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DEPARTMENT OF COMMERCE
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PROPAGATION OF AN ELLIPTICAL LASER BEAM THROUGH THE TURBULENT ATMOSPHERE (VERTICAL BEAMS)

ABSTRACT

We calculate the effect of a turbulence gradient on the mean irradiance profile of an elliptical beam. We conclude that in a turbulent atmosphere the peak irradiance can be shifted to a point further from the ground. Also, the vertical symmetry of the beam can be perturbed because of greater horizontal beam spreading at lower heights where turbulence is stronger. However, this effect is significant only for highly elliptical beams, and only when the horizontal beam divergence is determined by turbulence.

1. INTRODUCTION

The qualitative effects of atmospheric turbulence on the propagation of a laser beam can be described quite simply. Larger eddies (i.e., those larger than the beam diameter) cause the beam to wander in a random fashion, while smaller eddies cause it to break up and spread. Taken together, these two effects cause the mean irradiance near the center of the beam to be reduced. At the same time, the mean irradiance near the beam edge is enhanced as energy is scattered out from the center.

A quantitative description of this effect is more difficult. It depends on the beam geometry as well as the details of the turbulence. In this chapter, we derive an expression for the mean irradiance of an elliptical Gaussian laser beam after propagation through turbulence whose strength has an arbitrary power-law dependence on height. Over flat, but not necessarily level, terrain, the expression requires a single numerical integral. Rough terrain can be accommodated by a second numerical integral.

2. ANALYSIS

We use the extended Huygens-Fresnel principle (Lutomirski and Yura, 1971), to describe the effects of atmospheric turbulence on a propagating optical wave. The scalar optical field U at a transverse point $\vec{\varrho}$ along a slant path of length z, as shown in Fig. 1, is given by

$$U(\vec{\varrho}, z) = -\frac{ike^{ikz}}{2\pi z}\int d^2\vec{\varrho}_s U_0(\vec{\varrho}_s) e^{ik(\vec{\varrho}-\vec{\varrho}_s)^2/(2z)} e^{\psi(\vec{\varrho}_s,\vec{\varrho},z)} , \qquad (1)$$

where $k = 2\pi/\lambda$, λ is the optical wavelength, $U_0(\vec{\varrho}_s)$ is the source field at a transverse point $\vec{\varrho}_s$ in the transmitting plane, and $\psi(\vec{\varrho}_s, \vec{\varrho}, z) = \chi(\vec{\varrho}_s, \vec{\varrho}, z) + iS(\vec{\varrho}_s, \vec{\varrho}, z)$ is the sum of the log-amplitude and phase perturbations (Tatarskii, 1971), suffered by a spherical wave field

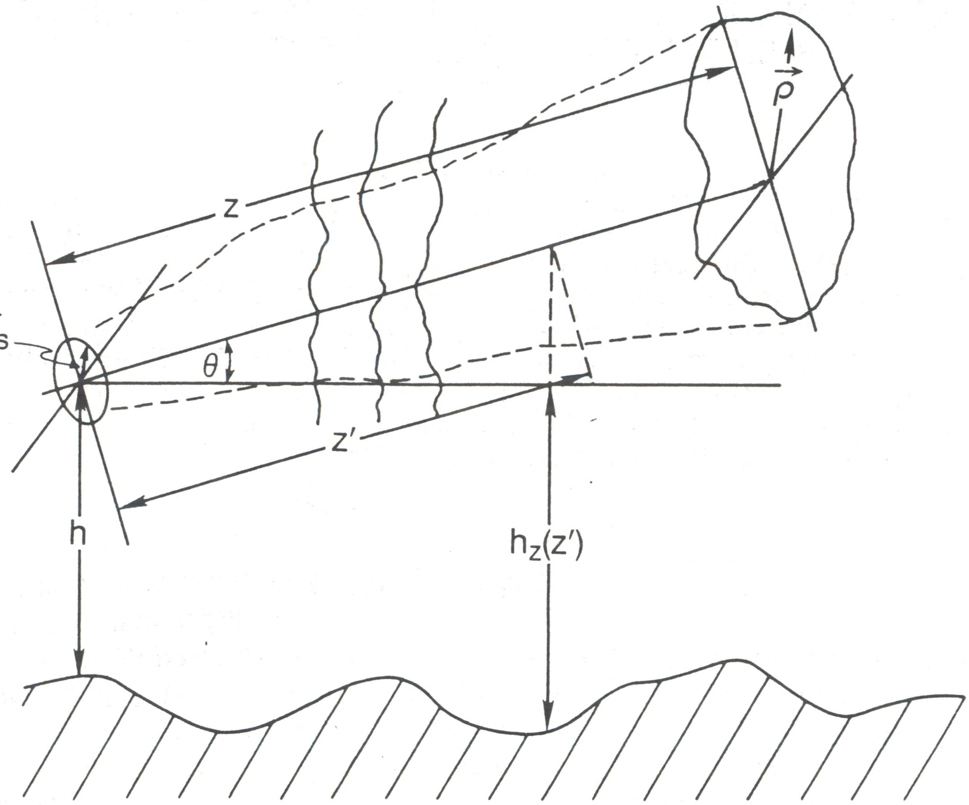

Figure 1. Propagation geometry for a slant path of length z at an angle θ relative to the horizontal over irregular terrain described by a height profile $h_z(z')$.

