NOAA Technical Report NOS 66 NGS 2

Effect of Geoceiver Observations Upon the Classical Triangulation Network

Rockville, Md.
June 1976

U.S. DEPARTMENT OF COMMERCE
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EFFECT OF GEOCEIVER OBSERVATIONS UPON THE CLASSICAL TRIANGULATION NETWORK *

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ABSTRACT. This paper investigates the use of Geoceiver observations as a means of improving triangulation network adjustment results. A test network of real data is used in this study, which is comprised of 32 separate projects and contains 838 first-order and 489 second-order stations in the States of Mississippi, Louisiana, and Alabama. Statistics are provided on a sequence of adjustments of this network in which the number of azimuth, base line, and Geoceiver observations were systematically varied. From an analysis of this sequence of adjustments, three important conclusions are made. First, the most effective separation for Geoceiver observations is about 250 km and greater. Second, there is a limit to the improvement in the a posteriori standard error that Geoceiver observations can effect in a triangulation network. Third, Geoceiver observations are an effective means of controlling distortions in the local network. The theory of how Geoceiver observations combine with the classical observations is explained.

1. INTRODUCTION

The National Geodetic Survey (NGS) is assembling data for the proposed readjustment of the North American network of triangulation. To correct the known areas of distortion and weakness in the existing network, the following additional observations are being considered: very long base lines (VLBI), satellite, Doppler, transcontinental traverse (TCT), geodimeter base lines, and astronomic azimuths. These observations will be included in the adjustment in order to strengthen the network.

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*Geoceiver is a trade name for the Doppler satellite tracking instrument manufactured by Magnavox Corporation. Doppler satellite tracking instruments made by other companies are available. While these results are based upon Geoceiver observations, there is no reason to suspect that any other comparable Doppler satellite tracking instrument would not give the same results.

One of the methods that will be used to improve the large scale configuration of the triangulation network is a planned network of approximately 150 Geoceiver positions. The Geoceiver, a relatively new technological development in geodesy, is used to measure the Doppler shift in the two coherently related signals transmitted by the Navy Navigation Satellite System (NNSS). The adjustment of a large number of these observations gives the position determination for the Geoceiver station used in the space rectangular coordinates of the Naval Weapons Laboratory (NWL) 9D system. The RMS difference residual for these position determinations is generally in the range from 0.15 m to 0.30 m in the X, Y, and Z directions. The sequence of steps to effect a transformation of this position in the geocentric NWL 9D system to the ellipsoidal coordinate system of the North American Datum (NAD) is given by Meade (1974). There is, first, a coordinate origin shift from the geocentric origin to the NAD origin. The XYZ space rectangular coordinates are then transformed into the ϕ, λ, and h ellipsoidal coordinates of the NAD. The origin shift that is applied is the mean of the origin shifts that were required at 36 reliable Geoceiver stations to correct the Doppler coordinates to the transcontinental traverse coordinates. Meade found that there is a small systematic difference of about 1.0 ppm between the NWL 9D coordinates and the TCT coordinates. It is common practice to bring the Doppler coordinates into closer agreement with the TCT coordinates by applying a correction to the 9D coordinate system. This correction is expressed by Anderle (1974) as a small scale change and a rotation, performed in ellipsoidal coordinates, on the NWL 9D coordinate system, giving a new coordinate system called NWL 10F. The correction transformation is:

$$\phi_{10F} = \phi_{9D}$$

$$\lambda_{10F} = \lambda_{9D} + 0''.260 \quad (\lambda \text{ east is positive})$$

$$h_{10F} = h_{9D} - 5.27 \text{ m} \quad (h \text{ height above a common ellipsoid})$$

The new origin shift parameters to transform into the TCT coordinate system are given by Vincenty (1975) as:

mean (meters)	std. error of mean (m)
$\Delta X = + 19.60$	0.22
$\Delta Y = - 155.02$	0.18
$\Delta Z = - 175.12$	0.18

Meade shows that these Doppler coordinates now have a mean difference from the TCT coordinates of only 1.03 m in latitude, 1.01 m in longitude, and 1.25 m in height. The contemplated system of Geoceiver stations includes about 65 stations that are also on the TCT. When all these stations are used, there will be a better determination of the systematic difference between the 9D and TCT coordinate systems. The agreement then between transformed Doppler and TCT coordinates is expected to be better.

The a priori positional standard error for a Geoceiver observation used in the adjustments in this paper is 0.9 m in latitude and 1.2 m in longitude with no correlation assumed to exist between the two components. NGS has adopted this standard error for use in Geoceiver observation evaluation studies only.

It is generally agreed that Geoceiver observations used as positional constraints will greatly improve the large scale configurations of the network.

On the smaller scale, weaknesses in the triangulation network have traditionally been strengthened by observing more lines, more distances, or more azimuths in the network. The strengthening of a network by observing additional lines is seldom done because of the expense of moving personnel and equipment back to the area and rebuilding the observation towers. Even though the distances between main scheme network stations may be easily and accurately measured with an electro-optical distance measuring instrument (EDM), using an EDM instrument for observing additional distances is often not a practical solution because the stations are not intervisible without observation towers. Strengthening a network by means of additional azimuth observations is expensive because the astronomic field party must not only observe the astronomic azimuth of a network line but also the astronomic position of the azimuth station. These observations require a skilled observer and quite often much time is lost due to overcast sky conditions. The Geoceiver has none of these drawbacks; it is portable and easy to operate. Observations are not expensive to obtain as the major item, the satellite, is provided by the U. S. Navy; intervisibility of network stations is not necessary, and since the instrument operates in the radio-frequency range, overcast skies are of no concern.

An obvious question is: Why can't additional Geoceiver observations be used to provide scale and azimuth constraints to the local system? Since Geoceivers are new, not much is known about the interaction of their observations with classical triangulation networks. It is toward alleviating this lack of understanding that this paper is directed. In general, we would like to know how Geoceiver observations affect the triangulation network, and particularly, how many Geoceiver observations (and their location) are needed to improve a weak triangulation network.

In this investigation, the test network is comprised of 32 separate projects, which contain 838 first-order and 489 second-order stations in the States of Mississippi, Louisiana, and Alabama. This is the same network used by Dracup (1975). One major difference, based on the suggestion of Dracup, has been the removal of the transcontinental traverse projects so that the test network would be similar to most of the triangulation in the United States. Within the combined network, there are five existing Geoceiver stations.

Station	Location	Transferred From	Doppler Number
Knob 1914	North		10022
Winn 1929	West	Greenville AFB 1957	10003
Little 1934	South	point near Little RM A	20016
Kelley 1971	East	point near Kelley 1971	51009
Webster 1929	Center	Webster 1939 RM 1	10023

Figure 1 shows the first-order, main scheme network and locations of the five stations where Geoceiver observations were made. Figure 2 shows the combined first-order, main scheme and second-order, main scheme networks. Figure 3 shows the locations of the 18 lengths, either base lines or geodimeter lines, in the first-order, main scheme network. Twenty-seven geodimeter lines in the first-order area project around station Kelley 1971 have been removed to give a more balanced system of length observations. Figure 4 shows the location of the 22 azimuth observations that orient the first-order network.

Table 1 shows the principal characteristics of the 13 test networks, formed from the observational data, that were adjusted and used in the analyses described later.

2. EFFECT OF GEOCEIVER OBSERVATIONS UPON THE LENGTH AND AZIMUTH STANDARD ERRORS

In this section, as well as sections 3 and 4, only the accidental errors which exist in the observations are considered. It is realized that systematic errors are probably present, but no means of detecting them were apparent to the authors.

An important question when considering the employment of Geoceiver observations as constraints in an existing triangulation network is: What is the best arrangement of Geoceiver stations? What quantity are we concerned with here? Since the local surveyor can directly observe the length or azimuth of any line in the National network, it is desirable that these observables in the National network be of such an accuracy that the local surveyor cannot detect discrepancies. Traditionally, therefore, the quality of a geodetic network has

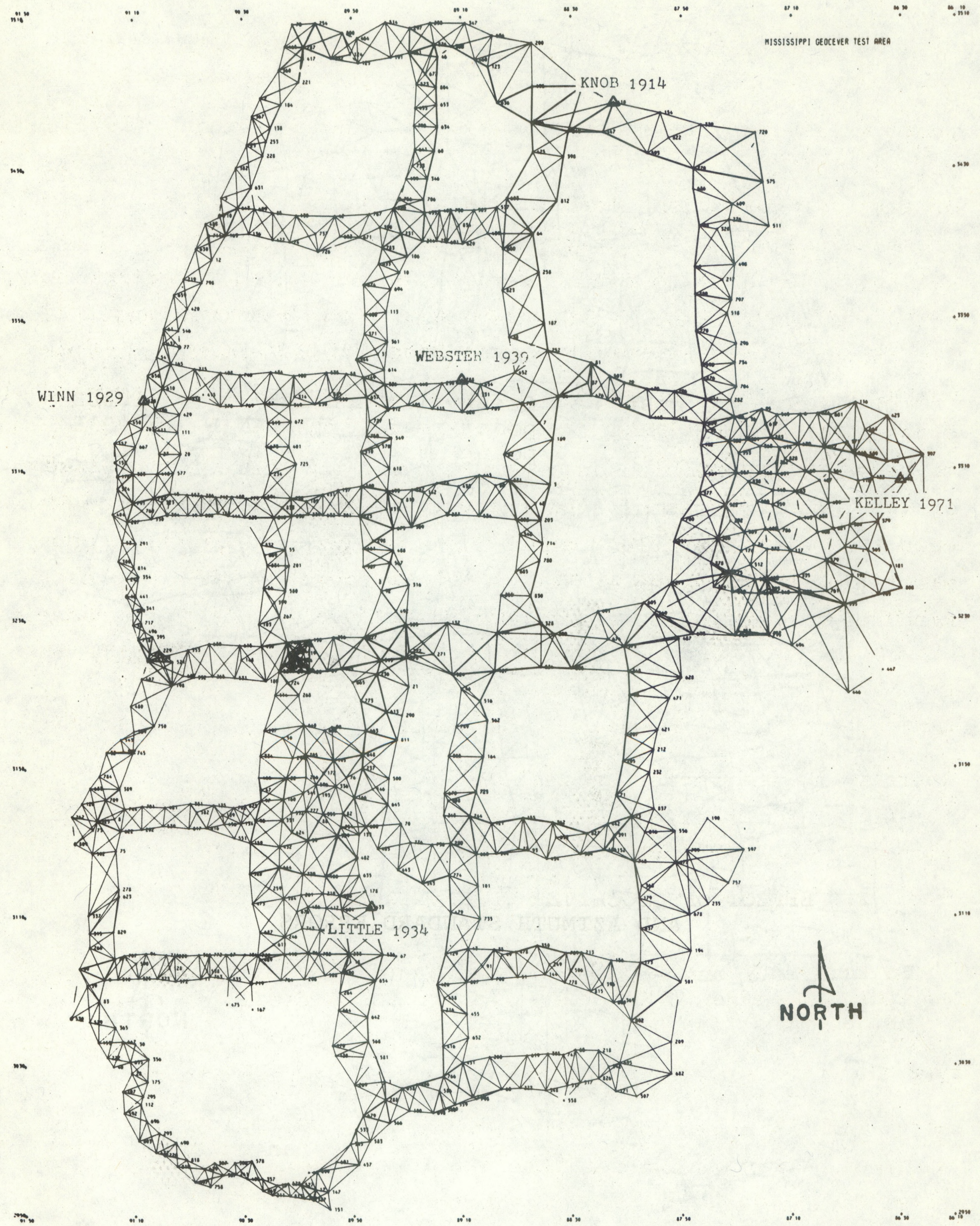

Figure 1.--Geoceiver test area first-order network.

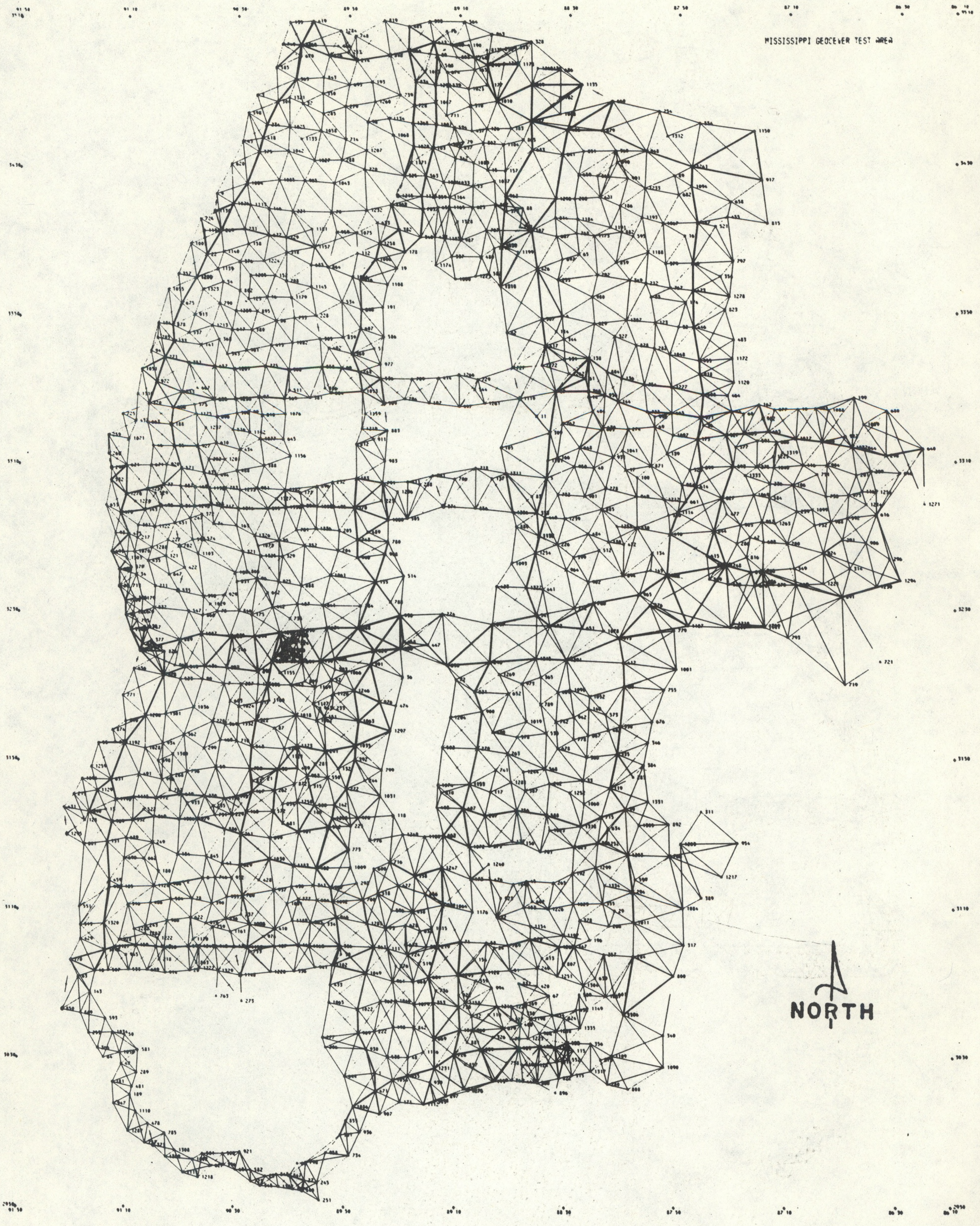

Figure 2.--Geoceiver test area first- and second-order networks.

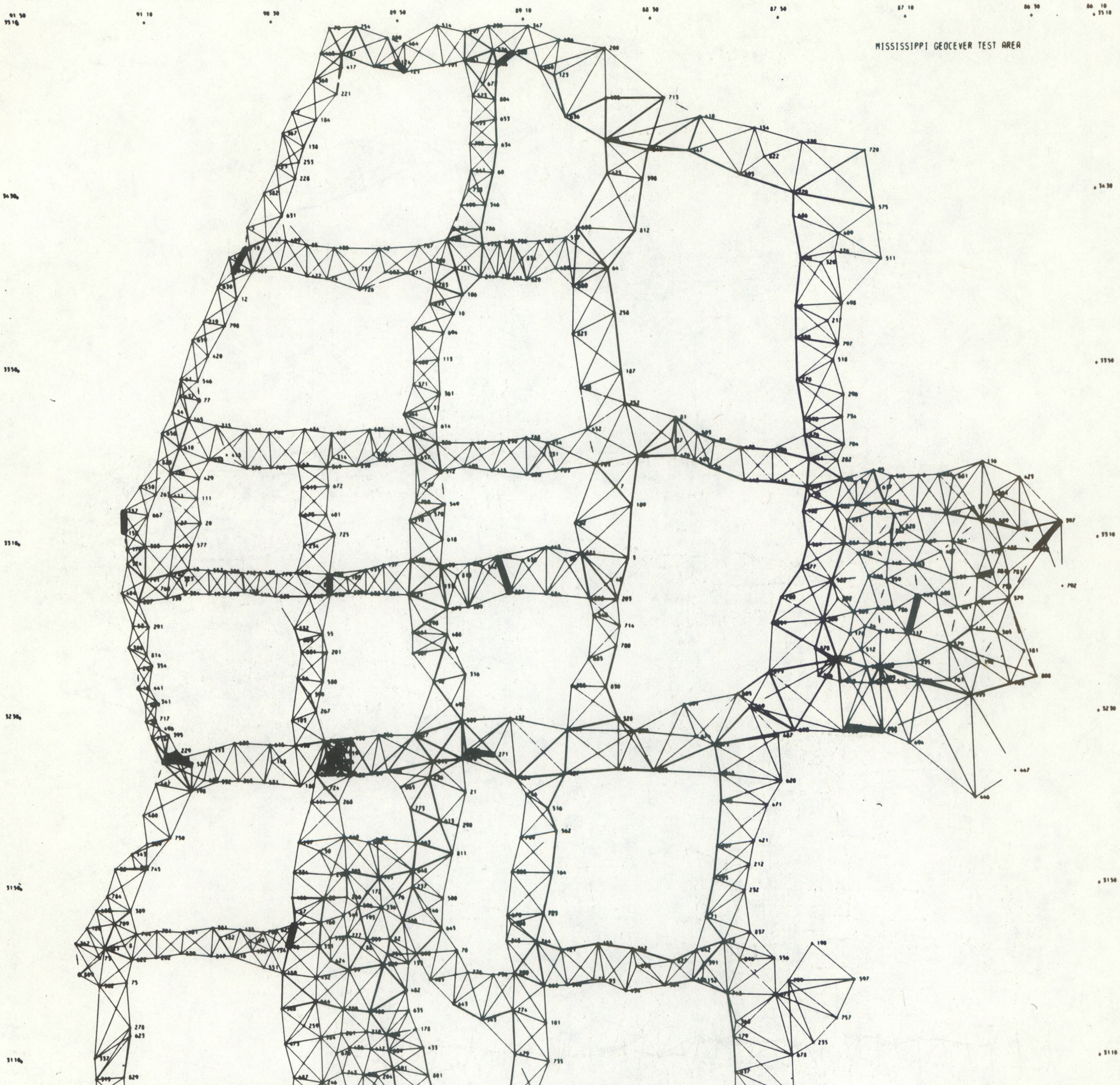

Figure 3.--Geoceiver test area first-order network base lines.

