QC
807.5
U6W6
no.61

NOAA Technical Memorandum ERL WPL-61

SIMULATIONS OF SURFACE TIDAL CURRENT CALCULATIONS FOR HF RADAR APPLICATIONS

R. S. Lyons
A. S. Frisch

Wave Propagation Laboratory
Boulder, Colorado
August 1980

NATIONAL OCEANIC AND ATMOSPHERIC ADMINISTRATION / Environmental Research Laboratories

A
QC
807.5
.U6W6
no. 61

NOAA Technical Memorandum ERL WPL-61

SIMULATIONS OF SURFACE TIDAL CURRENT CALCULATIONS FOR HF RADAR APPLICATIONS

R. S. Lyons
A. S. Frisch

Wave Propagation Laboratory
Boulder, Colorado
August 1980

SILVER SPRING
CENTER
NOV 17 1980
N.O.A.A.
U. S. Dept. of Commerce

UNITED STATES DEPARTMENT OF COMMERCE
Philip M. Klutznick, Secretary

NATIONAL OCEANIC AND ATMOSPHERIC ADMINISTRATION
Richard A. Frank, Administrator

Environmental Research Laboratories
Joseph O. Fletcher, Acting Director

80 3791

SIMULATIONS OF SURFACE TIDAL CURRENT CALCULATIONS FOR HF RADAR APPLICATIONS

R. S. Lyons and A. S. Frisch
NOAA/ERL/Wave Propagation Laboratory
Boulder, Colorado 80303

ABSTRACT

Coastal Ocean Dynamics Applications Radar (CODAR) data and possible data gaps were simulated to show the errors in mathematically recovered Fourier tidal coefficients when compared with the input coefficients. The modeled data consists of a Fourier series with three prominent tidal components of 12, 24, and 25.8 h, white noise with Gaussian distribution and a trend that simulates a storm surge and its recovery time. The baseline model (hourly sampled data, white Gaussian noise of standard deviation 1, and a 2-day trend) showed that recovery of the 12 h component with <20% error is possible with only 1 day's data, and all three components can be recovered in 5 days with <20% error. This model was then applied to data gaps that occurred during field experiments. Simulation of a German experiment with a possible data loss of 8 h daily showed data for 2 days is necessary to obtain <20% error for the 12 h component's recovery and 6 days for all three. In an Alaskan experiment of 8 days, data was missing for one of these days. The simulation for that situation showed <7% error for all three components. This simulation is also applied to different experimental data losses or interruptions, different sampling intervals besides hourly, different components, and different noise levels to find necessary sample sizes for specified confidence levels.

INTRODUCTION

In many oceanographic applications we need to determine the amplitude and phase of tidal coefficients. We can compute these tidal coefficients from measurements such as the time variation of sea surface elevation, bottom pressure, or current velocity. We are using an HF Doppler radar system (sometimes called CODAR, Coastal Ocean Dynamics Applications Radar), to measure ocean surface currents, and from these measurements we compute the tidal coefficients for selected sets of frequencies. However, we might have interruptions during the data taking process from equipment failure, or interference, for example, or changes in the flow caused by storm surge and transient wind conditions. All of these will cause errors in the calculation of the tidal coefficients. For this reason, we simulated raw CODAR data to explain these errors that arose during data gathering and reduction and to predict what sample sizes are necessary to obtain reasonable confidence.

SIMULATING CODAR DATA

Our computer program's variable parameters that simulate the CODAR data include time interval between data samples, number of components for the Fourier series, the components' amplitudes and frequencies, white noise level, sample length, and segments of no data. In addition, storm surges are simulated by adding a Fourier component. The output compares input amplitudes with the recovered amplitudes and the initial phase of 45° with the recovered phase in terms of the percentage error. The errors are graphed logarithmically by day according to the input prominent components, which we took as 12 h, 24 h, and 25.8 h. The amplitudes for these components of 2.570, 1.614, and 1.469 were taken from Defant's open ocean tidal coefficient table. The computer program forms a Fourier time series with the given amplitudes and frequencies, adds a Gaussian random noise with mean 0 and specified standard deviation for white noise, recovers the given amplitudes using an FFT and matrix inversion, and compares the recovered with the initial amplitudes by using

$$\text{phase error} = \frac{\left|\tan^{-1}\dfrac{B_i}{A_i} - \tan^{-1}\dfrac{B_i^*}{A_i^*}\right|}{\tan^{-1}\dfrac{B_i}{A_i}} \times 100 \text{ and}$$

$$\text{amplitude error} = \frac{\left|\sqrt{A_i^2 + B_i^2} - \sqrt{A_i^{*2} + B_i^{*2}}\right|}{\sqrt{A_i^2 + B_i^2}} \times 100 ,$$

where the signal at time t is $\sum_i [A_i \cos(\omega_i t) + B_i \sin(\omega_i t)] + N \times n$, a standard Fourier series with frequency ω_i with a noise factor N times noise, a random Gaussian generated number n. The recovered amplitudes are designated with *. The program starts with a given number of days and repeats with data for the last day removed in each successive iteration. When data for only one day are analyzed, the entire procedure is repeated with a new random number generator seed. The program then averages the error for 10 different signals, each using a different seed in the random number generator. The random number generator depends on its initial seeding. If the same seed is used each time, the results will be biased. Therefore, 10 different seeds are used and the resulting errors are averaged and plotted.

ANALYSES

First, we ran the program with hourly time intervals, no lost data, no trends, a noise level factor 1, and the three main Fourier coefficients for 20 days. The noise factor of 1 is an extreme example of noise relative to the actual data. The results of this run are shown in Figs. 1a and b. We can see the 12-h component recovered in 1 day, i.e., 24 data points, with an error of less than 20%. Recovery of the 12, 24, and 25.8 h components require data for 4 days for less than 20% error and data for 7 days for less than 5% error.

A low frequency disturbance was simulated with this model by adding a long period Fourier component. The simulation gave approximately the same errors with and without added trend, as seen in Figs. 2a and b for a 2-day trend and Figs. 3a and b for a 30-day trend. The errors oscillate with the period of the trend. Figures 2a and b show the errors with a 2-day trend of amplitude 4, and alternating daily perturbations resembling a cosine function can be seen. Figures 3a and b show the results with a 30-day trend of amplitude 10. The errors are higher than the no-trend data for small data samples and become lower at 15 days. If more data were computed, the errors would be expected to again be larger than the no-trend case at 30 days — again like a cosine function.

A 2-day trend with three components is a realistic case; it is used as a baseline for the comparisons that follow. We needed data for only 1 day to recover the 12-h component with less than 20% error. We needed data for 5 days to recover all three components with the same error; this is 2 days longer than the no-trend case because of the oscillations of the trend.

After establishing the 2-day trend as our baseline, we explored our white noise level. We found that changing the noise-level factor approximately multiplies the phase and amplitude errors by that amount as seen in Figs. 4a and b. This shows the logarithmic error graphs of the 12-h component with a 2-day trend when noise levels of .5, 1, and 2 are used. We can easily see that multiplying the noise by 2 multiplies the errors by $\sim$2.

We wrote this program to analyze errors resulting from data gaps. The program models various sampling intervals and gaps in data collection. The model indicates that increasing the sampling interval increases the error, (which implies that the number of days of the sample needs to be increased). Sampling every 4 hours increases the sample size of the 12-h component to 2 days for less than 20% error and of all three components to 6 days for the same error. Sampling every 6 h (Figs. 5a and b) would require 3 days for the 12-h component and 8 days for all with less than 20% error. We did not sample with >6-h intervals since sampling intervals >6 h exceeds the Nyquist frequency of the 12-h component. From this study, the sample size in days to recover all three components with $\leq$ 20% error seems to be the time interval in hours plus 2, which is a fairly small sample. The problem of missing data also shows slightly larger sample spaces necessary for reliable data.

A situation arose which we have used as an example for studying missing data. In an experiment in Helgoland, Germany, the CODAR signal sampled hourly had interference for the same 8-h period every day. This extreme case was simulated (Figs. 6a and b). Missing data are from the hours of 3 to 11 every day. To recover the 12-h component with less than a 20% error would require 2 days of data. To recover all three would require 6 days — not much more than the baseline sample sizes.

Another unique situation occurred during an Alaskan experiment where CODAR data were taken hourly for 8 days, but there were no data over a large ocean area for day 3. Figures 7a and b display the errors calculated by the model for the 8-day hourly sample with a data gap for all of day 3. The errors are all less than 7% for the 8-day period.