emitted at $(\vec{\varrho}_s, 0)$ and observed at $(\vec{\varrho}\,, z)$. The mean intensity of the optical field is given by

$$< I(\vec{\varrho}, z) > \;=\; \left(\frac{k}{2\pi z}\right)^2 \int\int d^2\vec{\varrho}_s d^2\vec{\varrho}_s{}' U_0(\vec{\varrho}_s) U_0^*(\vec{\varrho}_s{}') e^{ik(\varrho_s^2 - \varrho_s{}'^2)/(2z)}$$

$$\times\; e^{-ik\vec{\varrho}\cdot(\vec{\varrho}_s - \vec{\varrho}_s{}')/z} < e^{\psi(\vec{\varrho}_s, \vec{\varrho}, z) + \psi^*(\vec{\varrho}_s{}', \vec{\varrho}, z)} > \;, \qquad (2)$$

where the angle brackets describe an ensemble average, and the asterisk identifies a complex conjugate. The quantity in angle brackets on the right hand side of Eq. (2) is the two-point spherical-wave mutual coherence function. It can be evaluated by assuming the ψ terms are jointly Gaussian random variables and the turbulence is homogeneous, which leads to (Lee et al., 1977)

$$< \cdot \cdot \cdot > = e^{-\frac{1}{2} D_\psi (\vec{\varrho}_s - \vec{\varrho}_s{}', z)} \;, \qquad (3)$$

where D_ψ is the wave structure function defined by (Yura, 1972)

$$D_\psi(\vec{\varrho}_s - \vec{\varrho}_s{}', z) = 4\pi k^2 \int_0^z dz' \int d^2\vec{K}\Phi_n(\vec{K}, z')$$
$$\times \{1 - e^{i[1-(z'/z)]\vec{K}\cdot(\vec{\varrho}_s - \vec{\varrho}_s{}')}\} . \tag{4}$$

The first integral in this expression describes an integration along the propagation path z', and the second an integration over the transverse wavenumbers $\vec{K}$ of the intervening refractive index field described by the three-dimensional, path-dependent refractive index spectrum $\Phi_n(\vec{K}, z')$. For uniform isotropic turbulence and a Kolmogorov refractive index spectrum (Tatarskii, 1971),

$$\Phi_n(\vec{K}, z') = \Phi_n(K) = 0.033 C_n^2 K^{-11/3} \quad , \quad 2\pi/L_0 << K << 2\pi/\ell_0 , \tag{5}$$

where ℓ_o and L_o are the inner and outer scales of turbulence, respectively, and C_n^2 is the refractive index structure parameter, Eq. (4) reduces to

$$D_\psi(\vec{\varrho}_s - \vec{\varrho}_s{}', z) = 2\left(\frac{|\vec{\varrho}_s - \vec{\varrho}_s{}'|}{\varrho_0}\right)^{5/3} \quad , \quad \ell_0 << |\vec{\varrho}_s - \vec{\varrho}_s{}'| << L_0 , \tag{6}$$

where $\varrho_0 = (0.545 k^2 z C_n^2)^{-3/5}$ is the spherical wave coherence length (Clifford and Lataitis, 1985). The parameter ϱ_o describes the transverse distance over which a spherical wave field decorrelates due to turbulence. Equations (2), (3), and (6) can be used to calculate the irradiance profile of an optical beam after propagation through uniform, homogeneous, isotropic turbulence.

The assumption of uniform turbulence may not be reasonable for wider beams and longer paths. Variations of C_n^2 transverse to the beam and along the path can be accounted for by a more careful definition of the wave structure function. In general, the second-order wave structure function is defined by

$$D_\psi(\vec{\varrho}_s, \vec{\varrho}_s{}', \vec{\varrho}, \vec{\varrho}\,', z) \equiv < |\psi(\vec{\varrho}_s, \vec{\varrho}, z) - \psi(\vec{\varrho}_s{}', \vec{\varrho}\,', z)|^2 > \quad , \tag{7}$$

where $\psi(\vec{\varrho}_s, \vec{\varrho}, z)$ and $\psi(\vec{\varrho}_s{}', \vec{\varrho}\,', z)$ are the complex phase perturbations experienced by a spherical wave propagating along the two ray paths shown in Fig. 2. For uniform, homo–