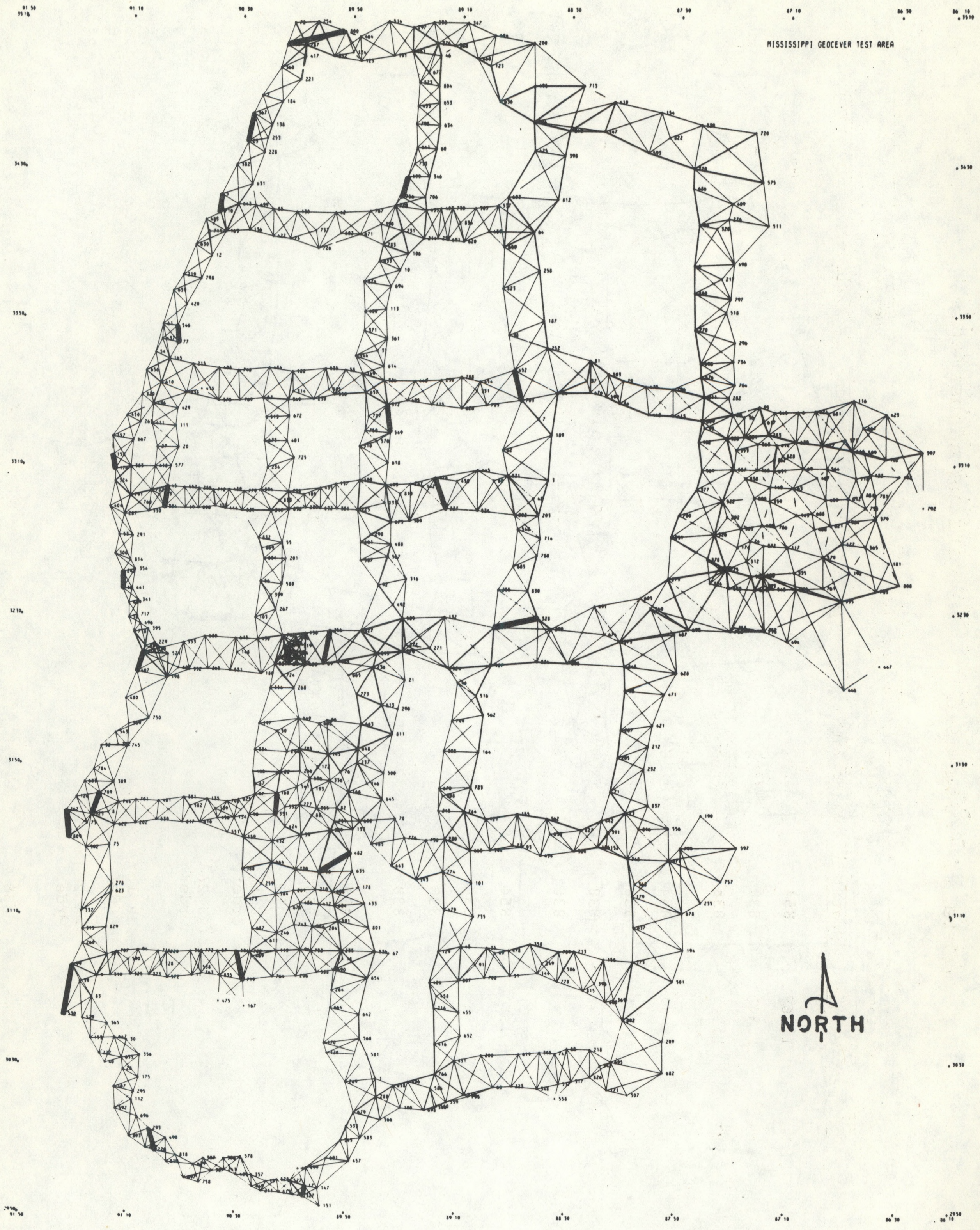

Figure 4.--Geoceiver test area first-order network azimuths.

Table 1.--Data used in the various adjustments.

Adjustment	Variance in Unit Weight	Triangulation Networks				Geoceiver		
		Stations	Directions	Distances	Azimuths	Number	Separation	Orientation
B	1.369	854	7297	0	0	5	-	NSEW
$B^{1/2}$	1.343	838	7202	8	8	5	-	NSEW
B^+	1.342	838	7202	15	18	5	-	NSEW
C	1.371	854	7297	0	0	2	426 km	N-S
C'	1.345	838	7202	0	0	2	266	N-S
C"	1.345	838	7202	0	0	2	350	N-S
C'^+	1.345	838	7202	15	18	2	266	N-S
D	1.371	854	7297	0	0	2	181	E-W
D'	1.344	838	7202	0	0	2	253	E-W
D"	1.345	838	7202	0	0	2	350	E-W
D'^+	1.345	838	7202	15	18	2	253	E-W
E*	1.347	838	7202	42	18	1	-	Center
E*	1.347	838	7202	42	18	--	-	Center
F*	1.273	838 1st Order 498 2nd Order	12195	63	22	1	-	Center
G*	1.272	838 1st Order 498 2nd Order	12195	63	22	5	-	NSEW
H	1.345	838	7202	0	0	2	578	N-S

been judged by the size of the length and azimuth standard error between nearby stations. The Geoceiver observations may then be thought of as being for the purpose of effecting a reduction in the size of the length and azimuth standard errors.

The investigation is carried out by performing a series of adjustments in which the number of base lines, azimuths, and Geoceiver stations in the first-order network is varied. The distance and azimuth standard errors in each of these solutions are computed at 44 selected lines in the network. The description of these lines is given in table 2. The lines, chosen so that they span the open areas between the arcs, are used to observe the movement of one arc relative to another. In general, each of the twenty-five areas (see figure 5) has a line oriented north-south and east-west. These lines are the same as those used by Dracup (1975).

The optimum arrangement and spacing of Geoceiver stations are investigated first. Adjustments C, D, C', D', C", D", and H are adjustments of the 838 station, first-order, main scheme network. These adjustments do not contain any base lines or azimuths. The scale and azimuth constraint are provided by two Geoceiver observations.

Adjustment	Stations		Orientation	Separation (km)
C	Knob 1914	Little 1934	north-south	426
C'	Webster 1939	Little 1934	north-south	266
C"	Little 1934	Thackers 1934*	north-south	350
D	Webster 1939	Winn 1929	east-west	181
D'	Webster 1939	Kelley 1971	east-west	253
D"	Rock 1939*	Arcola 1939*	east-west	350
H	Chalmette 2 1931*	Morris 1914*	north-south	578

The length and azimuth standard errors computed in these seven adjustments are given in table 3. As expected, the length and azimuth standard errors between pairs of stations vary in size depending upon the location of the stations in the network relative to the network constraints. Since only the orientation and spacing of the Geoceiver stations vary in this set of adjustments, the preferred arrangement would be the one in which the size of the length and azimuth standard errors is smallest. To find which adjustment has the better arrangement, the results in the D' adjustment are compared line for line to the results in the other six adjustments. These ratios are given in table 4.

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*Pseudo-Geoceiver stations. A Geoceiver observation was simulated at these stations to give the desired Geoceiver observation separation.

Table 2.--Description of test lines.

Station	Station	Area, Line	°/km
CAPLEVILLE SE BASE 1914	BATESVILLE 1956	1,1	10°/ 76.1
EVANSVILLE 1929	WEEKS 1934	1,2	265°/ 83.2
BOBO 1956	MEEKS 1939	2,1	10°/ 64.8
WHILKINSON 1929	KEATON 1934	2,2	280°/ 93.5
INDIANOLA 1939	STRAIGHT 1957	3,1	3°/ 44.8
SILENT SHADE 1957	SHIVERS 1929	3,2	90°/ 74.2
PALUSKA 1939	LEXINGTON 1958	4,1	9°/ 47.2
KEIRN 1957	MOORE 1934	4,2	274°/ 41.1
COUNTRY 1957	SLIKER 1931	5,1	5°/ 64.7
HOMESTEAD 1929	BENTONIA 1959	5,2	274°/ 71.5
RICHLAND 1958	FANNIN 1931	6,1	354°/ 60.7
PERSIMMON 1959	PINE 1934	6,2	255°/ 46.2
HAWKINS 1931	JEFF 1947	7,1	2°/ 63.2
TYLER 1929	CRYSTAL 1945	7,2	272°/ 63.5
BRANDON 1931	SHARP 1945	8,1	4°/ 29.6
FLORENCE 1945	SHILOH 1945	8,2	259°/ 31.7
CENTRAL 1945	BETHEL 1946	9,1	21°/ 42.7
CHOCTAW 1945	CLEM 1934	9,2	269°/ 41.1
FOSTER 1929	MCCOMB 1947	10,1	275°/ 77.8
TOLER 1946	BROCK 1939	11,1	355°/ 39.7
PIKE 1947	SMITH 1934	11,2	280°/ 34.2
MALONE 1914	THAXTON 1967	14,1	10°/ 75.0
RIDGE 1934	LEBANON 1935	14,2	283°/ 54.7
LOCHINVAR 1967	WEBSTER 1939	15,1	12°/ 73.2
RANDOLPH 1967	EUPORA 1939	15,2	10°/ 69.3
BUSH 1934	BARR 1935	15,3	280°/ 70.6
REFORM 1939	LOBUTCHA 1958	16,1	4°/ 36.4
PALMERTREE 1934	BEVEL 1935	16,2	277°/ 64.4
DRY 1958	CARSON 1930	17,1	0°/ 53.6
GRIMES 1934	SMITH 1935	17,2	264°/ 61.2
FOREST EAST BASE 1930	TISDALE 1939	18,1	1°/ 94.9
WILLIAMS 1934	GRANTHAM 1935	18,2	285°/ 30.9
LITTLE 1934	MCLAURIN 1935	19,1	273°/ 46.1
PLEASANT 1914	BOOG 1939	20,1	7°/113.3
KARR 1935	BRAKEFIELD 1939	20,2	294°/ 82.8
FEDERAL 1935	GALLOWAY 1939	20,3	277°/ 88.1
BRADSHAW 1939	MILL 1934	21,1	52°/162.6
WARREN 1935	EUTAW 1939	21,2	263°/ 77.0
WOLF 1930	HOUSE 1939	22,1	355°/ 77.9
CLAYBORN 1935	DANIELS 1938	22,2	272°/ 80.2
LITTLE 1939	WEDFORD 1942	23,1	347°/ 48.3
TINGLE 1935	COON 2 1938	23,2	264°/ 92.8
ROCK 1939	FULLER 1930	25,1	0°/ 72.9
MOUNDVILLE 1939	JAMISON 1887	25,2	257°/ 87.4

*The tabulation gives the azimuth and length of the line.
10°/76.1: 10° = azimuth of line from south, 76.1 = distance between points in kilometers.

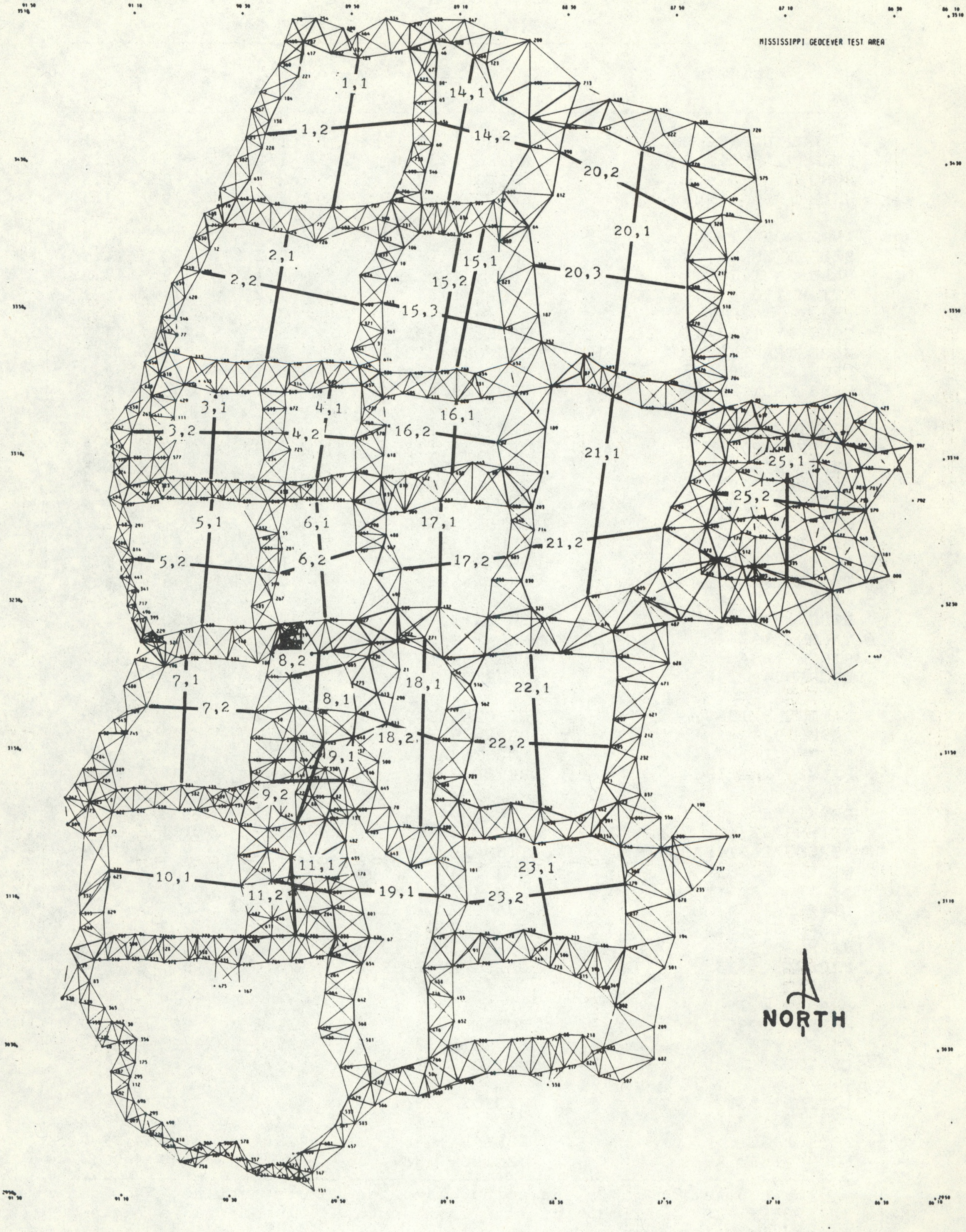

Figure 5.--Geoceiver test area line accuracies.

Table 3.--Distance and azimuth standard errors.

Area, Line	Adjustment C	D	C'	D'	C"	D"	H
1,1	0.419/181* 1.150	0.722/99 1.871	0.431/175 1.568	0.461/165 1.516	0.404/188 1.274	0.421/181 1.181	0.359/212 0.969
1,2	0.449/185 1.164	0.825/101 1.866	0.457/182 1.571	0.494/168 1.524	0.424/196 1.264	0.445/187 1.183	0.380/219 1.001
2,1	0.368/176 1.174	0.655/99 1.887	0.371/174 1.560	0.402/161 1.534	0.340/190 1.265	0.366/177 1.201	0.314/206 1.009
2,2	0.465/201 1.085	0.889/105 1.776	0.468/200 1.495	0.523/179 1.472	0.432/216 1.195	0.455/205 1.087	0.377/248 1.912
3,1	0.238/189 1.168	0.439/102 1.827	0.237/189 1.526	0.268/167 1.541	0.218/205 1.251	0.237/188 1.165	0.194/231 0.996
3,2	0.378/196 1.066	0.704/105 1.747	0.378/196 1.451	0.431/172 1.468	0.345/215 1.160	0.372/199 1.048	0.301/246 0.879
4,1	0.217/218 1.010	0.452/104 1.777	0.213/222 1.420	0.245/193 1.418	0.194/244 1.130	0.214/221 1.032	0.167/283 0.839
4,2	0.201/204 1.033	0.396/104 1.774	0.198/207 1.436	0.224/183 1.435	0.184/224 1.152	0.196/209 1.042	0.161/256 0.865
5,1	0.351/184 1.103	0.647/100 1.845	0.351/184 1.458	0.401/161 1.504	0.327/198 1.192	0.358/181 1.129	0.291/222 0.916
5,2	0.380/188 1.140	0.701/102 1.851	0.381/187 1.486	0.436/164 1.526	0.353/202 1.222	0.383/187 1.142	0.309/231 0.955
6,1	0.272/223 0.963	0.584/104 1.789	0.266/228 1.368	0.312/194 1.393	0.243/250 1.086	0.273/222 1.009	0.204/298 0.772
6,2	0.223/208 1.052	0.450/103 1.830	0.220/210 1.432	0.253/182 1.454	0.203/227 1.166	0.223/207 1.082	0.174/266 0.877
7,1	0.355/178 1.176	0.652/97 1.943	0.356/177 1.491	0.409/155 1.572	0.335/189 1.253	0.371/170 1.228	0.292/216 0.978
7,2	0.358/177 1.197	0.648/98 1.948	0.358/177 1.512	0.408/156 1.581	0.337/188 1.275	0.368/172 1.238	0.295/215 1.007

*Explanation of tabulation $\left\{\begin{matrix}0.419/181\\1.150\end{matrix}\right.$: 0.419 = σ for length in meters, 181 = proportional part in thousands or 181 = 1:181000, 1.150 = σ in azimuth.

Table 3.--Continued.

