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TURBULENCE EFFECTS ON REFLECTED OPTICAL PATTERNS (ATMOSPHERE AND STRUCTURE)

Richard J. Lataitis
James H. Churnside

Wave Propagation Laboratory
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ABSTRACT

The vacuum irradiance pattern produced by an optical beam illuminating a distant receiving plane is distorted by atmospheric turbulence. The distortion manifests itself as a wander and "breathing" of the beam spot. Reflected optical patterns exhibit the same type of distortion but to a degree that depends on the type of reflector. We describe the reflected irradiance pattern in terms of several length scales of interest. Numerically derived optical patterns for a spherical wave reflected from a plane retroreflector are also presented. Various reflector diameters and turbulence strengths are considered. These results indicate that the small-scale structure in the reflected optical pattern is no longer discernible when $\varrho_0 \leq D_r$, where ϱ_0 is the spherical wave coherence length and D_r is the reflector diameter.

1. INTRODUCTION

The near- and far-field vacuum diffraction patterns of any optical source operating in the atmosphere will be distorted by turbulence. It is difficult to describe the instantaneous distortion of the patterns; however, a qualitative discussion of the different types of distortion, when the distortions occur, and over what time scales is relatively straightforward. The length scales that are important in this problem are the Fresnel zone $\sqrt{\lambda L}$, where λ is the source wavelength and L is the propagation pathlength; the spherical wave coherence length $\varrho_0 = (0.545\, k^2\, L\, C_n^2)^{-3/5}$, where $k = 2\pi/\lambda$ and C_n^2 is the refractive index structure parameter (Fante, 1975); and the diameter D of the source or reflector aperture. We assume that the optical beam is very nearly collimated so that it also has a diameter of D. The one important time scale of interest is then $\tau = D/V$, where V is the wind speed transverse to the propagation path. In all our discussions we assume that the atmospheric turbulence is homogeneous and isotropic, and that it is adequately described by the Kolmogorov spectrum (Tatarskii, 1971)

$$\Phi_n(K) = 0.033\, C_n^2\, K^{-11/3}\ , \tag{1}$$

where K is the spatial wavenumber of the refractive irregularities. We also assume that all the length scales of interest are smaller than the turbulence outer scale L_0 and the larger than the inner scale ℓ_0. We first discuss direct patterns and then turn our attention to reflected patterns.

2. DIRECT PATTERN

Consider the instantaneous spot produced on some distant receiving plane by a laser source of diameter D. By instantaneous we mean the pattern that would be recorded on a

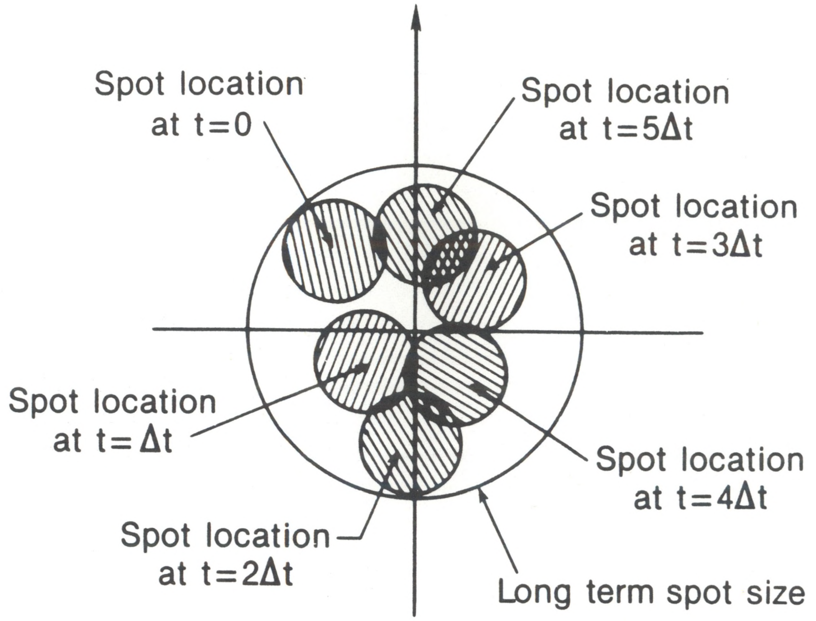

Figure 1. The time history of the wander of a laser beam spot in a receiving plane for propagation through a turbulent medium (from Fante, 1975).

