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USING THE DIVERGENCE EQUATION TO INITIALIZE A BAROTROPIC MODEL

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1. Introduction

The divergence equation has a long history in numerical modeling, atmospheric study and diagnosis. A form of the divergence equation, the balance equation, gives a unique relationship between the geopotential height field and wind field such that the divergence and vertical motion is zero. This equation was used to initialize early numerical models. Fankhauser (1974) used the full divergence equation to derive consistent fields of wind and geopotential height in "upper air" meso networks. Cram et. al. (1991) proposed to use the divergence equation to retrieve geopotential height information using the profiler winds.

In this paper, a simplified form of the divergence equation is used to initialize a barotropic model. The height field is determined by computing the laplacian of geopotential from the remaining terms of the divergence equation. The laplacian is solved numerically to yield the geopotential height information. A better initial analysis can be obtained by this method resulting in an improved 12-hour barotropic forecast.

2. The Divergence Equation

The divergence equation can be written as (Moore and Abeling 1988):

$$\frac{\partial D}{\partial t} = -\vec{V}\cdot\vec{\nabla}D - \omega\frac{\partial D}{\partial P} - D^2 - \frac{\partial \vec{V}}{\partial P}\cdot\vec{\nabla}\omega + 2\left(\frac{\partial u}{\partial x}\frac{\partial v}{\partial y} - \frac{\partial v}{\partial x}\frac{\partial u}{\partial y}\right) + f\zeta - \vec{\nabla}^2\phi - \beta U \qquad (1)$$

A B C D E F G H I

Here, D is divergence $\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}$, u, v, and w are the three components of the total wind velocity ($\vec{V}$), and beta is $\frac{\partial f}{\partial y}$, or the change in the Coriolis parameter in the y direction (the Beta-Plane approximation). The relative vorticity is represented by ζ, and the geopotential (g*z) is represented by Φ.

The local rate of change of divergence (Term A) is due to the following. Terms B and C are the advection of divergence in both the horizontal and vertical. Term D is the effect of the divergence field on its tendency. Term E describes the divergence tendency as a result of the interaction of the vertical wind shear with the horizontal gradients of vertical motion. The remaining terms (F-I) constitute a subset and specialized case of the divergence equation, called the "balance equation". The Jacobian term (F) relates the shear of the wind to the divergence tendency. It becomes significant in regions where the horizontal wind shear is large, such as in strongly sheared jets, or in curved jet streaks.

Term G is the relative vorticity times the Coriolis parameter. House (1961) relates this to term H, the laplacian of geopotential, in the following way. When the wind flow departs from the geostrophic value, in other words when ageostrophic motions are occurring, then the component of the vorticity due to the ageostrophic flow is defined as $f\zeta'=f(\zeta-\vec{\nabla}^2\phi/f_0)$, and represents the difference between the relative vorticity of the real wind and the geostrophic value. Therefore, by definition, the relative vorticity of the real wind and the geostrophic relative vorticity must be displaced from one another and different in magnitude any time ageostrophic motions are occurring. If the real wind is nearly geostrophic and nondivergent, then the geostrophic relative vorticity will be a good approximation to the relative vorticity of the actual wind. Likewise, in the presence of ageostrophic flow, the geostrophic vorticity may be a poor approximation to the actual relative vorticity. Finally, term I is the Coriolis term. Its magnitude is dependant on the u component (east-west) of the windspeed. It is relatively small in comparison with the other terms on the synoptic-scale.

The goal of this procedure is to force the height field to conform to the wind field. This should make the geostrophic relative vorticity a good approximation to the vorticity of the real wind, even in the presence of ageostrophic flow.

