NOAA Technical Memorandum NOS NGS-10

USE OF CALIBRATION BASE LINES

Rockville, Md.
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USE OF CALIBRATION BASE LINES

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ABSTRACT. During the early 1970's, the number and types of electronic distance measuring instruments (EDMI) dramatically increased. Their use was expanded to cover almost every conceivable surveying problem. Quality assurance became a pressing concern. But, unlike tape or wire standardization, no recognized agency or organization was responsible for calibration standards for EDMI. Therefore, in 1974, the National Geodetic Survey (NGS) of the National Ocean Survey (NOS) began establishing a series of calibration base lines for this purpose. This publication was prepared in conjunction with this program and is directed to the land surveyor who uses EDMI. General observing procedures are outlined, and an analysis of the observations is developed. Detailed formulas are given for determining the geometric transformation of distances. An analysis is made of error sources affecting the ambient refractive index.

INTRODUCTION

The land surveyor is rapidly moving into the age of electronics. One of the problems that must be overcome if the surveyor is to make full use of available instrumentation is the public's inherent distrust of electronic devices. Consider the usual reaction to a department store billing error. Very rarely does one consider the programer who wrote the billing software to be responsible for the error; it is always the "computer!" that is to blame.

For the past 2,000 years, the surveying profession has relied on physical methods to carry out its task. The surveyor's chain or tape is a physical instrument that can be seen, its operation is easily understood, and it can be compared with recognized standards. It has withstood the tests of both popularity and legality.

The "black box" of electronic distance measuring instruments is a very different thing. It is perceived quite differently by the surveyor and the general public. To the surveyor it is a panacea, while the public treats it with awe. It appears to measure distances by "magic." Press a button and a number appears. What is the relationship of the number to the distance being measured? How do we know when a "good" EDMI begins to provide "bad numbers"?

Surveyors have, for the most part, obtained excellent results from EDMI. This leads to the temptation to accept the instrument on faith. Such an approach, however, must be tempered with some systematic plan to ensure that a minimum accuracy requirement is maintained throughout the life of the instrument and, equally important, to provide legal documentation against possible lawsuits arising from its use.

The surveyor will always be held accountable for assuring that the EDMI provides acceptable results. Calibration base lines provide one method of monitoring the accuracy of EDMI.

SUGGESTED GUIDELINES FOR USING CALIBRATION BASE LINES

The solution to most complex problems can only be obtained by a thorough investigation of all its various facets. This approach is certainly true for the problem of calibrating EDMI. Because numerous variables, ranging from human intervention to atmospheric deviation, influence the effectiveness of EDMI, the theoretical basis and operation of each particular instrument should be fully understood. This document gives only the outlines of general procedures applicable to most EDMI. For detailed instructions, various professional papers, textbooks, and manufacturers' manuals should be consulted. (See bibliography.)

The calibration process can be considered as having two phases: (1) the acquisition of distance observations, and (2) the analysis of the observations. Valid observational procedures can be invalidated by a distorted analysis and vice-versa. Therefore, the full potential of the calibration base line can be realized only if great care is exercised in performing both phases of this process.

ACQUISITION OF OBSERVATIONS

Because observational procedures lay the foundation for acceptable results, this phase must be investigated and prepared for in detail. Accessory equipment, such as thermometers, barometers, psychrometers, tribachs, and tripods, should be checked and, where applicable, checked against a standard. In addition, the functional relationship of each accessory device to the distance measurement must be understood.

Perhaps the most important element in determining the operational limit and overall accuracy of EDMI is the maintenance of an accurate log of the entire observational procedure. Also, a continuous log provides a history of the instrument that may be used later either to isolate changes in instrument characteristics or for legal verification purposes.

It is suggested that the following information be recorded at the time each observation is made:

1. The names (or numerical designation) of the stations from and to which the observations are made.

2. Instrument/tape model and serial number.

3. Reflector model and serial number.

4. Date and time of observation (Local time - 24 hour-clock).

5. Instrument/reflector constants*.

6. Height of instrument/reflector above marks*.

7. Station elevations*.

8. Instrument/reflector eccentricity*.

9. Atmospheric observations*.

    a. Temperature
    b. Pressure
    c. Psychrometer readings.

10. Weather conditions (clear, cloudy, hazy, rain, snow, fog, etc.).

11. Any unusual or problematic conditions, e.g., dust blowing across line or measuring across a gulley 30 m wide and 3 m deep.

Suggested Procedures for Using Calibration Base Lines

The present configuration for calibration base lines has monuments located at 0 m, 150 m, 430 m, and 1,400 m; some variations may occur because of topographical restrictions at the base-line site. This configuration provides six distinct distances for testing EDMI. For cases where additional marks have been set, the number of distinct distances can be determined by $n(n-1)/2$ where "n" is the number of monuments.

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*Units of measurement and, if applicable, the reference datum should always be shown.

Before designing the calibration test, two questions must be answered:

1. For what order of work is the instrument going to be used?

2. Do the manufacturer's specifications indicate it is possible to obtain that order of work?

If most of the work falls into the second-order classification, then test procedures should be developed accordingly. If the manufacturer claims an accuracy of 1:10,000, then, regardless of the effort expended on the test, it is unlikely accuracies of 1:20,000 can be obtained.

For a complete calibration test, the recommended procedure is to perform distance observations both forward and backward over each section of the base line on two separate days. Care should be taken to obtain an as wide as possible range of weather conditions. For example, this can be done by starting observations in the early morning on one day and in the afternoon on the next day. The preferred method is to perform the observations on two successive days: once during daylight and once during the night.

A less accurate test, but one which is sufficient for most needs, consists of measuring all sections of the base line in every combination both forward and backward, i.e., 12 distances would be observed for a four-mark base line. This is recommended as the standard calibration test; the resulting higher confidence in the results far outweighs the extra effort involved.

If it is decided to observe fewer lines than is required for the standard calibration test, it is perhaps more orderly to begin the observing scheme at the "0 m" mark. Measurements should then be made to each of the other monuments in turn. However, regardless of which monument is chosen, the absolute minimum observing scheme is to measure the distances to all other points in the base line; i.e., for a four-mark base line, a minimum of three measurements must be made.

Observing Procedures

1. Set up the instrument and reflector directly over the points to which the published measurements are referred. Care must be taken to assure not only that the instrument and reflector are centered over the points, but also that the tripods are firmly set. Careless centering will defeat the entire purpose of using the base line. In general, there should be little difficulty in centering the equipment to 1 mm or less.

Note: If a quick test of an instrument is to be performed, it may be expedient to set the heights of the instrument and reflector (or slave unit) at approximately the same height. If the difference between the heights of the instruments above the marks is less than (0.001s/ΔH) m, where s = horizontal distance between marks, and ΔH = difference of elevation between marks, then no geometric corrections need be applied to compare the measured distance with the published mark-to-mark distance. For example, if s = 1,650 m, ΔH = 10 m, then the allowable difference between the heights of the instrument is 0.165 m.

2. Initial warmup of the instrument should be performed according to manufacturer's instructions.

3. Measure and record heights of instruments and reflectors above the marks.

4. Read and record meteorological observations (dry and wet bulb temperatures and barometric pressure). Since ambient meteorological conditions have a direct bearing on the results of the distance observations and the near-topography atmosphere is the most turbulent, all precautions should be taken to secure accurate meteorological observations. Ideally, temperatures and pressures should be observed along the entire line during the observation sequence. In most cases, this will not be feasible, so some compromise must be made. In decreasing order of preference, the following measurements should be made: (1) temperatures and pressures at both ends of the line, both prior to and following the distance observation, and (2) the temperature and pressure at the instrument site.

If the deviations in dry bulb (Δt) and wet bulb (Δt') temperatures are 1° C (1.8°F) and the deviation in barometric pressure (Δp) is 3 mm (0.1 in)* of Hg, then the following table gives the error (in parts per million for each component) that will be introduced into a distance observation.

Type of instrument & applicable temperature range		Δt = 1° C	Δt' = 1° C	Δp = 3 mm of Hg
		ppm	ppm	ppm
Lightwave, including infrared (0°C-30°C)		1	0	1
Microwave	0°	4.6	5.5	1
	10°	4.5	6.9	1
	20°	4.5	9.8	1
	30°	4.7	12.5	1

*0.1 in of Hg corresponds to a change in altitude of approximately 30 m (~100 ft).

Note: A combination of errors of the magnitude of those given in the previous table may yield significantly erroneous results. (For a thorough discussion of the meteorological effects on the measured distances, see appendix II.)

5. Perform the distance observations. The number of repetitions over each section should follow the manufacturer's recommendations or those suggested in the professional literature. Several instruments have been developed with one or more features that reduce the computational effort usually associated with electronic distance measurements. These features are:

a. A direct input facility for meteorological corrections.

b. A display that optionally gives results in feet or meters, or both.

c. A combination angular and distance instrument that reduces observed slope distances to horizontal distances.

