Impacts of the Phase Shift between Incident Radar Waves on the Polarization Variables from Ice Cloud Particles

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(Manuscript received 29 November 2019, in final form 4 June 2020)

ABSTRACT

The impacts of the differential phase of incident radar waves (\(\psi_i\)) on measured differential reflectivity (\(Z_{\text{DR}}\)), differential phase, and correlation coefficient from ice cloud particles are presented for radars employing simultaneous transmission and reception of orthogonally polarized waves (SHV radar design). The maximal values of \(Z_{\text{DR}}\) and the differential phase upon scattering (\(\delta\)) from ice particles are obtained as functions of \(\psi_i\). It is shown that SHV \(\delta\) from ice particles can exceed a dozen degrees whereas the intrinsic \(\delta\) is of a few hundredths of a degree. In melting layers, the \(\delta\) values from particles obeying the Rayleigh scattering law can be several degrees depending on \(\psi_i\), so that, to explain such \(\delta\) values, an assumption of resonance scattering is not necessary. The phase \(\delta\) affects the estimation of specific differential phase (\(K_{\text{DP}}\)) in icy media and, therefore, the phase \(\delta\) should be measured. The radar differential phase upon transmission \(\psi_t\) is a part of \(\psi_i\) and, therefore, affects the \(\delta\) values. A radar capability to alter \(\psi_t\) by varying \(\psi_t\) could deliver additional information about scattering media.

1. Introduction

Radar precipitation measurements and target recognitions are based on the intrinsic reflectivity, differential reflectivity (\(Z_{\text{DR}}\)), differential phase (\(\Phi_{\text{DP}}\)), and copolar correlation coefficient (\(p_{hv}\)). The intrinsic radar variables are measured with radars employing the alternate transmission of horizontally and vertically polarized waves (AHV radar design; e.g., Bringi and Chandrasekar 2001, sections 4.1.1 and 4.1.2). However, the vast majority of polarimetric weather radars transmit horizontally and vertically polarized waves simultaneously (SHV radar design). The SHV design is technically simpler than the AHV one (Doviak et al. 2000; Bringi and Chandrasekar 2001). On the other hand, the SHV variables are biased because the main backscattered waves interact with the depolarized waves. The differential phase \(\psi_i\) between the waves incident on scatterers plays a crucial role in this interaction. The general consensus is that the SHV polarimetric variables are sufficiently close to the AHV ones in measurements in rain (Doviak et al. 2000; Wang et al. 2006). In measurements in ice media, the difference between the SHV and AHV radar variables can be large (e.g., Doviak et al. 2000, section 6; Ryzhkov and Zrnić 2007; Hubbert et al. 2014a,b). The difference between the variables from AHV and SHV radars have been considered using intrinsic \(Z_{\text{DR}}\), \(p_{hv}\), and the linear depolarization ratio (Doviak et al. 2000; Wang et al. 2006; Bringi and Chandrasekar 2001, section 4.7), which are not measured with SHV radars. A different approach is utilized in this study: the impacts of \(\psi_i\) are obtained by using physical parameters of scatterers known from the literature.

Impacts of \(\psi_i\) on the SHV variables from icy areas of thunderstorms, where ice particles can be aligned by in-cloud electric fields, are considered in sections 2 and 3. The SHV variables for ice ellipsoids, hexagonal prisms, and dendrites are analyzed. Stratiform clouds do not typically have strong electric fields. In such clouds, ice particles flutter in the air and fall with their longest axes oriented horizontally in the mean (Pruppacher and Klett 1997, section 10.5). Impacts of \(\psi_i\) on radar variables from fluttering ice particles are presented in section 4.

A focus of this study is the differential phase \(\psi_i\) consisting of two addends:
\begin{equation}
\psi_i = \psi_i + \phi_{dp},
\end{equation}

where \( \psi_i \) is the phase shift between horizontally and vertically polarized waves upon transmission and \( \phi_{dp} \) is the one-way propagation differential phase. The \( \phi_{dp} \) value is used in calculations of the specific differential phase (\( K_{DP} \)). In snow and ice clouds, \( K_{DP} \) at S band is of a degree per a kilometer that requires precise measurements of the total differential phase \( \Phi_{DP} \), which depends on correct estimation of the phase upon scattering (\( \delta \)). The intrinsic \( \delta \) for ice particles at centimeter wavelengths is of a few hundredths of a degree whereas the SHV \( \delta \) values depend on \( \psi_i \) and can be of several degrees (section 4) and, therefore, the differential phase \( \psi_i \) plays an important role in \( K_{DP} \) estimations. Impacts of \( \psi_i \) on \( \delta \), \( \Phi_{DP} \), and \( K_{DP} \) in ice media should be thoroughly studied. Some aspects of these impacts are presented here.

The following questions are addressed in this study using known microphysical properties of ice particles:

- What is the maximal \( Z_{DR} \) and \( \delta \) from ice particles observed with SHV radar? How does \( \psi_i \) affect these maximal values? How does \( \psi_i \) impact \( Z_{DR} \) and \( \delta \) from ice particles of different habits (section 2)?
- The phase \( \delta \) from stratiform clouds can be linked to microphysical properties of scatterers. How does \( \psi_i \) affect measured \( \delta \) (section 4)?
- Measurements of \( \delta \) values in melting layers show some increase compared to values above the layers. This increase is explained with possible presence of large wet particles of resonance sizes (e.g., Trömel et al. 2013, 2014). Is it possible to explain such \( \delta \) values by scattering from Rayleigh ice particles (section 5)?

Another motivation for studying impacts of \( \psi_i \) on SHV variables is the possibility of obtaining additional information about scatterers. According to (1), \( \psi_i \) can be changed by varying the phase \( \psi_i \). In a conventional SHV radar, the phase \( \psi_i \) can be altered with phase shifters, which are mechanical devices featuring long switching times as compared to typical radar dwell times. A phased array radar is capable of changing \( \psi_i \) during the dwell time by varying the time delay between the transmitted waves that could be used for obtaining additional information about scatterers in real time.

