NOAA Technical Report EDS 3

Standard Deviation of Monthly Average Temperature in the United States

Washington, D.C.
Revised April 1978

U.S. DEPARTMENT OF COMMERCE
National Oceanic and Atmospheric Administration
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U.S. DEPARTMENT OF COMMERCE
Juanita M. Kreps, Secretary
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STANDARD DEVIATION OF MONTHLY AVERAGE TEMPERATURE IN THE UNITED STATES

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ABSTRACT. Maps of the distribution of standard deviation of monthly average temperature over the United States are given. These are based on the 1941-1970 period. A brief history of the derivation of United States normal temperatures and of past work on standard deviation distribution is presented. As an example, changes during this century in the standard deviation of January temperature at Lincoln, Neb., are discussed. A 1968 report by H.C.S. Thom is reprinted as an Appendix discussing confidence intervals, probabilities, quantiles of temperature functions, and approximation of normal monthly degree days as applications of standard deviation of temperature data.

INTRODUCTION

"Unusual" weather occurs far more often than many people realize. The individual's personal reference base is much too small both in area and time for one to be fully aware of the total variability of climate. The evidence from the past clearly reveals climatic variations on all scales. Short-term fluctuations and regional changes mask the long-term trends (World Meteorological Organization 1976). One of the applications of "normals" is to predict future conditions (Court 1967,1968). In such cases we use a 30-year period of record to determine an average temperature that will hopefully reflect conditions during the next 10 years. Ten years later another "normal" is computed for use during the next 10 years, etc. Now, while there is nothing mathematically incorrect about this procedure, such a method presumes a great deal in terms of climatic fluctuations--it seems to take for granted something not known.

The purpose of this article is to review briefly some of the ways temperature "normals" have been derived for weather stations in the United States since 1870 and to compare their standard deviation(s) over selected time periods. The first maps of standard deviations of monthly average temperatures (^{O}F) were based on the period 1921-50 and were published in 1968 (U.S. Dept. of Commerce 1968); the maps presented here are based on the 1941-70 period.

DISCUSSION

About 1935, members of the International Meteorological Organization, meeting in Geneva, Switzerland, agreed to compute normals for a 30-year period beginning with the years 1901-30. However, because few stations had sufficient record, a compromise was agreed upon that permitted a sliding scale (Landsberg 1972). At the present time, the United States has computed and published temperature "normals" for the following periods: 1873-1903 (U.S. Dept. of Agriculture 1908), 1875-1921 (U.S. Dept. of Agriculture 1925), 1921-50 (U.S. Dept. of Commerce 1956), 1931-60 (U.S. Dept. of Commerce 1963), and 1941-70 (U.S. Dept. of Commerce 1973).

According to Landsberg (1972), the term "normal" was first noted in meteorological literature as early as 1840; however, it was not until 1956 that temperature normals were finally computed for a 30-year period for stations in the United States. A great deal of discussion has centered around the definition and use of the word "normal" (U.S. Dept. of Agriculture 1919) as well as the optimum number of years required to compute a "normal" (Court 1967,1968). The methods and the lengths of record used in the earlier computations (U.S. Dept. of Agriculture 1908,1925) differed vastly from present methods. Blodgett (1857) followed the practice of earlier investigators (Landsberg 1972) and obtained differences from a "normal" computed for latitude 43°40'N. Techniques also varied depending on the investigator, and the question of homogeneity must remain in doubt for some of the early periods (U.S. Dept. of Agriculture 1895,1906,1907, 1908,1909,1919,1925; U.S. Dept. of Commerce 1956). Numerous adjustments and corrections were computed (U.S. Dept. of Agriculture 1909) that were intended to be applied to account for such things as topography, distance from large bodies of water, etc.; however, due to their small magnitude (often computed to hundredths of a degree), these corrections were not always used (U.S. Dept. of Agriculture 1925).

Corrections in the early years were also made to account for the differences in observation times, but this must have been a perilous task indeed, since tri-daily observations in the 1873-1888 period were taken at 0700, 1500, and 2300 EST. Stations in California were, therefore, taking their observations at 0400, 1200, and 2000 PST (U.S. Dept. of Agriculture 1925). In any case, several methods were employed to develop daily temperature normals (U.S. Dept. of Agriculture 1908,1925). In some cases, the 12 monthly means were plotted and curves were drawn representing the annual march of temperature. Later normals (U.S. Dept. of Agriculture 1925) were derived from curves constructed from weekly averages of temperature. This process continued as late as 1963 (U.S. Dept. of Commerce 1963), although by this time some computer processing of normals had begun. It was not until 1973 (U.S. Dept. of Commerce 1973) that all normals were prepared by computer for stations in the United States.

A primary goal of climatological analysis is to extract the underlying patterns that characterize our weather. The varied collections of observations often appear to contain oscillations and variations that evidence some trend or cycle when examined over brief periods. On a second look they may turn out to be random, or do not apply to other stations or regions. Non-randomness may be a result of changes in station location, instruments, exposure of instruments, or a change in observing practices. Man-made changes of the local environment can introduce inhomogeneity in a climatological record; however, the most important and unavoidable component of non-randomness in the climatological time series is that due to climatic fluctuations (World Meteorological Organization 1967).

While the earlier work is of interest, this report concentrates on the 30-year periods: 1921-50, 1931-60, and 1941-70. All climatological normals are some form of mean based on samples; in this instance, 30 years. Some are the simple means of the 30 monthly values, while others are adjusted to compensate for station moves (U.S. Dept. of Commerce 1963). Such values are only estimates of the "true" population values which one would like to know, but can really never know (U.S. Dept. of Commerce 1968). They are subject to the usual variation of random sampling. The "true" value will be covered by an interval with a preassigned confidence according to statistical theory. An extensive discussion of confidence intervals is in the Appendix.

The standard deviation of the monthly average temperature is a measure of its variability. It is estimated by taking the square root of the unbiased estimate of the variance using the formula

$$s = \sqrt{\Sigma(t-\overline{t})^2/(n-1)}$$

where s is the standard deviation, t is the average temperature for a month, $\overline{t}$ is the mean of all the t's and n is the sample size, 30 in this case.

There have been three sets of charts showing standard deviations of monthly average temperature for stations in the United States. Nagao's (1951) work covered the entire Northern Hemisphere, but was based on only 147 stations. Although his general period of record was 1921-40, many of the stations had less than 20 years while others had up to 60 years. The monthly maps have been greatly reduced in size which makes them difficult to read.

Sumner (1953) computed his values for about 140 stations in North America (excluding Mexico) covering periods from 20 to 50 years (1921-40 and 1896-1945). Standard deviations are not listed by station-months, but are in the form of maps with shading on a $2^{o}F$ scale; therefore, only generalized comparisons can be made to later periods.

Thom's article (U.S. Dept. of Commerce 1968) and maps are based on some 150 first-order weather stations in the United States from 1921-50, and he finds general agreement with the two earlier reports in spite of the differences in record length and the non-standard periods used. All of the studies included data from the 1930's and none used more than 150 stations. Kendall and Anderson (1966) published monthly and annual standard deviations of temperature for 195 stations in Canada based on the 1931-60 period.