3. The Balance Equation and Initialization of the Model

When all the terms relating to the divergence and vertical motion are assumed to be zero, the resulting equation is called the balance equation. It represents a unique windflow in which the height field and wind field are in a state of balance such that no divergence or vertical motions are occurring. In the real atmosphere, when terms (F-I) are summed and are non-zero, the atmosphere is said to be non-balanced or out-of-balance to a certain extent. When this condition is present, divergence and vertical motions form in response to the unbalanced flow. These ageostrophic motions then act to bring the atmosphere closer to a state of balance:

$$0=-\vec{\nabla}^2\phi+f\zeta+2\left(\frac{\partial u}{\partial x}\frac{\partial v}{\partial y}-\frac{\partial v}{\partial x}\frac{\partial u}{\partial y}\right)-\beta U \qquad (2)$$

Early numerical models were initialized with the balance equation when it was desired to supress fast moving gravity waves that were not meteorologically significant. The terms and definitions are the same as in the divergence equation. Here, ageostrophic flow is allowed, so the relative vorticity can be displaced from the geostrophic value. Either the height field can be forced into balance with the wind field or vice-a-versa. This is done by computing the appropriate term as a residual to the other three terms. In the case of the barotropic model, Φ the geopotential, is solved for by the method of simultaneous relaxation (Holton 1979) by computing it as a residual of the right three terms. The single boundary condition which must be specified is the value of the 500 mb height along the grid boundary. For this quantity, the actual 500 mb observed height was used. Once the height field has been solved for, it is input into the barotropic model as a more realistic initialization.

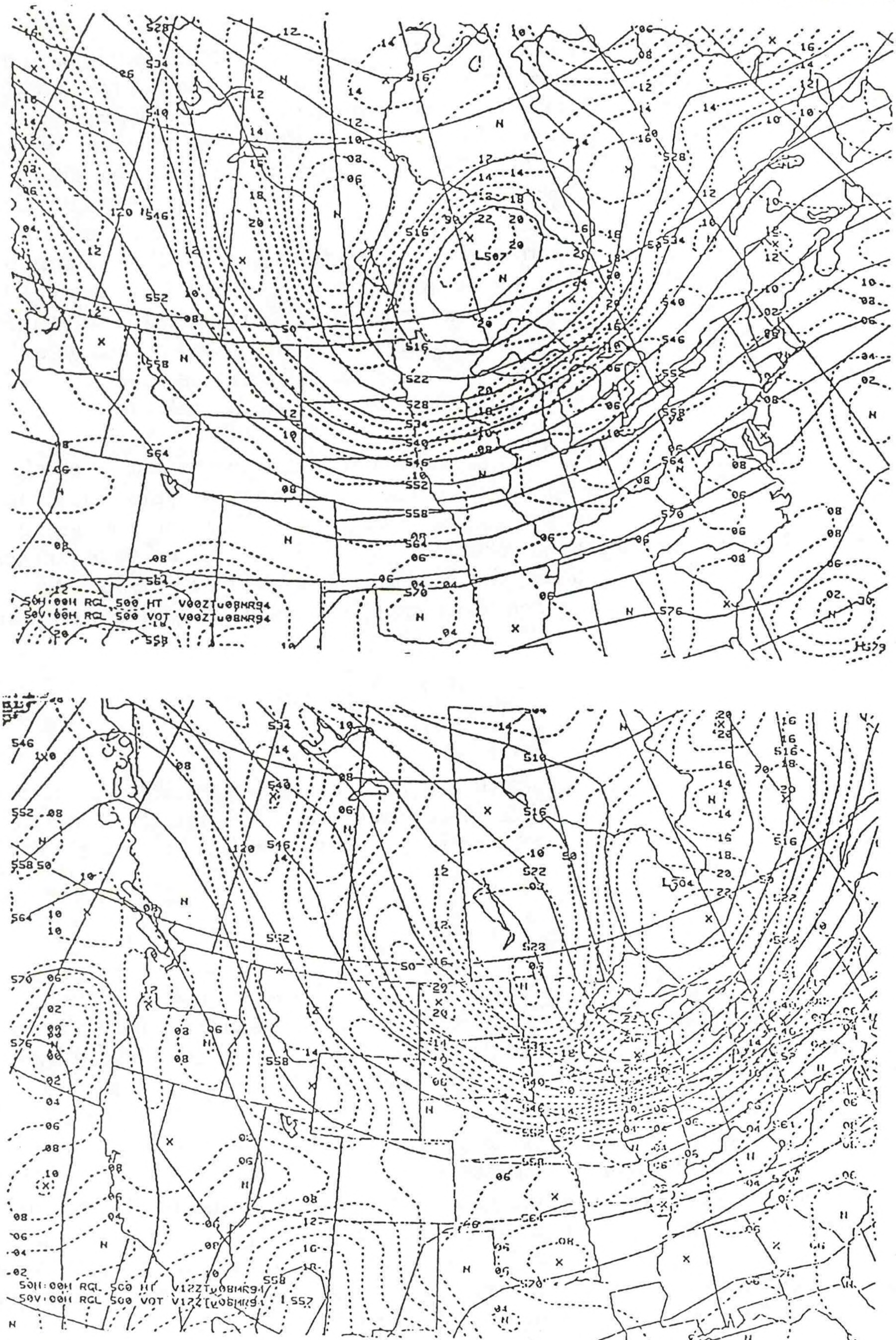
However, as was done in this study more information can be obtained about the state of the atmosphere by using the entire divergence equation. In this case, information about the vertical motion as well as the divergence would be needed. Since the observed vertical motion field is difficult to obtain operationally, to simplify the process, the final equation used will neglect the vertical motion terms and the divergence tendency. The resultant equation is then

