Ice Sheets and Fresh Water Reservoirs as Natural Dielectric Resonators

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***Abstract*— One proxy for global climate change is the change in the total mass of the Greenland and Antarctic ice sheets. Several complementary techniques have been used to estimate these changes with varying degrees of success. In this article, we describe a new approach that relies on the resonant behavior of ice masses. For very low electromagnetic (EM) frequencies (i.e., ≤ 2 kHz), pure ice acts like a strong dielectric resonator. Resonances can be excited in ice sheets by ambient EM noise from, for example, distant thunderstorms. The EM frequency spectrum measured in the vicinity of the ice mass should exhibit corresponding resonant frequencies. The evolution of the resonant modes over time can be used to monitor changes in ice mass and shape. The same approach can be, in principle, applied to monitor changes in the volume of fresh water reservoirs.**

***Index Terms*— Geophysical measurement techniques, ice, remote sensing.**

I. Introduction

THE importance of monitoring the change in the total mass of ice in the Greenland and Antarctic ice sheets as a proxy for global climate change is broadly recognized ([1], [2], and references therein). Several techniques have been developed for this purpose, including satellite altimetry and gravimetry [3], using ambient seismic noise [4], and radio echo-sounding [5]–[12]. All of these approaches have their relative strengths and weaknesses and are generally complementary.

In particular, radio-echo sounding of ice sheets is a well-developed technique. The range of frequencies used is very broad. For example, the suite of instruments developed at the University of Kansas Center for Remote Sensing of Ice Sheets (CReSIS) span frequencies of 14, 35, 140–160, 180–210, 165–215, and 150–600 MHz as well as a set of gigahertz frequencies [5]–[7]. Other systems [British Antarctic Survey (BAS) Ice-Sounding, High-Capability Radar Sounder (HICARS), Polar Operational Limit Assessment Risk Index System (POLARIS), and Warm Ice Sounding Explorer (WISE)] operate at 150, 60, and 435 MHz [8], [9]. The detailed history of radio-echo soundings of ice prior to 2008 with an extensive list of references can be found in [10].

The same radio-echo sounding approach was applied to the remote sensing of ice masses on other planets, for example, Mars [Mars Advanced Radar from Subsurface and Ionospheric Sounding (MARSIS) system], for which the frequencies used spanned 1.3–5.5 MHz [11]. The design of a corresponding subsurface/ice penetrating radar for the sounding of Jovian moons is presented in [12] with preferable frequencies on the order of tens of megahertz.

The list of systems and frequencies noted above is admittedly very far from being complete. Nevertheless, one can conclude that all frequencies used to date fall within the megahertz and higher frequency ranges. The lowest frequency used seems to be 1 MHz.

In this article, we propose to extend the range of applications to include ultralow electromagnetic (EM) frequencies below 2 kHz. The EM properties of ice at such low frequencies are fundamentally different from the properties in the megahertz range, and instead of echo-sounding, another approach based on the detection of EM resonances of large ice masses is presented. In addition, our approach is fundamentally different in intent from radio-echo sounding. At low frequencies, the EM wavelength is very long, and the local, fine structure of the ice sheet, such as the temperature and conductivity structure with depth, englacial water, etc., which are the primary variables of interest for radio-echo sounding, is averaged out and cannot be resolved. On the other hand, low-frequency resonances are sensitive to the spatially averaged, global properties of the ice sheets, such as total volume or characteristic value of dielectric constant, which are the primary variables of interest in this study.

For frequencies less than $\sim$2 kHz, pure ice possesses a very large dielectric constant and propagation losses are very small. This makes large masses of ice natural dielectric resonators. The associated resonant frequencies depend on the size and shape of the ice mass. Therefore, by monitoring resonant frequencies with a relatively simple *in situ* receiver configuration, one can, in principle, detect the evolution of ice sheets as they respond to a changing climate [13].

The same idea can be applied to fresh water reservoirs. Because the dielectric constant of water remains very large and losses small for frequencies up to hundreds of megahertz, the corresponding resonances can also be used to monitor changes in water volume.

