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GRAVIMETRIC TIDAL LOADING COMPUTED FROM INTEGRATED GREEN'S FUNCTIONS

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Gravimetric Tidal Loading Computed from Integrated Green's Functions

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ABSTRACT. The usual method of predicting the effects of ocean tides on geodetic measurements is to use impulse response functions (called Green's functions) by convolving them with the desired ocean tide model. Because ocean tide representations are usually expressed as areas or cells of constant amplitude and phase, it has been found that the integrals of Green's functions are more desirable for use with tidal loading calculations.

Predictions are presented of the loading effects on Earth-tide gravimeter measurements using a global M2 ocean tide model developed by E. W. Schwiderski. Souriau's calculations for loading effects on tidal gravity data available for western Europe are confirmed to an accuracy of ±0.2 microgal for most sites. Other results are available from California, Australia, and Japan.

INTRODUCTION

For several decades prediction of the response of the Earth to variable loads has captured the interest of geophysicists, oceanographers, and geodesists. During the past decade, improvements in Earth modeling enabled researchers to determine realistic response functions given the elastic properties of the Earth. It is only natural to use these tools to predict the response of the Earth to the loading of the ocean tides. In this paper an alternate form has been developed for the response functions. This method requires that the ocean tide height be given in terms of areas of constant amplitude and phase, the way global

tide models are usually computed. When such representations of the tide height are available over any cell or area, the amplitude times the sine or cosine of phase is constant and can be taken outside the integral. Then the integral of the impulse response function (or Green's function) remains to be evaluated. This integral is stable (by removal of one singularity) and enables one easily to use any set of load deformation coefficients in the study of Earth gravity or displacement response. Previously, one had to use published Green's functions (Farrell 1972) or compute the functions, requiring the evaluation of infinite series which do not converge when the angular argument is small. The technique presented here also allows one properly to account for the distance of the instrument above sea level. Of course, this method is not restricted to global representations. It also can be used in any region where the tide is represented by areas of constant amplitude and phase.

The results presented in this paper are based on the global representation of the M2 tide developed by E. W. Schwiderski of the Naval Surface Weapons Center, Dahlgren, Va. This model is constrained to agree with coastal and island tide data, and is used here to predict the effect of ocean loading on gravity data taken at several areas of the Earth. The results look promising. For example, in Australia, the tidal perturbations caused by the oceans appear to be predicted to the 0.5-microgal level, slightly better than the 1-microgal level reported by Breteger and Mather (1978). When the tidal gravimeter stations are more than 1° from the land-water boundaries, excellent agreement is demonstrated with Souriau (1979) who used 0.25° grids for nearby seas when correcting many tidal gravity observations taken in western Europe.

RESPONSE FUNCTIONS

Loading Potential

For the special case of the ocean tides, let the mass distribution be represented by a constant density layer with varying height, h, covering a large sphere of radius, a. The gravitational potential becomes

$$U' = G\rho a^2 \iint \frac{h(\theta,\alpha)\ \sin\theta\ d\theta\ d\alpha}{\sqrt{a^2 + r^2 - 2ar\cos\theta}} \ . \tag{1}$$

The quantities θ and α are the central angle and azimuth, r is the distance from the center of the Earth, G is the gravitational constant, and ρ is the density of sea water. Noting the presence of the generating function for the Legendre polynomials $P_n(\cos\theta)$, the integral becomes

$$U'_e = G\rho a^2 \sum_{n=o}^{\infty} \iint h(\theta,\alpha)\ P_n(\cos\theta)\ \frac{a^n}{r^{n+1}}\ \sin\theta\ d\theta\ d\alpha \tag{2a}$$

for the solution outside the sphere (exterior), and

$$U'_i = G\rho a^2 \sum_{n=o}^{\infty} \iint h(\theta,\alpha)\ P_n(\cos\theta)\ \frac{r^n}{a^{n+1}}\ \sin\theta\ d\theta\ d\alpha \tag{2b}$$

for the interior solution. The solution at the spherically coated surface is obtained from eq. (1) by letting r = a. However, measurements with Earth tidal gravimeters normally take place above sea level, and thus eq. (1) or (2a) must be used.