The fact that the 1941-70 normals were the first to be computed without the 1931-40 period makes them of special interest. Comparisons of standard deviations between the different periods show significant differences at many stations. The Midwest, for example, experienced bitter cold winters during the 1930's but it also had a number of very warm Januarys at many stations. Lincoln, Neb., had average January temperatures much above normal in 1931, 33, 34, 35, 38, and 39, but much below normal in 1930, 32, 36, 37, and 40. January and February 1936 still rank as the two coldest consecutive months at a number of stations in the Great Plains. The coldest Januarys of record at Lincoln were those of 1886 (7.0^{o}F) and 1888 (8.0^{o}F). During the 89 years from 1889 to 1977, the average January temperature exceeded (±) two standard deviations from the long-term mean only twice at Lincoln, and both occurred in the 1931-40 decade. This extreme variability is shown in figure 1. Also shown is the sharp decrease in variability at three stations from 1941 to 1977.

A comparison of the standard deviations of average January temperatures is shown in figure 2 for the periods 1921-50 and 1941-70. The difference at Lincoln is striking--a change of 3.0 F (7.5 to 4.5) occurred between these periods partly as a result of excluding the 1931-40 decade. Table 1 shows standard deviations for selected 10-, 30-, and 50-year periods for 17 stations in 10 States. Standard deviations are greatest during the 1931-40 period at 14 stations and in the 1911-20 decade at three stations. For the 30-year periods, the 1911-40 period shows the highest standard deviations for most stations. The greatest difference between any two decades was 7.64 at Lincoln where it decreased from 10.10 (1931-40) to 2.46 (1951-60).

The extremes of January average temperature at Lincoln during the period 1886-1977 are 7.0^{o}F (1886) and 36.6^{o}F (1933). The number of average monthly temperatures at Lincoln that have exceeded ± one and two standard deviations during the period 1887-1977 are shown in table 2 (although temperature records began in January 1886 at Lincoln, the values are missing for several months during that year. Since the earliest year of record at Kansas City was 1889, figure 1 is based on that date). According to theoretical expectations based on a Gaussian (normal) distribution, the average monthly temperature can be expected to depart more than ± one standard deviation 28 times and more than ± two standard deviations four times during a period of this length. Figure 3 shows the normal, or bell-shaped, distribution curve with the percent of observations included within ± one, two, and three standard deviations from the mean. Since average monthly temperatures for January are approximately normally distributed, an estimate of future distributions can be made. Over the past 91 years at Lincoln the average January temperature is 23.4^{o}F with a standard deviation of 6.43. Using these estimates and figure 3(b), we find that 68 percent of the time the average January temperature is expected to fall between 17.0^{o} and 29.8^{o}F. By expanding the coverage as shown in 3(c) to two standard deviations, the range is increased so that 95 percent of all average January temperatures at Lincoln can be expected to fall between 10.5^{o} and 36.3^{o}F. Finally, 3(d) shows that in four years out of a hundred, on the average, the average January temperature will fall outside these limits. Since the extremes of average January temperature at Lincoln

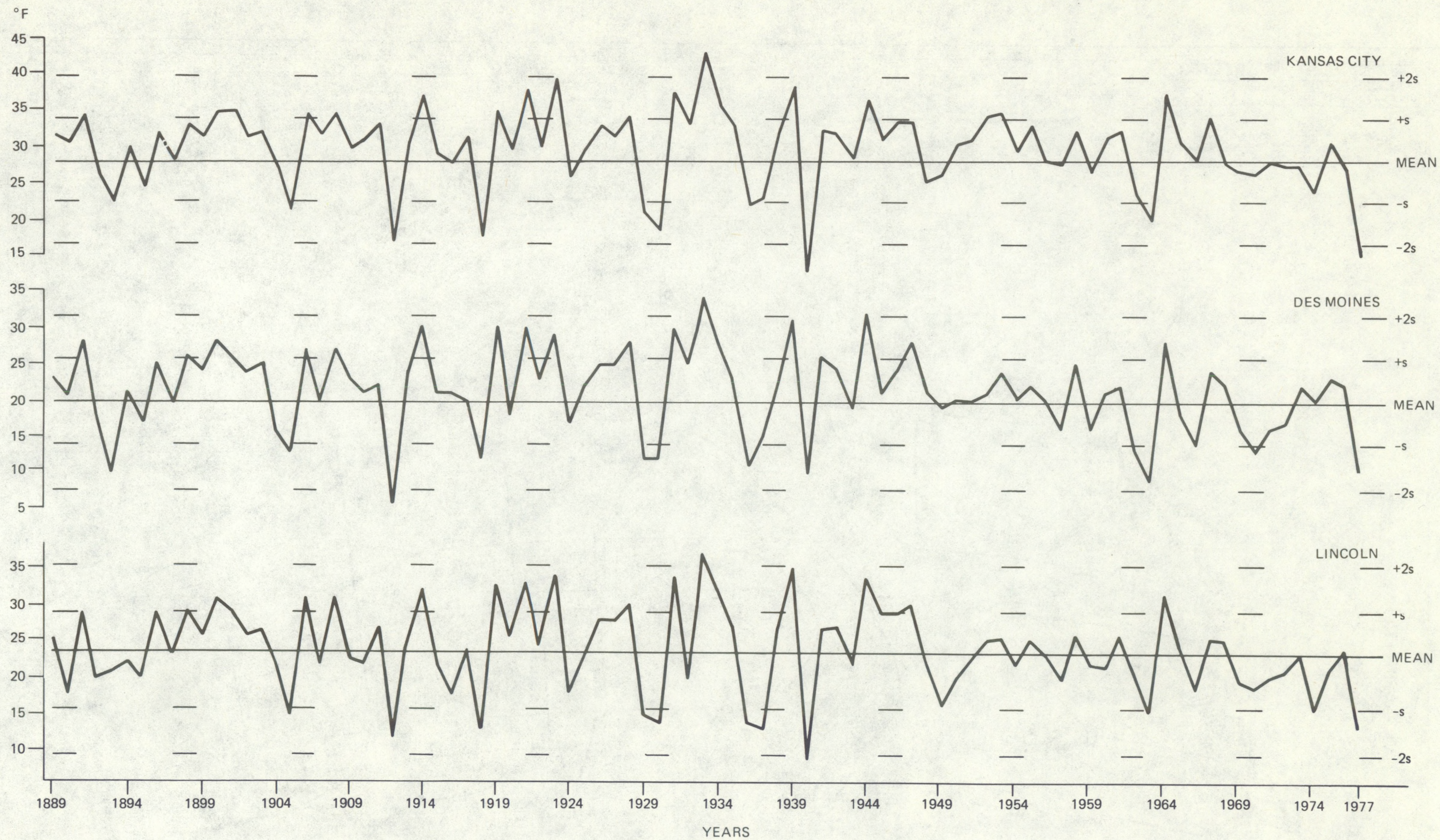

Figure 1.--Average January air temperatures and standard deviations for the period 1899-1977.