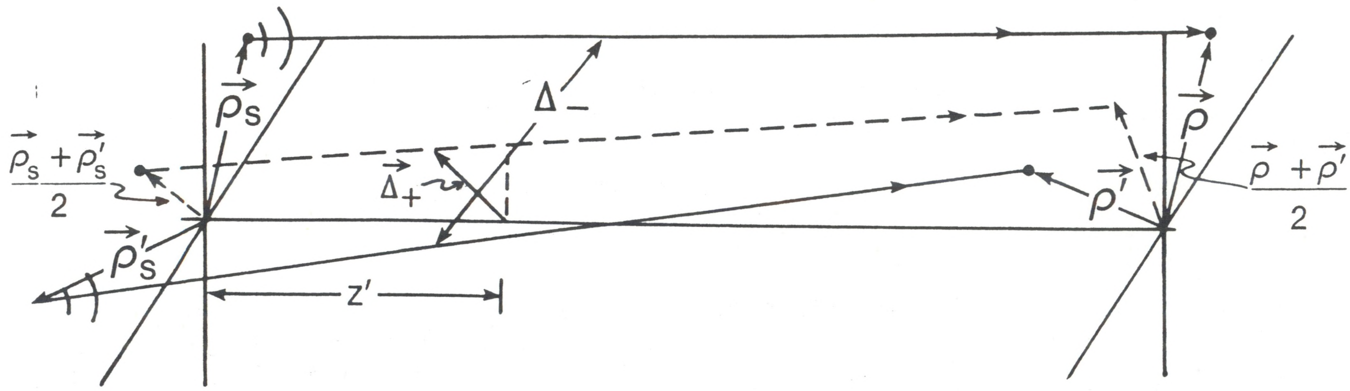

Figure 2. Propagation geometry for the two-source, two-receiver wave structure function calculation. The parameter Δ_- describes the ray separation and Δ_+ the transverse location of the mean ray path (dashed line) at a path position z'.

geneous, isotropic turbulence, Eqs. (4) and (5) can be used to express the wave structure function as

$$D_\psi(\vec{\varrho}_s - \vec{\varrho}_s{}', \vec{\varrho} - \vec{\varrho}\,', z) = \int_0^z dz' d_\psi(\Delta_-) \ , \tag{8a}$$

where $\Delta_- = |(1 - (z'/z))(\vec{\varrho}_s - \vec{\varrho}_s{}') + (\vec{\varrho} - \vec{\varrho}\,')z'/z|$ is the separation of the two ray paths at z' and

$$d_\psi(\Delta_-) = 8\pi^2(0.033)k^2C_n^2 \int_0^\infty dK\, K^{-8/3}[1 - J_0(K\Delta_-)] \tag{8b}$$

describes the local contribution (Flatte, 1979), to D_ψ at a range z'. Here J_o is the zero order Besel function of the first kind. Equation (8b) can be extended to include non-uniform turbulence by allowing C_n^2 in d_ψ to vary with the mean position of the two rays (Tatarskii, 1971), described by the center-of-mass coordinate

$$\vec{\Delta}_+ = \frac{\left(1 - \frac{z'}{z}\right)(\vec{\varrho}_s + \vec{\varrho}_s{}')}{2} + \frac{\frac{z'}{z}(\vec{\varrho} + \vec{\varrho}\,')}{2} \ ,$$

and with z'. Equation (8a) need only be modified by replacing $d_\psi(\Delta_-)$ with

$$d_{\psi'}(\vec{\Delta}_+, \Delta_-, z') = 8\pi^2(0.033)k^2 C_n^2(\vec{\Delta}_+, z') \int_0^\infty dK\, K^{-8/3}[1 - J_0(K\Delta_-)] \ . \tag{9}$$

The integration over K in Eq. (9) can be easily evaluated to give, in analogy to Eq. (6),

$$D_\psi\left(\frac{\vec{\varrho}_s + \vec{\varrho}_s{}'}{2}, \vec{\varrho}_s - \vec{\varrho}_s{}', \vec{\varrho}, z\right) = 2\left[0.545k^2 z|\vec{\varrho}_s - \vec{\varrho}_s{}'|^{5/3}\alpha\left(\frac{\vec{\varrho}_s + \vec{\varrho}_s{}'}{2}, \vec{\varrho}\right)\right], \tag{10a}$$

where

$$\alpha\left(\frac{\vec{\varrho}_s + \vec{\varrho}_s{}'}{2}, \vec{\varrho}\right) = \frac{8}{3}\int_0^1 du(1 - u)^{5/3} C_n^2(\vec{\Delta}_+, u) \ , \tag{10b}$$

$u = z'/z$ is the normalized path position with $u = 0$ describing the location of the transmitter and $u = 1$ the receiver, and $\vec{\Delta}_+ = (1 - u)(\vec{\varrho}_s + \vec{\varrho}_s{}')/2 + u\vec{\varrho}$. Equation (6) is recovered if C_n^2 is constant. Equations (2), (3), and (10) represent a general formulation for the irradiance profile of an optical beam after propagating through nonuniform turbulence.