If the instrument being tested has one or more of the above features, additional observations should be taken to ensure the accuracy of the features. For instance, if the instrument being tested permits encoding meteorological data, two complete sets of distance observations should be observed. One set should be observed with the values set at zero and a second set observed with the actual atmospheric data entered into the EDMI. Distances determined using zero meteorological values should then be reduced independently and compared with distances determined when meteorological data were encoded into the EDMI.

MATHEMATICAL REDUCTION

After observations are made, they should be reduced to a common datum. The reduction can be divided in two stages: one dependent on meteorological conditions and the other dependent on geometrical configurations. (This is true only if no corrections were applied during or because of the observing sequence.)

Reductions for Meteorological Conditions

The correction (ΔD) to the measured distance (D) for actual atmospheric conditions is given by

$$\Delta D = (n-n_a)\ D \tag{1}$$

and the corrected distance by

$$D_o = D + \Delta D \tag{2}$$

where

n = nominal index of refraction as recommended by the manufacturer,

n_a = actual index of refraction, and

n_a is dependent on whether the EDMI has a lightwave source (including infrared) or a microwave source.

Various computational and mechanical methods have been used for determining the refractive index for ambient conditions of the atmosphere. The National Geodetic Survey currently (1977) uses the following equations for the computation of n_a.

Lightwave (including infrared) source

The group refractive index (n_g) for modulated light in the atmosphere at 0° Celsius, 760 mm of mercury (Hg) pressure, and 0.03% carbon dioxide is:

$$n_g = 1 + \left(2876.04 + \frac{48.864}{\lambda^2} + \frac{0.680}{\lambda^4}\right) \times 10^{-7} \qquad (3)$$

where λ is the wavelength of the light expressed in micrometers (μm).

The index of refraction of the atmosphere at the time of observations due to variations in temperature, pressure, and humidity can be computed from:

$$n_a = 1 + \frac{n_g - 1}{1+\alpha t} \cdot \frac{p}{760} - \frac{5.5\ e}{1+\alpha t} \times 10^{-8} \qquad (4)$$

where

$\alpha = 0.003661$

e = vapor pressure in mm of Hg

p = atmospheric pressure in mm of Hg

t = dry bulb temperature in degrees Celsius (°C).

Microwave source

The refractive index of the atmosphere for radiowaves differs from that of lightwaves. This is given by:

$$n_a = 1 + \left[\frac{103.49\ p}{(273.2+t)} + \frac{495{,}882.48\ e}{(273.2+t)^2} - \frac{17.23\ e}{(273.2+t)}\right] \times 10^{-6} \qquad (5)$$

where all variables are as defined for lightwaves.

A modified form of this equation is:

$$n_a = 1 + \left[\frac{103.46\ p}{273.2+t} + \frac{490{,}814.24\ e}{(273.2+t)^2}\right] \times 10^{-6} . \quad (6)$$

Tables for e may be found in the Smithsonian Meteorological Tables (List 1963). For the temperature range usually encountered in actual practice, the following equations provide sufficiently accurate results:

$$e = e' + de$$

where

$$e' = 4.58 \times 10^{a}$$
$$a = (7.5\ t')/(237.3 + t')$$
$$de = -0.000660(1+0.00115\ t')\ p\ (t-t')$$
$$t' = \text{wet bulb temperature in } °C.$$

Note: See Meade (1972) for a comprehensive discussion of various equations for computing refractive index. This article also contains tabular values for e' at 1° F intervals.

Reductions for Geometric Configurations

After applying the meteorological correction, the observed distance should be corrected for any eccentricities of the instrument or reflector (or slave unit) and their constants. In the following analysis, distance D_1 should then be reduced to the horizontal distance by:

$$D_H = \left(D_1^2 - \Delta h^2\right)^{1/2} \quad (7)$$

where $\Delta h = (H_j + \Delta H_j) - (H_i + \Delta H_i)$

H_i = elevation of station i

ΔH_i = height of instrument/reflector above station i

H_j = elevation of station j

ΔH_j = height of instrument/reflector above station j .

ANALYSIS OF CALIBRATION BASE-LINE OBSERVATIONS

A prerequisite to analyzing the observations is an awareness of the numerous possibilities for introducing errors into the distance observations. Some of these sources are:

1. Centering errors .

2. Improper pointing, voltage,or readings .

3. Errors in height of instruments or reflectors .

4. Measuring under extreme conditions or in areas where external factors unpredictably affect the instrument.

5. Unfamiliarity with the operating condition of the EDMI.

6. Incorrect meteorological data .

7. Improper alignment of optics .

8. Incorrect values for the constants of the reflectors or instruments .

9. Changes in the frequency of the instrument.

Of the above, most can be minimized by following proper procedures and exercising care in obtaining the observations. The others are predominantly attributable to natural aging or to mechanical changes in the structure of the instrument.

These latter errors can be determined only by frequent and periodic observations over a calibration base line and then only through proper evaluation of those observations.

There are no hard and fast rules that govern the analysis of calibration base-line observations. Almost every case must be treated individually. Of prime importance is the original intent for making these observations.

In the introduction,we stated that the surveyor will always be held accountable for providing acceptable results. Therefore, acceptability must be the goal. However, to prove a measurement is acceptable it must be demonstrated that the measuring instrument is reliable and accurate. Tests for reliability and accuracy are not easy. Such conclusions at best are based on arbitrary methods.

Most EDMI manufacturers routinely attribute certain accuracies to their instruments. Although these accuracies should reflect the instrument's ability to measure a "true value," they may, in fact, indicate only the repeatability (precision) of the instrument or test results performed under laboratory conditions. Theoretically, if the accuracy statistic is given in terms of a standard error (σ), 68.3% of the differences between a "true value" and an observed value should fall within the stated specification. Therefore, this value could be used for decision purposes, i.e., as a test statistic. However, the above is true only for large samples and for known standard errors. Both of these requirements are rarely satisfied. In addition, by us ng this test statistic for rejection purposes, another type of error may be committed, i.e., the rejection of

valid observations. To reduce the possibility of rejecting a valid observation, a limit of 3σ (three times the standard error value) is usually chosen for deciding if an observation is acceptable or not acceptable. Theoretically, 99.7% of the differences should fall within the 3σ range.

The sequence of operations to perform an analysis of the baseline observation is:

1. Compute the differences between observed values and published values.

2. Analyze these differences. If 99.7% of the observations fall within three times the manufacturer's stated accuracy and 68.3% fall within the manufacturer's stated accuracy, the instrument can be accepted as working accurately and reliably.

If the differences do not agree within above specifications, then a different method must be used to determine an instrument's acceptability. Various approaches can be designed for this purpose.

One such approach is to examine the differences between observed values and published values and determine if the difference is a constant or is proportional to the distance being measured (scale error).

If the differences appear systematic, the instrument constant can be redetermined over the 150-m length and the distances recomputed. If the comparison now shows agreement with the published values (within the above specifications), the solution is considered to be complete and the instrument accepted.

If the differences become significantly larger or smaller as the distances increase, the proper approach is to determine this scale correction. Caution should be exercised in applying the scale correction to other measured distances. Tests have shown that atmospheric sampling techniques in near-topographic situations (i.e., at ground level) can introduce errors in the range of 5 to 6 parts per million.

Therefore, a scale correction should be applied only when an instrument has historically shown a similar scale error under various meteorological conditions.

THE LEAST-SQUARES METHOD

Most calibration tests do not show a pattern of differences as clear as those outlined above. Also, many methods rely on a hit-or-miss approach. The preferred approach is a least-squares solution that simultaneously determines a scale and a constant correction. This solution is based on the supposition that

the differences can be attributed either to a scale correction or to a constant correction, or both. The basic equation for this solution is:

$$V = D_A - D_H - SD_A - C \tag{8}$$

where

S = a scale unknown

C = a constant unknown

D_A = the published horizontal distance corresponding to the distance observation

D_H = the observed distance reduced to the horizontal

V = the residual to the observed horizontal distance.

One equation of the above type is written for each observation.

The solution to this system of equations is very similar to the fit of a straight line to a series of points. The theory behind this process is given in many elementary statistics and calculus texts, and will not be presented here.