Area, Line	Adjustment C	D	C'	D'	C"	D"	H
8,1	0.145/204 1.079	0.294/101 1.885	0.143/206 1.432	0.167/177 1.487	0.133/222 1.181	0.149/198 1.137	0.114/260 0.887
8,2	0.150/211 1.016	0.312/102 1.846	0.149/213 1.390	0.175/181 1.437	0.138/230 1.129	0.155/205 1.072	0.115/276 0.818
9,1	0.194/220 0.960	0.419/102 1.839	0.193/221 1.335	0.233/183 1.414	0.178/240 1.072	0.206/208 1.043	0.140/305 0.716
9,2	0.186/220 0.968	0.403/102 1.842	0.186/221 1.341	0.224/183 1.419	0.171/240 1.079	0.198/208 1.050	0.135/304 0.727
10,1	0.452/172 1.189	0.814/96 1.974	0.452/172 1.495	0.518/150 1.587	0.429/182 1.268	0.473/164 1.258	0.367/212 0.968
11,1	0.198/201 1.042	0.399/99 1.893	0.198/200 1.386	0.234/169 1.478	0.185/214 1.139	0.210/188 1.127	0.148/268 0.785
11,2	0.169/202 1.042	0.344/99 1.893	0.170/202 1.386	0.201/170 1.477	0.158/216 1.139	0.180/190 1.126	0.127/270 0.786
14,1	0.374/201 1.057	0.748/100 1.856	0.384/195 1.524	0.412/182 1.448	0.359/209 1.224	0.373/201 1.119	0.316/237 0.913
14,2	0.281/195 1.129	0.548/100 1.907	0.284/192 1.580	0.304/180 1.496	0.266/206 1.292	0.277/198 1.188	0.240/228 1.015
15,1	0.338/216 0.976	0.723/101 1.827	0.332/220 1.475	0.359/204 1.377	0.300/244 1.149	0.324/226 1.040	0.271/270 0.851
15,2	0.326/213 0.994	0.687/101 1.830	0.321/216 1.478	0.347/199 1.389	0.290/239 1.153	0.314/221 1.052	0.263/264 0.861
15,3	0.326/217 1.032	0.681/104 1.807	0.319/221 1.458	0.355/198 1.409	0.295/239 1.173	0.313/226 1.055	0.258/273 0.887
16,1	0.195/187 1.223	0.369/99 1.957	0.181/201 1.558	0.198/184 1.552	0.172/211 1.343	0.183/199 1.264	0.161/227 1.110
16,2	0.296/218 0.953	0.624/103 1.782	0.283/228 1.397	0.316/204 1.345	0.262/246 1.107	0.278/232 0.977	0.228/282 0.801
17,1	0.247/217 0.993	0.526/102 1.833	0.225/238 1.397	0.257/208 1.391	0.211/254 1.132	0.231/232 1.046	0.186/288 0.837

Table 3.--Continued.

	Adjustment						
Area, Line	C	D	C'	D'	C"	D"	H
17,2	0.275/223 0.985	0.593/103 1.827	0.258/237 1.395	0.293/208 1.368	0.239/256 1.130	0.258/238 1.019	0.205/299 0.830
18,1	0.410/232 0.930	0.924/103 1.826	0.401/237 1.326	0.482/197 1.371	0.369/257 1.060	0.423/224 1.005	0.293/323 0.703
18,2	0.194/159 1.303	0.330/94 2.027	0.190/163 1.613	0.206/150 1.631	0.183/169 1.397	0.191/162 1.335	0.168/184 1.151
19,1	deleted	deleted	0.258/179 1.532	0.293/157 1.624	0.247/187 1.327	0.269/171 1.321	0.213/216 1.060
20,1	0.524/216 1.007	1.132/100 1.881	0.507/223 1.513	0.539/210 1.390	0.474/239 1.227	0.493/230 1.094	0.437/260 0.951
20,2	0.438/189 1.061	0.851/97 1.875	0.434/191 1.514	0.453/182 1.396	0.412/201 1.223	0.420/197 1.091	0.387/214 0.939
20,3	0.434/203 1.075	0.889/99 1.880	0.407/216 1.504	0.428/206 1.361	0.385/229 1.221	0.391/225 1.067	0.356/248 0.949
21,1	0.639/254 0.861	1.548/104 1.764	0.545/298 1.334	0.634/256 1.273	0.492/331 1.043	0.542/300 0.906	0.403/404 0.711
21,2	0.371/208 1.020	0.773/100 1.869	0.340/226 1.443	0.368/209 1.334	0.321/240 1.184	0.333/231 1.028	0.290/265 0.904
22,1	0.369/211 1.031	0.779/100 1.894	0.358/218 1.399	0.415/188 1.415	0.337/231 1.156	0.373/209 1.089	0.291/267 0.843
22,2	0.386/208 1.045	0.804/100 1.900	0.371/216 1.407	0.422/190 1.433	0.350/229 1.166	0.380/211 1.100	0.304/264 0.855
23,1	0.300/161 1.299	0.522/93 2.065	0.301/160 1.595	0.334/144 1.662	0.291/166 1.395	0.312/155 1.376	0.264/183 1.128
23,2	0.488/190 1.093	0.955/97 1.939	0.486/191 1.425	0.556/167 1.498	0.463/200 1.197	0.508/183 1.171	0.404/230 0.869
25,1	0.395/184 1.053	0.760/96 1.904	0.145/502 1.476	0.146/501 1.259	0.144/507 1.232	0.143/508 1.038	0.142/514 0.979
25,2	0.472/185 1.047	0.910/96 1.900	0.170/515 1.471	0.171/512 1.254	0.168/520 1.225	0.168/521 1.033	0.166/528 0.971

Table 4.--Standard errors, ratios of σ_ℓ and σ_α.

Area, Line	C'/D'	D/D'	C/D'	D"/D'	C"/D'	H/D'
1,1	0.935	1.675	0.909	0.913	0.876	0.779
	1.034	1.234	0.758	0.779	0.840	0.639
1,2	0.925	1.670	0.908	0.901	0.858	0.769
	1.031	1.224	0.764	0.776	0.829	0.657
2,1	0.923	1.629	0.916	0.910	0.846	0.781
	1.017	1.230	0.766	0.783	0.825	0.658
2,2	0.895	1.700	0.890	0.870	0.826	0.721
	1.016	1.206	0.738	0.738	0.812	0.620
3,1	0.884	1.638	0.888	0.884	0.813	0.724
	0.990	1.186	0.757	0.756	0.812	0.646
3,2	0.877	1.633	0.877	0.863	0.801	0.698
	0.988	1.190	0.726	0.714	0.790	0.599
4,1	0.870	1.845	0.886	0.873	0.792	0.682
	1.001	1.253	0.712	0.728	0.797	0.592
4,2	0.884	1.768	0.897	0.875	0.821	0.719
	1.001	1.236	0.720	0.726	0.803	0.603
5,1	0.875	1.613	0.875	0.893	0.815	0.726
	0.969	1.227	0.733	0.751	0.793	0.609
5,2	0.874	1.608	0.871	0.878	0.810	0.709
	0.974	1.213	0.747	0.748	0.801	0.626
6,1	0.853	1.872	0.873	0.875	0.779	0.654
	0.982	1.284	0.691	0.724	0.780	0.554
6,2	0.869	1.779	0.881	0.881	0.802	0.688
	0.985	1.259	0.724	0.744	0.802	0.603
7,1	0.870	1.594	0.867	0.907	0.819	0.714
	0.948	1.236	0.748	0.781	0.797	0.622
7,2	0.877	1.588	0.877	0.902	0.826	0.723
	0.956	1.232	0.757	0.783	0.806	0.637
8,1	0.856	1.760	0.868	0.892	0.796	0.683
	0.963	1.268	0.726	0.765	0.794	0.597
8,2	0.851	1.783	0.857	0.886	0.789	0.657
	0.967	1.285	0.707	0.746	0.786	0.569

Table 4.--Continued.

Area, Line	C'/D'	D/D'	C/D'	D"/D'	C"/D'	H/D'
9,1	0.828 0.944	1.798 1.301	0.832 0.679	0.884 0.738	0.764 0.758	0.601 0.506
9,2	0.830 0.945	1.799 1.298	0.830 0.682	0.884 0.740	0.763 0.760	0.603 0.512
10,1	0.872 0.942	1.571 1.244	0.872 0.749	0.913 0.793	0.828 0.799	0.708 0.610
11,1	0.846 0.938	1.705 1.281	0.846 0.705	0.897 0.762	0.791 0.771	0.632 0.531
11,2	0.846 0.938	1.711 1.282	0.841 0.705	0.895 0.762	0.786 0.771	0.632 0.532
14,1	0.932 1.052	1.815 1.282	0.908 0.730	0.905 0.773	0.871 0.845	0.767 0.631
14,2	0.934 1.056	1.803 1.275	0.924 0.755	0.911 0.794	0.875 0.864	0.789 0.678
15,1	0.925 1.071	2.014 1.327	0.942 0.709	0.902 0.755	0.836 0.834	0.755 0.618
15,2	0.925 1.064	1.980 1.317	0.940 0.715	0.905 0.757	0.836 0.830	0.758 0.620
15,3	0.898 1.035	1.918 1.282	0.918 0.733	0.882 0.749	0.831 0.832	0.727 0.630
16,1	0.914 1.004	1.864 1.261	0.984 0.788	0.924 0.814	0.869 0.865	0.813 0.715
16,2	0.896 1.039	1.975 1.325	0.937 0.709	0.880 0.726	0.829 0.823	0.722 0.596
17,1	0.875 1.004	2.047 1.318	0.961 0.714	0.899 0.752	0.821 0.814	0.724 0.602
17,2	0.880 1.020	2.024 1.335	0.938 0.720	0.880 0.745	0.816 0.826	0.700 0.607
18,1	0.832 0.967	1.917 1.332	0.850 0.678	0.878 0.733	0.766 0.773	0.608 0.513
18,2	0.922 0.989	1.602 1.243	0.941 0.799	0.927 0.818	0.888 0.856	0.816 0.706

Table 4.--Continued.

Area, Line	C'/D'	D/D'	C/D'	D"/D;	C"/D'	H/D'
19,1	0.880	--	--	0.918	0.843	0.727
	0.943			0.813	0.817	0.653
20,1	0.941	2.100	0.972	0.915	0.879	0.811
	1.088	1.353	0.725	0.787	0.883	0.684
20,2	0.958	1.879	0.967	0.927	0.909	0.854
	1.084	1.343	0.760	0.781	0.876	0.673
20,3	0.951	2.077	1.014	0.914	0.899	0.832
	1.105	1.381	0.790	0.784	0.897	0.697
21,1	0.860	2.442	1.008	0.855	0.776	0.636
	1.048	1.386	0.676	0.712	0.819	0.559
21,2	0.924	2.100	1.008	0.905	0.872	0.788
	1.082	1.401	0.765	0.771	0.888	0.678
22,1	0.863	1.877	0.890	0.899	0.812	0.701
	0.989	1.338	0.729	0.770	0.817	0.596
22,2	0.879	1.905	0.914	0.900	0.829	0.720
	0.982	1.326	0.730	0.768	0.814	0.597
23,1	0.901	1.563	0.898	0.934	0.871	0.790
	0.960	1.242	0.781	0.828	0.839	0.679
23,2	0.874	1.718	0.877	0.914	0.833	0.727
	0.951	1.294	0.729	0.782	0.799	0.580
25,1	0.993*	5.206*	2.705*	0.979	0.986	0.973
	1.172*	1.512*	0.836*	0.824	0.979	0.778
25,2	0.994*	5.322*	2.759*	0.982	0.982	0.971
	1.173*	1.515*	0.835*	0.824	0.977	0.774
$\bar{\sigma}_\ell$	0.8880	1.8136	0.9061	0.9003	0.8348	0.7344
s_ℓ	0.0342	0.1873	0.0481	0.0253	0.0494	0.0807
$\bar{\sigma}_\alpha$	1.0015	1.2812	0.7324	0.7658	0.8248	0.6224
s_α	0.0472	0.0534	0.0310	0.0304	0.0474	0.0614

*These values were not used in subsequent computations because for some unknown reason they differed too much from the mean.

The means of the ratios with respect to D' are listed below:

Adjustment	Distance Standard Error	Azimuth Standard Error
D'	1.00	1.00
D	1.81	1.28
C	0.90	0.73
C'	0.88	1.00
D"	0.90	0.77
C"	0.83	0.82
H	0.73	0.62

The above table may be interpreted as follows. On the average, the distance standard error of a line in adjustment D will be 1.81 times greater than the distance standard error of the same line in adjustment D'

To test whether there is any orientation bias, pseudo-Geoceiver stations are used in the C" and D" adjustments to achieve a set of adjustments with the same station separation but with different orientation. The orientation of the C" stations is north-south and the D" stations is east-west. The distance standard errors in the C" adjustment are 8 percent smaller than in the D" adjustment. This is probably due to the smaller a priori latitude standard error of the north-south Geoceiver stations. The azimuth standard errors in the D" adjustment are 6 percent smaller than in the C" adjustment. This again is probably due to the smaller a priori latitude standard error. There is then a small preference in orientation of Geoceiver stations depending upon whether one wants to improve the distance standard errors or azimuth standard errors the most.

The variation of the distance standard error in these seven adjustments is shown in figure 6.

There is rapid reduction in distance standard error as the separation between Geoceiver stations increases to about 250 km. At this point, there is a dramatic change in the effectiveness of further separation to reduce the standard error.

The graph of the variation of the azimuth standard error is shown in figure 7.

The azimuth standard errors in this set of data continued to decrease as the distance between the two Geoceiver stations increased. However, the rate of decrease became less and less.

Thus we conclude that Geoceiver stations need to be separated by at least 250 km, to most effectively improve the scale accuracy of a network. The azimuth accuracies are dependent only upon distance; therefore, the most effective way of

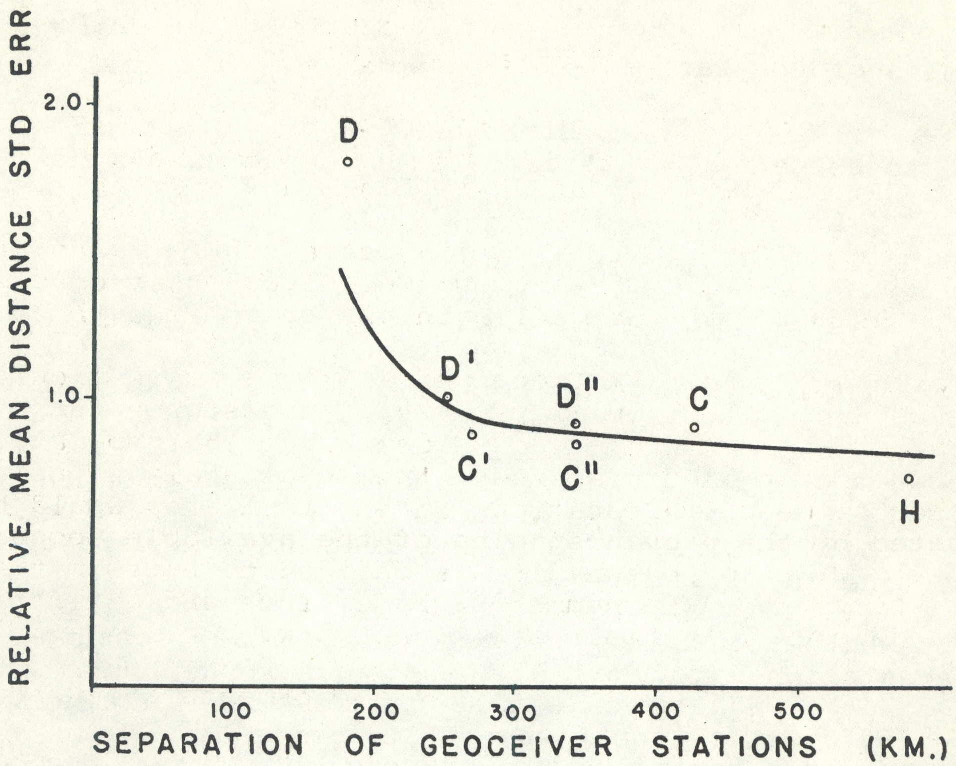

Figure 6.--Variation of the distance standard error.

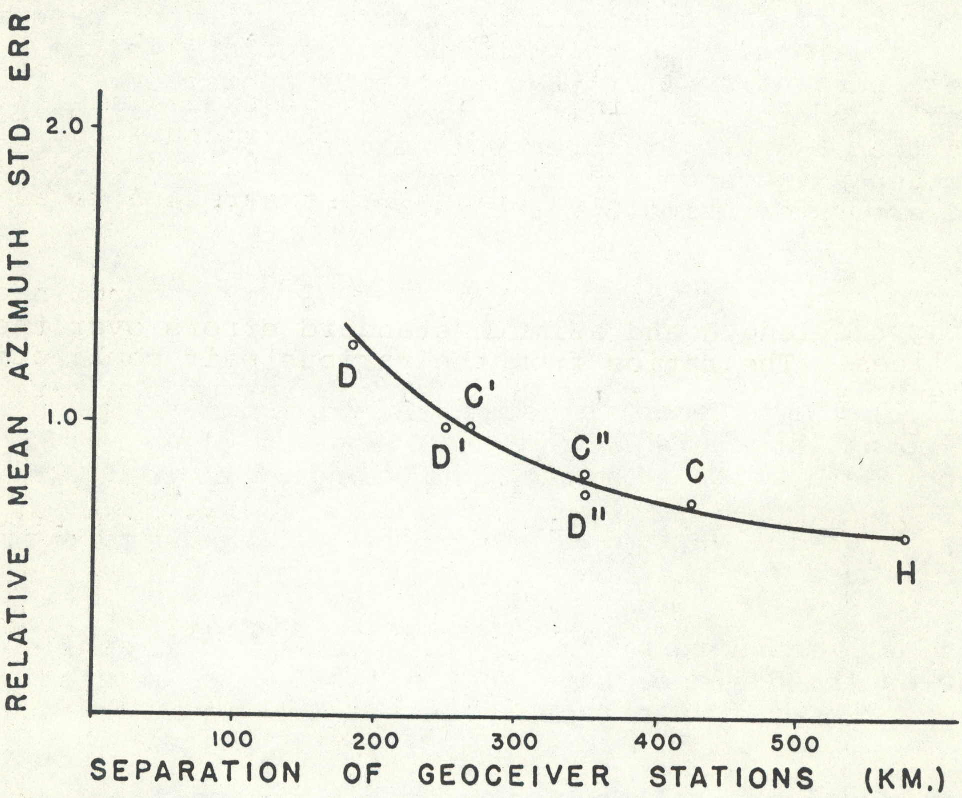

Figure 7.--Variation of the azimuth standard error.

improving azimuth accuracies with Geoceiver stations is to separate the stations as much as possible.

Another important question is: What is the density of Geoceiver observations that can benefit an existing network by reducing the standard errors of distance and azimuth?

One Geoceiver station is a trivial case; there is no effect upon the accuracies. At least two Geoceiver position observations are needed to effect a length and/or an azimuth constraint.

