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Quantifying Systematic Error When Using Axial Rotation Rates Rather Than Geographic Euler Pole Parameters When Describing Tectonic Plate Rotation

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Versions

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1 Introduction

The National Geodetic Survey (NGS) is modernizing the National Spatial Reference System (NSRS). As part of that modernization, NGS will define four terrestrial reference frames (TRFs) which will be mathematically defined relative to the International Terrestrial Reference Frame of 2020, ITRF2020 (NGS, 2020). Those four frames are NATRF2022, PATRF2022, CATRF2022 and MATRF2022 (named for the North American, Pacific, Caribbean and Mariana plates, respectively).

As mentioned by NGS (ibid.), the defining relationship between time-dependent Earth-Centered, Earth-Fixed Cartesian coordinates in ITRF2020 and time-dependent Cartesian coordinates in each of the four terrestrial reference frames will be through a 3 × 3 rotation matrix, generically written as:

$$\begin{bmatrix} X(t) \\ Y(t) \\ Z(t) \end{bmatrix}_F = R_{F,I}(t, t_0) \begin{bmatrix} X(t) \\ Y(t) \\ Z(t) \end{bmatrix}_I \tag{1}$$

Where *F* is a general variable meaning "some plate fixed frame", which could be NATRF2022, PATRF2022, CATRF2022 or MATRF2022 in the modernized NSRS. The *I* stands for ITRF2020. The value "t_0" will be set to 2020.00 (ibid.).

Although the *R* matrix in Equation 1 can be derived into a non-approximate form (NGS 2020, Equation 21), it is quite complex. NGS has stated that they intend to use an approximate form of the *R* matrix instead. This document attempts to quantify how much systematic error NGS will introduce through this approximation.

2 The Euler pole parameters

The *R* matrix in Equation 1 is built to replicate the rotation of a particular tectonic plate within the ITRF2020 frame. If the Earth were a spherical surface upon which the tectonic plates are rotating smoothly at some constant rate about some constant point, the most accurate way to describe that rotation is through "geographic" Euler pole parameters, or EPPs.

The Euler pole is the point about which the plate rotates, and geographic Euler pole parameters (EPPs) are the location of that pole (latitude or co-latitude[1] and longitude) and the rotation rate about that pole (usually in milliarcseconds per year or degrees per million years). Thus for each plate there are three EPPs, called "geographic" in this paper to distinguish them from a different set of EPPs, namely "rotation rate" EPPs.

The definition of all rotation matrices used throughout this report will be consistent with a positive rotation in the counterclockwise direction of a right-handed coordinate system, when viewed down the axis from the viewpoint of its positive end (Leick and van Gelder, 1975).

[1] In NGS (2020), the co-latitude was used, to simplify the derivation.

If written out fully, using geographic EPPs, Equation 1 reads (see also Equation 21 in NGS (ibid.):

$$\begin{bmatrix} X(t) \\ Y(t) \\ Z(t) \end{bmatrix}_F = \begin{bmatrix} \cos\lambda_0 & \sin\lambda_0 & 0 \\ -\sin\lambda_0 & \cos\lambda_0 & 0 \\ 0 & 0 & 1 \end{bmatrix}^{-1} \begin{bmatrix} \cos\theta_0 & 0 & -\sin\theta_0 \\ 0 & 1 & 0 \\ \sin\theta_0 & 0 & \cos\theta_0 \end{bmatrix}^{-1} \begin{bmatrix} \cos[\dot{\omega}_0(t-t_0)] & -\sin[\dot{\omega}_0(t-t_0)] & 0 \\ \sin[\dot{\omega}_0(t-t_0)] & \cos[\dot{\omega}_0(t-t_0)] & 0 \\ 0 & 0 & 1 \end{bmatrix}^{-1} \begin{bmatrix} \cos\theta_0 & 0 & -\sin\theta_0 \\ 0 & 1 & 0 \\ \sin\theta_0 & 0 & \cos\theta_0 \end{bmatrix} \begin{bmatrix} \cos\lambda_0 & \sin\lambda_0 & 0 \\ -\sin\lambda_0 & \cos\lambda_0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} X(t) \\ Y(t) \\ Z(t) \end{bmatrix}_I \tag{2}$$