The solution is given by:

$$S = \frac{n\ \Sigma(D_A\Delta) - \Sigma\ D_A\ \Sigma\ \Delta}{n\ \Sigma\ D_A^{\,2} - (\Sigma\ D_A)^2} \tag{9}$$

$$C = \frac{\Sigma\ D_A^{\,2}\ \ \Sigma\Delta - (\Sigma\ D_A)\ \Sigma(D_A\ \Delta)}{n\ \Sigma\ D_A^{\,2} - (\Sigma\ D_A)^2} \quad \text{or} \tag{10}$$

$$C = \bar{\Delta} - S\ \bar{D}_A \tag{10a}$$

where

n = number of distances observed

$\Delta = D_A - D_H$ = difference between the published horizontal distance and the observed horizontal distance

Σ = the summation of values. For example, $\Sigma\ D_A$ = the sum of the published distances involved

$\bar{\Delta} = \Sigma\Delta/n$

$\bar{D}_A = \Sigma D_A/n$.

In addition to solving for S and C, it is also useful to compute four additional statistics to assist in analyzing the acceptability of the test: the estimated standard error of S ($\hat{\sigma}_S$), a test statistic t_S, the estimated standard error of C ($\hat{\sigma}_C$), and a second test statistic t_C. These values can be computed using the following:

$$\hat{\sigma}_S = \left[\hat{\sigma}_0^2 \quad \frac{n}{n \Sigma D_A^2 - (\Sigma D_A)^2}\right]^{\frac{1}{2}} \tag{11}$$

$$\hat{\sigma}_C = \left[\hat{\sigma}_0^2 \quad \frac{\Sigma D_A^2}{n \Sigma D_A^2 - (\Sigma D_A)^2}\right]^{\frac{1}{2}} \tag{12}$$

where

$$\hat{\sigma}_0^2 = \frac{\Sigma V^2}{n-2} \qquad \text{or} \tag{13}$$

$$\hat{\sigma}_0^2 = \frac{\Sigma(\Delta-\bar{\Delta})^2 - \frac{S}{n}\left[n \Sigma(D_A \Delta) - \Sigma D_A \Sigma \Delta\right]}{n-2} \tag{13a}$$

Eq. (13a) gives results that are computationally more correct. However, eq. (13) will give equal results if sufficiently significant digits are carried throughout the computations.

$$t_S = \frac{S}{\hat{\sigma}_S} \tag{14}$$

$$t_C = \frac{C}{\hat{\sigma}_C} \tag{15}$$

It can be shown that t_S and t_C follow the Student's t distribution, which is useful in analyzing small sample tests. For a more thorough explanation of the t statistic, see Mendenhall (1969, pp. 189-220).

Using eqs. (8) through (15), the following procedure may be used to analyze the calibration base-line test results:

(1) Compute S and C from (9), and (10) or (10a).

(2) Compute the residuals (V) from (8).

(3) Compute $\hat{\sigma}_0^2$ from (13) or (13a).

(4) Compute $\hat{\sigma}_S$ and $\hat{\sigma}_C$ from (11) and (12).

(5) Compute t_S and t_C from (14) and (15).

(6) Test the significance of S and C. For this we test the hypothesis (or supposition) that S and C are

statistically equal to 0 by comparing the values of t_S and t_C against the critical values of $t_{0.01\ d.f.}$ (d.f. = degrees of freedom = n-2) as given in table 1. There are four possible results:

(a) The absolute value of t_S is less than $t_{0.01\ d.f.}$ Then it can be said that S is statistically equal to 0, and S need not be applied.

(b) The absolute value of t_S is greater than $t_{0.01\ d.f.}$ This implies S is statistically not equal to 0. However, because the determination of the refractive index at ground level is very difficult, the instrument should be retested at another time under considerably different atmospheric conditions.

(c) The absolute value of t_C is less than $t_{0.01\ d.f.}$ As above for S, C is statistically equal to 0 and need not be applied.

(d) The absolute value of t_C is greater than $t_{0.01\ d.f.}$ The value of C should be applied to all observations made with the instrument. Note: The constant determined by means of these procedures should not be confused with an instrument constant. For example, the observations could contain a constant error from the instrument, the reflector, or a miscentering. This error source cannot be specifically identified or divided into individual components. For these reasons, without additional independent observations, the constant determined should more properly be called a system constant.

Table 1.--Critical values of t for "degrees of freedom" (d.f.) at 0.01 significance level.

d.f. = n-2	$t_{0.01}$	d.f. = n-2	$t_{0.01}$
1	63.657	9	3.250
2	9.925	10	3.169
3	5.841	11	3.106
4	4.604	12	3.055
5	4.032	13	3.012
6	3.707	14	2.977
7	3.499	15	2.947
8	3.355	20	2.845
		25	2.787

The preceding can best be illustrated by a few examples. However, before proceeding to the examples, a brief description of the published data will be given.

DESCRIPTION OF PUBLISHED DATA

As stated earlier, the present (1977) recommended configuration for calibration base lines consists of four monuments located at 0 m, 150 m, 430 m, and 1,400 m. This layout provides six distinct distances, as listed in the following format. (Where additional monuments are set, the number of distinct distances can be determined by the formula $n(n-1)/2$ where "n" is the number of monuments. See fig. 1 for an example.)

FROM STATION	ELEVATION (M)	TO STATION	ELEVATION (M)	ADJUSTED DISTANCE HORIZONTAL (M)	ADJUSTED DISTANCE MARK-MARK (M)	S.E. (MM)
XXX.....XXX	XXX.XX	XXX....XXX	XXX.XX	XXXX.XXXX	XXXX.XXXX	X.XX

The following should be noted:

1. The FROM and TO station names have been arbitrarily assigned and may not agree with the stamping on the disk.

2. Although the differential elevations are considered to be sufficiently accurate for the reduction of the measured distance, the elevations will not be integrated into the National Vertical Control Network, and therefore, should not be treated as bench marks.

3. The adjusted distances listed are the horizontal distances and the mark-to-mark distances. These distances are defined as the distance at the mean elevation of the two stations and the spatial chord distance between the centers of the disks, respectively.

4. The standard error is an estimated value determined from the adjustment and may be more of an indication of the repeatability of the instruments used for measuring the base line than of the actual accuracy of the base line. In this sense, the standard error may be optimistic.

US. Dept. of Commerce - NOAA
NOS - Natl. Geodetic Survey
Rockville, Maryland 20852

******FINAL******
CALIBRATION BASE LINE DATA
Beltsville Base Line
Source No. 13204

Quad - 390763
State - Maryland
County - Prince Georges

List of Adjusted Distances

From Sta. Name	Elev. (m)	To Sta. Name	Elev. (m)	Adj. Dist. (m) Horizontal	Adj. Dist. (m) Mark - Mark	Std. Error (mm)
BELTSVILLE 150	47.44	BELTSVILLE 300	46.21	149.9929	149.9979	0.2
BELTSVILLE 150	47.44	BELTSVILLE 600	44.38	449.9990	450.0094	0.2
BELTSVILLE 150	47.44	BELTSVILLE 1800	50.54	1649.9959	1649.9988	0.2
BELTSVILLE 300	46.21	BELTSVILLE 600	44.38	300.0061	300.0117	0.3
BELTSVILLE 300	46.21	BELTSVILLE 1800	50.54	1500.0030	1500.0093	0.3
BELTSVILLE 600	44.38	BELTSVILLE 1800	50.54	1199.9969	1200.0128	0.3

Figure 1.--An example of the adjusted results.