Newtonian Tidal Attraction Evaluation

Several numerical solutions of the Laplace tidal equations have recently become available. Previously, solutions were given by cotidal charts which showed contours of constant amplitude and phase, or low degree spherical harmonic expansions. Now the solutions are almost always provided in gridded form where the tidal amplitude and phase are given as constants over small areas (e.g., 1° geographic squares). Let A_i and σ_i be the amplitude and Greenwich phase, respectively, over the i-th region. Then the tide height, $h_i(t)$, at time t is given by

$$h_i(t) = A_i \cos(\bar{n} \cdot \bar{\beta} + \sigma_i) \tag{3}$$

where $\bar{n}$ is the coefficient vector for a given constituent and $\bar{\beta}$ is a vector of six astronomical angles (Cartwright and Tayler 1971). Decomposing eq. (3) with multiple angle identities yields the sinusoidal and cosinusoidal terms

$$h_i(t) = h_i^c \cos(\bar{n} \cdot \bar{\beta}) - h_i^s \sin(\bar{n} \cdot \bar{\beta}). \tag{4}$$

Substituting eq. (4) into eq. (1), and assuming that the h_i^c and h_i^s are constant over the i-th sector bounded by azimuthal angles α_{1i} and α_{2i} and central angles θ_{1i} and θ_{2i}, yields a rather simple representation of the direct loading potential

$$U' = \sum_i \frac{G\rho a}{r} (\alpha_{2i} - \alpha_{1i}) \left[h_i^c \cos(\bar{n} \cdot \bar{\beta}) - h_i^s \sin(\bar{n} \cdot \bar{\beta}) \right] \cdot \left[\sqrt{a^2 + r^2 - 2ar\cos\theta} \right]_{\theta_{1i}}^{\theta_{2i}} . \tag{5}$$

Differentiation of eq. (5), or either 2a or 2b, with respect to r yields gravity above or below the tidal layer. The

mathematical model of an infinitesimally thin layer covering the surface of a sphere exhibits a discontinuity in gravity as one crosses the boundary of the tidal sheet. Thus gravity readings made very close to coasts or on islands can be significantly perturbed. (See Pekeris (1978) for a further discussion of the problem.)

Load Deformation Coefficients h'_n, k'_n, and l'_n

Because the Earth is not a perfectly rigid body, a deformation will occur due to the application of the tidal load. The response of the Earth to the load is described by the load deformation coefficients h'_n, k'_n, l'_n (Munk and MacDonald 1960). Let U'_n represent the n-th degree contribution to the loading potential in eq. (2). Then the change in the distance from the center of the Earth (or height) is given by $h'_n U'_n/g$. The change in the n-th degree potential caused by the redistribution of mass is represented by the coefficients k'_n. The actual potential after deformation is given by $(1+k'_n)U'_n$. Similarly to h'_n, the l'_n represents horizontal displacements of degree n. Numerical values of load deformation coefficients used in this study have been taken from Farrell (1972) and Zschau (1978).

Gravity Change Caused by Deformation

Since the direct or Newtonian contribution to gravity has been given, one must now determine the contribution to gravity resulting from deformation. For the external problem differentiation of $\Sigma k'_n U'_n$ with respect to r gives

$$- \frac{1}{r} \sum_{n=o}^{\infty} k'_n \, (n+1) \, U'_n .$$

The change in gravity resulting from height changes is given by

$$\frac{2g}{r} \sum_{n=o}^{\infty} h'_n \, U'_n/g.$$

The combination of these two terms along with the Newtonian contribution gives the total change in gravity caused by the tidal mass layer

$$\Delta g_{tide} = \frac{1}{r} \sum_{n=o}^{\infty} \left[(n+1)(1+k'_n)-2h'_n\right] U'_n \qquad (6)$$

where the sign is reversed so a downward attraction is positive (as is measured by a gravimeter). The term for the attraction inside the brace should be used rather than the corresponding term for attraction under the tidal sheet given by Farrell (1972) and Longman (1963).

