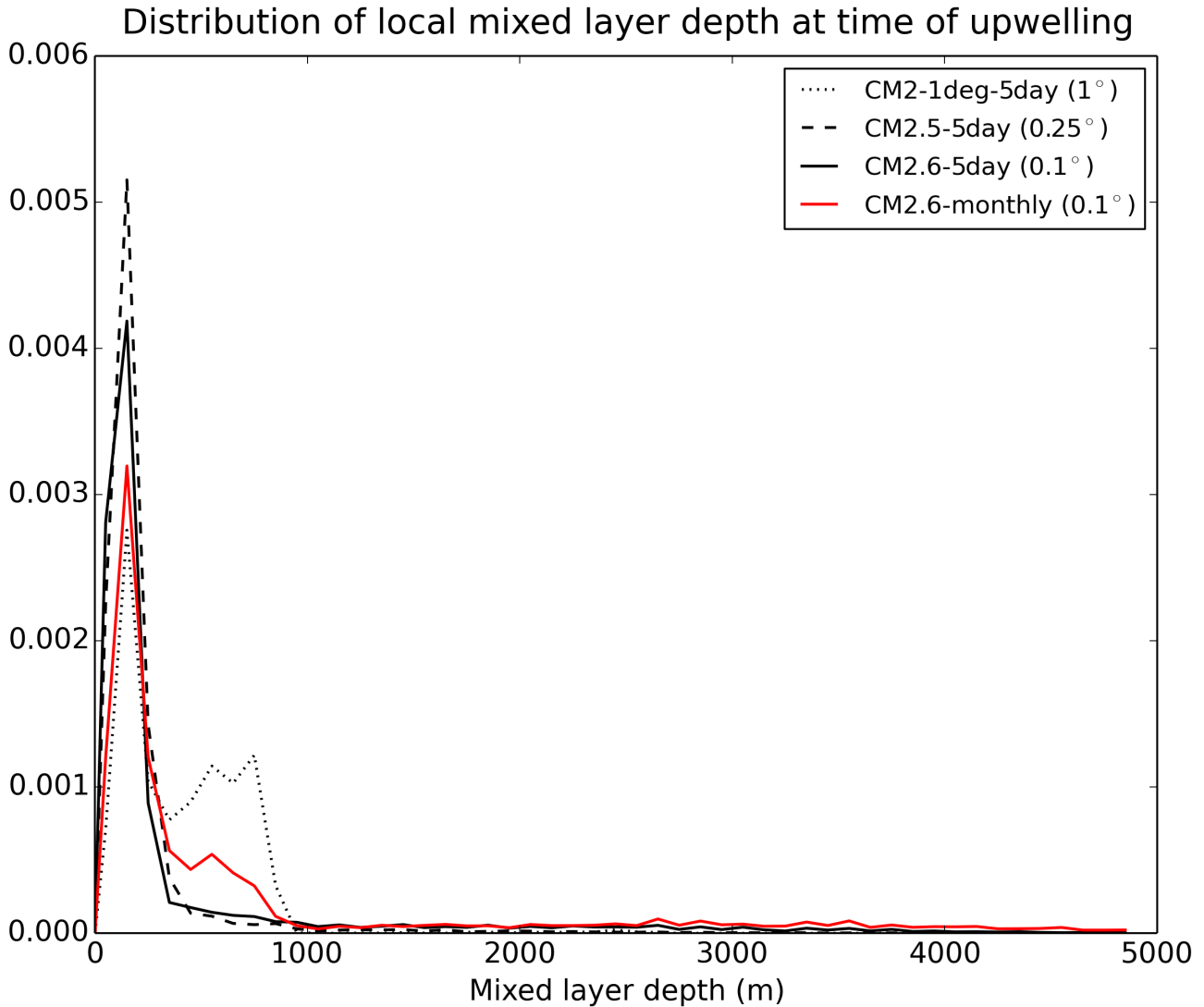


**Figure S4.** The three larger plots above are identical to the three panels of Figure 1. The plots below show decompositions of the above three plots by release section at 30°S (blue line).