SUMMARY

This model simulates situations that arose during CODAR field experiments. The basic data model includes a Fourier series with three prominent components as well as noise and a trend. The white noise factor multiplied the phase and amplitude errors by that factor, i.e., doubling the noise doubled the error; throughout these studies we used a noise level of 1. A trend is input as a 2-day Fourier component of amplitude about double that of the prominent components. This simulates a storm (frequent during experiments) and its recovery time off-shore. The errors using a trend were similar in magnitude to the no-trend case except for oscillations that fluctuate with the period of the trend. Surprisingly, with or without a trend, the 12-h component's recovery is relatively error free. In data for only 1 day sampled hourly the phase and amplitude errors were less than 20%, so using that amount gives fairly accurate results. Recovery of the 24- and 25.8-h components requires 7 days of data to obtain errors less than 20%. This is because the method has a difficult time differentiating between the closely spaced components. In fact, modeling with a 24.5-h component instead of the 25.8 nearly doubled the errors. This proves that the closer the components are together, the more difficult is recovery. By varying the time intervals between samples we found that increasing the time interval increased the sample size necessary for confidence in the data. If we sampled every 8 h instead of hourly, we needed data for 8 days to obtain the 12-h component and for 11 days to obtain all three components with less than 20% error. For an Alaskan experiment of hourly data for 8 days with one missing day the simulation using day 3 as the data gap gave errors of less than 7% for recovery of all three components. Simulating another situation of an 8-h daily data gap, showed that the 12-h component would be recovered in 2 days with a 10% error and all three amplitudes could be recovered in only 6 days with less than a 20% error. This modeling process can be useful in the future to simulate similar data problems so data interpretation will be easier.

The results of this study give sample size for various errors of specific simulated conditions, which lends credibility to the previous CODAR experiments. These results can help plan the lengths of future experiments with reasonable error limits.

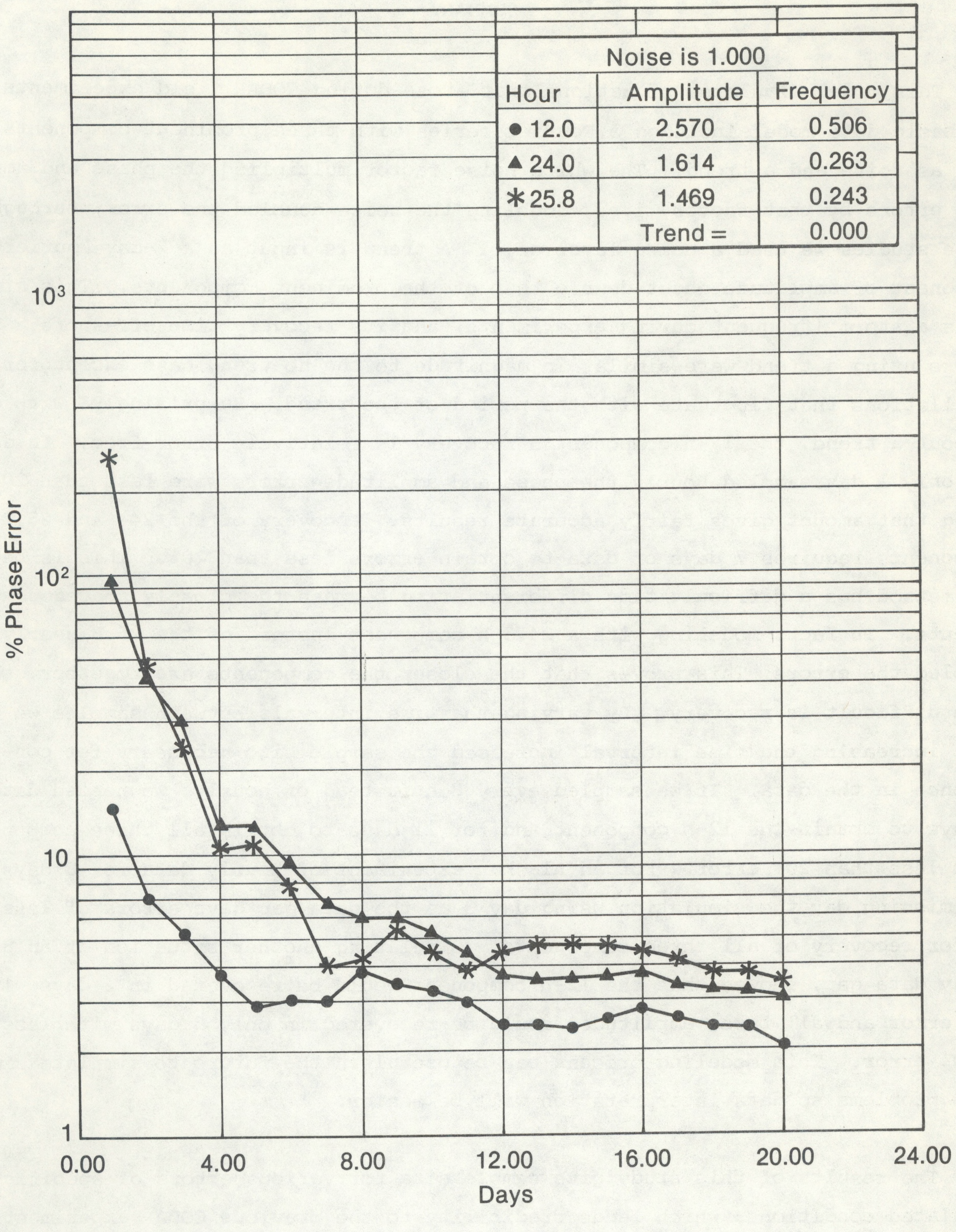
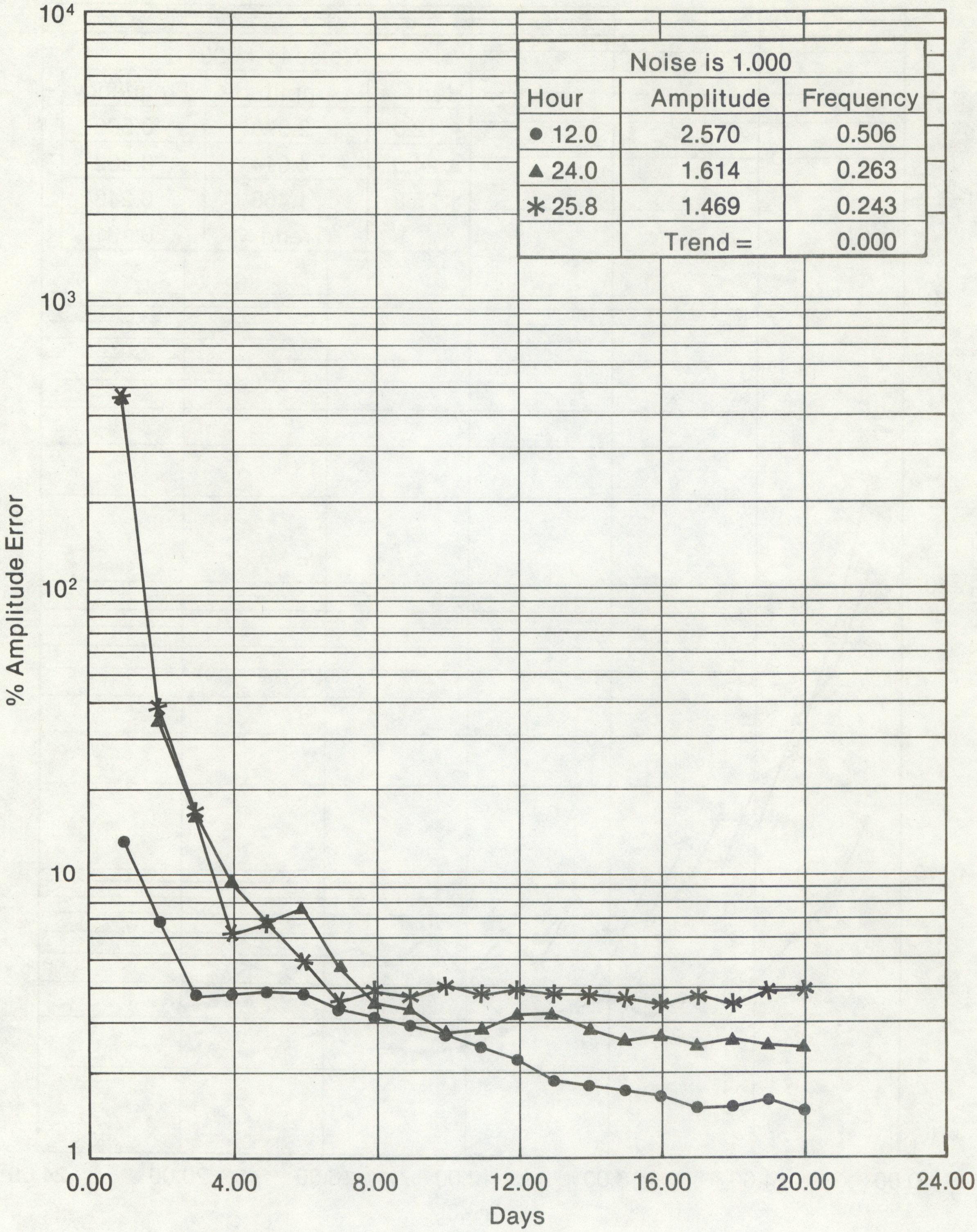


Figure 1a & b. Phase and amplitude errors of three tidal components plotted logarithmically over the number of days of hourly theoretical data with no trend and noise level 1. These graphs can be used to determine the number of days needed to obtain data with a specific error.


