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JGR Atmospheres

Supporting Information for

Characterizing the information content of cloud thermodynamic phase retrievals from the Notional PACE OCI shortwave reflectance measurements

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Contents of this file

Supplemental Text S1, and Figure S1.

Introduction

This supporting information summarizes entropy and information relationships in a list of useful principles. In our analysis, we tested these principles at each iteration of the algorithm to ensure the robustness of our diagnostic results.

Summary of entropy and information relationships.

The conditional entropies and conditional information contents do not have symmetric properties, which means the conditional information gain in one parameter is not necessarily equivalent to the conditional information gain in another. This is in contrast to the mutual information, which does have symmetric properties.

The mathematical relationships between the joint, $H(x,y)$, marginal, $H(x)$ or $H(y)$, conditional, $H(x|y)$, and mutual, $I(x;y)$, entropy for cloud optical thickness (τ) and droplet effective radius (r_{eff}) parameters are:

$$I(\tau; r_{eff}) = H(\tau) + H(r_{eff}) - H(\tau, r_{eff})$$
$$= H(\tau) - H(\tau|r_{eff})$$
$$= H(r_{eff}) - H(r_{eff}|\tau)$$

Figure S1 is a Venn diagram that depicts an example of these relationships for τ and r_{eff} parameters. In Figure 1, we have depicted a different uncertainty in the optical thickness and effective radius parameters by using circles of different sizes is to represent a hypothetical case where the τ retrieval has smaller entropy and correspondingly larger information content than the retrieval of r_{eff}. This choice emphasizes the non-symmetry in the conditional entropies of the parameters, $H(\tau| r_{eff})$ and $H(r_{eff}| \tau)$. The mutual information, $I(\tau; r_{eff})$, however, has symmetric properties. The choice of using a circle in Figure 1 denotes symmetrically distributed uncertainties, such as occurs with Gaussian distributions. However, we note that the entropy and information relationships derived in this section are valid regardless of how the uncertainties are distributed.

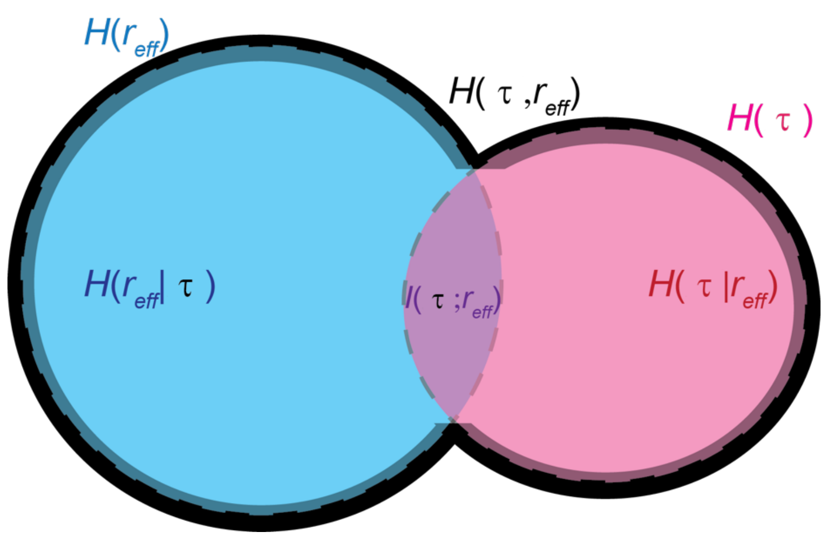

Figure S1. The information in a spectral measurement can be shared amongst parameters. The generalized inverse problem (Equation 1 of main paper) provides the mapping from measurement space to the parameter space(s) for this relationship. This Venn diagram depicts a hypothetical example of the relationships in the marginal, joint, and conditional entropies for cloud optical thickness, τ, and droplet effective radius, r_{eff}, and the mutual information shared by the parameters after a spectral measurement of cloud radiation provides information on both τ and r_{eff} parameters. The sum of the marginal information in optical thickness, ($H(\tau)$: pink circle encircled by dashed line) and effective radius

($H(r_{eff})$: blue circle encircled by dashed line) is not equal to the joint information of the parameters $H(\tau, r_{eff})$: solid black curve at the outer boundaries of the pink and blue circles) because optical thickness and effective radius share mutual information ($I(\tau; r_{eff})$: purple shaded region at the intersection of the blue and pink shaded circles).

List of Useful Principles.

- Entropy is non-negative. The marginal entropy is equal to zero if and only if a parameter is completely determined. The joint entropy of more than one parameter is equal to zero if and only if all parameters are completely determined.

- Entropy has a theoretical upper bound that is achieved when the parameter is uniformly distributed. A number of studies have utilized this theoretical upper bound in order to present Shannon information content results on a scale ranging from zero to unity [*Vukicevic et al.*, 2010; *Coddington et al.*, 2012, *Coddington et al.*, 2013].

- The joint entropy is always at least equal to the entropies of the individual parameters alone (i.e., the joint entropy cannot be less than any of the individual marginal entropies). In other words, adding a new parameter can never reduce the uncertainty.

- The joint entropy is never larger than the sum of the marginal entropies in each individual parameter. *Coddington et al.* [2012] illustrated this principle for cloud optical properties using hyperspectral shortwave cloud albedo measurements.

- Mutual entropy is non-negative. This provides a theoretical lower bound to the mutual entropy.

- Mutual entropy has a theoretical upper bound that occurs in cases where the parameters are identical (i.e., when all information in parameter 'X' is conveyed by parameter 'Y' or vice versa). In this case, the mutual entropy is bounded at the upper end by the smaller of the theoretical maxima in either parameter when the parameters are uniformly distributed.