Summation of Series

As previously discussed, the Newtonian contribution is best evaluated by using eq. (5) which avoids the evaluation of an infinite sum. The important difference between this approach and that given by other investigators is that the integrals of Green's functions are computed rather than Green's functions themselves. Again noting that the amplitude and phase of the ocean tide are constant over limited areas (as was done for the Newtonian attraction), the tide height term can be taken outside the integral to simplify the process. This technique then reduces to an evaluation of the expression $\int P_n(\cos\theta) \sin\theta \, d\theta$ which can be obtained by using recursive expressions. Let $T_n(\theta) = \int P_n(\cos\theta) \sin\theta \, d\theta$. Then the T's are given by

$$T_n(\theta) = \frac{\sin\theta}{n(n+1)} P_{n1}(\cos\theta) \qquad (7)$$

where $P_{n1}(\cos\theta)$ is the associated Legendre function of degree n and order one. The recursive expressions for $P_{n1}(\cos\theta)$ are used to obtain

$$T_n(\theta) = \frac{2n-1}{n+1} \cos\theta \; T_{n-1}(\theta) - \frac{n-2}{n+1} \; T_{n-2}(\theta). \qquad (8)$$

The functions T_n are very desirable in that the infinite sum $\sum_{n=0}^{\infty} T_n$ exhibits no singularities as does $\sum_{n=0}^{\infty} P_n$ when the central angles are small. These are essentially the disk factors that Farrell (1972, 1973) used to improve the convergence characteristics of Green's functions. Although not required, Kummer's method (Farrell 1972) can also be used to facilitate the evaluation of the infinite series because h_n' and nk_n' approach constants as n gets large. The terms involving height above sea-level, $(a/r)^n$, remain in the infinite sums. These can be important especially if the gravimeter is placed in rather high locations. Pekeris (1978) has shown that as one approaches the surface from above or below the tidal sheet, the infinite sum becomes a composition of two terms. One term represents the solution in the center of the surface or boundary, and the other is a delta function accounting for the attraction of the mass directly above or below.

The technique of using the integral of Green's functions rather than Green's functions themselves does not limit itself to gravity calculations only. The same technique can be used for all effects such as displacement, tilt, and strain calculations to remove one singularity. It is not only limited to global tide models, but can be used regionally if the regional representations are given as areas of constant amplitude and phase.

All ocean tidal contributions calculated in this study omitted the degree zero term (n=0) in order to impose mass conservation.

Because some of the observations were taken near the ocean, a small nonzero initial central angle was used.

M2 OCEAN TIDE

The global 1° square representation of the M2 constituent was obtained from E. W. Schwiderski, of the Naval Surface Weapons Center, Dahlgren. This M2 model was generated by "hydrodynamical interpolation" (Schwiderski 1978). That is, in solving the Laplace tidal equations, more than 2,000 empirical tide gage observations from continental and island stations were used to constrain the solution height amplitude and phase inside the grid compartments where tide gage observations were available. This feature is very important for studying ocean loading. Frequently, modification of global tide models is undertaken using more realistic local models because gravity measurements taken near coasts are sensitive to the local tide. However, for the results quoted here, no modifications of the Schwiderski M2 model were made. The fine mesh size (1°x1°) was also an important consideration in choosing this particular model for this study.

RESULTS

Tidal Observations

Normally, analyses of tidal gravity series are given in terms of amplitude factor δ and phase ψ. The amplitude factor is the ratio of actual tidal response amplitude to the theoretical gravity value for a rigid Earth. Solutions of Earth modeling yield an amplitude factor of δ=1.16 and phase ψ=0°. The M2

signal in microgals at any time, t, can be generated from the expression

$$\Delta g_{M2}\ (\phi,\lambda,t) = -\delta\ \frac{6x10^8}{a}\ \sqrt{5/96\pi}\ g\ C_{M2}\ \cos^2(\phi) \cdot \cos\ (\bar{n}\cdot\bar{\beta} + 2\lambda + \psi) \qquad (9)$$

where $\bar{\beta}$ was defined earlier, λ is east longitude, $\bar{n}=(2,0,0,0,0,0)$, g is magnitude of gravity in m/sec^2, a is the Earth semimajor axis in meters, and ϕ is latitude. The numerical value for C_{M2} is taken from Cartwright and Edden (1973). C_{M2} is equal to 0.63192. Differing sign conventions for the local phase angle, ψ, are found in the literature. For this reason (9) is explicitly shown. When the phase angle, ψ, takes on negative values, it is regarded as a lag.

The most precise tidal gravimeter results available are from the superconducting gravimeter studies of Warburton et al. (1975). These results are reported to have an accuracy of ±0.2% Tidal series with this gravimeter are available for La Jolla and Piñon Flat, Calif.