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II. Dielectric Resonators

EM waves are ubiquitous in the atmosphere. This ambient EM noise field includes both a natural broadband component associated with tropical and midlatitude thunderstorms and various magnetospheric processes and narrowband man-made components. It is reasonable to expect some coupling between these ambient fields and natural dielectric resonators such as ice sheets and fresh water reservoirs. The EM field measured as a function of frequency in the vicinity of such a resonator should exhibit resonant peaks at frequencies that depend on the shape and the volume of the resonator. This opens the possibility of monitoring ice masses and water reservoirs by detecting and tracking associated resonant frequencies. In general, only natural noise which has relatively flat spectrum can serve this purpose. Man-made noise typically includes numerous spikes that could mask the resonances of interest.

Dielectric resonators with a relative permittivity (dielectric constant) $\varepsilon = \varepsilon' + i\varepsilon''$, such that ε' is large ($\varepsilon' \gg 1$) and ε'' is relatively small ($\varepsilon'' << \varepsilon'$) are well known and widely used in microwave technology [14]. A high value of dielectric constant ε' results in a trapping of EM radiation within the dielectric due to total internal reflection at the boundaries. For the case of small losses ($\varepsilon'' << \varepsilon'$), the dielectric would exhibit resonances with a high Q-factor.

For very low frequencies below ~2kHz, pure ice (without impurities) has a dielectric constant that is very close to the static value for water ($\varepsilon' \sim 90$). For higher frequencies, the dielectric constant of ice rapidly tends to $O(1)$ [15]. At these low frequencies, ice is also a relatively low loss medium with $\varepsilon'' \sim O(1) \ll \varepsilon'$ making large ice masses near-ideal, natural dielectric resonators.

Similarly, fresh water up to frequencies on the order of a few gigahertz has $\varepsilon' \sim 90$. To evaluate associated losses, we note that the imaginary part of the dielectric constant can be expressed in terms of a conductivity σ as follows:

$$\varepsilon'' = \frac{4\pi\sigma}{\omega} = \frac{2\sigma}{f} \tag{1}$$

where $\omega = 2\pi f$. For frequencies $f < 1$ MHz, the conductivity of pure water equals its static (dc) value $\sigma_{\rm dc} \sim 10^{-6}\ \Omega^{-1}\mathrm{m}^{-1} \sim 10^4\mathrm{s}^{-1}$ [15]. For frequencies 1 MHz $< f < 10^3$ MHz, the conductivity increases in proportion to the square of frequency ($\sigma \sim f^2$) and one finds

$$\frac{\varepsilon''}{f} = \frac{2\sigma}{f^2} \sim 2 \cdot 10^{-8}\ \mathrm{s}. \tag{2}$$

Therefore, for frequencies below 100 MHz, the ratio $\varepsilon''/\varepsilon'$ remains small and water is a relatively low-loss medium. Thus, reservoirs of fresh water at these frequencies can also be considered near-ideal, natural dielectric resonators.

III. Resonances in Planar Waveguides

Realistic EM models of glaciers and water reservoirs that take into account their complex shape and composition as well as the dielectric properties of the underlying bedrock would be very complex, and even their numerical treatment would represent a significant challenge. A not too unrealistic but tractable model of natural dielectric resonators would be a planar waveguide for which reasonable order-of-magnitude estimates of corresponding resonant frequencies can be computed.