To proceed further, we need to specify the functional form of C_n^2 in Eq. (10b). We do this by decomposing the argument $\vec{\Delta}_+$ of C_n^2 into its horizontal Δ_{+x} and vertical Δ_{+y} components and considering the following separable form for C_n^2:

$$C_n^2(\vec{\Delta}_+, u) = C_{n0}^2 f_1(\Delta_{+y}, u) f_2(\Delta_{+x}, u) \ , \tag{11}$$

where C_{n0}^2 is some reference C_n^2 value and the functions f_2 and f_1 describe the variation of C_n^2 in the horizontal and vertical directions, respectively, at a path position u. We assume, for now, that the propagation axis is horizontal (i.e., $\theta = 0$). The change in C_n^2 with altitude a in the surface layer over flat ground is best described by a power law; –2/3 at night and –4/3 during the day (Clifford, 1978). In mountainous regions, a –1/3 dependence may be more nearly correct day or night (Belen'kiy et al., 1986). For the sake of generality, we leave the particular power law dependence n unspecified and let $f_1(\mathrm{a}, u) = (\mathrm{a}/h)^{-n}$; therefore $f_1(\Delta_{+y}, u) = \{[h_z(u) + \Delta_{+y}]/h\}^{-n}$, where h is the height of the source above the ground, and $h_z(u)$ describes the height profile along the propagation axis as a function of the normalized path position u. We assume that $f_2(0,0) = 1$, so C_{n0}^2 is the on-axis C_n^2 value at the transmitter. The corresponding variation in C_n^2 with the horizontal coordinate is then described by $f_2(x, u) = [h_x(x, u)/h_x(0, u)]^{-n}$, and therefore

$$f_2(\Delta_{+x}, u) = [h_x(\Delta_{+x}, u)/h_x(0, u)]^{-n} \ ,$$

where $h_z(x, u)$ describes the terrain profile transverse to the propagation axis at u and $h_z(0,u) = h_z(u)$. The effect of nonuniform heating of the ground due to irregular ground cover or shading, which produces different initial C_n^2 values at the ground, can be accounted for by multiplying the right side of Eq. (11) by an additional factor $g(x,u) = g(\Delta_{+x},u)$. This function can be used to scale the C_n^2 values at the ground relative to the C_n^2 value just below the transmitter. We note that $g(0,0) = 1$. To generalize this discussion to a slant path at an angle θ relative to the horizontal, we need only use the rotational coordinate transformation

$$\Delta_{+x} \rightarrow \Delta_{+x},\ \Delta_{+y} \rightarrow \Delta_{+y} \cos\theta + zu \sin\theta, \text{ and } z' = zu \rightarrow -\Delta_{+y} \tan\theta + zu$$

with the terrain profile defined as in Fig. 1.

We consider the specific case of a slant path over uniformly heated flat ground [i.e., $f_2(\Delta_{+x}, u) = g(\Delta_{+x}, u) = 1$] for which

$$C_n^2(\vec{\Delta}_+, u) = C_{n0}^2\left(1 + \frac{\Delta_{+y}\cos\theta + zu\sin\theta}{h}\right)^{-n} . \tag{12}$$

Substituting this result into Eqs. (10), Eqs. (10) into Eq. (3), and Eq. (3) into Eq. (2), we obtain

$$I(x, y, z) = \left(\frac{k}{2\pi z}\right)^2 \iiiint_{-\infty}^{\infty} dx_s\, dy_s\, dx_s'\, dy_s'\; U_0(x_s, y_s)U_0^*(x_s', y_s')$$

$$\times\, e^{ik(x_s^2 - x_s'^2)/(2z)}\, e^{ik(y_s^2 - y_s'^2)/(2z)}$$

$$\times\, e^{-ikx(x_s - x_s')/z}\, e^{-iky(y_s - y_s')/z}$$

$$\times \exp\left(-\frac{\sqrt{(x_s - x_s')^2 + (y_s - y_s')^2}}{\varrho_0}\right)^{5/3} a_0\left(\frac{y_s + y_s'}{2}, y\right), \tag{13a}$$

where we have expressed the vectors $\vec{\varrho}_s$ and $\vec{\varrho}_s{}'$ in terms of their (horizontal, vertical) components (x_s, y_s) and (x_s', y_s'), respectively, and

$$a_0\left(\frac{y_s + y_s'}{2}, y\right) = \frac{8}{3}\int_0^1 du(1 - u)^{5/3}\left\{1 + \frac{\left[(1 - u)\dfrac{y_s + y_s'}{2} + uy\right]\cos\theta + zu\sin\theta}{h}\right\}^{-n} . \tag{13b}$$

