NOAA Technical Memorandum ERL NHEML-1

DEPARTMENT OF COMMERCE
UNITED STATES OF AMERICA

THE DISTRIBUTION OF TURBULENT KINETIC ENERGY DISSIPATION IN HURRICANES OVER A LIMITED RANGE OF WIND SPEEDS

Francis J. Merceret

National Hurricane and Experimental Meteorology Laboratory
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noaa NATIONAL OCEANIC AND ATMOSPHERIC ADMINISTRATION / Environmental Research Laboratories

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THE DISTRIBUTION OF TURBULENT KINETIC ENERGY DISSIPATION IN HURRICANES OVER A LIMITED RANGE OF WIND SPEEDS

Francis J. Merceret

The distribution of turbulent kinetic energy dissipation in three hurricanes is found to be an approximately lognormal random variable nearly independent of altitude (below 650 m) and distance from storm center for wind speeds from 20 to 30 m s^{-1}.

1. INTRODUCTION

During 1975, an extensive set of small-scale turbulence measurements was made with a hot-film anemometer that sensed the streamwise velocity fluctuations u' from a DC-6 aircraft in Hurricanes Caroline, Eloise, and Gladys. The experiments, the instrumentation, and the data processing techniques are discussed in detail in Moss and Merceret (1976) and Merceret (1976a) and will not be redescribed here. This paper is concerned with a detailed empirical description of the distribution of the kinetic energy dissipation rate ε of the turbulence measured in hurricanes.

Before the experiments it was expected that ε would be found to be a function of the wind speed (U), altitude (Z), and distance (R) from the storm center. The preliminary results from Hurricane Caroline (Merceret, 1976a) indicated that any such dependence was weak and that ε was more accurately described as an independent random variable. The more detailed analysis presented here confirms that indication for the variation of ε with R and Z (below 650 m) and suggests a model probability distribution which may be used with considerable accuracy as a description of the behavior of ε. The model selected is an extension of the lognormal hypothesis of Kolmogorov (1962). The range of wind speeds in the samples used here was too small to determine the variation of ε with U, and nearly all of the data were collected below the altitude of 1 km, thus limiting our resolution of variation of ε with Z.

2. THE LOGNORMAL HYPOTHESIS AND ITS EXTENSION

It has long been recognized that the intensity of turbulence at high Reynolds numbers is intermittent when averaged over small regions in time and space. In back-to-back papers, Oboukhov (1962) and Kolmogorov (1962) suggested a specific description of the intermittency. They proposed that the logarithm of the turbulent kinetic energy dissipation rate ε is a Gaussian random variable and that its variance σ^2 is determined by the gross flow characteristics and size of the region over which observations are averaged. Specifically, Kolmogorov (1962) suggested

$$\sigma^2 \sim A + k' \log L/\ell \quad (1)$$

provided that $L >> \ell >> \eta$ where A $(\underline{x},t)$ is a function of order unity determined by the gross characteristics of the flow, $\underline{x}$ is vector position, t is time, L is an integral scale of the flow, η is the Kolmogorov microscale, ℓ is the length scale characteristic of the averaging volume, and k' is expected to be universal constant of order unity. An excellent exposition of the hypothesis and some of its consequences may be found in Tennekes (1973).

Measurements of the probability distribution of log ε reported from experiments in GATE (Merceret, 1976b) and preliminary results from Hurricane Caroline (Merceret, 1976a) seemed to be close to lognormal for averaging scales $\ell \approx 15$ m. The Caroline observations of the rate of change of velocity power spectra with frequency (f) further supported the lognormal hypothesis. The steepening of the slope from $f^{-5/3}$ to $f^{-1.80 \pm 0.17}$ observed in Caroline (ibid.) could result from a lognormal distribution of ε obeying (1) if k' had the value 1.2 ± 1.5 (Wyngaard and Tennekes, 1970). This value is within the high end of the range of value measured by several investigators as reported by Gibson and Masiello (1972). A more detailed examination of the data seemed in order to determine how well the lognormal distribution works in hurricanes, especially since the averaging scale ℓ selected for hurricane measurements is large enough that (1) is not expected to apply. For a variety of reasons, including the scales of analysis used by other investigators who will use these results, and hardware and software limitations of our data processing equipment, it was decided that the minimum practical averaging scale for use in this study would be $\ell = 300$ m (3 s of record at true airspeed = 100 m s^{-1}). This violates the condition $L >> \ell$ for which (1) is specified since $L \approx 300$ m (Moss, 1977). It was the purpose of this experiment to determine whether ε remains lognormally distributed when $\ell/L \approx 1$. In addition, for possible application to numerical models using a 10-km grid (see, e.g., Rosenthal, 1974), three runs were composited from the 3-s data where $\ell = 10$ km. Here $\ell/L >> 1$. Based on results described in this paper, it is suggested that the extension of Kolmogorov's (1962) hypothesis to larger averaging scales provides a useful description of the probability distribution of ε, provided we no longer constrain σ^2 to obey (1), since no significant difference was found in the value of σ when ℓ was changed from 300 to 10,000 m.