### 2. Maximal \( Z_{DR} \) and \( \delta \) from ice particles as functions of \( \psi_i \)

Positive and negative \( Z_{DR} \) and \( K_{DP} \) are frequently observed in thunderstorms, where strong in-cloud electric fields orient ice particles along the fields. Cloud areas with aligned particles can be observed with radars (e.g., Hendry and McCormick 1976; Caylor and Chandrasekar 1996). SHV radars detect particles having common vertical tilts of their major axes. Vertically aligned ice particles produce negative \( Z_{DR} \) and \( K_{DP} \). In some cases, fields of \( Z_{DR} \) and \( \Phi_{DP} \) exhibit stripe patterns (e.g., Ryzhkov and Zrnić 2007; Hubbert et al. 2018). Maximal values of \( Z_{DR} \) and \( \delta \) from ice particles are of interest.

Ice cloud crystals in forms of plates and columns frequently have shapes of hexagonal prisms (Fig. 1). Scattering properties of such particles can be obtained with the discrete dipole approximations (Purcell and Penyapacker 1973; Draine and Flatau 1994), scattering models based on the methods of moments (e.g., WIPL-D 2020; FEKO 2020; Chobanyan et al. 2015), and the finite element method (e.g., ANSYS 2020) to name a few. Thin ice particles can be satisfactorily approximated with spheroids or ellipsoids that are characterized with the axis ratio of width/length \( = b/a \) (Fig. 1; e.g., Bringi and Chandrasekar 2001; Hogan et al. 2012; Matrosov 2015). The shapes of particles’ edges are not important for thin particles with \( a/b > 20 \). Scattering properties of ellipsoids can be derived analytically in the Rayleigh limit (e.g., Bohren and Huffman 1983, section 5.3) or obtained numerically at any size/wavelength ratio (e.g., Mishchenko et al. 2002) that makes ellipsoids convenient model scatterers. Scattering geometry can be described using the scattering plane (e.g., Holt 1984; Vivekanandan et al. 1991; Ryzhkov and Zrnić 2007); here, the laboratory frame affixed to the ground is used (frame OXYZ in Fig. 1). This frame naturally describes orientations of cloud particles relative to the ground and to a radar beam. In this frame, \( \theta \) is the canting angle of a particle and the incident waves are horizontally and vertically polarized at low antenna elevation angles.

The \( Z_{DR} \) and \( \delta \) values from pristine cloud particles depend on their shape, axis ratio, dielectric permittivity, and orientation relative to the polarization planes of incident waves. In thunderstorms, ice particles can be aligned at any \( \theta \) and \( \phi \) by in-cloud electric fields and, therefore, these angles can lie in intervals \( 0^\circ \leq \theta \leq 180^\circ \) and \( 0^\circ \leq \phi \leq 360^\circ \).

Scattering of horizontally (subscript \( h \)) and vertically (subscript \( v \)) polarized electromagnetic waves by a single scatterer is described by the scattering matrix \( S \) with the elements \( S_{mn} \) (\( m \) and \( n \) are any of \( h \) and \( v \)). Let \( E_{hi} \) and \( E_{vi} \) be the amplitudes of the incident waves (Fig. 1); then the scattered (subscript \( s \)) waves \( E_{hs} \) and \( E_{vs} \) are written as

\begin{equation}
\begin{pmatrix}
E_{hs} \\
E_{vs}
\end{pmatrix} =
\begin{pmatrix}
S_{hh} & S_{hv} \\
S_{vh} & S_{vv}
\end{pmatrix}
\begin{pmatrix}
E_{hi} \\
E_{vi} e^{i\psi_i}
\end{pmatrix},
\end{equation}
the received polarization channels may also be different. Corrections for the system amplitude imbalances are done in $Z_{\text{DR}}$ calibration, but the transmitted and received waves remain shifted in phase. The one-way propagation phase adds to the phase shifts between the incident waves [see (1)]. After $Z_{\text{DR}}$ calibration, the transmitted wave amplitudes can be considered equal; i.e., $E_{vi} = E_{hi} = E$ at negligible differential attenuation. The scattered waves acquire $\delta$ at scattering and the propagation differential phase $\phi_{dp}$ on their way back to the radar. The received waves also get shifted by $\psi_i$ by the radar chains so that the total phase shift at a radar signal processor is $\psi_i + \phi_{dr} + \delta$. The received waves from a single scatterer can be written as

$$
\begin{pmatrix}
E_{hr} \\
E_{vr}
\end{pmatrix} = C_R \begin{pmatrix} 1 & 0 \\ 0 & e^{i(\phi_r + \phi_{dp})} \end{pmatrix} \begin{pmatrix} S_{hh} & S_{hr} \\ S_{hv} & S_{vv} \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & e^{i\phi_i} \end{pmatrix} \begin{pmatrix} E \\ E \end{pmatrix},
$$

(3)

where $C_R$ is the radar constant with the range dependence included. Note that $\delta$ does not appear in (3) explicitly and originates from the scattering matrix $S$. The first and third matrices on the right-hand side in (3) describe propagation of the waves from radar to the resolution volume and back. The zero off-diagonal terms in the propagation matrices signify negligible depolarization in the propagation medium in which differential attenuation is also negligible. The radar constant $C_R$ and the amplitude $E$ will be omitted in the following discussion because $Z_{\text{DR}}$, $\rho_{hv}$, and $\delta$ are relative values and do not depend on those; therefore,

$$
E_{hr} = S_{hh} + S_{hv} e^{i\phi_i} \quad \text{and} \quad E_{vr} = (S_{vv} e^{i\phi_i} + S_{hv}) e^{i(\phi_r + \phi_{dp})}.
$$

(4)