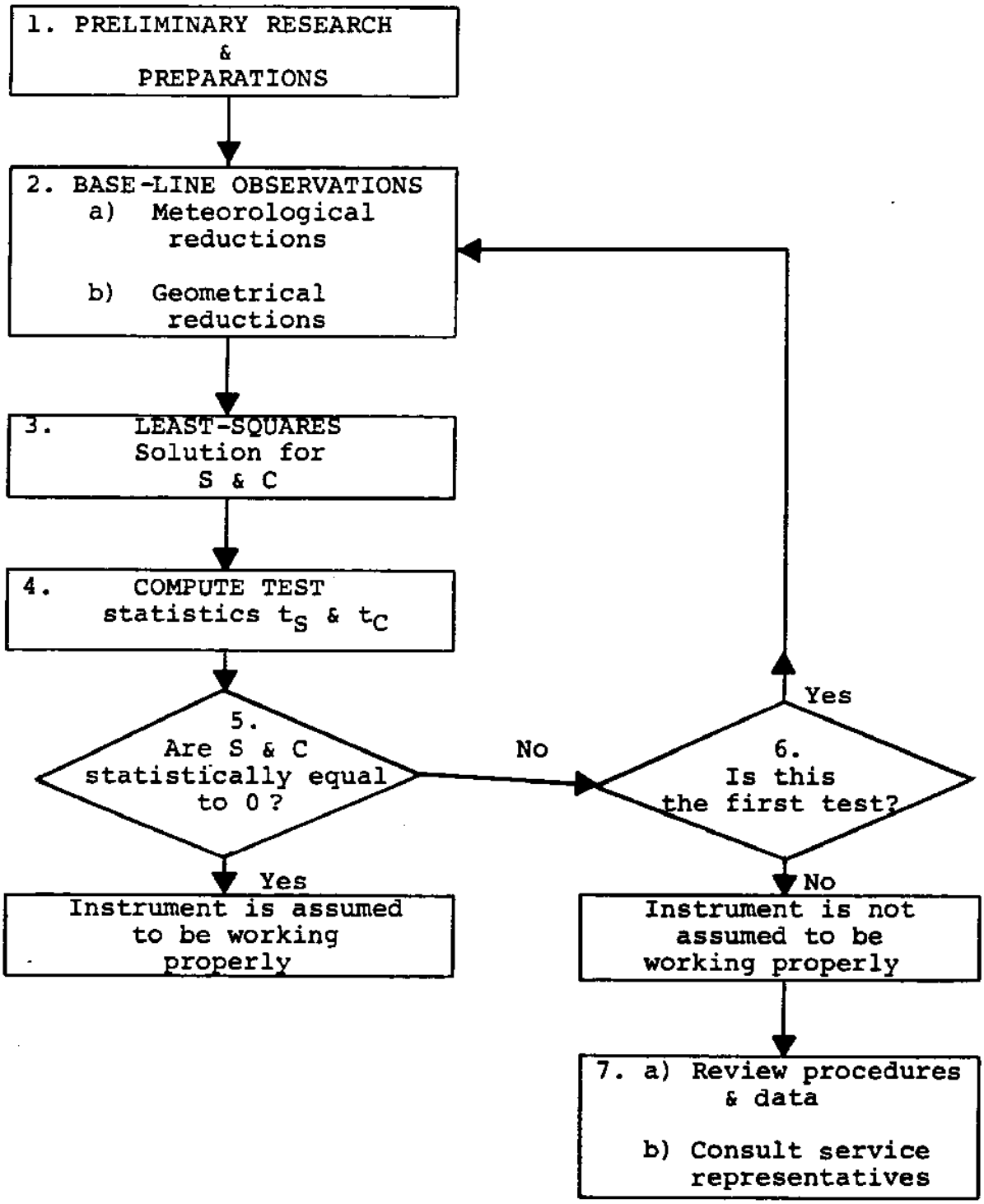

Figure 2.--Schematic of test procedure for EDMI at calibration base line.

EXAMPLES OF EDMI CALIBRATION TESTS

Example #1. The following example is an actual set of test observations performed by a private surveyor over the National Geodetic Survey's calibration base line at Beltsville, Maryland (see fig. 1 for the published data). The instrument to be tested was a short-range infrared EDMI with n = 1.0002782 and λ = 0.9100 μm. The instrument and reflector constant were assumed to be equal in magnitude but opposite in algebraic sign. The resultant system constant is thus assumed equal to zero. (Only one set of prisms was used throughout the test.) The manufacturer's stated accuracy for this instrument is $\pm$ 0.01 m $\pm$ D $\times$ 10^{-5} m. The observed distances and corresponding meteorological data are given below. The estimated accuracy of the temperature observations is "within a few degrees."

From Sta.	Height of Inst. (m)	To Sta.	Height of Inst. (m)	Mn Temp. (°C)	Mn Pressure (mm of Hg)	Obs. Distances D (m)
150	0.20	300	1.53	20.0	760.7	149.9892
300	1.58	150	0.145	21.7	760.7	149.9897
150	0.20	600	1.56	20.0	760.7	449.9927
600	1.61	150	0.145	21.1	761.0	449.9851
150	0.20	1800	3.23	20.0	760.7	1649.9635
1800	3.24	150	0.145	18.9	760.7	1649.9783
300	1.58	600	1.56	21.7	760.7	300.0041
600	1.61	300	1.53	21.1	761.0	300.0018
300	1.58	1800	3.23	21.7	760.7	1499.9763
1800	3.24	300	1.51	18.9	760.7	1499.9972
600	1.61	1800	3.23	21.1	761.0	1200.0050
1800	3.24	600	1.54	18.9	760.7	1200.0070

The distances were corrected for atmospheric refraction using the following:

From eqs. (3) and (4)

$$n_g = 1 + \left[2876.04 + \frac{48.864}{(0.91)^2} + \frac{0.680}{(0.91)^4}\right] \times 10^{-7}$$

$$= 1.0002936$$

and

$$n_a = 1 + \frac{0.0002936}{1 + \alpha t} \cdot \frac{p}{760} - \frac{5.5\ e}{1 + \alpha t} \times 10^{-8}\ .$$

Then from eq. (1)

$$\Delta D = (1.0002782 - n_a)\ D.$$

The distances were corrected for eccentricities, instrument constant, and reflector constant (the sum of which was equal to zero). They were then reduced to the horizontal distance using eq. (7).

The following equations were written in accordance with eq. (8).

$$V_1 = (149.9929 - 149.9899) - S \cdot 149.9929 - C.$$

$$V_2 = (149.9929 - 149.9905) - S \cdot 149.9929 - C.$$

$$V_3 = (449.9990 - 449.9916) - S \cdot 449.9990 - C.$$

$$V_4 = (449.9990 - 449.4849) - S \cdot 449.9990 - C.$$

$$V_5 = (1649.9959 - 1649.9600) - S \cdot 1649.9959 - C.$$

$$V_6 = (1649.9959 - 1649.9728) - S \cdot 1649.9959 - C.$$

$$V_7 = (300.0061 - 300.0003) - S \cdot 300.0061 - C.$$

$$V_8 = (300.0061 - 299.9984) - S \cdot 300.0061 - C.$$

$$V_9 = (1500.0030 - 1499.9739) - S \cdot 1500.0030 - C.$$

$$V_{10} = (1500.0030 - 1499.9906) - S \cdot 1500.0030 - C.$$

$$V_{11} = (1199.9969 - 1199.9866) - S \cdot 1199.9969 - C.$$

$$V_{12} = (1199.9969 - 1199.9858) - S \cdot 1199.9969 - C.$$

Using eqs. (9) and (10), S and C are then solved.

For any computational purposes it may facilitate operations to rearrange the above equations in the tabular form shown below.

(1) Obs.	(2) From	(3) To	(4) D_A (m)	(5) D_H (m)	(6) Δ (m)	(7) $D_A \cdot \Delta$ (m^2)	(8) V (m)*
1	150	300	149.9929	149.9899	+ 0.0030	0.44997870	- 0.0007
2	300	150	149.9929	149.9905	+ 0.0024	0.35998296	- 0.0013
3	150	600	449.9990	449.9916	+ 0.0074	3.32999260	- 0.0004
4	600	150	449.9990	449.9849	+ 0.0141	6.34498590	+ 0.0063
5	150	1800	1649.9959	1649.9600	+ 0.0359	59.23485281	+ 0.0119
6	1800	150	1649.9959	1649.9728	+ 0.0231	38.11490529	- 0.0009
7	300	600	300.0061	300.0003	+ 0.0058	1.74003538	0.0000
8	600	300	300.0061	299.9984	+ 0.0077	2.31004697	+ 0.0019
9	300	1800	1500.0030	1499.9739	+ 0.0291	43.65008730	+ 0.0071
10	1800	300	1500.0030	1499.9906	+ 0.0124	18.60003720	- 0.0096
11	600	1800	1199.9969	1199.9866	+ 0.0103	12.35996807	- 0.0076
12	1800	600	1199.9969	1199.9858	+ 0.0111	13.31996559	- 0.0068

*The residuals (V) are computed after solving for S and C. They may be computed by using the above equations or by using the tabular entries in: col. 6 - (S × col. 4) - C. As a check on the computation, the sum of the residuals should also be computed; assuming no round-off error, the result should be equal to zero.

The following results are then computed:

ΣD_A = the sum of the elements in column 4.
= 10499.9876 m.

$(\Sigma D_A)^2$ = the square of the above result
= 110249739.6 m^2.

ΣD_A^2 = the sum of the square of the elements in column 4.
= 13454977.32 m^2.

$\Sigma \Delta$ = the sum of the elements of column 6.
= + 0.1623 m.

$\Sigma D_A \Delta$ = the sum of the products of the elements in column 4 and column 6 taken on a row-by-row basis (sum of column 7).
= 199.8148389 m^2.

n = the number of observations.
= 12.