The coherence length ϱ_o in Eq. (13a) corresponds to the C_n^2 level at the transmitter. We note that if $n = 0$ (i.e., if C_n^2 is constant) $a_0 = 1$. Equation (13b) can be evaluated in terms of special functions (Gradshteyn and Ryzhik, 1980), if we write it in the more suggestive form

$$a_0\left(\frac{y_s + y_s'}{2}, y\right) = \frac{8}{3}\int_0^1 du(1-u)^{5/3}(\gamma + \delta u)^{-n} \tag{14a}$$

$$= \frac{8}{3}\gamma^{-n} B\left(\frac{8}{3}, 1\right) {}_2F_1\left(n, 1; \frac{11}{3}; -\frac{\delta}{\gamma}\right) \tag{14b}$$

$$= \gamma^{-n} {}_2F_1\left(n, 1; \frac{11}{3}; -\frac{\delta}{\gamma}\right), \tag{14c}$$

where $\gamma = 1 + (y_s + y_s')\cos\theta/(2h)$, $\delta = h^{-1}\{(z \sin\theta + [2y - (y_s + y_s')]\cos\theta/2\}$, B is the beta function, and ${}_2F_1$ is the hypergeometric function.

To evaluate Eq. (13a) we also need to specify the source field distribution $U_0(x_s, y_s)$. The actual distribution will have a truncated Gaussian form

$$U_0(x_s, y_s) = \begin{cases} U_0 \exp\left[-\frac{x_s^2}{2}\left(\frac{1}{d^2} + \frac{ik}{F_x}\right)\right]\exp\left[-\frac{y_s^2}{2}\left(\frac{1}{d^2} + \frac{ik}{F_y}\right)\right], & \sqrt{x_s^2 + y_s^2} \le \frac{D}{2} \quad (15a) \\ 0, & \sqrt{x_x^2 + y_s^2} > \frac{D}{2}, \quad (15b) \end{cases}$$

where U_o is the field strength at the center of the aperture, d is the standard deviation of the Gaussian amplitude distribution, F_x and F_y are the horizontal and vertical focal lengths respectively, and D is the aperture diameter. This, however, makes the computation numerically intensive, and we replace it with the more tractable Gaussian form with an e^{-1} intensity decay at $\sqrt{x_s^2 + y_s^2} = D/2$:

$$U_0(x_s, y_s) = U_0 \exp\left[-2x_s^2\left(\frac{1}{D^2} + \frac{ik}{4F_x}\right)\right]\exp\left[-2y_s^2\left(\frac{1}{D^2} + \frac{ik}{4F_y}\right)\right]. \tag{16}$$

which, when substituted into Eq. (13a) under the coordinate transformation

$$\xi_x = x_s - x_s', \; 2\eta_x = x_s + x_s', \; \xi_y = y_s - y_s', \; \text{and } 2\eta_y = y_s + y_s'$$

yields

$$
I(x, y, z) = \left(\frac{k}{2\pi z}\right)^2 I_0 \iiiint_{-\infty}^{\infty} d\xi_x\, d\eta_x\, d\xi_y\, d\eta_y
$$

$$
\times\, e^{-(4\eta_x^2+\xi_x^2)/D^2} e^{-(4\eta_y^2+\xi_y^2)/D^2}
$$

$$
\times\, e^{ik\xi_x[\eta_x(1-z/F_x)-x]/z}
$$

$$
\times\, e^{ik\xi_y[\eta_y(1-z/F_y)-y]/z}
$$

$$
\times\, e^{-[(\xi_x^2+\xi_y^2)/\varrho_0^2]^{5/6}\alpha_0(\eta_y, y)} \,. \tag{17}
$$

Equation (17) can be somewhat simplified by evaluating the η_x integral; however, the remaining three integrals remain analytically intractable. To obtain a simpler result, we use the quadratic structure function approximation (Wandzura, 1980), which replaces the 5/3 exponent in Eq. (6) by 2, or equivalently raises the exponent of the last term in Eq. (17) to the 6/5 power. The impact of this approximation is that it implies that the only effect of turbulence is a random tilt of the propagating wave at each path position. This is a reasonable approximation for the mean intensity calculation and allows us to evaluate the ξ_x and ξ_y integrals, resulting in

$$
< I(x, y, z) >= \frac{k^2 D^3 I_0}{8\sqrt{\pi}\, z^2} \int_{-\infty}^{\infty} d\eta_y\, AB\, e^{-4\eta_y^2/D^2}\, e^{-[kDA/(2z)]^2 x^2}
$$