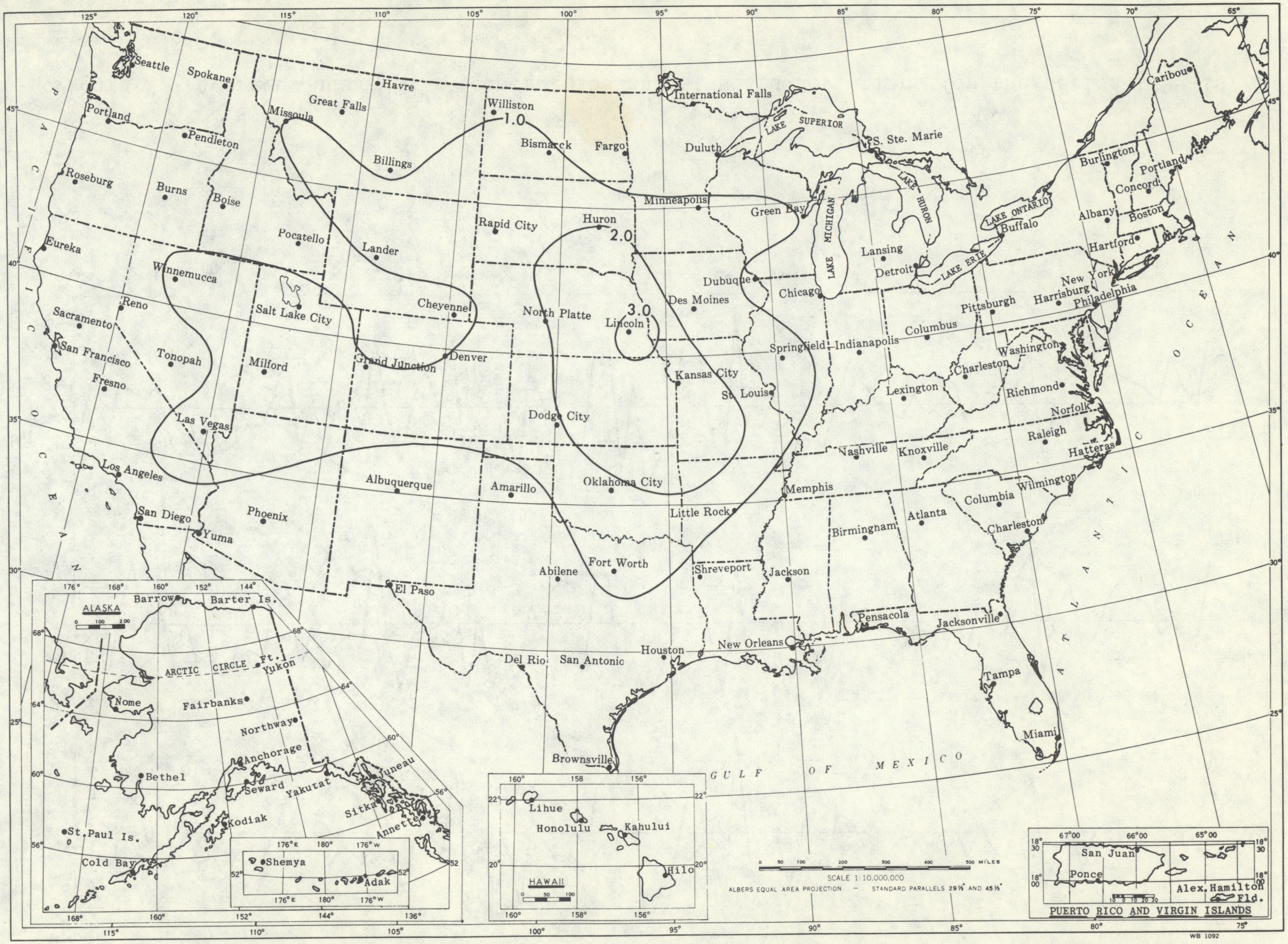

Figure 2.--Difference in standard deviation of average January temperature between 1921-50 and 1941-70.

TABLE 1 STANDARD DEVIATIONS OF JANUARY MONTHLY MEAN TEMPERATURE

		10 Years							30 Years					50 Years		
	Airport from	1901-10	1911-20	1921-30	1931-40	1941-50	1951-60	1961-70	1901-30	1911-40	1921-50	1931-60	1941-70	1901-50	1911-60	1921-70
Fort Smith AR	9/45	4.30	6.03	5.49	6.16	3.11	2.76	3.70	5.18	5.73	4.94	4.17	3.33	4.98	4.76	4.43
Little Rock, AR	3/42	4.34	6.00	4.48	5.89	3.75	2.92	3.65	4.87	5.33	4.64	4.22	3.79	4.80	4.58	4.36
Amarillo, TX	3/41	3.32	5.69	5.54	5.26	3.91	4.02	5.01	4.89	5.41	4.95	4.47	4.29	4.80	4.87	4.74
Fort Worth, TX	12/39	4.35	5.08	5.61	5.67	3.34	3.67	3.93	4.91	5.31	4.94	4.35	3.77	4.79	4.66	4.59
Kansas City, MO	3/35	4.10	6.57	6.54	9.10	3.39	3.00	5.10	5.72	7.27	6.49	5.67	3.90	6.09	5.97	5.68
Springfield, MO	12/39	4.07	6.59	5.55	8.05	3.21	2.96	4.32	5.35	6.63	5.76	5.13	3.57	5.57	5.43	5.06
Columbia, MO	12/50	4.22	7.15	5.89	8.50	3.31	3.21	5.04	5.72	7.13	6.12	5.43	4.09	5.98	5.86	5.49
Dodge City, KS	6/42	4.23	6.61	5.85	7.91	3.68	4.39	4.47	5.54	6.67	5.86	5.45	4.07	5.68	5.70	5.26
Topeka, KS	8/46	4.23	6.50	6.79	8.77	4.03	3.49	4.95	5.79	7.24	6.61	5.78	4.27	6.12	6.04	5.84
Des Moines, IA	12/49	4.93	7.20	6.71	8.43	4.05	2.87	6.00	6.20	7.31	6.42	5.60	4.90	6.28	6.08	6.06
Dubuque, IA	1/51	4.56	7.70	6.17	7.53	4.33	3.43	6.24	6.15	7.18	6.07	5.57	5.38	6.19	6.13	6.08
Denver, CO	12/34	3.09	5.17	6.03	7.41	4.98	4.69	5.74	4.82	6.06	6.06	5.71	5.10	5.42	5.57	5.68
Cheyenne, WY	8/35	2.36	5.59	5.78	6.29	4.48	4.46	5.24	4.70	5.70	5.38	5.04	4.65	4.95	5.20	5.14
Rapid City, SD	7/42	5.69	8.23	7.90	9.28	8.26	5.95	6.03	7.19	8.30	8.31	7.74	6.63	7.72	7.73	7.38
Tulsa, OK	11/38	5.09	6.77	6.38	7.26	3.02	2.98	4.48	6.11	6.72	5.72	4.78	3.66	5.82	5.49	5.12
North Platte, NE	12/48	4.44	6.65	6.93	8.66	6.37	4.65	5.15	6.12	7.40	7.17	6.59	5.49	6.63	6.65	6.42
Lincoln, NE	12/71	4.98	7.11	7.17	10.10	5.25	2.46	4.88	6.32	8.01	7.52	6.57	4.43	6.91	6.68	6.33

TABLE 2 FREQUENCY OF AVERAGE MONTHLY TEMPERATURE DEPARTURES EXCEEDING (±) ONE AND TWO STANDARD DEVIATIONS FROM THE LONG TERM MEAN FOR SELECTED PERIODS (1887–1977)