Then

$$S = \frac{n \Sigma(D_A \Delta) - \Sigma D_A \Sigma \Delta}{n \Sigma D_A^2 - (\Sigma D_A)^2}$$

$$= \frac{12(199.8148389\ m^2) - (10499.9876\ m)\ (+\ 0.1623\ m)}{12(13454977.32\ m^2) - 110249739.6\ m^2}$$

$$= \frac{693.63008\ m^2}{51209988.20\ m^2}$$

$$= 1.354482015 \times 10^{-5}$$

$$\simeq 0.0000135,$$

and using eq. (10)

$$C = \frac{\Sigma D_A^2\ \Sigma\Delta - \Sigma D_A\ \Sigma(D_A\ \Delta)}{n \Sigma D_A^2 - (\Sigma D_A)^2}$$

$$= \frac{(13454977.32\ m^2)\ (0.1623\ m) - (10499.9876\ m)\ (199.8148389\ m^2)}{12(13454977.32\ m^2) - 110249739.6\ m^2}$$

$$= \frac{85689.488\ m^3}{51209988.20\ m^2}$$

$$= 1.673296 \times 10^{-3}\ m$$

$$\simeq + 0.0017\ m.$$

Using eq. (10a),

$$C = \bar{\Delta} - S\,\bar{D}_A$$

$$= 0.1623/12 - 0.000013545 \times 10499.9876/12$$

$$= 1.673296 \times 10^{-3}\,m$$

$$\approx 0.0017\ m.$$

From eq. (13a),

note: $(n\ \Sigma\ D_A\ \Delta - \Sigma\ D_A\ \Sigma\Delta)$ is the numerator from eq. (9),

$$\hat{\sigma}_0^2 = \frac{\Sigma(\Delta - \bar{\Delta})^2 - \frac{S}{n}\left[n\ \Sigma\ D_A\ \Delta - \Sigma\ D_A\ \Sigma\Delta\right]}{(n-2)}$$

$$= \left[0.001218442500 - \frac{1.354482015 \times 10^{-5}}{12} \times 693.63008\right] \div 10$$

$$= 4.355191077 \times 10^{-5}$$

$$\approx 0.0000436.$$

From eq. (11),

$$\hat{\sigma}_S = \left[\hat{\sigma}_0^2\ \frac{n}{n\ \Sigma\ D_A^2 - (\Sigma\ D_A)^2}\right]^{\frac{1}{2}}.$$

Note: The denominator is the same as in eqs. (8) and (9).

$$\hat{\sigma}_S = \left[4.355191077 \times 10^{-5}\ \frac{12}{51209988.20}\right]^{\frac{1}{2}}$$

$$= 3.194602582 \times 10^{-5}$$

$$\approx 0.0000032.$$

From eq. (12),

$$\hat{\sigma}_C = \left[\hat{\sigma}_0^2\ \frac{\Sigma\ D_A^2}{n\ \Sigma\ D_A^2 - (\Sigma\ D_A)^2}\right]^{\frac{1}{2}}$$

$$= \left[4.355191077 \times 10^{-5} \times \frac{13454977.32}{51209988.20}\right]^{\frac{1}{2}}$$

$$= 3.382732845 \times 10^{-3}$$

$$\approx 0.0034\ m.$$

From eqs. (14) and (15),

$$t_S = \frac{S}{\hat{\sigma}_S}$$

$$= \frac{1.354482015 \times 10^{-5}}{3.194602582 \times 10^{-6}}$$

$$= 4.240$$

$$t_C = \frac{C}{\hat{\sigma}_C}$$

$$= \frac{1.673296 \times 10^{-3}}{3.382732845 \times 10^{-3}}$$

$$= 0.495.$$

Following step 6 of the procedure for analyzing the data, we can now decide the validity of the results. From figure 2, with d.f. = 10, the critical value of t is 3.169. It can be seen that t_S is greater than $t_{0.01,10}$. Therefore, at the 1% significance level, it is possible to reject the hypothesis that S is statistically equal to 0. However, for reasons mentioned previously, a retesting should be performed.

However, t_C is less than $t_{0.01,10}$. Therefore, we cannot reject the hypothesis that C is equal to 0 at the 1% significance level.

Note: If the same sequence and number (n) of observations are performed for each test, then ΣD_A, $(\Sigma D_A)^2$, ΣD_A^2, and n will be constants for a particular base line. The values $\Sigma\Delta$ and $\Sigma D_A \Delta$ only need be computed for each calibration test.

Assume that instead of observing 12 observations, only station 150 or station 1800 was occupied. Both situations are given below. The observations are taken from the previous example.

Example #2.

Station 150

Obs.	From	To	D_A (m)	D_H (m)	Δ (m)	$D_A \Delta$ (m^2)	V (m)
1	150	300	149.9929	149.9899	0.0030	0.44997870	0.0010
2	150	600	449.9990	449.9916	0.0074	3.32999260	- 0.0013
3	150	1800	1649.9959	1649.9600	0.0359	59.23485281	0.0003

$\Sigma D_A = 2249.9878$ m $\Sigma \Delta = +0.0463$ $\Sigma D_A \Delta = 63.01482411$ $\Sigma V = 0$

$(\Sigma D_A)^2 = 5062445.1$ m^2 $\Sigma(\Delta-\bar{\Delta})^2 = 6.380066667 \times 10^{-4}$

$\Sigma D_A^2 = 2947483.44$ m^2

As above

$$n \Sigma D_A \Delta - \Sigma D_A \Sigma\Delta = 84.87003720$$

$$\Sigma D_A^2 \Sigma\Delta - \Sigma D_A \Sigma D_A \Delta = -5.3141022 \times 10^3$$

$$n \Sigma D_A^2 - (\Sigma D_A)^2 = 3.780005220 \times 10^6$$

$S = 2.245235979 \times 10^{-5}$

≈ 0.0000225

$\hat{\sigma}_0^2 = 2.829129700 \times 10^{-6}$

$\hat{\sigma}_S = 1.498445171 \times 10^{-6}$

≈ 0.0000015

$t_S = 14.984$

$C = -1.405845201 \times 10^{-3}$

≈ -0.0014 m

$\hat{\sigma}_C = 4.184181198 \times 10^{-3}$

≈ 0.0042 m

$t_C = -0.336$

From figure 2, $t_{0.01,1} = 63.657$. Therefore, statistically we cannot reject the hypothesis that both S and C are zero.

Example #3.

Station 1800

Obs.	From	To	D_A (m)	D_H (m)	Δ (m)	$D_A \Delta$ (m^2)	V (m)
1	1800	600	1199.9969	1199.9858	+ 0.0111	13.31996559	- 0.0021
2	1800	300	1500.0030	1499.9739	+ 0.0291	43.65008730	+ 0.0065
3	1800	150	1649.9959	1649.9728	+ 0.0231	38.11490529	- 0.0043

$\Sigma D_A = 4349.9958$ m $\Sigma \Delta = 0.0633$ m $\Sigma D_A \Delta = 95.0849518$ m^2 $\Sigma V = 0$

$(\Sigma D_A)^2 = 18922463.5$ m^2 $\Sigma(\Delta-\bar{\Delta})^2 = 1.68 \times 10^{-4}$

$\Sigma D_A^2 = 6412488.0$ m^2

As given previously,

$$n \Sigma D_A \Delta - \Sigma D_A \Sigma\Delta = 9.900140400$$

$$\Sigma D_A^2 \Sigma\Delta - \Sigma D_A \Sigma D_A \Delta = -7.708678300 \times 10^3$$

$$n \Sigma D_A^2 - (\Sigma D_A)^2 = 3.15005 \times 10^5$$

$$S = 3.142851828 \times 10^{-5} \approx 0.0000314$$

$$C = -2.447160616 \times 10^{-2} \approx -0.0024 \text{ m}$$

$$\hat{\sigma}_0^2 = 6.42844188 \times 10^{-5}$$

$$\hat{\sigma}_S = 2.47431372 \times 10^{-5} \approx 0.0000247$$

$$\hat{\sigma}_C = 3.617490672 \times 10^{-2} \approx 0.0362 \text{ m}$$

$$t_S = 1.270$$

$$t_C = 0.676$$

As in example 2, $t_{0.01,1} = 63.657$. Again, on the basis of statistics we cannot reject the hypothesis that both S and C are zero.

APPENDIX I. THE GEOMETRICAL TRANSFORMATION OF ELECTRONICALLY MEASURED DISTANCES

Notation:

α = Mean azimuth of line (clockwise from south).

ϕ = Mean latitude of line.

H_i = Elevation of station above mean sea level.

ΔH_i = Height of instrument (or reflector) above mark.

N_i = Geoidal undulation.

k = Index of refraction (for lightwave instruments $k \approx 0.18$, for microwave instruments $k \approx 0.25$).

D_0 = Observed slope distance corrected for ambient atmospheric conditions and mode of measurements (e.g., eccentricities, instrument constant, mirror (or reflector) constant, etc.).

D_1 = $D_0 \pm$ the correction for second velocity (see Höpcke, W., "On the curvature of electromagnetic waves and its effect on measurement of distance," Survey Review, No. 141, pp. 298-312, July 1966). (See eq. (I-7) on page 27.)

D_2 = Chord distance at instrument elevations.