The scattering matrix elements can be represented as

$$
S_{hh} = \alpha_a + \Delta \alpha \sin^2 \theta \sin^2 \varphi, \quad S_{vv} = \alpha_a + \Delta \alpha \cos^2 \theta, \quad S_{hv} = \Delta \alpha \sin \theta \cos \varphi, \quad \text{with} \quad \Delta \alpha = \alpha_b - \alpha_a,
$$

(5)

where $\alpha_a$ and $\alpha_b$ are polarizabilities along the major particle’s axes ($\alpha_a$ refers to the longer axis), [e.g., Bringi and Chandrasekar 2001, their Eq. (2.53)]. Note that the antenna angle $\gamma$ is absent in (5); i.e., $\gamma = 0$ is set. For nonzero $\gamma$, (5) can be considered relative to the angle $\gamma$ that will not change the maximal values in question. Substitution of (5) into (4) yields

$$
E_{hr} = \alpha_a + \Delta \alpha A \sin \theta \sin \varphi,
$$

(6)

$$
E_{vr} = (\alpha_a e^{i\phi_i} + \Delta \alpha A \cos \theta) e^{i(\phi_r + \phi_{dp})},
$$

(7)

with $A = \sin \theta \sin \varphi + \cos \theta e^{i\phi_i}$.

Fig. 1. Geometry of scattering by (a) platelike and (b) columnar hexagonal ice particles. $E_{hi}$ and $E_{vi}$ are the amplitudes of incident horizontally and vertically polarized waves propagating in direction $k$. The plane OXY is horizontal and ON is orthogonal to the plate’s face. The angles $\theta$ and $\gamma$ are the canting and antenna elevation angles, respectively. OM is horizontal, $k$ lies on the plane OXZ, and $\varphi$ is orientation angle of the particle’s axis.
Also, $Z_{\text{DR}}$ and $\delta$ from a single scatterer (or a collection of scatterers equally oriented) are

$$Z_{\text{DR}} = 10 \log(|E_{\text{br}}^\text{c}|^2 / |E_{\text{vr}}^\text{c}|^2) \quad \text{and} \quad \delta = \arg(E_{\text{br}}^\text{c} E_{\text{vr}}^\text{c} - \psi_t - \psi_t - 2\varphi_{\text{dp}}),$$

where the asterisk (*) stands for complex conjugate and $E_{\text{br}}^\text{c}$ is the correlation function. The first addend on the right-hand side of (9) is the total measured differential phase $\Phi_{\text{DP}}$. To obtain $\delta$, one has to subtract the propagation and system differential phases from the measured phase. One can see from (6)–(9) that measured $Z_{\text{DR}}$ and $\delta$ depend on $\psi_t$.

To make (9) clearer, consider scattering by a sphere for which $\alpha_a = \alpha_a$ and, therefore, $S_{\text{hv}} = 0$. Then $E_{\text{br}} = \alpha_a$. $E_{\text{vr}} = \alpha_a e^{i(\psi_t + \psi_t + \varphi_{\text{dp}})}$, and $R_{\text{hv}} = |\alpha_a|^2 e^{i(\psi_t + \psi_t + \varphi_{\text{dp}})}$, the argument of which is $\psi_t + \psi_t + 2\varphi_{\text{dp}}$. According to (9), $\delta = 0$ as it should be for a sphere. So SHV $\delta$ emerges at scattering by nonspherical particles and depends on $\psi_t$. To calculate $\delta$ from model particles, one can set $\psi_t = \varphi_{\text{dp}} = 0$ and $\delta$ is obtained as the argument of the correlation function $R_{\text{hv}}$ with $\psi_t$ as an argument.

It is seen from (4) that the scattered waves have two contributions: the primary ones containing $S_{\text{hh}}$ and $S_{\text{vv}}$ and depolarized ones depending on $S_{\text{uv}}$. These mixed contributions are sometimes referred to as wave coupling upon scattering. Due to the coupling, $Z_{\text{DR}}$ and $\delta$ values measured with SHV radar can significantly deviate from those measured with AHV radar. For AHV radar, (2) can also be used by setting $S_{\text{hv}} = 0$ and $\psi_t = 0$. For such radar, the amplitudes of received waves are $E_{\text{br}}^{\text{AHV}} = S_{\text{hh}}$ and $E_{\text{vr}}^{\text{AHV}} = S_{\text{vv}}$, i.e.,

$$E_{\text{br}}^{\text{AHV}} = \alpha_a + \Delta\alpha \sin^2\theta \sin^2\varphi \quad \text{and} \quad E_{\text{vr}}^{\text{AHV}} = \alpha_a + \Delta\alpha \cos^2\theta.$$  

(10)

For plates, $\Delta\alpha < 0$ and the maximal $Z_{\text{DR}}^{\text{AHV}}$ and $\delta_{\text{max}}^{\text{AHV}}$ are attained at $\theta = 0$, i.e., at horizontal orientation of the scatterer:

$$Z_{\text{DRmax}}^{\text{AHV}} = 10 \log(|\alpha_a|^2 / |\alpha_b|^2) \quad \text{and} \quad \delta_{\text{max}}^{\text{AHV}} = \arg(\alpha_a^* \alpha_b).$$

(11)

For a columnar scatterer, the maximal $Z_{\text{DR}}^{\text{AHV}}$ and $\delta_{\text{max}}^{\text{AHV}}$ are attained at $\theta = \varphi = 90^\circ$, i.e., at its horizontal orientation (Fig. 1b). For scatterers that are much smaller than the radar wavelength, i.e., in the Rayleigh scattering limit, and approximated with a spheroid, polarizabilities $\alpha_{a,b}$ are (e.g., Doviak and Zrnić 2006, section 8.5.2.4)

$$\alpha_{a,b} = V \frac{\varepsilon - 1}{1 + L_{a,b}(\varepsilon - 1)},$$

where $V$ is the scatterer’s volume, $\varepsilon$ is dielectric permittivity of ice ($\varepsilon = 3.17 - 0.0015j$ at S frequency band), and $L_{a,b}$ are the shape factors. For a thin plate, $L_a = 0$ and $L_b = 1$, so (Vivekanandan et al. 1994; Hogan et al. 2002)

$$Z_{\text{DRmax}}^{\text{AHV}} = 20 \log(|\varepsilon|) = 10 \text{ dB.}$$

(13)