photographic plate using an exposure time much shorter than τ. Eddies larger than the beamwidth tend to deflect the spot from its on-axis position, while those smaller than the beamwidth cause the spot to "breathe," that is, expand and contract as a function of time. This latter effect tends to distort or warp the small-scale structure within the pattern. If we could watch the behavior of the spot it would look something like that shown in Fig. 1. The spot would be continuously deflected over time intervals of order $\Delta t = \tau$ as different eddies were advected through the beam. The details and small-scale structure within the spot would also change over the same time scale. These are some of the questions we want to answer: How distorted is the instantaneous spot? What is the relative amount of spot wander to spot spreading? Can we quantify the degree of distortion?

Let us first consider what happens in very weak turbulence, that is when $\varrho_0 >> D >> \sqrt{\lambda L}$ In this limit, turbulence has essentially no effect and the observed optical pattern is the undistorted vacuum diffraction pattern. As the turbulence strength increases, ϱ_0 decreases and the instantaneous irradiance pattern wanders more and more in the receiving plane. Provided $\varrho_0 > D >> \sqrt{\lambda L}$, the pattern, although it wanders, is still undistorted from its vacuum value. As the turbulence strength continues to increase so that $D > \varrho_0 >> \sqrt{\lambda L}$, the pattern wanders less and begins to breathe, thereby distorting the details in the diffraction pattern. When $D >> \varrho_0 >> \sqrt{\lambda L}$, the pattern no longer wanders appreciably but is increasingly distorted.

Up to this point in our discussion amplitude fluctuations or scintillations have been neglected. As ϱ_0 decreases relative to $\sqrt{\lambda L}$ these become more important. In the limit of very strong turbulence, when $D >> \sqrt{\lambda L} >> \varrho_0$, the optical pattern will have deep intensity fluctuations across it. In this regime the pattern tends to break up into multiple patches. There is no discernable wander of the instantaneous pattern, and it is severely distorted from its vacuum value.

These different regimes are discussed in detail by Fante (1975), and some examples of numerically generated turbulence degraded irradiance patterns are presented by Lutomirski and Yura (1971).

The literature describing beam wander and breathing is always couched in terms of short- and long-term effects. It may be worthwhile at this point to discuss the differences between the two. The effects of turbulence, because they are of an inherently random nature, need to be quantified in terms of ensemble averages. Two parameters, for example, that can be used to quantify beam distortion are the short- and long-term spot size. The short-term spot size is the average (ensemble) diameter of the smaller (instantaneous) spot shown in Fig. 1. The ergodic hypothesis says that this can be obtained from a time series of the instantaneous spot diameter as the spot moves around in the receiving plane. This short-term spot size represents the effect of eddies that are smaller than the beam diameter. The long-term spot size is the diameter of the larger circle in Fig. 1. It is the average (ensemble) beam diameter including beam wander. If we invoke the ergodic hypothesis, it is the diameter of the spot that would be recorded on a photographic plate exposed for a time $t >> \tau$ to the optical pattern. It includes the effect of both large and small eddies. This discussion can easily be extended to include the short- and long-term optical patterns. The short-term optical pattern would be an average of all the instantaneous patterns observed after the displacement has been removed. It would therefore be a smoothed version of the instantaneous pattern. The long-term pattern would be that recorded on a photographic plate for a long ($>> \tau$) exposure time.

3. REFLECTED PATTERN

The general concepts presented in the previous section also apply for reflected beams. The exact details, however, depend on the type of reflector and correlated nature of the turbulence on the outgoing and return path.

The specific case we consider is that of a retroreflector of diameter D_r. The one obvious modification to the previous discussion is that for this case the spot will tend to wander less than for a direct beam provided the retroreflector is large enough. This beam wander suppression is due to the tilt-compensation effect of the retroreflector (Lutomirski and Warren, 1975).

To quantify the effect of turbulence on the reflected optical pattern, we numerically evaluate the long-term reflected irradiance pattern for different turbulence strengths. Although the long-term pattern does not really give us any information about the instantaneous distortion, it should give a rough idea of the impact of turbulence on the reflected pattern.