D_3 = Chord distance at station elevation (mark-to-mark).

D_4 = Geoidal or sea level distance.

D_5 = Chord distance at the sea level surface.

D_6 = Ellipsoidal or geodetic distance.

D_7 = Chord distance at the ellipsoidal surface.

D_H = Horizontal chord distance at mean elevation of instruments.

a = Semi-major axis = 6378206.4, Clarke Spheroid 1866.

b = Semi-minor axis = 6356583.8, Clarke Spheroid 1866.

Classically, observed distances have been reduced to one of two surfaces, either the geoid (sea level) or the ellipsoid. To which surface the distances were reduced depended on available information. Generally, in the United States distances were

reduced to the geoid. However, with the acquisition of more accurate information on geoidal undulations, the present trend is to reduce the distances to the ellipsoid.

With the introduction of satellite positioning systems, very long base line interferometry (VLBI) or for special purpose surveys, the term "reduction" will no longer suffice. Therefore, we should think in terms of the transformation of distances.

Generally, this transformation can be divided into two procedures:

1. The transformation of the distance along an arc to a chord distance or its inverse.

2. The transformation of a chord distance at one altitude to a chord distance at another altitude.

The general equations for these transformations are:

Chord Distance to Chord Distance:

$$D_1^2 = \frac{D^2 - (H_2 - H_1)^2}{\left(1 + \frac{H_1}{R}\right)\left(1 + \frac{H_2}{R}\right)} \left(1 + \frac{H_1'}{R}\right)\left(1 + \frac{H_2'}{R}\right) + \left(H_2' - H_1'\right)^2 \qquad \text{(I-1)}$$

where D is the spatial chord distance at elevations H_1 and H_2, and D_1 is the desired spatial chord distance at elevations H_1' and H_2', and R is the radius of curvature.

Arc to Chord:

$$D_1 = 2R \sin \frac{D}{2R} \cdot \qquad \text{(I-2)}$$

Here

D_1 is the desired chord distance, and
D is the distance along an arc.

Normally eq. (I-2) is a small correction that amounts to a change in distance of 1.5 mm for a line 10,000 m in length.

The following specific equations for various geometric distances were derived from the above two equations. (See figure I-1 for a graphic representation of the geometric relationships.)

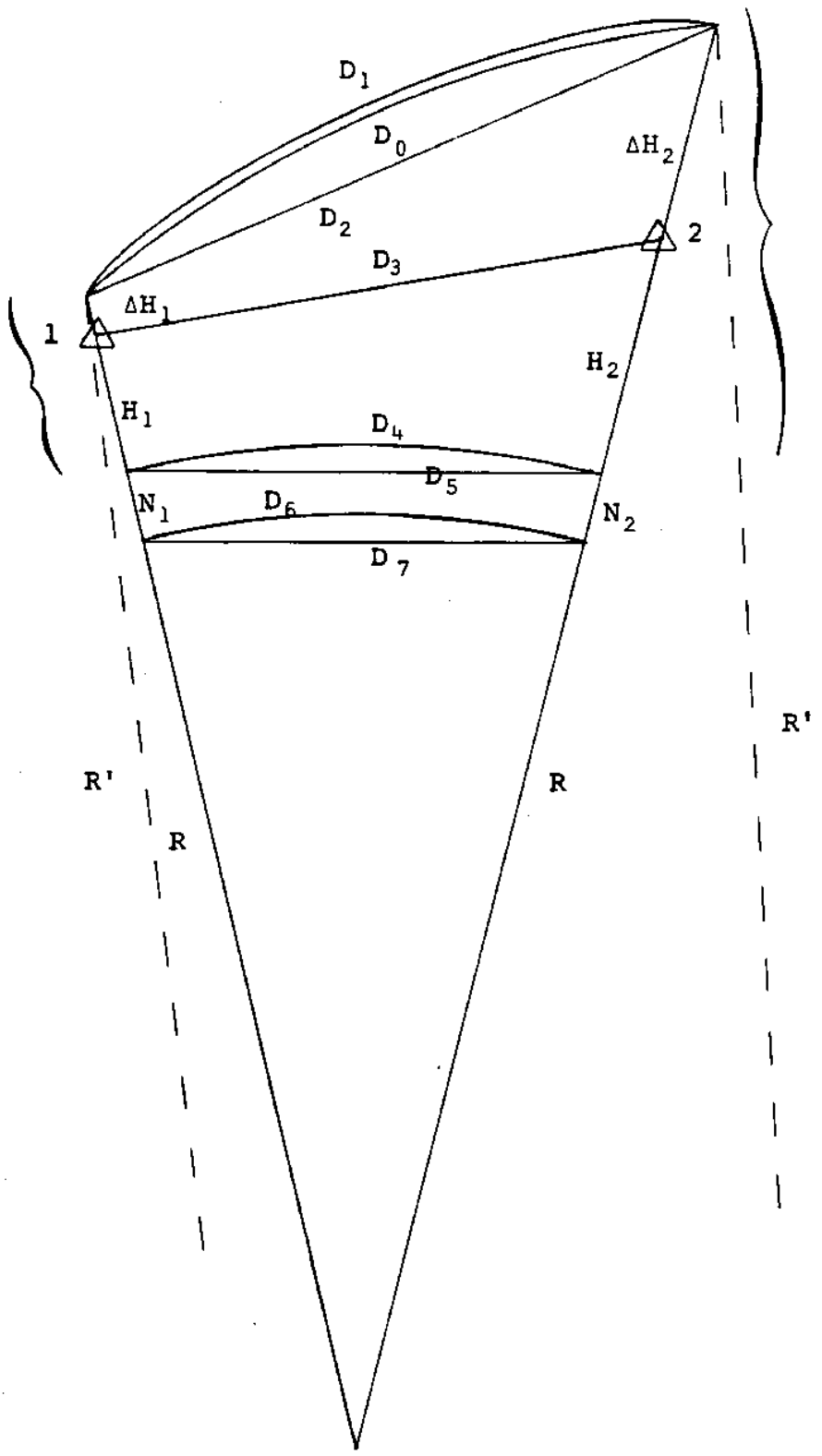

Figure I-1.--Graphic representation of the geometric relationship between distances.

Equations for the transformation of electronically measured distances:

$$e'^2 = \frac{a^2 - b^2}{b^2} \tag{I-3}$$

$$c = \frac{a^2}{b} \tag{I-4}$$

$$N = \frac{c}{(1 + e'^2 \cos^2\phi)^{1/2}} \tag{I-5}$$

$$R = \frac{N}{1 + e'^2 \cos^2\phi \cos^2\alpha} \tag{I-6}$$

$$D_1 = D_0 - (k - k^2)\, D_0^3/12\, R^2 \tag{I-7}$$

$$R' = \frac{R}{k} \tag{I-8}$$

$$D_2 = 2\, R' \sin\left(D_1/2\, R' \ \frac{180}{\pi}\right) \tag{I-9}$$

$$H_1' = H_1 + \Delta H_1 \tag{I-10}$$

$$H_2' = H_2 + \Delta H_2 \tag{I-11}$$

$$\Delta H = H_1' - H_2' \tag{I-12}$$

$$D_5 = \left[(D_2^2 - \Delta H^2) / \left\{(1 + H_1'/R)(1 + H_2'/R)\right\}\right]^{1/2} \tag{I-13}$$

$$D_4 = 2\, R \left[\sin^{-1} (D_5/2R)\right] \frac{\pi}{180} \tag{I-14}$$

$$D_3 = \left[D_5^2 (1 + H_1/R)(1 + H_2/R) + (H_1 - H_2)^2 \right]^{1/2} \tag{I-15}$$

$$h_1 = H_1' + N_1 \tag{I-16}$$

$$h_2 = H_2' + N_2 \tag{I-17}$$

$$\Delta h = h_1 - h_2 \tag{I-18}$$

$$D_7 = \left[(D_2{}^2 - \Delta h^2) / \left\{(1+h_1/R)(1+h_2/R)\right\}\right]^{1/2} \tag{I-19}$$

$$D_6 = 2R \left[\sin^{-1}(D_7/2R)\right] \frac{\pi}{180} \tag{I-20}$$

$$H_m = (H_1 + H_2)/2 \tag{I-21}$$

$$D_H = \left[\left\{(D_3^2 - \Delta H^2)(1 + H_m/R)^2\right\} / \left\{(1 + H_1/R)(1 + H_2/R)\right\}\right]^{1/2} \tag{I-22}$$

$$\simeq (D_3^2 - \Delta H^2)^{1/2}$$

Note: In eqs. (I-9), (I-14), and (I-20) the terms $\pi/180$ or $180/\pi$ were added to convert from angular measure to radian measure (or vice versa).