This value varies insignificantly at centimeter wavelengths. For thin needles, $L_a = 0$ and $L_b = 0.5$; therefore,

$$Z_{\text{DRmax}}^{\text{AHV}} = 20 \log(|\varepsilon + 1|/2) = 6.4 \text{ dB.}$$

(14)

To obtain maximal $Z_{\text{DR}}$ for thin needles randomly oriented on the horizontal plane, one has to average $|E_{\text{br}}^{\text{AHV}}|^2$ and $|E_{\text{vr}}^{\text{AHV}}|^2$ from (10) over the angle $\varphi$ at $\theta = 90^\circ$. The imaginary part of $\varepsilon$ is very small compared to its real part and can be neglected, so (Hogan et al. 2002)

$$Z_{\text{DRmax}}^{\text{AHV}} = 10 \log \{1 + \left[ (|\varepsilon| - 1)/2 \right] + 3(|\varepsilon| - 1)^2/32 \} = 4.0 \text{ dB.}$$

(15)

The phase $\delta_{\text{max}}^{\text{AHV}}$ is calculated from (11). For ice plates, $\delta_{\text{max}}^{\text{AHV}} = - \arg(\varepsilon)$ and for needles, $\delta_{\text{max}}^{\text{AHV}} = - \arg(\varepsilon + 1)$, i.e., very small values. Thus, the differential phase upon backscattering by ice particles is of a few hundredths of a degree for AHV radar in the Rayleigh limit. Pristine ice cloud particles are better represented with hexagonal prisms (Westbrook 2014). Values of $\alpha_{a,b}$ for very thin prisms and ellipsoids are equal so that $Z_{\text{DRmax}}^{\text{AHV}}$ and $\delta_{\text{max}}^{\text{AHV}}$ for these shapes are also equal.

Now consider $Z_{\text{DR}}$ and $\delta$ measured with SHV radar. Figure 2 presents $Z_{\text{DR}}$ and $\delta$ as functions of $\theta(0^\circ-180^\circ)$ and $\varphi(0^\circ-360^\circ)$ at $\psi_t = 90^\circ$ for thin ice plates and needles. The values $\theta_{\text{max}}$ and $\varphi_{\text{max}}$ are angles at which $Z_{\text{DR}}$ and $\delta$ attain their maximum. These variables for other $\psi_t$ are shown in the online supplemental material (Fig. S1). The maximal $Z_{\text{DR}}$ depends on $\psi_t$. A value of $Z_{\text{DR}} = 12.3 \text{ dB}$, the absolute maximum (maximum maximorum) for ice plates, is attained at $\psi_t = 0$, $\theta = 16^\circ$, and $\varphi = 90^\circ$ (or $\psi_t = 180^\circ$, $\theta = 106^\circ$, and $\varphi = 270^\circ$, Fig. S1a). A tilt in the canting angle $\theta$ increases $Z_{\text{DR}}$ over 10 dB obtained for AHV radar. For ice needles, the absolute maximum of 6.9 dB is attained at $\theta = 10^\circ$ or $170^\circ$, $\varphi = 90^\circ$, and $\psi_t = 0^\circ$ (Fig. S1d).

The phase $\delta$ exhibits a strong dependence on $\psi_t$. At $\psi_t = 0^\circ$, $\delta$ is very small for all $\theta$ and $\varphi$. The absolute maximum of $\delta_{\text{max}}$ is $62.3^\circ$ for plates is attained at $\theta = 50^\circ$, $\varphi = 90^\circ$, and $\psi_t = 60^\circ$. For ice needles, the absolute maximum of $\delta_{\text{max}}$ is $40.9^\circ$ is attained at $\theta = 50^\circ$, $\varphi = 90^\circ$, and $\psi_t = 70^\circ$. These results for maximal $Z_{\text{DR}}$ and $\delta$ illustrate strong impacts of depolarized waves on the
It is easy to intuitively accept that horizontally oriented particles produce positive $Z_{DR}$ and $d$ and vertically oriented particles produce negative $Z_{DR}$ and $d$; i.e., these variables should be of the same sign. Figure 2 shows that the opposite situations can occur. For nearly horizontal particles, for instance, at an area about $u = 10^\circ$ and $\phi = 270^\circ$ for plates (Fig. 2a), the $Z_{DR}$ values are positive, but the $\delta$ values are negative (Fig. 2b; see also Fig. S2 showing radar observations). Positive $d$ values can be produced at negative $Z_{DR}$, for instance, at $u = 70^\circ$ and $\phi = 90^\circ$ for plates (Figs. 2a,b). To reveal the source of these counterintuitive situations, examine the waves scattered by an ice plate oriented at $\theta = 10^\circ$ and $\phi = 270^\circ$ at $\psi_i = 90^\circ$. For short, in this paragraph, H and V stand for horizontal and vertical polarizations, respectively. Obtain the contribution to the scattered waves in terms of the main H backscattered wave; i.e., let $\alpha_a = 1$. The imaginary parts of $\alpha_a, \beta$ can be neglected for ice. Then the contribution to the H wave from depolarization [see (4)] is $0.00 + 0.12j$ and the sum of the primary and depolarized waves is $1.00 + 0.12j$. The latter shows that the depolarized wave substantially increases the imaginary part of the scattered wave. Recall that the imaginary parts of waves are responsible for phase shifts; the small imaginary parts of scattered AHV waves lead to the small $\delta$ discussed above. The main V backscattered wave is $0.00 + 0.34j$; i.e., the differential phase in incident radiation contributes to a strong imaginary part of the main backscattered wave. The contribution to V wave from depolarization is $0.12 + 0j$ and their sum is $0.12 + 0.34j$. From the two amplitudes $1.00 + 0.12j$ and $0.12 + 0.34j$, the argument of correlation function is $64^\circ$. Since $\psi_i = 90^\circ$, the phase upon scattering is $\delta = 64^\circ - 90^\circ = -26^\circ$, i.e., a large negative number. The negative phase $\delta$ is caused by strong depolarization and the incident differential phase plays a critical role in the origin of $\delta$. For a smaller $\psi_i$, for instance, $\psi_i = 40^\circ, \delta = -14^\circ$; i.e., $\delta$ remains negative and large. Negative $\delta$ values of $-6^\circ$ to $-8^\circ$ have been observed in smoke plumes containing fluttering scatterers oriented horizontally in the mean (Melnikov et al. 2008, 2009). Note that the situation of positive $Z_{DR}$ and negative $\delta$ has its mirror counterpart; i.e., negative $Z_{DR}$ can exhibit positive $\delta$.