$$
\times\, e^{-\{kDB[\eta_y(1-z/F_y)-y]/(2z)\}^2} \,, \tag{18a}
$$

where

$$
A = A(\eta_y, y) = \left[1 + \frac{k^2 D^4}{16 z^2}\left(1 - \frac{z}{F_x}\right)^2 + \frac{D^2}{\varrho_0^2}\alpha_0^{6/5}\right]^{-1/2} , \tag{18b}
$$

$$
B = B(\eta_y, y) = \left[1 + \frac{D^2}{\varrho_0^2}\alpha_0^{6/5}\right]^{-1/2} , \tag{18c}
$$

and

$$
\alpha_0 = \alpha_0(\eta_y, y) = \left[1 + \frac{\eta_y}{h}\cos\theta\right]^{-n} {}_2F_1\left[n, 1; \frac{11}{3}; -\frac{z\sin\theta + (y - \eta_y)\cos\theta}{(h + \eta_y\cos\theta)}\right] . \tag{18d}
$$

In performing the integral in the x-dimension, turbulence levels were implicitly assumed to be constant along a line perpendicular to the beam and parallel to the ground.

The expressions should still be fairly good if this condition is not satisfied (due to uneven terrain in this direction, for example) as long as the variation across the beam is not too large. In this case, one must decide what value to use in the calculation. Two obvious choices are the value at the center of the beam and the value along the line of sight to the observation point. Because scattering by turbulence tends to produce very small scattering angles, we expect the irradiance at a point to be largely determined by turbulence along the line of sight to that point. We therefore favor using the value along the line of sight for most calculations. In the y-dimension integral, the turbulence may be changing very rapidly, and therefore the constant turbulence assumption was not used.

3. EXAMPLES

To illustrate the magnitude of these effects, a typical laser system was considered under strong turbulence conditions. The laser was assumed to produce 1 Wm^{-2} at the center of the transmitted beam. The wavelength is near the center of the visible region of the spectrum at 500 nm. A 10–cm–diameter transmitter was used. It was located 2 m above the ground and pointed at a 15 mrad elevation angle. A range of 2 km was assumed. (These parameters are meant to be typical of a generic laser system and should not be taken as indicative of any particular system design.) A daytime turbulence profile over flat, level terrain was assumed to provide the steepest gradient of turbulence. At the transmitter, a fairly strong value of C_n^2 of $10^{-12}m^{-2/3}$ was used.

Figure 3 is a vertical profile of the irradiance through the center of the beam for a collimated transmitter. The solid line represents a case with no turbulence. In this case, the beam at 2 km is almost identical to the transmitted beam, with an irradiance of 1 Wm^{-2} at the center and an e^{-1} radius of 5 cm. It is, of course, circularly symmetric. The dashed line in the figure includes the effects of turbulence. For the narrow beam considered here, the effect of turbulence is to spread the beam by the same amount in all directions. The resulting beam width is 21 cm and the on-axis irradiance is 56 m Wm^{-2}.

In the PHI propagation model used by the U.S. Army's Atmospheric Sciences Laboratory, the turbulence-induced beam spread is calculated from

$$\alpha_{TT} = 0.6\lambda/r_0 \ , \tag{19}$$

where the Fried coherence length (Fried, 1967), is given by

$$r_0 = 2.1\left[1.45k^2 \int_0^z dzC_n^2(z)\left(1 - \frac{z'}{z}\right)^{5/3}\right]^{-3/5} , \tag{20}$$

For the irradiance profile of Fig. 3, r_0 is 3.8 mm. Adding the turbulence-induced beam spread to the transmitted beam size produces an e^{-1} beam radius of 21 cm, and the PHI code is seen to work very well for narrow circular beams.

