CRH SSD
OCTOBER 1992

CENTRAL REGION TECHNICAL ATTACHMENT 92-21

TRANSFORMATION EQUATIONS FOR MAPPING THE NMC OCTAGONAL GRID AND NORTHERN HEMISPHERE MAP BACKGROUND TO A VGA SCREEN

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Rapid advances in PC technology have made many new data sets available to the operational line forecaster. One such data set is the complete record of upper air observations interpolated to the NMC Octagonal Northern Hemisphere Grid. Historical data from 1946 to the present, covering many levels and parameters, is available on CD-ROM. Because the data contains only the values at the NMC gridpoints, users must generate their own map background and perform their own display analysis. This programming note will explain how to generate a map background by use of polar stereographic and scaling transformation equations. The author has a file for distribution that contains the latitude and longitude of all the points needed to draw a map of the Northern Hemisphere. The borders of the lower forty eight states are also in this file.

The NMC Octagonal Grid (Figure 1) is enclosed by a rectangle whose dimensions are 47 x 51 gridpoints. Note that the Greenwich Meridian is offset to the west by 10 degrees. To employ a Cartesian Coordinate System, we will use east longitude values. We also will need to subtract 10 degrees from each longitude.

The VGA screen contains 640 x 480 pixels. If we assign a scale of 9 pixels per NMC gridpoint our map background will be 423 pixels wide and 459 pixels tall. This will leave ample space to the right of the map for program use and sufficient space at the bottom for a date and title. We will label the gridpoints as 1 through 47 in the x direction and 1 through 51 in the y direction. The North Pole, which lies at the center of our map, is at gridpoint (24,26) or pixel (216,234).

Figure 2 shows the geometry of the polar stereographic projection. The map lies tangent to the earth's surface at the North Pole, B. Projection rays such as line (A,F) originate at the south pole, F. Point C on the earth's surface has a latitude of $(90^\circ - \epsilon)$ and lines (C,E) and (E,F) are equal in length to the earth's radius.

Assume that we have the latitude and longitude of point C and we want to find the pixel location of its projection, A, on our screen. First we must find the length of line (A,B), which is the distance that point A lies from the map position of the North Pole.

The triangle (A,B,F) is a right triangle and thus:

$$(\overline{A,B}) = \tan(\gamma) * (\overline{B,F})$$

Note that (B,F) is twice the radius of the earth, or 2R. Our equation now becomes:

$$(\overline{A,B}) = \tan(\gamma) * 2R$$

Since $\alpha + \epsilon$ must equal 180°:

$$\alpha = 180^{\circ} - \epsilon$$

And because (C,E,F) is an isosceles triangle whose 2 acute angles, β and γ, are equal (recall that the sum of the angles of a triangle is 180°):

$$\gamma = \frac{180^{\circ} - (180^{\circ} - \epsilon)}{2} \quad \text{or} \quad \gamma = \frac{\epsilon}{2}$$

Now note that the latitude (ϕ) of point C is $90^{\circ} - \epsilon$ or likewise ϵ equals $90^{\circ} - \phi$. Making this substitution we get:

$$\gamma = \frac{90^{\circ} - \phi}{2}$$

Because (A,B,F) is a right triangle we can solve for the length (A,B):

$$(\overline{A,B}) = \tan\left(\frac{90^{\circ} - \phi}{2}\right) * 2R$$

Figure 3 gives the view from directly above the North Pole. This is how the projection would normally appear on a computer screen. Since we know the pixel location of the North Pole, all that remains is to find the length of lines (B,G) and (A,G). These, when added to the location of B, will give us the x and y pixel coordinates of point A. Recall that we already know the length of line (A,B). Noting that the angle ψ is the longitude (λ) of point A minus the 10° offset of the NMC grid we get:

$$(\overline{B,G}) = (\overline{A,B}) * \cos(\lambda - 10^{\circ})$$

$(\overline{G,A}) = (\overline{A,B}) * \sin(\lambda - 10°)$

(B,G) represents the x offset from the pole of point A and (G,A) represents the y offset. Now making the substitution of the equation for (A,B) we get:

$$x \text{ offset} = \tan\left(\frac{90° - \phi}{2}\right) * 2R * \cos(\lambda - 10°)$$

$$y \text{ offset} = \tan\left(\frac{90° - \phi}{2}\right) * 2R * \sin(\lambda - 10°)$$