The values of $\delta_{\text{max}}$ are large for thin ice particles (Fig. 2) and can be positive and negative depending on $\psi_i$ and orientation of the scatterers. This contrasts with very small $\delta$ values at alternate polarizations. The differential phase $\psi_i$ depends on the radar phase shift in transmit $\psi_i$ [see (1)], so to interpret the differential phases measured with a SHV radar, $\psi_i$ should be known.

The measurable SHV differential phase $\Phi_{\text{DP}}$ is

$$\Phi_{\text{DP}} = 2\psi_{dp} + \psi_i + \psi_z + \delta.$$
The sum $\psi_i + \psi_r$ is called the radar system differential phase $\psi_{sys}$. This phase can be measured at the closest-to-radar edge of precipitation, where $\varphi_{dp}$ is negligible and, therefore, $\Phi_{DP} = \psi_{sys} + \delta$. For light rain at S band $\delta \approx 0$ because the droplets are almost spherical; therefore, $\psi_{sys} = \Phi_{DP}$. Because $\delta$ can reach larger values in ice clouds, measurements of $\psi_{sys}$ could be incorrect. The SHV $\delta$ depends on $\psi_r$, yet only the sum $\psi_i + \psi_r$ can be readily obtained. Separate measurements of $\psi_i$ and $\psi_r$ are challenging on such systems.

The positive absolute maximum $Z_{DR}$ values as a function of $\psi_i$ (Fig. 3a) have been obtained as maximum maximorum values on surfaces as those in Fig. 2. Positive and negative $Z_{DR}$ and $\delta$ values are connected as $Z_{DR}(\psi_i) = -Z_{DR}(180^\circ - \psi_i)$ and $\delta(\psi_i) = -\delta(180^\circ - \psi_i)$.

Hexagonal prisms are a better representation of the pristine ice plates and columns than spheroids. The previous results are valid for very thin spheroids and hexagonal prisms, i.e., at $a/b \gg 1$. The maximal $Z_{DR}$ and $\delta$ for a moderate $a/b$ exhibit dependence on the particle’s shape. The $Z_{DR_{max}}$ values for spheroids are about 1 dB larger than those for prisms at a moderate axis ratio of $b/a = 0.3$ (Fig. 4a). The $\delta_{max}$ values are noticeably different for prisms and spheroids (Fig. 4b). The polarization properties of hexagonal prisms have been obtained with the NOAA release of WIPL-D package.

Dendrites and stellars are frequent habits of ice particles. The dendrite branches can have various shapes and mass. Auer and Veal (1970) documented relations between the length and width of natural dendrites. $Z_{DR}$ and $\delta$ as functions of $\theta$ and $\varphi$ for the dendrite of these dimensions (Figs. 5b,c) are close in shape to those for plates (Fig. 2 and Fig. S1), but the maximal $Z_{DR}$ value is 4.1 dB and the maximal $\delta$ is 25.5°. Dendrites of the same lengths and width can have more subbranches and, therefore, more mass. For such dendrites, $Z_{DR}$ and $\delta$ increase, but cannot exceed the values for the thin ice plates obtained above. It is quite obvious that, in the Rayleigh limit, an ice particle of any shape cannot produce $Z_{DR}$ and $\delta$ values larger than those obtained for thin ice plates at the same $\psi_i$.

3. An example of $\delta$ in a thunderstorm

Positive and negative SHV $\delta$ values are frequently observed in thunderstorms. The case in Fig. 6 was observed with S-band SHV WSR-88D KOUN radar located at Norman, Oklahoma [the radar parameters can be found in Doviak and Zrnić (2006), their Tables 3.1 and 6.1]. It can be seen as an increase in $\Phi_{DP}$ values at the cloud top (left panel) at heights of 12–15 km. A range profile of $\Phi_{DP}$ through this area (Fig. 7a) shows a gradual increase to a distance of 46 km and a strong bump at distances of 46–49 km. If the gradual increase is attributed to the propagation effect, then the sharp increase of about 5° should be due to $\delta$. Similar $\Phi_{DP}$ peaks are also observed at the adjacent radials that indicates that the peak is not caused by signal fluctuations.
A large area of negative $Z_{\text{DR}}$ is seen at heights of 9–14 km, where $\Phi_{\text{DP}}$ drops by about 16° when compared to the values at the edge of the area closer to the radar. The negative $Z_{\text{DR}}$ values indicate the presence of ice particles oriented primarily vertically. A $\Phi_{\text{DP}}$ range profile through this area (Fig. 7c) exhibits a drop of about 7° at distances from 35 to 47 km. This drop can be attributed to propagation of the waves in areas with vertically oriented particles. Between distances of 47 and 67 km, $\Phi_{\text{DP}}$ increases at negative $Z_{\text{DR}}$ that could be due to positive $\delta$ as it was discussed in the previous section. Therefore, it is possible that positive $\delta$ values contribute to the increase in $\Phi_{\text{DP}}$ of about 16° from a range of 47 km to the end of the radial. KOUN’s unknown $\psi_i$ does not allow for a more certain interpretation. A case in Fig. S2, where a decrease in $\Phi_{\text{DP}}$ in the cloud top occurs at positive $Z_{\text{DR}}$, is another example of difficulties in interpretations of $\Phi_{\text{DP}}$ and $\delta$ without knowing $\psi_i$. To narrow down uncertainties in the interpretation of $\Phi_{\text{DP}}$, variations in $\psi_i$ could be helpful. Such variations can be accomplished by varying $\psi_i$. This radar capability could deliver additional information about scattering media.