APPENDIX II. THE INFLUENCE OF METEOROLOGICAL DATA ON THE ACCURACY OF ELECTRONICALLY MEASURED DISTANCES

The determination of the refractive index of the ambient atmosphere has a critical influence on the accuracy of distances measured with EDMI. These effects can be evaluated by varying the parameters in the equations for n_a (refractive index) and computing their influence. Alternately, their influence may be computed by evaluating the partial derivatives of the refractive index equation at nominal values. The partial derivatives of the refractive index equation for microwave and lightwave sources are discussed below.

Microwave Source EDMI

From eq. (6) (see page 8) we have

$$(n_a-1) \times 10^6 = \frac{103.46p}{273.2+t} + \frac{490,814.24e}{(273.2+t)^2}$$

where

$$e = e' + de$$

$$e' \simeq 4.58 \times 10^a$$

$$a = 7.5t'/(237.3+t)$$

$$de = -0.000660\ (1 + 0.00115t')\ p\ (t-t').$$

Then, letting

$$n = (n_a-1) \times 10^6$$

the partial derivatives with respect to t, t', and p are:

$$\frac{\partial n}{\partial p} \simeq \frac{103.46}{273.2+t} - \frac{323.94}{(273.2+t)^2}\ (1 + 0.00115t')\ (t-t') \qquad \text{(II-1)}$$

$$\frac{\partial n}{\partial t} \simeq \frac{-103.46p}{(273.2+t)^2} - \frac{981628.48e}{(273.2+t)^3} - \frac{323.94}{(273.2+t)^2}\ (1 + 0.00115t')\ p \qquad \text{(II-2)}$$

$$\frac{\partial n}{\partial t'} \simeq \frac{490814.24}{(273.2+t)^2}\left[\frac{4098.026e'}{237.3+t')^2} + \right. \qquad \text{(II-3)}$$

$$\left. 0.00066p\ (1 + 0.00230t' - 0.00115t)\right].$$

The above derivatives when evaluated yield results in units of ppm, when t and t' are in degrees Celsius and pressures are in mm of Hg.

Evaluating eq. (II-1) for $0° C \leq t \leq 30° C$

$$t-t' = 10° C$$

(a) $t = 0°C$: $\frac{\partial n}{\partial p} = 0.34$

(b) $t = 10°C$: $\frac{\partial n}{\partial p} = 0.33$

(c) $t = 20°C$: $\frac{\partial n}{\partial p} = 0.31$

(d) $t = 30°C$: $\frac{\partial n}{\partial p} = 0.30$

If we assume the error of observing pressure is approximately 3 mm (0.1 in) of Hg, then for a mean value of $\partial n/\partial p$ equal to 0.32, the error introduced into the computation of refraction and, thus, the distance is:

$$\Delta n = 0.32 \Delta p$$

$$\Delta n = 0.32 \ (3) = 1.0 \text{ ppm.}$$

Evaluating eq. (II-2) for $0° C \leq t \leq 30° C$

$$t' = t$$

$$p = 760 \text{ mm of Hg}$$

and e' given by the following:

t'(°C) =	0°	10°	20°	30°
e'(mm of H_g) =	4.58	9.20	17.53	31.81

(a) $t = 0°C$: $\frac{\partial n}{\partial t} = -4.57$

(b) $t = 10°C$: $\frac{\partial n}{\partial t} = -4.52$

(c) $t = 20°C$: $\frac{\partial n}{\partial t} = -4.52$

(d) $t = 30°C$: $\frac{\partial n}{\partial t} = -4.75$.

Assuming an error in observing dry bulb temperatures on the order of 0.5° C and using the mean value from above, the effect on the refractive index is:

$$\Delta n = -4.58\,\Delta t$$

$$\Delta n = (-4.58)(0.5)$$

$$\Delta n = -2.3 \text{ ppm.}$$

Evaluating eq. (II-3) for $0° \text{ C} \leq t \leq 30° \text{ C}$

$t' = t$

$p = 760$ mm of H_g

e' as above

(a) $t = 0° \text{ C}$: $\frac{\partial n}{\partial t'} = 5.49$

(b) $t = 10° \text{ C}$: $\frac{\partial n}{\partial t'} = 6.92$

(c) $t = 20° \text{ C}$: $\frac{\partial n}{\partial t'} = 9.08$

(d) $t = 30° \text{ C}$: $\frac{\partial n}{\partial t'} = 12.51.$

Again one can assume an error in determinations of the wet bulb temperature to be approximately 0.5° C. However, a mean of the above values would not be very indicative. Therefore, the range of the effect will be given.

For 0° C: $\Delta n = 5.49(0.5)$
$= 2.74$

For 30° C: $\Delta n = 12.51(0.5)$
$= 6.26$

or for $0° \text{ C} \leq t' \leq 30° \text{ C}$

$2.7 \text{ ppm} \leq \partial n \leq 6.2 \text{ ppm.}$

It should be noted that previously some authors have stated that a change of 1° C in t produces a change of 1 ppm in the distances. From the evaluation of eq. (II-2) above, the effect is approximately 5 ppm. Perhaps the confusion arises because of a failure to evaluate the third term in this equation or because of an alternate approach to these differentials. If the partial derivatives are taken with respect to p, t, e (instead of p, t, t'), consider the following:

$$\frac{\partial n}{\partial p} = \frac{103.46}{273.2+t} \qquad \text{(II-4)}$$

$$\frac{\partial n}{\partial t} = \frac{-103.46p}{(273.2+t)^2} - \frac{981628.48e}{(273.2+t)^3} \qquad \text{(II-5)}$$

$$\frac{\partial n}{\partial e} = \frac{490814.24}{(273.2+t)^2} \; . \qquad \text{(II-6)}$$

Comparing eqs. (II-1) and (II-2), the difference is the second term of (II-1). This term evaluated for nominal values contributes less than 0.1 ppm and thus has no real effect.

Evaluating eq. (II-5) for values as in eq. (II-2) we have:

(a) $t = 0°$ C: $\frac{\partial n}{\partial t} = -1.27$

(b) $t = 10°$ C: $\frac{\partial n}{\partial t} = -1.38$

(c) $t = 20°$ C: $\frac{\partial n}{\partial t} = -1.59$

(d) $t = 30°$ C: $\frac{\partial n}{\partial t} = -1.98$.

However, e (the vapor pressure) is determined from observations of t, t', and p. From

$$e = e' + de$$

$$e' = 4.58 \times 10^{a}$$

$$a = (7.5t')/(237.3+t')$$

$$de = -0.000660\ (1 + 0.00115t')\ p\ (t-t') \; ,$$

the following partials are determined:

$$\frac{\partial e}{\partial p} = -0.000660\ (1 + 0.00115t')\ (t-t') \qquad \text{(II-7)}$$

$$\frac{\partial e}{\partial t} = -0.000660\ (1 + 0.00115t')\ p \qquad \text{(II-8)}$$

$$\frac{\partial e}{\partial t'} = \frac{4098.764e'}{(237.3+t')^2} + 0.00066p\ (1 + 0.00230t' - 0.00115t). \qquad \text{(II-9)}$$

Combining with eq. (II-6) and evaluating eqs. (II-8) and (II-9) for $0° C \leq t \leq 30° C$

$$t' = t$$

$$p = 760 \text{ mm of Hg} \cdot$$

Then

$t = 0° C$: $\Delta n = -3.29\Delta t + 5.46\Delta t'$

$t = 10° C$: $\Delta n = -3.12\Delta t + 6.89\Delta t'$

$t = 20° C$: $\Delta n = -2.91\Delta t + 9.14\Delta t'$

$t = 30° C$: $\Delta n = -2.78\Delta t + 12.50\Delta t'$.

From eq. (II-5) the impression is given that the effect of 1°C change in dry bulb is in the magnitude of 1 ppm. However, when combined with the above, the results are similar to those obtained using eqs. (II-1) through (II-3).