All that remains to complete our transformation equations is the scaling of R, the earth's radius in pixels. This can be accomplished by solving a special case of the above equation for the x offset from the North Pole. Refer back to Figure 1. Note that the midpoint of the line from NMC coordinates (1,37) to (1,15) lies at 17.21 N, 190.0 E. The x offset from the North Pole of this point is -23 grid lengths or -207 pixels. Inserting these values into the transformation equation we get:

$$-207 = \tan\left(\frac{90° - 17.21°}{2}\right) * 2R * \cos(190° - 10°)$$

The solution for R is 140.41 pixels. The equations give a pixel location relative to the North Pole's position on the screen. Consequently, we must add the pole's horizontal and vertical offsets from the lower left origin of the screen to the equations. Their final form now becomes:

$$x \text{ pixel} = \tan\left(\frac{90° - \phi}{2}\right) * 280.82 * \cos(\lambda - 10°) + 216$$

$$y \text{ pixel} = \tan\left(\frac{90° - \phi}{2}\right) * 280.82 * \sin(\lambda - 10°) + 234$$

Where: ϕ is the north latitude and λ is the east longitude.

These two equations will transform any latitude and longitude location to a pixel location on a VGA screen. Many compilers format the screen from top to bottom. These equations assume the traditional coordinate system where the y axis increases in value from bottom to top. The user must make the proper adjustment in program logic to handle this. To locate the NMC gridpoints over this screen simply multiply their x and y values by 9 to get the pixel locations. For example, gridpoint (20,30) is at (x,y) pixel location (180,270). Be sure to number the NMC gridpoints as starting with a one instead of a zero.