3. DESCRIPTION OF THE DATA BASE AND METHOD OF ANALYSIS

Sections of records (hereafter called "runs" or "legs") from relatively homogeneous regions of Hurricanes Caroline, Eloise, and Gladys were selected for analysis. In these regions, time series of the logarithm of the power spectral density of u' at 648 Hz (12-Hz bandwidth) with 2-s smoothing (32 degrees freedom per spectral estimate) were sampled at 3-s intervals and converted to dissipation values (Merceret, 1976a). The first four raw moments

(moments about zero) of the dissipation (ε) were computed. From these, the corresponding central moments were computed along with the skewness coefficient Γ_1 and coefficient of excess (kurtosis) Γ_2 for each run. The latter are defined by

$$\Gamma_1 = \overline{\varepsilon'^3} \Big/ \left[\overline{\varepsilon'^2}\right]^{3/2} \tag{2}$$

and

$$\Gamma_2 = \left\{ \overline{\varepsilon'^4} \Big/ \left[\overline{\varepsilon'^2}\right]^2 \right\} - 3 \tag{3}$$

where $\varepsilon' = \varepsilon - \overline{\varepsilon}$ and an overbar denotes the average over a run. The data base is presented in table 1.

If the distribution of ε is lognormal with log ε having mean value μ and standard deviation σ, then the n^{th} moment of ε is given by

$$\varepsilon^n = \exp\ (n^2\sigma^2/2 + \mu n) \tag{4}$$

and given any pair of moments we can solve (4) for σ^2 and μ. This technique is called the "method of moments" by Aitchison and Brown (1976). By applying the method to the six possible combinations of the first four moments in pairs, we obtain six estimates for μ and σ for each run. The degree of consistency of these estimates is a measure of the lognormality of the observed distribution. Using the mean values obtained for μ and σ from the six estimates, we can employ (4) to compute model values for the first four moments and compare them with the observed values. Their degree of agreement is another measure of the lognormality of the observed distribution. In this study, both measures are used.

The results of the application of the method of moments to the data base are presented in table 2. To concisely represent the lognormality of the distribution in a quantitative fashion, a "goodness-of-fit" factor GF was prepared for each run from four factors, f_1, f_2, f_3, and f_4. The first, f_1, is 1 minus the square of the ratio of the standard deviation of the six estimates of μ to their mean. This measure will fail if ε is large enough that μ approaches zero, but the values observed in the data considered here are all significantly different from zero so that f_1 is a useful measure of dispersion. The second, f_2, is 1 minus the square of the ratio of the standard deviation of the six estimates of σ to their mean. The third, f_3, is the square of the correlation coefficient between the observed values and the model values of the first four raw moments of the distribution. The last, f_4, is obtained by computing the ratio of each observed moment to the model value, squaring it, inverting it, if necessary, so that it is less than 1, and averaging the results for each of the four moments. Clearly, each of these measures is bounded from above by 1, and the closer each is to unity, the better the fit of the observations to lognormality. The value of GF is given by

$$GF = 1000\ f_1\ f_2\ f_3\ f_4 \tag{5}$$

Table 1. Data base (mks units)