3.1 Theory

The incident spherical wave field U_i at a transverse point $\vec{\varrho}_r'$ in a reflecting plane a distance L from the source can be expressed as

$$U_i(\vec{\varrho}_r') = U_s \frac{e^{ik[L+\varrho_r'^2/(2L)]}}{L} e^{\vec{\psi}(0,\vec{\varrho}_r')} , \tag{2}$$

where U_s is the complex, spherical-wave, vacuum source amplitude and $k = 2\pi/\lambda$. The parameter $\vec{\psi}(\vec{\varrho}_s, \vec{\varrho}_r')$ is the complex phase perturbation (Tatarskii, 1971) suffered by a spherical wave emitted from a transverse point $\vec{\varrho}_s$ in the source plane as observed at a transverse point $\vec{\varrho}_r'$ in the reflecting plane. The arrow above ψ identifies the direction of propagation; in this case from $\vec{\varrho}_s$ to $\vec{\varrho}_r'$. The reflected field U at a transverse point $\vec{\varrho}$ back in the source plane is given by the extended Huygens-Fresnel integral (Lutomirski and Yura, 1971),

$$U(\vec{\varrho}) = \frac{ik}{2\pi L} e^{ik[L+\varrho^2/(2L)]} \int d^2\vec{\varrho}_r e^{ik\varrho_r^2/(2L)} e^{-ik\vec{\varrho}_r\cdot\vec{\varrho}/L}$$
$$\times\, e^{\overleftarrow{\psi}(\vec{\varrho}_r,\vec{\varrho})} \int d^2\vec{\varrho}_r' R(\vec{\varrho}_r, \vec{\varrho}_r') U_i(\vec{\varrho}_r') , \tag{3}$$

where $\vec{\varrho}_r$ is a transverse vector in the reflecting plane and $R(\vec{\varrho}_r,\vec{\varrho}_r')$ is the reflection coefficient. $R(\vec{\varrho}_r,\vec{\varrho}_r')$ links the incident field at a transverse point $\vec{\varrho}_r'$ on the reflector to the reflected field at a transverse point $\vec{\varrho}_r$ on the reflector. For a retroreflector $R(\vec{\varrho}_r,\vec{\varrho}_r') = A(\vec{\varrho}_r)\, \delta(\vec{\varrho}_r + \vec{\varrho}_r')$, where $A(\vec{\varrho}_r')$ is the complex retroreflector reflectance and δ is the Dirac delta function. Substituting this expression and Eq. (2) into Eq. (3), we have

$$U(\vec{\varrho}) = i\frac{U_s k}{2\pi L^2} e^{ik[2L+\varrho^2/(2L)]} \int d^2\vec{\varrho}_r A(\vec{\varrho}_r) e^{ik\varrho_r^2/L} e^{-ik\vec{\varrho}_r\cdot\vec{\varrho}/L}$$
$$\times\, e^{\vec{\psi}(0,-\vec{\varrho}_r)+\overleftarrow{\psi}(\vec{\varrho}_r,\vec{\varrho})} . \tag{4}$$

The ensemble or long-term mean irradiance pattern is then given by

$$\langle I(\vec{\varrho})\rangle = \langle |U(\vec{\varrho})|^2\rangle$$
$$= \left(\frac{k}{2\pi L^2}\right)^2 I_s \int d^2\vec{\varrho}_r \int d^2\vec{\varrho}_r' A(\vec{\varrho}_r) A^*(\vec{\varrho}_r') e^{ik(\varrho_r^2-\varrho_r'^2)/L} e^{-ik\vec{\varrho}\cdot(\vec{\varrho}_r-\vec{\varrho}_r')/L}$$
$$\times \left\langle e^{\psi(0,-\vec{\varrho}_r)+\psi^*(0,-\vec{\varrho}_r')+\psi(\vec{\varrho},\vec{\varrho}_r)+\psi^*(\vec{\varrho},\vec{\varrho}_r')}\right\rangle , \tag{5}$$

where $I_s = |U_s|^2$, the asterisk denotes a couple conjugate, and we have used reciprocity to set $\tilde{\psi}(\vec{\varrho}_r, \vec{\varrho}) = \tilde{\psi}(\vec{\varrho}, \vec{\varrho}_r) = \psi(\vec{\varrho}, \vec{\varrho}_r)$ (Clifford and Wandzura, 1981).