Station: Lincoln, Nebraska

Period:		Jan.	Feb.	Mar.	Apr.	May	Jun.	Jul.	Aug.	Sep.	Oct.	Nov.	Dec.	Avg.	Total
1887–1916	± s	7	4	12	8	6	11	13	5	3	5	5	8	7.3	87
	± 2s	1	1	1			1	1	2				1	.7	8
1917–1946	± s	12	9	10	7	9	8	5	9	6	7	1	8	7.6	91
	± 2s	2	3	3	1	1	3	3			1			1.4	17
1947–1977	± s	4	8	9	3	7	9	12	4	5	4	4	8	6.4	77
	± 2s		1	2		1	1		1		1			.6	7
1887–1977	± s	23	21	31	18	22	28	30	18	14	16	10	24	21.3	
	± 2s	3	5	6	1	2	5	4	3	0	2	0	1	2.7	

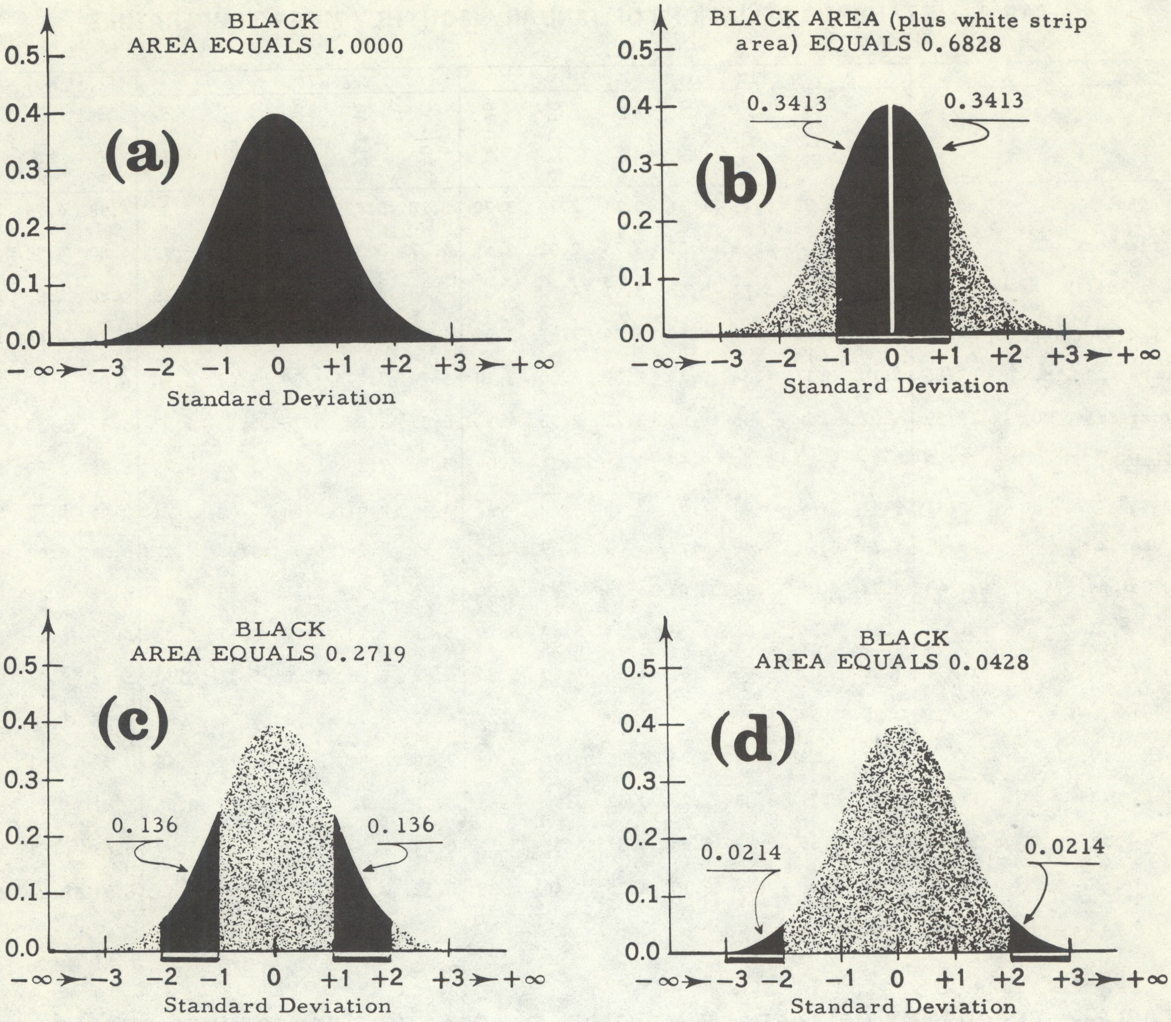

Figure 3.--a) Normal distribution curve; the percent of observations included within b) one standard deviation c) two standard deviations and d) three standard deviations are shown.

during the period 1886-1977 are 7.0^{o}F and 36.6^{o}F, we see that the actual results match the theoretical distribution very well.

We can also perform the same operation on heating degree days (HDDs) to find the likelihood, or probability, of exceeding certain values. When the average monthly temperature is less than about 40^{o}F, the standard deviation is not a factor in determining HDDs (Thom 1954a). Under these conditions, the monthly HHDs are approximately equal to the product of the number of days in the month and the difference between the average monthly temperature and the base (65^{o}F). For example, a very cold January of 7^{o}F would have (65-7) (31) = 1798 HDDs at Lincoln, while a very warm one (36^{o}F) would have (65-36) (31) = 899 HDDs. Assuming a normal distribution with a monthly mean and standard deviation of 1300 and 173 HDDs respectively, we again refer to figure 3. We would expect a January range from 1473 to 1127 HDDs (within ± one standard deviation of the mean) 68 percent of the time and a range from 1646 to 954 HDDs (or ± 2 S.D.) about 95 percent of the time. We find good agreement between the actual and the theoretical distributions in this case, as expected.

There has been a pronounced decrease in variability of the average January temperature since 1940 at the stations examined (McQuigg 1976). In spite of this relative lull, the variability measured over the last 92 years at Lincoln, Neb., fits very well with expected variability based on statistical theory. There have been remarkable differences in standard deviations from one decade to another (10°F) and differences of 3.0°F between 30-year periods. Lincoln is just one of many stations that show similar patterns, although others had less variability in January temperatures. The variability of average temperatures for winter months is not unusual, nor even unexpected at inland stations north of about 35°N latitude. One "unusual" feature at Lincoln is that during the years 1951-77 only three had average January temperatures that departed by as much as one standard deviation from the mean; this is unmatched in its 92-year history. Perhaps departures should not be thought of as being "unusual" unless they fall outside the theoretical expectation of two standard deviations from the mean or average value.

Changing the base period for normals from 30 to 40, 50, or 60 years would reduce the standard deviation of monthly temperatures as seen in table 1. The problem os how best to use "normal" values in relation to future weather probably should be considered on an individual basis. Certainly, it is evident that variability is a built-in and elusive feature of climate that has, thus far, evaded capture.

The maps of standard deviations of monthly average temperatures that follow are based on the 1941-70 period. They reflect the variability of that period only. Just as inclusion of the 1931-40 data in computing standard deviations for 30-year periods caused them to be higher than the long-term values, the inclusion of the 1951-60 data in the 1941-70 study has produced standard deviations lower than the long-term averages. This lower figure will also be reflected in the next set of normals which will be based on the 1951-80 period.