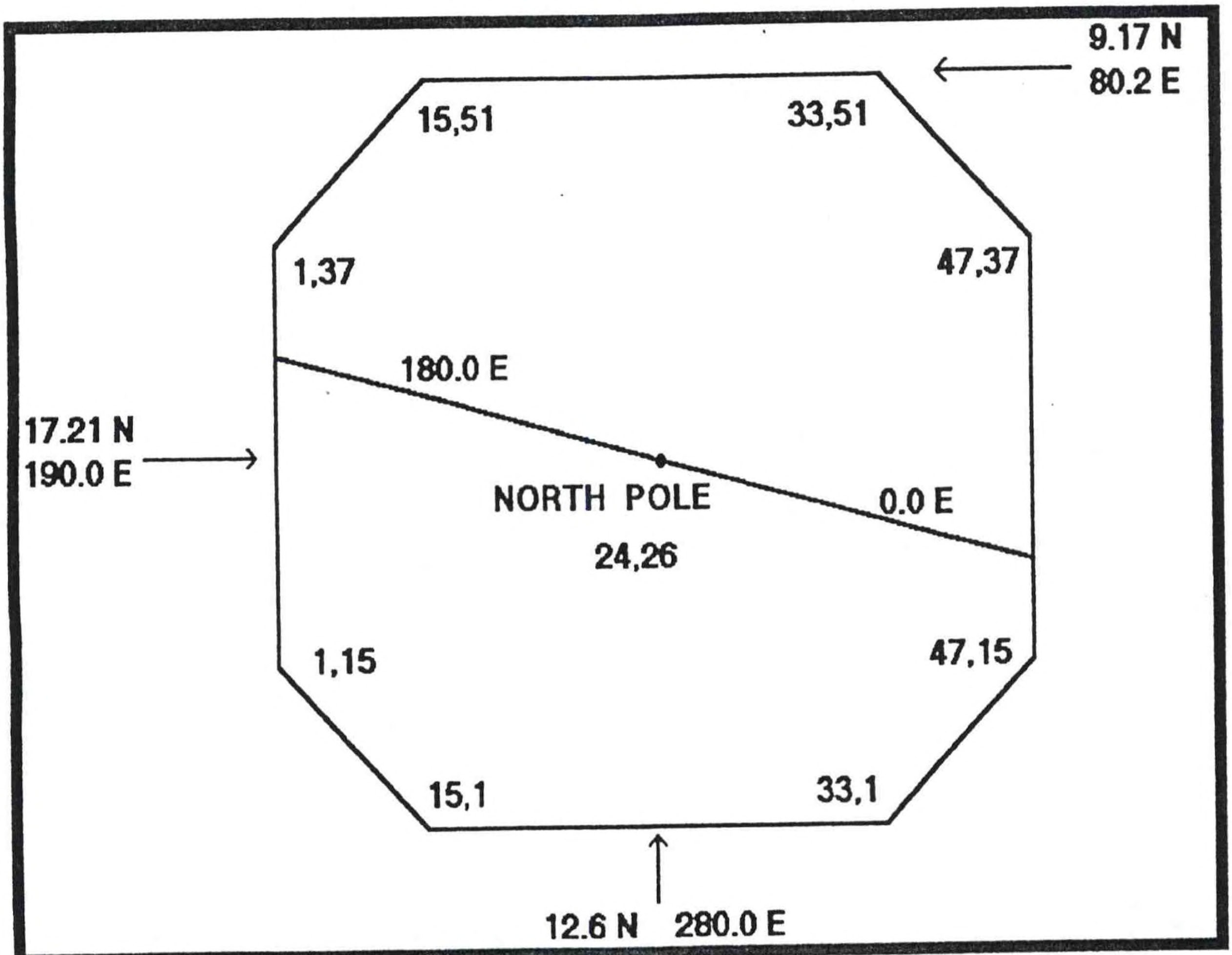

Figure 1. NMC OCTAGONAL GRID

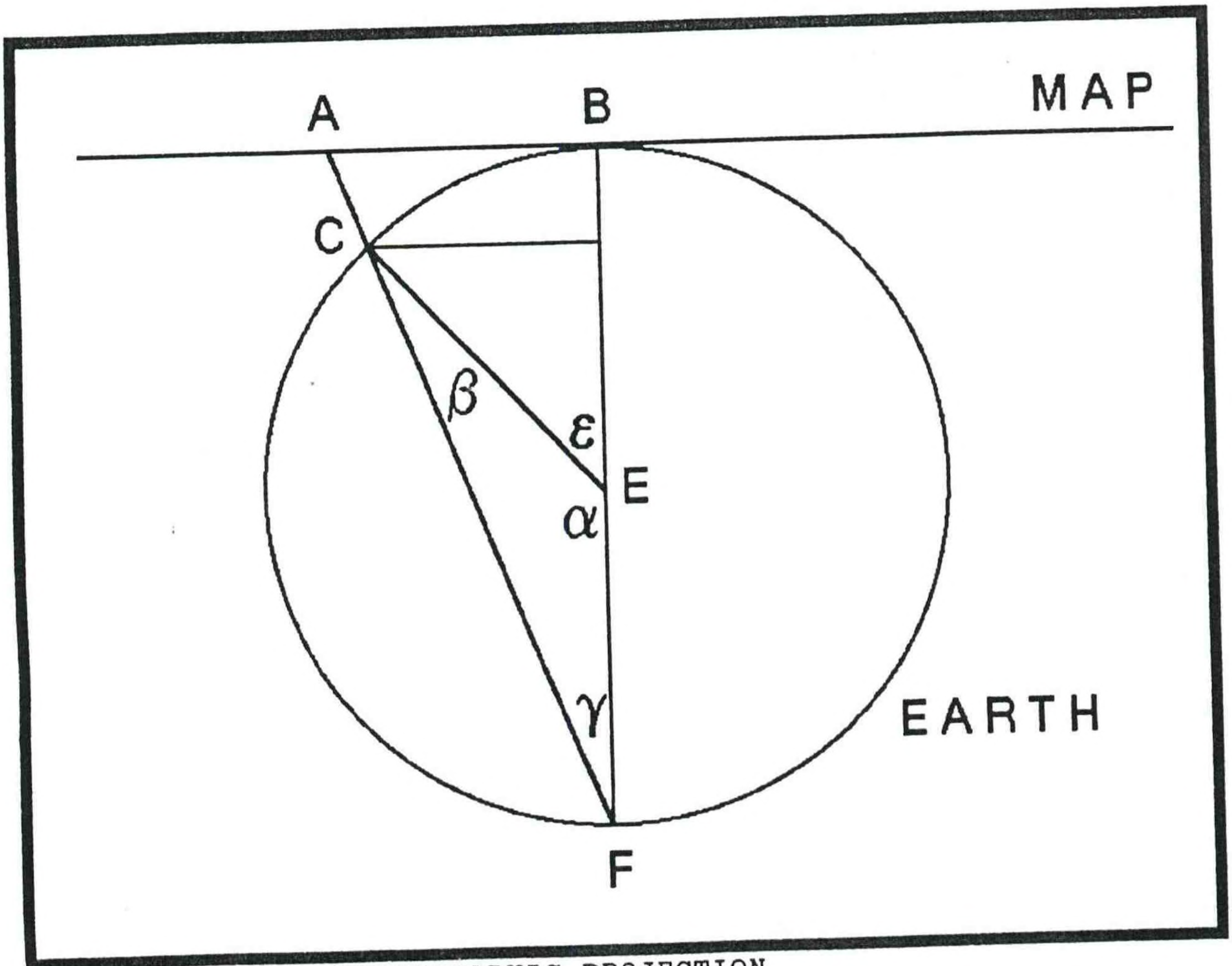