Where the geographic EPPs for frame *F* are θ_0 = co-latitude of the Euler pole, λ_0 = longitude of the Euler pole and $\dot{\omega}_0$ = rotation rate about the Euler pole. As mentioned in NGS (ibid.), this equation is exact, in that it exactly produces the time-dependent coordinates in frame *F* of a point whose only movement inside of ITRF2020 is a fixed rotation rate of $\dot{\omega}_0$ around a fixed Euler pole at (θ_0, λ_0).[2] As mentioned earlier, Equation 2 is complex, and so NGS has chosen a simpler form to relate their terrestrial reference frames to ITRF2020, which will be investigated shortly.

In NGS (ibid.), it was hypothesized that the rotation matrix in Equation 1 could be derived, not from the geographic EPPs, but rather by treating Equation 1 as an alternative 14 parameter Helmert transformation. In this way, all translations, scales and time-independent rotations became zero, and only time dependent rotation rates about the ITRF axes remained (leaving 3 of the 14 parameters as non-zero). Those three axial rotation rates will be called the "rotation rate" EPPs. In that hypothesis, the rotations were chosen in the specific order of Z, then Y, then X, which led to the following equation:

$$\begin{bmatrix} X(t) \\ Y(t) \\ Z(t) \end{bmatrix}_F \stackrel{?}{=} R_X R_Y R_Z \begin{bmatrix} X(t) \\ Y(t) \\ Z(t) \end{bmatrix}_I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos[\dot{\omega}_X(t-t_0)] & \sin[\dot{\omega}_X(t-t_0)] \\ 0 & -\sin[\dot{\omega}_X(t-t_0)] & \cos[\dot{\omega}_X(t-t_0)] \end{bmatrix} \begin{bmatrix} \cos[\dot{\omega}_Y(t-t_0)] & 0 & -\sin[\dot{\omega}_Y(t-t_0)] \\ 0 & 1 & 0 \\ \sin[\dot{\omega}_Y(t-t_0)] & 0 & \cos[\dot{\omega}_Y(t-t_0)] \end{bmatrix} \begin{bmatrix} \cos[\dot{\omega}_Z(t-t_0)] & \sin[\dot{\omega}_Z(t-t_0)] & 0 \\ -\sin[\dot{\omega}_Z(t-t_0)] & \cos[\dot{\omega}_Z(t-t_0)] & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} X(t) \\ Y(t) \\ Z(t) \end{bmatrix}_I \tag{3}$$

Note the "?" over the equals sign, indicating that this is a hypothesis, and not a proven equivalence.

The order of rotations matters at this point: there are actually 6 alternative forms of Equation 3, each reflecting one of the 6 choices of order of the rotations (ZYX, ZXY, YXZ, YZX, XYZ and XZY). For now, those 5 other forms are ignored, in lieu of asking the larger question: is the alternative 14 parameter Helmert transformation (Equation 3) capable of exactly replicating the time-dependent motions of multiple points rotating around an Euler pole? Or to put it more succinctly, is Equation 3 truly equivalent to Equation 2 for all possible $\{[X(t),Y(t),Z(t)]_I\}^T$?

In order to answer this question, the two rotation matrices would need to be set equal to one another, and a proof given that they remain equal for all times. That is, it must be proven that for

[2] Interesting side note: if such a point truly existed, that is to say, if a plate were perfectly rigid and perfectly rotated around a constant Euler pole upon a sphere with a constant rate, then $[X(t),Y(t),Z(t)]_F$ would actually *not* change over time (see Equation 20 and associated discussion in NGS (ibid.).