The quantity in the angle brackets $\langle \cdot \cdot \cdot \rangle$ can be readily evaluated if we assume the ψ terms are jointly Gaussian random variables. If in addition we assume that the phase fluctuations dominate any effects produced by amplitude fluctuations, we have (Lee et al., 1977)

$$\langle \cdot \cdot \cdot \rangle = \exp\left\{-\frac{1}{2}\left[D(0, \vec{\varrho}_r - \vec{\varrho}_r{}') - D(\vec{\varrho}, 2\vec{\varrho}_r) + 2D(\vec{\varrho}, \vec{\varrho}_r + \vec{\varrho}_r{}') - D(\vec{\varrho}, 2\vec{\varrho}_r{}') + D(0, \vec{\varrho}_r{}' - \vec{\varrho}_r)\right]\right\}, \tag{6}$$

where $D(\vec{x}, \vec{y})$ is the two-source spherical-wave structure function. For homogeneous isotropic turbulence and the Kolmogorov refractive index spectrum defined in Eq. (1), $D(\vec{x}, \vec{y})$ can be expressed as

$$D(\vec{x}, \vec{y}) = 2.91 k^2 L\, C_n^2 \int_0^1 du |u\vec{x} + (1-u)\vec{y}|^{5/3}, \tag{7}$$

where $\vec{x}$ and $\vec{y}$ are the separation vectors between the two source and receiver points respectively. Here the integration is over the normalized path position $u = z/L$, where z is the distance from the source. From Eq. (7) it is clear that $D(\vec{x}, \vec{y}) = D(-\vec{x}, -\vec{y})$ so that $D(0, \vec{\varrho}_r{}' - \vec{\varrho}_r)$ in Eq. (6) can be written as $D(0, \vec{\varrho}_r - \vec{\varrho}_r{}') = 2(|\vec{\varrho}_r - \vec{\varrho}_r{}'|/\varrho_0)^{5/3}$. Equation (6) can then be expressed as

$$\langle \cdot \cdot \cdot \rangle = \exp\left\{-2\left(\frac{|\vec{\varrho}_r - \vec{\varrho}_r{}'|}{\varrho_0}\right)^{5/3}\right\} \exp\left\{\frac{1}{2}\left[D(\vec{\varrho}, 2\vec{\varrho}_r) + D(\vec{\varrho}, 2\vec{\varrho}_r{}') - 2D(\vec{\varrho}, \vec{\varrho}_r + \vec{\varrho}_r{}')\right]\right\}. \tag{8}$$

We note that if the second exponential in Eq. (8) is set equal to unity, the result is what would have been obtained had we assumed that the outgoing and reflected waves propagated through independent regions of turbulence. The second exponential therefore describes the effect of the correlated nature of the turbulence on the outward and return paths.

Equation (8) in its present form is relatively complicated because of the dependence of the structure functions in the second exponential on $\vec{\varrho}$, $\vec{\varrho}_r$, and $\vec{\varrho}_r{}'$. What we want to do then is to simplify the second exponential. To this end we consider two limits. The first is $D \ll \varrho_0$. In this limit the dominant effect of the atmosphere is to randomly tilt the propagating wavefront. This phenomenon is accurately described by a quadratic structure function (Clifford and Wandzura, 1981; Wandzura, 1980). The quadratic approximation to the structure function in Eq. (7) for common source points ($\vec{x} = 0$) is

$$D(0,\vec{y}) = 2\left(\frac{|\vec{y}|}{\varrho_0}\right)^{5/3} \approx 2\left(\frac{|\vec{y}|}{\varrho_0}\right)^2 . \tag{9}$$

For separated source points ($\vec{x} \neq 0$), a quadratic structure function is less straightforward to generate. To fabricate such a function we note that Eq. (9) is obtained from

$$D(0,\vec{y}) = 2\left\{\left[\frac{8}{3}\int_0^1 du|(1-u)\vec{y}|^{5/3}\right]/\varrho_0^{5/3}\right\}^{6/5} . \tag{10}$$