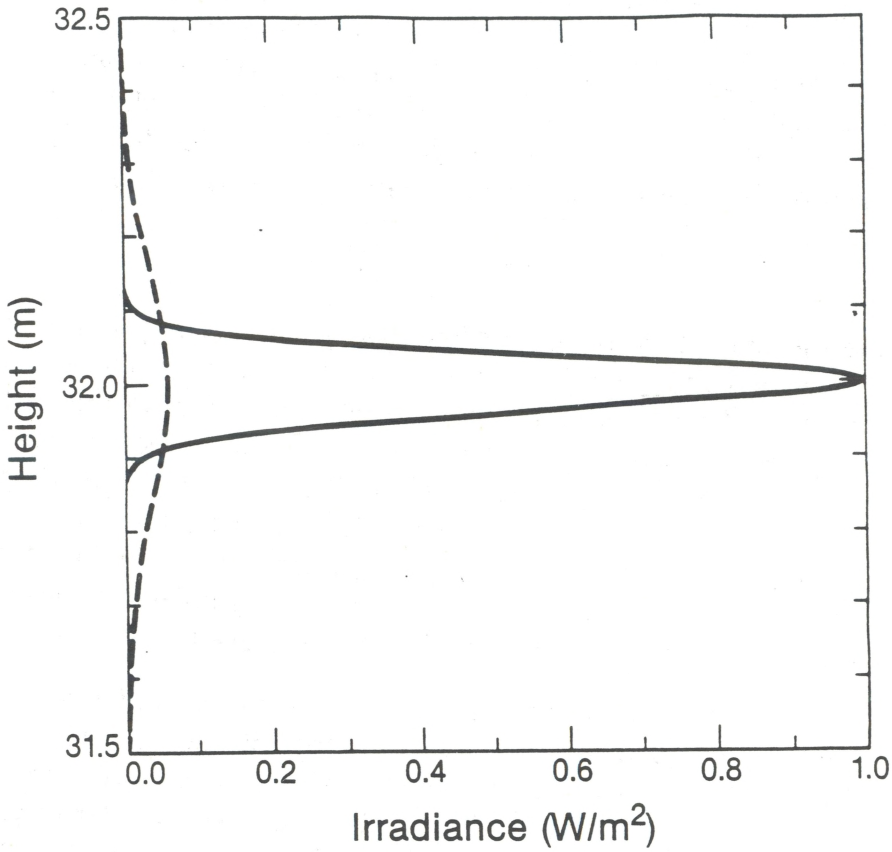

Figure 3. Vertical profiles of irradiance through the center of a collimated beam 2 km from the transmitter for no turbulence (solid line) and strong daytime turbulence (dashed line).

Figure 4 is a plot of the vertical profile of irradiance for a broad fan beam. In this case, the beam was diverged 30 μrad in the vertical and 12 mrad in the horizontal. The turbulence case and the no turbulence case are identical. This seems reasonable since the turbulence-induced beam spread from Eq. (19) is only about 80 μrad.

Figure 5 represents the highly elliptical case of a beam diverged 30 mrad in the vertical, but not at all in the horizontal. With no turbulence, the peak irradiance is 1.7 m Wm^{-2} and the vertical beam width is 30 m. In the presence of turbulence, the peak irradiance is 0.41 mWm^{-2}, and it is shifted to a position 5 m higher than the peak in the no-turbulence case. This shift is due to the greater horizontal spreading of the beam at lower elevation angles where the turbulence is greater. For the same reason, the distance from the peak to the e^{-1} irradiance point is also asymmetric. Below the center of the beam, this distance is 27 m and above the center it is 29 m.

In the case of a narrow circular beam, turbulence spreads the laser energy out in two dimensions, and the irradiance is reduced by the square of the ratio of the spot size without turbulence to the spot size with turbulence. For a diverged beam, this ratio is

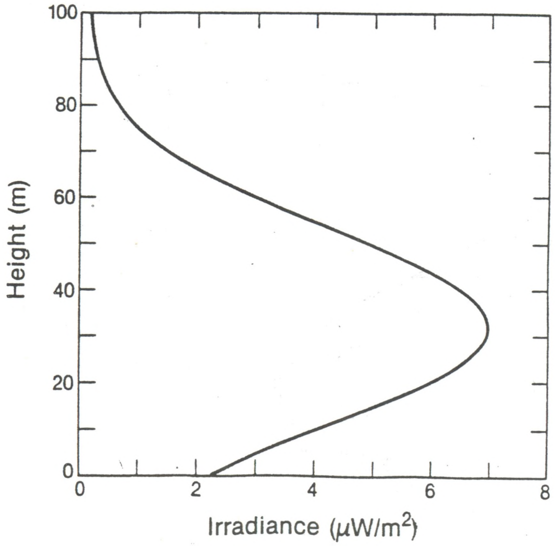

Figure 4. Vertical profile of irradiance through the center of an elliptical beam 2 km from the transmitter. No-turbulence and strong-daytime-turbulence cases are indistinguishable.

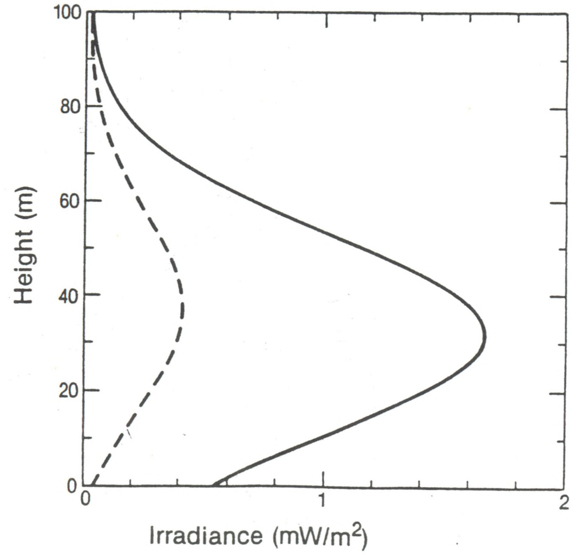

Figure 5. Vertical profiles of irradiance through the center of a highly elliptical beam 2 km from the transmitter for no turbulence (solid line) and strong daytime turbulence (dashed line).

unity, and turbulence has no effect. For a beam that is narrow in horizontal dimension but broad in vertical dimension, turbulence should spread the laser energy out in only one dimension, and the irradiance is reduced by the width of the beam without turbulence divided by the width of the beam in the presence of turbulence. If turbulence is not uniform vertically, that ratio will be a function of height.

