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Managing Populations Under the Marine Mammal Protection Act of 1994: A Strategy for Selecting Values for $\mathrm{N}_{\text {MIN }}$, the Minimum Abundance Estimate, and $\mathrm{F}_{\mathrm{R}}$, the Recovery Factor.

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#### Abstract

The 1994 amendments to the Marine Mammal Protection Act require specifying a minimum abundance estimate, $\mathrm{N}_{\text {MIN }}$, and a recovery factor, $\mathrm{F}_{\mathrm{R}}$, to calculate a potential biological removal level, PBR. A two-step strategy is proposed for selecting values for $\mathrm{N}_{\text {MIN }}$ and $\mathrm{F}_{\mathrm{R}}$, using computer simulations. First, the percentile of the abundance estimate for estimating $\mathrm{N}_{\text {MIN }}$ is found that ensures that marine mammal populations, in the absence of serious problems, will recover to or remain within the Optimum Sustainable Population level (OSP). Second, using that percentile for $\mathrm{N}_{\text {MIN }}$, a default recovery factor for unknown status populations, $\mathrm{F}_{\mathrm{R}}(\mathrm{u})$. is found that ensures populations will recover to or remain within the OSP level even under significant but plausible "problem" scenarios, such as biased mortality estimates. Results of the simulations indicated that to meet the above criteria, it was necessary to use the $20^{\text {th }}$ percentile of the abundance estimate for $\mathrm{N}_{\mathrm{MIN}}$, and that it was necessary to use values for $\mathrm{F}_{\mathrm{R}}(\mathrm{u})$ of 0.50 for pinnipeds and 0.65 for cetaceans. Additionally, a value for $F_{R}(e)$ of 0.15 was found to be sufficient to not delay the percentage increase in recovery time to OSP by more than $10 \%$. Finally, it is suggested that populations shown to be within OSP could have higher values for $\mathrm{F}_{\mathrm{R}}$ than the default, $\mathrm{F}_{\mathrm{R}}(\mathrm{u})$. However, before such action is taken, reasonable assurance in the form of scientific justification should be provided to ensure that the estimates of abundance, mortality, and $\mathrm{R}_{\mathrm{M}}$ are not severely biased, and that the coeffecients of variation of the abundance and mortality estimates are within the range used in these simulations ( $<0.8$ for the abundance estimate, $<0.30$ for the mortality estimates).


## Introduction

A National Marine Fisheries Service proposal to manage populations of marine mammals which are incidentally killed in commercial fisheries was incorporated into the 1994 amendments to the re-authorized Marine Mammal Protection Act. It is based on the calculation of a level of incidental fisheries mortality that will allow a population to remain at, or recover to, Optimum Sustainable Population level (OSP), defined to be a level between the maximum net productivity level (MNPL) and carrying capacity (Gerrodette and DeMaster 1990). The procedure is based on the equation

$$
\begin{equation*}
P B R=N_{M I N} \frac{1}{2} R_{M} F_{R} \tag{1}
\end{equation*}
$$

where: $\quad \mathrm{PBR}=\quad$ Potential Biological Removal, $\mathrm{N}_{\text {MIN }}=\quad$ A minimum abundance estimate of the population, $1 / 2 R_{M}=\quad$ one-half the maximum net recruitment rate.
$\mathrm{F}_{\mathrm{R}}=\quad$ a recovery or safety factor between 0.0 and 1.0 .
Taylor (1993) considered a method for determining the PBR, using a lower, 2-tailed, $95 \%$ confidence limit for $\mathrm{N}_{\text {MIN }}$, which represents the $2.5^{\text {th }}$ percentile of the distribution, and an $F_{R}$ of 0.5 , with $1 / 2 R_{M}$ set as 0.02 to represent cetaceans and 0.06 to represent pinnipeds. The results of Taylor (1993) showed that using the lower confidence limit was superior to using the point estimate (the $50^{\text {th }}$ percentile), when evaluated by the status of given simulated populations in 100 years. When subjected to "robustness trials", involving significant but plausible problems (such as bias in the abundance estimates), using the $2.5^{\text {th }}$ percentile still resulted in populations being significantly above MNPL in 100 years.