To extend this expression to include two source points we need to replace $(1-u)\vec{y}$ in Eq. (10) by $u\vec{x} + (1-u)\vec{y}$. This, however, still leaves us with an intractable integral. To obtain a tractable result we use

$$D(0,\vec{y}) \approx 2\left[3\int_0^1 du|(1-u)\vec{y}|^2\right]/\varrho_0^2 , \tag{11}$$

which gives a result equivalent to Eq. (10). Extending this to include two source points, we obtain

$$D(\vec{x},\vec{y}) \approx 2\left(\frac{x^2 + \vec{x}\cdot\vec{y} + y^2}{\varrho_0^2}\right) . \tag{12}$$

Using this expression in Eq. (8), we obtain

$$\langle\cdot\cdot\cdot\rangle = \exp\left\{-2\left(\frac{\xi}{\varrho_0}\right)^{5/3}\left[1-\left(\frac{\xi}{\varrho_0}\right)^{1/3}\right]\right\}, \quad D << \varrho_0 , \tag{13}$$

where $\xi = |\vec{\xi}| = |\vec{\varrho}_r - \vec{\varrho}_r{}'|$. The second limit we consider is $\varrho_0 << D$. In this limit the first exponential in Eq. (8) falls off rapidly to zero when $\vec{\varrho}_r \neq \vec{\varrho}_r{}'$, which limits the region of integration in Eq. (5) to $\vec{\varrho}_r \approx \vec{\varrho}_r{}'$. Substituting $\vec{\varrho}_r = \vec{\varrho}_r{}'$ into the second exponential in Eq. (8), we have

$$\langle\cdot\cdot\cdot\rangle = \exp\left\{-2\left(\frac{\xi}{\varrho_0}\right)^{5/3}\right], \quad \varrho_0 << D , \tag{14}$$

which is the independent path result.

Equations (13) and (14) can now be used in Eq. (5) to obtain an expression for the long-term irradiance pattern. Using the change of variables $\vec{\xi} = \vec{\varrho}_r - \vec{\varrho}_r'$ and $2\vec{\eta} = \vec{\varrho}_r + \vec{\varrho}_r'$, we find that

$$\langle I(\vec{\varrho})\rangle = I_s\left(\frac{k}{2\pi L^2}\right)^2 \int d^2\vec{\xi} e^{-ik\vec{\varrho}\cdot\vec{\xi}/L} T(\xi) F(\vec{\xi}) \ , \tag{15a}$$

where

$$T(\xi) = \begin{cases} \exp\left\{-2\left(\dfrac{\xi}{\varrho_0}\right)^{5/3}\left[1 - \left(\dfrac{\xi}{\varrho_0}\right)^{1/3}\right]\right\}, & D << \varrho_0 \ , \\ \exp\left[-2\left(\dfrac{\xi}{\varrho_0}\right)^{5/3}\right], & \varrho_0 << D \ , \end{cases} \tag{15b}$$

describes the effects of turbulence and

$$F(\vec{\xi}) = \int d^2\vec{\eta} A(\vec{\eta} + \vec{\xi}/2) A^*(\vec{\eta} - \vec{\xi}/2) e^{i2k\vec{\eta}\cdot\vec{\xi}/L} \tag{15c}$$

describes the effect of the reflector. For a circular reflector with amplitude transmittance

$$A(\vec{\varrho}_r) = \begin{cases} 1, & |\vec{\varrho}_r| \le D_r/2 \ , \\ 0, & |\vec{\varrho}_r| > D_r/2 \ , \end{cases} \tag{16}$$