Consider a three-layer, planar (i.e., horizontally stratified) waveguide with an upper half-space with dielectric constant ε_1, a middle layer of thickness h with dielectric constant ε, and a lower half-space with dielectric constant ε_2. Although the upper layer in our case will be air with $\varepsilon_1 = 1$, we will not specify ε_1 at this point to preserve the symmetry of the equations. In the general case, a dispersion relation that links the horizontal wavenumber of a mode k and its angular frequency ω follows from the equation:

$$V_1(\omega,k)V_2(\omega,k)\exp[2iq(\omega,k)h] = 1 \tag{3}$$

where $V_{1,2}(\omega,k)$ are Fresnel reflection coefficients of the corresponding plane wave from the upper and lower boundaries of the intermediate layer, respectively, and $q(\omega,k)$ is the vertical wavenumber of the plane wave within the layer

$$q(\omega,k) = \left(\varepsilon\frac{\omega^2}{c^2} - k^2\right)^{1/2}, \quad \mathrm{Im}q(\omega,k) \geq 0. \tag{4}$$

Physically, (3) and (4) state that after a plane wave is reflected from both the upper and lower boundaries (after passing twice through the middle layer), the corresponding reflected wave reproduces itself. Equation (4) does not necessarily have a real solution for any specific ω. The frequencies for which unique solutions of (4) with real k appear first are the critical frequencies of the waveguide.

The reflection coefficients $V_{1,2}(\omega,k)$ depend on the polarization of the incident plane wave. Consider the case of vertical (TM, or p) polarization first (i.e., the electric vector lies in the vertical plane). In this case, the Fresnel reflection coefficient can be expressed as

$$V_1(\omega,k) = \frac{\varepsilon_1(\varepsilon\omega^2/c^2 - k^2)^{1/2} - \varepsilon(\varepsilon_1\omega^2/c^2 - k^2)^{1/2}}{\varepsilon_1(\varepsilon\omega^2/c^2 - k^2)^{1/2} + \varepsilon(\varepsilon_1\omega^2/c^2 - k^2)^{1/2}} \tag{5}$$

and $V_2(\omega,k)$ follows from (5) after the substitution $\varepsilon_1 \to \varepsilon_2$. Let us assume that all dielectric constants are real and positive, and $\varepsilon_1 < \varepsilon_2 < \varepsilon$. The condition for trapped waves (i.e., inhomogeneous waves in the two half-spaces and a propagating wave in the middle layer) requires that

$$\frac{\omega}{c}\sqrt{\varepsilon_2} < k < \frac{\omega}{c}\sqrt{\varepsilon}. \tag{6}$$

Applying (6) to (5), we note that the second terms in the numerator and denominator of (5) are purely imaginary. For this case, the dispersion relation (3) reduces to

$$h(\varepsilon\omega^2/c^2 - k^2)^{1/2} - \arctan\frac{\varepsilon(k^2 - \varepsilon_1\omega^2/c^2)^{1/2}}{\varepsilon_1(\varepsilon\omega^2/c^2 - k^2)^{1/2}} - \arctan\frac{\varepsilon(k^2 - \varepsilon_2\omega^2/c^2)^{1/2}}{\varepsilon_2(\varepsilon\omega^2/c^2 - k^2)^{1/2}} = \pi n \tag{7}$$

where n is an integer. The lowest possible value of n is -1, which occurs when $k = \sqrt{\varepsilon}\omega/c$. This limit corresponds to a plane wave propagating strictly horizontally within the middle layer. However, for this case, $V_1 = V_2 = -1$, and the reflected wave cancels the incident one, and such a mode, in fact, does not exist. Therefore, the only nontrivial values of n are $n = 0, 1, \ldots$

One can easily see that the left-hand side (LHS) of (7) is a monotonously decreasing function of k. Therefore, the maximum value of the LHS of (7) is achieved for the minimal possible k (i.e., $k = \sqrt{\varepsilon_2}\omega/c$). Substituting $n = 0$ into (7), one finds the following expression for the critical frequency:

$$f_{cr} = \frac{\omega_{cr}}{2\pi} = \frac{1}{2\pi}\frac{c}{h}\frac{1}{\sqrt{\varepsilon - \varepsilon_2}} \arctan \frac{\varepsilon\sqrt{\varepsilon_2 - \varepsilon_1}}{\varepsilon_1\sqrt{\varepsilon - \varepsilon_2}}. \tag{8}$$