Flight	Leg	Time (Z)	$\overline{\epsilon}$ x10^2	$\overline{\epsilon^2}$ x10^4	$\overline{\epsilon^3}$ x10^6	$\overline{\epsilon^4}$ x10^8	Standard Deviation (ε) x10^2	Γ_1	Γ_2	N
750917A	PBL 1	2048-2050	7.445	59.417	500.95	4400	2.00	-0.101	-1.42	44
Eloise	2	2053½-2055½	1.891	3.8732	8.4772	19.544	0.545	0.176	-1.16	45
	3	2058-2100	5.1233	27.825	159.1	950.92	1.256	0.196	-0.714	46
	4	2103-2105	1.598	2.946	6.181	14.3133	0.626	0.891	-0.537	44
	5	2108½-2110½	5.568	33.704	218.7	1501	1.644	0.215	-0.794	51
	6	2116-2118	2.331	6.3404	19.889	71.145	0.952	1.021	1.663	46
	7	2121-2123	2.887	8.859	28.91	99.09	0.763	0.168	-0.567	46
	X1	2127-2128	1.925	3.848	7.973	17.08	0.377	0.325	-0.612	21
	X2	2130½-2131½	0.866	0.785	0.7437	0.734	0.187	0.487	-0.710	21
	X3	2134½-2135½	1.77	3.357	6.759	14.309	0.437	0.224	-0.751	23
	Xcombo	X1+X2+X3	1.528	2.685	5.208	10.817	0.592	0.169	-0.999	65
	1	1850-1857	0.47383	0.69778	2.01621	7.2334	0.688	3.80	15.76	133
	2	1933-1939	9.55214	400.124	26244	11.962x10^6	17.56	3.043	9.09	118
	3	1939-1944	1.97373	7.4469	60.437	746.422	1.884	4.74	28.54	96
	4	1950-1959	3.12428	14.8727	96.2783	732.46	2.261	1.55	1.381	185
	5	1959-2006	2.38779	6.13542	16.73	48.1314	0.659	0.028	0.75	136
750929A	1	2036-2044	11.463	508.59	40933	4.068x10^6	19.42	3.611	14.86	170
Gladys	2	2044-2052	1.9304	4.6828	13.459	44.7774	0.978	0.777	1.26	163
	3	2052-2059½	3.4592	15.5595	83.3884	505.611	1.896	0.691	4.65	142
	4	2126-2130	42.9531	4289.17	561992	82.746x10^6	49.44	1.369	0.927	81
	5	2146-2154	2.56273	8.0264	28.6986	114.735	1.208	0.370	0.493	162
750830A	1	1948-1954	2.7231	10.3911	51.0153	294.972	1.725	1.27	1.14	122
Caroline	2	2000-2007	4.52705	38.5311	803.776	26614.6	4.247	6.08	44.76	129
	3	2014-2021	2.46273	9.1709	51.6	375.441	1.762	2.51	6.38	132
	4	2156-2159	45.68	2170.4	107240	5.515x10^6	9.151	0.593	1.21	59
	PBL 3	2251-2253	0.63589	0.58916	0.67933	0.89334	0.43	0.877	0.054	41
	PBL 4	2254-2256	1.01895	1.61362	3.24641	7.29601	0.759	0.985	-0.334	40
	PBL 5	2302-2305	1.32036	2.0223	3.60214	7.31533	0.528	1.326	1.196	58
	PBL 6	2306½-2308½	1.00152	1.6366	3.4059	7.8776	0.796	0.987	-0.348	31
	PBL 7	2310-2312	0.85371	0.79431	0.78943	0.8257	0.256	-0.03	-0.705	40
	PBL 8	2314-2316	0.64916	0.48211	0.39993	0.361933	0.246	0.545	-0.369	40
	5	2321-2324	1.54425	5.9769	44.879	421.202	1.895	3.607	13.46	59
Long base	Pkg#1	*	6.395	113.05	2694	67322	8.49	1.711	1.058	6
composites	Pkg#2	**	4.665	64.82	1750.7	55822	6.56	3.704	13.29	23
	Pkg#3	***	3.074	11.447	50.49	258.77	1.413	1.070	1.781	16

* 750917A 1933-1944
** 750917A 1950-2006 and 750929A 2036-2059
*** 750929A 2146-2154 and 750830A 2000-2007, 2014-2021

NOTE: $\Gamma_1 = \overline{\epsilon'^3}/\left[\overline{\epsilon'^2}\right]^{3/2}$. $\Gamma_2 = \left\{\overline{\epsilon'^4}/\left[\overline{\epsilon'^2}\right]^2\right\}-3$. N is the number of data points in each run.

Table 2. Model values of statistical quantities (mks units) based on μ and σ of lognormal distribution derived from observed values.