Noise is 1.000
Hour
Amplitude
Frequency
12.0
2.570
0.506
24.0
1.614
0.263
25.8
1.469
0.243
Trend =
0.000
10^4
10^3
10^2
10
1
% Amplitude Error
0.00
4.00
8.00
12.00
16.00
20.00
24.00
Days

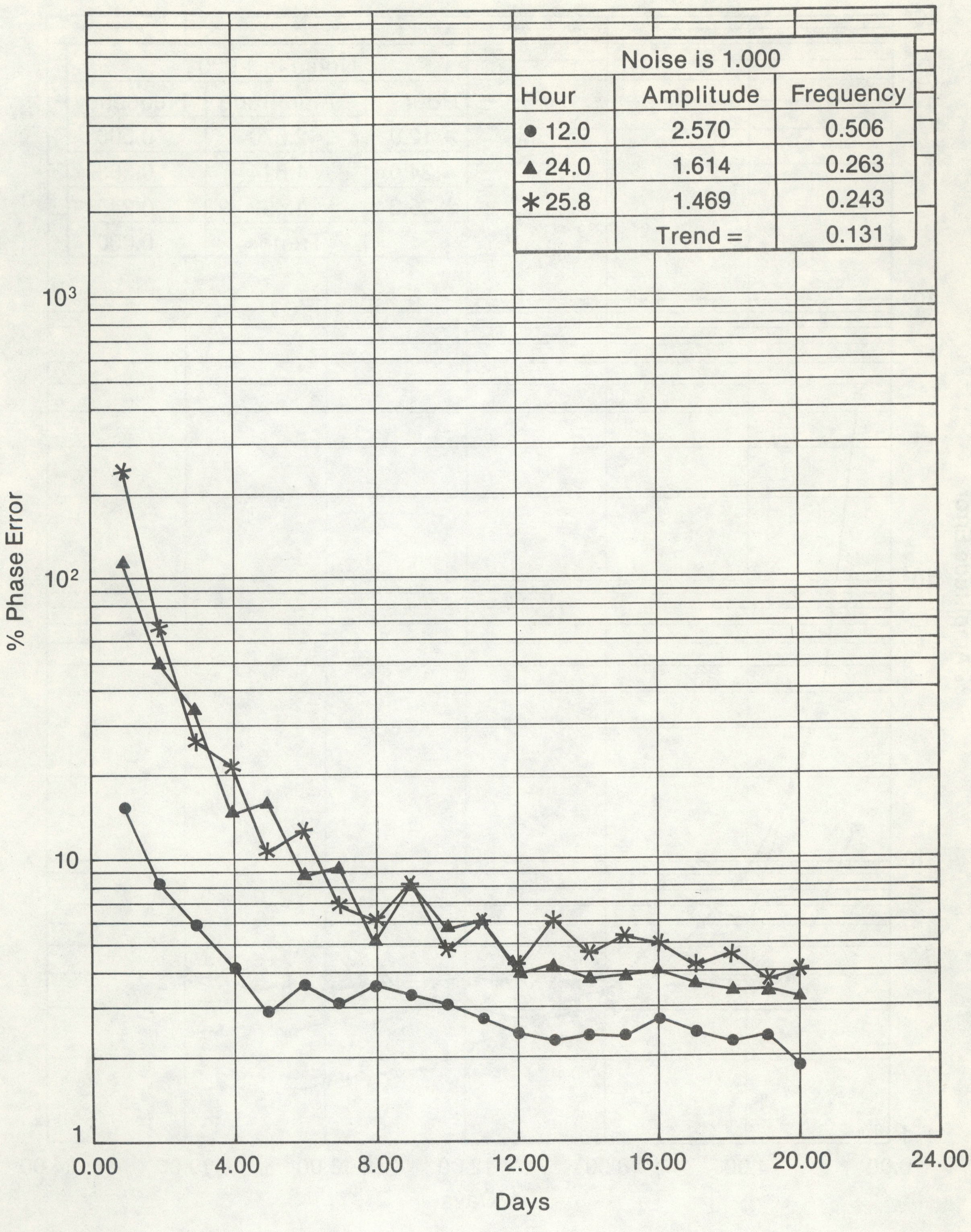
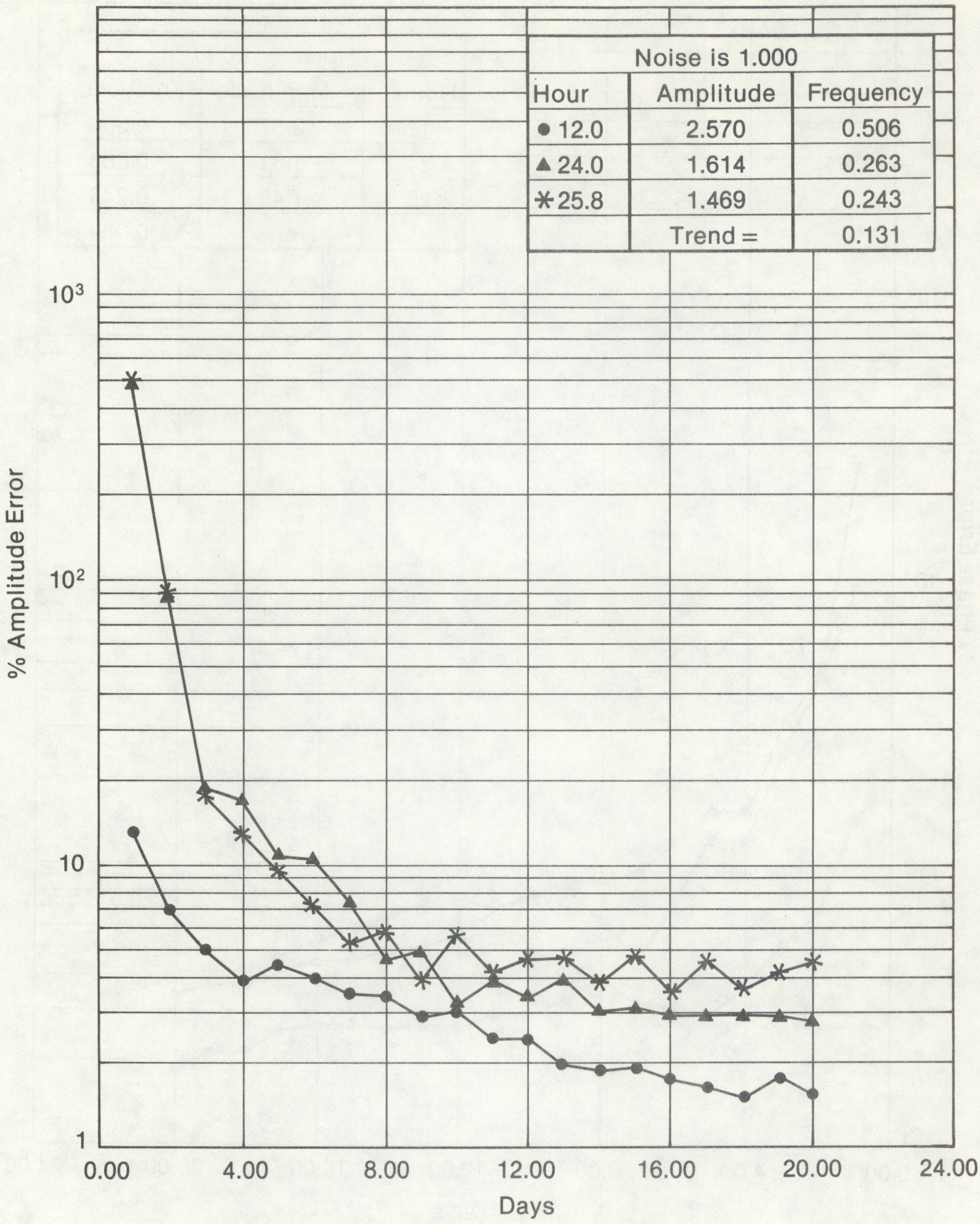


Figure 2a & b. A 2-day trend of amplitude 4 was added to the three tidal components plotted logarithmically over the number of days sampled hourly for the phase and amplitude error. This graph which is used as our baseline for other studies shows that the 12-h component can be recovered in one day and all three in 5 days with less than a 20% error. The daily fluctuations caused by this 2-day trend (compare with no-trend case, Fig. 1.)