The case of two or more Geoceiver stations is difficult to analyze because, as shown previously, the distance and azimuth standard errors are directly dependent upon the separation between Geoceiver stations. Any attempt at analysis be varying the number of Geoceiver stations in the test area would be complicated by the uneven spacing of the available Geoceiver stations. For this reason, it was decided to perform the analysis by varying the number of base lines and azimuths in the basic first-order, main scheme network.

The B, $B^{1/2}$, B^+, D', D'^+, and C'^+ adjustments are used in the analysis.

The B series of adjustments contain all five of the Geoceiver observations while the D series contain only two. Reference may be made to table 1 for the complete makeup of the data sets. The point at which Geoceiver observations ceased to have an appreciable effect upon the solution was sought by first adjusting the network using no observed distances and azimuths then using one-half of the distances and azimuths, and finally using all of the distances and azimuths. The distance and azimuth standard errors from these adjustments are given in tables 3 and 5.

As in the previous section, the analysis is accomplished by comparing the length and azimuth standard errors over the 44 sample lines. The ratios from the various pair combinations of adjustments are given in table 6. The means of the ratios with respect to adjustment B^+ are:

Adjustment	Distance Standard Error	Azimuth Standard Error
B^+	1.00	1.00
$B^{1/2}$	1.07	1.13
B	1.39	1.37

Table 5.--Distance and azimuth standard errors.

Area, Line	Adjustment B	B^{+}	$B^{1/2}$	C'^{+}	D'^{+}
1,1	0.370/205* 0.946	0.299/254 0.745	0.308/247 0.767	0.309/246 0.782	0.307/248 0.779
1,2	0.389/214 0.952	0.264/315 0.732	0.314/265 0.767	0.270/207 0.773	0.269/309 0.772
2,1	0.326/199 0.979	0.268/242 0.757	0.271/239 0.841	0.272/238 0.782	0.271/239 0.781
2,2	0.385/243 0.840	0.254/368 0.612	0.278/337 0.675	0.258/362 0.654	0.258/362 0.652
3,1	0.202/221 0.940	0.139/323 0.724	0.150/299 0.787	0.140/320 0.752	0.140/320 0.752
3,2	0.309/240 0.801	0.201/369 0.535	0.223/333 0.614	0.205/361 0.574	0.206/361 0.574
4,1	0.178/265 0.772	0.129/365 0.580	0.136/346 0.653	0.131/359 0.614	0.132/358 0.613
4,2	0.169/243 0.793	0.121/341 0.614	0.136/302 0.678	0.122/338 0.649	0.122/338 0.649
5,1	0.302/214 0.890	0.230/281 0.638	0.251/258 0.722	0.233/278 0.660	0.234/277 0.661
5,2	0.321/223 0.925	0.209/342 0.652	0.243/295 0.740	0.211/339 0.674	0.211/339 0.674
6,1	0.222/273 0.729	0.160/379 0.511	0.170/358 0.596	0.162/374 0.543	0.164/370 0.543
6,2	0.187/247 0.838	0.127/363 0.634	0.150/308 0.718	0.128/361 0.660	0.128/360 0.659
7,1	0.314/201 0.997	0.216/293 0.678	0.241/262 0.811	0.217/291 0.691	0.218/290 0.694
7,2	0.314/202 0.018	0.217/292 0.734	0.237/268 0.860	0.218/291 0.747	0.219/290 0.749

*Explanation of tabulation {0.370/205 / 0.946: 0.370 = σ for length in meters, 205 = proportional part in thousands or 205 = 1:205000. 0.946 = σ in azimuth.

Table 5.--Continued.

Area, Line	Adjustment B	B^+	$B^{1/2}$	C'^+	D'^+
8,1	0.124/238 0.886	0.093/318 0.672	0.099/299 0.758	0.094/316 0.689	0.094/314 0.691
8,2	0.127/250 0.806	0.085/371 0.565	0.096/331 0.669	0.086/368 0.587	0.087/366 0.587
9,1	0.161/265 0.744	0.091/470 0.417	0.101/421 0.563	0.092/462 0.439	0.094/457 0.443
9,2	0.155/265 0.754	0.088/466 0.428	0.098/418 0.574	0.090/459 0.449	0.091/454 0.453
10,1	0.403/193 1.022	0.248/314 0.658	0.273/285 0.811	0.249/313 0.666	0.249/312 0.669
11,1	0.170/233 0.850	0.100/396 0.495	0.110/362 0.645	0.101/391 0.508	0.102/388 0.512
11,2	0.146/235 0.850	0.086/396 0.505	0.094/364 0.650	0.087/391 0.518	0.088/388 0.522
14,1	0.323/232 0.855	0.242/310 0.658	0.254/296 0.683	0.253/296 0.706	0.251/299 0.703
14,2	0.243/225 0.943	0.193/283 0.784	0.201/272 0.808	0.202/271 0.839	0.200/274 0.832
15,1	0.287/255 0.761	0.231/316 0.605	0.239/306 0.637	0.242/303 0.660	0.240/304 0.650
15,2	0.278/249 0.781	0.226/307 0.627	0.233/298 0.661	0.235/295 0.677	0.234/297 0.668
15,3	0.270/261 0.806	0.197/358 0.644	0.206/343 0.689	0.202/349 0.681	0.201/351 0.678
16,1	0.171/212 1.053	0.145/251 0.929	0.150/242 0.977	0.147/248 0.952	0.147/247 0.952
16,2	0.244/264 0.704	0.160/402 0.534	0.165/391 0.578	0.162/397 0.578	0.162/397 0.570
17,1	0.207/259 0.775	0.162/331 0.595	0.170/314 0.679	0.164/326 0.631	0.165/324 0.628

Table 5.--Continued.

Area, Line	Adjustment B	B^+	$B^{1/2}$	C'^+	D'^+
17,2	0.225/272 0.751	0.143/429 0.576	0.156/393 0.630	0.144/424 0.616	0.144/424 0.608
18,1	0.335/283 0.700	0.216/440 0.430	0.236/401 0.541	0.221/429 0.464	0.223/425 0.463
18,2	0.177/175 1.142	0.144/215 0.999	0.156/198 1.048	0.144/215 1.017	0.144/215 1.015
19,1	deleted	0.183/252 0.881	0.194/238 0.950	0.185/249 0.895	0.185/249 0.900
20,1	0.440/258 0.810	0.358/316 0.669	0.367/309 0.700	0.386/293 0.742	0.382/297 0.729
20,2	0.379/218 0.844	0.332/249 0.672	0.338/245 0.707	0.351/236 0.724	0.345/240 0.713
20,3	0.361/244 0.839	0.305/288 0.697	0.312/282 0.732	0.322/274 0.754	0.315/280 0.734
21,1	0.481/338 0.584	0.290/561 0.392	0.311/522 0.458	0.302/539 0.459	0.302/538 0.446
21,2	0.307/251 0.771	0.258/299 0.657	0.265/290 0.696	0.266/290 0.725	0.264/292 0.698
22,1	0.315/247 0.818	0.239/326 0.616	0.256/304 0.686	0.244/318 0.653	0.245/317 0.648
22,2	0.327/245 0.833	0.255/314 0.625	0.268/299 0.692	0.263/306 0.660	0.262/307 0.655
23,1	0.276/175 1.147	0.236/204 0.974	0.246/196 1.036	0.239/202 0.990	0.240/202 0.992
23,2	0.431/264 0.904	0.337/275 0.656	0.360/258 0.744	0.346/268 0.680	0.345/269 0.681
25,1	0.309/236 0.744	0.130/559 0.641	0.131/557 0.672	0.133/550 0.762	0.132/554 0.703
25,2	0.368/237 0.736	0.153/571 0.634	0.154/568 0.665	0.156/562 0.756	0.155/565 0.697

Table 6.--Standard errors, ratios of σ_ℓ and σ_α.

Area, Line	$^B/_{B^+}$	$^{B^{1/2}}/_{B^+}$	$^{D'}/_{B}$	$^{D'^+}/_{B^+}$	$^{C'^+}/_{D'^+}$
1,1	1.24 1.27	1.030 1.029	1.24 1.60	1.027 1.046	1.006 1.004
1,2	1.47 1.30	1.189 1.048	1.27 1.60	1.019 1.055	1.004 1.001
2,1	1.22 1.29	1.011 1.111	1.23 1.57	1.011 1.032	1.004 1.001
2,2	1.51 1.37	1.094 1.103	1.36 1.75	1.016 1.065	1.000 1.003
3,1	1.45 1.30	1.079 1.087	1.33 1.41	1.007 1.039	1.000 1.000
3,2	1.54 1.50	1.109 1.148	1.39 1.83	1.025 1.073	0.995 1.000
4,1	1.38 1.33	1.054 1.126	1.38 1.84	1.023 1.057	0.992 1.002
4,2	1.40 1.29	1.124 1.104	1.33 1.81	1.008 1.057	1.000 1.000
5,1	1.31 1.40	1.091 1.132	1.33 1.69	1.017 1.036	0.996 0.998
5,2	1.54 1.42	1.163 1.135	1.36 1.65	1.010 1.034	1.000 1.000
6,1	1.38 1.43	1.060 1.166	1.41 1.91	1.025 1.063	0.988 1.000
6,2	1.47 1.32	1.181 1.132	1.35 1.74	1.008 1.039	1.000 1.001
7,1	1.45 1.47	1.116 1.196	1.30 1.58	1.009 1.024	0.995 0.996
7,2	1.45 1.39	1.092 1.172	1.30 1.55	1.009 1.020	0.995 0.997
8,1	1.33 1.32	1.064 1.128	1.35 1.68	1.011 1.028	1.000 0.997
8,2	1.49 1.43	1.129 1.184	1.38 1.78	1.023 1.039	0.988 1.000

Table 6.--Continued.

Area, Line	B/B^+	$B^{1/2}/B^+$	D'/B	D'^+/B^+	C'^+/D'^+
9,1	1.77	1.110	1.45	1.033	0.979
	1.78	1.350	1.90	1.062	0.991
9,2	1.76	1.114	1.44	1.034	0.989
	1.76	1.340	1.88	1.058	0.991
10,1	1.62	1.101	1.28	1.004	1.000
	1.55	1.232	1.55	1.017	0.995
11,1	1.70	1.100	1.38	1.020	0.990
	1.72	1.303	1.90	1.034	0.992
11,2	1.70	1.093	1.38	1.023	0.989
	1.68	1.287	1.74	1.034	0.992
14,1	1.34	1.050	1.28	1.037	1.008
	1.30	1.038	1.69	1.068	1.004
14,2	1.26	1.041	1.25	1.036	1.010
	1.20	1.031	1.59	1.061	1.008
15,1	1.24	1.035	1.25	1.039	1.008
	1.26	1.053	1.81	1.074	1.015
15,2	1.23	1.031	1.25	1.035	1.004
	1.25	1.054	1.78	1.065	1.013
15,3	1.37	1.046	1.31	1.020	1.005
	1.25	1.070	1.75	1.053	1.004
16,1	1.18	1.034	1.16	1.014	1.000
	1.13	1.052	1.47	1.025	1.000
16,2	1.52	1.031	1.29	1.012	1.000
	1.32	1.082	1.91	1.067	1.014
17,1	1.28	1.049	1.24	1.018	0.994
	1.30	1.141	1.79	1.056	1.005
17,2	1.57	1.091	1.30	1.007	1.000
	1.30	1.094	1.82	1.056	1.013
18,1	1.55	1.093	1.44	1.032	0.991
	1.63	1.258	1.95	1.077	1.002
18,2	1.23	1.083	1.16	1.000	1.000
	1.14	1.049	1.43	1.016	1.002

Table 6.--Continued.

Area, Line	B/B^+	$B^{1/2}/B^+$	D'/B	D'^+/B^+	C'^+/D'^+
19,1	--	1.060	--	1.011	1.000
		1.078		1.022	0.994
20,1	1.23	1.025	1.23	1.067	1.010
	1.21	1.046	1.71	1.090	1.018
20,2	1.14	1.018	1.19	1.039	1.017
	1.26	1.052	1.65	1.061	1.015
20,3	1.18	1.023	1.19	1.033	1.022
	1.20	1.050	1.62	1.053	1.027
21,1	1.66	1.072	1.32	1.041	1.000
	1.49	1.168	2.18	1.138	1.029
21,2	1.19	1.027	1.20	1.023	1.008
	1.17	1.059	1.73	1.062	1.039
22,1	1.32	1.071	1.32	1.025	0.996
	1.33	1.114	1.73	1.052	1.008
22,2	1.28	1.051	1.29	1.027	1.004
	1.33	1.107	1.72	1.048	1.008
23,1	1.17	1.042	1.21	1.017	0.996
	1.18	1.064	1.45	1.018	0.998
23,2	1.28	1.068	1.29	1.024	1.003
	1.38	1.134	1.66	1.038	0.998
25,1	2.38*	1.008*	0.47*	1.015	1.008
	1.16*	1.049*	1.69*	1.097	1.084
25,2	2.40*	1.006*	0.46*	1.013	1.006
	1.16*	1.049*	1.70*	1.099	1.085
$\bar{\sigma}\ell$	1.400	1.0749	1.303	1.0215	1.0000
$s\ell$	0.175	0.0431	0.076	0.0128	0.0080
$\bar{\sigma}\alpha$	1.365	1.1264	1.717	1.0524	1.0079
$s\alpha$	0.164	0.0839	0.157	0.0247	0.0196

*These values were not used in subsequent computations because for some unknown reason they differed too much from the mean.

These data are plotted on figures 8 and 9. The location of the D' and D'+ solution relative to the B series of adjustments is also shown.

These figures show that the distance and azimuth standard errors become smaller as the number of observed distances and azimuth increases.

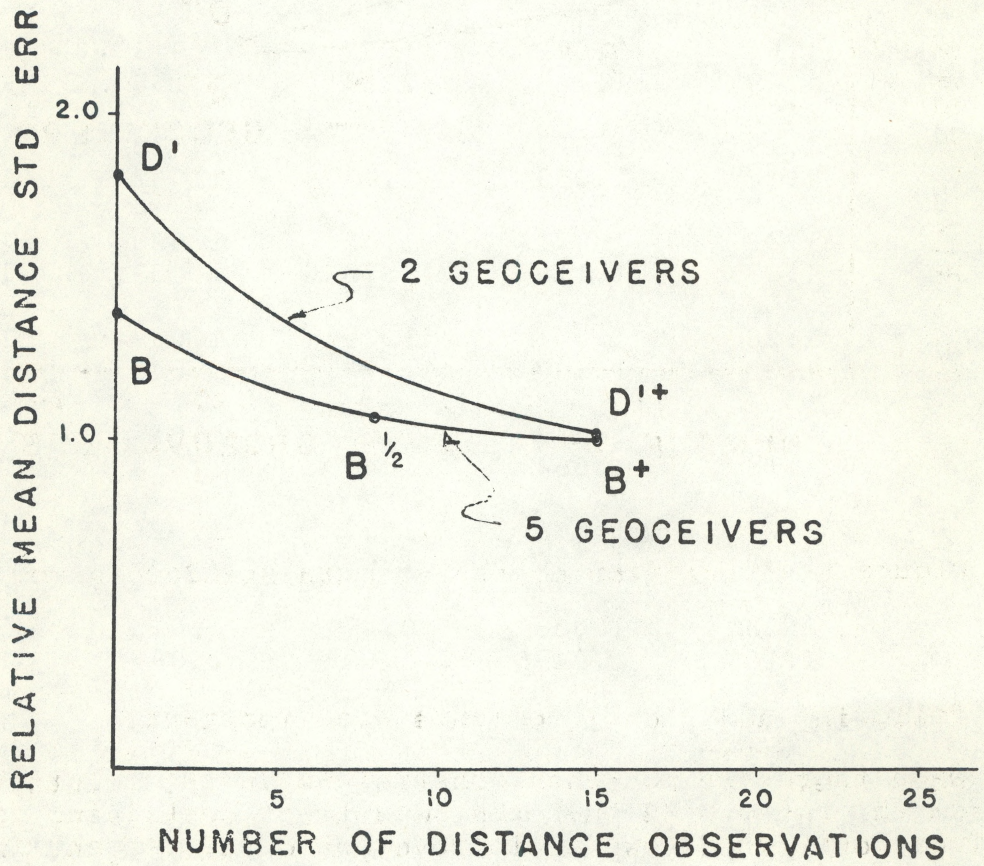

Figure 8.--Variation of the distance standard error.

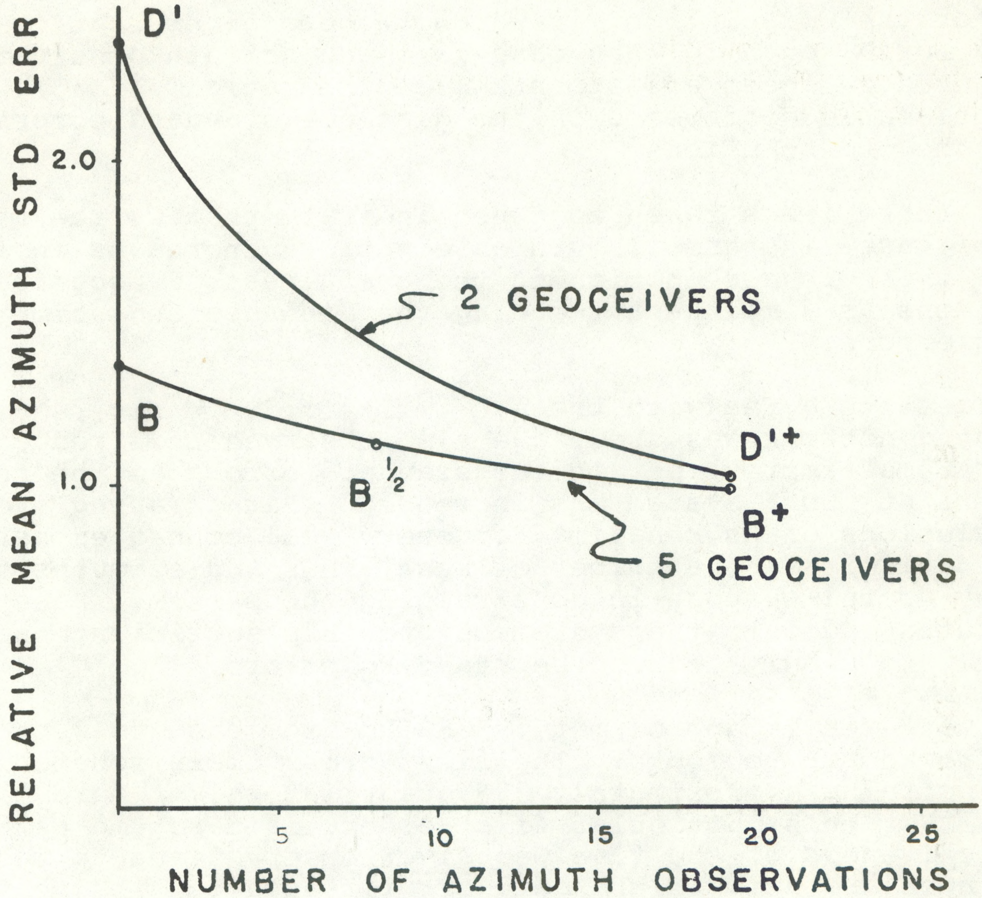

Figure 9.--Variation of the azimuth standard error.