Figure 2. POLAR STEREOGRAPHIC PROJECTION

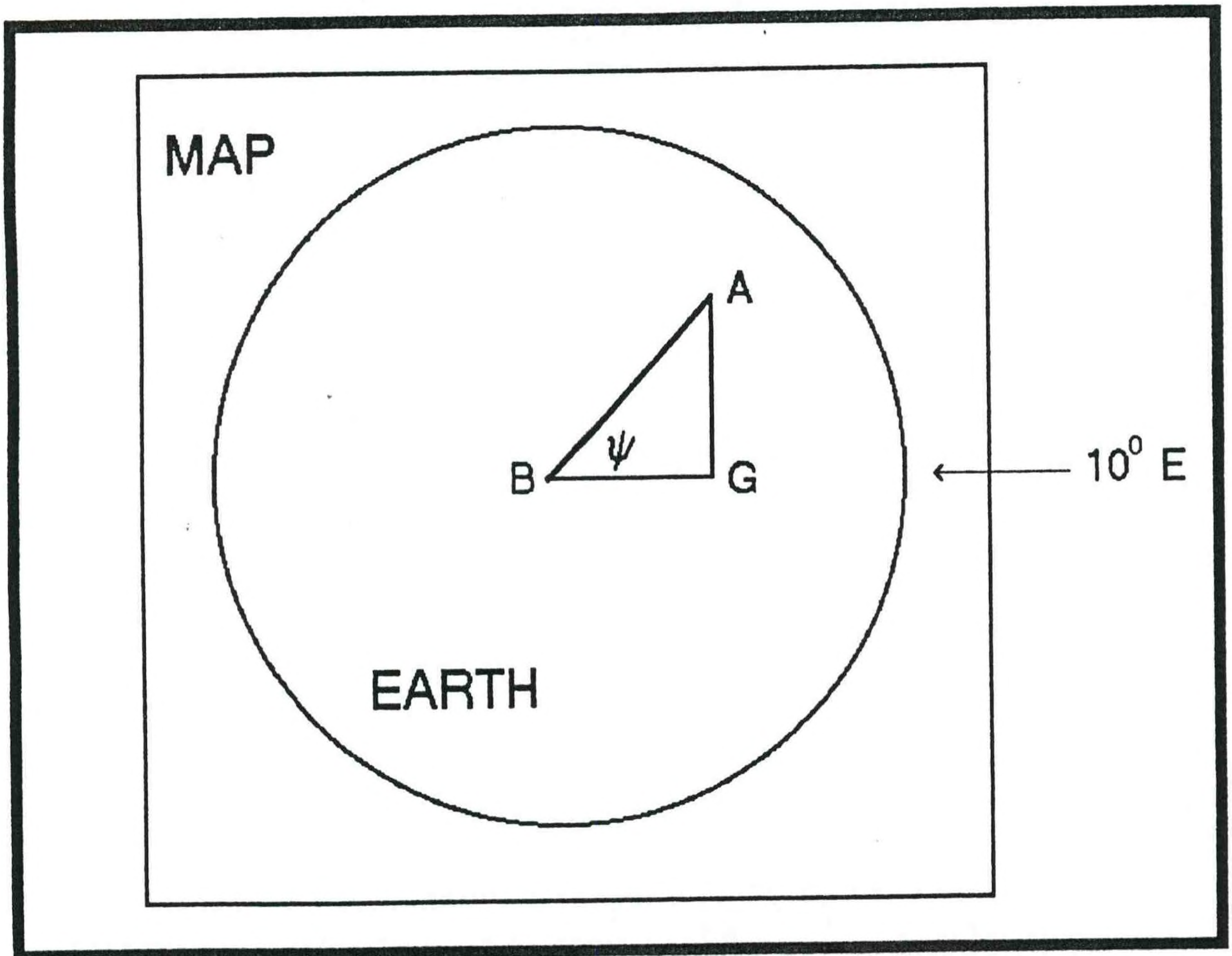

FIGURE 3. VIEW FROM ABOVE THE NORTH POLE