Noise is 1.000
Hour
Amplitude
Frequency
● 12.0
2.570
0.506
▲ 24.0
1.614
0.263
✳ 25.8
1.469
0.243
Trend =
0.131
10^3
10^2
10
1
0.00
4.00
8.00
12.00
16.00
20.00
24.00
Days
% Amplitude Error

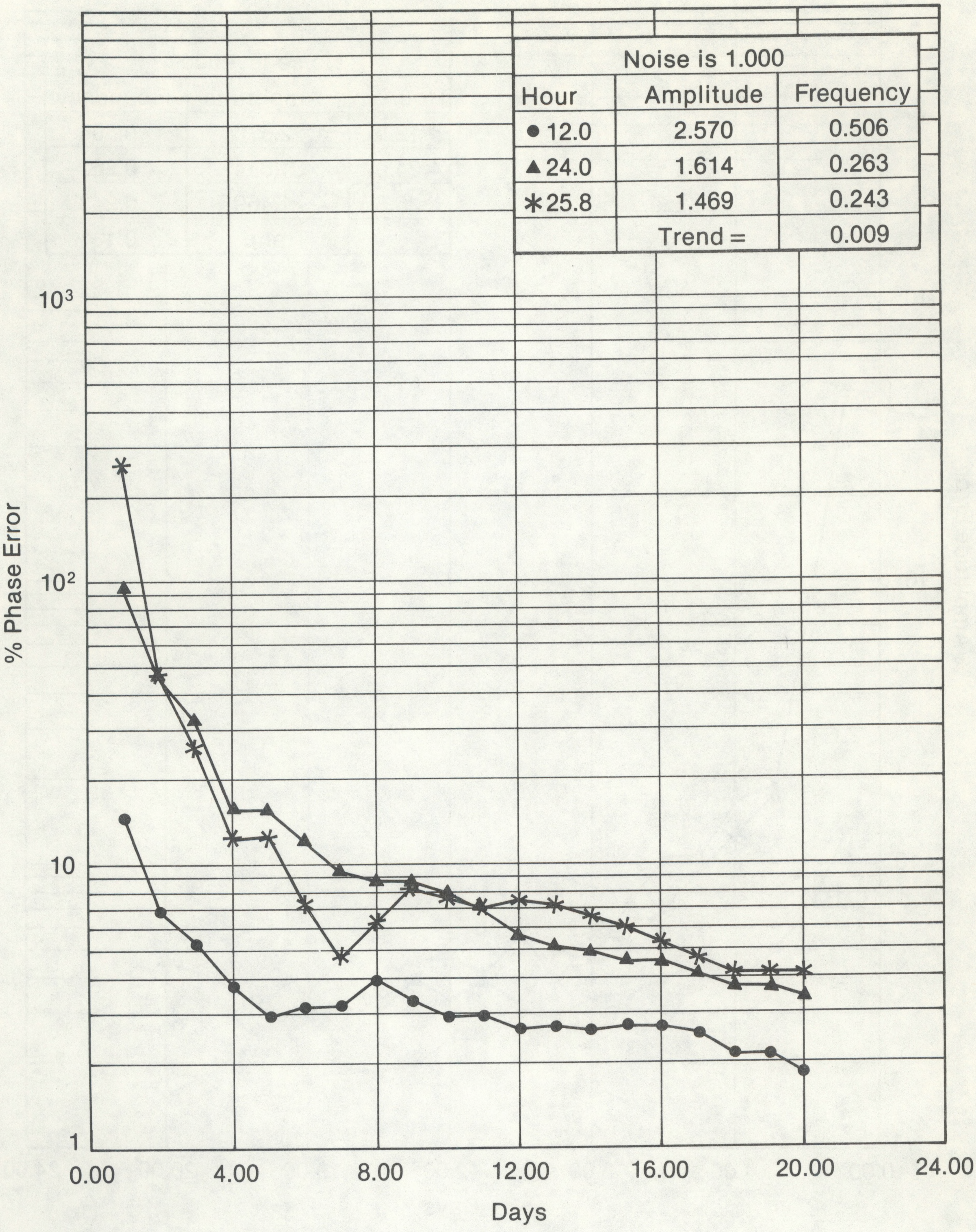
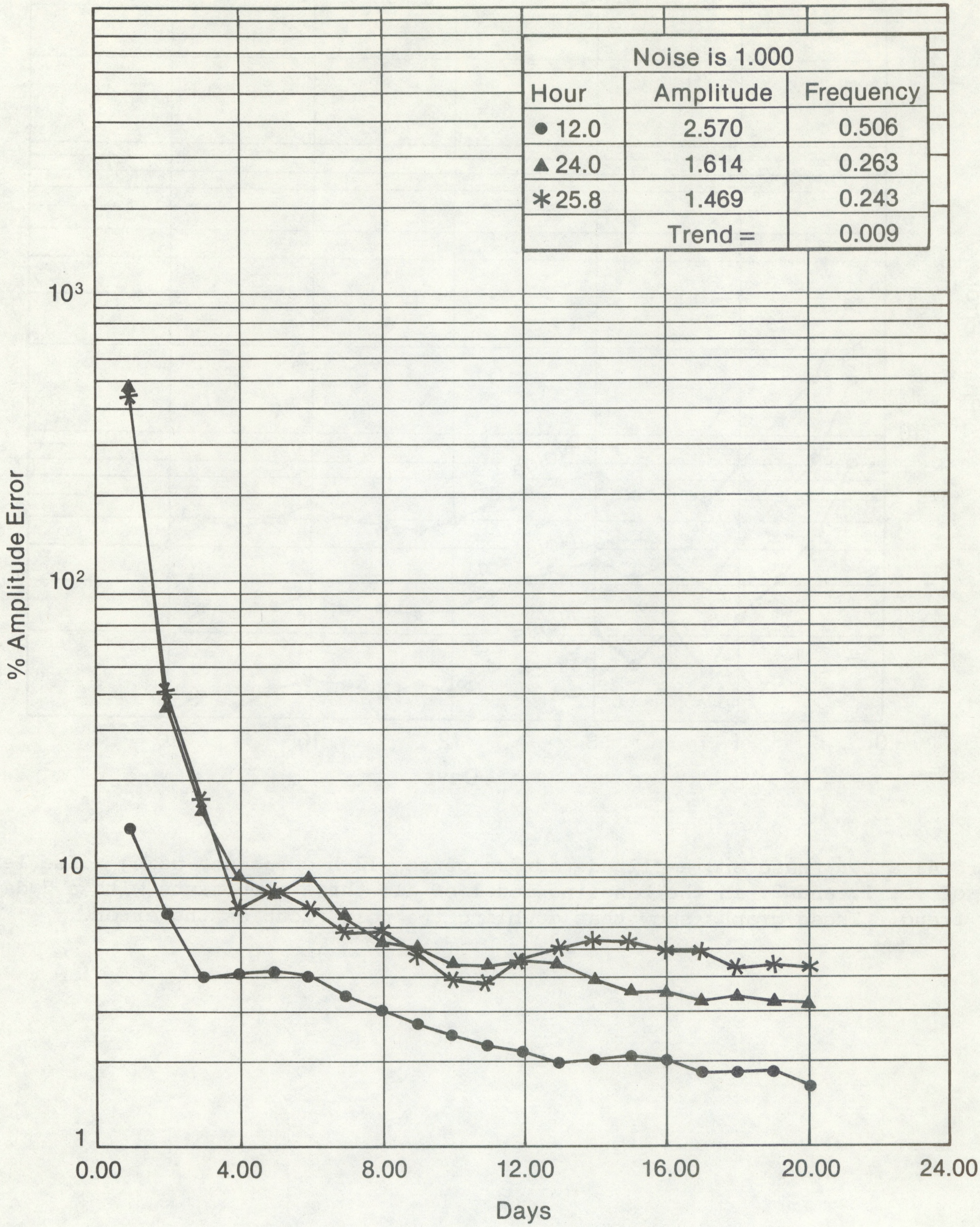


Figure 3a & b. A 30-day trend of amplitude 10 was added to the three tidal components plotted over the number of hourly sampled days for the phase and amplitude errors. This case when compared with Fig. 1, the no trend case, and Fig. 2 with a 2 day trend, a more logical case, shows the differences of a long period trend.