any given (θ_0, λ_0, $\dot{\omega}_0$) combination, there exists a ($\dot{\omega}_X$,$\dot{\omega}_Y$,$\dot{\omega}_Z$) combination such that Equation 4 is always true:

$$\begin{gathered}\begin{bmatrix}\cos\lambda_0 & \sin\lambda_0 & 0\\ -\sin\lambda_0 & \cos\lambda_0 & 0\\ 0 & 0 & 1\end{bmatrix}^{-1}\begin{bmatrix}\cos\theta_0 & 0 & -\sin\theta_0\\ 0 & 1 & 0\\ \sin\theta_0 & 0 & \cos\theta_0\end{bmatrix}^{-1}\begin{bmatrix}\cos\dot{\omega}_0(t-t_0) & -\sin\dot{\omega}_0(t-t_0) & 0\\ \sin\dot{\omega}_0(t-t_0) & \cos\dot{\omega}_0(t-t_0) & 0\\ 0 & 0 & 1\end{bmatrix}^{-1}\begin{bmatrix}\cos\theta_0 & 0 & -\sin\theta_0\\ 0 & 1 & 0\\ \sin\theta_0 & 0 & \cos\theta_0\end{bmatrix}\begin{bmatrix}\cos\lambda_0 & \sin\lambda_0 & 0\\ -\sin\lambda_0 & \cos\lambda_0 & 0\\ 0 & 0 & 1\end{bmatrix}\\ =\begin{bmatrix}1 & 0 & 0\\ 0 & \cos[\dot{\omega}_X(t-t_0)] & \sin[\dot{\omega}_X(t-t_0)]\\ 0 & -\sin[\dot{\omega}_X(t-t_0)] & \cos[\dot{\omega}_X(t-t_0)]\end{bmatrix}\begin{bmatrix}\cos[\dot{\omega}_Y(t-t_0)] & 0 & -\sin[\dot{\omega}_Y(t-t_0)]\\ 0 & 1 & 0\\ \sin[\dot{\omega}_Y(t-t_0)] & 0 & \cos[\dot{\omega}_Y(t-t_0)]\end{bmatrix}\begin{bmatrix}\cos[\dot{\omega}_Z(t-t_0)] & \sin[\dot{\omega}_Z(t-t_0)] & 0\\ -\sin[\dot{\omega}_Z(t-t_0)] & \cos[\dot{\omega}_Z(t-t_0)] & 0\\ 0 & 0 & 1\end{bmatrix}\end{gathered} \tag{4}$$

It is not immediately obvious that the proposed proof can be found. To do so requires setting each of the 9 elements of the left-hand side (LHS) matrix equal to its corresponding element in the right-hand side (RHS) matrix. Because the matrices are not symmetric, that means solving 9 possibly independent equations for 3 variables ($\dot{\omega}_X$,$\dot{\omega}_Y$,$\dot{\omega}_Z$). This redundancy might make a unique solution impossible. Even if a unique solution were found, it is possible that it would be unique only for the ZYX order of rotations, and that each of the other 5 possible ways to order the rotations might yield its own unique solution.

For now, further pursuit of an algebraic proof of Equation 4 will be set aside. However, it should be emphasized that the hypothesis that Equation 4 is valid (that is, that tectonic rotation can be mathematically expressed through an alternative 14 parameter Helmert transformation) is the first (of two) assumptions which will be made in this paper. The second will be addressed in a moment.

Using Equation 3 rather than Equation 2 when representing Equation 1 does not do much to increase simplicity. NGS (ibid.) pointed out that with "small angle" approximations, both Equation 2 and Equation 3 can be simplified, provided that the rotation rates are small and t and t_0 are sufficiently close to one another. In order to justify using the small angle approximation consider the fastest rotating plate for the NSRS, which is the Mariana plate, rotating at approximately 2.8 degrees per million years. Thus as long as t and t_0 are within one million years of one another, the total angle the plate rotates will remain under 2.8 degrees (or 0.017 radian). In the small angle approximation, this leads to:

$$\cos 0.017 = 0.99986 \;\approx 1 \tag{5}$$
$$\sin 0.017 = 0.0169992 \;\approx 0.017 \tag{6}$$

In a real situation, t and t_0 would be much closer than 1 million years. Plus all plates of the NSRS have smaller rotation rates than the Mariana, so it is reasonable to say that, for any of the four plates within the modernized NSRS over time spans under 1 million years, the total possible tectonic rotation angle is small enough to consider invoking the small angle approximation. Furthermore, as the combined effect of the three axial rotations must simulate the total rotation about the Euler pole, they must be of similar magnitude. This use of the small angle approximation is the second (of two) assumptions being made in this paper.