The critical frequency f_{cr} identifies the lower bound for observable resonant frequencies f_{res}. We can estimate the resonant frequencies as $f_{res} \sim f_{cr} + i\Delta f_{res}$, where $i = 1, \ldots, L_{hor}/\lambda_\varepsilon$ is an integer representing a horizontal mode index, L_{hor} is the characteristic horizontal scale of the dielectric, and $\lambda_\varepsilon < \lambda = c/\sqrt{\varepsilon} f$ is the wavelength within the dielectric, which is shorter by a factor of $\sqrt{\varepsilon}$ than the wavelength λ in air. For $L_{hor} \gg \lambda_\varepsilon$, we can estimate the interval between resonant frequencies Δf_{res} as follows:

$$\Delta f_{res} \sim \frac{c}{\sqrt{\varepsilon} L_{hor}}. \tag{9}$$

Note that for microwave applications usually $\varepsilon_1 = \varepsilon_2$ and then from (8) one finds $f_{cr} = 0$. In geophysical applications, however, one has $\varepsilon_2 > \varepsilon_1 = 1$, and the critical frequency in (8) becomes nonzero. For $\varepsilon \gg \varepsilon_2$, and a not too small value of $\sqrt{\varepsilon_2 - \varepsilon_1} > 1$, one obtains

$$f_{cr} \approx \frac{1}{4\sqrt{\varepsilon}}\frac{c}{h}. \tag{10}$$

Now let us consider the case of horizontal (TE, or s) polarization (i.e., the electric vector lies in the horizontal plane). The Fresnel reflection coefficient in this case reads

$$V_1(\omega, k) = \frac{(\varepsilon\omega^2/c^2 - k^2)^{1/2} - (\varepsilon_1\omega^2/c^2 - k^2)^{1/2}}{(\varepsilon\omega^2/c^2 - k^2)^{1/2} + (\varepsilon_1\omega^2/c^2 - k^2)^{1/2}} \tag{11}$$

and as a result, (3) becomes

$$h(\varepsilon\omega^2/c^2 - k^2)^{1/2} - \arctan\frac{(k^2 - \varepsilon_1\omega^2/c^2)^{1/2}}{(\varepsilon\omega^2/c^2 - k^2)^{1/2}} - \arctan\frac{(k^2 - \varepsilon_2\omega^2/c^2)^{1/2}}{(\varepsilon\omega^2/c^2 - k^2)^{1/2}} = \pi n. \tag{12}$$

Just as in the case of vertical polarization, the condition $k = \sqrt{\varepsilon_2}\omega/c$ corresponds to $V_1 = V_2 = -1$, and such a mode does not exist. For $n = 0$, (8) is replaced by

$$f_{cr} = \frac{1}{2\pi}\frac{c}{h}\frac{1}{\sqrt{\varepsilon - \varepsilon_2}} \arctan\frac{\sqrt{\varepsilon_2 - \varepsilon_1}}{\sqrt{\varepsilon - \varepsilon_2}}. \tag{13}$$

Assuming $\varepsilon \gg \varepsilon_2$, one finds

$$f_{cr} \approx \frac{1}{2\pi}\frac{c}{h}\frac{\sqrt{\varepsilon_2 - \varepsilon_1}}{\varepsilon - \varepsilon_2}. \tag{14}$$

A comparison of (10) and (14) indicates that the critical frequency for horizontal polarization for the $n = 0$ mode is smaller by a factor of $\sqrt{\varepsilon}$ than for vertical polarization. In what follows we consider only the horizontal polarization case for which the wavelength in the dielectric can be approximated by

$$\lambda_\varepsilon = \frac{c}{f_{cr}\sqrt{\varepsilon}} \sim 2\pi\sqrt{\frac{\varepsilon}{\varepsilon_2 - \varepsilon_1}}h. \tag{15}$$