Noise is 1.000
Hour
Amplitude
Frequency
12.0
2.570
0.506
24.0
1.614
0.263
25.8
1.469
0.243
Trend =
0.009
% Amplitude Error
10^3
10^2
10
1
0.00
4.00
8.00
12.00
16.00
20.00
24.00
Days

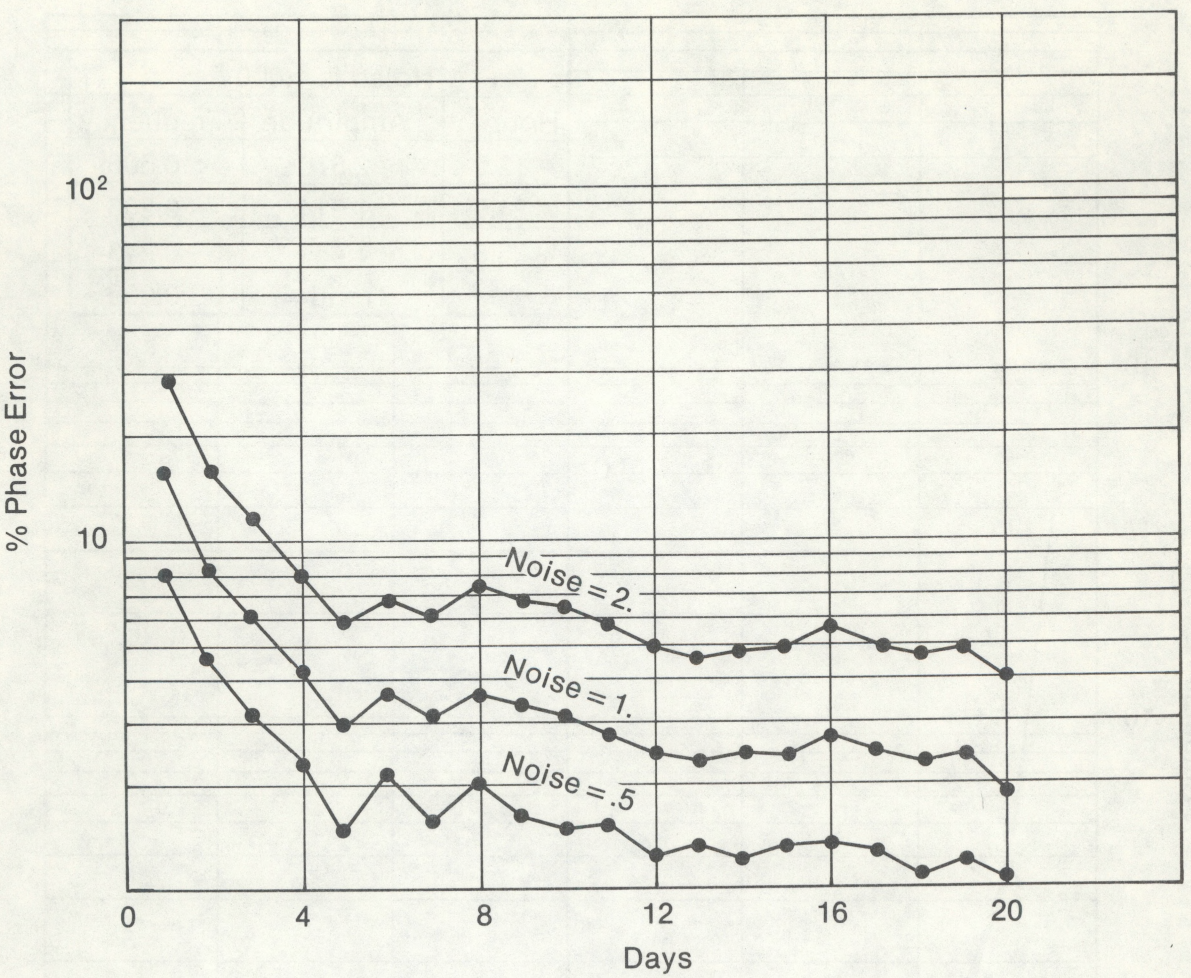
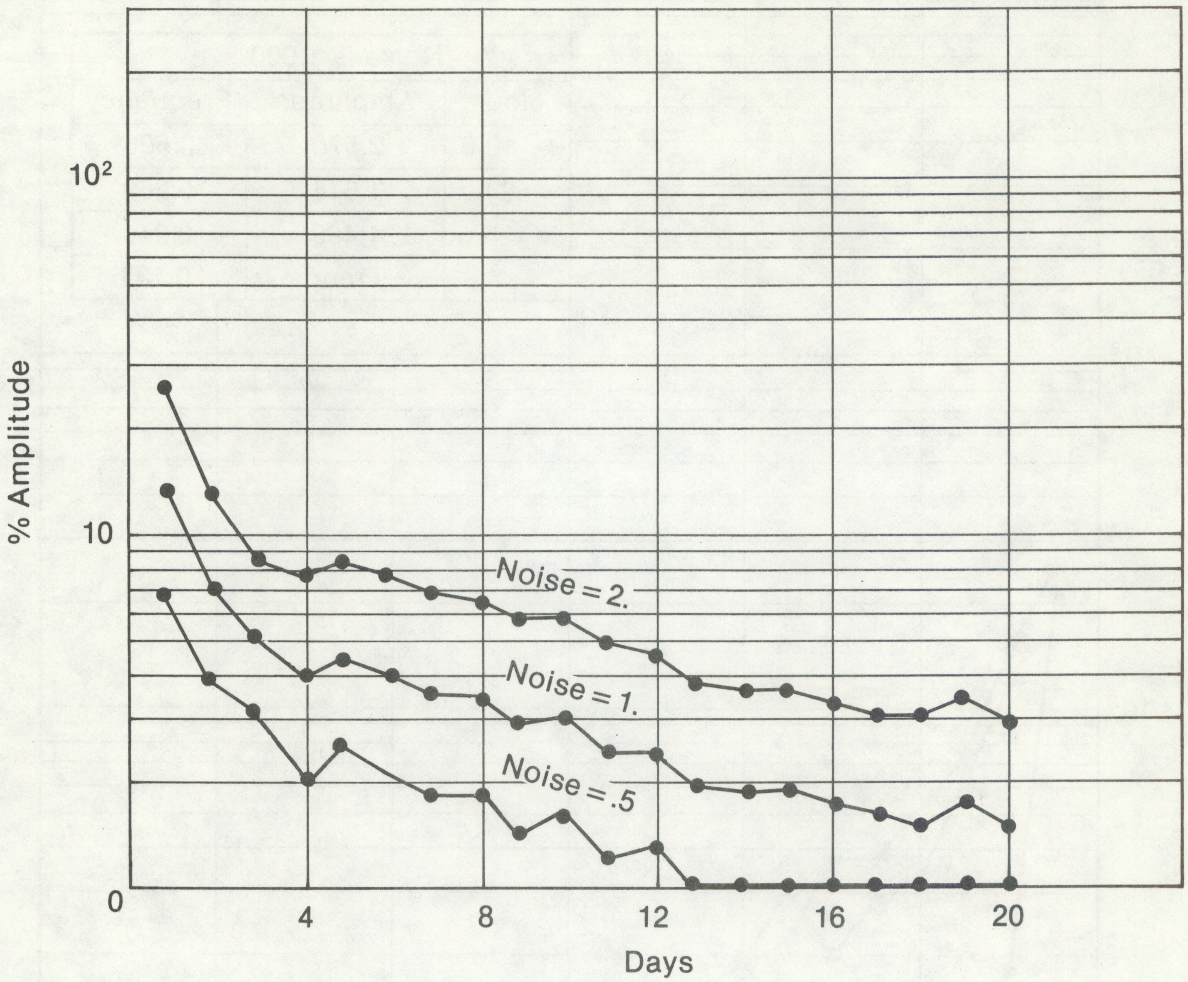


Figure 4a & b. Phase and amplitude errors of the 12-h component using noise levels of .5, 1, and 2, in the baseline model of all three components with a 2-day trend. These graphs show that doubling the noise doubles the error.