$$\vec{\nabla}^2\phi = -\vec{V}\cdot\vec{\nabla}D - D^2 + 2\left(\frac{\partial u}{\partial x}\frac{\partial v}{\partial y} - \frac{\partial v}{\partial x}\frac{\partial u}{\partial y}\right) + f\zeta - \beta U \quad (3)$$

Here, it is assumed the vertical motion terms are small in comparison with the divergence terms, and the local time rate of change of divergence is small. These are fairly good assumptions on the synoptic scale for short time periods.

4. A Case Example

Data from 0000 UTC 8 March 1994 was chosen as a case example. A strong and complex 500-mb cyclone, with several absolute vorticity maxima rotating around it was located southwest of Hudson Bay, as depicted by the NGM initial analysis (Figure 1a). The initial- and 12-hour forecast of the barotropic model using the actual height field are shown in Figures 2a and 2b. The initial- and 12-hour barotropic model forecast using the geopotential height derived from the divergence equation is shown in Figures 3a and 3b. For comparison, the NGM initial analysis valid 1200 UTC March 8 is presented in Figure 1b. The derived height from the divergence equation provides greater detail and a closer fit to the initial NGM analyses the latter, which is much more sophisticated. The model was able to capture the vorticity maxima better and also able to make an improved 12-hour forecast (as can be seen from comparisons of Figures 3b to 1b).



(1a)

(1b)

Figure 1. (1a) 500 mb NGM analysis 0000 UTC 8 March and (1b) 1200 UTC NGM analysis.