Figure 6 is a profile of the highly elliptical beam case with the turbulence effects calculated three ways. The solid line represents the exact theory and is reproduced from the dashed line in Fig. 5. The long dashed line was calculated using a single value of the Fried coherence length. This is the calculation used by the current PHI code. Even though this is an extremely elliptical beam, the single-coherence-length calculation is generally within a factor of 2 of the correct result except very close to the ground. For less elliptical beams, the agreement will be better. The short-dashed line was made using a beam-spread calculation based on a coherence length computed at each height. This results in a very good approximation to the correct values at all heights.

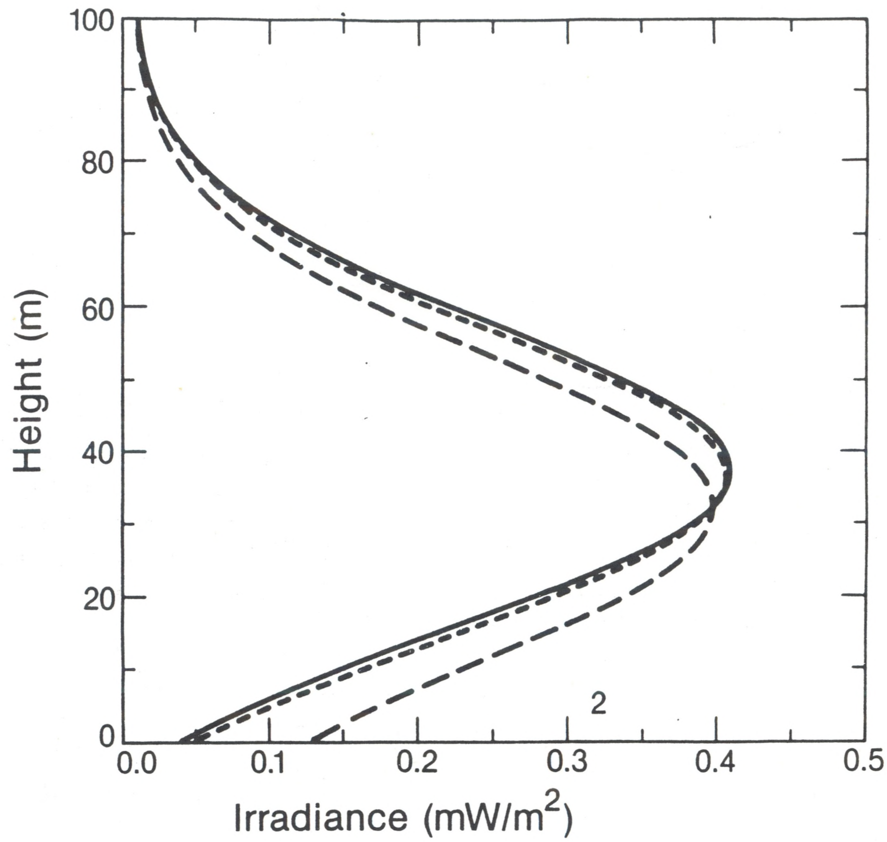

Figure 6. Vertical profile of irradiance through the center of a highly elliptical beam in strong daytime turbulence: exact calculation (solid line), calculation using r_0 at beam center (long-dashed line), and calculation using r_0 at each height (short-dashed line).

4. CONCLUSIONS

For a highly elliptical laser beam, a gradient in turbulence strength can shift the peak irradiance to a greater height in the atmosphere than the peak is in the absence of turbulence. This is due to greater horizontal beam spreading at the lower heights where turbulence is greater. However, this effect is significant only for highly elliptical beams (ellipticities of the order of 100 or greater) and only when the turbulence-induced horizontal beam divergence is greater than the intentional divergence or the diffraction induced divergence if there is no intentional horizontal divergence.

Under these conditions, the effect can be adequately modeled by calculating the Fried coherence length at each height and using that to find the horizontal divergence at each height. The vertical divergence will generally be negligible for such a highly elliptical beam. A more sophisticated model is probably not necessary since the magnitude of the effect is generally small.

Acknowledgments

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