Tidal gravimeter results for the Australian stations Alice Springs and Canberra were taken from Melchior (1978). Results for Bruxelles were taken from Melchior et al. (1976). Observation results for Walferdange (Torge and Wenzel 1977), Potsdam (Altmann et al. 1977), and Mizusawa (Hosoyama 1977) were presented at the Eighth International Symposium on Earth Tides, Bonn, September, 1977.

The results of this study are given in table 1. The observations in terms of amplitude factor and local phase are given in columns A. The observed values are reduced by the theoretical solid Earth values δ=1.16 and ψ=0° and are given in columns B. Columns C show the predicted ocean tide contribution to the

Table 1.--Global tidal gravity

Station	Latitude (deg)	Longitude (deg)	Height (m)	A Observed Amplitude factor	A Observed Phase (deg)	B Observed minus theoretical load[1] Amplitude (μgal)	B Observed minus theoretical load[1] Phase (deg)	C Computed ocean load[2] Amplitude (μgal)	C Computed ocean load[2] Phase (deg)	D Observed minus computed load Amplitude factor	D Observed minus computed load Phase (deg)
La Jolla[3]	32.87	242.73	123.	1.1722	-3.33	3.64	-81.	3.35	-96.	1.1765	-0.25
								3.30	-96.	1.1764	-0.30
Piñon Flat[3]	33.59	243.54	1280.	1.1678	-1.29	1.42	-74.	1.74	-106.	1.1770	0.29
								1.74	-107.	1.1771	0.28
Alice Springs[4]	-23.72	133.83	590.	1.1656	-0.31	0.53	-48.	0.02	41.	1.1653	-0.29
								0.03	-146.	1.1660	-0.30
Canberra[4]	-35.32	149.00	663.	1.215	-2.02	3.57	-41.	2.37	-55.	1.1872	-0.37
								2.31	-56.	1.1879	-0.41
Bruxelles[5]	50.80	4.39	101.	1.1910	2.80	2.02	60.	2.04	66.	1.1663	-0.19
								2.00	66.	1.1662	-0.13
Walferdange[6]	49.62	6.15	295.	1.1910	2.61	1.81	55.	1.91	59.	1.1584	0.12
								1.87	59.	1.1588	0.17
Potsdam[7]	53.38	13.07	82.	1.1834	1.09	0.86	44.	1.30	38.	1.1450	-0.39
								1.28	38.	1.1456	-0.36
Mizusawa[8]	39.08	141.08	61.	1.1884	1.39	1.82	46.	2.26	56.	1.1599	-0.61
								2.20	56.	1.1606	-0.56

[1]Theoretical amplitude 1.16, phase 0°.
[2]Load deformation coefficients are taken from: Farrell (1972) for first line, Zschau (1978) (real part only) for second line.
[3]Warburton et al. (1975).
[4]Melchior (1978).
[5]Melchior et al. (1976).
[6]Torge and Wenzel (1977).
[7]Altmann et al. (1977).
[8]Hosoyama (1977).

gravity measurements using the Schwiderski M2 ocean tide model, the loading deformation coefficients of Farrell (1972) and Zschau (1978), and the integral Green's function technique presented in this paper. By comparing columns B and C, one can see that the predicted ocean contribution to the gravity signal agrees with the theoretical values (1.16,0°) to the 0.5-microgal level. These results are slightly better than the Australian results obtained by Bretreger and Mather (1978) using global ocean tide models of Hendershott and Zahel.

Columns D represent corrected amplitude factors and phases under the assumption that the predicted ocean tide contributions do indeed properly model the ocean load. Except for Piñon Flat and Walferdange, the phases seem to show a negative trend. The magnitude of these phases is contrary to that predicted by Zschau (1978) for an imperfectly elastic Earth. His modeling shows that the lag in gravity measurements should be very small (order of 1/1,000°). These corrected results should not be taken too seriously, however. These phases represent measurement accuracies of 0.5 microgal or less, which is not the case. They may also be subject to common calibration errors. Further improvement is also possible in modeling the ocean tide in the open oceans where direct measurements of the ocean tidal amplitude and phase are sparse.