The following additional comments are important.

1. When there are no other constraints in the solution, we see from the ratio D'/B that the adjustment containing five Geoceiver positions shows an improvement of 30 percent in distance standard errors and 71 percent in azimuth standard errors over the adjustment constrained by two Geoceiver positions.

2. When the adjustments contain base line and azimuth observations, as in the ratio D'^{+}/B^{+}, the five-Geoceiver adjustment shows only a 2 percent improvement in distance standard errors and a 5 percent improvement in azimuth standard errors over the adjustment that contains only two Geoceiver observations.

3. In section 2 where no distances and azimuth observations were involved in the adjustments (see ratio C'/D' - table 4), there was a 13-percent improvement in the distance standard errors for the C' solution (in which two Geoceiver stations are oriented north-south). When base line and azimuth observations are included in these adjustments (see the ratio C'^{+}/D'^{+}), there is no noticeable difference in the distance standard errors between the two solutions.

All of these items taken together indicate that as the number of base line and azimuth observations increases in a network, there is a reduction in the usefulness of Geoceiver observations as a means of reducing the distance and azimuth standard errors.

These results agree with those of Ashkenazi and Cross (1975) when a simulated network was used. He also observed this reduction in the rate of improvement in the standard errors as the number of constraints in a system was increased. Paraphrasing from the conclusions of Ashkenazi, "For every well connected network there is a limit to the number of base lines and azimuths that serve any useful purpose in constraining the system. Base lines and azimuths added to the system beyond this sufficient number serve only to slowly reduce the standard errors."

The study area is approximately 350 km from east to west and 550 km from north to south. The first-order, main scheme network contains approximately 386 quadrilateral or more complex figures. For this particular first-order, main scheme network, the effectiveness of the five Geoceiver observations to reduce the distance and azimuth standard errors seems to disappear when the network contains about 20 distance and 25 azimuth observations. In other words, this arrangement of five Geoceiver stations would cause a reduction in the distance and azimuth standard errors of this triangulation network, only if there is less than one base line observation per 20 quads and one azimuth observation per 15 quads. A general guideline for using Geoceiver observations may now be stated: "If the first-order, main scheme network in a given area contains less than one base line observation per 20 quads and one azimuth observation per 15 quads, then Geoceiver observations may be used to improve the internal accuracy."

In adjustment B^{+}, the distance standard errors range from 0.085 to 0.358 meter with a mean of 0.197 meter, and the azimuth standard errors range from 0".392 to 0".999 with a mean of 0".645. The standard errors can be reduced only slightly beyond this point by additional length, azimuth or position observations in the adjustment.

As pointed out by Ashkenazi and Cross (1975), the controlling factors are the large number of observed directions, their standard errors, and how "well connected" the network is. In this adjustment the a posteriori mean standard error for the 7,202 observed directions is 0".385. The length and azimuth standard errors in this network are thus most dependent upon the set of direction observations and their standard errors. Any other set of observations, Geoceiver, for instance, would have to be large and well-connected to appreciably reduce the network length and azimuth standard errors.

3. EFFECT OF GEOCEIVER OBSERVATIONS UPON THE POSITIONAL ACCURACIES

In an adjustment where the new network is appended to the existing network, the new stations have a positional uncertainty that is due in part to the uncertainty in the position of the station or stations in the existing network used as constrained positions in the new adjustment and, in part, because of the observational errors in the new network. Geoceiver observations are a means of obtaining geodetic positions independent of the triangulation system. In this experiment the analysis method used was to vary the amount of observational data and the number of Geoceiver stations in each of four adjustments and to note the change in the 95 percent positional error ellipses at 44 selected first-order stations (see figure 10).

A series of four adjustments of the first-order network and the first-order, second-order combined networks are performed in which the constraints are different for each adjustment. The name of each station at which a positional error ellipse is computed and the dimension, in meters of the semi-major and semi-minor axes, are given in table 7.

Adjustment $\hat{E}*$ is an adjustment of the 838 stations in the first-order, main scheme network. The network contains 42 geodimeter lines, or base lines, and 18 azimuth observations. Station Webster 1939 was heavily constrained. The a priori variance allowed in latitude and longitude was $1''.0 \times 10^{-20}$.

Adjustment E* is the same as adjustment $\hat{E}*$, except that the Geoceiver determined position for station Webster 1939 was constrained in latitude to a standard deviation of 0.9 meter and in longitude to a standard deviation of 1.2 meters.

Adjustment F* is an adjustment of the 838 stations in the first-order, main scheme network and 498 stations in the second-order, main scheme networks. The combined networks contained 63 geodimeter lines, or base lines, and 22 azimuth observations. The Geoceiver-determined position for station Webster 1939 was again the constrained position.

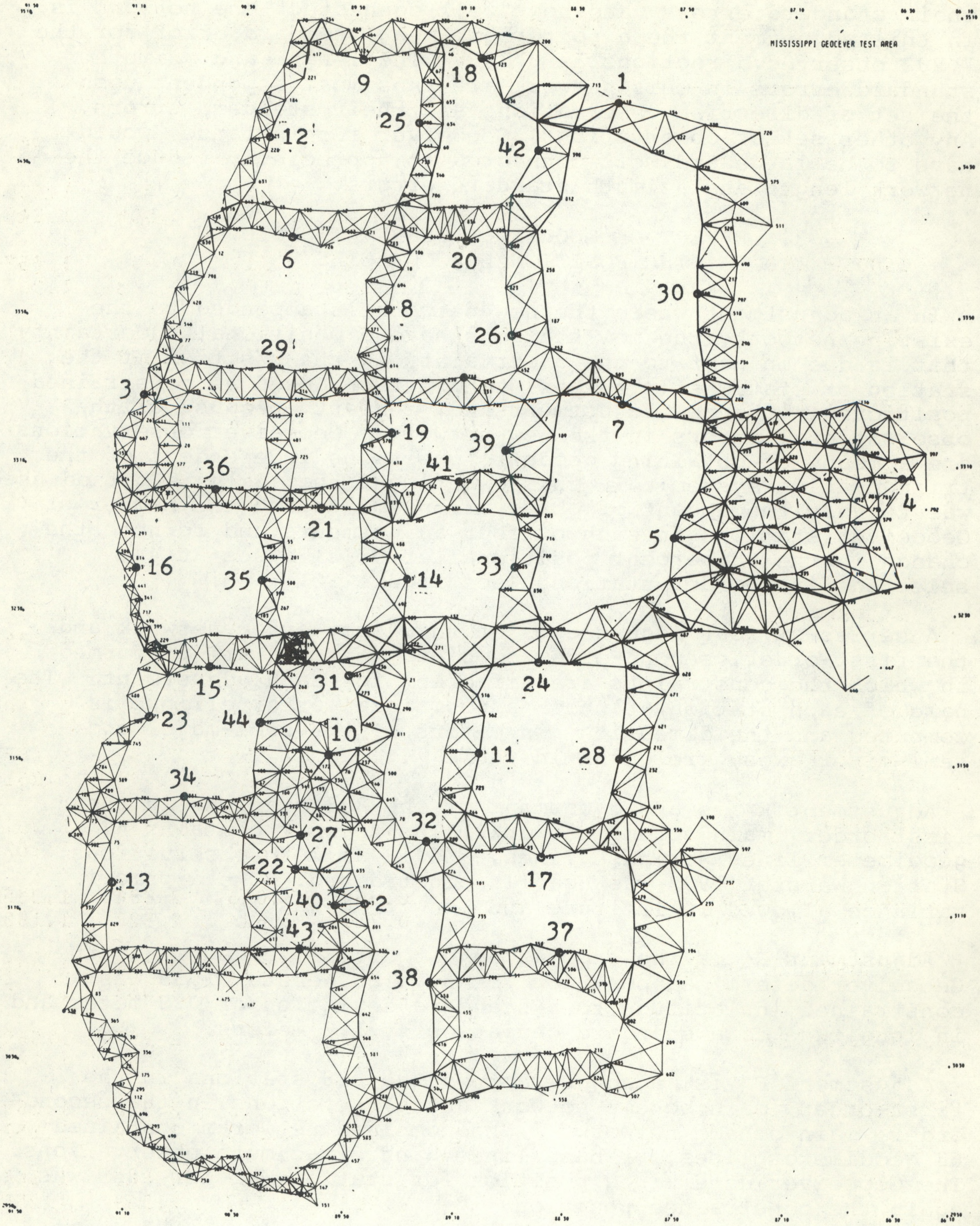

Figure 10.--Geoceiver test area position accuracies.

Table 7.--Error ellipse, with semi-major and semi-minor axes in meters.

Station	No.	Adjustment Ê*	E*	F*	G*
KNOB 1914	1	1.170	3.080	2.886	1.429
		0.958	2.727	2.555	1.248
LITTLE 1934	2	1.255	3.111	2.950	1.457
		0.943	2.727	2.583	1.267
WINN 1929	3	0.997	2.956	2.835	1.331
		0.757	2.734	2.587	1.302
KELLEY 1971	4	1.733	3.081	2.845	1.499
		0.779	2.960	2.791	1.297
EUTAW 1939	5	0.942	2.977	2.828	1.304
		0.696	2.675	2.544	1.174
BOBO 1956	6	0.826	2.942	2.813	1.328
		0.646	2.663	2.530	1.219
BRADSHAW 1939	7	0.597	2.912	2.797	1.270
		0.545	2.611	2.515	1.144
BUSH 1934	8	0.491	2.889	2.800	1.286
		0.415	2.591	2.502	1.168
CAPLEVILLE SE BASE 1914	9	1.167	3.059	2.892	1.483
		0.957	2.748	2.544	1.261
CENTRAL 1945	10	0.975	3.007	2.878	1.361
		0.707	2.657	2.542	1.202
CLAYBORN 1935	11	0.964	3.015	2.886	1.366
		0.711	2.644	2.527	1.183
EVANSVILLE 1929	12	1.077	3.011	2.854	1.431
		0.797	2.715	2.563	1.275
FOSTER 1929	13	1.478	3.172	2.987	1.569
		1.135	2.839	2.664	1.388
GRIMES 1934	14	0.616	2.921	2.823	1.288
		0.472	2.593	2.504	1.151
HAWKINS 1931	15	1.051	3.007	2.865	1.364
		0.785	2.707	2.569	1.250
HOMESTEAD 1929	16	1.071	2.988	2.857	1.363
		0.788	2.737	2.586	1.287

Table 7.--Continued.

Station	No.	Adjustment $\hat{E}^*$	E^*	F^*	G^*
LITTLE 1939	17	1.285	3.132	2.963	1.475
		0.939	2.714	2.567	1.254
MALONE 1914	18	1.061	3.046	2.892	1.460
		0.941	2.715	2.527	1.229
PALMERTREE 1934	19	0.372	2.880	2.796	1.266
		0.322	2.569	2.492	1.149
RANDOLPH 1967	20	0.557	2.907	2.810	1.305
		0.536	2.606	2.497	1.153
RICHLAND 1958	21	0.590	2.908	2.810	1.275
		0.451	2.598	2.509	1.166
TOLER 1946	22	1.196	3.082	2.931	1.440
		0.876	2.711	2.579	1.258
TYLER 1929	23	1.199	3.046	2.893	1.418
		0.870	2.751	2.601	1.296
WOLF 1930	24	0.839	2.976	2.855	1.328
		0.577	2.614	2.510	1.151
MEEKS 1939	25	0.646	2.911	2.802	1.279
		0.559	2.627	2.524	1.206
BARR 1935	26	0.354	2.877	2.793	1.273
		0.331	2.570	2.491	1.138
BETHEL 1946	27	1.132	3.058	2.914	1.414
		0.828	2.695	2.568	1.241
DANIELS 1938	28	1.216	3.100	2.934	1.441
		0.860	2.693	2.550	1.226
WEEKS 1934	29	0.866	2.974	2.849	1.390
		0.782	2.676	2.513	1.198
GALLOWAY 1939	30	1.018	3.003	2.827	1.331
		0.746	2.687	2.557	1.210
SHILOH 1945	31	0.828	2.966	2.849	1.321
		0.599	2.627	2.522	1.174
TISDALE 1939	32	1.131	3.069	2.923	1.417
		0.841	2.687	2.556	1.228

Table 7.--Continued.

Station	No.	Adjustment			
		$\hat{E}^*$	E*	F*	G*
SMITH 1935	33	0.615	2.918	2.821	1.285
		0.474	2.596	2.501	1.139
JEFF 1947	34	1.211	3.065	2.914	1.439
		0.873	2.735	2.595	1.281
BENTONIA 1959	35	0.793	2.941	2.829	1.301
		0.576	2.639	2.531	1.196
STRAIGHT 1957	36	0.812	2.929	2.820	1.295
		0.589	2.660	2.542	1.223
WEDFORD 1942	37	1.570	3.259	3.040	1.586
		1.235	2.830	2.626	1.355
LUMBERTON 1943	37A			2.988	1.510
				2.609	1.314
LEE 1935	38	1.463	3.203	3.008	1.539
		1.149	2.801	2.625	1.342
BEVEL 1935	39	0.383	2.881	2.797	1.266
		0.306	2.567	2.491	1.133
SMITH 1934	40	1.253	3.107	2.950	1.461
		0.934	2 726	2.586	1.272
LOBUTCHA 1958	41	0.450	2.892	2.806	1.275
		0.347	2.571	2.492	1.134
LEBANON 1935	42	0.824	2.970	2.841	1.360
		0.725	2.650	2.511	1.180
BROCK 1939	43	1.346	3.143	2.977	1.507
		1.016	2.759	2.610	1.309
CRYSTAL 1945	44	1.006	3.007	2.871	1.360
		0.737	2.676	2.552	1.291
Major axis mean		0.964	3.009	2.873	1.381
Major axis s.d.*		0.333	0.094	0.065	0.090
Minor axis mean		0.730	2.684	2.554	1.226
Minor axis s.d.*		0.228	0.081	0.055	0.065

*s.d. - standard deviation.

Adjustment G* contains, in addition to the data contained in adjustment F*, four more Geoceiver stations, Winn 1929, Little 1934, Knob 1914, and Kelley 1971, entered as constrained positions.

The first adjustment, $\hat{E}$*, where station Webster 1939 is heavily constrained, was used to determine the error propagation characteristics of the network. The positional error ellipses computed in this adjustment are relative to the Geoceiver position of station Webster 1939. The computed 95 percent positional error ellipses are shown in figure 11. The positional error varies with distance from 0.3 meter at Barr 1935, near the fixed station, to 1.5 meters at Wedford 1942, near the edge of the area.

The first test performed was adjustment E*. This adjustment produced error ellipses quite consistent in size and orientation. The mean error ellipse had a semi-major axis of 3.009 meters with a standard deviation of 0.094 meter, and a semi-minor axis of 2.684 meters with a standard deviation of 0.081 meter. The error ellipses for this adjustment are shown in figure 12. The orientation of the major axis of the error ellipses was 90° (east-west) with very little variation. There is a small systematic variation in the size of the error ellipses with distance from the constrained station. The variation between station Barr 1935, located near the constrained station, and station Wedford 1942, near the edge of the network, was 0.3 meter.

In the next test, adjustment F*, only a 1-percent reduction occurred in the size of the error ellipses when the second-order network was included. These error ellipses are shown in figure 13. The error ellipses were again consistent in size and orientation. The mean of the semi-major axis was 2.873 meters, with a standard deviation of 0.065 meter, and the mean of the semi-minor axis was 2.55 meters, with a standard deviation of 0.066 meter. The orientation of the major axis of the error ellipses had only a slightly larger variation from 90° than the first-order network alone. From this test, based on this particular set of data, it appears that the inclusion of the second-order projects does not significantly improve the positional accuracies at the 44 selected stations over that which was obtained from the adjustment of only the first-order projects.

The third test, adjustment G*, was an adjustment of the first- and second-order networks with the five Geoceiver stations, Webster 1939, Knob 1914, Little 1934, Winn 1929, and Kelley 1971, as constrained positions. The addition of the five Geoceiver

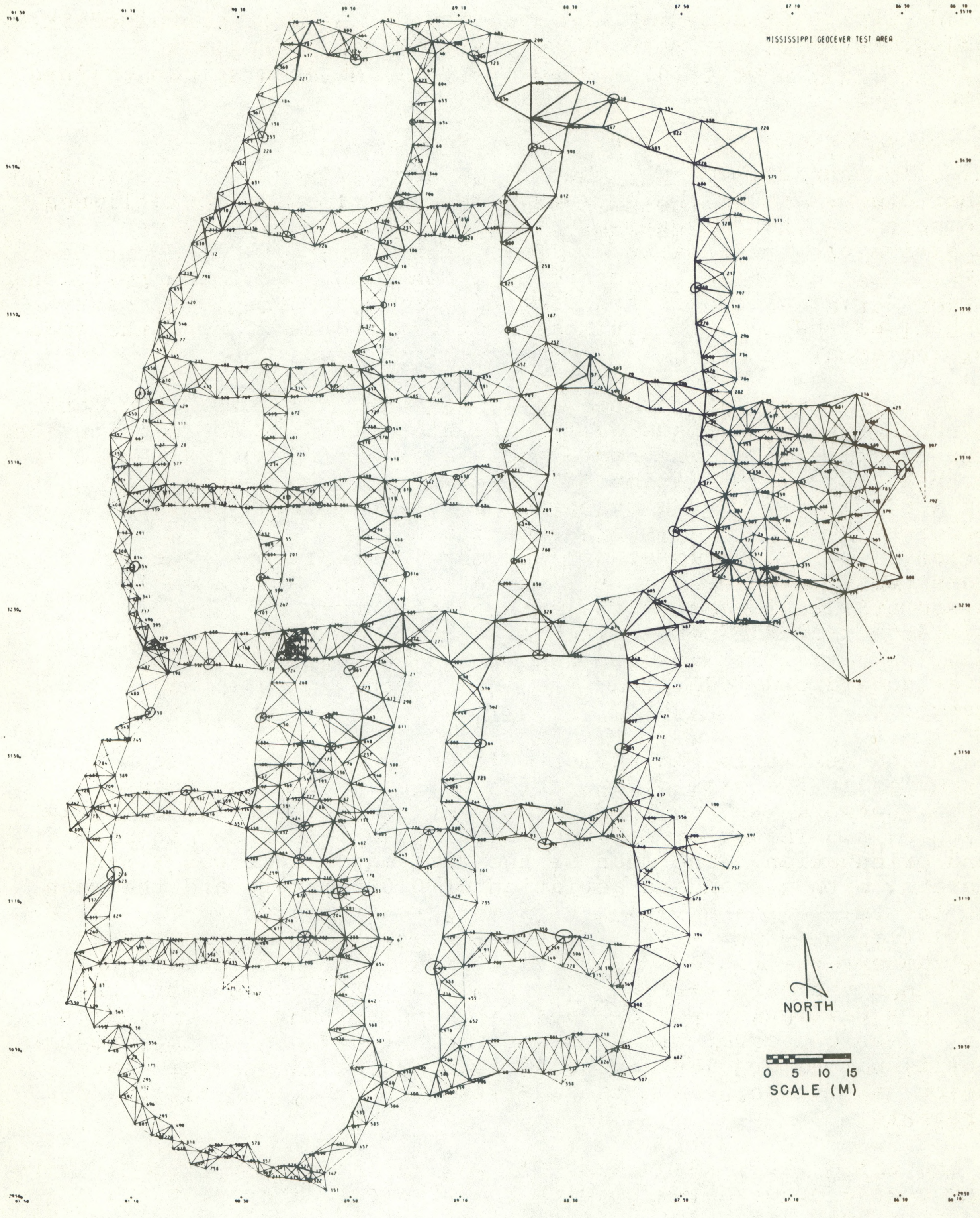

Figure 11.--Geoceiver test area $\hat{E}^*$ adjustment error ellipses.