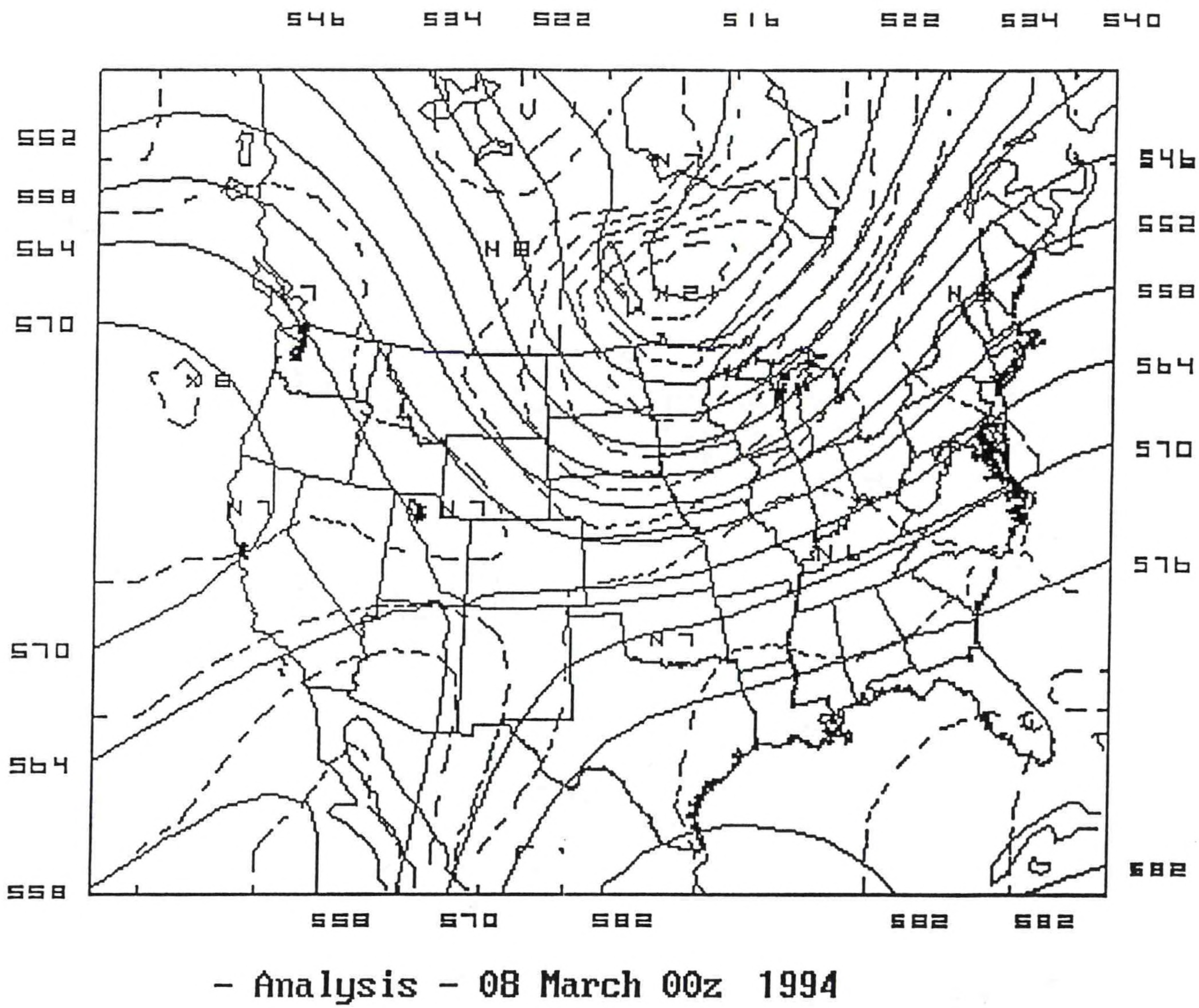


Figure 2a. 500 mb barotropic analysis initialized with geopotential height.