Flight	Leg	μ Mean	μ Std. Dev.	σ Mean	σ Std. Dev.	$\overline{Y}$ x10^2	$\overline{Y^2}$ x10^4	$\overline{Y^3}$ x10^6	$\overline{Y^4}$ x10^8	Std. Dev. (y) x10^2	Γ_{1y}	Γ_{2y}
750917A	PBL 1	-2.623	0.009	0.2443	0.014	7.478	59.37	500.2	4474	1.857	0.724	1.262
Eloise	PBL 2	-4.00	0.007	0.2665	0.011	1.898	3.867	8.458	19.86	0.514	0.838	1.255
	PBL 3	-2.996	0.004	0.2317	0.0071	5.135	27.82	159.0	959.3	1.205	0.706	1.258
	PBL 4	-4.196	0.013	0.3621	0.0133	1.608	2.946	6.157	14.67	0.600	1.207	2.510
	PBL 5	-2.921	0.009	0.2723	0.0119	5.591	33.67	218.4	1525	1.553	0.853	1.177
	PBL 6	-3.829	0.007	0.3828	0.0071	2.338	6.331	19.84	72.02	0.930	1.237	2.988
	PBL 7	-3.576	0.006	0.2478	0.0091	2.886	8.856	28.90	100.3	0.726	0.783	1.074
	X1	-3.968	0.001	0.1907	0.0026	1.926	3.846	7.966	17.11	0.369	0.649	0.174
	X2	-4.771	0.002	0.2107	0.0028	0.866	0.784	0.743	0.735	0.185	0.810	-1.500
	X3	-4.063	0.006	0.2506	0.0089	1.775	3.353	6.747	14.46	0.450	0.846	0.937
	Xcombo	-4.227	0.022	0.3369	0.0259	1.545	2.673	5.182	11.25	0.535	1.102	2.071
	1	-5.626	0.284	0.9031	0.121	0.5419	0.6637	1.8376	11.499	0.608	4.783	58.52
	2	-2.558	0.4524	0.9326	0.192	11.964	341.6	23272	3.78E6	14.09	5.165	70.70
	3	-4.241	0.076	0.809	0.032	1.997	7.675	56.75	807.34	1.920	3.772	33.06
	4	-3.598	0.079	0.5815	0.0515	3.242	14.74	93.97	840.17	2.056	2.158	9.284
	5	-3.764	0.0065	0.2554	0.0101	2.396	6.127	16.73	48.735	3.223	-2.141	0.198
750929A	1	-2.426	0.3704	0.9409	0.1546	13.76	458.9	37095	7.27E6	16.42	5.278	74.60
Gladys	2	-4.031	0.0271	0.4406	0.0244	1.9376	4.653	13.432	47.08	0.948	1.0922	3.82
	3	-3.446	0.0446	0.4567	0.0382	3.538	15.417	82.766	547.38	1.704	1.556	4.595
	4	-0.9767	0.2461	0.7145	0.1365	48.60	3936	531032	1.19E8	39.67	2.992	19.28
	5	-3.729	0.0299	0.4021	0.0297	2.603	7.963	28.64	121.07	1.0903	1.330	3.302
750830A	1	-3.717	0.0536	0.528	0.0388	2.794	10.318	50.345	324.6	1.584	1.883	6.904
Caroline	2	-3.454	0.0871	0.8366	0.0389	4.4877	40.55	738	27040	4.518	4.041	38.94
	3	-3.882	0.0462	0.6235	0.0255	2.5026	9.240	50.32	404.28	1.725	2.395	11.68
	4	-0.803	0.0002	0.1984	0.00026	45.68	2170.5	107271	5.52E6	9.1552	0.608	0.774
	PBL 3	-5.158	0.0789	0.5284	0.0585	0.6614	0.5784	0.6687	1.022	0.3754	1.885	6.921
	PBL 4	-4.691	0.1053	0.5597	0.0734	1.0731	1.5752	3.1625	8.685	0.6508	2.042	8.231
	PBL 5	-4.399	0.0052	0.3830	0.0045	1.3224	2.0249	3.5906	7.373	0.5256	1.255	2.927
	PBL 6	-4.710	0.1238	0.5793	0.0837	1.0647	1.5856	3.303	9.624	0.6723	2.146	9.172
	PBL 7	-4.795	0.0110	0.2706	0.0158	0.8584	0.7928	0.7879	0.8425	0.2366	0.848	1.306
	PBL 8	-5.089	0.0147	0.3442	0.0163	0.6538	0.4812	0.399	0.3719	0.2318	1.108	2.261
	5	-4.433	0.2091	0.8443	0.0932	1.696	5.867	41.40	595.8	1.729	4.119	40.76
Long base	Pkg#1	-2.884	0.323	0.766	0.167	7.499	101.06	2448	1106580	6.696	3.391	25.74
composites	Pkg#2	-3.325	0.283	0.885	0.123	5.321	61.976	1580.1	88176.6	5.802	4.567	52.31
	Pkg#3	-3.564	0.013	0.421	0.012	3.094	11.428	50.391	265.3	1.362	1.406	3.710