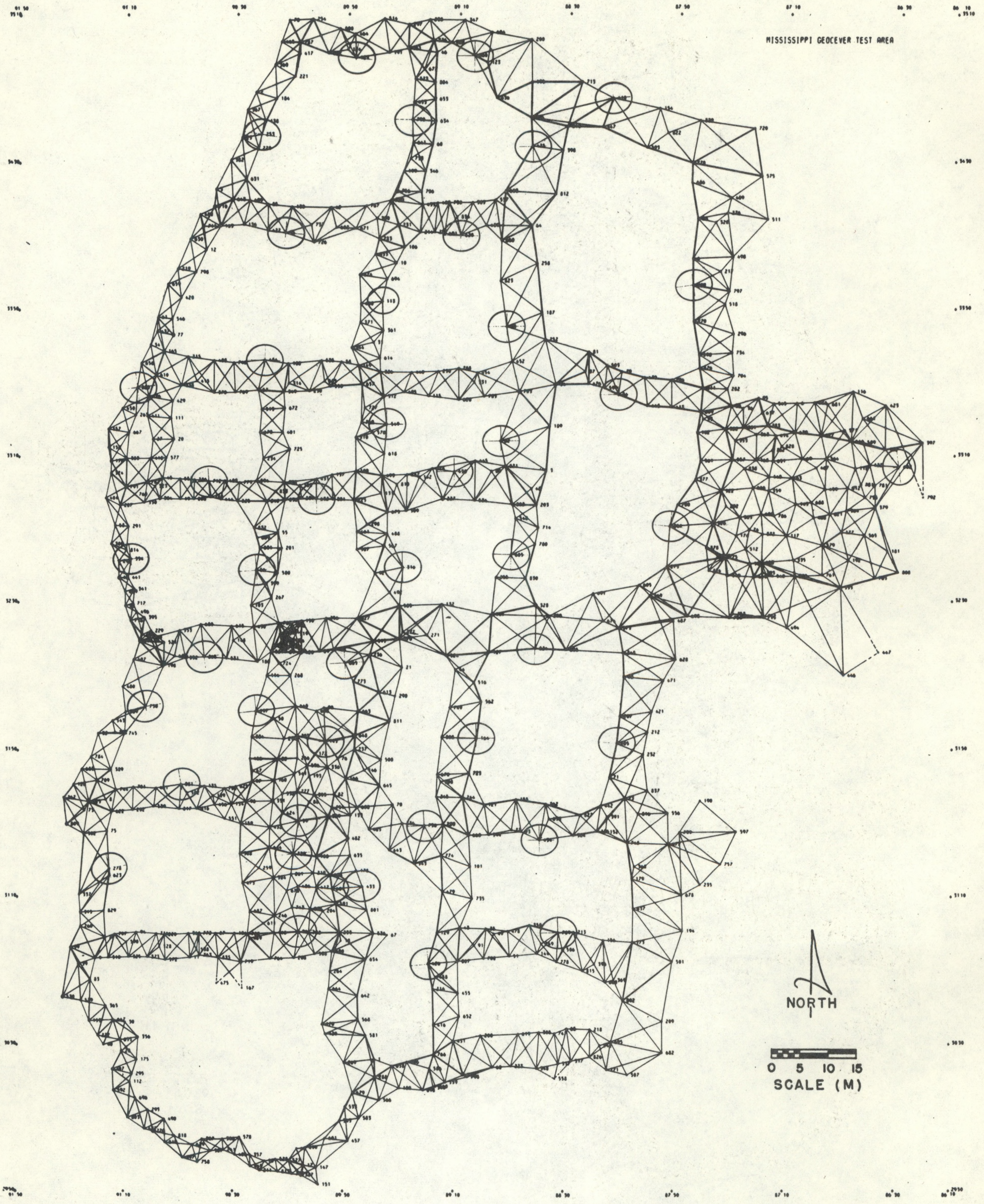

Figure 12.--Geoceiver test area E* adjustment error ellipses.

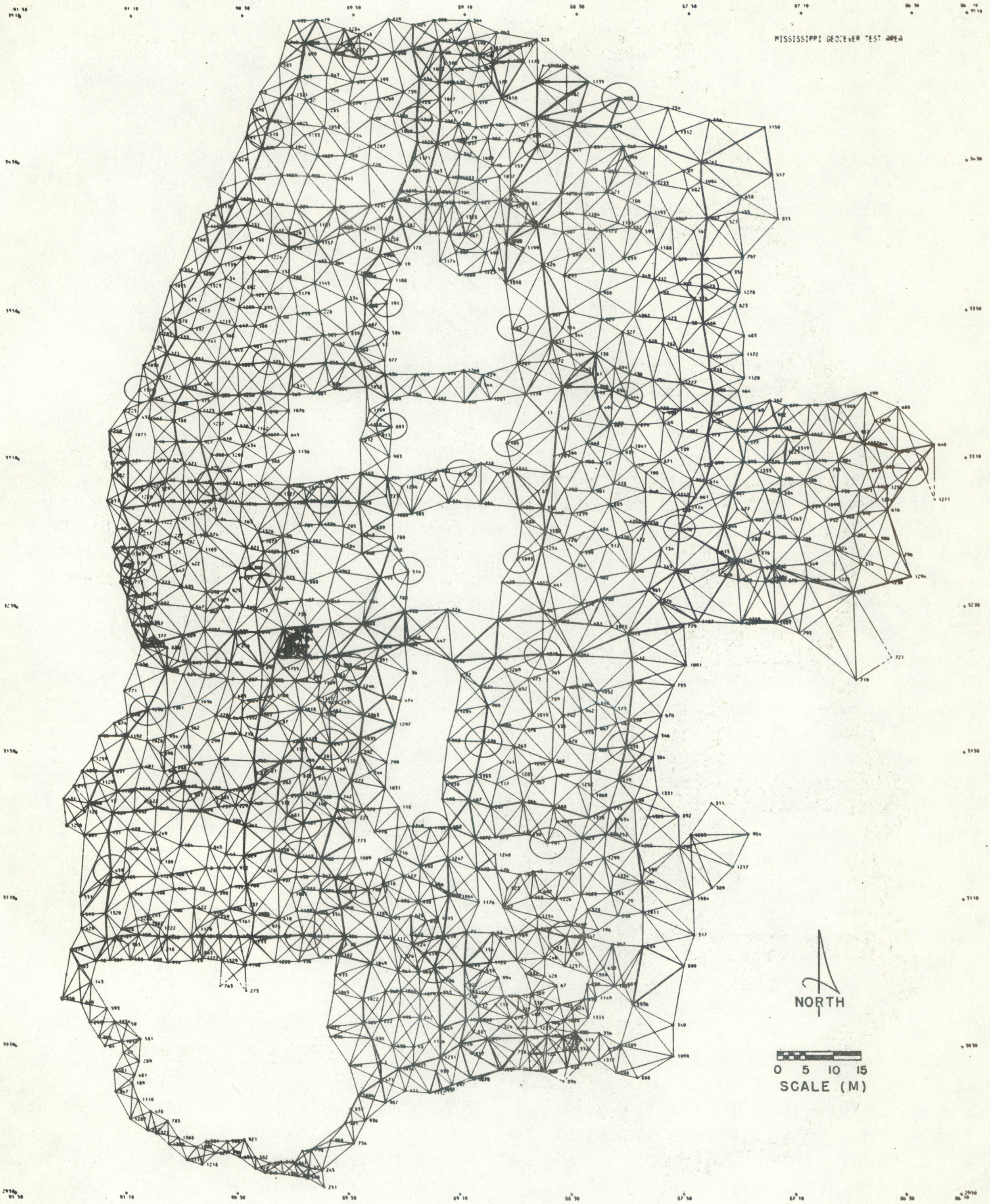

Figure 13.--Geoceiver test area F* adjustment error ellipses.

stations produced an appreciable reduction in the size of the error ellipses. The relative size and orientation of the error ellipses throughout the whole network were again much the same (figure 14). The mean of the semi-major axis dimension was 1.381 meters with a standard deviation of 0.090 meter. The mean of the semi-minor axis dimension was 1.226 meters with a standard deviation of 0.065 meter. This improvement in the accuracy of the positions over those obtained in the adjustment where one Geoceiver position was used seems to be in direct proportion to the increase in the square root of the number (n) of Geoceiver stations. The actual improvement was 2.08, which is close to the square root of 5(2.24). The variation between stations Barr 1935 and Wedford 1942 was again 0.3 meter.

4. EFFECT OF GEOCEIVER OBSERVATIONS UPON THE FINAL POSITIONS

This experiment was performed to determine if the Geoceiver observations have a significant effect upon the final positions. The four adjustments, E*, $\hat{E}$*, F*, and G*, in which the amount of observational data or the number of Geoceiver stations is different were considered. The differences in these four data sets are given in table 1. The analysis was based upon the final positions at the 44 first-order stations shown in figure 10. The adjusted positions at these stations are given in table 8.

The first comparison was of the final positions produced by the adjustment of the first-order network when the position of a centrally located station, Webster 1939, was rigidly constrained to 3.1×10^{-9} meters in latitude and 2.4×10^{-9} in longitude (adjustment $\hat{E}$*), and when the same station was constrained to 0.9 meter in latitude, 1.2 meters in longitude (adjustment E*). Table 8 shows that these changes are not appreciable; however, no change was expected. It is felt that the mean changes of -0.93 mm in latitude and -2.25 mm in longitude should not have occurred. This problem is being investigated.

The second comparison (F*-E*, table 9) isolated the contribution of the second-order observations. The final positions from the adjustment of the first-order network (adjustment E*) was compared to the final positions from the adjustment of the first- and second-order networks (adjustment F*). In both of these adjustments, station Webster 1939 was constrained to 0.9 meter in latitude and 1.2 meters in longitude.

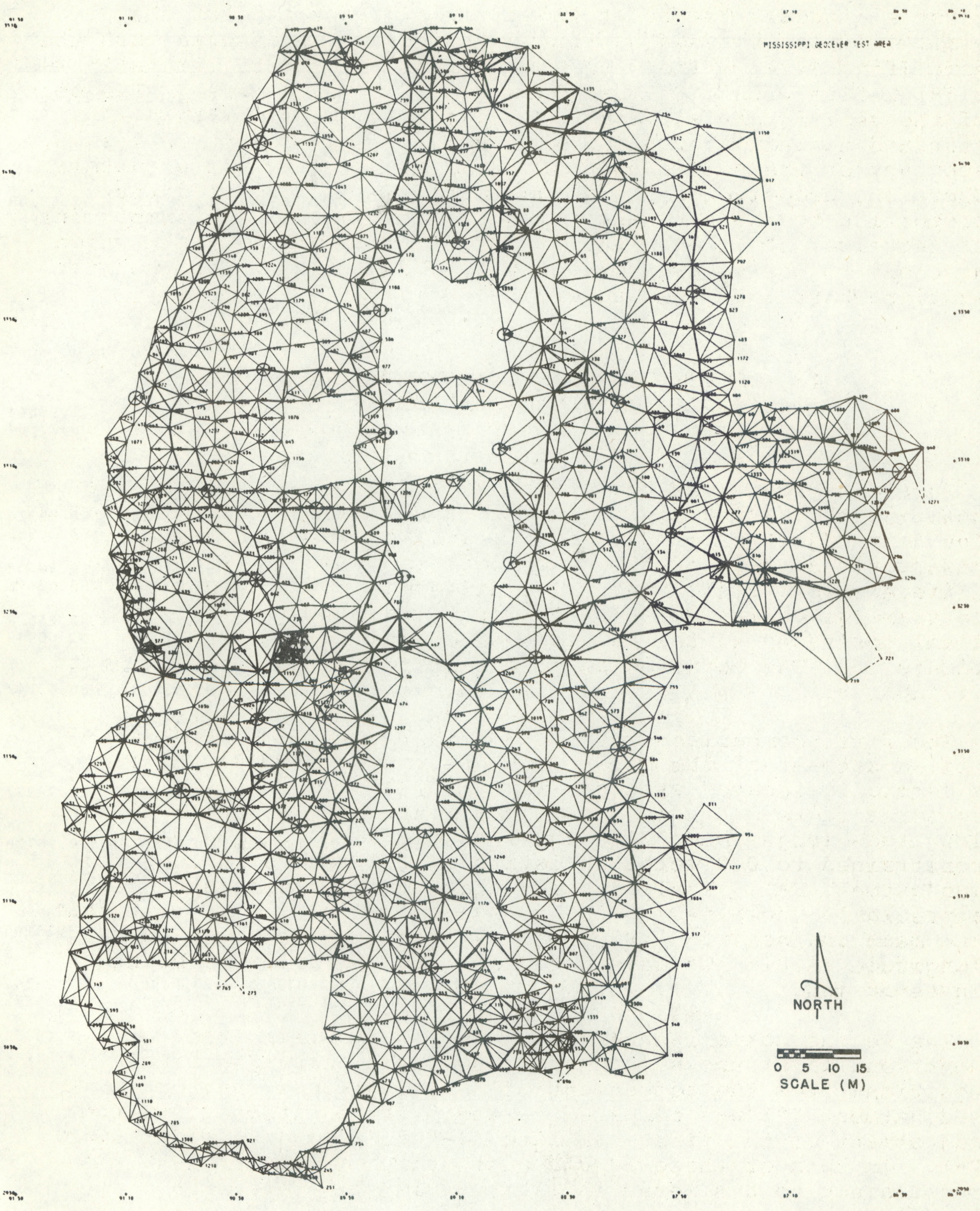

Figure 14.--Geoceiver test area G* adjustment error ellipses.

Table 8.--Adjusted positions, final seconds of ϕ and λ.

Name	No.	Preliminary Position	Adjustment $\hat{E}$*	E*	F*	G*
KNOB 1914	1	15.54700 29.70900	15.44372 29.79028	15.44375 29.79037	15.45625 29.80112	15.46069 29.79247
		15.4591* 29.7816*				
LITTLE 1934	2	42.81400 32.79600	42.68631 32.87719	42.68634 32.87728	42.68513 32.87981	42.69108 32.87487
		42.6907* 32.8949*				
WINN 1929	3	51.66800 30.47800	51.55880 30.54665	51.55882 30.54675	51.54979 30.55215	51.55642 30.54470
		51.5586* 30.5265*				
KELLEY 1971	4	02.09247 00.21970	01.93772 00.28646	01.93775 00.28655	01.92660 00.27834	01.93044 00.27154
		01.9356* 00.2739*				
EUTAW 1939	5	46.71900 54.22200	46.58564 54.26251	46.58567 54.26260	46.57762 54.25592	46.58214 54.24935
BOBO 1956	6	51.34000 55.75400	51.28017 55.82297	51.28020 55.82306	51.28053 55.83467	51.28646 55.82671
BRADSHAW 1939	7	42.38600 01.94400	42.26163 02.01473	42.26166 02.01482	42.26052 02.01014	42.26518 02.00303
BUSH 1934	8	03.14800 38.52000	03.05619 38.62372	03.05622 38.62381	03.05815 38.62908	03.06370 38.62142
CAPLEVILLE SE BASE 1914	9	12.54900 30.09700	12.48872 30.17597	12.48875 30.17606	12.48454 30.19373	12.49018 30.18499
CENTRAL 1945	10	22.56500 17.72200	22.44018 17.79536	22.44020 17.79545	22.43765 17.79492	22.44369 17.78927
CLAYBORN 1935	11	12.82800 28.83700	12.69081 28.91245	12.69083 28.91254	12.68758 28.91293	12.69296 28.90730
EVANSVILLE 1929	12	34.91300 24.84000	34.86452 24.90492	34.86454 24.90502	34.85605 24.92563	34.86205 24.91728
FOSTER 1929	13	47.86600 00.23800	47.72906 00.27816	47.72909 00.27825	47.72401 00.27671	47.73106 00.27164
GRIMES 1934	14	27.17500 29.71500	27.04935 29.79204	27.04938 29.79213	27.04808 29.78918	27.05369 29.78275
HAWKINS 1931	15	57.57700 21.49800	57.45082 21.57031	57.45085 21.57040	57.44641 21.56906	57.45305 21.56293

*Geoceiver-determined positions.

Table 8.--Continued.