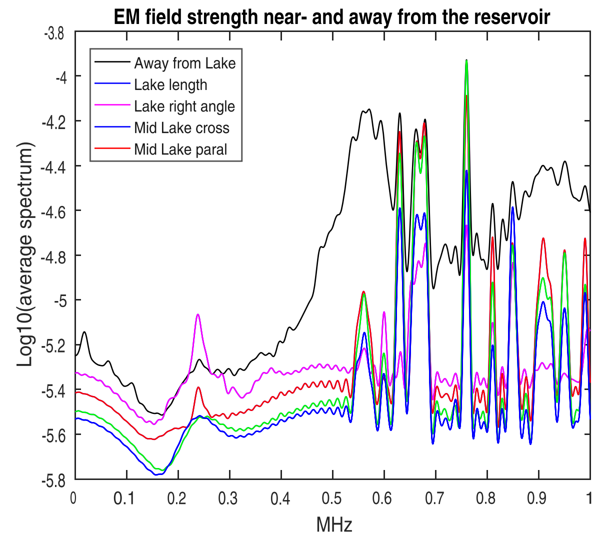

Fig. 1. Frequency spectra of the EM field measured in the vicinity of Barker Reservoir near Boulder, CO, USA. The peaks for frequencies exceeding 0.5 MHz are due to man-made noise. The maxima around 0.24 MHz could possibly be due to the dielectric resonator effect.

For reference, we note that the critical frequency for the $n = 1$ mode in (12) gives

$$f_{cr}^{(1)} \approx \frac{1}{2\sqrt{\varepsilon}}\frac{c}{h}. \tag{16}$$

(The terms associated with $\tan^{-1} \sim \varepsilon^{-1/2} \ll \pi$ in (12) are neglected above).

IV. Numerical Estimates for Water Reservoirs

Let us first consider the case of a shallow water reservoir with $h \sim 10$ m and $\varepsilon \sim 90$, and let us set for an estimate somewhat arbitrarily $\varepsilon_2 \sim 5$ (being a not too large and not too small). From (14), one obtains for the lowest ($n = 0$) mode

$$f_{cr} \sim 0.11 \text{ MHz}. \tag{17}$$

According to (15), the wavelength within the reservoir will be $\lambda_\varepsilon \sim 300$ m, and the horizontal scale of the reservoir L_{hor} should exceed λ_ε. The critical frequency for the $n = 1$ mode estimated according to (16) gives $f_{cr}^{(1)} \sim 1.6$ MHz.

We have made an attempt to measure fresh water reservoir resonances at the Barker Reservoir near Boulder, CO, USA. The intensity of the EM field as a function of frequency is plotted in Fig. 1. The EM field was measured using different orientations of a loop antenna, and at points both near (within 10 m) of the reservoir (the color lines on the figure) and at a point 800 m distant (the black line). Fig. 1 exhibits numerous peaks spanning 0.51 MHz. These peaks most likely correspond to man-made signals. One can also see a local maxima near 0.24 MHz. This frequency is not too far away from the estimate (17) (a perfect match would require setting $\varepsilon_2 \sim 17$, or a corresponding reduction of the average depth of the reservoir). Based on the observation that the corresponding maxima were observed only when the antenna was placed near the reservoir, it is possible that this peak is due to dielectric resonator effect; however, unambiguous identification of such resonances would require comprehensive measurements and development of robust techniques for separating man-made noise from the resonant signatures.

V. Numerical Estimates for Glaciers

Measuring EM resonances associated with water reservoirs, although intriguing in principle, may be of limited practical interest. The rudimentary experimental results presented here are meant only to illustrate the concept using accessible measurements. The possibility of monitoring changes in glacier volume using resonant techniques and a relatively simple and inexpensive *in situ* receiving system is much more compelling.