The trials in Taylor (1993) resulted in nearly all trajectories being well above the maximum net productivity level. Therefore, it is worth considering whether other combinations of $\mathrm{N}_{\text {MIN }}$ (using percentiles of the abundance estimate) and $\mathrm{F}_{\mathrm{R}}$ are sufficient to maintain populations within OSP. The intent of the proposed management scheme is to provide a conservative level for the PBR that will allow populations to recover to or remain within OSP in spite of uncertainty, whether in the form of imprecise or biased information. However, it is obvious from equation 1 that conservatism could be built into any of the three parameters that determine the PBR. If the default values for $1 / 2 R_{M}$ are close to the true values, then using an unbiased point estimate ( $\mathrm{N}_{\text {MEAN }}$ ) for $\mathrm{N}_{\text {MIN }}$ and an $\mathrm{F}_{\mathrm{R}}$ of 1.0 would result in a population equilibrating exactly at the maximum net productivity level. It is thus intuitively satisfying to keep a reasonable estimate for $1 / 2 R_{M}$ and then allow populations to equilibrate above MNPL by ensuring that the product of $N_{\text {MIN }}$ and $F_{R}$ is less than $N_{\text {MEAN }}$. Here I propose a specific strategy for setting values of those two parameters, $\mathrm{N}_{\text {MIN }}$ and $\mathrm{F}_{\mathrm{R}}$.

## Strategy

(1) The percentile of the abundance estimate is chosen such that (A) any population, in the base
case of an absence of significant problems, will be within OSP with $95 \%$ probability in 100 years, with an $F_{R}$ equal to 1.0 , and such that (B) a population starting at the lower bound of OSP $(0.5 \mathrm{~K})$ will still be within OSP in 20 years with $95 \%$ probability.
(2) A default value for $F_{R}$ for unknown status populations, called here $F_{R}(u)$, is chosen such that the above criteria ( 1 A and 1 B ) are also met during robustness trials, in which the data are assumed to have unknown problems, such as significant bias.
(3) A value of $F_{R}$ for populations listed as endangered, called here $F_{R}(e)$, will be chosen such that the time to recovery of a depleted population is not more than $10 \%$ greater than populations that experience no incidental kill with $95 \%$ probability.

## Methods

## Simulation Trials

Methods nearly identical to Taylor (1993) were used here for the simulations. The underlying population dynamics model was a discrete form of the generalized logistic equation,

$$
\begin{equation*}
N_{t+1}=N_{t}+N_{t} R_{M}\left[1-\left(\frac{N_{t}}{K}\right)^{\ominus}\right] \tag{2}
\end{equation*}
$$

where: $\quad N_{t}=$ population size at time t
$\mathrm{R}_{\mathrm{M}}=$ the maximum net recruitment rate
$\mathrm{K}=$ the equilibrium population size
$\theta=$ the shape parameter, set to 2.345 which sets the maximum net productivity level at approximately $60 \%$ of K .

The procedure and sequence of each base case simulation was:
(1) The population was projected from year $t$ to year $t+1$ using equation 2 , with $R_{M}$ equal to either 0.04 or 0.12 . In each simulation, $K=10000$, and $\theta=2.345$, for an approximate maximum net productivity level of 0.6 K , or 6000 .
(2) Every $\mathrm{i}^{\text {ith }}$ year (starting in year 1), an estimate of abundance was "surveyed" by randomly drawing from a distribution with a specified coefficient of variation, $\mathrm{CV}(\mathrm{N})$.
(3) A PBR was then calculated from equation 1, using the most recent survey.
(4) Incidental fisheries mortality was simulated by subtracting from the current population a gaussian random deviate from a distribution with a mean equal to the PBR with a coefficient of variation, $\mathrm{CV}(\mathrm{M})$, of 0.30 .
(5) This sequence was repeated until the population was projected from year 0 to year 100. Each trajectory was initiated in year 0 at a population size equal to a specified fraction of K . The first survey occurred in year 1.
(6) For each trial, 1000 trajectories were simulated, and performance statistics (described below) were calculated.

The sampling error of the survey was assumed to follow a log-normal distribution with a mean equal to the true population size, with a specified CV of either 0.2 or 0.8 . Each abundance estimate, or "survey", was generated by

$$
\begin{equation*}
\hat{N}_{t}=\exp \left[\ln \left(\frac{N_{t}}{\sqrt{\left(1+C V^{2}\right)}}\right)+X \sqrt{\ln \left(1+C V^{2}\right)}\right] \tag{3}
\end{equation*}
$$

where $x=a$ gaussian random deviate with $a$ mean of zero and $a$ variance of 1 .
$\mathrm{N}_{\text {min }}$ was calculated as the lower percentile of a log-normal distribution, as

$$
\begin{equation*}
N_{M I N}=\frac{\hat{N}}{\exp \left(z \sqrt{\ln \left(1+C V(N)^{2}\right)}\right)} \tag{4}
\end{equation*}
$$

where $\mathrm{z}=\quad 1.96$ for the $2.5^{\text {th }}$ percentile, 1.645 for the $5^{\text {th }}, 1.282$ for the $10^{\text {th }}$, and 0.842 for the $20^{\text {th }}$, et cetera.