10²
10
0
% Amplitude
Noise = 2.
Noise = 1.
Noise = .5
4
8
12
16
20
Days

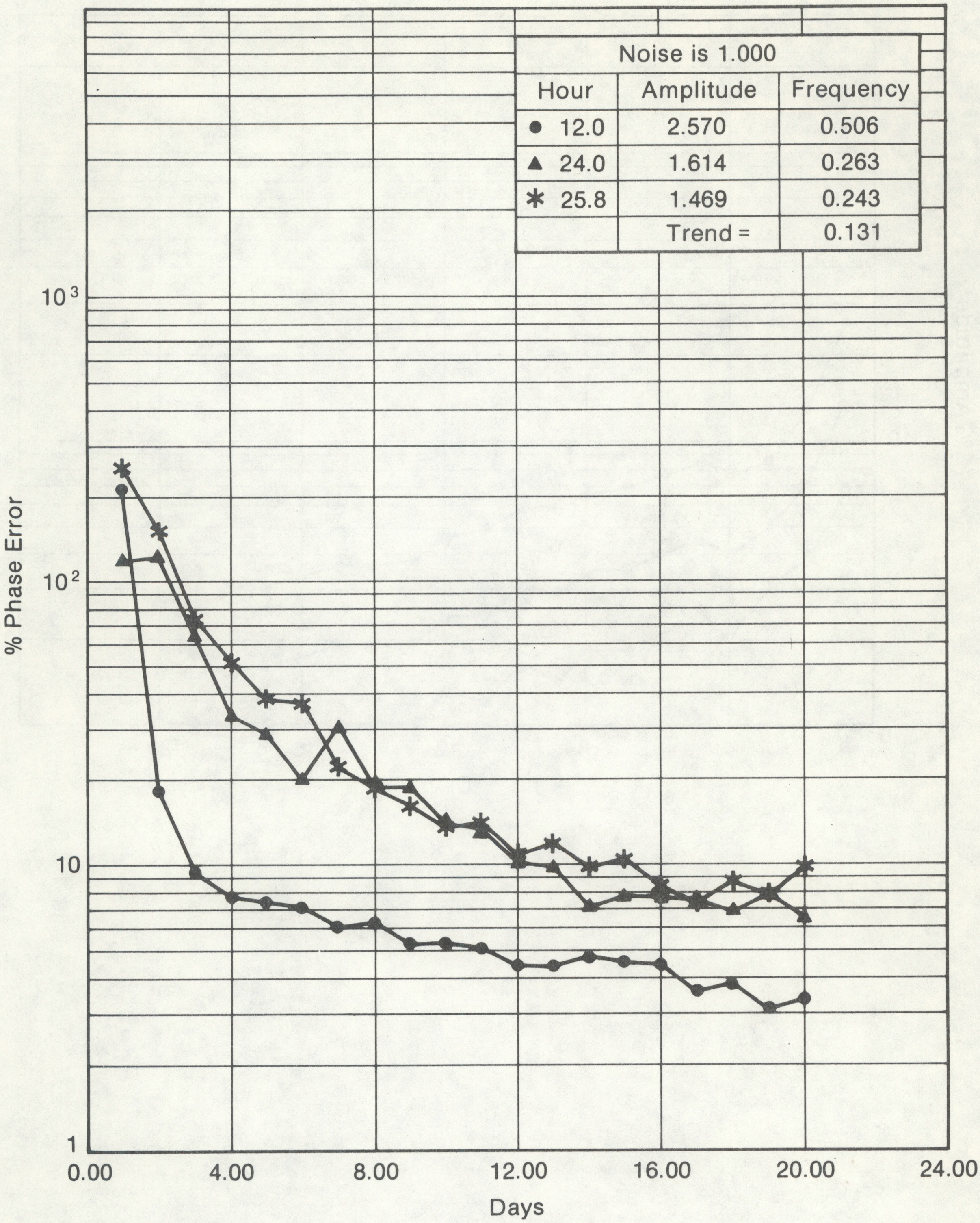
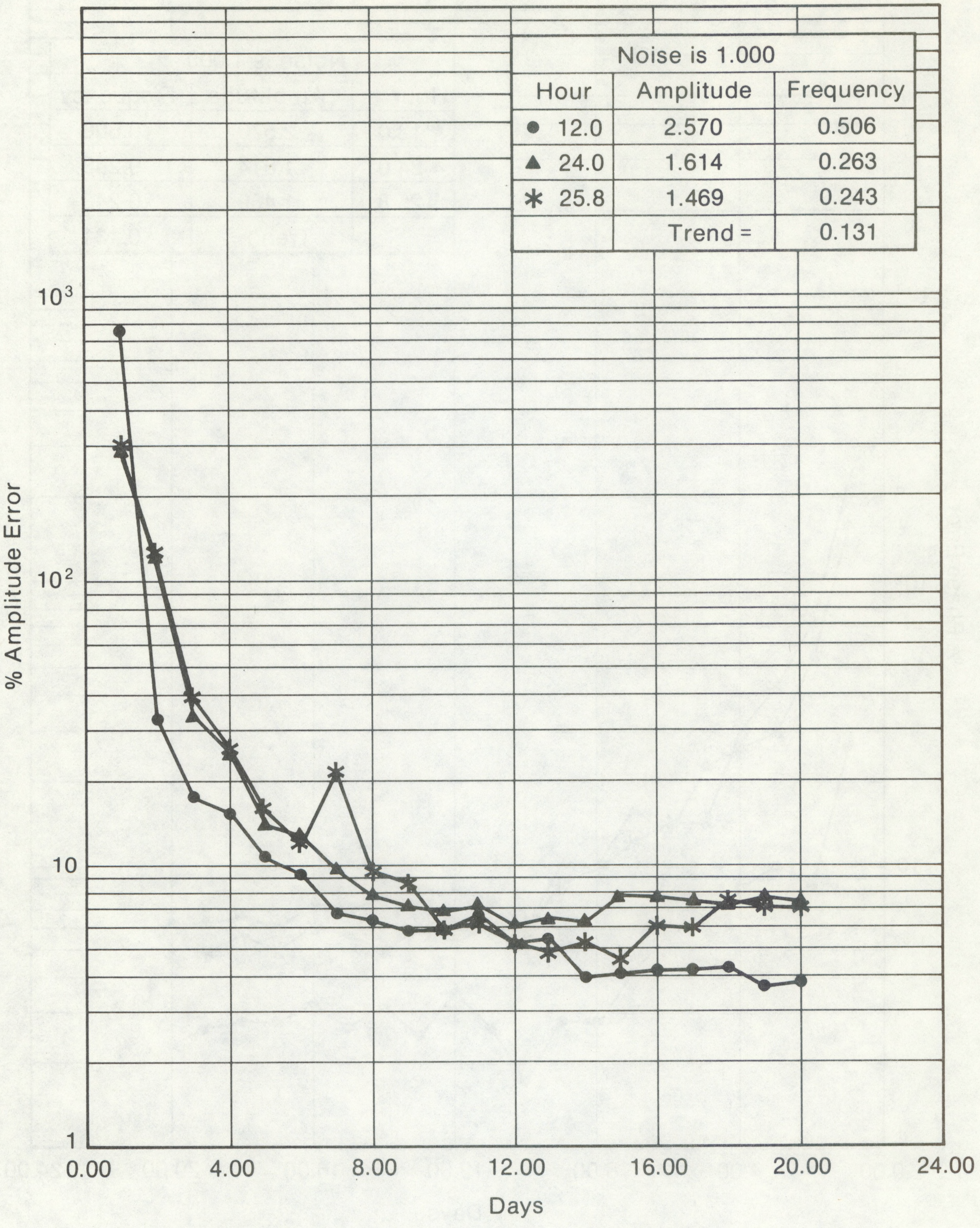


Figure 5a & b. The phase and amplitude errors of sampling every 6 h instead of hourly, using the baseline parameters of noise level 1 and a 2-day trend. Note the longer sample sizes necessary to recover the components: 3 days for the 12-h and 8 days for all three.