## Transit-Time Distributions of upwelling particle-transport

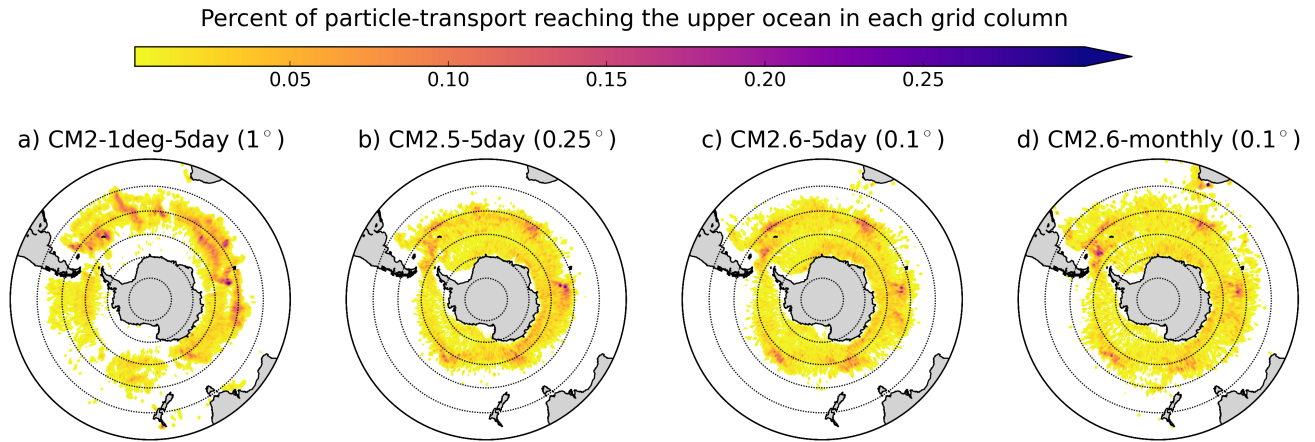


**Figure S5.** a), b), c), and d) are decompositions of Figure 2a into the six individual TTDs: a western and eastern TTD for each of the major ocean basins: the Atlantic Ocean, the Indian Ocean, and the Pacific Ocean. The analytical inverse Gaussian TTD fits are plotted with slightly transparent yearly-binned raw data overlaid. e) shows the sum of the raw data (slightly transparent) and sums of the inverse Gaussian fits (opaque) in a), b), c), and d). The opaque curves in e) are those reproduced in Figure 2a of the main text.



**Figure S6.** Percent of particle-transport upwelling into the surface mixed layer per year in experiments a) CM2-1deg-5day ( $1^\circ$ ), b) CM2.5-5day ( $0.25^\circ$ ), c) CM2.6-5day ( $0.1^\circ$ ), and d) CM2.6-monthly ( $0.1^\circ$ , temporally degraded). We note in particular that upwelling into 500-1000 m deep mixed layers mostly corresponds to lateral advection into mid-latitude deep mixed layers (see also Supplementary Fig. 7), which suggests fixed depth cut-offs are a more useful criteria than the spatially-varying depth of the mixed layer for determining interior upwelling timescales.





**Figure S7.** Percent of particle-transport upwelling into the surface mixed layer in each  $1^\circ$  by  $1^\circ$  grid column in experiments a) CM2-1deg-5day ( $1^\circ$ ), b) CM2.5-5day ( $0.25^\circ$ ), c) CM2.6-5day ( $0.1^\circ$ ), and d) CM2.6-monthly ( $0.1^\circ$ , temporally degraded). We note in particular that the mid-latitude upwelling in a) and d) mostly reflect lateral advection into deep mixed layers, which suggests fixed depth cut-offs are a more useful criteria than the spatially-varying depth of the mixed layer for determining interior upwelling timescales.