Name	No.	Preliminary Position	Adjustment $\hat{E}^*$	E^*	F^*	G^*
HOMESTEAD 1929	16	53.15100 21.09800	53.01600 21.16775	53.01603 21.16784	53.00769 21.16765	53.01452 21.16104
LITTLE 1939	17	31.29400 25.62400	31.14408 25.67611	31.14411 25.67620	31.13380 25.66666	31.13893 25.66153
MALONE 1914	18	47.58504 50.01443	47.51091 50.09420	47.51094 50.09430	47.51153 50.10818	47.51666 50.09937
PALMERTREE 1934	19	39.64000 45.73000	39.54174 45.82250	39.54177 45.82260	39.53900 45.82436	39.54460 45.81726
RANDOLPH 1967	20	01.52080 28.95731	01.42908 29.05229	01.42911 29.05238	01.43629 29.05964	01.44152 29.05170
RICHLAND 1958	21	12.96410 59.45200	12.84950 59.53510	12.84953 59.53520	12.84629 59.53764	12.85224 59.53085
TOLER 1946	22	45.18900 49.33300	45.06304 49.41429	45.06307 49.41438	45.06191 49.41551	45.06815 49.41038
TYLER 1929	23	54.87700 58.88000	54.75489 58.93921	54.75492 58.93930	54.75234 58.93722	54.75920 58.93136
WOLF 1930	24	19.69200 28.59700	19.55924 28.66637	19.55927 28.66646	19.55169 28.66767	19.55678 28.66164
MEEKS 1939	25	19.96500 06.21600	19.86313 06.26692	19.86316 06.26701	19.85721 06.26925	19.86327 06.26182
BARR 1935	26	09.96200 40.49200	09.86381 40.57422	09.86384 40.57432	09.86467 40.57402	09.86975 40.56656
BETHEL 1946	27	47.03900 58.32100	46.91529 58.39825	46.91531 58.39834	46.91365 58.39874	46.91985 58.39346
DANIELS 1938	28	42.69100 38.00000	42.54932 38.01832	42.54934 38.01841	42.53822 38.00569	42.54295 38.00010
WEEKS 1934	29	23.56200 09.47700	23.48918 09.57357	23.48921 09.57366	23.49039 09.58548	23.49580 09.57701
GALLOWAY 1939	30	27.35000 01.30500	27.21822 01.37769	27.21825 01.37779	27.21768 01.37320	27.22191 01.36568

Table 8.--Continued.

Name	No.	Preliminary Position	Adjustment E*	E*	F*	G*
SHILOH 1945	31	26.96900 14.59200	26.83712 14.66855	26.83714 14.66864	26.83529 14.66655	26.84120 14.66055
TISDALE 1939	32	32.50900 15.87300	32.37682 15.94502	32.37685 15.94511	32.37267 15.94554	32.37833 15.94032
SMITH 1935	33	49.22000 32.45700	49.09322 32.53772	49.09325 32.53781	49.08729 32.53460	49.09246 32.52814
JEFF 1947	34	47.38200 49.35600	47.25478 49.41039	47.25481 49.41048	47.25547 49.41234	47.26218 49.40686
BENTONIA 1959	35	45.86930 43.97770	45.73965 44.05457	45.73967 44.05467	45.74186 44.05446	45.74811 44.04798
STRAIGHT 1957	36	10.27530 11.36170	10.15277 11.42431	10.15280 11.42440	10.14661 11.42446	10.15302 11.41753
WEDFORD 1942	37	04.99800 25.53900	04.87460 25.55116	04.87463 25.55125	04.86477 25.54041	04.86986 25.53573
LUMBERTON 1943	37A	52.01800 17.35100	--	--	51.88174 17.41792	51.88752 17.41323
LEE 1935	38	43.71500 37.65500	43.58025 37.71459	43.58027 37.71468	43.57546 37.71555	43.58115 37.71096
BEVEL 1935	39	02.15000 39.97900	02.03768 40.07201	02.03771 40.07210	02.03503 40.06896	02.04018 40.06199
SMITH 1934	40	04.09000 37.13600	03.96199 37.22074	03.96202 37.22083	03.96063 37.22259	03.96671 37.21764
LOBUTCHA 1958	41	37.64280 14.32790	37.52578 14.41633	37.52581 14.41643	37.52137 14.41586	37.52672 14.40900
LEBANON 1935	42	17.25650 43.51347	17.16617 43.60217	17.16620 43.60226	17.17247 43.60891	17.17734 43.60055
BROCK 1939	43	21.84500 49.13100	21.70385 49.21721	21.70387 49.21730	21.70215 49.21936	21.70841 49.21458
CRYSTAL 1945	44	35.76000 41.18000	35.62935 41.25508	35.62938 41.25517	35.62624 41.25105	35.63258 41.24524

Table 9.--Position differences in meters of ϕ and λ.

No.	F* - E*	G* - E*	F* - G*	$\hat{E}$* - G*
1	0.38750	0.52514	-0.13764	-0.52607
	0.26875	0.05250	0.21625	-0.05475
2	-0.03751	0.14694	-0.18445	-0.14787
	0.06325	-0.06025	0.12350	0.05800
3	-0.27993	-0.07440	-0.20553	0.07378
	0.13500	-0.05125	0.18625	0.04875
4	-0.34565	-0.22661	-0.11904	0.22568
	-0.20525	-0.37525	0.17000	0.37300
5	-0.24955	-0.10943	-0.14012	0.10850
	-0.16700	-0.33125	0.16425	0.32900
6	0.01023	0.19406	-0.18383	-0.19499
	0.29025	0.09125	0.19900	-0.09350
7	-0.03534	0.10912	-0.14446	-0.11005
	-0.17700	-0.29475	0.17775	0.29250
8	0.05983	0.23188	-0.17205	-0.23281
	0.13175	-0.05975	0.19150	0.05750
9	-0.13051	0.04433	-0.17484	-0.04526
	0.44175	0.22325	0.21850	-0.22550
10	0.07905	0.10819	-0.18724	-0.10881
	-0.01325	-0.15450	0.14125	0.15225
11	-0.10075	0.06603	-0.16678	-0.06665
	0.00975	-0.13100	0.14075	0.12875
12	-0.26319	-0.07719	-0.18600	0.07657
	0.51525	0.30650	0.20875	-0.30900
13	-0.15748	0.06107	-0.21855	-0.06200
	-0.03850	-0.16525	0.12675	0.16300
14	-0.04030	0.13361	-0.17391	-0.13454
	-0.07375	-0.23450	0.16075	0.23225
15	-0.13764	0.06820	-0.20584	-0.06913
	-0.03450	-0.18675	0.15225	0.18450
16	-0.25854	-0.04681	-0.21173	0.04588
	-0.00475	-0.17000	0.16525	0.16775

Table 9.--Continued.

No.	F* - E*	G* - E*	F* - G*	Ê* - G*
17	-0.31961 -0.23850	-0.16058 -0.36675	-0.15903 0.12825	0.15965 0.36450
18	0.01829 0.34700	0.17732 0.12675	-0.15903 0.22025	-0.17825 -0.12925
19	-0.08587 0.04400	0.08773 -0.13350	-0.17360 0.17750	-0.08866 0.13100
20	0.22258 0.18150	0.38471 -0.01700	-0.16213 0.19850	-0.38564 0.01475
21	-0.10044 0.06100	0.08401 -0.10875	-0.18445 0.16975	-0.08494 0.10625
22	-0.03596 0.02825	0.15748 -0.10000	-0.19344 0.12825	-0.15841 0.09775
23	-0.07998 -0.05200	0.13268 -0.19850	-0.21266 0.14650	-0.13361 0.19625
24	-0.23498 0.03025	-0.07719 -0.12050	-0.15779 0.15075	0.07626 0.11825
25	-0.18445 0.05600	0.00341 -0.12975	-0.18786 0.18575	-0.00434 0.12750
26	0.02573 -0.00750	0.18321 -0.19400	-0.15748 0.18650	-0.18414 0.19150
27	-0.05146 0.01000	0.14074 -0.12200	-0.19220 0.13200	-0.14136 0.11975
28	-0.34472 -0.31800	-0.19809 -0.45775	-0.14663 0.13975	0.19747 0.45550
29	0.03658 0.29550	0.20429 0.08375	-0.16771 0.21175	-0.20522 -0.08600
30	-0.01767 -0.11475	0.11346 -0.30275	-0.13113 0.18800	-0.11439 0.30025
31	-0.05735 -0.05225	0.12586 -0.20225	-0.18321 0.15000	-0.12648 0.20000
32	-0.12958 0.01075	0.04588 -0.11975	-0.17546 0.13050	-0.04681 0.11750

Table 9.--Continued.

No.	F* - E*	G* - E*	F* - G*	Ê* - G*
33	-0.18476	-0.02449	-0.16025	0.02356
	-0.08025	-0.24175	0.16150	0.23950
34	0.02046	0.22847	-0.20801	-0.22940
	0.04650	-0.09050	0.13700	0.08825
35	0.06789	0.26164	-0.19375	-0.26226
	-0.00525	-0.16725	0.16200	0.16475
36	-0.19189	0.00682	-0.19871	-0.00775
	0.00150	-0.17175	0.17325	0.16950
37	-0.30566	-0.14787	-0.15779	0.14694
	-0.27100	-0.38800	0.11700	0.38575
37A	--	--	-0.17918	--
			0.11725	
38	-0.14911	0.02728	-0.17639	-0.02790
	0.02175	-0.09300	0.11475	0.09075
39	-0.08308	0.07657	-0.15965	-0.07750
	-0.07850	-0.25275	0.17425	0.25050
40	-0.04309	0.14539	-0.18848	-0.14632
	0.04400	-0.07975	0.12375	0.07750
41	-0.13764	0.02821	-0.16585	-0.02914
	-0.01425	-0.18575	0.17150	0.18325
42	0.19437	0.34534	-0.15097	-0.34627
	0.16625	-0.04275	0.20900	0.04040
43	-0.05332	0.14074	-0.19406	-0.14136
	0.05150	-0.06800	0.11950	0.06575
44	-0.09734	0.09920	-0.19654	-0.10013
	-0.10300	-0.24825	0.14525	0.24600
$\overline{\Delta\phi}$	-0.09000	0.08514	-0.17523	-0.08600
s	0.14902	0.14919	0.02296	0.14921
$\Delta\lambda$	0.02869	-0.13485	0.16251	0.13254
s	0.17202	0.15420	0.03115	0.15422

There was a resulting mean change of 0.090 meter, with a standard deviation of 0.149 meter in latitude and 0.172 meter in longitude. The addition of the second-order observations resulted in a mean shift of 0.9 meter in the positions for the 44 selected first-order analysis stations. This suggests that there were one or more second-order projects that have an appreciable influence upon the final positions to those first-order stations that are in the vicinity of the second-order projects.

The shifts in final positions produced by adding more Geoceiver observations to the basic set of triangulation data were investigated next.

The third comparison (G*-E*, table 9) was of the final positions from the adjustment of the first-order network with station Webster 1939 constrained (adjustment E*) with the final positions from the adjustment of the first- and second-order networks with five constrained stations (Webster 1939, Knob 1914, Little 1934, Winn 1929, and Kelley 1971) using adjustment G*. This comparison resulted in mean changes of 0.085 meter, with a standard deviation of 0.149 meter in latitude, and 0.135 meter, with a standard deviation of 0.154 meter in longitude.

The fourth comparison (F*-G*, table 9) isolated the influence of the addition of Geoceiver observations upon the final positions of the 44 selected first-order analysis stations. The final positions from the adjustment of the first- and second-order networks, with station Webster 1939 as the constrained station (adjustment F*), was compared with the final positions from the adjustment of the first- and second-order networks with the five constrained stations listed in the third test, adjustment G*. The mean change in the comparison of the final positions was -0.175 meter with a standard deviation of 0.023 meter in latitude and 0.162 meter with a standard deviation of 0.031 meter in longitude. The small standard deviation of 0.02 and 0.03 meter indicates that a rather uniform shift of -0.175 meter in latitude and 0.162 meter in longitude has occurred throughout the test area.

The mean final positions in the four adjustments, $\hat{E}$*, E*, F*, and G*, are plotted relative to the initial preliminary position in figure 15.

The previous method of analysis, i.e., the comparison of the final positions from adjustments involving Geoceiver stations, is very much dependent upon the agreement of the Geoceiver-determined positions and the positions at the same stations obtained through the triangulation network. A similar analysis in another area would not necessarily give the previous results.

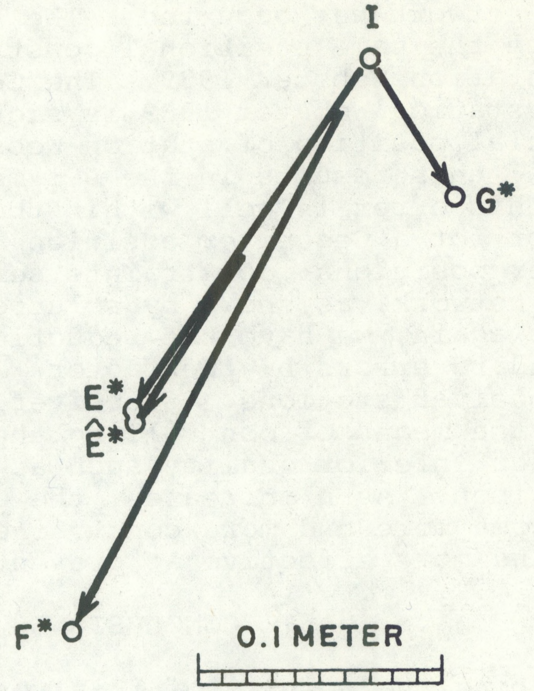

Figure 15.--Movement vectors.

Explanation of figure:

		(meters)
F* - E*	$\overline{\Delta\phi}$	-0.09000
	$\overline{\Delta\lambda}$	0.02869
G* - E*	$\overline{\Delta\phi}$	0.08514
	$\overline{\Delta\lambda}$	-0.13485
F* - G*	$\overline{\Delta\phi}$	-0.17523
	$\Delta\lambda$	0.16251
Ê* - G*	$\overline{\Delta\phi}$	-0.08600
	$\overline{\Delta\lambda}$	0.13254

Adjustment Ê* - First-order network, one fixed station.
Adjustment E* - First-order network, one constrained station.
Adjustment F* - First- and second-order network, one constrained station.
Adjustment G* - First- and second-order network, five constrained stations.
I is the preliminary position in all of the adjustments.

It is now possible to state additional conclusions. The $\hat{E}$*, E*, and F* solution seems to form a set in which a similar mean shift of the network has occurred. The $\hat{E}$*, E*, and F* adjustments all have the same positional constraint, the Geoceiver position of station Webster 1939. The Geoceiver-determined position for station Webster 1939 is significantly different from the initial position of station Webster 1939 obtained by triangulation; this results in the 0.1 meter shift seen in figure 15. This error is well within the 1 meter a priori standard error for a Geoceiver position. The addition of four more Geoceiver positional constraints serves to reduce the mean shift of the network, as shown by solution G* in the same figure. Here again, we have the reduction in the Geoceiver position standard errors by the factor $\sqrt{n}$, where n = 5, the number of Geoceiver stations. Geoceiver observations would appear to be the means of controlling the distorting influence of a project of inferior quality such as the one detected in the second comparison. As n increases, the Geoceiver station positions become more and more constrained and consequently become more and more effective at preventing network distortions.

5. CONCLUSIONS

In interpreting the results of this report, the reader must remember that while the theory is applicable to any network, the inferences from the data are strictly speaking applicable only to the particular network with which the authors worked. This is probably not a severe limitation because the full network was first skeletonized to what might be considered a representative first-order network. This network was further abstracted by the removal of most of the measured distances, the removal of still more measured distances, and finally the removal of all measured distances. A solution was obtained for each of these generalized networks. For most of these networks, solutions were also obtained in which there were different numbers of Geoceiver positions. It should, therefore, be possible to make a comparison, without too much error, between one or more of our generalized networks and other networks generalized in the same way. Then, the results can be extrapolated to the fuller network, except in situations where the network is pathological.

Perhaps the most important conclusion (see section 2) is that as soon as a network contains a small number of distance and azimuth observations (20 and 25 respectively in our example), the network becomes "rigid." The effect of adding Geoceiver observations is almost entirely to decrease slowly the standard error of the network scale and orientation. The shape and size of the network remain practically unchanged, until a large number of Geoceiver observations are added. The standard deviation in location or orientation is inversely proportional to the square root of the number of stations at which Geoceiver positions were observed.

Secondly, in a network not containing measured distances, the scale, location, and orientation of the network are determined by the Geoceiver observations. Behavior of the standard deviations of location and orientation is the same as for a network containing measured distances, but behavior of the standard deviations of shape and scale is less clear. According to theory (appendices 1 and 2), the error in scale is inversely proportional to the distance between Geoceiver positions (when only two Geoceiver observations are involved); while the error in shape is determined almost entirely by the measured directions.

The data for networks containing only two Geoceiver observations and no measured distances agree only approximately with what would be expected from theory. While theory (appendix 2) predicts that the standard deviation in a coordinate should be inversely proportional to the distance between Geoceiver positions, the agreement of this conclusion between the data presented in Dracup's paper (1975) and this paper (section 2 and table 4) is shown to be only approximate. The average value of the standard deviation (in length of a side) changes by about 50 percent when the distance between Geoceivers is increased from 181 km to 426 km, while the change from 181 km to 256 km, is still about 50 percent.

There are two ways of accounting for this anomaly. One is to accept the variation of average standard deviation with distance as an empirical fact. An alternative (appendix 2) to note that while the theory assumes that the standard deviations are computed with one Geoceiver at the origin of coordinates, this assumption does not hold true for the cases used. It appears that the adjustment is about a different center in each case. Then a comparison of standard deviations at the same points in the different networks would not give information on the variation of standard deviations with distance.

While base line and azimuth observations are preferred, the adjustments which were run using this test network indicate that Geoceiver observations may be used in lieu of the more traditional base lines and azimuths to provide scale and orientation in the local network.

APPENDIX 1. MATHEMATICAL ANALYSIS OF A GEODETIC NETWORK CONTAINING MEASURED DIRECTIONS, DISTANCES, AND COORDINATES

Since three different kinds of quantities--directions, distances, and coordinates--were to be combined into one set of equations, it seemed best to use a set of unknowns more closely related to the observables than the conventional coordinates of stations. The set adopted was the dimensionless ratio, x_i, y_i, of each coordinate, X_i, Y_i, to coordinate X_2; coordinates X_1, Y_1 were left out. Three new unknowns were introduced to make up for the three dropped. These were f, the scale of the unknowns with respect to length, and Δ_X, Δ_Y, the coordinates of point P_1 with respect to a selected origin.

Then we have as the set of observation equations

$$\overset{Y}{\begin{bmatrix} dY_x \\ dY_\ell \\ dY_\theta \end{bmatrix}} = \overset{A}{\begin{bmatrix} A_{x1} & A_{x2} & [1] \\ A_\ell & A_{\ell 2} & O \\ A_\theta & O & O \end{bmatrix}} \overset{X}{\begin{bmatrix} dx \\ df \\ dg \end{bmatrix}}$$

where [1] denotes a vector of 1's, g is the vector Δ_X, Δ_Y, and the subscripts θ, ℓ, and x refer to directions, distances, and coordinates respectively.