Lightwave source EDMI

From eq. (4) (see page 7) we have

$$(n_a - 1) \times 10^6 = \left[\frac{n_g - 1}{1+\alpha t} \times \frac{p}{760} - \frac{5.5e10^{-8}}{(1+\alpha t)^2}\right] \times 10^6$$

Again, letting

$$n = (n_a - 1) \times 10^6$$

the partial derivatives with respect to p, t, and t' are:

$$\frac{\partial n}{\partial p} = \frac{(n_g - 1)}{(1+\alpha t) \cdot 760} \times 10^6 + \frac{0.0000363}{(1+\alpha t)^3} (1 + 0.00115t') (t-t') \quad \text{(II-10)}$$

$$\frac{\partial n}{\partial t} = \frac{-\alpha (n_g - 1) p \times 10^6}{(1+\alpha t)^2 \, 760} + \frac{0.11 \, e\alpha}{(1+\alpha t)^3} + \quad \text{(II-11)}$$

$$\frac{0.0000363 \, (1 + 0.00115t') \, p}{(1+\alpha t)^2}$$

$$\frac{\partial n}{\partial t'} = \frac{-0.055}{(1+\alpha t)^2} \left[\frac{4098.764}{(237.3+t')_2} + \right. \quad \text{(II-12)}$$

$$\left. 0.00066p \, (1 + 0.0023t' - 0.00115t)\right].$$

Remembering

$$(n_g - 1) \times 10^6 = \left[2876.04 + \frac{48.864}{\lambda^2} + \frac{0.680}{\lambda^4}\right] \times 10^{-1}$$

then for $\lambda = 0.6328$ μm

$$(n_g-1) \times 10^6 = 300.2308$$

and for $\lambda = 0.9300$ μm

$$(n_g-1) \times 10^6 = 293.3446.$$

Evaluating eq. (II-10) for $0° C \leq t \leq 30° C$

$$t-t' = 10° C$$

and $\lambda = 0.6328$ μm.

we have

(a) $t = 0° C$: $\frac{\partial n}{\partial p} = 0.40$

(b) $t = 10° C$: $\frac{\partial n}{\partial p} = 0.38$

(c) $t = 20° C$: $\frac{\partial n}{\partial p} = 0.37$

(d) $t = 30° C$: $\frac{\partial n}{\partial p} = 0.36$.

For $\lambda = 0.9300$ μm

(a) $t = 0° C$: $\frac{\partial n}{\partial p} = 0.39$

(b) $t = 10° C$: $\frac{\partial n}{\partial p} = 0.37$

(c) $t = 20° C$: $\frac{\partial n}{\partial p} = 0.36$

(d) $t = 30° C$: $\frac{\partial n}{\partial p} = 0.35$.

Using the mean value of $\partial n/\partial p$ equal to 0.37 and an error of 3 mm (0.1 in) of Hg, the error introduced into the refractive index is:

$$\Delta n = (0.37)(3)$$
$$= 1.1 \text{ ppm.}$$

Evaluating eq. (II-11) for $0° C \leq t \leq 30° C$

$t' = t$

$p = 760$ mm of Hg

$\lambda = 0.6328$ μm

e' (see values on page 30).

then

(a) $t = 0°$ C: $\frac{\partial n}{\partial t} = -1.07$

(b) $t = 10°$ C: $\frac{\partial n}{\partial t} = -1.00$

(c) $t = 20°$ C: $\frac{\partial n}{\partial t} = -0.93$

(d) $t = 30°$ C: $\frac{\partial n}{\partial t} = -0.86$

For $\lambda = 0.9300$ μm:

(e) $t = 0°$ C: $\frac{\partial n}{\partial t} = -1.04$

(f) $t = 10°$ C: $\frac{\partial n}{\partial t} = -0.97$

(g) $t = 20°$ C: $\frac{\partial n}{\partial t} = -0.90$

(h) $t = 30°$ C: $\frac{\partial n}{\partial t} = -0.84$.

The mean from above is 0.95. Using an error in t of 0.5° C, then the effect on the refractive index is:

$$\Delta n = (0.95)(0.5) = 0.5 \text{ ppm} .$$

Evaluating eq. (II-12) for $0° \text{ C} \leq t \leq 30° \text{ C}$

$t' = t$

$p = 760$ mm of Hg

$\lambda = 0.6328$ μm and 0.9300 μm

(a) $t = 0°$ C: $\frac{\partial n}{\partial t'} = -0.05$

(b) $t = 10°$ C: $\frac{\partial n}{\partial t'} = -0.06$

(c) $t = 20°$ C: $\frac{\partial n}{\partial t'} = -0.08$

(d) $t = 30°$ C: $\frac{\partial n}{\partial t'} = -0.10$

From the above, it can be seen that the effect of nominal errors in the wet bulb temperature on the determination of refractive index is minimal.

In addition to errors in temperature and pressure, the refractive index of light is affected by errors in the assigned angstrom rating of the light source. From eqs. (II-3) and (II-4),

$$\frac{\partial n}{\partial \lambda} = \frac{-9.7728}{\lambda^3} - \frac{0.272}{\lambda^5} \quad \frac{p}{(1+\alpha t)(760)} \qquad \text{(II-13)}$$

Evaluating eq. (II-13) for $0° C \leq t \leq 30° C$

$$p = 760 \text{ mm of Hg}$$

For $\lambda = 0.6328$ μm

(a) $t = 0° C$: $\frac{\partial n}{\partial \lambda} = -41.25$

(b) $t = 10° C$: $\frac{\partial n}{\partial \lambda} = -39.79$

(c) $t = 20° C$: $\frac{\partial n}{\partial \lambda} = -38.43$

(d) $t = 30° C$: $\frac{\partial n}{\partial \lambda} = -37.17$

For $\lambda = 0.9300$ μm

(e) $t = 0° C$: $\frac{\partial n}{\partial \lambda} = -12.54$

(f) $t = 10° C$: $\frac{\partial n}{\partial \lambda} = -12.10$

(g) $t = 20° C$: $\frac{\partial n}{\partial \lambda} = -11.68$

(h) $t = 30° C$: $\frac{\partial n}{\partial \lambda} = -11.30$

An error of 0.01 μm in λ introduces a change in the refractive index of 0.4 ppm for instruments having a light source in the range of 0.6328 μm and 0.1 ppm for instruments having a light source in the range of 0.9300 μm.

For instruments using a red laser light, the light source wavelengths are around 0.6328 μm. Infrared wavelengths are around 0.9 μm.

APPENDIX III. TABLE OF SELECTED CONVERSION FACTORS

Temperature:

$°C = 5/9\ (°F - 32)$
$°F = 9/5\ °C + 32$

where

°C = degrees Celsius
°F = degrees Fahrenheit

Pressure:

1 in of mercury (H_g) = 33.86389 mb = 0.3386389 kPa
1 mm of H_g = 1.333224 mb = 0.0133224 kPa
1 in of H_g = 33.86389 mb
1 mb = 0.02952998 in of H_g
1 mb = 0.7500616 mm of H_g
1 in of H_g = 25.4 mm of H_g

Pressure in mm of $H_g = 25.4 \times e^a$

where

$a = 3.3978 - Alt\ (3.6792 \times 10^{-5})$

and

Alt = altimeter reading in feet

e = base of natural logarithm

= 2.718281828 . . .

NOTE: If, as in some altimeters, zero feet does not equal sea level, then the altimeter reading will have to be modified accordingly.

Length:

1 m = 39.37 in
1 m = 3.28083333 ft
1 ft = 0.30480061 m
1 in = 25.400051 mm

BIBLIOGRAPHY

Dracup, J. F., Fronczek, C. J., and Tomlinson, R. W., 1977: Establishment of calibration base lines. NOAA Technical Memorandum NOS NGS-8, U.S. Department of Commerce, National Oceanic and Atmospheric Administration, Rockville, Md., 25 p.

Dracup, J. F., Kelley, C. F., Lesley, G. B., and Tomlinson R. W., 1973: Surveying instrumentation and coordinate computation workshop lecture notes. Control Surveys Division, American Congress on Surveying and Mapping, Falls Church, Virginia, 127 p.

Holdridge, R. H., 1976: EDM equipment--use, care, calibration. Presented at the Twenty-seventh Annual Surveyors' Institute, Madison, Wisconsin. Wisconsin Department of Transportation, Madison.

Höpcke, W., 1966: On the curvature of electromagnetic waves and its effect on measurement of distance. Survey Review no. 141, 298-312.

List, Robert J. (Editor), 1963: Smithsonian Meteorological Tables (6th edition). Smithsonian Institution, Washington, D. C., 527 p. (publication no. 4014).

Meade, B. K., 1972: Precision in electronic distance measuring. Surveying and Mapping, XXXII (1), 69-78.

Mendenhall, William, 1969: Introduction to Probability and Statistics. Wadsworth Publishing Company, Inc., Belmont, California, 393 p.

O'Quinn, C. A., 1976: Florida's electronic distance measuring equipment base lines. Proceedings of the American Congress on Surveying and Mapping, 36th Annual Meeting, Washington, D. C., February 22-28, 118-127.

Robertson, K. D., 1975: A method for reducing the index of refraction errors in length measurements. Surveying and Mapping, XXXV (2), 115-129.

Saastamoinen, J. J. (Editor), 1967: Surveyor's Guide to Electromagnetic Distance Measurement. University of Toronto Press, Canada, 193 p.

Tomlinson, R. W., Burger, T. C. (1971) revised 1975: Electronic distance measuring instruments. Technical Monograph No. CS-2, Control Surveys Division, American Congress on Surveying and Mapping, Falls Church, Virginia, 68 p.