4. Impacts of $\psi_i$ on $\delta$ and $\rho_{hv}$ from ice stratiform clouds

Stratiform ice clouds typically do not have strong electric fields capable of aligning ice particles; therefore, ice particles fall with their major axes being horizontal in the mean (e.g., Pruppacher and Klett 1997, section 10.5). In such clouds, $\delta$ values typically vary spatially (more often in height) by about 2°–3°. For instance, compare the $\Phi_{\text{DP}}$ values at the top and bottom of a cloud in Fig. 8 at horizontal distances beyond 20 km. The median $\Phi_{\text{DP}}$ values are 3.1° and 5.4° at heights of 3–4 and 4–5 km, respectively. The propagation contribution to $\Phi_{\text{DP}}$ is negligible at S band at these distances in such clouds and, therefore, the variation in $\Phi_{\text{DP}}$ are caused by $\delta$. Thus, the increase in $\delta$ from the top to bottom is 2.3° in the mean.

Ice particles flutter in the air. Fluttering particles produce lower $Z_{\text{DR}}$ and $\rho_{hv}$. The dependences of $\delta$ values upon the flutter intensity are more complicated because the $\delta$ values are also affected by $\psi_i$. The $\delta$ values typically increase with the flutter intensity to some intermediate value and then decrease at stronger flutter. This initial increase is caused by the increase in $\delta$ at some nonzero canting angles (section 2). When flutter changes the canting angles close to the one that produces enhanced $\delta$, observed $\delta$ values are larger than those for horizontal particles.

The phase $\delta$ from fluttering particles is obtained from the correlation function $R_{hv}$ (Melnikov 2017):

$$R_{hv} = e^{i\phi_i + \psi_i + 2\alpha_i \omega_t \left[ \langle \alpha \rangle^2 \right]} + \Re(\langle \alpha_i \Delta \alpha \rangle)C_4 + \Im(\langle \alpha_i \Delta \alpha \rangle)C_5 + (\langle \alpha_i \Delta \alpha \rangle C_6, \tag{17}$$

where the angle brackets stand for the size averaging and $C_4$, $C_5$, and $C_6$ are functions of $\psi_i$ and the standard
deviation $\sigma_\theta$ in canting angles of the particles. Alternations in the canting angles can be caused by flutter or/and asymmetry in the particles’ shapes. Asymmetrical ice particles have nonzero mean canting angles and are frequently observed in clouds (e.g., Pruppacher and Klett 1997, section 2.2; Wolde and Vali 2001; Hogan et al. 2012).

Figure 9 presents $\rho_{hv}$ and $\delta$ values for Rayleigh ice plates fluttering in the air. The standard deviation in the canting angles is 15°, which is the value obtained by Garrett et al. (2015) and Melnikov and Straka (2013). Figures 9a and 9b present results for a very thin ice plates. At $\psi_i = 0^\circ$, the $\delta$ values are very small. The maximal $\delta$ of 7° is attained at $\psi_i = 90^\circ$ at horizontal incidence. Note also that $\delta$ is not equal to 0° at vertical incidence for nonzero $\psi_i$. Nonzero $\psi_i$ makes scattering geometry asymmetrical even though particles viewed from bottom are symmetrical. For an ice plate having the axis ratio of 0.3 (Figs. 9c,d), the $\rho_{hv}$ values are much higher than those for the very thin plates and the $\delta$ values drop to about 2°. Note that such $\delta$ values have been observed in the case in Fig. 8.

The following conclusions can be drawn from the presented results. 1) Values of $\delta$ can be used as a property parameter of ice particles because $\delta$ depends on their axis ratio. 2) To use $\delta$ values quantitatively, the differential phase $\psi_i$ should be measured. Since $\psi_i$ is a part of $\psi_t$, the system phase upon transmission must be known (measured). 3) The phase $\delta$ should be accounted for in obtaining $K_{DP}$ in icy media because $K_{DP}$ is obtained from $\Phi_{DP}$, which depends on $\delta$ [see (16)]. 4) The phase $\delta$ depends on the properties of a particle and on $\psi_i$ (and, correspondingly, on $\psi_t$); therefore, additional information about particles could be obtained by varying $\psi_t$.

5. Impacts of $\psi_i$ on radar variables from melting layers

Values of radar variables in melting layers (ML) frequently are quite different from those measured in areas located above and below MLs. The supplemental material contains radar images illustrating some typical situations described here. Small $Z_{DR}$ values above the
ML can be accompanied by greater $Z_{DR}$ values below the ML (e.g., Fig. S4) and vice versa (Figs. S5 and S7). Sometimes, a layer of low $Z_{DR}$ values is observed just above MLs (Fig. S6) that is explained by the aggregation processes there (e.g., Griffin et al. 2020). In some situations, a layer of reduced $Z_{DR}$ above MLs is not observed (Figs. S4, S5, and S7). Vertical stripes of enhanced $Z_{DR}$ can be observed above MLs (Fig. S8) that could point to the presence of strong ice generating cells at the cloud tops. The $Z_{DR}$ values inside the shown MLs span an interval from about 0.5 dB (Fig. S4) to about 5 dB (Fig. S9). Values of $Z_{DR}$ in MLs should be analyzed at distances close to the radar because data at longer distances are smeared by increased beam broadening in range (e.g., Figs. S5 and S6 beyond a distance of 100 km).