A major difference between water and ice is that for frequencies exceeding 2 kHz, the real part of dielectric constant of ice drops off significantly to values of O(1) for $f > 10$ kHz. For $f < 2$ kHz, however, the dielectric constants of water and pure ice are nearly equal. Because detecting a resonance requires $f < f_{\max} = 2$ kHz, (14) puts a limit on the smallest depth of the ice layer that can support a resonance

$$h_{\min} \approx \frac{1}{2\pi}\frac{c}{f_{\max}}\frac{\sqrt{\varepsilon_2-\varepsilon_1}}{\varepsilon-\varepsilon_2} \approx 560 \text{ m} \tag{18}$$

where we have as before set $\varepsilon_2 = 5$ and $\varepsilon = 90$. Thus, the resonances considered above could be observed for very large glaciers only. The typical depth of the Greenland and Antarctic ice sheets can be approximated by $h \sim 1.8$ km, so that criterion (18) is satisfied. For this depth, one finds

$$f_{\text{cr}} \approx \frac{1}{2\pi}\frac{c}{h}\frac{\sqrt{\varepsilon_2-\varepsilon_1}}{\varepsilon-\varepsilon_2} \approx 625 \text{ Hz}. \tag{19}$$

According to (15), the wavelength of an EM wave in ice for this frequency is $\lambda_\varepsilon \sim 54$ km. A horizontal scale of a planar waveguide can be estimated as $L_{\text{hor}} \sim S^{1/2}$, where S is the horizontal area. Then, from (9), one finds for Greenland ($S \sim 1.8 \cdot 10^6$ km^2), and the interval between consecutive resonances is

$$\Delta f \sim \frac{c}{\sqrt{\varepsilon S}} \sim 24 \text{ Hz} \tag{20}$$

and for the Antarctic ($S \sim 14 \cdot 10^6$ km^2) $\Delta f \sim 8$ Hz. These resonances should lie between the critical frequency given in (19) and frequencies of the order of 2 kHz. Of course, these numerical results are based on a rather crude planar model (and for a dielectric constant of bedrock set arbitrarily to $\varepsilon_2 = 5$), so that (18)–(20) should be considered as rough estimates only.

Let us evaluate the sensitivity of the resonant frequencies to changes in the ice sheet volume by considering the lowest order vertical mode only, i.e., by setting in (12) $n = 0$. (Variations of the dielectric constant due to temperature effects, for example, should be restricted to rather narrow layers and most likely can be neglected.) Let us first assume that the horizontal extent L_{hor} can be written as $L_{\text{hor}} + \delta L_{\text{hor}}$. Since the horizontal wavenumber $k \sim L_{\text{hor}}^{-1}$, we have $\delta k/k \sim -\delta L_{\text{hor}}/L_{\text{hor}}$, and the corresponding variation of the resonant frequency ω becomes

$$\frac{\delta\omega}{\omega} = \frac{1}{\omega}\frac{d\omega}{dk}\delta k = \frac{k}{\omega}\frac{d\omega}{dk}\frac{\delta k}{k} \sim -\frac{k}{\omega}\frac{d\omega}{dk}\frac{\delta L_{\text{hor}}}{L_{\text{hor}}}. \tag{21}$$

A simple analysis of (12) shows that for resonant frequencies close to the critical frequency ω_{cr}, the group velocity is close to phase velocity: $d\omega/dk \approx \omega/k = c/\sqrt{\varepsilon_2}$, and since $L_{\text{hor}} \sim S^{(1)/(2)}$, one finds

$$\frac{\delta\omega}{\omega} \sim -\frac{1}{2}\frac{\delta S}{S} \sim -\frac{1}{2}\frac{\delta V}{V} \tag{22}$$

where $V \sim hS$ is the ice sheet volume. A slightly more involved analysis of the derivative $d\omega/dh$ for constant L_{hor} (and, consequently, constant horizontal wavenumber k) yields

$$\frac{\delta\omega}{\omega} \approx -\frac{\varepsilon}{\sqrt{\varepsilon_2}}\frac{h}{c}(\omega-\omega_{\text{cr}})\frac{\delta V}{V}. \tag{23}$$

The resonances close to the critical frequency are not sensitive to variations of the ice sheet thickness. If, however, they are separated from f_{cr} by a few hundred hertz, the sensitivity becomes significant. Let us assume as above $\varepsilon_2 = 5$, $\varepsilon = 90$, $h = 1.8$ km, and $\omega - \omega_{\text{cr}} \sim 2\pi \cdot 350$ Hz. Then, the coefficient in front of $\delta V/V$ in the right-hand side of (23) becomes 0.5, and the estimate (22) holds again.