No adjustments were made to the average temperatures in this study. The standard deviations for the maps were based on observations from cooperative stations where any change in location was adjudged not significant. The standard deviations in table 1 were computed from the unadjusted published temperatures from city offices and airports. According to Thom (U.S. Dept. of Commerce 1968), the variance will always be increased by station location changes which change the mean, and the standard deviation for a long record will always have a larger positive bias than for a short record. This does not seem to be the case with the stations examined in this report. The effect of non-random climatic fluctuations appears to be by far the major cause of shifts in the standard deviations of average monthly temperatures. It also appears that increasing the record length to 50 years results in a more stabilized value for this statistic.

Four other aspects of standard deviation of temperature, discussed by Thom (1968) are reprinted from that report in the Appendixes.

APPENDIX 1. CONFIDENCE INTERVALS

All climatological normals are some form of mean based on samples. These are estimates of the "true" population expected values which one would like to know but can never know. They are, therefore, statistics which are always subject to the usual variation of random sampling. If it were possible to take several sample records, these estimates of normals would vary from sample to sample. It is useful to know the magnitude of this variability, since this would give an indication as to how close this value is to the "true" value. Statistical analysis has provided several techniques for depicting this variability; the most appropriate for our purpose is the confidence interval. The confidence interval consists of two values of a random variable which determines the interval between them. It is essential to keep in mind that it is determined by two random end points which are estimated from the sample. These are determined in a manner which ensures that the "true" value or population value will be covered by the interval with a preassigned probability called the confidence. When one sees the familiar notation: a parameter has been estimated to be a with a probable error e', often written as a ± e', it should be remembered that this really means $a - e' \leqq \alpha \leq a + e'$, with a confidence of 0.50, or 50 percent; i.e., the random interval (a - e', a + e') will cover or enclose the "true" value in 50 percent of the samples. Hence, the probable error determines a 50 percent confidence interval which says that in 50 percent of the samples the interval will contain α, and in 50 percent of the samples it will not contain α.

The theory of confidence intervals has been highly developed by statisticians (Mood 1950). The essential problem is to determine the distribution of (α-a) where α is the parameter or population true value and a is a statistic for estimating α from a sample. Given the distribution of (α-a), the probability of (α-a) lying on an interval (-b,b) may be expressed by:

$$P(-b < (\alpha - a) < b) = p.$$

Adding a to each member of the inequality which, in effect, is an equiprobability transformation gives

$$P(a - b, < \alpha < a + b) = p.$$

This is the expression for the confidence interval with confidence probability p. It gives the probability that α will be on the interval (a-b,a+b) in sampling to estimate α.

If τ is the true monthly mean temperature, $\bar{t}$ is an estimate of it based n years or record, and s is the standard deviation of monthly average temperatures given by the charts, $\sqrt{n}(\tau - \bar{t})/s$ has a "Student" t distribution. The interval on the t-distribution for 0.90 probability is then

$$P(-t_{0.05} < \sqrt{n}\,(\overline{t} - \tau)/s < t_{0.05}) = 0.90.$$

Multiplying by $s/\sqrt{n}$ and adding $\overline{t}$ gives the 0.90 confidence interval

$$P(\overline{t} - t_{0.05}\, s/\sqrt{n} < \tau < t + \overline{t}_{0.05}\, s/\sqrt{n}) = 0.90. \quad (2)$$

From Table IV (Mood 1950) for 29 degrees of freedom $t_{0.05} = 1.699$, hence for $n = 30$, $t_{0.05}/\sqrt{30} = 0.310$ and the confidence interval is

$$\overline{t} - 0.310 \leqq \tau \leqq \overline{t} + 0.310. \quad (3)$$

Examination of the t-table shows that for large n, the t-distribution approaches normality. Hence, it follows that for large n, $\sqrt{n}\,(\overline{t} - \tau)/s$ is approximately normally distributed and even for n = 30 gives a good approximation of the confidence interval. Substituting the 0.05 quantiles from the normal table for $t_{0.05}$ in equation (2) and simplifying gives

$$\overline{t} - 0.300 \leqq \tau \leqq \overline{t} + 0.300. \quad (4)$$

This will be satisfactory for most purposes (see discussion in Thom 1952).

As an example we may estimate the 0.90 confidence interval for the December normal temperature at Minneapolis Airport: From U.S. Dept. of Commerce (1956) we find $\overline{t} = 19.4^{o}F$; from the December chart $s = 5.0^{o}F$. Substituting in equation (3) gives the 0.90 confidence interval

$$17.9 \leqq \tau \leqq 21.0.$$

Thus, the interval (17.9,21.0) will enclose the "true" normal with a probability 0.90 which gives an estimated normal 19.4.

It is clear that since the charts only give estimates of the true standard deviation σ, s is also a statistic and will, therefore, have a confidence interval. This is found by following a procedure similar to that for the normal temperature except that the distribution of the variance s^2 is used, viz., the well-known χ^2 distribution. The confidence limits on s then are

$$\sqrt{(n-1)s^2/\chi^2_p} \leqq \sigma \leqq \sqrt{(n-1)s^2/\chi^2_{1-p}} \quad (5)$$

where p is the probability of $\chi^2 < \chi_p^2$ and 1 - p is the probability of a $\chi^2 < \chi_{1-p}^2$ and the confidence is 2p.

For 0.90 confidence intervals, 1 - p = 0.95 and p = 0.05. Hence, from Table III of Mood (1950) with 29 degrees of freedom $\chi_p^2 = 42.6$ and $\chi_{1-p}^2 = 17.7$. Substituting in equation (5) gives the 0.90 confidence interval for σ

$$0.83s \leq \sigma \leq 1.28s. \qquad (6)$$

For Minneapolis, again s = 5.0; so, the required confidence interval for the "true" standard deviation is

$$4.1 \leq \sigma \leq 6.4.$$

APPENDIX 2. PROBABILITIES

It is well known that the average monthly temperature is approximately normally distributed. Some have overemphasized the significance of its departure from normality, but this has been mostly a result of a lack of understanding of the difference between statistical significance and practical significance as well as some lack of understanding of statistical tests themselves.

As to the first, there may be small skewness and kurtosis in the distribution, but if one is interested in obtaining probabilities, this is usually of little importance. In most instances, the fit of the normal curve is indeed remarkable as compared to the fit of even the best meteorological theories to atmospheric observations. Secondly, previous investigators have often failed to take into account the correlations between stations. These tend to produce many so-called departures from normality which are areally distributed around a station with a large departure. This same effect is found at a single station through a sequence of months because there is also a month-to-month correlation. Hence, the significance tests on such data must be interpreted differently than has generally been done in the past. Over and above all this heuristic argument there is the powerful mathematical theorem, the central-limit theorem, which says that averages tend to be normally distributed as the number of their sample elements increases, subject to only weak conditions on the original distribution which are almost always met in meteorological data. This is not a mere physical "law" but a mathematical theorem.

Sumner (1953) shows the smallness of the difference between expected frequency for several months and stations. Totaling of his frequencies gives an expected value of 555 as compared to an actual value of 558 at one standard deviation from the means, a remarkable agreement. Assuming the actual value to be distributed in a binomial distribution with n = 555 and the probability corresponding to his one standard deviation, the 0.95

acceptance region for a test of hypothesis on the actual frequency is $533 \leq N \leq 577$ where N is the true frequency. It is seen that the actual frequency total 558 lies well within this interval, in fact somewhat too well because of the correlation. To remove this correlation effect, one may consider only the January values for Havre, Mont.; Concord, N.H.; and New Orleans, La., which are far enough apart so that the correlation is low. For these stations the expected frequency totals to 102 while the actual is 105. The binomial distribution for n = 102 gives a 0.95 acceptance region of $92 \leq N \leq 112$. Hence, even with the correlation removed, the observed total lies well within the variation due to random sampling and is a further verification of the validity of the assumption of normality.