Noise is 1.000
Hour Amplitude Frequency
12.0 2.570 0.506
24.0 1.614 0.263
25.8 1.469 0.243
Trend = 0.131
% Amplitude Error
10^3
10^2
10
1
0.00
4.00
8.00
12.00
16.00
20.00
24.00
Days

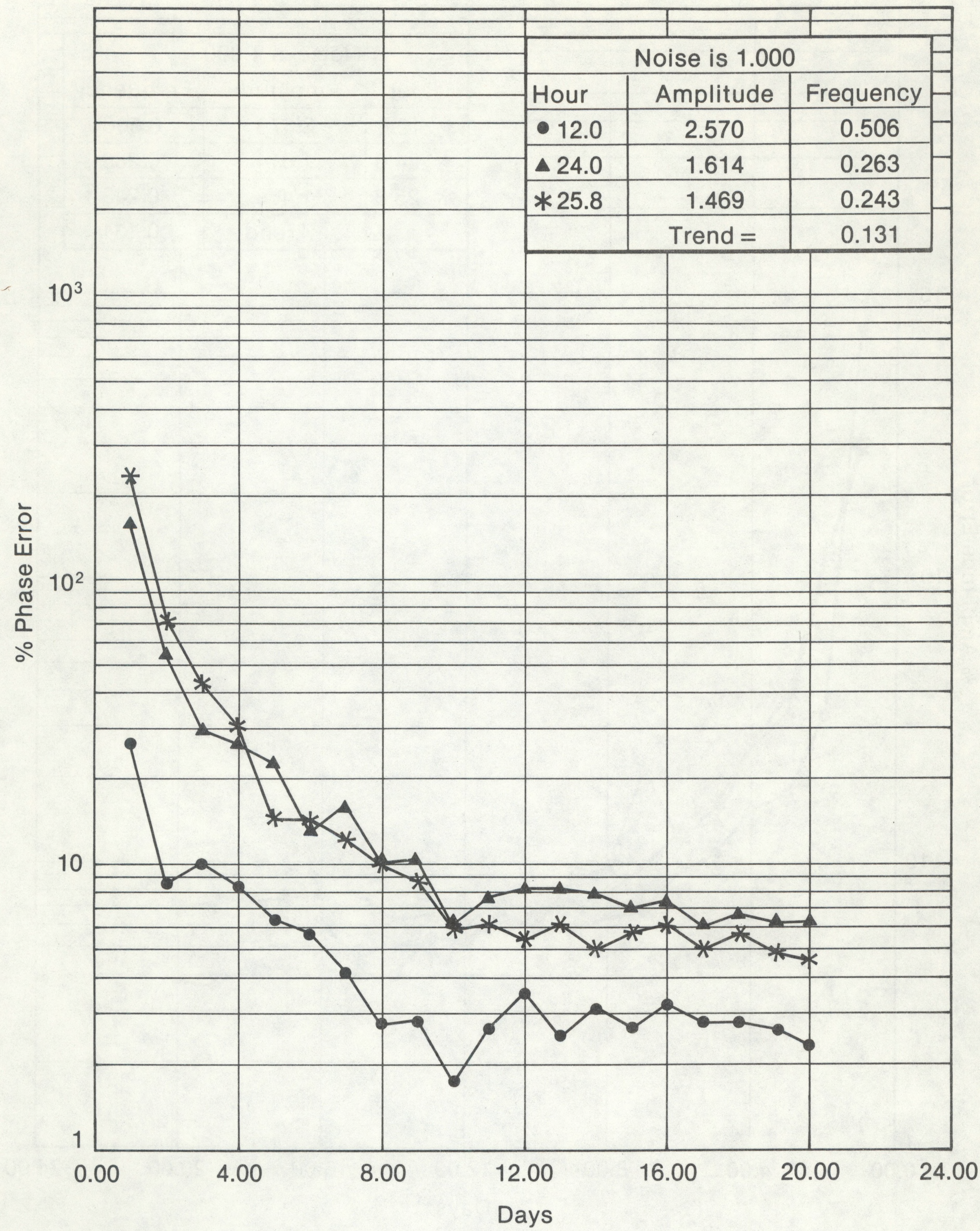
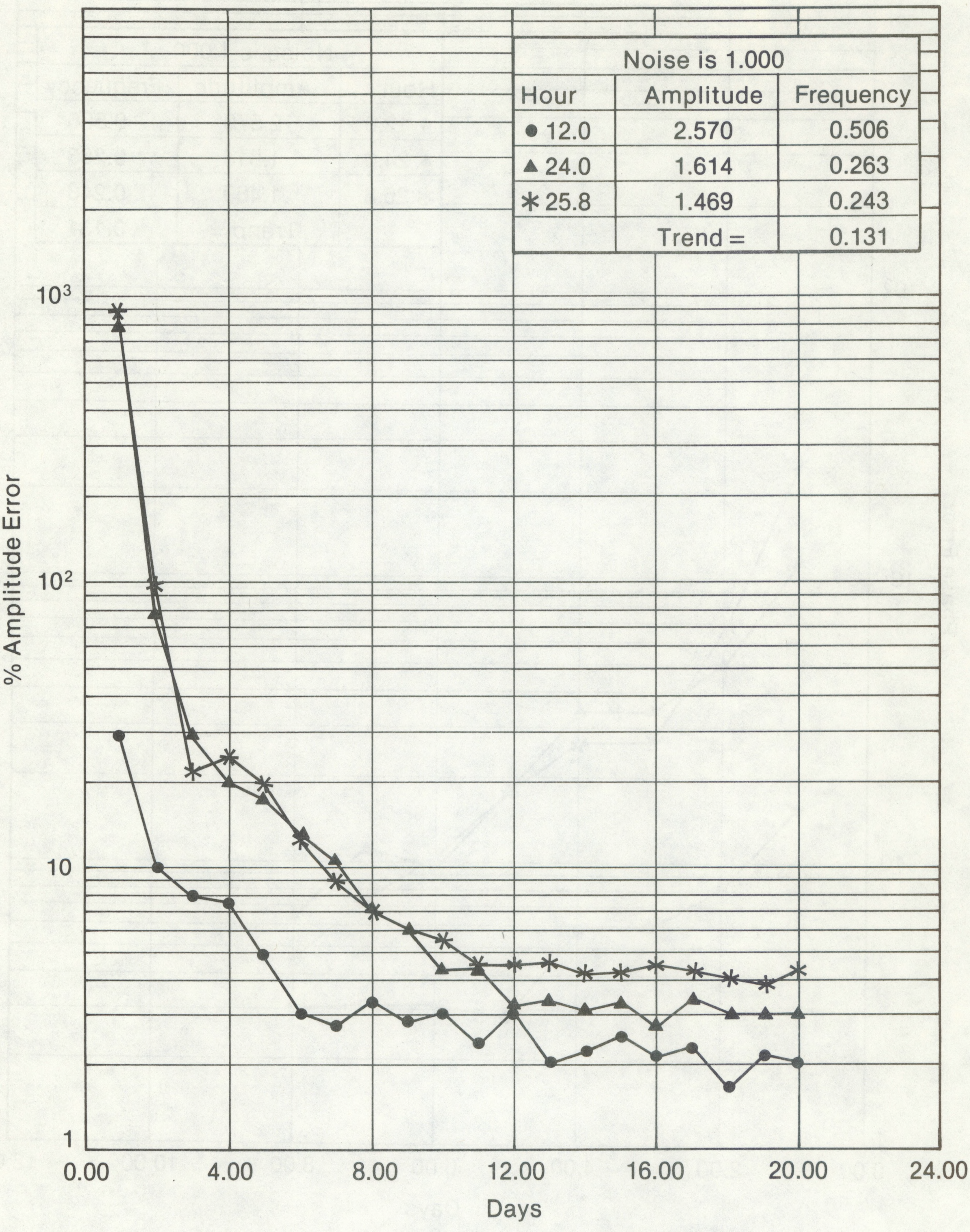


Figure 6a & b. Simulation of Helgoland data. The necessary sample sizes for phase and amplitude errors less than 20% for the case of 8 h of missing hourly data every day using the baseline parameters are increased. (Compared with Fig. 2a & b.) Recovery of the 12-h component needs 2 days of data; recovery of all three needs 6 days of data.