The covariance Σ^2 of the unknowns is then related to the covariance Σ_y^2 with components Σ_x^2, Σ_ℓ^2, and Σ_θ^2 of the observations Y_x, Y_ℓ, and Y_θ by

$$\Sigma^2 = \left[A^T \Sigma_y^{-2} A \right]^{-1}$$

where Σ_y^{-2} denotes the inverse of Σ_y^2.

Breaking the matrix A into its components and multiplying out,

$$\Sigma^2 = \left[\begin{array}{c|c|c} A^T_{x_1}\Sigma_x^{-2} A_{x_1} + A^T_\ell \Sigma_\ell^{-2} A_\ell + A^T_\theta \Sigma_\theta^{-2} A_\theta & A^T_{x_1}\Sigma_x^{-2} A_{x_2} + A^T_\ell \Sigma_\ell^{-2} [Y_\ell] & A^T_{x_1}\Sigma_x^{-2}[1] \\ A^T_{x_2}\Sigma_x^{-2} A_{x_1} + [Y_\ell]^T \Sigma_\ell^{-2} A_\ell & A^T_{x_2}\Sigma_x^{-2} A_{x_2} + [Y_\ell]^T \Sigma_\ell^{-2} [Y_\ell] & A^T_{x_2}\Sigma_x^{-2}[1] \\ [1]^T \Sigma_x^{-2} A_{x_1} & [1]^T \Sigma_x^{-2} A_{x_2} & N\Sigma_x^{-2} \end{array}\right]^{-1}$$

$$= \begin{bmatrix} B_{11} & B_{12} & B_{13} \\ B_{21} & B_{22} & B_{23} \\ B_{31} & B_{32} & B_{33} \end{bmatrix}$$

where $[Y_\ell]$ has been substituted for A_{ℓ_2}, to which it is approximately equal. N is the number of Geoceivers in the network.

We now take a look at those elements of Σ^2 that lie along the main diagonal, and separate them into three kinds of variance: $\Sigma^2{}_{dx}$, pertaining to the shape of the network; $\Sigma^2{}_{df}$, pertaining to the scale of the network; and $\Sigma^2{}_{dg}$, pertaining to the location of the network. Note that in this analysis no attention has been paid to the network's orientation. The orientation was ignored because its standard deviation behaves so much like the standard deviation of location that the conclusions about standard deviations of location can be applied immediately to standard deviation of orientation without having to complicate the analysis. (Orientation, like location, is determined by two quantities, for example, the ratio of northing and easting of a particular point with respect to a fixed point. Since the equations have been linearized, the fact that orientation is a ratio is irrelevant; the errors appear linearly.)

Using Schur's well-known lemma, we obtain

$$\Sigma^2_{dx} = \left[B_{11} - F_1 b_{11}^{-1} F_1^T\right]^{-1}$$

$$\Sigma^2_{df} = \left[B_{22} - F_2 b_{22}^{-1} F_2^T\right]^{-1}$$

$$\Sigma^2_{dg} = \left[B_{33} - F_3 b_{33}^{-1} F_3^T\right]^{-1}$$

where b_{jj} denotes the adjoint of B_{jj} and F_j denotes the off-diagonal submatrix coupling B_{jj} and b_{jj}. A number of facts are immediately obvious on comparing these three equations with the equation for Σ^2.

In the equation Σ^2_{dg} we see that both B_{33} and the accompanying term contain the factor N, the number of Geoceivers in the network. The standard deviation of g, the location of the origin of the network, is therefore inversely proportional to $\sqrt{N}$. It is also independent, obviously, of the locations of the Geoceivers or the distances between them and of the scale and shape of the network.

In the second equation the variances of the data from the Geoceivers and of the measured distances are coupled. Where the variance of the measured distances is considerably smaller than the variance Σ^2_x of the coordinates of the Geoceivers, $[Y^2_\ell]$, the sum of the squares of measured distances will have a predominating influence. In the network investigated the standard deviations of the measured distances appear to be smaller than the standard deviations of the equivalent distances from the Geoceivers. In addition, the measured distances are more numerous by a factor of at least five to ten. Hence we would expect the data from the Geoceivers to have little effect on scale of the network and, in fact, this is what the results show. There is one exception to this, and that is when there are very few or no measured distances. In this case the second equation shows that the factor on the right contains the quantity $[X^2]$, the sum of the squares of the distances of the Geoceivers from the origin. When only two Geoceivers are present, and one is at the point of origin, the standard deviation of scale is inversely proportional to the distance between the two Geoceivers. This prediction is only approximately supported by the computations; the reason for this discrepancy is discussed further in appendix 2.

Finally, looking at the first equation, we see that the term B_{11} does not contain directly either the number N or the distances between Geoceivers or the sum of lengths of measured distances. These quantities do enter indirectly into B_{11} through the parts A_{x_1} and A_{x_2} of the observation matrix A. But for a network in which the standard deviation of the coordinates of the Geoceivers and the measured distances are not much smaller than the equivalent standard deviation of the directions (multiplying the standard deviation of a direction by the length of the line), the Geoceiver coordinates and the measured distances obviously will not have much effect on the shape of the network. This conclusion is fully supported by the results. Directions were compared before and after adding Geoceivers to networks that contained no measured distances, and a few measured distances were compared. It was found that where about 20 measured distances were already present, the shape (directions) did not change at the 0."01 level. When no measured distances were used, most of the changes were still below the 0."01 level, while those above that level tended to cluster in parts of the network which were suspected to be weak.

All in all, the theory fully supports the experimental results. However, there has been sufficient interest in the way standard errors of a network are affected by varying the spacing between Geoceivers that a more detailed examination of this point seems worthwhile; appendix 2 provides this information.

APPENDIX 2. EFFECT OF INCREASING DISTANCE BETWEEN GEOCEIVERS ON STANDARD DEVIATION OF COORDINATES

What happens to a network containing only directions if two points in the network are occupied by Geoceivers, so that the coordinates of these two points are "measured"? The answer is immediately derivable from the analysis in appendix 1 by dropping all terms involving length ℓ and setting N equal to 2. The result is that (putting one point at the origin for convenience) the scale is proportional to the distance between the two points, while the ratios between coordinates are not affected; that is, the shape of the network remains unchanged.

Another more graphic way of showing the effect of increasing distance between two Geoceivers is to conceive of a geodesic being drawn connecting the two stations P and P'. This geodesic is the shortest distance; its standard error, being determined by the standard errors of the end points, is constant regardless of the length of the geodesic. But P and P' can also be thought of as being connected by a large number of other paths that start at P and proceed along the sides of the triangles of the network to end at P'. Now the standard error in any one of these alternative paths is determined by the standard errors in the lengths of the sides that make up this path. With a little ingenuity, we can formulate the relationship between the standard error of the geodesic, the length of the geodesic, and the standard lengths of the sides making up a particular alternative path. Since the error in the distance from P to P' must be the same whether calculated along the geodesic or from an alternative path, and since increasing the distance increases the number of sides in alternative paths, the standard errors in the sides must decrease to keep the total effect constant. This conclusion is intuitively obvious from the geometric picture presented. To formulate the procedure algebraically is somewhat tricky although straightforward, and will not be given here.

A final variation of proof goes as follows: Let Z be the vector from P_1 and P_2, the two points at which Geoceivers are placed. Consider any sequence of sides (of triangles) forming a continuous path from P_1 to P_I and consider these sides as vectors Z_i, i = 1 to I. If the coordinates of the ends of vector Z_i are X_i, Y_i and X_{i+1}, Y_{i+1}, we have for the length r of vector Z

$$r^2 = Z^T Z$$

and

$$dr = dZ = \left[\frac{\Sigma(X_{i+1} - X_i)}{r} \quad \frac{\Sigma(Y_{i+1} - Y_i)}{r} \right] \begin{bmatrix} \Sigma d(X_{i+1} - X_i) \\ \Sigma d(Y_{i+1} - Y_i) \end{bmatrix}$$

The standard deviation σ^2 of r is then

$$\sigma^2 = [\cos\theta \ \sin\theta] \begin{bmatrix} \sigma^2_{\Sigma X} & 0 \\ 0 & \sigma^2_{\Sigma Y} \end{bmatrix} \begin{bmatrix} \cos\theta \\ \sin\theta \end{bmatrix}$$

where $\sigma^2_{\Sigma X}$ and $\sigma^2_{\Sigma Y}$ pertain to $\Sigma d(X_{i+1} - X_i)$ and $\Sigma d(Y_{i+1} - Y_i)$, respectively. Putting this in terms of the standard deviations of the individual segments, we have

$$\sigma^2 = [\cos\theta \ldots\ldots \cos\theta \ \sin\theta \ldots\ldots \sin\theta] \begin{bmatrix} \sigma^2_{X1,2} & & & & & & \\ & \sigma^2_{X2,3} & & & & & \\ & & \ddots & & & & \\ & & & \sigma^2_{XI,I-1} & & & \\ & & & & \sigma^2_{Y1,2} & & \\ & & & & & \ddots & \\ & & & & & & \sigma^2_{YI,I-1} \end{bmatrix} \begin{bmatrix} \cos\theta \\ \vdots \\ \cos\theta \\ \sin\theta \\ \vdots \\ \sin\theta \end{bmatrix}$$

where $\sigma_{i,i+1}$ is the standard deviation of the segment from point P_i to point P_{i+1}.

We now write

$$\sigma^2_{X2,1} = - \sigma^2_{X1,2} \text{ , etc.}$$

Then

$$\sigma^2 = \cos^2\theta \sum_k \sigma^2_{X_{k+1,k}} + \sin^2\theta \sum_\ell \sigma^2_{Y_{\ell+1,\ell}}$$

where the sum is now over a set of intervals on the X and Y axes that are all positive.

Then, since σ^2 must be constant, regardless of the size of k and ℓ, i.e., of distance between P_1 and P_2, it follows that as k and ℓ increase $\sum\sigma^2_{X_{k+1,k}}$ and $\sum\sigma^2_{Y_{\ell+1,\ell}}$ must decrease. This in turn implies that corresponding variances of the end points of the segments must decrease, since

$$\sigma^2_{X21} = \sigma^2_{X2} + \sigma^2_{X1}, \text{ etc.}$$

It will be noted that this conclusion does not agree exactly with Dracup's (1975) results.

Dracup's results

Case	No. of Geoceivers	Distance between Geoceivers	Relative error (average)
D	2	181	1:90,000
C	2	436	1:156,000
B	5	436 (maximum)	1:178,000

No computations were carried out specifically to identify the cause of the lack of agreement. However, an analysis of tables 3 and 5 giving ratios of standard deviations for the various cases shows that the ratios not only do not obey the

$(\text{distance})^{-1}$ law but vary from point to point in the network. A glance at the plots of error ellipses shows that the program apparently adopted, for each different configuration of Geoceivers, a different center from which to compute standard deviations. With this being the case, it follows that comparing standard deviations of the same sides, for varying arrangements of Geoceivers, is comparing data which are affected by more than just different distances between Geoceivers.

APPENDIX 3. STATISTICAL CONSIDERATIONS

All the adjustments in this report were carried out using assumed values for the standard errors of the observations. These values are based on extensive experience of NGS in the analysis of errors in other smaller nets. We do not know, of course, that these values actually apply in the present case. It would be helpful to be able to find from the network being investigated better values for the standard errors of the observations. If all the observations are of one kind, this is no problem. But where, as in the present case, there are several kinds and classes of observations, finding improved values is not easy. It may even be impossible, as, for instance, if there are only one or two observations of a particular kind. In general, however, each kind of observation can be expected to be present in considerable numbers, and estimates of the standard error of each kind made. The following formula is suggested for the purpose:

$$\sigma_i^2 = \frac{v_i^T v_i - k}{n_i - \mathrm{Tr}\left\{\left[A_i^T A_i\right]\left[A^T A\right]^{-1}\right\}}$$

where v_i is the vector of residuals of observation of type i, n_i the number of observations of type i, A the matrix of observations, and A_i that submatrix of A relevant to observations of type i. k is a constant, whose value can be found to be

$$k = \frac{1}{I} \sum_i \mathrm{Tr}\left\{\left[A_i^T A_i\right]\left[A^T A\right]^{-1}\right\}$$

where i = 1 to I.

Derivation of the formula is easy enough that the authors have not troubled to search the literature for earlier derivations. A somewhat similar, but different, formula was apparently derived by Thiel (1963), and is quoted by Bossler (1972).

APPENDIX 4. COMPARISON OF ADJUSTED NETWORKS WITH AND WITHOUT DATA FROM GEOCEIVERS: ADJUSTMENTS B, C, AND D

The networks used by Dracup (1975) in his analysis did not contain any measured lengths. They derived their scale solely from the data of the Geoceivers. Hence the coordinates of points in these networks cannot be compared with the coordinates of points in a Geoceiver-free network. But some guesses can be made as to the behavior of the Geoceiver-containing networks with respect to a Geoceiver-free network containing a true scale.

The coordinates of points in networks B, C, and D are the same, after adjustment,to within 3 cm. Assuming an average length of 20 km for the sides of the triangles in the networks, this 3 cm corresponds to about 0".2 maximum difference in directions. The networks, therefore, could be considered essentially the same in all three cases. But the case for considering the networks practically unchanged by introducing the data from the Geoceivers is even stronger if one examines the trend of these differences. The difference of about 3 cm is nearly constant in longitude between networks C and D; it decreases in latitude to 0 from about 3 cm. Comparing solutions C and D with the solution for network B, we find that B agrees to within a centimeter or so in latitude with C and to within a centimeter or so in longitude with D, while there is a 3-cm nearly constant difference between B and C and D in longitude and latitude, respectively.

It seems clear from these numbers that the basic network remains similar under all introductions of Geoceiver data; that is, it changes size but not shape. Furthermore, on examining the lists of residuals in directions we find that for all three networks (B, C, D) the residuals are within 0".02 of each other. Comparing these residuals with those obtained by adjusting the network containing measured distances, we find agreement to within 0".04 for the most part, with a few discrepancies as high as 0".2 and a very few higher than this.

APPENDIX 5. COMPARISON OF ADJUSTMENTS ON NETWORKS WITH AND WITHOUT DATA FROM GEOCEIVERS

The coordinates of the adjusted coordinates of each station were compared for the cases:

a. No Geoceiver stations in the network.

b. Central and southern Geoceiver stations in the network.

c. Central and eastern Geoceiver stations in the network.

d. All five Geoceiver stations in the network.

It was found that the adjusted positions were the same for all four stations, to within 1 to 2 cm. Consequently, we can conclude that the data from the Geoceivers did not affect the geometry of the network in any way, but merely affected the standard error of location of the network as a whole. In other words, the network is rigid with respect to action on it from Geoceiver data.

APPENDIX 6. VARIATION OF VARIANCE OF SHAPE WITH LOCATION OF GEOCEIVERS

The shape portion of the matrix given in appendix 1 is in general quite difficult to invert. However, there are several simple cases which are interesting for this investigation. In particular, for instance, when the network contains only directions and one or two Geoceivers without any measured distances. In these cases, the resulting matrices are easily inverted. The problem can be simplied still further by assuming that one of the Geoceivers is at the origin of coordinates or, what is almost the same thing, by placing the origin at one of the Geoceivers. The data from that Geoceiver do not contribute to the shape submatrix at all. The contributions from the data of the other Geoceiver appear only as additions of $k^2\sigma_{jj}^2$ to consecutive elements on the main diagonal. (k is the scale factor and σ_{jj}^2 the variance of each given coordinate.) To get a clear picture of what is happening, we look first at the effect of specifying only the second X-coordinate, then at the effect of specifying both X- and Y-coordinates of the second Geoceiver. The variance $\bar{\sigma}_i^2$ of pseudo-coordinate i is

$$\bar{\sigma}_i^2 = \bar{B}_{ii} / \bar{B}$$

where the bar ($^-$) indicates that data from Geoceivers are included, while absence of a bar refers to the Geoceiverless network. We expand numerator and denominator in elements and cofactors of the j*th* row (the second Geoceiver being at point Pj). We get

$$\bar{\sigma}_i^2 = \frac{B_{ii} + k^2\bar{\sigma}_{jj}^{2}B_{jj}^{i}}{B + k^2\bar{\sigma}_{jj}^{2}B_{jj}}$$

where the superscripts indicate that the designated rows and columns have been deleted. Using a self-evident notation this becomes

$$\bar{\sigma}_i^2 = \frac{\sigma_i^2 + k^2\sigma_{jj}^{-2}\sigma_j^{2,i}}{1 + k^2\sigma_{jj}^{-2}j^2} \tag{1}$$

Extension to the case where data on both X- and Y-coordinates are given is somewhat more complicated, but the end result is

$$\bar{\sigma}_i^2 = \frac{\sigma_i^2 + k^2\sigma_{jj}^{-2}\left(\sigma_j^{2,i} + \sigma_{j+1}^{2,i}\right) + k^4\sigma_{jj}^{-4}\ \sigma_{j+1}^{2,\,i,j}}{1 + k^2\sigma_{jj}^{-2}\left(\sigma_j^2 + \sigma_{j+1}^2\right) + k^4\sigma_{jj}^{-4}\ \sigma_{j+1}^{2,j}} \tag{2}$$

This formula can be generalized fairly easily to the case where neither Geoceiver is at the origin. It will look very much like equation 2 but will go up to the fourth power of $(k^2\sigma_{jj}^{-2})$ and the sum of four (two pairs) of variances of the original matrix. For our purposes equation 1 is sufficient, since equation 2 is analogous to it. For actual networks we can assume that the second term in the numerator is small compared to the first. Then, $\bar{\sigma}_i^2$ is inversely proportional to $(1 + k^2\sigma_{jj}^{-2}\sigma_j^2)$, which is a linear function of σ_j^2. The value of σ_j^2 depends on the position of P_j in the network and cannot be expected to vary monotonically as the distance between P_1 and P_j increases. Hence, the portion

$$A_1^T\sigma_X^{-2}\ A_1 + A_\theta^T\ \sigma_\theta^{-2}A_\theta$$

of the normal matrix will contribute a rather irregular variation of overall variance of a particular coordinate as the distance between P_i and P_j.

Note that the contributions of other parts of the normal matrices have been ignored. If we include them, we find that the actual distance $X_j^2 + Y_j^2$ occurs in the denominator. This is over and above whatever is contributed by the shape portion of the matrix.

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