## Performance Statistics

Two types of statistics will be calculated to address the performance of the simulations for ensuring the safety of marine mammal populations. To measure long-term behavior, the mean and the $5^{\text {th }}$ and $95^{\text {th }}$ percentiles of the distribution of population sizes at the end of the 100 year will be calculated. If the lower percentile (representing a two-tailed $90 \%$ confidence limit) value is within OSP, it can be concluded that more than $95 \%$ of the trajectories are within OSP.

To measure short-term behavior, the population size after 20 years of the trajectory will be compared to a reference level. For depleted populations ( $\mathrm{N}_{0}=0.3 \mathrm{~K}$ ), the reference level is what the population size of an identical population would be if there was no incidental mortality (no PBR). For a population starting within OSP $\left(\mathrm{N}_{0}=0.6 \mathrm{~K}\right)$ the reference level is that starting point, to easily determine whether a population is still within OSP after 20 years.

To measure the impact on fisheries, sample plots of the trajectories of PBR levels are shown. Additionally, the mean PBR level is calculated, as well as a measure of the year-to-year variability in the PBR level, the root mean square error of the PBR between years, calculated as

$$
\begin{equation*}
R M S E=\sqrt{\sum_{i=1}^{99} \frac{\left(P B R_{i}-P B R_{i+1}\right)^{2}}{99}} \tag{5}
\end{equation*}
$$

## Base Cases and Robustness Trials

A total of eight "base cases" were considered (Table 1). To represent cetacean life history, four cases used a $1 / 2 \mathrm{R}_{\mathrm{M}}$ of 0.02 for the PBR calculation (eq. 1) with a $\mathrm{R}_{\mathrm{M}}$ of 0.04 specified in the population model (eq. 2). The four cases represented all combinations of starting the population at 0.3 K and 0.6 K , and specifying $\mathrm{CV}(\mathrm{N})$ at 0.2 and 0.8 . To represent pinniped life history, another four base cases used a $1 / 2 R_{M}$ of 0.06 for the PBR calculation with a $R_{M}$ of 0.12 specified in the population model.

A total of seven "robustness trials" were considered (Table 2). Trial 1 simply represents the base case. Trials 2, 3, and 4 represent bias in the estimates of mortality, abundance, and $1 / 2 \mathrm{R}_{\mathrm{M}}$, respectively. Trials 5 and 6 represent situations where the variance of an estimate is severely under-estimated. Trial 7 represents carrying out a survey every 8 years rather than every 4 years.

## Results and Discussion

For the base case trials, the $20^{\text {th }}$ percentile results in all populations equilibrating within OSP (Figure 1). With low variance $(\mathrm{CV}(\mathrm{N})=0.2)$, all percentiles are within OSP. However, with higher variance $(\mathrm{CV}(\mathrm{N})=0.8)$, at least the $40^{\text {th }}$ percentile is required for cetaceans $\left(1 / 2 R_{M}=0.02\right)$ and the $20^{\text {th }}$ for pinnipeds. The initial population level is not important to where the population is in 100 years, as all base case trials started at 0.6 K (Figure 2) are virtually identical in 100 years to starting at 0.3 K (Figure 1).

In the low variance cases, even the $2.5^{\text {h }}$ percentile does not allow depleted populations to be within $10 \%$ of a population with no incidental mortality (no pbr), but with high variance, the $10^{\text {th }}$ percentile results in the mean of the simulations being within $10 \%$ (Figure 3). For populations starting at 0.6 K , the $20^{\text {th }}$ percentile is sufficient to keep $95 \%$ of the trajectories within OSP in 20 years (Figure 4).