As for the estimates of the normals assumed in Section 2 to be normally distributed, this is a fortiori true by the central-limit theorem. This may be readily shown by a consideration of the behavior of the skewness β_1 and the kurtosis β_2 when the random variable is averaged as it is to obtain an estimate of the monthly normal. If $\beta_1(t)$ and $\beta_2(t)$ are for a series of monthly averages, the skewness and kurtosis for the estimate of the normal will be $\beta_1(t)/n$ and $[\beta_2(t) - 3]/n$. It is readily seen from these relations that for our sample size 30, the effect of nonnormality will be one-thirtieth as important for the distribution of the estimates of the normals as for the original series and, hence, small indeed.

EDS's conclusion, then, is that satisfactory estimates of probabilities may be obtained using the standard deviation charts in conjunction with the normals (U.S. Dept. of Commerce 1956) through use of the quantity $(\tau - t)/s$ as an entry in any normal probability table. A convenient short table is given in Thom (1952) and is repeated below:

Normal Probability Table

$P[x < (\bar{x} - zs)]$	z Standard Deviation Units	$P[x < (\bar{x} + zs)]$
.02	2.05	.98
.05	1.64	.95
.10	1.28	.90
.15	1.04	.85
.20	.84	.80
.25	.67	.75
.30	.52	.70
.35	.39	.65
.40	.25	.60
.45	.13	.55
.50	.00	.50
$P[x > (\bar{x} + zs)]$	z	$P[x > (\bar{x} - zs)]$

Read down for headings and up for footings.

APPENDIX 3. QUANTILES OF TEMPERATURE FUNCTIONS

By a temperature function we mean any function of the monthly average temperature or any other available temperature statistic. Only functions of the monthly average temperature are discussed here. These methods are quite general and arise from the fact that the location and scale (mean and standard deviation for the normal distribution) of the distribution of the function are related to the location and scale of the available statistic. Often this is a simple linear relationship. The relationship will in general be uniform over a climatic region; i.e., over a region in which the meteorological conditions have the same causes, e.g., a meteorologically homogeneous region (see Thom 1940).

These regions may often be established by delimiting them through meteorological analysis of a few well-distributed stations and then adjusting back and forth between the results of the analysis and the degree of uniformity of the functional relationship found until the region fits both theory and observation. The relationship may then be applied to any station in the region. The methods are often quite useful since they readily provide estimates of quantiles of the function from available statistics which would ordinarily be very costly and sometimes even impossible to obtain by direct methods.

The basic relationship for meteorological elements where location and scale parameters only are involved (e.g., temperature) arises from the simple standardized variate $k = [g_p(t) - \overline{g(t)}]/s[g(t)]$ where p designates the probability associated with the quantile on the distribution of g, g(t) is its location, s(g(t)) its scale, and k is the standardized variate. Solving for the quantile measured from the location,

$$g_p(t) - \overline{g(t)} = k\, s[g(t)].$$

Hence, if

$$\overline{g(t)} = \overline{t}$$

and

$$s[g(t)] = ms(t)$$

is a linear function then

$$g_p(t) - \overline{t} = kms(t) = bs(t). \qquad (7)$$

If the mean and standard deviation are substituted for the location and scale, we have the relationship which was suggested for obtaining the design temperatures in the Climatological Atlas of Canada (Thomas 1953).

If $\overline{g(t)}$ is a linear function of $\bar{t}$ this may be established independently from the available data. Since equation (7) will still be a linear function, it may also be established simply by plotting and fitting a line. The relationship to scale will usually be a linear function as given unless g is nonlinear. Should neither the location nor scale relationships be linear then the relationships could be established on the available data or, again one could resort to plotting and fitting equation (7). An excellent example of the application of these methods is discussed by Thomas (1955).

APPENDIX 4. DEGREE DAYS

The approximation of normal monthly degree days from temperature normals and standard deviation has been discussed in detail in Thom (1954a,b, 1966). The standard deviation charts presented here have found their greatest use in such approximations. The conservatism of the standard deviation and its relative independence of difference between stations, factors which greatly affect normals, make it possible to use the charts to obtain standard deviations at stations where only means or normals are available. This in turn makes it possible to obtain quickly degree-day normals for stations where only means are available, thus resulting in a considerable saving in computation.

The conversion from temperature to degree days may be expressed by the generalized formula

$$\bar{\bar{D}}^{\pm} = N[\pm(\bar{t}-b) + \ell(\mp x_0)\sqrt{N}\ s]. \qquad \begin{array}{l}(+) = \text{degree days above b}\\(-) = \text{degree days below b}\end{array}$$

Here $\bar{\bar{D}}$ is the normal degree days for the month concerned, N is its number of days, s is its standard deviation obtained from the charts, b is the base, and $x_0 = (b - \bar{t})/\sqrt{N}\ s$). $\ell(x_0)$ is a universal factor (Thom 1966) which applies to all degree days above or below any base with the conditions that for degree days below the base the negative sign is to be taken in the formula and for degree days above the base the positive sign. It is obtained from the following formulas or the table in Thom (1954a):

$$\ell(x_0) = 0.34\ e^{-4.7x_0} - 0.5\ e^{-7.8x_0}$$

and

$$\ell(-x_0) = \ell(x_0) + x_0.$$

APPENDIX 5. SCALE CHARACTERISTICS

Nagao (1951) and Sumner (1953) have both discussed certain aspects of their charts in general, attempting to relate them to geographical and meteorological parameters. In view of the central position of statistical analysis in analytical climatology we prefer to use statistical concepts in discussing more or less the same properties. Thom (1954a) used such concepts in developing the relationship between degree days and temperature.

The climatological distribution, like the statistical frequency distribution, may be said to have three general properties: location, scale, and shape. These are determined by the mathematical form of the distribution and measured by its parameters. The location of the distribution is determined by the position of the distribution curve along the climatological variable scale. In the case of temperature it is measured by the normal. The scale of the distribution is determined by the comparative values of the units in which the climatological variable is measured. If the scale is larger, the frequency distribution will be spread along a considerable range of the climatological variable; if it is small, it will be spread only along a narrow range of the variable. For temperature, the scale is determined by the standard deviation of the distribution. The last property, shape, is determined by the position of the mode or modes in respect to the location of the distribution. It is measured by the skewness and kurtosis or flatness of the distribution, but largely by its skewness and kurtosis as geometrical rather than analytical properties. The temperature distribution is essentially symmetrical and, hence, has what may be called the zero shape or the standard shape from which the departure of other shapes is measured. In general, the skewness is related to the boundedness of the climatological variable. If bounded below, the distribution will be skewed to the right, if bounded above--skewed to the left. Thus the precipitation distribution, for example, is skewed to the right because it has a zero bound. Since the standard deviation is the scale property of the temperature distribution, we may discuss this aspect of monthly average temperature in terms of the standard deviation exclusively.