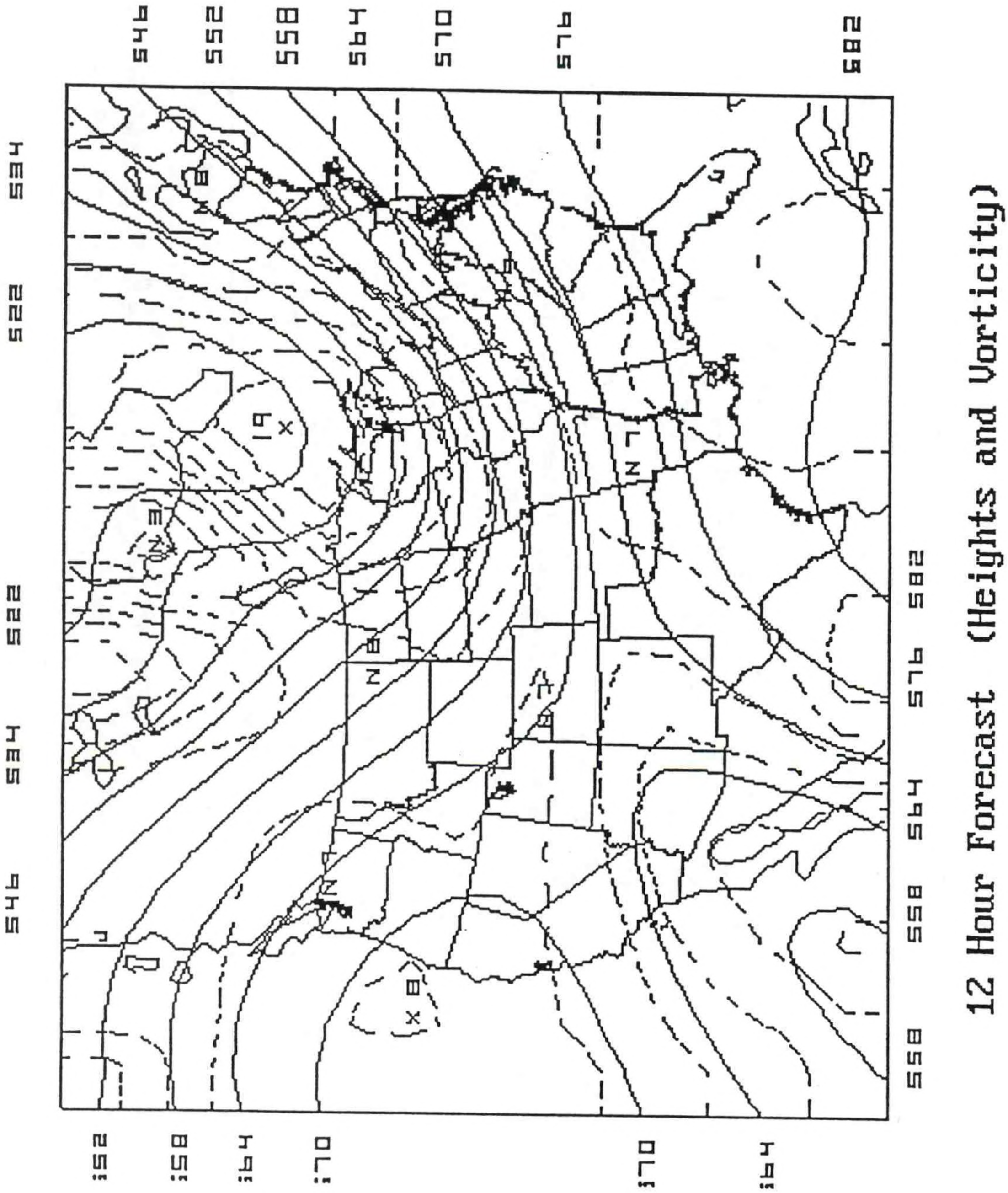


Figure 2b. 500 mb forecast initialized with observed geopotential height.