Applying the small angle approximation to $\dot{\omega}_0(t-t_0)$, Equation 2 reduces to:

$$\begin{bmatrix} X(t) \\ Y(t) \\ Z(t) \end{bmatrix}_F \approx I + \dot{\omega}_0(t - t_0) \begin{bmatrix} 0 & cos\theta_0 & -sin\lambda_0 sin\theta_0 \\ -cos\theta_0 & 0 & cos\lambda_0 sin\theta_0 \\ sin\lambda_0 sin\theta_0 & -cos\lambda_0 sin\theta_0 & 0 \end{bmatrix} \begin{bmatrix} X(t) \\ Y(t) \\ Z(t) \end{bmatrix}_I \quad (7)$$

Applying the small angle approximation to $\dot{\omega}_X(t - t_0)$, $\dot{\omega}_Y(t - t_0)$ and $\dot{\omega}_Z(t - t_0)$, Equation 3 reduces to:

$$\begin{bmatrix} X(t) \\ Y(t) \\ Z(t) \end{bmatrix}_F \approx I + (t - t_0) \begin{bmatrix} 0 & \dot{\omega}_Z & -\dot{\omega}_Y \\ -\dot{\omega}_Z & 0 & \dot{\omega}_X \\ \dot{\omega}_Y & -\dot{\omega}_X & 0 \end{bmatrix} \begin{bmatrix} X(t) \\ Y(t) \\ Z(t) \end{bmatrix}_I \quad (8)$$

The "?" over the equals has been dropped in favor now of the approximation sign, though the underlying (unproven) hypothesis remains in place. Interestingly, no matter which of the 6 orders of rotations was chosen in the alternative 14 parameter Helmert transformation, the application of the small angle approximation will always yield Equation 8.

An obvious parallelism can now be seen between Equations 7 and 8 which was hidden in the complexity of Equations 2 and 3. This parallelism can lead to a relation between the three geographic EPPs and the three rotation rate EPPs, but only under the two assumptions of this paper. Therefore, before proceeding, a solution for $\dot{\omega}_X$, $\dot{\omega}_Y$ and $\dot{\omega}_Z$ in terms of θ_0, λ_0 and $\dot{\omega}_0$ will be provided. Using Equations 7 and 8 as a guide, one can write this equivalency, which is a simplification of Equation 4:

$$I + \dot{\omega}_0(t - t_0) \begin{bmatrix} 0 & cos\theta_0 & -sin\lambda_0 sin\theta_0 \\ -cos\theta_0 & 0 & cos\lambda_0 sin\theta_0 \\ sin\lambda_0 sin\theta_0 & -cos\lambda_0 sin\theta_0 & 0 \end{bmatrix} = I + (t - t_0) \begin{bmatrix} 0 & \dot{\omega}_Z & -\dot{\omega}_Y \\ -\dot{\omega}_Z & 0 & \dot{\omega}_X \\ \dot{\omega}_Y & -\dot{\omega}_X & 0 \end{bmatrix} \quad (9)$$

Like Equation 4, if one wishes to solve for $\dot{\omega}_X$, $\dot{\omega}_Y$ and $\dot{\omega}_Z$ in terms of θ_0, λ_0 and $\dot{\omega}_0$, that requires setting each of the 9 elements of the LHS matrix equal to each of the 9 elements of the RHS matrix. Unlike Equation 4, however, this does not lead to 9 unique equations in 3 variables. Instead, due to zeroes on the diagonal and the skew symmetric nature of matrices in both Equations 7 and 8, this leads to 3 independent equations in 3 variables, with an easily found unique solution. The solution to Equation 9 as derived in NGS (ibid.) is:

$$\dot{\omega}_X = \dot{\omega}_0 cos\lambda_0 sin\theta_0 \quad (10)$$
$$\dot{\omega}_Y = \dot{\omega}_0 sin\lambda_0 sin\theta_0 \quad (11)$$
$$\dot{\omega}_Z = \dot{\omega}_0 cos\theta_0 \quad (12)$$

while solving in the opposite direction yields:

$$\theta_0 = \frac{\pi}{2} - ArcTan\left[\frac{\dot{\omega}_Z}{\sqrt{(\dot{\omega}_X^2 + \dot{\omega}_Y^2)}}\right] \quad (13)$$

$$\lambda_0 = ArcTan\left[\frac{\dot{\omega}_Y}{\dot{\omega}_X}\right] \tag{14}$$

$$\dot{\omega}_0 = \sqrt{\dot{\omega}_X^2 + \dot{\omega}_Y^2 + \dot{\omega}_Z^2} \tag{15}$$