Perhaps the most noticeable property of the standard deviation of monthly average temperature is its conservatism relative to the location and mean. The range in mean for January from Florida to Montana is more than 60°F, but the range of the standard deviation is only about 8°F. Thus, the standard deviation changes only about one-eighth as fast geographically as the mean. Experience in drawing the charts also indicated that the scale is relatively more independent of local changes. This is important in the application of the charts for it makes interpolation much more effective than it would be on charts of the means. For example, if a station is moved in such a fashion as to cause a considerable shift of the mean because of local effect, it may be assumed that the population standard deviation will be essentially unchanged. Thus, the standard deviation charts should continue to be useful even though changes in station location make the revision of normals necessary.

Another striking property of climatological scale is its correlation with the location. In statistical analysis the normal distribution has the well-known important property that the mean and standard deviation are uncorrelated. This, however, is for sampling from the same population. In climatology we are often sampling successively from a number of populations or at different stations such as the stations on the standard deviation charts. Here the standard deviation is negatively correlated with the normal, i.e., the lower the normal the higher the standard deviation. This correlation also expresses itself in a similar fashion seasonally. Over a meteorologically homogeneous region this relationship may be assumed to be linear and invariant as has been done in Section 4. For climatological variables which are bounded below we have found the standard deviation to be positively correlated with the mean because of the effect standard deviation has on the shape in bounded distributions.

Proximity to bodies of water, as would be expected, has a striking effect in stabilizing the scale. For example, the temperature scale is practically identical from Maine to Florida in January while the range of the mean is more than 50°. This effect tends to extend farther inland on the west coasts of the continents in middle latitudes because of the prevailing westerlies. In contrast to this, one observes the large-scale change from south to north in the continental part of the United States. This scale-measure of continentality has long been recognized by climatologists.

The centers of large scale are also interesting, being associated with air mass sources. In the winter months there is a large closed center of high scale or variability, the lower part of which is in the north-central part of the country. As the season progresses into spring, this center moves southeast until in midsummer there is a flat closed center in the central plains area. When the sun moves south again in autumn the center moves northwest, returning to its typical winter position. This movement is probably related closely to the intensity of radiation together with the length of day. In winter, the long night in the north coupled with intense nocturnal radiation produces a center of large scale. In summer, the longer day together with the intense radiation through the clear skies of the central plains causes the high center in that area.

There are other aspects of scale which are of interest, and it is hoped that this brief discussion will stimulate others to study this interesting problem of dynamic climatology in greater detail.

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STANDARD DEVIATION OF MONTHLY AVERAGE TEMPERATURE (°F), JANUARY (1941–70)

STANDARD DEVIATION OF MONTHLY AVERAGE TEMPERATURE (°F), FEBRUARY (1941–70)

STANDARD DEVIATION OF MONTHLY AVERAGE TEMPERATURE (°F), MARCH (1941–70)

STANDARD DEVIATION OF MONTHLY AVERAGE TEMPERATURE (°F), APRIL (1941–70)

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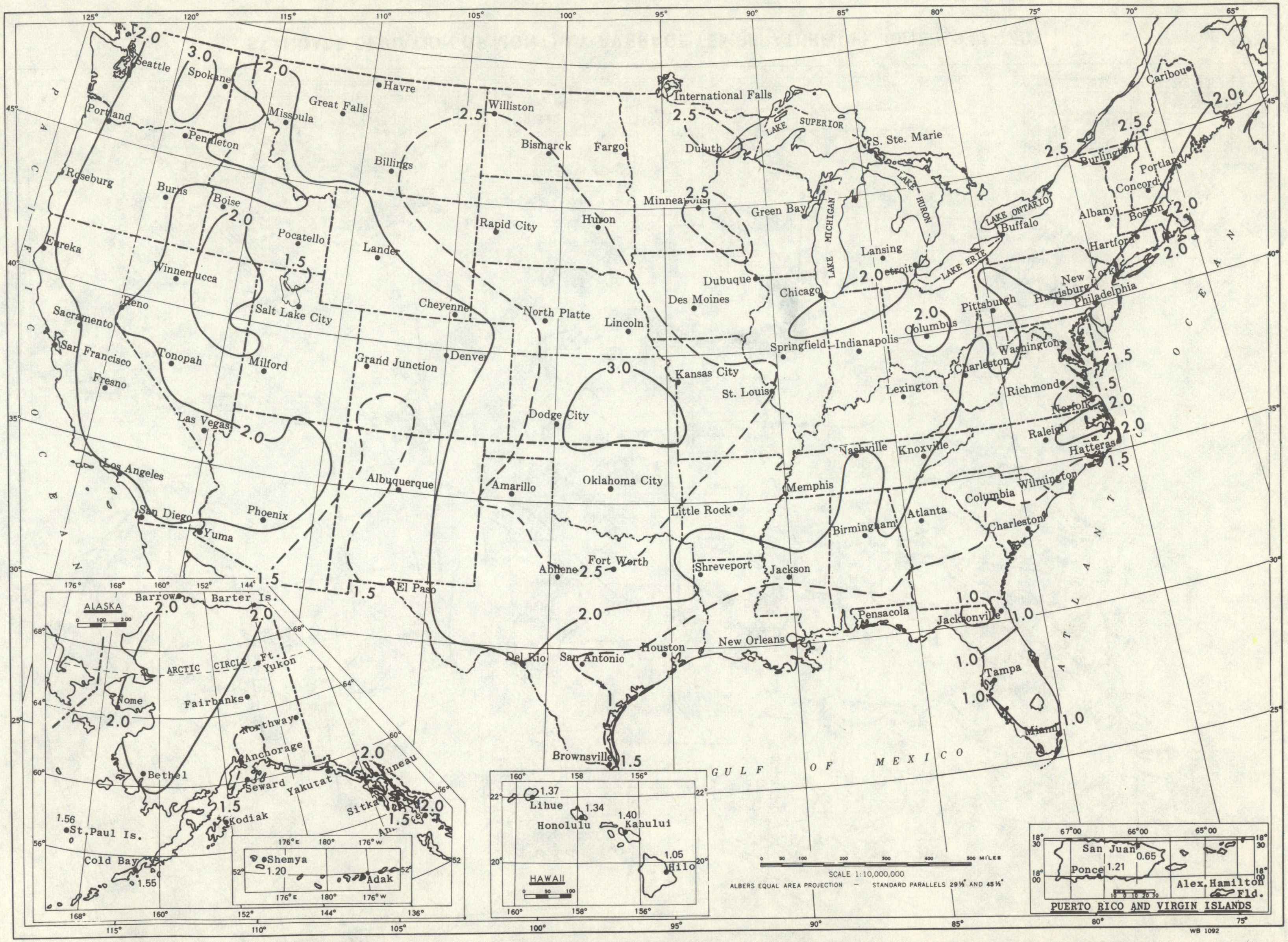