NOTE: Y denotes model value of ε. $\Gamma_{1y} = \overline{Y'^3}/\left[\overline{Y'^2}\right]^{3/2}$. $\Gamma_{2y} = \left\{\overline{Y'^4}/\left[\overline{Y'^2}\right]^2\right\} - 3$.

and ranges between 0 and 1000. This measure has no significance in any formal statistical theory. It was selected for its ease of computation and empirical utility. Distributions for which GF $\geq$ 850 appear to be "good" fits in the author's judgment, while those for which GF < 850 do not. The reader may wish to establish some different value for the selection of fits that are to be called "good." Whatever level is used, the closer the value of GF to 1000, the more closely the data obey the lognormal model. Actual distributions with the corresponding model values and GF's are shown in figure 1. An attempt to improve upon the objective technique for finding μ and σ described above was made by trying a subjective fit to the data as shown in the figure. The result was counterproductive with f_4 decreasing from 0.6053 to 0.4106.

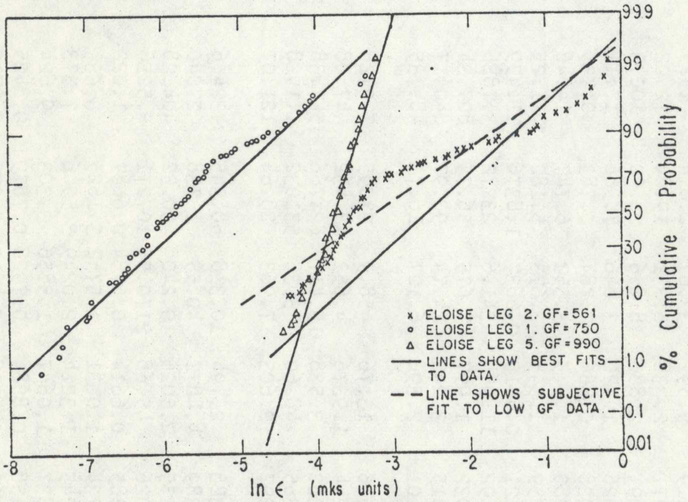

Figure 1. Typical probability distributions of log ε for a range of values of μ and σ, and a trial subjective distribution.

A summary of the data, model values, GF's, and the environments for each run is presented in table 3.

4. RESULTS-UTILITY OF THE LOGNORMAL MODEL AND CORRELATION OF THE DISTRIBUTION OF ε WITH ENVIRONMENTAL PARAMETERS

An examination of the values of GF in table 3 and a comparison of the moments in tables 1 and 2 demonstrate that the lognormal model is a good approximation to the distribution of ε in the atmosphere in hurricanes for wind speeds below 30 m s^{-1}. The mean value of GF for the 35 runs is 900. Since GF is in essence the product of the squares of four kinds of correlation coefficients, the geometric mean equivalent correlation coefficient to the lognormal model is 0.987. The ratios of the observed moments to the model moments are near unity and the standard deviations of μ and σ are small compared with their means. See table 4.

Table 3. Summary of environmental parameters and results for all runs used in the study