The $\Phi_{DP}$ patterns above and below MLs can be about the same (Figs. S7 and S8), but can be quite different (Figs. S5, S6, S9, and S10). The $\Phi_{DP}$ patterns above and below MLs can have similar (Figs. S7 and S8) or quite different characteristics (Figs. S5, S6, S9, and S10). Because of short signal paths in clouds at distances shorter than 15 km, the increase in $\Phi_{DP}$ above the ML in Figs. S5 and S10, can be largely attributed to greater $\delta$ values. Significant propagation $\Phi_{DP}$ contributions are observed in icy parts of clouds in Figs. S5 and S6 whereas those contributions are very small below the MLs. In contrast, no difference in the $\Phi_{DP}$ range dependences are seen in Figs. S7 and S8. The $\Phi_{DP}$ values inside MLs typically exhibit some enhancements compared to values outside MLs. The mean enhancements typically are $2^\circ$–$5^\circ$ (e.g., Figs. S5 and S7), which can be largely attributed to increase in $\delta$. Griffin et al. (2020) report on an increase in $\delta$ up to $10^\circ$ in MLs. Such $\delta$ values are sometimes explained with the presence of large particles of resonance sizes (e.g., Trömel et al. 2014). It is shown in this section that the observed $\delta$ can be produced by Rayleigh particles if the incident waves are shifted in phase.

The $\rho_{hv}$ values in MLs lie in a wide interval. In the supplemental materials, one can see $\rho_{hv}$ values from 0.7 (Fig. S9) to 0.93 (Fig. S7) inside MLs. In areas above MLs, correlation is typically observed between $\rho_{hv}$ and $Z_{DR}$ values: $Z_{DR}$ exhibit lower $\rho_{hv}$, and vice versa (Griffin et al. 2018). Such a correspondence is evident in Fig. S5, although almost no difference in $\rho_{hv}$ can also be observed (Fig. S6).

The described features can be observed simultaneously in various parts of a weather system (e.g., Fig. S11). Strong influence of the $\delta$ phase on the $\Phi_{DP}$ field is evident in Fig. S11: note significant variations in

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**FIG. 7.** (a),(b) Range profiles of $\Phi_{DP}$ and $Z_{DR}$, respectively, from the fields in Fig. 6 at an antenna elevation of 17.4°. (c),(d) As in (a) and (b), but for an antenna elevation angle of 11.3°.

**FIG. 8.** As in Fig. 6, but the data were collected at 1825 UTC 2 Feb 2014 at an azimuth of 230°.
\[ \Phi_{DP} \] at distances within 30 km and above the ML, the decrease in \[ \Phi_{DP} \] at ranges 40–50 and 60–80 km in rain areas and at ranges about 60–70 and 130–140 km in the cloud tops.

This short review of the radar variables measured inside and above MLs point to a variety of shapes of particles falling into MLs and various compositions of ice and water in particles inside MLs. Water in ice particles can be a result of their melting and can also be acquired at a collision of an ice particle with water droplets emerged from already melted particles. To study the impacts of the differential phase \( \psi_i \) on the radar variable from MLs, simple models of wet ice particles have been analyzed and the results follow.

If no aggregation above an ML takes place, an ice particle inside an ML retains the shape it had above the ML and can get coated with a thin water film in the ML. Let ice particles be approximated with oblate spheroids having an axis ratio of 0.6. This axis ratio has been obtained from the in situ studies by Hogan et al. (2002). Above an ML, such an ice spheroid produces \( Z_{DR} \) in an interval of 1.5–2.5 dB depending on the intensity of flutter (Fig. S12, the corresponding \( \alpha_{\phi} \) interval is 20°–5°).

This \( Z_{DR} \) interval is a typical one in situations with no aggregation above MLs (e.g., Figs. S5 and S7). If a water film of 0.03 \( \alpha \) uniformly covers the ice core, the \( \rho_{hv} \) drops and \( \delta \) increases (Fig. 10) compared to the values for the ice core (Fig. S12). The parameter \( \alpha = 1 \) mm was used in the calculations. Polarizabilities \( \alpha_{\alpha} \) and \( \alpha_{\beta} \) have been calculated using equations for layered scatterers (e.g., Bohren and Huffman 1983, section 5.4). Zero mean canting angle has been assumed in the calculations because of symmetry of the particle. For zero mean canting, the \( Z_{DR} \) values are not affected by \( \psi_i \), but depend on \( \alpha_{\phi} \). The corresponding \( Z_{DRv} \) values are shown in Fig. 10a with numbers. The \( Z_{DR} \) values are about equal to those observed in the melting layer (Figs. S5 and S7). One can see that the differential phase \( \psi_i \) strongly affects \( \rho_{hv} \) and \( \delta \) values (Fig. 10). The maximal impact on \( \delta \) is about 2.6° at \( \psi_i = 90° \). For the ice particle without the water cover, the maximal \( \delta \) is 0.5° (Fig. S12).

Water can cover a particle nonuniformly. Due to moderate fall velocities associated with small particles, the larger density of water compared to the ice core can cause the water to form a hanging drop at the bottom of the falling particle (Fig. S13). The scattering properties of such particles cannot be obtained analytically even in the Rayleigh limit; therefore, the WPL-D software has been used. The ice core diameter was assumed to be 1 mm, and the bottom water droplet maximal width was assumed to be 0.15 mm (Fig. S13).

If a layer of low \( Z_{DR} \) exists above an ML (e.g., Figs. S6 and S9), particles falling into MLs can be represented as aggregates of ice columns (e.g., Fabry and Zawadzki 1995; Fabry and Szyrmer 1999). Water in such particles can occupy a part of the aggregate or can cover ice crystals completely. To examine this situation, a particle has been modeled as one having two parts: an inner part consisting of a mixture of ice and water and an outer part consisting of ice and air (Fig. S14). Both particle’s parts are treated as the Maxwell Garnett mixtures with the inner part containing 80% of ice and 20% of water and the outer layer consisting of 10% of ice and 90% of air. Dielectric permittivities of the inner and outer parts have been calculated via the Maxwell Garnett equation (e.g., Bohren and Huffman 1983, section 8.5; Ishimaru 1991) and the whole particle has been treated as a shielded particle (Bohren and Huffman 1983, section 5.4). The axis ratio of the particle assumed to be 0.6 and equal for the inner part and whole particle. Let the major axis of the whole particle be 1.5 times the major axis of its inner part. The \( \delta \) values from this particle are
significantly lower than those for the previous models (previous models are shown in Figs. 10 and 11; the current model is shown in Fig. 12). The radar variables from the model particles are close to those shown in the supplemental material (Fig. S6). The models show that the influence of $\psi_i$ and $\sigma_0$ on radar variables is significant.