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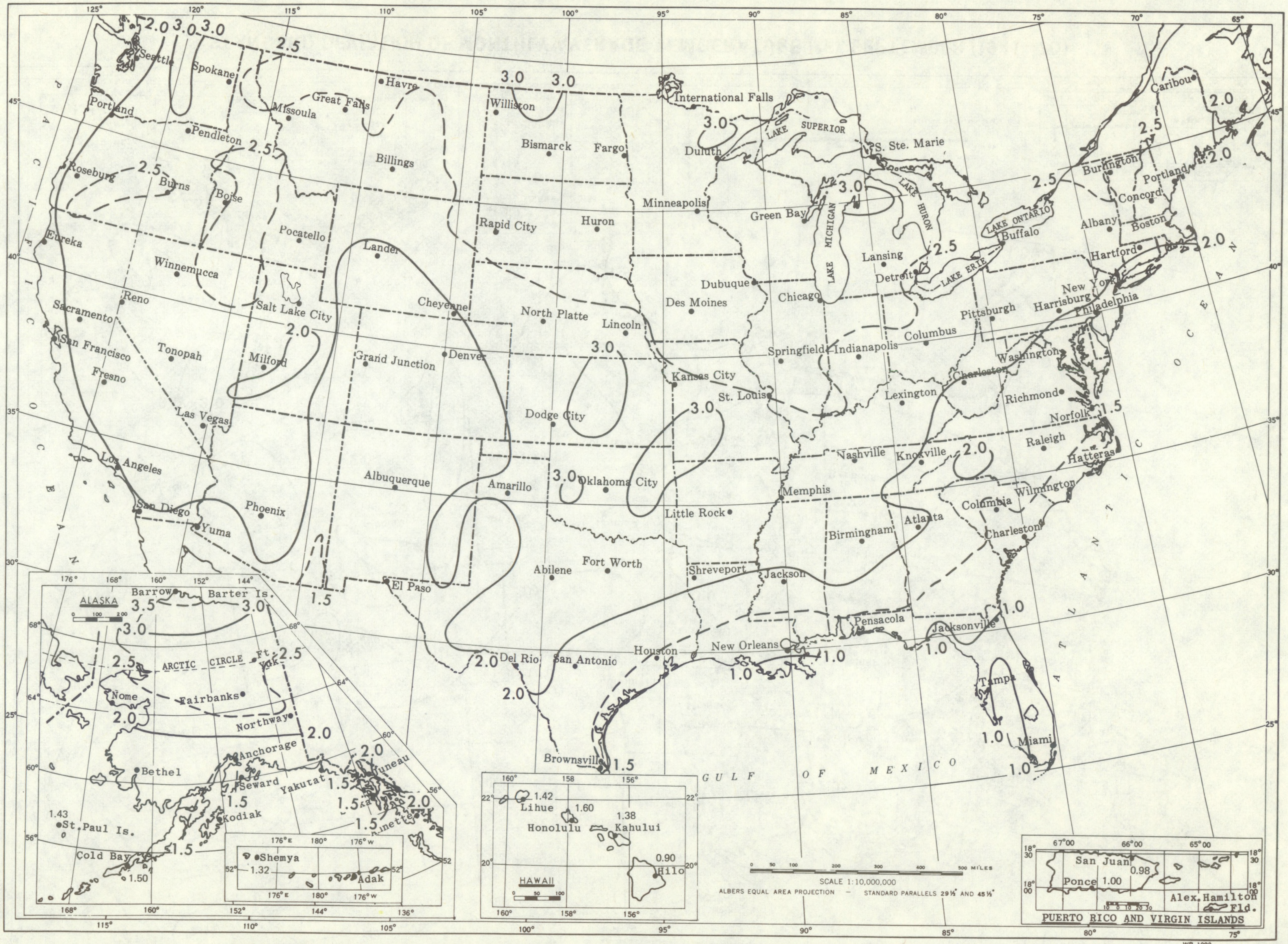

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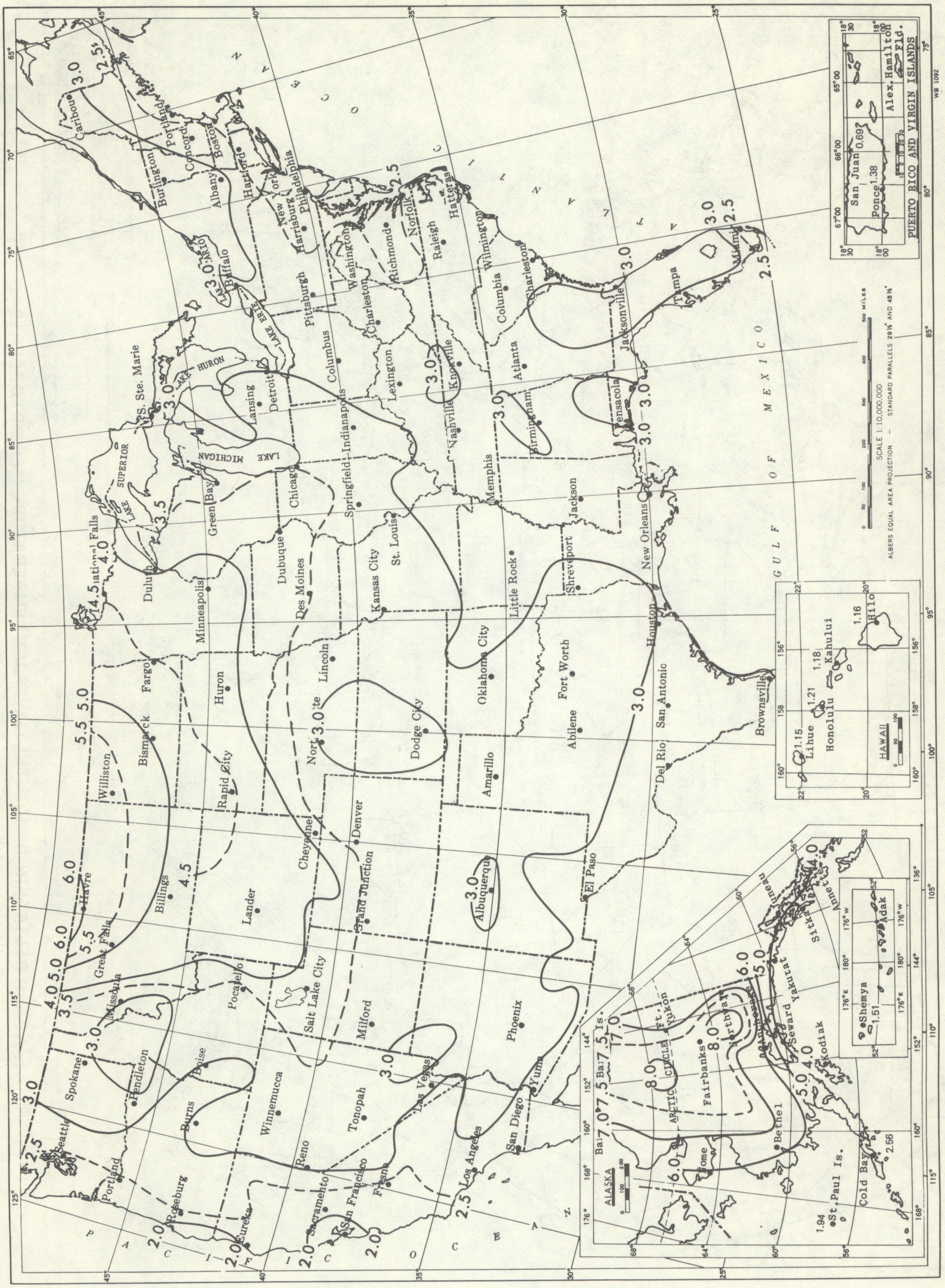

STANDARD DEVIATION OF MONTHLY AVERAGE TEMPERATURE (°F), NOVEMBER (1941–70)

STANDARD DEVIATION OF MONTHLY AVERAGE TEMPERATURE (°F), DECEMBER (1941–70)

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