Flight	Leg	Z m	U m/s	R km	Notes	X	N	$10^2\ \bar{\varepsilon}$ m^2s^{-3}	μ	σ	GF
750917A Eloise	PBL 1	1200	14	120±10	Parallel to wind	-1	44	7.5	-2.62	0.244	985
	PBL 2	900	18	120±10	Parallel to wind	-1	45	1.9	-4.00	0.267	987
	PBL 3	650	21±1	120±10	Parallel to wind	-1	46	5.1	-3.00	0.232	993
	PBL 4	550	21±1	120±10	Parallel to wind	-1	44	1.6	-4.20	0.362	982
	PBL 5	350	21±1	120±10	Parallel to wind	-1	51	5.6	-2.92	0.272	987
	PBL 6	150	21±1	120±10	Parallel to wind	-1	46	2.3	-3.83	0.383	990
	PBL 7	100	21±1	120±10	Parallel to wind	-1	46	2.9	-3.58	0.248	991
	X1	350	21±1	120±10	Crosswind	1	21	1.9	-3.97	0.191	998
	X2	350	21±1	120±10	Crosswind	1	21	0.87	-4.77	0.211	998
	X3	350	21±1	120±10	Crosswind	1	23	1.8	-4.06	0.251	991
	Xcombo	350	21±1	120±10	Crosswind	1	65	1.5	-4.23	-.337	965
	1	250	<10	≤30	"eye"	0	133	0.5	-5.63	0.903	750
	2	250	20-25	240±20	Parallel	-1	118	11.0	-2.56	0.933	561
	3	250	20-25	240±20	Parallel	-1	96	2.0	-4.24	0.809	912
	4	150	20-25	280±10	Crosswind	1	185	3.1	-3.60	0.582	898
	5	150	20-25	280±10	Crosswind	1	136	2.4	-3.76	0.255	990
750929A Gladys	1	150	20-25	180±20	Crosswind	1	170	12.0	-2.43	0.941	628
	2	150	23±2	140±20	Crosswind	1	163	1.9	-4.03	0.441	967
	3	150	27±3	90±20	Crosswind	1	142	3.5	-3.45	0.457	937
	4	300	30-25	60±5	Parallel	-1	81	46.0	-0.977	0.715	645
	5	300	25	100±10	Parallel	-1	162	2.6	-3.73	0.402	956
750830A Caroline	1	300	25	80		0	122	2.7	-3.72	0.528	928
	2	300	20	80		0	129	4.5	-3.45	0.837	920
	3	300	20	90-95		0	132	2.5	-3.88	0.624	940
	4	3150	25	60-80	Outbound radial	1	59	46.0	-0.803	0.198	1000
	PBL 3	500	20±2	130±10	Parallel	-1	41	0.64	-5.16	0.528	881
	PBL 4	400	20±2	130±10	Parallel	-1	40	1.00	-4.69	0.560	880
	PBL 5	300	20±2	130±10	Parallel	-1	58	1.3	-4.40	0.383	993
	PBL 6	150	20±2	130±10	Parallel	-1	31	1.00	-4.71	0.579	833
	PBL 7	300	20±2	130±10	Crosswind	1	40	0.85	-4.79	0.271	941
	PBL 8	300	20±2	130±10	Crosswind	1	40	0.65	-5.09	0.344	977
	5	300	20	135-150	Outbound radial	1	59	1.6	-4.43	0.844	774
Long base composites	Pkg#1	250	20-25	240±20	Crosswind	1	6	7	-2.88	0.766	626
	Pkg#2	150	20-25	50-200	Crosswind	1	23	5	-3.33	0.885	619
	Pkg#3	300	20-25	90±10	Parallel to Wind	-1	16	3.1	-3.56	0.421	982

NOTE:

Z is aircraft altitude. U is mean wind speed. R is distance from storm center. X is 1 for crosswind runs, -1 for runs parallel to the wind, otherwise 0. N is number of samples in the record. $\bar{\varepsilon}$ is the mean dissipation. μ, σ, and GF are the mean, standard deviation and goodness of fit of log ε to the normal distribution.

Table 4. Statistics of ratios of observed to model moments and of model parameters for the complete data set

Quantity	Mean for 35 Runs	Standard deviation for 35 runs
1st moment ratio	0.965	0.059
2nd moment ratio	1.017	0.042
3rd moment ratio	1.029	0.040
4th moment ratio	0.889	0.144
Std. dev. μ/mean μ	0.030	0.058
Std. dev. σ/mean σ	0.075	0.058

It is not surprising that the fourth moment is least well represented. The higher moments are the hardest to measure because they are most sensitive to the few extreme values which occur in a highly skewed distribution and which may easily be missed during sampling. This appears much more dramatically when central moments are examined. Figure 2 shows that the skewness coefficient of excess Γ_2 predicted by the model is several times larger than that computed from the observations. This behavior of Γ_2 was also observed by Stewart, Wilson, and Burling (1970), although Gibson, Stegen, and Williams (1970) found much closer agreement between theory and observation.