Therefore, the $20^{\text {th }}$ percentile of the abundance estimate for $\mathrm{N}_{\text {MIN }}$ appears sufficient to allow populations to recover to or remain within OSP, in the absence of problems such as biased estimates of abundance or mortality, and meets both the 100-year and 20-year specified criteria. Using the $20^{\text {th }}$ percentile, a value of 0.50 for $F_{R}$ for pinnipeds and 0.65 for cetaceans results in all populations equilibrating within OSP during robustness trial 2 , when the incidental mortality was twice the estimated mortality (Figure 5). Additionally, a similar $\mathrm{F}_{\mathrm{R}}$ is required to keep a population starting at 0.6 K from being substantially below where it would be in 20 years under
trial 1, with no bias (Figure 6).
The full robustness trials confirm that the combination of the $20^{\text {th }}$ percentile and a $F_{R}$ of 0.50 for pinnipeds and 0.65 for cetaceans resulted in all populations having $95 \%$ (or close to $95 \%$ ) of their trajectories recovered to OSP in 100 years (Figure 7). Again, populations starting at 0.6 K have very similar distributions for $\mathrm{N}_{100}$ (Figure 8), indicating that the initial conditions are not important. Thus, these values for $\mathrm{N}_{\text {MIN }}$ and $\mathrm{F}_{\mathrm{R}}$ should allow populations at any level to recover to or remain within OSP. The robustness trials further indicate that those values are sufficient to meet the 20 year criteria, also (Figures 9-10).

To investigate the impact on recovery time for a depleted population, the percent increase in time to OSP $(0.6 \mathrm{~K})$ for a population starting at 0.3 K was calculated for a range of values of $\mathrm{F}_{\mathrm{R}}$ (Figure 11). A value of 0.15 resulted in all cases having $95 \%$ of their trajectories not delayed in time to recovery by more than $10 \%$ (i.e., the upper confidence limit is below the $10 \%$ line).

A sample of 30 trajectories of the PBR level in each year show that the PBR is more variable when the variance of the abundance estimate is higher (Figures 12-13). A sample of 30 of the simulation trajectories gives a visual representation of the performance of the chosen values of the $20^{\text {th }}$ percentile for $\mathrm{N}_{\text {MIN }}$ and of 0.50 for pinnipeds and 0.65 for cetaceans for $\mathrm{F}_{\mathrm{R}}(\mathrm{u})$ (Figures 14-15). The desired properties of the management scheme are evident -- depleted populations steadily recover to OSP and stay there, in spite of uncertainty in the estimates of abundance and mortality. Additionally, appropriate motivation exists to improve the precision of the estimates of abundance, as the mean PBR is higher when the CV of the abundance estimate is lower (Fig. 16). If highly variable PBR levels are also seen to be unecessarily restrictive on commercial fishing operations, more precise estimates of abundance also lead to less variable PBR levels, as lower CV's lead to lower RMSE's (Fig. 17).

Adjusting, or "tuning", the values of $\mathrm{N}_{\text {MIN }}$ and $\mathrm{F}_{\mathrm{R}}$ to meet specific criteria should allow for a robust management procedure than ensures the safety of marine mammal populations while allowing for the impact on commercial fisheries to be minimized. Other adjustment criteria could be specified which would also work, such as putting all the conservatism in to just 1 of the parameters by adjusting $\mathrm{N}_{\text {MIN }}$ to pass the robustness trials. However, the two-part procedure suggested has some desirable qualities. First of all, using a lower percentile for $\mathrm{N}_{\text {min }}$ meets the intent of the 1994 ammendments, which state that $\mathrm{N}_{\text {MIN }}$ " $(\mathrm{A})$ is based on the best available scientifica information on abundance, incorporating the precision and variability associated with such information; and, (B) provides reasonable assurance that the stock size is equal to or greater than the estimate." Second, using a lower confidence limit encourages improving the precision of abundance estimates, as lower CV's result in higher PBR's.

Tuning $\mathrm{F}_{\mathrm{R}}$ to pass the robustness trials ensures a robust management procedure that will work for populations of unknown status, even under conditions of fairly severe bias in the collection of data. Specifying a default value of $F_{R}(u)$ of less than 1.0 also builds flexibility into the management scheme. Populations meeting specified criteria could have $F_{R}$ increased from the default value for $\mathrm{F}_{\mathrm{R}}(\mathrm{u})$, again encouraging the collection of information. Suggested criteria
could include populations known to be within OSP, or known to have been increasing while experiencing known levels of incidental mortality. Before such action is taken, reasonable assurance in the form of scientific justification should be provided to ensure that the estimates of abundance, mortality, and $R_{M}$ are not severely biased, and that the coeffecients of variation of the abundance and mortality estimates are within the range used in these simulations ( $<0.8$ for the abundance estimate, $<0.30$ for the mortality estimates).