As stated in [16], current estimates of the total loss of ice mass are based on sparse mass balance measurements representing only a small fraction of the glaciers and ice cap area. Based on GRACE gravimetric measurements, we estimate the total volume of ice loss in Greenland and Antarctica to be of order ~100 km^3/year, or ~2 km^3/week (according to [8], the ice loss in Greenland during 2005–2010 was even larger: ~252 km^3/year). Using this ice loss estimate in (22), the corresponding rate of drift of the lowest resonant frequency for the Greenland ice sheet becomes

$$\Delta f \sim \frac{f}{2}\frac{\Delta V}{V} \sim \frac{625 \text{ Hz}}{2}\frac{2 \text{ km}^3 \text{ /week}}{3 \cdot 10^6 \text{ km}^3} \sim 2 \cdot 10^{-4}\frac{\text{Hz}}{\text{week}}. \tag{24}$$

To measure resonant frequencies with such precision in the ideal case, it is sufficient to have a record length on the order of $T \sim 1/\Delta f \sim 1.5$ h. Of course, in practice, the accuracy of the measurements of the resonance location will be most likely limited by noise. Note that the variation of the noise level as a function of frequency within a width of the resonant curve without change of its shape will not result in a shift of the resonant peak and is thus irrelevant. Only variations of the noise spectrum as a function of frequency within the resonant curve matter. A quantitative description of such variations (which also depend on an unknown width of the resonant curves) is currently unavailable, as well as an overall effect of such variations on the accuracy of locating the resonance. Still, one can argue that there is a potential of tracking ice mass changes on a week-long time scales.

VI. Comment on Measurements of Ice Sheet Resonances

Measurements of weak EM signals in the range 300 Hz − 30 kHz are of interest for investigation of magnetospheric and ionospheric processes and lightning activity. The hardware that is used for measurements of this type exists and is described in detail, in particular, in [17] and [18]. It utilizes standard electronics (not superconducting devices) and performs measurements in the 50 Hz − 30 kHz frequency range, which includes the frequencies considered above. If, however, using or building such hardware is not an option, in this section, we present some estimates of signal parameters to be measured and associated thermal noise levels. These estimates can be used to construct a rudimentary receiving system that should be able to perform the required measurements.

The magnetic field for horizontally polarized waves propagating at grazing angles is directed nearly vertically. It can be measured by a loop antenna lying directly on the ice. Because the resonant wavelength is so long, it does not matter where on the ice sheet the antenna is placed as long it is not located at a mode's null, which seems highly unlikely. The spectral density of the magnetic field of the ambient EM noise in the frequency range 1 kHz − 100 kHz can be estimated in [13] as

$$\frac{\mathrm{dB}}{d(\sqrt{f})} \sim 10^{-15} T \cdot \mathrm{Hz}^{-1/2}. \tag{25}$$

Let us consider a loop antenna consisting of one circular turn of diameter D of copper wire. The voltage υ induced in the loop by the EM noise around a central frequency f within bandwidth Δf according to Faraday's law of induction will be

$$\upsilon = 2\pi f \frac{\mathrm{dB}}{d(\sqrt{f})} \sqrt{\Delta f} \frac{\pi D^2}{4}. \tag{26}$$

The resistance R of the wire can be expressed as $R = 4\rho D/d^2$, where d is the diameter of the wire and $\rho = 1.7 \cdot 10^{-8} \Omega \cdot \mathrm{m}$ is the specific resistance of copper. The level of thermal noise in the loop is given by $N = (k_B T_0 R \Delta f)^{1/2}$, where $k_B = 1.4 \cdot 10^{-23} J \cdot K^{-1}$ and T_0 is an absolute temperature. The signal-to-noise ratio at $T_0 = 300\ K$ is then given by

$$\frac{\upsilon}{N} = \frac{\pi^2}{4} \frac{\mathrm{dB}}{d(\sqrt{f})} (k_B T_0 \rho)^{-1/2} f D^{3/2} d \approx 0.3 f D^{3/2} d. \tag{27}$$