When the possible sampling problems are considered, the lognormal model seems to be a good descriptive representation of the distribution of ε in hurricanes, even though it may not predict higher moments well enough to be used in sophisticated theoretical analyses in which the higher moments play a significant role. The value of σ computed from the long base composites is not radically different from the other runs. Since this represents a change by a factor of 33 in the value of L/ℓ (a change of 3.5 in log L/ℓ), it supports the conclusion that (1) does not apply outside of the region of scales for which it was specified by Kolmogorov (1962).

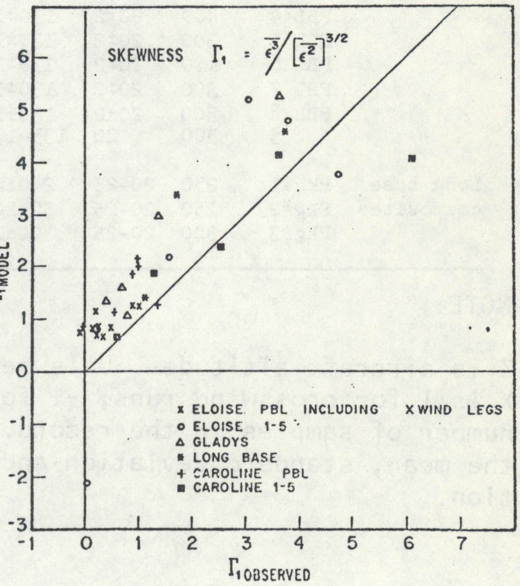

Figure 2. Model values of the skewness coefficient Γ_1 as a function of the observed values.

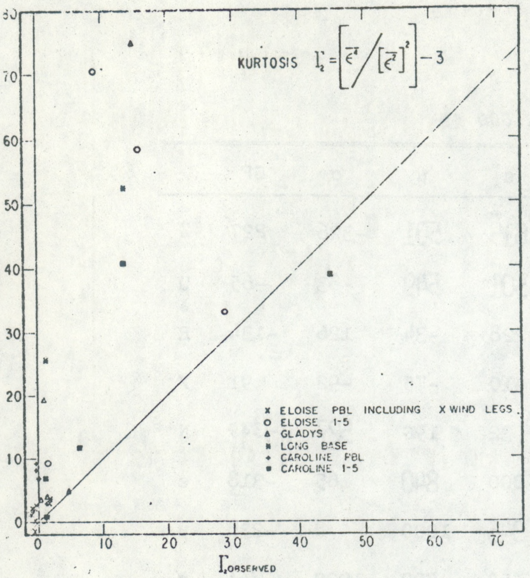

Figure 3. Model values of the coefficient of excess Γ_2 as a function of the observed values.

As mentioned in the introduction, it was hoped that we might be able to at least roughly predict $\overline{\varepsilon}$ from the wind speed, altitude, and distance from the storm center. If $\overline{\varepsilon}$ could not be predicted, it was hoped that perhaps μ and σ could be. To determine whether $\overline{\varepsilon}$, μ, or σ were correlated with environmental variables, cross correlation coefficients r_{xy} were computed for two sets of data. The first set included all runs except the long-base composites. The second set excluded all composites, all runs for which GF < 850 (equivalent mean correlation coefficient to lognormal distribution < 0.98), and the runs taken at altitudes above 650 m where the data sample was too small to guarantee representativeness. The results are presented in table 5.

Given (4), it is comforting that $\overline{\varepsilon}$ and μ are correlated, and disappointing that $\overline{\varepsilon}$ and σ are not, although σ is small enough that the result is not surprising. The correlation between $\overline{\varepsilon}$ and μ has no useful significance, though. Somewhat more interesting is the negative correlation between GF and σ. This tends to confirm the failure of the model at the higher moments since they are more dependent on σ than is the first moment and the divergence between the model and observation becomes greater as σ becomes larger. Most interesting is that the only statistically significant correlation remaining which appears in both sets of data is that between U and μ. The dissipation rate does not appear to be a function of altitude (below 650 m) or position, although a wide range of both was sampled. The dependence on altitude indicated when the runs above 650 m are included may be accidental, since only three legs at Z > 650 were taken, and the leg at Z = 3150 m was taken from a region with unusual spectral properties that were not representative of the bulk of the storm (Merceret, 1976a). The value of ε is weakly dependent on wind speed in the expected manner, the correlation being positive. The dependence is termed "weak" because with $r_{\mu U} = 0.5$ only 25% of the variance of μ can be explained by its dependence on U. Even this weak dependence is not physically significant, since the observations are so strongly clustered in the narrow range $20 \text{ m s}^{-1} \leq U \leq 30 \text{ m s}^{-1}$. The remaining significant correlations appear to be accidental and appear in only one data set.