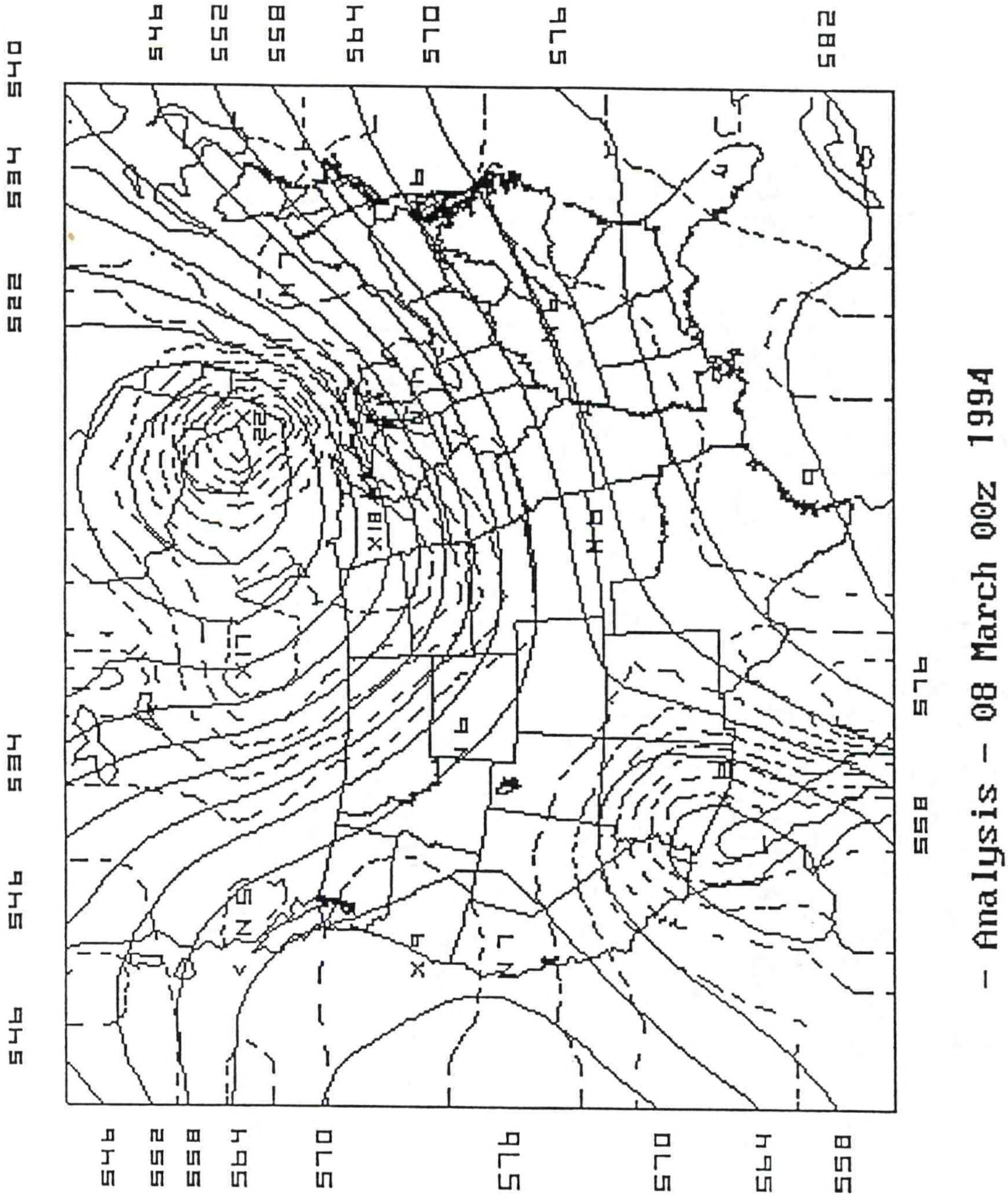


Figure 3a. 500 mb barotropic analysis initialized with the divergence equation.