Noise is 1.000
Hour
Amplitude
Frequency
● 12.0
2.570
0.506
▲ 24.0
1.614
0.263
✱ 25.8
1.469
0.243
Trend =
0.131
% Amplitude Error
10³
10²
10
1
0.00
4.00
8.00
12.00
16.00
20.00
24.00
Days

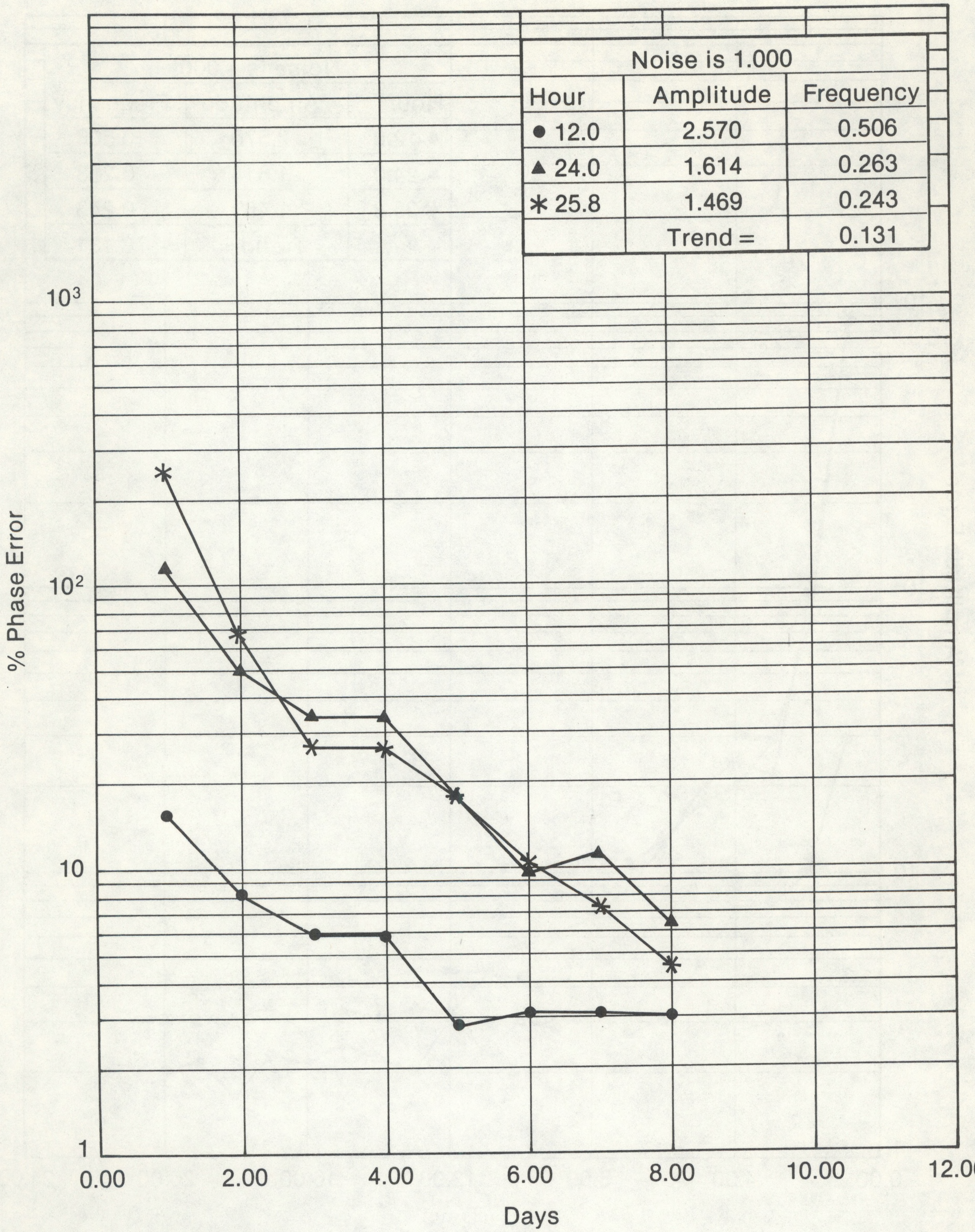
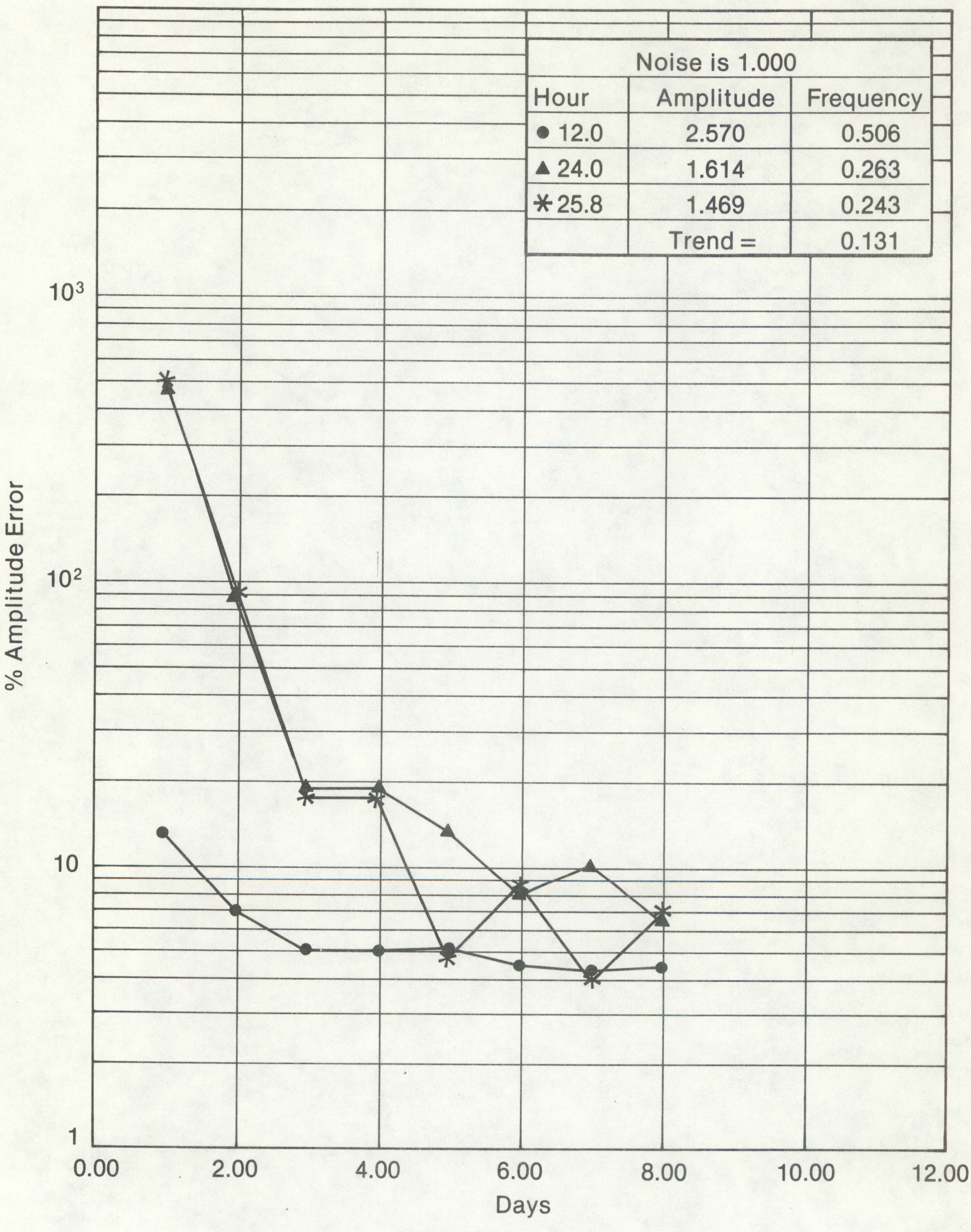


Figure 7a & b. Simulation of Alaska data. The phase and amplitude errors of hourly data for 8 days, [using our baseline model] for the case where there was a 24-h data gap at day 3. This data gap situation happened for large areas of an Alaskan experiment. When compared with Fig. 2, and the hourly-2 day trend case, this simulation shows that the sample size will increase 1 day to 5 days for three-component recovery with less than a 20% error.


Noise is 1.000
Hour
Amplitude
Frequency
12.0
2.570
0.506
24.0
1.614
0.263
25.8
1.469
0.243
Trend =
0.131
% Amplitude Error
10^3
10^2
10
1
0.00
2.00
4.00
6.00
8.00
10.00
12.00
Days