Note that Equations 10-15 are *not* solutions of Equation 3, but rather its simplified version, Equation 9. These equations and their derivations are given in Appendix A in NGS (ibid.)

3 Quantifying the approximations

Four possible ways to apply Equation 1 have been presented. Two (Equations 2 and 3) are complicated and two (Equations 7 and 8) are much simpler. In fact, Equation 8 has an advantage over Equation 7, in that it invokes a linear relation with the three rotation rate EPPs rather than invoking trigonometric functions of geographic EPPs. For this reason, when solving for EPPs, it is much easier mathematically to solve for the rotation rate EPPs, and convert to geographic EPPs than the other way around.

Again, this is only possible because of the dual assumptions that an alternative 14 parameter Helmert transformation is appropriate to describe tectonic rotation, and that the small angle approximation is valid, but within those two approximations Equations 9-16 *are* exact. Thus, this 3-way equation can be formed:

$$\begin{aligned}
&\begin{bmatrix} \cos\lambda_0 & \sin\lambda_0 & 0 \\ -\sin\lambda_0 & \cos\lambda_0 & 0 \\ 0 & 0 & 1 \end{bmatrix}^{-1} \begin{bmatrix} \cos\theta_0 & 0 & -\sin\theta_0 \\ 0 & 1 & 0 \\ \sin\theta_0 & 0 & \cos\theta_0 \end{bmatrix}^{-1} \begin{bmatrix} \cos[\dot{\omega}_0(t-t_0)] & -\sin[\dot{\omega}_0(t-t_0)] & 0 \\ \sin[\dot{\omega}_0(t-t_0)] & \cos[\dot{\omega}_0(t-t_0)] & 0 \\ 0 & 0 & 1 \end{bmatrix}^{-1} \begin{bmatrix} \cos\theta_0 & 0 & -\sin\theta_0 \\ 0 & 1 & 0 \\ \sin\theta_0 & 0 & \cos\theta_0 \end{bmatrix} \begin{bmatrix} \cos\lambda_0 & \sin\lambda_0 & 0 \\ -\sin\lambda_0 & \cos\lambda_0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \\
&\approx I + \dot{\omega}_0(t-t_0) \begin{bmatrix} 0 & cos\theta_0 & -sin\lambda_0 sin\theta_0 \\ -cos\theta_0 & 0 & cos\lambda_0 sin\theta_0 \\ sin\lambda_0 sin\theta_0 & -cos\lambda_0 sin\theta_0 & 0 \end{bmatrix} \\
&\approx I + (t-t_0) \begin{bmatrix} 0 & \dot{\omega}_Z & -\dot{\omega}_Y \\ -\dot{\omega}_Z & 0 & \dot{\omega}_X \\ \dot{\omega}_Y & -\dot{\omega}_X & 0 \end{bmatrix}
\end{aligned} \tag{16}$$

The first approximation sign in Equation 16 reflects the small angle approximation, while the second reflects both the hypothesis that an alternative 14 parameter Helmert transformation is appropriate to describe tectonic rotation *and* the small angle approximation.

NGS (ibid., equation 60) states that Equation 8 will be used (with an equal sign, not an approximation sign) to relate the four terrestrial reference frames of the modernized NSRS to ITRF2020, rather than Equation 2 (or 3 or 7). That mean using the third row of Equation 16, rather than the first. The goal of this paper is to quantify how much systematic error builds up due to those approximation signs in Equation 16.