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## References

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Table 1. Specifications for the 8 base case trials for the simulations.

| Base Cases | Starting N | $1 / 2 \mathrm{R}_{\mathrm{M}}$ | Survey CV |
| :--- | :--- | :--- | :--- |
| A | 0.3 K | 0.02 | 0.2 |
| B | 0.3 K | 0.02 | 0.8 |
| C | 0.3 K | 0.06 | 0.2 |
| D | 0.3 K | 0.06 | 0.8 |
| E | 0.6 K | 0.02 | 0.2 |
| F | 0.6 K | 0.02 | 0.8 |
| G | 0.6 K | 0.06 | 0.2 |
| H | 0.6 K | 0.06 | 0.8 |

Table 2. Specifications for the robustness trials for the simulations.

| Trial | Description |
| :--- | :--- |
| 1 | Base case |
| 2 | Estimated mortality $1 / 2$ actual mortality |
| 3 | Estimated N twice actual N |
| 4 | Estimated $1 / 2 \mathrm{R}_{\mathrm{M}}$ twice actual $1 / 2 \mathrm{R}_{\mathrm{M}}$. If estimated to be 0.02 , actual $1 / 2 \mathrm{R}_{\mathrm{M}}$ <br> should have been 0.01 as $\mathrm{R}_{\mathrm{M}}$ is set to 0.02 . For estimated $1 / 2 \mathrm{R}_{\mathrm{M}}$ of 0.06, <br> actual $1 / 2 \mathrm{R}_{\mathrm{M}}$ should have been 0.03 as $\mathrm{R}_{\mathrm{M}}$ is set to 0.06. |
| 5 | Estimated abundance $\mathrm{CV}<$ actual CV (estimated CV of 0.2 actually <br> 0.8 , estimated CV of 0.8 actually 1.6 ) |
| 6 | Estimated mortality CV $1 / 4$ actual $\mathrm{CV} . \mathrm{CV}(\mathrm{M})$ is set to 1.20 rather than <br> 0.30. |
| 7 | Abundance estimated every 8 years rather than every 4 years. |

Figure 1. Population size in 100 years, with confidence limits capturing $90 \%$ of the simulations, versus the percentile of the abundance estimate used to calculate $\mathrm{N}_{\mathrm{MIN}}$, with $\mathrm{F}_{\mathrm{R}}=1.0$ and the initial population size equal to $30 \%$ of K .


Figure 2. Population size in 100 years, with confidence limits capturing $90 \%$ of the simulations, versus the percentile of the abundance estimate used to calculate $\mathrm{N}_{\mathrm{MIN}}$, with $\mathrm{F}_{\mathrm{R}}=1.0$ and the initial population size equal to $60 \%$ of K .


Figure 3. For depleted populations ( $\mathrm{N}_{0}=0.3 \mathrm{~K}$ ), population size in 20 years with incidental mortality, $\mathrm{N}_{20}$ (pbr), relative to population size in 20 years for a population with no incidental mortality, $\mathrm{N}_{20}$ (no pbr) versus the percentile of the abundance estimate used to calculate $\mathrm{N}_{\text {MIN }}$, with $\mathrm{F}_{\mathrm{R}}=1.0$.


Figure 4. For populations within OSP $\left(\mathrm{N}_{0}=0.6 \mathrm{~K}\right)$, population size in 20 years relative to 0.6 K versus the percentile of the abundance estimate used to calculate $\mathrm{N}_{\mathrm{MIN}}$, with $\mathrm{F}_{\mathrm{R}}=1.0$.


Figure 5. For depleted populations ( $N_{0}=0.3 \mathrm{~K}$ ), population size in 100 years versus the recovery factor, $\mathrm{F}_{\mathrm{R}}$, using the $20^{\text {th }}$ percentile of the abundance estimate to calculate $\mathrm{N}_{\text {MIN }}$.


Figure 6. For populations within $\operatorname{OSP}\left(\mathrm{N}_{0}=0.6 \mathrm{~K}\right)$, population size in 20 years with incidental mortality, $\mathrm{N}_{20}(\mathrm{pbr})$, relative to population size in 20 years for a population with no incidental mortality, $\mathrm{N}_{20}$ (no pbr) versus the recovery factor, $\mathrm{F}_{\mathrm{R}}$, using the $20^{\text {th }}$ percentile of the abundance estimate to calculate $\mathrm{N}_{\text {MIN }}$.