The presented results demonstrate that the radar variables from melting layers can be satisfactorily represented at S band by Rayleigh particles and the SHV $\delta$ values can emerge at scattering by incident waves shifted in phase. If adjacent radars feature different $c_i$, they can measure different polarimetric variables from same ice cloud areas.

The results in this section have been obtained for fluttering particles having a zero-mean canting angle. Asymmetrical ice particles have nonzero-mean canting angles. A wet ice particle can be asymmetrical if water is concentrated not at the center of a symmetrical ice part. Figure S15 shows a symmetrical ice dendrite with a small droplet at one of its branches. The particle acquires a nonzero equilibrium canting angle and its scattering characteristics can significantly deviate from those obtained for zero canting angle.

6. Conclusions

Weather radars employing simultaneous transmission and reception of orthogonally polarized waves (SHV design) deliver biased radar variables compared to those from alternate transmission radars (AHV design). In the SHV mode, the incident waves are typically shifted in phase ($\psi_i$) thereby affecting measured $Z_{\text{DR}}$, $\rho_{hv}$, and $\delta$ values due to interference between the scattered primary and depolarized waves. Maximal $Z_{\text{DR}}$ values from thin ice plates measured with AHV radar is 10 dB, whereas for SHV radar it is 12.3 dB. Values of $\delta$ from thin ice cloud particles measured with AHV radar are of a few hundredths of a degree, whereas for SHV radar, maximal $\delta$ is 62.3° (section 2). For ice particles of moderate axis ratios, $\delta$ can reach 20° (for a width/length ratio = 0.3). $Z_{\text{DR}} = 12.3$ dB and $\delta = 62.3^\circ$ are maximal possible values for Rayleigh ice particles other than thin plates. The maximal possible $Z_{\text{DR}}$ value of 12.3 dB is much larger than measurement $Z_{\text{DR}}$ limits on some radars. For example, the $Z_{\text{DR}}$ span on WSR-88D is ±7.9 dB circa 2020 (WSR-88D Radar Operations Center 2018). Since 12.3 dB is the mean value and radar signals experience natural fluctuations, the maximal measurable $Z_{\text{DR}}$ limit should be at least 15 dB for measurements in clouds.

The SHV $\delta$ values can be positive and negative depending on orientation of particles and the differential phase $\psi_i$. Typically, $Z_{\text{DR}}$ and $\delta$ have the same sign that is intuitively clear. The analysis in section 2 shows that

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**Fig. 10.** Dependences of (top) $\rho_{hv}$ and (bottom) $\delta$ on the incident differential phase and flutter intensity ($\sigma_0$) for an ice spheroid uniformly covered with a water film. The axis ratio of the spheroid is 0.6 and the water cover thickness is 0.03 of its longest axis. The legends in the panels are applicable for all curves. The insert in the top panel shows half of the ice core (yellow) covered with a water film (blue).
particles producing positive $Z_{DR}$ can exhibit negative $\delta$ and vice versa depending on their orientation relative to the radar beam and $\psi_i$.

The SHV $\delta$ values are of a few to several degrees depending on the axis ratio of a particle. Therefore, measured $\delta$ can be used in retrieval microphysical procedures if $\psi_i$ is known (measured). Ice particles in shapes of ellipsoids and hexagonal prisms of the same axis ratios (width/length $= b/a$) produce different $Z_{DR}$ and $\delta$ at $b/a > 0.1$ that should be taken into account in the retrieving of particles' parameters from radar data. The $\delta$ is a part of the measured total differential phase from which the specific differential phase ($K_{DP}$) is obtained. In precise $K_{DP}$ measurements in snow and ice clouds, the contribution from $\delta$ should be accounted for.

Fluttering of ice particles significantly affects the polarimetric variables. Flutter of particles is an additional
unknown parameter, which complicates the interpretation of radar data. The intensity of flutter is characterized with the standard deviation in the canting angles. The latter parameter depends not only on flutter, but also on the spread of canting angles due to asymmetry in particles’ shapes.

The phase $\psi_l$ contributes to the $Z_{DR}$, $\rho_{hv}$, and $\delta$ values observed in melting layers. Presented results show that such values of the radar variables can be produced by Rayleigh scatterers and there is no need to assume the presence of large particles of resonance sizes. The presented results show impacts of $\psi_l$ on the $\delta$ and $\rho_{hv}$ values in melting layers for some simple models of particles; the results correspond to data collected with S-band KOUN.

Polarization properties of melting ice particles can be modeled using WIPL-D, a software tool allowing composition and analysis of ice and water together in a particle.

Dependences of $Z_{DR}$, $\rho_{hv}$, and $\delta$ values on the radar system differential phase upon transmission ($\psi_l$) point to additional measurement opportunities. Variations in $\psi_l$ during the radar dwell time can deliver additional information about scatterers in real time. Fast $\psi_l$ variations can be available on phased array radars, where alternations in $\psi_l$ can be accomplished by changing a time delay between transmitted polarized waves.

Acknowledgments. The author thanks Dr. Terry Schuur for valuable comments and the reviewers for their suggestions, which helped to improve the presentation. Mr. Tomislav Milosevic from the WIPL-D company provided helpful advices on the WIPL-D modeling. Funding for this study was provided in part by the NOAA/Office of Oceanic and Atmospheric Research under NOAA–University of Oklahoma Cooperative Agreement NA110OAR4320072, U.S. Department of Commerce.

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