In order to quantify the differences between Equation 2 and Equation 8, it was first necessary to determine the points being evaluated. While NATRF2022, PATRF2022, CATRF2022 and MATRF2022 are all *global* reference frames, the expectation is that use of these four frames are likely to be *limited* to areas of interest that more or less incorporate the tectonic plates for which each frame is named. Therefore, a regular grid of points, falling explicitly within the boundaries of each plate (as defined in Bird, 2003) was tested. The spacing of the grid was 1 × 1 degree for the North American and Pacific plates, and 0.25 × 0.25 degree for the Caribbean and Mariana plates.

A computer program was written to do the following:

- Pick a plate/frame.
- Take the latest known ($\dot{\omega}_X$,$\dot{\omega}_Y$,$\dot{\omega}_Z$) for this plate/frame and use Equations 13-15 to compute (θ_0, λ_0, $\dot{\omega}_0$). See Table 1.
- Form a grid of points on each plate, using the plate boundaries of Bird (2003).
- Assuming zero ellipsoid height, compute X, Y, and Z for these grid points.
- Set $t_0 = 2020$.
- Begin looping t forward from t_0 at 10 year intervals. At every interval, assume the X,Y,Z values are the ITRF2020 values at t. Then apply Equation 2 and also Equation 8 to yield the (N/P/C/M)ATRF2022 coordinates also at t.
- Compare the X,Y and Z coordinates coming from Equation 2 with those from Equation 8. Compute the absolute linear distance of the difference (since the change in ellipsoid height is essentially zero, this will give the horizontal distance).
- If any of the differences exceeds 0.5 millimeter, stop and report.

The RMS is used as it reflects to total linear disagreement between the two XYZ positions, and is therefore reflective of achieving a total unacceptable error (0.5 mm), rather than waiting for one particular direction to achieve said error.

Parameter	North America	Pacific	Caribbean	Mariana
$\dot{\omega}_X$ (mas/yr)	0.024	-0.409	-0.072	-8.089
$\dot{\omega}_Y$ (mas/yr)	-0.694	1.047	-0.933	5.937
$\dot{\omega}_Z$ (mas/yr)	-0.063	-2.169	0.596	2.159
θ_0 (deg) co-lat.	95.184	152.605	57.507	77.857
ϕ_0 (deg) lat.	-5.184	-62.605	32.493	12.143
λ_0 (deg) lon.	271.981	111.338	265.587	143.723
$\dot{\omega}_0$ (mas/yr)	0.697	2.443	1.109	10.264
Reference Frame	ITRF2014	ITRF2014	IGS14	ITRF2014
Source	Altamimi et al, 2017	Altamimi et al, 2017	Snay and Saleh, 2020	Smith, 2020

As NGS is interested in positioning accuracy to the millimeter level, exceeding 0.5 millimeter was seen as an unacceptable level of systematic error. And by using a search that spans the entire plate, this meant that if anywhere on the plate the approximation in Equation 8 exceeded 0.5 millimeter, it would bound how long Equation 8 could justifiably be used.

With this strategy, the following values were found:

0.5 mm of systematic error

Plate/Frame	Years past 2020 when Equation 8's error exceeds 0.5 mm
North American / NATRF2022	3710
Pacific / PATRF2022	1070
Caribbean / CATRF2022	2910
Mariana / MATRF2022	550

First off, Table 2 confirms that there is, in fact, a difference between using Equation 2 and Equation 8. But more importantly it states how long Equation 8 can be used before its inherent approximations begin to break down at a level unacceptable for the NSRS.

Also, as a point of interest, equation 7 yields identical results as equation 8. This isn't surprising, since the values of $\dot{\omega}_X$,$\dot{\omega}_Y$,$\dot{\omega}_Z$ and θ_0, λ_0, $\dot{\omega}_0$ are not determined independently, but rather are directly related through equations 13-15. However, while interesting, this equivalence of values does not make equation 7 useful, since it still requires knowing geographic EPPs which are contained within trigonometric functions.

In conclusion, with even the worst-case scenario being a 0.5 mm breakdown in the approximations at a timeline of 5.5 centuries past 2020, NGS can confidently continue to use Equation 8 as the official definition between ITRF2020 and the four terrestrial reference frames of the modernized NSRS.

4 Bibliography

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