Eq. (15c) can be evaluated to give (Lutomirski and Yura, 1971)

$$F(\vec{\xi}) = F(\xi) = D^2 \int_0^{\cos^{-1}(\xi/D_r)} \left\{ \frac{\sin\left[\dfrac{kD_r\xi}{L}\left(\cos\theta - \dfrac{\xi}{D_r}\right)\right]}{\dfrac{kD_r\xi}{L}\cos\theta} - \frac{1 - \cos\left[\dfrac{kD_r\xi}{L}\left(\cos\theta - \dfrac{\xi}{D_r}\right)\right]}{\left(\dfrac{kD_r\xi}{L}\cos\theta\right)^2} \right\} \tag{17}$$

for $\xi \leq D$, and 0 otherwise. Here $\xi = |\vec{\xi}|$. To simplify these results a bit we note that the expression for $T(\xi)$ in Eq. (15) can be extended in an *ad hoc* manner to include arbitrary ratios of D/ϱ_0:

$$T(\xi) \approx \exp\left\{-2\left(\frac{\xi}{\varrho_0}\right)^{5/3}\left[1 - e^{-(D/\varrho_0)^2}\left(\frac{\xi}{\varrho_0}\right)^{1/3}\right]\right\}, \tag{18}$$

which has the correct limits for large and small D relative to ϱ_0. Since $F(\vec{\xi})$ depends only on ξ, the angular integration in Eq. (15c) can be performed to finally give

$$\langle I(\varrho)\rangle = I_s \frac{k^2}{2\pi L^4} \int_0^\infty d\xi \xi J_0(k\varrho\xi/L)T(\xi)F(\xi) , \tag{19}$$

where $\varrho = |\vec{\varrho}|$ and J_0 is the zero-order Bessel function (Gradshteyn and Ryzhik, 1980). We note that in terms of the transverse coordinates x and y, $\varrho = \sqrt{x^2 + y^2}$. Equations (17), (18), and (19) can now be used to generate the long-term reflected optical patterns for a retroreflector. We caution that these equations are approximations and therefore should only be used to *estimate* the effects of turbulence, and that these expressions are strictly valid only when $\varrho_0 > \sqrt{\lambda L}$

3.2 Discussion

Equation (19) normalized to its on-axis value in the absence of turbulence (i.e., $T(\xi) = 1$) is shown plotted in Fig. 2 as a function of the transverse coordinate x normalized by the reflector diameter D_r. The vacuum patterns are shown together with the long-exposure ($>> \tau$) irradiance patterns for $D_r = \varrho_0$ and $D_r = 10\ \varrho_0$. Several different reflector diameters are considered. Clearly, the optical patterns tend to lose their fine structure when $\varrho_0 \leq D_r$.