If "typical" values of μ and σ are desired for order of magnitude estimates of $\overline{\varepsilon}$ and its variation, these values are best determined from a scatter diagram, because μ and σ seem to be completely independent of each other and of the environment, except as noted above. This is presented in figure 4. Fortunately, the range of variation is small enough that the designated "typical" values $\mu = -4.0$ and $\sigma = 0.4$ will probably be satisfactory for most purposes.

*Table 5. Cross Correlation Coefficient Matrix r_{xy} X1000 ***

	Z	U	R	X	N	ε	μ	σ	GF	
Z	1000	27	-261	44	-240	**615**	**501**	-326	227	Z
U	-448	1000	96	65	138	**501**	**549**	-83	-65	U
R	-345	155	1000	111	267	-228	-34	126	-134	R
X	-362	220	216	1000	226	-10	-75	-92	91	X
N	*-526*	**799**	*488*	213	1000	32	190	**527**	*-349*	N
ε	11	361	20	-229	259	1000	**840**	65	-318	ε
μ	-147	*466*	63	-155	355	**946**	1000	8	-238	μ
σ	-156	284	407	-273	430	-159	-200	1000	**-831**	σ
GF	-12	-135	-286	124	-299	235	335	**-797**	1000	GF
	Z	U	R	X	N	ε	μ	σ	GF	
Mean (top)	428	21.1	131	-0.63	80.9	5.65	-3.77	0.473	912	
Std dev.(top)	537	4.18	56.0	0.933	49.9	10.8	1.05	0.237	115	
Mean (bot)	305	21.7	144	-0.053	73.7	2.21	-4.07	0.378	961	
Std.dev.(bot)	145	1.84	54.6	0.999	53.2	1.35	0.623	0.154	39.7	
Units	m	$m\ s^{-1}$	Km	___	___	___	___	___		

** Data above the diagonal (top) were computed using all runs except long-base composites (set 1). Below the diagonal (bot) are results obtained using only runs for which $Z \leq 650$ m, $GF \geq 850$, crosswind factor (X) $\neq 0$ and where all composites were excluded (set 2). Top data have 30 degrees of freedom and the bottom data have 17. The 5% levels of significance (*italics*) are respectively $r_{xy} = \pm 0.349$ and ±0.456; the 1% levels (**BOLDFACE**) are ±0.449 and ±0.575 (Snedecor, 1946, p. 146). Example: $r_{\mu u}$ (set 1) = 0.549 (1% significance), $r_{\mu u}$ (set 2) = 0.466 (5% significance).

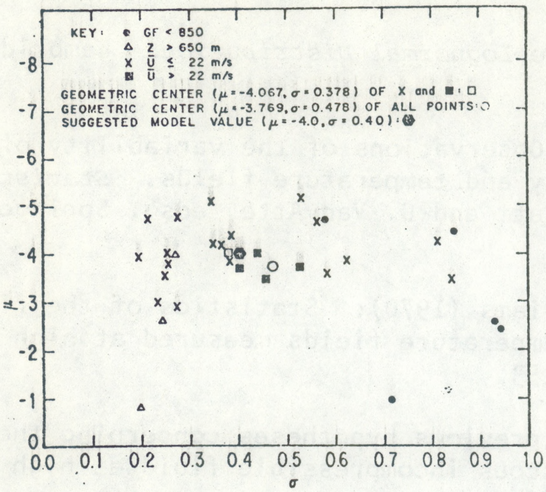

Figure 4. Scatter diagram of μ versus σ for the observations with suggested values of μ and σ for use in numerical models.

5. CONCLUSION

While it is disappointing to find that we cannot expect to predict $\overline{\overline{\varepsilon}}$ with reasonable accuracy from the bulk meteorological variables, it is comforting to know that enough order is evident in the flow to allow us to use a convenient statistical model as a working approximation for describing the distribution of ε as an independent random variable. Given a much larger data base with a wider range of U, eventually it may be possible to determine the significance of the weak correlation between U and μ, and the variation, if any, with altitude above the boundary layer. Some form of statistical prediction may become feasible.

6. ACKNOWLEDGMENTS

The author thanks Drs. Lloyd Shapiro and Stanley Rosenthal for reading earlier drafts of this paper and for their helpful criticism.

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