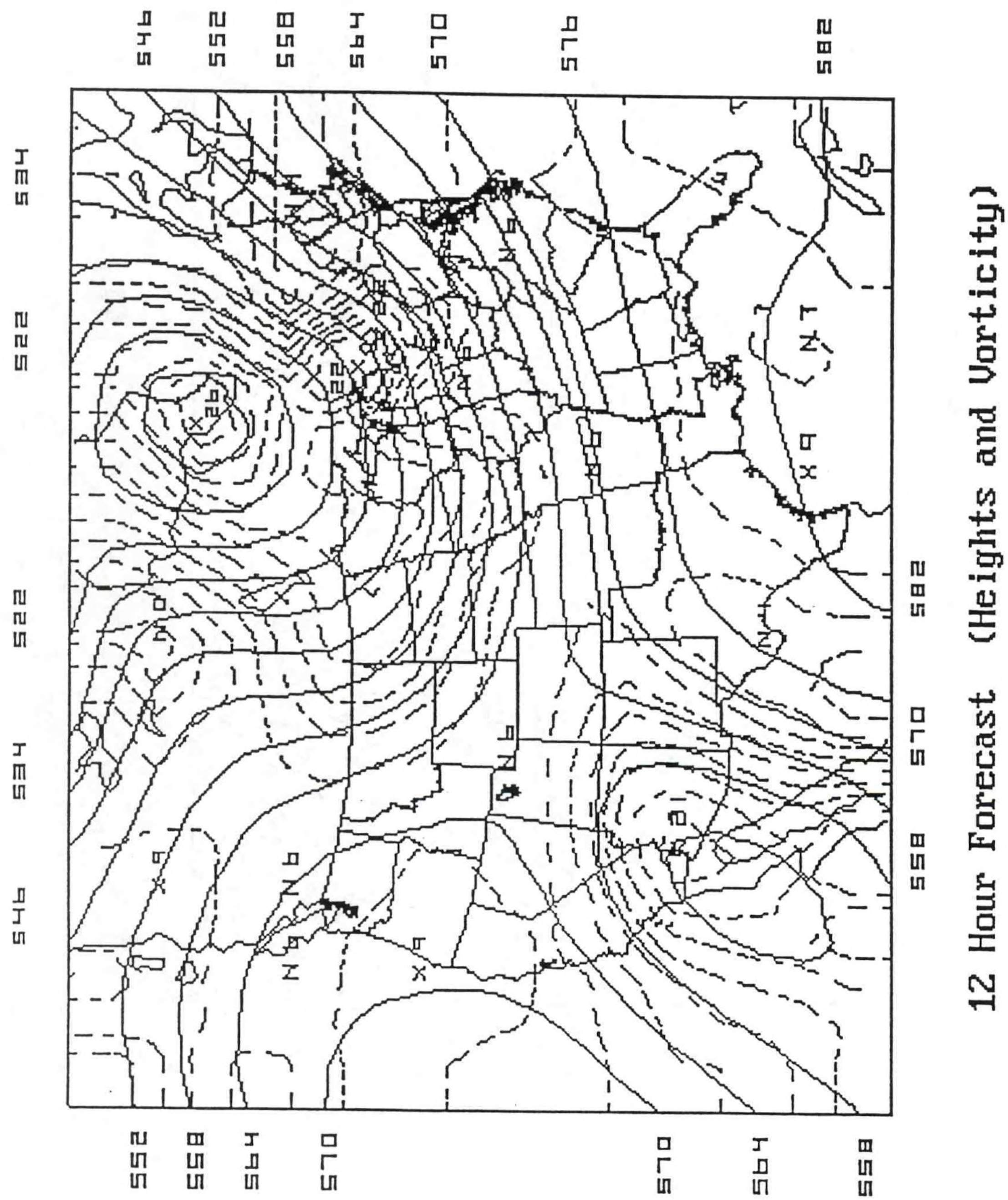


Figure 3b. 500 mb barotropic forecast initialized with the divergence equation.

Scattered snow showers developed across western and central North Dakota by 1800 UTC 8 March 1994 was likely associated with the short wave trough/maximum in absolute vorticity that was moving southeastwards. The barotropic forecast using the actual 500-mb height field placed the short wave trough north of Minnesota with anticyclonic vorticity advection across the Dakotas. The model utilized with the divergence equation, had greater detail with the features, and was able to predict the passage of an "important" short wave trough/vorticity maximum across North Dakota.

5. Conclusion

The barotropic model is a simple and fast numerical integration that can forecast 500 mb features with enough skill to be useful as a backup to NCEP numerical guidance in the 12 to 24 hour time frame (Abeling 1992). The model can run on a 486 class PC in 4-to-6 minutes (and on the new SACs), making the analysis and forecast suitable as a quick look at 500 mb initial and forecast features. That can be done as soon as the 50A plotfiles arrive. The model can be run utilizing the observed geopotential height and vorticity, or can be initialized with data from the divergence equation. The latter may more realistically resolve the short wavelength features that are divergent and ageostrophic. While the barotropic model still cannot forecast cyclogenesis, the improved analysis can result in an improved short-term forecast.

6. References

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