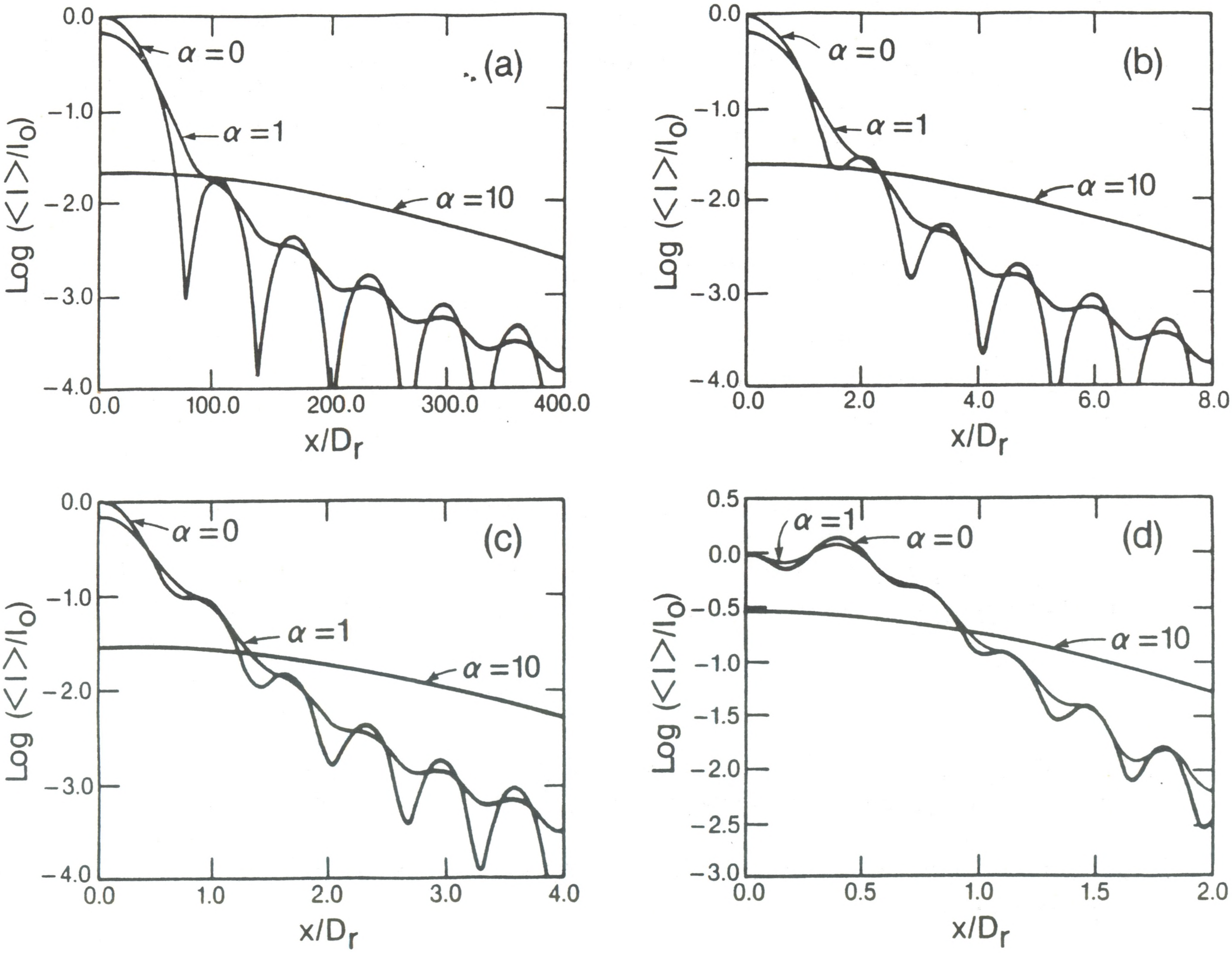

Figure 2. The long-term mean intensity $<I>$ normalized by its on-axis vacuum value I_o is plotted on a log-linear scale as a function of the transverse distance from the propagation axis normalized by the retroreflector diameter D_r, for (a) $D_r = 4.47\sqrt{L/k}$, (b) $D_r = 3.16\sqrt{L/k}$, (c) $D_r = 2.24\sqrt{L/k}$, and (d) $D_r = 0.32\sqrt{L/k}$. Each plot has three curves, each for a different level of turbulence as described by $\alpha = D_r/\varrho_0$. Here L is the path length, $k = 2\pi/\lambda$, λ is the source wavelength. $\varrho_0 = (0.545\, k^2 L\, C_n^2)^{-3/5}$ is the spherical-wave lateral coherence length, and C_n^2 is the refractive index structure parameter.

4. CONCLUSIONS

For typical operating conditions in the atmosphere we expect the propagation parameters to fall primarily into one of two regimes. These are $D_r >> \varrho_0 >> \sqrt{\lambda L}$ or $D_r >> \sqrt{\lambda L} \gtrsim \varrho_0$, with the latter being the most probable. For the former regime the dominant effect of turbulence on the reflected optical pattern is an expansion and contraction or breathing of the beam spot, which tends to warp the details within the pattern. Wander of the beam spot is negligible. This effect occurs over a time scale on the order of D_r/V. When $D_r >> \sqrt{\lambda L} \gtrsim \varrho_0$ the pattern also contains deep amplitude fluctuations over spatial scales on the order of ϱ_0. In this regime the pattern is severely distorted and probably unrecognizable. In the unlikely case of very weak turbulence ($\varrho_0 >> D_r >> \sqrt{\lambda L}$), the reflected spot wanders in the receiving plane but is otherwise undistorted.

In an attempt to quantify the distortion we numerically evaluated the long-term reflected irradiance pattern for different levels of turbulence. Once again, the long-term pattern is what would be recorded on a photographic plate exposed to the pattern for a time long compared with D_r/V. Although it does not describe the instantaneous distortion it does give us a feeling for how strong the turbulence needs to be before the details of the pattern are smeared out. We find that when $\varrho_0 \lesssim D_r$, the details in the pattern are no longer clearly discernible.

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