Figure 7. For depleted populations ( $N_{0}=0.3 \mathrm{~K}$ ), population size in 100 years for 7 robustness trials, using a recovery factor, $\mathrm{F}_{\mathrm{R}}$, of 0.65 for cetaceans and 0.50 for pinnipeds, and the $20^{\text {th }}$ percentile of the abundance estimate to calculate $\mathrm{N}_{\text {min }}$. Trial 1 is the base case.


Figure 8. For populations within $\operatorname{OSP}\left(\mathrm{N}_{0}=0.6 \mathrm{~K}\right)$, population size in 100 years for 7 robustness trials, using a recovery factor, $\mathrm{F}_{\mathrm{R}}$, of 0.65 for cetaceans and 0.50 for pinnipeds, and the $20^{\text {th }}$ percentile of the abundance estimate to calculate $\mathrm{N}_{\text {MIN }}$. Trial 1 is the base case.


Figure 9. For depleted populations ( $\mathrm{N}_{0}=0.3 \mathrm{~K}$ ), population size in 20 years with incidental mortality, $\mathrm{N}_{20}$ (pbr), relative to population size in 20 years for a population with no incidental mortality, $\mathrm{N}_{20}$ (no pbr) for 7 robustness trials, using a recovery factor, $\mathrm{F}_{\mathrm{R}}$, of 0.65 for cetaceans and 0.50 for pinnipeds and the $20^{\text {th }}$ percentile of the abundance estimate to calculate $\mathrm{N}_{\text {MIN }}$. Trial 1 is the base case.


Figure 10. For populations within $\operatorname{OSP}\left(\mathrm{N}_{0}=0.6 \mathrm{~K}\right)$, population size in 20 years relative to 0.6 K for 7 robustness trials, using a recovery factor, $\mathrm{F}_{\mathrm{R}}$, of 0.65 for cetaceans and 0.50 for pinnipeds and the $20^{\mathrm{hh}}$ percentile of the abundance estimate to calculate $\mathrm{N}_{\text {MIN }}$. Trial 1 is the base case.


Figure 11. Mean percent increase in recovery (from 0.3 K to 0.6 K ) time of a population relative to the recovery time of a population with no incidental mortality (no PBR), versus a range of values for $F_{R}$, the safety or recovery factor.


Figure 12. A sample of 30 trajectories of Potential Biological Removal level (PBR) from year 1 to 100, for the cetacean example ( $1 / 2 R_{M}=0.02$ ).


Figure 13. A sample of 30 trajectories of Potential Biological Removal level (PBR) from year 1 to 100, for the pinniped example ( $1 / 2 \mathrm{R}_{\mathrm{M}}=0.06$ ).


Figure 14. A sample of 30 population trajectories from year 1 to 100 , for the cetacean example ( $1 / 2 \mathrm{R}_{\mathrm{M}}=0.02$ ).


Figure 15. A sample of 30 population trajectories from year 1 to 100 , for the pinniped example $\left(1 / 2 \mathrm{R}_{\mathrm{M}}=0.06\right)$.


Figure 16. Mean of 1000 mean Potential Biological Removal (PBR) levels over 100 year trajectory versus the coefficient of variation of abundance, $\mathrm{CV}(\mathrm{N})$, using $\mathrm{N}_{\text {MIN }}=20^{\text {th }}$ percentile and $\mathrm{F}_{\mathrm{R}}=0.50$ for pinnipeds and 0.65 for cetaceans for $N_{0}=0.3 \mathrm{~K}$, and $\mathrm{F}_{\mathrm{R}}=1.0$ for both cases for $\mathrm{N}_{0}=0.6 \mathrm{~K}$.


Figure 17. Mean of 1000 root mean squared errors (RMSE) of adjacent Potential Biological Removal (PBR) levels over 99 year-to-year transitions, versus the coefficient of variation of abundance, $\mathrm{CV}(\mathrm{N})$, using $\mathrm{N}_{\text {MIN }}=20^{\text {th }}$ percentile and $F_{R}=0.50$ for pinnipeds and 0.65 for cetaceans for $N_{0}=0.3 K$, and $F_{R}=1.0$ for both cases for $N_{0}$ $=0.6 \mathrm{~K} . \mathrm{RMSE}$ is a measure of the year-to-year variability in PBR level.