Differences in Tidal Gravity

Because of the quality of the observations at La Jolla and Piñon Flat, further investigation is indicated as a result of the disagreement between the observed tidal values at these locations after correcting for the ocean contribution. The ocean predictions here seem to be slightly worse than those calculated by Warburton et al. (1975). Elimination of ocean tide effects from far afield is accomplished by subtracting the ocean effects between the two sets of observations. (La Jolla and Piñon Flat

are only 1° apart.) Table 2 shows the results of such differencing. One immediately notices that the differential predictions between Warburton et al. (1975) and the technique used here, along with the Schwiderski M2 model, are almost identical.

Table 2.--M2 ocean tidal differences in gravity between La Jolla and Piñon Flat

	Amplitude (μgal)	Phase (degree)
Observed	2.25	39.0
Goad (this study)	1.65	38.3
Warburton et al. (1975)	1.61	39.6

Western European Gravity Comparisons

Many tidal observations have been taken in western Europe during the past several years. Many of these observations were published by Melchior et al. (1976). Since then, Souriau (1979) published a set of corrections to the Melchior, Kuo, Ducarme observation set for the effects of the ocean tidal loading. His procedure was to use the Green's functions of Farrell (1972) in conjunction with ocean tidal information obtained from digitized cotidal charts at 0.25° spacing for the neighboring seas. Cotidal charts of several investigators were used to model the large water bodies. Table 3 gives these comparisons for western Europe. Notice that the predictions by Souriau and those computed using the Schwiderski M2 model with the load deformation coefficients of Zschau are very similar. The major differences occur at Bordeaux and Cambridge where the effects of the oceans are rather large. Obviously the 0.25° resolution of nearby seas was an important ingredient in the tidal corrections

Table 3.--Western European gravity

Station	Latitude (deg)	Longitude (deg)	Observed		Ocean load[1]		Ocean load[2]		Corrected[1]		Corrected[2]	
			Amplitude factor	Phase (deg)	Amplitude (μgal)	Phase (deg)	Amplitude (μgal)	Phase (deg)	Amplitude factor	Phase (deg)	Amplitude factor	Phase (deg)
Clermont-Ferrand	45.75	3.10	1.2092	3.60	3.02	69.	2.84	72.	1.1768	-0.04	1.1830	0.09
Bordeaux	44.83	-0.53	1.2119	7.03	5.66	80.	4.88	84.	1.1754	0.07	1.1900	0.95
Grasse	43.75	6.93	1.1884	2.13	1.98	61.	2.07	61.	1.1628	0.02	1.1583	-0.12
Strasbourg	48.58	7.77	1.1883	1.59	2.03	57.	1.87	54.	1.1543	-0.91	1.1545	-0.65
Walferdange	49.67	6.17	1.1910	2.61	2.15	60.	2.00	55.	1.1559	-0.24	1.1536	0.10
Witteveen	52.82	6.67	1.2136	2.15	1.98	49.	1.58	41.	1.1650	-0.41	1.1692	0.38
Bruxelles	50.80	4.37	1.1946	2.80	2.18	69.	2.02	63.	1.1669	-0.45	1.1659	-0.19
Hannover	52.38	9.70	1.1931	1.23	1.75	51.	1.67	46.	1.1534	-1.12	1.1515	-0.86
Graz	47.06	15.43	1.2120	1.12	1.41	40.	1.44	37.	1.1807	-0.10	1.1788	-0.06
Chur	46.85	9.53	1.1934	2.02	1.84	53.	1.78	51.	1.1615	0.02	1.1606	0.14
Torino	45.67	7.55	1.1975	1.35	2.05	58.	1.95	58.	1.1682	-0.94	1.1692	-0.84
Cambridge	52.20	0.12	1.1961	3.99	2.77	64.	2.54	53.	1.1510	-0.24	1.1388	0.57

[1]Souriau (1979).
[2]Goad (this study).

at these two sites. Nevertheless, predictions of this quality using a global M2 model without any modifications are very good.

CONCLUSIONS

Integrals of Green's functions are more suitable for the special case of tidal loading where the tide is represented by areas or cells of constant amplitude and phase. Their use allows for the inclusion of the height of the instrument above sea level. This method also directly uses sequences of load deformation coefficients which is advantageous for making comparisons. The 1° square Schwiderski M2 ocean tide model predicts accurately ocean load perturbations of Earth tidal gravity observations. Global calculations in this paper seem to be good to the 0.5-microgal level. Hopefully, with improvements in surface and space techniques, we are approaching a period when accurate checks between strain, tilt, gravity, and displacement observations will be possible for the prediction of effects of ocean tide models.

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