Selecting $D = 100$ m, $d = 10^{-3}$ m, and $f = 625$ Hz yields a signal-to-noise ratio of $\upsilon/N = 187$ or 23 dB, which is sufficient for detection of the resonant signal. If needed for practical reasons one can make the loop antenna smaller by reducing its diameter D by a factor a and increasing the number of turns by a^2, which would leave the induced voltage υ unchanged while only slightly increasing the thermal noise N by a factor of $a^{1/2}$. Another practical concern is matching the low impedance of the loop antenna with a higher impedance amplifier, which can be accomplished using a relatively simple transformer configuration. The design of such a transformer is presented in detail in [17].

VII. Conclusion

The dielectric constant of ice at very low frequencies up to about 2 kHz and of water up to frequencies on the order of 1 GHz, is very large ($\varepsilon \sim 90$). This makes large ice masses and water reservoirs strong dielectric resonators. Such resonators can be excited by the ambient EM noise field. Natural noise possesses a relatively flat spectrum. The EM field measured in the vicinity of such resonators will exhibit maxima at resonant frequencies. The resonant frequencies depend on the volume and the shape of the resonator. The evolution of these resonances can be used to monitor the shape and the size of a resonator. Of particular interest would be the monitoring of the Greenland and Antarctic ice sheets using resonant techniques.

The conductivity of and therefore the losses associated with pure ice are relatively small. If losses due to radiation from the resonators can be neglected, the Q-factor of the resonances will be relatively large, and resonant peaks would be correspondingly sharp and narrow and easily detectable. The major question is whether the Greenland and Antarctic ice sheets have sufficiently low losses for resonance peaks to be detected. For resonant modes to exist, it is necessary that the trapped EM waves pass without significant attenuation throughout both the vertical and horizontal extents of the ice sheet. The horizontal extent of the Greenland and Antarctica ice sheets is of the order of 20 wavelengths (since $\lambda_\varepsilon \sim 54$ km). Note that at megahertz frequencies, EM waves penetrate ice down to bedrock, what is essentially thousands of wavelengths.

Measurements of ice conductivities in the natural glaciers of Colorado and Canada [20] indicated that they exceeded the theoretical values characteristic of pure ice, suggesting that natural glaciers are more lossy. To what extent the measured values are applicable to the Greenland or Antarctica ice sheets is not clear at this point, although these are much more pristine environments with conductivities that may more closely approximate that of pure ice.

Resonances of the same type should also exist in the fresh water reservoirs, and they should extend to much higher frequencies. An attempt to measure fresh water resonances as an accessible demonstration of the concept was made and a candidate peak was observed. However, unambiguously identifying a resonant peak in the presence of man-made spectral noise would require a more comprehensive data set.

A realistic model of the Greenland and/or Antarctic ice sheet should take into account the composition and complex geometry of the corresponding ice masses and underlying bedrock. The development of such a model represents a significant theoretical and numerical challenge that might still yield inconclusive results given the uncertainty in many of the associated parameters. To validate that such resonant modes exist in nature, we suggest that an experiment designed to detect resonant modes might do more to advance the science than a detailed theoretical and numerical analysis.

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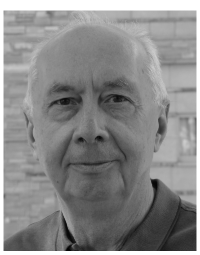

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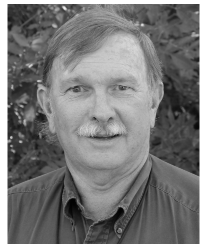
Scott W. Abbott, photograph and biography not available at the time of publication.



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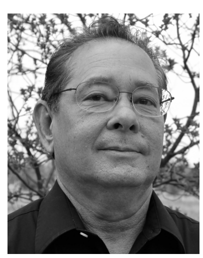

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